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ALFRED P. SLOAN SCHOOL OF MANAGEMENT

THE VALUATION OF CONVERTIBLE BONDS

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The Valuation of Convertible Bonds

Convertible Bonds are bonds that are convertible into another security at the option of the holder subject to conditions specified in the indenture. For our paper we will restrict the term 'convertible' to mean exchangeable for 'the common stock of the issuing corporation.' The restriction is not a stringent one: the author in examining publicly traded bonds issued between 1948 and 1963 by companies that are traded on an organized stock exchange (or over the counter) found no bonds which were excluded by that definition. The vast majority of nation-wide traded convertible bonds is not only unsecured, but even subordinated to prior or even after-acquired debt.¹ Deducing from cum hoc to ergo propter hoc this has led many writers to state or hypothesize that one of the reasons, if not the principal one, to attach to the bond the convertibility feature was the necessity to have a sweetener make an otherwise unpalatable instrument acceptable to the investor.²

The conversion price indicates how many dollars of face value must be given up at conversion for each common share. Occasionally, we find a conversion ratio instead, stating into how many shares one debenture of \$1,000

¹The converse statement also holds: subordinated bonds are usually convertible, see K. L. Browman, "The Use of Convertible Subordinated Debentures by Industrial Firms" 1949-59, Quart. Rev. of Econ. Business (Spring 1963) Vol. 3 no. 1, p. 65-75. The same point is made by R. Johnson in "Subordinated Debentures: The Debt That Serves as Equity," Journal of Finance (March 1955) Vol. 10, no. 1, p. 16.

²See Pilcher, op. cit. p. 57 ff. for sources.

(or \$500 etc.) is convertible. In the case of a \$1,000 bond, a conversion ratio¹ of twenty thus would be equivalent to a \$50 conversion price. We will use almost exclusively the term 'conversion price,' since it is both customary and independent of the denomination of the bond. The conversion price is usually well above the stock price at the time of issue.²

The indenture will also specify between which dates the conversion price applies. There are many bonds for which one conversion price applies from the date of issue to maturity; occasionally conversion is not permitted before a certain time, say two or three years, have elapsed since flotation. More often there is a clause stating that the conversion privilege expires after ten, twelve, fifteen or so years, well in advance of maturity. An additional complication, that is even more frequent (and applied to almost 40 per cent of a sample of 165 postwar bonds), is a conversion price that changes at specified dates, usually in five-year intervals, invariably increasing. Finally, in a few cases, the indenture requires the owner of the bond to surrender not only his bond at conversion but also a specified amount of cash per share received.

¹The conversion ratio is not to be confused with the word 'ratio' in another frequently found statement such as "The corporation is offering to the holders of its outstanding Common Stock of record...rights to subscribe for the above debenture in the ratio of \$100 in principal amount of debentures for each 28 shares of Common Stock then held of record. "Such a statement has nothing to do with conversion but indicates that the issue is a preemptive one, i.e., is offered to stockholders before it is offered to the public at large.

²With A. T. and T.'s eight postwar convertible issues furnishing a notable exception.

The conversion privilege is somewhat like a nondetachable warrant with the conversion price taking the place of the exercise price. If conversion price and exercise price are the same, a convertible bondholder could after conversion achieve the position the owner of a bond-plus-non-detachable warrant is in after exercising his right by buying a bond of similar quality, maturity, and coupon rate costing the amount the bond-plus-warrant holder spends when exercising his right. Cash, bond, stock positions then would be identical. Vice versa, the bond-plus-warrant holder could reach the position of the convertible bond owner after conversion by simply selling the bond after exercising the warrant.

A difference, however, is that a conversion price of \$ y does not mean that something worth \$ y is given at conversion. If, e.g., the conversion price is \$25 while the straight debt value of a \$100 face value bond is \$90 then at conversion a consideration of \$25 times $\frac{90}{100}$ is paid for the share received. This is perfectly normal, the coupon rate for convertible bonds usually is below the going rate for bonds of that quality, the full price of \$100 is paid because of the conversion privilege.¹ The difference between straight debt value and par value

¹For sample of 165 bonds we found as average figures for a \$100 face value bond shortly after its flotation a straight debt value of \$93, a market price of \$106, making for an average value placed on the conversion privilege of \$13. Average subscription price was closer to \$101, but since about half of them were preemptive, this figure understates the value of the bond for the investor.

(discount) is gradually amortized as maturity approaches. A change in the quality of the bond will also change the straight debt value. The same factors that push share price above conversion price may also increase the straight debt value.

Evaluation of the Convertible Bond in Toto

We shall start by looking at the convertible bond in toto--straight debt part and conversion option. We select arbitrarily a point t in time when we will either sell the bond at its straight debt value or convert it into common stock and sell the shares. If, at time t, the stock price x exceeds the straight debt value y, then we convert and receive stock worth x; otherwise, we keep the straight debt value y. For a given y the expected value of the stock is the sum of all values of x for which $x > y$ multiplied by their probability of occurrence $h(x | y)$ and summed:

$$\int_y^{\infty} x h(x | y) dx$$

To this expression we have to add the straight debt value y times the probability that the stock price x falls short of y, in which case we do not convert

$$y \int_0^y h(x | y) dx.$$

Bringing the terms together, we state that the expected value of the

convertible bond given a straight debt value y is

$$y \int_0^y h(x|y) dx + \int_y^{\infty} x h(x|y) dx$$

Since bond yields vary just as prices do, we must do our analysis for all possible values of y , multiply by the probability of the occurrence $g(y)$ and integrate from 0 to ∞ . The expected value of the bond thus is

$$E(p) = \int_0^{\infty} \left[y \int_0^y h(x|y) dx + \int_y^{\infty} x h(x|y) dx \right] g(y) dy.$$

