WORKING PAPER
ALFRED P. SLOAN SCHOOL OF MANAGEMENT

UNDEPRICING OF SEASONED ISSUES

by:

John Parsons and Artur Raviv
Revised April, 1984

WP #1580-84

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
50 MEMORIAL DRIVE
CAMBRIDGE, MASSACHUSETTS 02139
UNDEPRICING OF SEASONED ISSUES

by

John Parsons and Artur Raviv
Revised April, 1984

WP #1580-84
Abstract

In this paper we provide a model of the underwritten offerings of new shares of seasoned securities. Our purpose is to explain why the offering price chosen by the underwriter is lower than the market price of the firm's shares. Our model recognizes the interdependence between the markets surrounding the announcement and sale of the new issue and recognizes as well the effect which asymmetric information regarding investor demands has upon the prices in these markets.
I. **Introduction**

Two methods exist for raising equity capital on financial markets: the rights and underwritten offerings. In the rights offering current shareholders receive a "right" from the firm giving them an option to purchase additional shares at a pre-specified exercise price. Under this method the firm receives capital from the sale of new shares to holders of the rights. Current shareholders either use their rights to purchase new shares or sell the rights in the financial markets. In a firm commitment underwritten offering the investment banker purchases the new shares from the firm and then offers the shares for sale to the public. The investment banker announces an initial offering price. If the issue is oversubscribed at this price then the shares are rationed among buyers. In the event that the shares cannot be sold at the initial offering price, the underwriter is forced to lower the price. In the firm commitment arrangement the firm's proceeds are guaranteed and the risk is borne by the underwriter. Under this arrangement current shareholders may sell their shares in the market after announcement of the new issue. Like other investors they may buy more shares from the underwriter.

Recent research has noted several interesting features regarding issues of new shares. Smith (1977) found that in the underwritten issue the initial offering price is below both the market price prevailing prior to arrival of the new issue and the price following the distribution of the new issue. He views this underpricing as a cost imposed upon shareholders after an underwritten cash offer is used. This
underpricing has, therefore, contributed to the paradox that underwriters are employed in over 90 percent of the offerings while the costs of issuing via rights appear to be lower.

Current finance literature provides very little in the way of explaining these empirical facts. A recent paper by Kevin Rock (1982) provides a theoretical model explaining why unseasoned issues may be sold at a price below the market value. Hess and Frost (1982) provide no suggestion for an explanation of the above mentioned price effects. Baron (1982) provides a theory of underpricing based upon asymmetric information between the issuer and the investment banker and the need to motivate the banker to reveal his superior information. Hansen and Pinkerton (1982) have provided new estimates for the costs of both underwritten and rights issues, but have not addressed the problem of underpricing.

In this paper we provide a model of underwritten offerings of new shares of seasoned securities, the purpose of which is to explain the phenomena mentioned above. That is, we explain why the offering price chosen by the underwriter is lower than the market price of the firm's shares. In doing that we contribute to the discussion of why corporations choose the underwritten offerings instead of rights.

In our model an existing firm has a new project and chooses a method of financing. The firm faces a population of asymmetrically informed investors. In particular each investor knows his own valuation but not that of any other investor, except, of course, the incomplete information revealed through the markets. Neither the firm nor its investment banker knows the valuation of any investor. In the formulation we consider two distinct market stages. First, following the firm's announcement of the
new issue and the method of sale there exists a competitive market in the old shares. Second, the new issue enters the market by sales on the part of the underwriter. The equilibrium solution recognizes the interdependence between the markets in the two stages. Therefore the first stage competitive solution results from investor's demands/supplies derived in anticipation of the events in the second stage. This interdependence between the markets is strongest in the dependent valuations model in which the source for the divergence in valuations and its resolution are explicitly incorporated. However, our analysis begins with an independent valuations model in which the heterogeneity among investors is exogenously imposed.

Our results are:

1) In underwritten offerings, for both the dependent and the independent model, the equilibrium price in the competitive market operating prior to the arrival of the new issue will always be higher than the price at which the new issue will be sold.

2) In the independent valuations model this "underpricing" is not a "cost" to current shareholders, since it does not reduce the expected revenue from the sale when compared with a rights issue. In the dependent valuations model the results are ambiguous.

These results help in explaining some of the empirical observations discussed above. The underwriter chooses the initial price attempting to attract investors with a high valuation of the firm's new project. This price is chosen sufficiently low so as to encourage high valuation investors to purchase at this initial price rather than to attempt to buy at a subsequently lowered price. The high valuation investors recognize that they will be successful in purchasing at a lower price only in the
event that the issue is undersubscribed at the initial price. The underwriter uses this threat to charge an initial price which extracts some of the surplus from high valuation investors, but due to asymmetric information cannot set it so high as to extract all of the surplus. Therefore, since in the market for old securities taking place before the new issue arrives investors can purchase a share with certainty, the competitive price will be driven to a level higher than the initial offering price. This explains why, in empirical studies, it is observed that the initial offering price is below the market price prevailing prior to arrival of the new issue. According to our theory this phenomenon should not be interpreted as irrational setting of the offering price below the market price. Rather, it is a reaction of the market to an anticipated offering price.

