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Abstract

A detailed study on the lateral displacements of a TM wave transmitted and reflected from a symmetric gyrotropic slab is presented. We give the analytic formulas for the transmission coefficient and the reflection coefficient, as well as the corresponding lateral displacements. It is found that due to the external magnetic field, the displacement of a transmitted beam is different from that of reflected, even for a lossless symmetric configuration. Furthermore, within the chosen frequency band, when the incident angle is near the Brewster angle, the shift of a reflected wave can be large with nonzero reflectance, and can be positive or negative depending on the direction of the applied magnetic field and the incident wave.

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1. Introduction

The Goos-Hänchen (GH) lateral shift effect [1, 2], which refers to the spatial displacement of the reflected wave from the position expected by geometrical reflection, has been studied for many years. Traditionally, GH effect has been most often referred to the shift occurred during total reflection, when a beam in a transparent medium is reflected at the interface adjacent to a medium of lower reflective index. In this situation, the displacement is usually positive and in the forward direction, i.e., in the same direction of the component of the incident wave vector along the interface. Recently there are some studies on negative GH shifts in different systems: strongly reflecting and attenuating media such as metal at IR frequencies [3, 4], non-absorbing [5], weakly absorbing interfaces [6-10], slabs [11], metallic gratings [12], transparent dielectric slabs [13], dielectric slabs backed by a metal [14], photonic crystals [15], and left-handed materials [16-18]. Among them there are some reports on large lateral shift near Brewster or pseudo-Brewster angle upon reflection from a weakly absorbing medium [9, 10, 17, 18].

In this present paper, we consider the GH lateral displacements from a symmetric lossless gyrotropic slab in the Voigt configuration. Due to the effect of the applied magnetic field, the lateral displacement of a TM wave reflected from the slab is not the same as that of the transmitted wave. It is shown that within a chosen frequency band, when the incident angle is near the Brewster angle, a large displacement of the reflected wave with nonzero reflectance can be observed. Moreover, we can make the lateral displacement be positive or negative by adjusting the direction of the incident wave or the applied magnetic field.

This paper is arranged as follows. In section 2, the theory is presented and it includes the analytic formulas for the reflection and transmission coefficients and mathematical result for the lateral displacement which is obtained by the stationary phase approach. In section 3, we give examples that illustrate the lateral displacements of TM waves transmitted and reflected by the symmetric gyrotropic slab. Finally, concluding remarks are given in section 4.

2. Lateral displacements using the stationary phase approach

We consider the geometric problem in Fig. 1, where a plane wave is incident from an isotropic medium into a infinite gyrotropic slab at an oblique angle θ_i with respect to the normal of the interface. The gyrotropic slab of thickness d is arranged in the Voigt configuration, where the external magnetic field $\bar{B}_0$ is in +z direction. The isotropic media in region 1 and region 3 are the same, with permeability μ_1 and permittivity ε_1. Region 2 is gyrotropic medium with permeability μ_2 and permittivity tensor $\overline{\overline{\varepsilon}}_2$, which takes the following form:

$$\overline{\overline{\varepsilon}}_2 = \begin{bmatrix} \varepsilon_{xx} & i\varepsilon_g & 0 \\ -i\varepsilon_g & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix}, \qquad (1)$$

where elements are given by

$$\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{\infty}(1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2}), \tag{2.1}$$

$$\varepsilon_{zz} = \varepsilon_{\infty}(1 - \frac{\omega_p^2}{\omega^2}), \tag{2.2}$$

$$\varepsilon_g = \varepsilon_{\infty}[-\frac{\omega_p^2\omega_c}{\omega(\omega^2 - \omega_c^2)}]. \tag{2.3}$$

Here, $\omega_p = \sqrt{Nq_e^2 / m_{eff}\varepsilon_{\infty}}$ and $\bar{\omega}_c = q_e \bar{B}_0 / m_{eff}$ are the plasma and cyclotron frequencies respectively, ε_{∞} is the background permittivity, N is the electron density, m_{eff} is the effective mass, and q_e is the electron charge.

It is known that in the Voigt configuration, waves can be decoupled into TE and TM modes and only the TM mode is affected by the gyrotropy [19, 20]. Hence we only focus on the TM wave here. For TM waves, with wave vectors $\bar{k}_1 = \pm\hat{x}k_{1x} + \hat{y}k_y$ in the isotropic medium and $\bar{k}_2 = \pm\hat{x}k_{2x} + \hat{y}k_y$ in the gyrotropic medium, the dispersion relations can be expressed as

$$k_y^2 + k_{1x}^2 = \omega^2\mu_1\varepsilon_1, \tag{3}$$

$$k_y^2 + k_{2x}^2 = \omega^2\mu_2\varepsilon_V. \tag{4}$$

Here $\varepsilon_V = (\varepsilon_{xx}^2 - \varepsilon_g^2)/\varepsilon_{xx}$, is the equivalent permittivity of the gyrotropic medium in the Voigt configuration for the waves being studied.