Writing $h(x,y)$ for $h(x|y) g(y)$ we get

$$E(p) = \int_0^{\infty} \left[y \int_0^y h(x,y) dx + \int_y^{\infty} x h(x,y) dx \right] dy$$

$$E(p) = \underbrace{\int_0^{\infty} \int_0^{\infty} x h(x,y) dx}_{\text{expected stock value}} + \underbrace{\int_0^{\infty} \int_0^y (y-x) h(x,y) dx}_{\text{value of floor guarantee}} dy$$

In words, owning a convertible bond is like owning a stock with the guarantee that a loss due to a fall below a floor y will be made up. Rearranging the terms in $E(p)$ in the original form differently and using

$$\int_0^{\infty} h(x|y) dx = 1$$

we may write

$$E(p) = \underbrace{\int_0^{\infty} y g(y) dy}_{\text{expected straight debt value}} + \underbrace{\int_0^{\infty} \int_0^{\infty} (x-y) h(x,y) dx dy}_{\text{expected value of the conversion option}}$$

This is simply the other side of the same coin. The second term is the gain from conversion--value of stock received minus value of straight debt given up--weighted by the probability density and summed over the range where conversion occurs, $x \geq y$, and the range of possible straight debt values, $0 \leq y < \infty$.

We have succeeded in splitting the convertible bond into a straight bond and conversion option. The present value of the straight bond part, inclusive of all interest payments on the convertible bond, is equal to the price of a bond of comparable quality without the convertibility feature. This value has been computed for us by Moody's Investors' Service. This organization regularly rates large, widely held U. S. convertible bonds as to their investment quality.¹ They specifically assign equivalent straight debt yields and straight debt values to the bonds. These last two measures have, of course, a one-to-one correspondence once the coupon rate and maturity are given.

Baumol, Malkiel, and Quandt in a paper presented at an earlier TIMS meeting went most of the way in deriving these formulae without realizing that they are in all cases fully equivalent. D. E. Farrar in an unpublished paper did just that up to a small mathematical mistake.

You will realize that the last part of the second equation gives the value of the somewhat more simple warrant. The exercise price, y , is not

¹As well as R. H. M. Associates, New York.

a stochastic variable and we can write

$$\text{Expected value of the warrant } EPW = \int_y^{\infty} (x-y) h(x) dx$$

This is known since Bachelier's work at the turn of the century. Part of the notation here is taken from a thesis by Case Sprenkle on warrants. But back to the convertible bond. Inspection of our second form of the equation for the convertible bond shows that while the straight debt value is not affected by the stock price movements x , the option will fluctuate with the straight debt value y , which in turn is a function of fluctuating bond yields. We see the effect of this clearer if we substitute for $h(x,y)$ a specific joint density function, e.g., the lognormal one. I do not want to use my time defending this choice, much less do I want to become involved in the Mandelbrot-Cootner arguments to whether the Pareto-Levy distribution is more appropriate but rather refer you to the articles collected by Paul H. Cootner in a book entitled "The Random Character of Stock Market Prices." If we, then, make the above-mentioned substitution we get:

$$EPW = \int_0^{\infty} \int_y^{\infty} \frac{x-y}{2\pi\sigma_x\sigma_y} e^{-\frac{1}{2} \frac{1}{1-R^2} \left[\left(\frac{\ln x - \mu_x}{\sigma_x}\right)^2 - 2R \frac{\ln x - \mu_x}{\sigma_x} \frac{\ln y - \mu_y}{\sigma_y} + \left(\frac{\ln y - \mu_y}{\sigma_y}\right)^2 \right]} dx dy$$

is the correlation coefficient between x and y .

This integral is exceedingly cumbersome to evaluate and, before we take the trouble, we want to make sure that the effort is well spent. In other words, we would like to know that, by letting the value of y

vary, we are significantly improving our estimation of EPW. To test this, we shall observe empirically the effect of changes in bond yields, y , on the value of the conversion option, EPW. We first test, assuming perfect correlation between y and x , the value of stock for which the bond is convertible, then for zero correlation which is closer to our estimate of the actual correlation between the two.¹ If changes in the value of y do not have a significant effect on the estimate of EPW, then we could safely fix y at its expected value, μ_y , with $\sigma_y=0$, and return to our simple model of options,

$$EPW = \int_y^{\infty} (x-y)f(x)dx$$

Let me simply state the results:

If we have perfect positive (negative) correlation, neglect of bond price variability results in an overestimate (underestimate) of the conversion option by 36%^{2,3} and we would not be justified in neglecting it. Now these calculations assumed perfect correlation between stock price and bond yield. In reality the correlation is much closer to zero than to one as we shall see in a moment.

¹The coefficient of correlation between the inverse of Standard and Poor's stock price index and Standard and Poor's index of yields of AAA bonds in the year 1946-61 was $R=-.840$. Trend removal reduces this figure to .0258.

²This is for yearly standard deviation of stock price of 14%, i.e., low stock price volatility. The standard deviation of bond prices is ca. 3.4% per year - which is about average.

³It may be surprising at first blush that a positive correlation of bond prices and stock prices should decrease the value of the option. I have given the strict proof elsewhere, let me here just state some considerations to make this plausible to you:

If bonds and stocks move together, there is little to be gained by switching (converting) from one to the other. For negative correlation the reasoning is the converse of this.