The remainder of this paper is organized as follows. In section II we provide the details of the model. Sections III and IV analyze the price effects of underwritten offerings in the independent and dependent valuations models. The rights and the underwritten offerings are compared in Section V. Concluding remarks are contained in the last section.

II The Model

We consider a model in which an existing firm seeks to undertake a new investment project. The firm has \( j \) outstanding (seasoned) shares with total market value \( V_0 \). The new project requires an initial investment of \( I \) and yields an uncertain present value of returns denoted by \( \tilde{V} \). To finance the new project the firm issues \( \ell \) new shares which must be sold to investors. the issue will raise total revenue \( R \);
the amount of revenue raised will depend upon the method of issue and will be specified below.\footnote{We take as given the decision to undertake the project, the decision to finance the project with an equity issue, and the number of securities to be issued. The incorporation of these decisions into the analysis would be interesting, but is beyond the scope of this paper.}

We distinguish between two critical events. At date 1, the firm announces its decision to raise additional equity, the method to be used to float this issue, and its plans for a new project to be financed by this issue. At date 2, the firm executes the sale of the new issue. A competitive market in the firm's outstanding shares exists between these two dates. Our model consists of an analysis of the equilibrium prices in this market and the sale of the new issue when investors and the firm anticipate the full sequence of events.

In the general cash offer (hereafter referred to as an underwritten issue) the firm announces that is has entered a firm commitment arrangement, the revenue received as a result of the arrangement, and the number of shares in the issue. The underwriter will set an initial offering price at the time of distribution and this decision may be conditioned upon market events occurring subsequent to the firm's announcement of the new issue. Investors are, of course, aware of the options available to the underwriter, including feasible initial offering prices and the possibility that he might drop the price below the initial offering price were the issue to go unsold. Rational investors will anticipate the optimal strategy for the underwriter.

The market for this firm's shares consists of $N$ potential investors. We assume that short sales are not possible in this market. In the
"traditional" analysis the quantity of this firm's shares demanded by an investor would be derived from a portfolio optimization problem including the use of diversification. In order to simplify the analysis and to focus on our problem we have opted for risk neutrality, thus avoiding the need to consider diversification into other securities. An investor, therefore, is willing to purchase this share whenever the share price does not exceed his valuation. Without further restrictions this would have resulted in an infinite demand at any price not exceeding an investor's valuation of the share. We have, therefore, bounded the demand, and without loss of generality, have assumed demand for the share by each investor to be of at most one unit. We have also assumed that current owners each possess some fraction of a share. That is, each potential investor $i$, $i = 1, \ldots, N$ is willing to purchase at most one unit at any price $p < v_i$, where $v_i$ denotes the $i$'th investor's valuation of the share. Thus the utility function for investor $i$ purchasing quantity $q$ at price $p$ is:

$$ U(p,q,v_i) = (v_i - p) \cdot \min(q,1) \quad (1) $$

The firm's announcement at date 1 also includes the specifics of the new project. Consequent to this announcement the value of the firm to investor $i$ is the sum of the initial value of the firm, the expected value of the new project net of the investment required, and the expected revenue raised through the issue of new shares. Thus the value of a share to an investor is:

$$ v_i = \frac{V_i + R}{j + \xi} \quad \text{where} \quad V_i = E(V_0 + \tilde{V} - I), \quad R = E(R). \quad (2) $$

Note that the expected value of the new project may differ across agents, and that since agents assume that the future market price will reflect the value of the firm including the new project, the expected value of a share will
also differ across agents. In particular we assume that upon announcement of the new project there are two types of investors: those with high expected valuations, \( V_1 \), and those with low expected valuations, \( V_2 \), where \( 0 > V_2 < V_1 \). Thus, in this part of the paper we take as exogenously given each investor's valuation of the firm. Moreover, investors do not adjust their valuation upon learning the valuation of other investors for the same security. We refer to this as the independent valuations model. In section IV this assumption will be changed to take into account the attempt by each investor to learn the market's valuation of a share and to reevaluate the share based on this information. This later model will be referred to as the dependent valuations model.

To complete the description of the model we must specify the information available to the issuer and to the investors. The issuer is uncertain as to how many investors value this issue highly, and he does not know the valuation of any particular investor. Similarly each investor is uncertain as to the valuation of any other investor. In the independent valuations model this environment is formalized by the assumption that the issuer views the type of an investor as an independent draw, with probability \( \rho \) and \((1 - \rho)\) of being type 1 and 2 respectively. The total number of investors of type 1 is therefore viewed as a random variable with a binomial

\[ 2 \text{ According to expression (2), these different expectations may be with regards to the valuation of the existing asset, } V_0, \text{ or with regards to the valuation of the new project, } V - 1. \text{ Our results depend exclusively upon } V_1 \text{ and so are independent of the source of the different expectations. In our view the most appropriate interpretation is that the disagreement about existing assets has been resolved through the operation of the market in outstanding shares. It should also be noted that we do not consider the inferences made concerning the value of existing assets which may be due exclusively to the decision to issue new shares. This type of an announcement of an offering effect has recently been analyzed by Myers and Majluf (1982) and by Miller and Rock (1982).} \]
distribution. Other than the identity of each investor, the issuer has complete knowledge of the environment discussed above. Upon announcement of the new project each investor knows his own type, but is uncertain regarding the type of any other investor, and views it as a random variable in the same manner as does the issuer.

In the market between dates 1 and 2 the price of the outstanding shares is determined by supply and demand which are affected by the realizations of the number of type 1 investors and the method of sale. At date 2 the firm or the underwriter executes the new issue. In the competitive market for the firm's shares which operates subsequent to date 2, the divergence between the valuations of any two investors disappears, and the new equilibrium market price reflects this common valuation.

III. Price Effects of Underwritten Issue

In this section we analyze the results of issuing new shares through an investment banker. In our model the investment banker purchases the new issue from the firm and then sells shares to interested investors so as to maximize his expected revenue. The purchase price paid by the underwriter to the firm is assumed to equal the expected revenue generated by the sale of the issue. The distinguishing characteristic of this method of sale is that the underwriter can set an initial offering price, \( p_1 \), and may plan to drop the price to a new price, \( p_2 \) in the case of unsold shares. Each investor chooses his demand at each of the two prices knowing the allocation rule used by the underwriter: 1) investors requesting to purchase at \( p_1 \) are allocated their full demands when the total demand at \( p_1 \) does not exceed the number of new shares, \( Z \), 2) in case of oversubscription, investors are allocated a prorated
fraction of a share, 3) only after all demands at \( p_1 \) are fulfilled are investors waiting to purchase at \( p_2 \) allocated a share or fraction thereof. Thus investors waiting to purchase at \( p_2 \) succeed in purchasing a share only in the case of underscription at the initial offering price, \( p_1 \). It will be shown that an optimal pricing strategy for the underwriter is to set the prices so that type 1 investors will be induced to purchase at the initial offering price while type 2 investors will wait and purchase only if the price is lowered. Since this method of sale is known at the outset, investors' anticipation of the results of the sale affect the equilibrium price of old shares in the market functioning after the announcement but prior to the sale of the new issue. Therefore our analysis will involve the simultaneous treatment of the market in old shares and the sale of new issues.

Our first step is to define an equilibrium for every choice of an offering price and fallback price made by the underwriter.

**Definition:** An equilibrium is

- a share price, \( p_m \), in the market for old shares which takes place subsequent to the announcement of the new issue, and
- a set of the quantities of the old shares demanded/supplied at \( p_m \), and
- a set of quantities of the new shares demanded at \( p_1 \)
- a set of quantities of the new shares demanded at \( p_2 \)

such that

1) \( p_m \) clears the market for old shares
2) each investor's demand/supply is optimal at every stage given his rational expectations regarding the behavior of other investors and given information revealed through the operation of the market.
We now discuss the optimal pricing strategy for the underwriter, \( p_1, p_2 \). The fallback price, \( p_2 \), must be low enough to guarantee that the issue sells out under all conditions. Thus, \( p_2 \) cannot be greater than the valuation of type 2 investors, \( p_2 \leq v_2 \). Moreover, the underwriter has no reason to set the price strictly below \( v_2 \) and thus \( p_2 = v_2 \). Knowing \( p_2 \) we can determine the choice of the optimal initial offering price, \( p_1 \). The underwriter realizes that for any price \( p_1 \) he sets, a type 1 investor has two options: Either he purchases a share at the offer price, or he refuses, thus betting on the event that the issue will not be completely subscribed and he will be able to purchase a share at the lower price, \( p_2 \). This consideration places an upper limit on the price the underwriter is able to charge while expecting type 1 investors to purchase. A type 1 investor purchasing one unit at \( p_1 \) obtains a surplus of \( v_1 - p_1 \). However the issue may be oversubscribed at the initial offering price so that each investor requesting a share receives only a fraction of his order. Denoting by \( B \) the expected number of shares allocated to an investor requesting a share at \( p_1 \), the expected surplus is \( (v_1 - p_1)B \). Consider the other alternative available to a type 1 investor. By waiting to purchase in the event that the price drops to \( p_2 \) he realizes a larger surplus per share, \( v_1 - p_2 \); however, he is less likely to have his order fulfilled. We denote by \( \gamma \) the probability of receiving a share at \( p_2 \) and so the expected surplus is \( (v_1 - p_2)^* \gamma \). For the underwriter to succeed in selling at the initial price, \( p_1 \), it must be low enough to ensure that the expected surplus from an order at \( p_1 \) is higher then the expected surplus from buying at \( p_2 \), \( (v_1 - p_1)B < (v_1 - p_2)^* \gamma \). It should also be clear that the underwriter in his
attempt to maximize receipts will choose \( p_1^* \) such that this constraint is satisfied as an equality. Figure 2 illustrates the choice of \( p_1^* \). Note that the constraint requires that area A equal area B.