We mention, as mildly interesting aside, that even for zero correlation between stock prices and bond yields, bond yield variability has some effect. A somewhat stronger statement can be proved: if the expected bond price is equal to the expected stock price and both are normally distributed with finite variance, the value of the conversion option increases with the bond yield variance. However, numerical computations have shown that for values of stock price volatility and bond yield volatility as actually prevail, such influence is negligible.

Next, we checked where between zero and perfect correlation the actual one is to be found.

As a first step, we ran a correlation and regression analysis of the yields of bonds rated AAA, AA, A, BBB by Standard and Poor and time against the inverse of the price index for 425 industrial stocks. Observations are taken monthly from January 1946 to December 1963. As we all know, bond yields and stock prices all rose sharply during that period. But since we cannot expect bond yields to rise simply as they have risen in the past, we eliminated the time trend from the fluctuations of bond yields and stock prices. The principal results are:

First, that common stock yield differentials have an extremely low positive correlation with all other variables--including preferred stock yield differentials (the correlation with AA, A, BBB bonds is actually higher);

Second, that all zero order regressions have insignificant coefficients;

And third, although the various bond yield differentials are significantly correlated, the correlation is smaller than expected. The coefficient of determination, i.e., the percentage of variance of one bond which is explained by that of another, typically is only 60% or so;

Finally, standard deviations fall monotonically from AAA to BBB bonds.¹ The fall is even more pronounced if we normalize by dividing standard deviation by average yield in each case.² If AAA bonds on the average have a longer life than BBB bonds, AAA bonds fluctuate more in price than BBB bonds (or AA or A bonds), even more than indicated by the difference in the fluctuation of yields.³

Having found that for zero correlation between bond yield variability and stock prices, bond yield variability has no significant influence on

¹If data from 1946 to 1963 are considered, A bonds are an exception.

²Professor Cootner has suggested a possible explanation of this phenomenon. The variability of bond prices (or yields) is due to two causes, first, variation of the pure rate of interest, second, change in the risk of default (or cessation of interest payments) due to change in prospects or the company or change of the investor's view thereof. Yield variation among the very high grade corporate bonds would be due mainly to the first cause, while to explain yield variation of low grade bonds, the second cause would have to be added. If bond yields fall (or bond prices rise) in a recession and rise during a boom, while the risk of default increases during a recession and falls during a boom, then for a low grade bond whenever one factor pushes up bond prices, the other would push them down. For high grade bonds the countervailing influence is much weaker. This then leads to larger bond price variance for high grade than for low grade bonds (especially if recessions are slight). This hypothesis would seem to be empirically verifiable. If true, it means that investors in high grade bonds satisfy the yield differential between high grade and low grade bonds exclusively for greater safety, not for greater bond price stability.

³We obtain similar results by holding time constant in the un-detrended un-differenced regression analysis.

the expected value of the conversion option, having found furthermore that no significant correlation exists, we happily simplify the complete expression for the conversion option to

$$EPW = \int_y^{\infty} (x-y) f(x) dx$$

In the case of lognormally distributed prices we write

$$EPW = \int_y^{\infty} \frac{1}{\sqrt{2\pi} \sigma_x} (x-y) e^{-\frac{1}{2} \left(\frac{\ln x - \mu_x}{\sigma_x} \right)^2} dx$$

The Investor's Horizon

Thus far we have dealt with the expected value of the option at a given time t from now, $EPW(t)$. If the investor requires a rate of return i^1 on his investment, the present worth, PW , or price he is willing to pay at most for the option is

$$PW = e^{-it} EPW$$

If the investor at time 0 is told he can make his choice of converting at any time T between now and expiration of the conversion privilege and if he is also told (unrealistically) that he must say, now, at what T he will decide to convert, then the value of the conversion option, PW^2 , is

$$PW = \max_T e^{-iT} EPW$$

or $\left(\frac{dPW}{dT} \right)_T = 0$; $\frac{d^2PW}{dT^2} < 0$

$$\frac{dPW}{dT} = -ie^{-iT} EPW + e^{-iT} \frac{dEPW}{dT} = 0$$

¹We shall discuss what determines i later on. We might mention factors such as the differential interest rate-dividends, risk associated with the stock and the like.

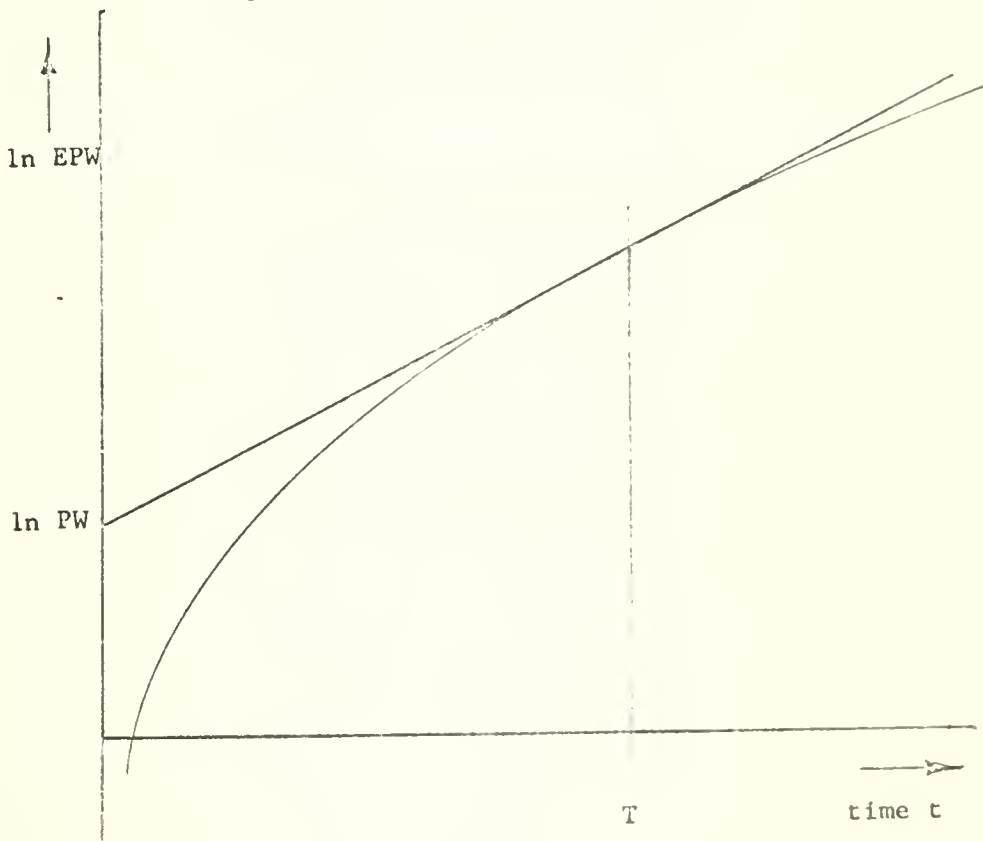
²Assuming i is not predicted to change in a systematic way.

$$\frac{dEPW/EPW}{dT} = i$$

This is the time-worn prescription that the value of an asset is equal to the discounted value of the asset at the moment when its growth rate per dollar invested, $\frac{dEPW/EPW}{dT}$ is equal to the required rate of return. If the option were held longer than to T , the return would fall below the required one. Therefore, nothing that happens beyond that point matters. The example usually adduced is that of the growing tree.

Figure 1

Expected Value and Investor Horizon



We now have two equations

$$PW = e^{-iT} EPW (T)$$

$$\frac{dEPW/EPW}{dT} = i$$

for our two unknowns PW and T.

Now we know, that when the investor buys his convertible bond he is not told to decide when he will make his choice whether to convert or not. He is free to do so any time up to the expiration date of the conversion feature and he has available for his decision the knowledge of what happened up to the point of conversion.

We claim, however, that to neglect this will not change the results materially.

Neglecting Risk: A Single Rate of Return

To bring out the importance of risk and its various measures in explaining conversion option prices, we neglect risk as a first step, in order to have a standard of reference. Thus we make the assumption that the investor is interested only in the expected gain accounting properly for the horizon. In this case, every bond should yield the same rate of return.

$$PK_k = e^{-iT_k} EPW [r_k, \sigma_k, T_k] + u_k \quad i = 1, 2, \dots, n$$

$$= PW_k + u_k \quad , \quad \text{where}$$

- k : running variable of the observations (bonds)
- PK : observed price of the conversion option of bond k in our cross-section shortly after their flotation
- i : required rate of return (to be estimated)
- T : length of horizon of bond k
- EPW : expected value of bond R at horizon T_k given the expected growth rate R and volatility σ_k of the underlying stock
- u : error term

We estimate i by the maximum likelihood method as explained in some detail below. If this naive model is a good explanation of investor behavior then the variance of u is small relative to the variance of PK.¹

If u_k is normally distributed then the probability of getting the observed u_k is

$$p(u_1, u_2, \dots, u_n) = \frac{1}{(\sigma_u^2 \cdot 2\pi)^{n/2}} e^{-\frac{1}{2\sigma_u^2} \sum_{k=1}^n u_k^2} du_1 du_2 \dots du_n$$

As pointed out by Sprengle² we cannot really expect u_k to be distributed normally since $PK > 0$, or $u_k > PW$ is not admissible. But if our model is good, we expect u_k to be small, in which case the truncation at the left end does not matter.

It is a different question whether bias is introduced into the estimate of i by other imperfections. Heteroscedasticity may be expected but does not impart a bias to i though impairing the efficiency of the estimation

¹This is, in essence, the approach taken by Sprengle, although as mentioned above his observations are not different warrants, but the same warrant taken at different points in time.

²Ibid, p. 228.

procedure. Serial correlation would seem to be a small danger in a cross-sectional sample, if the issues are not ordered with respect to an omitted variable. The likelihood function is

$$L = \frac{1}{(\sigma_u^2 \cdot 2\pi)^{n/2}} e^{-\frac{1}{2\sigma_u^2} \sum_{k=1}^n u_k^2}$$

We can demonstrate that there must be a minimum, but there is no guarantee that $i > 0$ (see Figure 2).

To find the maximum likelihood i we differentiate $\log L$ with respect to the desired parameter i and set it equal to zero in order to find the maximum of the likelihood L . We can demonstrate that the maximum exists. The process employed was one of iteration by Newton's method.

Results

The results such a naive approach gives can be described as dismal. The variance of the actual conversion premia (around their mean) is 69.96 ($\2), the variance of PW, the estimated conversion premia has a minimum of 73.00 (using $r = r_w$), i.e., the mean of all conversion premia is a better estimate of the conversion premium than the one derived with the complicated process outlined above, or our model explains nothing.

This is so for various estimates of the growth rate as expected by the investor.

At this point a word is in order about the parameters r , the stock price growth and σ , the stock price volatility we assume that the investor expects to prevail when he buys the conversion option.

We cannot inspect the mind of the inspectors as it was when they

bought the options nor can we hope that the same estimation procedure was employed for every bond or by every investor. We, therefore, experimented with various estimates.