Insert Figure 1 here

This discussion has so far been very intuitive. In what follows we make the argument more precise. This is done by first (Proposition 1) specifying the constraints facing the underwriter in his choice of offering prices, \( p_1 \) and \( p_2 \). We then (Proposition 2) derive the optimal pricing strategy for the underwriter.

Proposition 1: Suppose \((p_1, p_2)\), \(p_1 > p_2\), \(p_1 > v_2\), is such that equilibrium demands are non-zero at both prices, i.e., type 1 investors choose to purchase one new share at \( p_1 \) and type 2 investors purchase new shares at \( p_2 \) and not at \( p_1 \). Then \((p_1, p_2)\) must satisfy

\[
\begin{align*}
    p_1 &\leq v_1 \\
    p_2 &\leq v_2 \\
    (v_1 - p_1) \delta &\geq (v_1 - p_2) \gamma \\
    (v_2 - p_2) \gamma &\geq (v_2 - p_1) \delta \\
    (v_2 - p_2) \sum_{i=0}^{\ell} B_i \frac{\xi}{N-j} &\geq (v_2 - p_1) \sum_{i=0}^{j} B_i .
\end{align*}
\]

where

\[
\delta = \sum_{i=j+1}^{j+\ell-1} B_i + \sum_{i=j+\ell}^{N-1} B_i \frac{\ell}{1-j+1},
\]

and

\[
\gamma = \sum_{i=j+1}^{j+\ell-1} B_i \frac{j+\ell-i}{N-1}.
\]
The proof is in the appendix.³ The reader should be aware that the symbol \( v_1 \) in these conditions is not an exogenous variable. The \( v_1 \)'s were defined by equation (2). The expected revenue, \( R \), in equation (2) is a function of the prices \( p_1 \) and \( p_2 \), and so \( v_1 \) will be a function of \( p_1 \) and \( p_2 \).

In both (4) and (5) it is assumed that, in the market for old shares held before the new issue, type 2 shareholders will have sold their shares to type 1 investors.⁴ Therefore there will remain type 1 investors requesting to purchase the new issue in the event that the total number of type 1 investors is greater than the number of old shares, \( j \). The price in the pre-issue market will have revealed that this event has occurred. Since the market for old shares is competitive, investors make inferences from the market price based upon the assumption that their demands do not affect the market equilibrium. Therefore, the information obtained by each type of investor is the same. Hence, the indices for the number of type 1 investors begin with \( j + 1 \) in (4) and (5). Inequality (6) refers to the event that the number of type 1 investors has been revealed to be less than \( j \). The inequality implies that in this event a type 2 investor waits to purchase the share at \( p_2 \) rather than requesting to purchase at the initial offering price \( p_1 \).

We now derive the optimal pricing strategy for the underwriter.

---

³ \( B_i \) denotes the binomial coefficient: the probability of \( i \) type 1 investors among \( N-1 \) investors when \( p \) is the probability that a particular investor is type 1.

⁴ In the proof of Proposition 1 it is shown that this assumption is satisfied in equilibrium.
Proposition 2: The offering price $p_1^*$, and fallback price, $p_2^*$, which maximize the expected revenue received by the underwriter are given by

$$p_2^* = v_2 = \frac{V_2 + R^*}{j + \lambda}$$

$$p_1^* = (1 - \frac{\gamma}{\delta})v_1^* + \frac{\gamma}{\delta} p_2^* = (1 - \frac{\gamma}{\delta}) \frac{V_1 + R^*}{j + \lambda} + \frac{\gamma}{\delta} p_2^*$$

where $R^* = p_1^* E(n) + p_2^* (\lambda - E(n))$ and $E(n)$ is the expected number of new shares sold at the initial offering price $p_1^*$.