For the more critical expected rate of growth we mention three estimates plus a compound of two of these:

r_w weights the growth rates in the last six years before the issuance of convertible bonds with 1,2,...,6, the highest weight being assigned to the most recent year;

r_m is derived from the assumption that all stocks can be expected to yield the same return to their owner and the more is taken out in dividends the less will be received in the form of price appreciation;

r_a is the actual growth of the stock between the date of issue and four years later (or retirement or June 15, 1964, whatever comes first).

r_c is a compound of r_w and r_m ¹

σ is the variance of the logarithms of stock price ratios, we tried two estimates, one based on the stock prices in the last fifty-three weeks before the bond sale, the other on seven yearly stock prices before the

$$r_c = r_m + r_w \frac{r_w - r_m}{r_m}$$

If r_w is close to r_m the formula gives r_w ; the larger the difference $r_w - r_m$, the smaller the fraction of it that is added to r_m up to a maximum of $2r_m$.

flotation, the first estimate proved to be definitely superior and results mentioned in the following are based on it.

We now turn to the main purpose of the chapter, namely, to devise various measures of risk and to test their effectiveness in explaining observed discount rates (rates of return demanded by the investor) and option prices.

Value of the Option, Required Rate of Return, and Risk

As we expected beforehand, differences between option prices could not be explained with the differences in expected value at the horizon using the same rate of discount for all options.

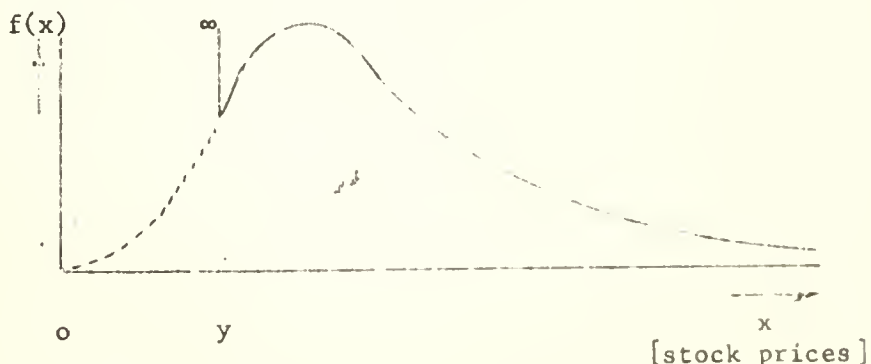
We shall, therefore, try to explain both option prices and rates of return by the investor by introducing measures of risk. Before talking about the results, let me introduce as a third and final model a more sophisticated one.

Unless the investor is risk-neutral, he will look not only at the expected value of outcome, i.e., at

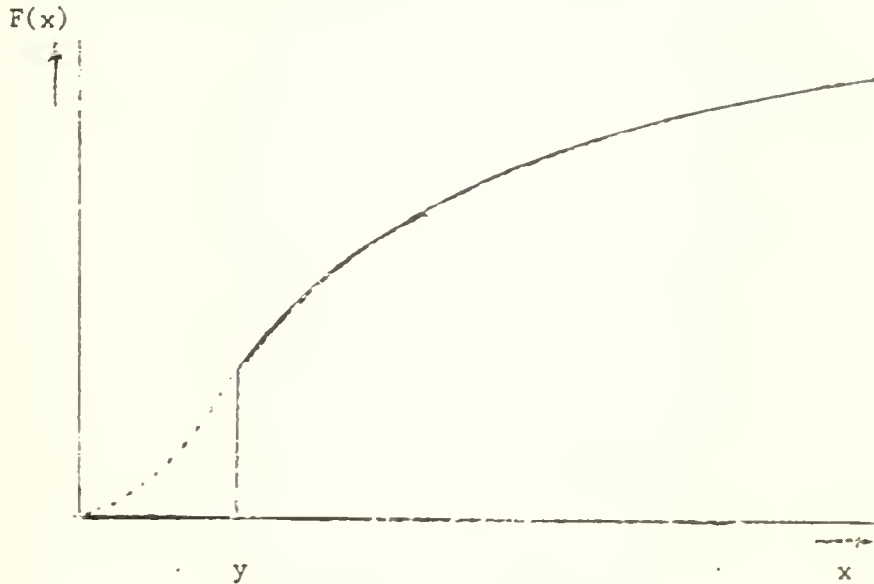
$$EPW = \int_y^{\infty} (x-y) f(x) dx$$

properly discounted to the present, but also at the shape of the distribution of outcomes, which in turn can be represented by the various moments.

The distribution of outcomes has this shape



and the cumulative distribution the one below



It is constructed from the lognormal distribution of stock prices (dotted line) by adding everything that is to the left of the exercise price y (below which conversion is unprofitable) to the distribution at point y and following the lognormal distribution from there.

We have pointed out earlier that simple operating with the lognormal distribution, i.e., neglecting the truncation appears unsatisfactory to us, furthermore, the strong skewness imparted to the distribution by the truncation allows us to test whether skewness is a significant factor in the mind of the investor or whether for him the distribution is adequately described by its first two moments.

We have used the word truncated for the above distribution . This, however, is not correct. In the case of a truncated distribution, we take the area under the lopped off tail and distribute it over the remainder of the distribution proportionately to the density there, here we concentrate it at y : every stock price below y for us is exactly the same as a stock price y . The censored distribution does not fit our bill either, it assumes no knowledge of the distribution below the point of censorship, y , and thus is not defined in its moments at all. We have not found anywhere our distribution and its moments and call it for lack of a better name 'truncated distribution of Class Two.' The method of moment computation is similar to that of the truncated one. The j -th moment around the origin is given by the Stieltjes integral

$$\begin{aligned} v_j &= \int_0^{\infty} x^j dP \\ &= y^j \Lambda(y|\mu, \sigma^2) + \int_y^{\infty} x^j d\Lambda(x|\mu, \sigma^2) \\ &= y^j \Lambda(y|\mu, \sigma^2) + e^{j\mu + \frac{1}{2}j^2\sigma^2} [1 - \Lambda(y|\mu + j\sigma^2, \sigma^2)] \end{aligned}$$

where

$\Lambda(y|\mu, \sigma^2)$ is the cumulative lognormal distribution of y with mean μ and variance σ^2 .

The first moment around the truncation point y instead of the origin gives the expected value EPW, which we will call mean, higher moments, namely, variance, skewness, and curtosis are computed around the mean with

the aid of the Taylor expansion.

The formula makes it clear that μ, σ^2, x and y define the moments completely, there is no need to go back to the original data to compute them. On the other hand, any three moments contain all information about the distribution. Now consider

$$-iT$$

$$PK = e^{-iT} m_1$$

$$\ln PK = -iT + \ln m_1$$

where m_1 , the first moment around the truncation point is the expected value (EPW) as set down earlier.

If we regress $\ln PK$ on $\ln m_1$ and T we get a poor fit and the coefficient of T will have a large error relative to its size; this much we know already from the attempt to work with a simple rate of return.¹

If, then, the valuation of the option depends on risk, and risk as felt by the investor is well described by the moments we worked out in the last few pages then

$$\begin{aligned} \ln PK = \text{const} + c_1 \ln m_1 + c_2 \ln m_2 + c_3 \ln m_3 \\ + c_4 \ln m_4 + c_5 \text{RATING} + c_6 \text{ISS} + c_7 T \end{aligned}$$

where

PK price of one option to acquire one share. It is calculated as price

¹ see Section 4.4

of the conversion privilege divided by $100/PC$, the number of shares that can be acquired through conversion of a \$100 bond if the conversion price is PC . The price of the conversion privilege, in turn, is the price of the bond the first time it is quoted on the market after the flotation, minus the estimated straight debt value at that time.

m_0 zero-th moment of the truncated lognormal distribution of the outcomes. This is the cumulative probability that the stock price will exceed at the horizon T the straight debt value per share (which in turn is roughly equal to the conversion price).

m_1 first moment around the truncation point, i.e., expected value of the option at the horizon T , mean

m_2 second moment around the mean m_1 , variance

m_3 third moment around the mean m_1 , skewness

m_4 fourth moment around the mean m_1 , kurtosis (excess)

$\ln m_1, \ln m_2, \ln m_3, \ln m_4$ are the natural logarithms of the above moments

ATING quality rating (Aa, A, Baa, Etc.) by Moody's Investors Service, Inc., a proxy measure for risk

ISS date of issue, introduced in order to detect any change in either

- a. investor attitude towards the convertible bond or
- b. terms of the conversion privilege

over time

T horizon¹

should give a much better regression and even the coefficient of T will be accompanied by a much smaller standard error relative to its size, its meaning now being the rate of return demanded after accounting for risk.

¹See also below

We shall also regress

$$\ln PK = c_1 + c_2 r + c_2 \ln y + c_4 \sigma^2 + c_5 \ln x_0 + c_6 \text{RATING} \\ + c_7 \text{ISS} + c_8 \sigma$$

where

r expected future stock price growth rate; we use alternatively

r_w weighted pre-issue stock price growth rate

r_m "market" rate of return

r_c a compound of the preceding two

r_a actual (post-issue) stock price growth rate

in order to investigate how the investor estimates future stock price growth and whether his estimate is close to growth actually realized

σ, σ^2 standard deviation and variance of the logarithms of stock prices in the last fifty-three weeks before the flotation of the bond

$\ln x_0$ logarithm of the stock price observed when the bond is quoted the first time after flotation, a possible proxy variable for risk, with higher priced stocks presumably being less risky

$\ln y$ logarithm of the straight debt value divided by the share price at the time of issue. (Since x_0 is set to 1 as explained below, $\ln y/x_0 = \ln y$). The variable is a measure of the distance of the stock price at the time of the bond issue to the conversion price.

in order to see how well we can predict or explain PK without the relatively complicated model developed earlier and the moments derived in one of the last sections. If we can get just as good an explanation from a few easily obtainable simple variables, there is no point in searching for elaborate models. We have tried to make this naive model as good as possible.

Next we combine the variables from both regression equations to see whether the second set contains elements we have not taken care of automatically by our moments.

One adjustment remains to be explained: all stock prices x_0 are normalized to one in the computation of the moments and everywhere else (except in $\ln x_0$, of course). PK is likewise divided by x_0 . Since the unadjusted option price PK presumably varies with the price of the optioned stock, x_0 , and the moments definitely vary with x_0 , not making the adjustment builds positive correlation (as well as heteroscedasticity) into the regression. After the adjustment, all variables are dimensionless or have only time as dimension.

We have room only for a summary of the results:

In the regression without moments r_c , $\ln x_0$, $\ln y$, σ^2 explain about 46% of the variance of $\ln PK$; adding RATING, DVYLD, and time of issue, ISS, boosts that figure to 58%. All variables are significant except σ^2 and RATING.

This is as far as analysis without moments carries us in explaining the value an investor puts on the option. Considering that we are dealing with a cross-sectional study, with the members being taken over a 15 year interval, the correlation is high and very highly significant. We may feel somewhat uneasy when we notice that the proxy variable for risk σ^2 has a positive sign, from a risk-averting investor we expect the opposite; furthermore, σ^2 is totally insignificant.

The behavior of σ^2 , however, is no riddle: in PK or $\ln PK$ it has two conflicting roles: it increases expected value, but also heightens risk. DVYLD has a negative coefficient--the investor apparently reasons that the higher the dividend, the lower retained earnings and expected

price increase. r_c and DVYLD are highly (negatively) intercorrelated.