The proof is given in the appendix. It consists of showing that the underwriter's expected revenue attains a maximum over the feasible set defined in Proposition 1 when constraints (3b) and (4) are binding. The statement of Proposition 2 provides two simultaneous equations of $p_1^*, p_2^*$. The explicit solution to this pair of equations is also given in the proof.

Having analyzed the underwriter's optimal pricing strategy we can now investigate the equilibrium arising in the pre-issue market when investors anticipate the conditions of the impending offering by the underwriter. The characterization of equilibrium involves the simultaneous treatment of the pre-issue market and the underwriter's distribution. In the competitive pre-issue market the $N_1$ type investors will attempt to purchase shares from type 2 current owners. The value of the market price, $p_m$, depends upon whether supply exceeds demand, or vice-versa. In the event that demand is low, i.e., $N_1 < j$, the price is forced down to the point at which a type 2 shareholder is indifferent between retaining or selling his fraction of a share. This indifference determines the market price $y$. In the event
that demand is high, \( N_1 > j \), the price is forced up to the point at which the type 1 shareholder is indifferent between buying and waiting to purchase from the underwriter at \( p_1^* \). This indifference determines the market price, \( x \). It is important to notice that the price which the type 1 investor is willing to pay in the competitive pre-issue market is less than his valuation, \( v_1^* \), due to his anticipation of the option available to him to purchase shares from the underwriter at \( p_1^* \).

The equilibrium is characterized in:

**Proposition 3:** Let \( (p_1^*, p_2^*) \) be the expected revenue maximizing prices. An equilibrium is given by;

\[
A) \text{ a share price } p_m = \begin{cases} 
y & \text{if } N_1 \leq j \\
x & \text{if } N_1 > j
\end{cases}
\]

where \( y = v_2^* = p_2 \)

\[
x = \delta v_1^* + (1 - \delta) p_1^*
\]

\[
\delta = 1 - \delta / \sum_{i=j+1}^{N-1} B_i
\]

and \( N_1 \) is the number of type 1 investors.

**B) type 1 investors demand and type 2 owners supply their old shares at \( p_m \). If \( p_m = y \) there exist type 2 owners that have not sold their fraction of a share. If \( p_m = x \) there exist type 1 investors that do not own any shares.
C) all type 1 investors that do not own one share after all transactions in the market for old shares are completed request one share at \( p_1^* \); type 2 investors who do not own a share demand one.

D) at \( p_2^* \) all type 2 investors who do not own a share demand one.

The proof is provided in the appendix. In the event that demand is low the market price is forced down to the valuation of type 2 investors and it is common knowledge that the underwriter will also be forced to sell the full offering at \( p_2^* = v_2^* \). The market price \( x \), in the event that the number of type 1 investors is high, must offer to type 1 investor a surplus, \( v_1^* - x \), as high as the expected surplus obtained through purchasing from the underwriter, \( (v_1^* - p_1^*)\beta \).

This is illustrated in figure 2, where area C is equated with area A.

**Insert Figure 2 here**

These results provide us with a prediction regarding the relation between the offering prices chosen and the equilibrium market prices that result from this choice. From Proposition 3 it follows that:

\[
x > p_1^* > y = p_2^*
\]

The market price \( x \) is strictly higher than the offering price \( p_1^* \). This is the result of the fact that the new issue may be over subscribed and therefore the investor will receive only a fraction of his demand (\( \beta < 1 \)). In the pre-issue market, however, an investor obtains a share at price \( x \) with certainty for which he is willing to pay a premium. The
equilibrium market price, whether it is $x$ or $y$, is higher than (or equal to) the price at which shares are sold by the underwriter. The empirical observation of this phenomenon should not be interpreted to imply irrational behavior on the part of the underwriter. As our theory demonstrates, this occurs when the offering prices are chosen optimally. The correct interpretation of this empirical observation is that, after the announcement of the terms of an underwritten offering, the optimal behavior of investors leads to an equilibrium market with the characteristic discussed above. Thus our analysis draws attention to the fact that the pre-issue market price is not independent with respect to the underwriter's decisions. The two are determined in equilibrium, and for any anticipated offer price the associated pre-issue market price is higher.

IV. Effects of Information Revealed Through the Market

In the model discussed so far we have assumed that each investor has an exogenously specified valuation of the firm and does not alter his valuation upon learning of other investors' valuations. This assumption may be criticized on the basis that the cause of a divergence in valuations by investors was not specified; moreover, the divergence was assumed to persist during trade in the market and to disappear immediately upon distribution of the new shares. There was no restriction placed upon the valuations made by investors to guarantee that they were consistent with the competitive prices in the post-issue market. We now alter the model so as to include information and learning through interaction in the market, and show that the insights regarding price effects derived in the simpler model carry over to the more complete model.
In this model the value of the firm is a random variable, \( \hat{V} \), characterized by a single parameter distribution, \( F(\cdot, \mu) \), where \( \mu \) is unknown (for example, \( \mu \) might be the mean of the distribution). Investors begin with identical priors regarding the value of \( \mu \). Simultaneous with the firm's announcement of the new issue, each investor receives a signal from a distribution also parameterized by \( \mu \). These signals are drawn independently from a common distribution and are private information to each investor. In order to maintain consistency with our model in which there are two types of investors, we assume that the signals are of two types, "high" or "low". For example, if \( \mu \) is the mean value of the distribution \( F(\cdot) \) and the messages are "the value of the firm is going to be high/low," then the probability of receiving a "high" message would be greater, the greater is \( \mu \). Based upon all information available to a particular investor, he forms a posterior regarding \( \mu \) (using Bayes theorem), and from this calculates the posterior expected value of the firm conditional on the total number of high signals being equal to \( i \), and by \( v(i) \) the corresponding value of a share. We also assume that \( V(i) \) increases with \( i \).

Although there is no disagreement regarding the posterior expected values conditional on each event, nevertheless, at the time when investors know only their private signal, there will be disagreement regarding the likelihood of each event. An investor who has received a high signal (to be referred to as a type 1 investor) assesses the
likelihood that the number of "high" signals other than his own is equal to $i$ as $B^1_i$, $i=0, \ldots, N-1$. \[ \sum_{i=0}^{N-1} B^1_i = 1, \quad B^1_i \geq 0. \] Similarly, an investor receiving a low signal (to be referred to as a type 2 investor) assesses the likelihood that the total number of "high" signals is equal to $i$ as $B^2_i$, $i=0, \ldots, N-1$. \[ \sum_{i=0}^{N-1} B^2_i = 1, \quad B^2_i \geq 0. \] Our intuitive notion of a type 1 investor is that he assigns greater probability to the event that the total number of type 1 investors is large. In particular, we assume that whenever the market reveals the number of high type investors to be between $s$ and $t$, $0 < s < t \leq N$, then a type 1 investor's expected value of the firm conditional upon this event exceeds that of a type 2.

As in the previous model, a competitive market in the j old shares operates prior to the new issue, and investors buy from the underwriter after observing the price in this market. The equilibrium price in a competitive market is $p_m$ and depends upon the number of type 1 investors as before. This information will be incorporated into the valuation of investors prior to the underwriter's offering.

We may now turn our attention to the constraints facing the underwriter in his choice of offering price. If the equilibrium price is $x$, then type 2 current owners will have sold their shares to type 1 investors, and it is common information to all investors that the number of type 1's is larger than $j$. Individual rationality on the part of type 1 and type 2 investors seeking to purchase a share from the underwriter implies

\[ \sum_{i=j+1}^{j+\ell-1} B^1_i (v(i+1)-p_1) + \sum_{i=j+\ell}^{N-1} B^1_i (v(i+1)-p_1) \frac{\ell}{1-j+1} \geq 0 \quad (7) \]
These equations are exactly analogous to (3a) and (3b). If, on the other hand, the equilibrium market price is y, then all type 1 investors will have bought a share, and it is commonly inferred that the number of type 1 investors is less than j. There are no type 1 investors seeking to purchase a share from the underwriter. The underwriter is, therefore, forced to offer a fallback price, \( p_3 \), and since the low number of type 1 investors has been revealed, the individual rationality constraint for type 2 now becomes

\[
\sum_{i=0}^{j} B_1^2 (v(i) - p_3) \frac{\lambda}{N-j} \geq 0.
\]

If the triplet \((p_1, p_2, p_3)\) is to be such that in equilibrium unsatisfied type 1 investors seek to purchase from the underwriter at \( p_1 \) and such that type 2's seek to purchase at \( p_2 \) when the competitive market price is \( x \), then the following two self selection constraints must be satisfied

\[
\sum_{i=j+1}^{j+k-1} B_1^1 (v(i) - p_1) + \sum_{i=j+k}^{N-1} B_1^1 (v(i) - p_1) \frac{\lambda}{1-j+1} \geq \sum_{i=j+1}^{j+k-1} B_1^1 (v(i+1) - p_2) \frac{j+k-1}{N-1}
\]

\[
\sum_{i=j+1}^{j+k-1} B_2^2 (v(i) - p_2) \frac{j+k-1}{N-1} \geq \sum_{i=j+1}^{j+k-1} B_1^2 (v(i) - p_1) + \sum_{i=j+k}^{N-1} B_1^2 (v(i) - p_1) \frac{\lambda}{1-j+1}
\]