Turning to the regression with the logarithms of the moments, we notice that the expected value as computed in our model taken alone explains the actual price paid for the option better than rate of return, stock price, ratio of conversion price to stock price, stock price taken together (80% vs. 59%). Adding the second through fourth moments adds 3% to explained variance.

The logarithms of the moments alternate in sign. This would seem to indicate risk aversion, preference for skewness and dislike of kurtosis. All variables are significant at the .01 level. Some caution in reading the results seems to be in order. The logarithms of the moments are high collinearity causes instability of the coefficients. If the moments themselves are added, unexplained variance is almost halved ($R^2 = .89$). Expected value, variance, etc. are now represented twice with the result that each appears once with a positive and once with a negative coefficient. Adding r_c , σ^2 , etc., increases R^2 only from .89 to .93, whereas adding the moments and their logarithms to these variables increases explained variance from 58% to 92%.¹

The Role of the Date of Issue

A question of interest is whether the bonds or the investors' attitude towards them has changed over the 15 years time span from 1948 to 1963.

¹Somewhat surprisingly, we got very much the same results in both regressions using PK as dependent variable (and the moments themselves as independent variables).

In the zero-order correlations time of issue is highly significant ($t=-6.38$); the more advanced the years, the higher the price paid for the option and the lower the required rate of return. When, however, we re-examine its t -coefficient as the moments are introduced, we discover that it drops below the 10% level of significance and when $\ln y$, etc. are added it drops below the 20% level ($t=-.34$). This combination of findings plus the discovery that there exists a modest but highly significant correlation ($R = .58$) between the ISS and the logarithms of the moment of expected value, which is not affected by the rate of return required by the investor, suggests to us a different explanation. While the investors' attitude may or may not have changed over the years, the corporations' attitude towards the bond has changed. Not only are there more bonds issued in later years, particularly the second half of the Fifties, but the corporations have also made the conversion privilege more attractive, more valuable. This is confirmed by the tendency to issue bonds of poorer quality (positive correlation RATING and ISS, $R = .38$) which nonetheless are positively correlated with the logarithm of expected value ($R = .12$). There is a strongly positive correlation ($R = .54$) between time of issue and price of the option in per cent of the total package named convertible bond. One method corporations have chosen to make the conversion privilege more attractive is to lower the conversion price in relation to the stock price at the time of issue; we thus find a negative correlation between $\ln y$ and ISS ($R = .23$). Second, corporations issuing convertible bonds had a significantly better past growth record (positive correlation between r_w and ISS is $R = .15$). Third, stock

price volatility σ grew with ISS ($R = .30$) making for greater expected value. It clinches our case to find that in the regression for the required rate of return, i , the time of issue, ISS is represented with a positive though insignificant coefficient once the coefficients of variance and of other variables are present.¹

The Rate of Return Demanded by the Investor

Consider again

$$PK = e^{-iT} m_1$$

$$\ln PK = iT + \ln m_1$$

Rates of return demanded by the investor i were computed as the rate that discounted expected value of the horizon T such as to equal the price PK paid for the option.

If i is a function of risk as expressed by variance, skewness, etc. then

$$\ln PK = (i_0 + c_2 \frac{m_2}{m_1} + c_3 \frac{m_3}{m_2} + c_4 \frac{m_4}{m_2} + \dots) T + \ln m_1$$

- where m_2/m_1^2 : coefficient of variation squared
 $m_3/m_2^{3/2}$: coefficient of skewness
 m_4/m_2 : coefficient of skewness

¹We also found that for a sample of 62 bonds issued between 1948 and 1956, the average option was priced at \$10 per \$100 bond vs. \$18 for a sample of 63 bonds issued between 1959 and 1963.

- i_0 : rate of return demanded on a riskless investment
+... : other factors one might like to introduce

Rearranging the above equation slightly

$$i = \frac{\ln PK - \ln m_1}{T} = i_0 + c_2 \frac{m_2}{m_1} + c_3 \frac{m_3}{m_2} + c_4 \frac{m_4}{m_2}$$

To introduce the moments themselves or their logarithms would be undesirable when considering the required rate of return which is dimension-free except for time. We have, therefore, computed magnitude-free measures of variance, skewness and kurtosis.

Starting again with the regression without the moments we find the rate of stock price growth is one of the variables with the greatest influence on the required rate of return i , the higher the one, the higher the other; this is well in keeping with earlier remarks that the investor may extrapolate past growth but that he is aware of the fact that the higher the growth rate of the past, the less reliable the extrapolation. If we use r_w which is only based on past performance, this finding is brought out even more strongly. The magnitude of the stock price at the time of issue of the convertible bonds turns out to make a significant contribution in explaining the variance of the dependent variable, too. The correlation is a negative one. Bigger, more stable corporations have higher priced shares than the more risky, smaller corporations. And, one does associate risk with penny-stocks. Unlike in the regression for $\ln PK$, σ^2 is a highly significant indicator of risk, since it no longer has to do double duty as a variable standing for high expected value and

high risk. Trying to explain i , the investor's discount rate, with the coefficients of variation, skewness and kurtosis alone is disappointing. Only 17% of total variance in the required rate of return are explained but taken together with r_c and σ^2 we get a substantial improvement not only over the coefficients of variation taken by themselves but also r_c , σ , etc., considered alone (47%). Although the number of independent variables has increased, the size of the coefficients relative to their standard errors has gone up over what they were in the separate regressions. The coefficient of skewness reduces unexplained variance highly significantly, so does the coefficient of kurtosis, which when introduced also increases the t-coefficient of the coefficient of skewness.

All the variables have the 'correct', i.e., the predicted sign--the required rate of return increases with σ^2 , coefficient of variance, kurtosis and decreases with skewness.¹

¹As these variables are highly intercorrelated, our confidence in the results is not as great as it would be without collinearity. It is comforting to notice, however, that when variables are added that, in our opinion, measure the same thing (e.g., range of the outcomes) which a variable already in the regression also measures, they tend to reduce the t-coefficient of the variables introduced earlier. When, however, to such a variable another is added, which is also collinear with it but in our opinion measures something different (e.g. skewness) something towards which according to our hypothesis the investor has the opposite attitude--then the t-coefficient of the variable introduced first is increased. If our hypothesis that the investor likes skewness and dislikes variance is correct, this is exactly what we expect. Suppose we introduce σ^2 first and then the coefficient of variance. The second variable will contribute some new information, but otherwise duplicate information already introduced through the first one. If the latter aspect is important, the t-coefficient of the first variable may well fall, especially, if the second variable specifies the information in a superior fashion. Since loosely speaking there is only so much 'significance' to distribute, the t-coefficient of the first must fall of needs if the second variable is assigned a high one. Now let us change the picture. Suppose we introduce first a measure for variance then one of skewness. The two measures are intercorrelated. Now assume that the investor's utility is increased through skewness but decreased through variance. When variance is introduced alone then due to collinearity it stands for both variance and skewness with low significance as the result. When, however, both variables are present, there is one for each job; to measure variance and to measure skewness and both emerge as what they are: highly significant.

Date of issue ISS is insignificant and positive. Thus, discrediting the hypothesis that the investor has become more familiar with the convertible bond and is satisfied with a low rate of return. It is interesting to notice that variable $\ln x_0$, the price of the stock at time of issue, is insignificant once other risk measuring variables are introduced, though highly significant in the zero-order correlation. This means the investor does not associate (demonstrably) a high priced share with low risk cet. par. or per se.

Anticipation of Objections

Several objections may have come to your mind.

Let me anticipate three of them.

1. One may ask whether in view of high intercorrelation between some of the 'independent' variables might not bias the coefficients, or, much worse, whether the signs of the coefficients of which we made so much are not built-in by this method of attack. The answer is that some bias quite likely may exist that, however, the change of signs from the coefficient of variance to that of skewness and back to that of kurtosis can be shown not to be built in. If there is an influence dragging the size of the coefficient of variance in one direction, it would be even stronger in the same direction for skewness. The same observation can be made with respect to the pair--skewness and kurtosis.

2. The second anticipated objection concerns the horizon. The horizon, it will be remembered, was found in a diagram of expected value as a function of time by tilting a straight line around the price actually paid for the option (as pivot) until it touched the expected value curve.

The slope of the curve gave the rate of return demanded by the investor, the abscissa at the point of contact, the horizon. Moments then were calculated at that horizon and regressed of the option price. It can be maintained that the dependent variable, the option price, is used in the computation of the independent variables and that correlation is to be expected. To remove that objection we employed a two-step procedure. First, we did the moment calculation and regression analysis with the same average horizon for all bonds, then we compute the option price as predicted by the regression equation. These option prices are then used to compute individual horizons and the moments associated with them. In a second step, the actual option prices are then regressed on those moments and other variables. (The procedure is somewhat reminiscent of two-stage least squares.)

The result is simple to state: correlation is reduced but all conclusions are unaffected, in particular, the coefficients still have the 'correct,' i.e., the predicted sign.

3. Third, it may be supposed that the high return on convertible bonds is incompatible with portfolio efficiency. Does there exist a portfolio of straight debt and conversion option yielding more than straight debt while not exceeding it in variance? While no attempt has been made to refute this for every bond individually, we have checked that using the average figures for the options and stocks concerned, the high return on conversion options is consistent with portfolio efficiency.

Investor Foresight

Our final point takes up the question of how well the investor at

the time of purchase of the option was able to foresee future development of stock prices. We can present the results only in very condensed form: Extrapolation of past growth used as the estimate of future growth in computing the moments used for the regression analysis explains option prices far better than moments based on the actual post-issue growth rate, i.e., on perfect foresight. Regressing the rates directly alone or together with other variables shows the same pattern: the higher preissue price growth, the higher the option price, the higher post-issue growth, the lower the option price. Furthermore, correlation is higher between price and pre-issue growth than between price and post-issue growth. Pre- and post-issue growth are negatively correlated. All this suggests that the investor, on the average, had incorrect expectations. The picture, however, is more complicated: the required rate of return is more highly correlated with post-issue growth rate than with the pre-issue rate, furthermore, the correlation is positive also for the latter one, indicating to us that though the investor was not able to predict which stocks would rise, he was quite well able to identify the risky ones.

Our findings throw a curious oblique light on John F. Muth's "Rational Expectations" hypothesis. To quote Muth

1. Averages of expectations in an industry are more accurate than naive models and as accurate as elaborate equation systems, although there are considerable cross-sectional differences of opinion.
2. Reported expectations generally underestimate

the extent of changes that actually take place.¹

While Muth's statement No. 1 may hold for firms as he claims, it is not borne out in our case for individual investors. On the other hand, our investor seems to have a good feeling for stocks where the extent of the change is likely to be great and classifies such stocks as risky, but we cannot quite maintain that this is in contradiction to statement No. 2 without further analysis. In any case, the investor does not seem to be able to forecast the change of direction of stock price growth.

¹John F. Muth, "Rational Expectations and the Theory of Price Movements," Econometrica (29, No. 3), July 1961, p. 316.