Notice that in this model the underwriter chooses 3 prices. \( p_2 \) and \( p_3 \) are both 'fallback' prices. \( p_2 \) is the price to which the underwriter is forced when the market has revealed the number of high signals to be greater than \( j \) but less than \( j+k \). \( p_3 \) is the fallback price when the market has revealed the number of high signals to be less than \( j \).
These equations are comparable to (4) and (5). Given that 
\((p_1, p_2, p_3)\) satisfy (7)–(10) we can give a precise expression for 
the market equilibrium price, \(p_m\). When \(N_1 > j\), type 1 investors 
will bid up the price, and \(x\) will be the most they are willing to pay 
where \(x\) is defined by

\[
\sum_{i=j+1}^{N-1} B_i^{1} (v(i+1)-x) = \sum_{i=j+1}^{j+\delta-1} B_i^{1} (v(i+1)-p_i) + B_j^{1} (v(j+1)-p_j) \frac{p}{1-j+1}
\]

When \(N_1 \leq j\), competition among type 2 sellers will drive the price 
down to their valuation, i.e. \(y\) will equal \(p_3\). The underwriter 
chooses \(p_1^*, p_2^*\) and \(p_3^*\) to maximize expected revenue 
subject to (7)–(10).

The sequence of prices about which the dependent valuations model 
yields predictions are: the competitive price in old shares, the price 
at which the underwriter offers the new shares, the fallback offering 
price, and the final competitive market price conditional upon the 
success (or lack thereof) of the offering. The predictions are 
summarized in:

**Proposition 4:**

1) \(x > p_1^* > p_2^*\)
2) \(E_1 (p_M^*) > x > p_1^*\)

\[
E_2 (p_M^* \mid \text{undersubscription}) > p_2^*
\]

where \(p_M^*\) is the competitive price in the market following distribution 
of the new issue, and

\[
E_1 (p_M) = \sum_{i=j+1}^{N-1} B_i^{1} v(i+1) / \sum_{i=j+1}^{N-1} B_i^{1}
\]
The proof of the first part follows directly from the definition of $x$, $p_1^*$, and $p_2^*$. The proof of the second part follows from the fact that type 1's individual rationality constraint is non-binding, and the fact that $x$ is defined so that type 1 investors purchasing in the competitive market receive the same surplus that they would receive purchasing from the underwriter. Since type 2 investor's individual rationality constraint, (8), is binding, and since he receives on average the 'worst' shares with higher probability, the last claim of portion 2 follows. This effect is known as the winner's curse.

These predictions regarding price effects are illustrated in Figure 3.

(Insert figure 3 here)

The first part of the proposition predicts that the competitive market price for the share immediately prior to the offering will exceed the offering price. Just as was explained in section III, this should not be interpreted to imply irrational behavior on the part of underwriters. The correct interpretation is that the underwriter sets the offering price optimally. Since the underwriter is not able to extract all the surplus from the investors, and since each investor faces the risk of being rationed, investors drive the price in the competitive market for all shares to a level above the initial offering price. The second part of the proposition predicts that the competitive market price for the shares in the post-offering period will exceed the price at which the new issue is distributed, and will exceed the price in the
competitive market for old shares immediately prior to the distribution
of the offering. In the distribution of the new issue there is a
surplus captured by type 1 buyers which results from the asymmetric
information that characterizes the environment. After the new offering,
when all information is revealed and common, the price fully reflects all
information and the surplus is fully captured by the owners of the
shares. Therefore, on average the post issue price is higher.

These results are strikingly consistent with the empirical data
reported in Smith (1977). He found that there was a negative return in
the period immediately preceding the new issue of the security, followed
by a positive post issue return. In particular, "for the 328 firms with
the requisite data, the average return from the close to the offer price
is -0.0054 and the average return from the offer price to the close on
the offer date is +0.0082." He concluded that "these figures are
significantly different from the average daily return." The proposition
indicates that when one measures share prices before the issue and after,
and compares them with the offering price, there will be a negative
return followed by a positive return.

---

For a firm, since all information will be revealed in the results of the
sale of the new issue, the market price will be a random variable,
v(N1), depending upon the number of type 1 investors. An empirical
analysis would provide a cross-sectional average of this variable across
firms. What the proposition yields is an expected value of this variable
from the perspective of type 1 or type 2 investors.
V. Rights vs. Underwritten Offerings

In the preceding sections it was shown that the optimal offering price for the underwritten issue is lower than the prevailing market price. It might therefore appear as if the use of an underwritten offering imposes an additional cost on shareholders through the revenue sacrificed relative to the revenue that could be raised in a rights offering. This intuition is not, in general, correct. To address the question of revenue raised under alternative methods of sale, it is useful to view them as different types of auctions. The underwritten issue as analyzed above is a "priority pricing" scheme, while the rights issue may be viewed as a competitive (or single-price) auction. A comparison of these two methods was made in Harris and Raviv (1981).

To construct a model of the rights issue comparable to the independent valuation model in Section III, we view the firm as distributing one right to each shareholder: \( j/\lambda \) of these rights will entitle the shareholder to buy one additional share. It is known at the time of announcement that the competitive equilibrium price for an ex-rights share will be either \( v_1 \) or \( v_2 \) depending upon whether the number of type 1 investors is above or below the total supply of shares, \( j + \lambda \). This is identical to the price that would obtain in a competitive auction for the shares. Therefore, the expected total revenues received for the sale of the \( \lambda \) new shares is equal to the expected revenue from a competitive auction. This expected revenue and the revenue obtained in a "priority pricing" scheme are equal (in the independent valuation model), and therefore the underwritten and rights offerings are equivalent with respect to the expected revenue raised.

Dependent valuations introduce two new considerations into the comparison. First, the fact that each investor's valuation rises with an
increase in the valuation of other investors generates the phenomenon known as the "winner's curse." In other words, a type 1 investor who is bidding for a share from the underwriter at \( p_1 \) recognizes that the fraction of the share he receives is inversely related to the value of the share. Whenever the issue is undersubscribed, he receives a whole share, but the undersubscription arises precisely because there are a few investors with favorable information and this implies that the share should not be valued highly. On the other hand, when the share is most highly valued, that is when the number of positive signals is high, then the issue is oversubscribed and he receives a small fraction of a share. These considerations lower the prices \( p_1^* \) and \( x \) as compared to what they would have been otherwise. The winner's curse does not exist in the rights market, however, and therefore makes the rights issue relatively more attractive with respect to revenue raised.

The second new consideration stems from the fact that information is acquired through the observation of market prices. In the rights issue, the competitive market price reveals whether the number of 'high' messages is above or below the total supply of shares, \( j+\mathcal{E} \). Therefore, the competitive price will be a weighted average of values conditional on the appropriate event. On the other hand, in the underwritten issue the competitive market operating prior to arrival of the new issue reveals whether the number of 'high' messages is above or below the supply of old shares, \( j \). Therefore the prices that the underwriter can charge, \( (p_1^*, p_2^*, p_3^*) \), will be averages of values conditioned on this event. Thus the comparison between the two methods of sale is very sensitive to the assumptions made regarding priors and the updating of the priors that occurs as a result of the different signals the investors receive.
Current shareholders are, of course, concerned with more than just the revenue raised by the new issue. A complete treatment of their welfare would have to consider that some shareholders will want to purchase additional shares while others will want to sell out their current positions. Therefore, the different price effects induced by the two methods will have different influences upon shareholders' welfare. We have not been able to derive conditions for which these effects are unambiguous.

VI. Conclusion

We believe that this paper has explained the price effects that have been empirically documented in new issues of seasoned securities. In particular, we have shown that the market price before a new issue will be higher than the initial offer price, and that the market price following the new issue should be expected to rise above the offer price. The major insight gained regarding the pricing of a new issue is that the market prices and the offer price are jointly determined in the equilibrium. The investment banker, then, cannot simply set the initial offer price to correspond to the current market price. In our model the banker sets an optimal market price, and the market reacts to this choice as outlined above. The "underpricing" of new issues is a direct consequence of the potential for oversubscription and the rationing that will follow. Since the relation is explicit in our results it lends itself to an empirical test. With regards to the robustness of the results, we feel that the critical assumptions are the restriction against short sales and the special nature of the demands of individual investors. We assumed that each investor was risk neutral and had a
bounded, horizontal demand function. We believe that the results on price effects can be obtained in a more general setting since they depend only upon the fact that each investor recognizes the possibility of over-subscription and the rationing that would follow from that.

Two related areas of interest are the price effects associated with block trades and those associated with interfirm tender offers. The pattern of prices around the block sale documented by Kraus and Stoll (1972) shows remarkable similarity to the pattern of prices depicted in Figure 3. Similarly, the price pattern around the announcement and execution of a tender offer analyzed by Bradley (1980) are comparable to those discussed in this paper. We believe that an analysis similar to the one employed here can shed additional light upon the price effects and we are currently pursuing this line of research.
Fig. 1: Underwriter's Choice of Offer Prices

Area A

Area B
Fig. 2: Pre-Issue Market Price
Fig. 3a: Price Behavior when there is Undersubscr.

Fig. 3b: Price Behavior when there is Oversubscr.

\[ E(P_M | P_m = x^*) \]

\[ E(P_M | \text{undersubscr.}) \]

\[ P_1^* \]

\[ P_2^* \]

\[ x^* \]

\[ \text{time} \]