Determined by the Maxwell equations and the boundary conditions, the transmission and reflection coefficients can be written as

$$T = \frac{1}{\cos(k_{2x}d) - i\sin(k_{2x}d)\left[(k_{2x}^2 + \sigma^2k_{1x}^2 + \tau^2k_y^2)/(2\sigma k_{1x}k_{2x})\right]}, \tag{5}$$

$$R = \frac{\sin(k_{2x}d)\left[\tau k_y/k_{2x} - i(\sigma^2k_{1x}^2 - \tau^2k_y^2 - k_{2x}^2)/(2\sigma k_{1x}k_{2x})\right]}{\cos(k_{2x}d) - i\sin(k_{2x}d)\left[(k_{2x}^2 + \sigma^2k_{1x}^2 + \tau^2k_y^2)/(2\sigma k_{1x}k_{2x})\right]}, \tag{6}$$

where the parameters σ and τ are dimensionless, defined as

$$\sigma = \frac{\varepsilon_V}{\varepsilon_1}, \qquad \text{and} \qquad \tau = \frac{\varepsilon_g}{\varepsilon_{xx}}. \tag{7}$$

Here we can see that τ is a direct manifestation of the applied magnetic field $\bar{B}_0$, arising from the off-diagonal element ε_g in the permittivity tensor. Without an external magnetic field, both ε_g and τ are zero.

The lateral shift has the well-known expression $S = -d\phi/dk_y$, which was proposed by Artmann using the stationary phase method [21]. After some algebra manipulations, and by noting that the denominators of both coefficients are the same, the lateral displacements of the reflection and transmission can be expressed as

$$S_t = -\frac{1}{|T|^2}\left\{ \mathrm{Re}[T]\frac{\partial\,\mathrm{Im}[T]}{\partial k_y} - \mathrm{Im}[T]\frac{\partial\,\mathrm{Re}[T]}{\partial k_y} \right\}$$

$$= 2\sigma d\left|\tan\theta_i\right| \frac{\frac{\sin(2k_{2x}d)}{2k_{2x}d}\left[-\sigma^2 k_{1x}^4 - k_{2x}^4 + k_{1x}^2 k_{2x}^2(\sigma^2+1-2\tau^2) - \tau^2 k_y^2(k_{1x}^2+k_{2x}^2)\right] + k_{1x}^2(\sigma^2 k_{1x}^2 + k_{2x}^2 + \tau^2 k_y^2)}{4\sigma^2 k_{1x}^2 k_{2x}^2\cos^2(k_{2x}d) + (\sigma^2 k_{1x}^2 + k_{2x}^2 + \tau^2 k_y^2)^2\sin^2(k_{2x}d)}$$

, (8)

$$S_r = -\frac{1}{|R|^2}\left\{ \mathrm{Re}[R]\frac{\partial\,\mathrm{Im}[R]}{\partial k_y} - \mathrm{Im}[R]\frac{\partial\,\mathrm{Re}[R]}{\partial k_y} \right\} = \Delta S_{rt} + S_t\,, \tag{9}$$

where

$$\Delta S_{rt} = -2\sigma\tau\frac{k_y\left|\tan\theta_i\right|\left[k_{1x}^2(\sigma^2+\tau^2-2) + k_{2x}^2 + \tau^2 k_y^2\right] + k_{1x}(\sigma^2 k_{1x}^2 - k_{2x}^2)}{4\sigma^2\tau^2 k_{1x}^2 k_y^2 + (\sigma^2 k_{1x}^2 - k_{2x}^2 - \tau^2 k_y^2)^2}\,. \tag{10}$$

When there is no applied magnetic field, τ=0, then the result above is consistent with those in reference [22]. Furthermore, when the inequality $k_y^2 \ge \omega^2\mu_2\varepsilon_V$ is satisfied, total reflection occurs at the boundary x=0, and k_{2x} becomes imaginary. Let $k_{2x}=i\alpha$, the dispersion relation of gyrotropic medium becomes $k_y^2 - \alpha^2 = \omega^2\mu_2\varepsilon_V$, and Eqs. (8) and (10) can be rewritten as

$$S_t' = 2\sigma d\left|\tan\theta_i\right| \frac{\frac{\sinh(2\alpha d)}{2\alpha d}\left[\sigma^2 k_{1x}^4 + \alpha^4 + k_{1x}^2\alpha^2(\sigma^2+1-2\tau^2) + \tau^2 k_y^2(k_{1x}^2 - \alpha^2)\right] - k_{1x}^2(\sigma^2 k_{1x}^2 - \alpha^2 + \tau^2 k_y^2)}{4\sigma^2 k_{1x}^2\alpha^2\cosh^2(\alpha d) + (\sigma^2 k_{1x}^2 - \alpha^2 + \tau^2 k_y^2)^2\sinh^2(\alpha d)}$$

(11)

,

$$\Delta S_{rt}' = -2\sigma\tau\frac{k_y\left|\tan\theta_i\right|\left[k_{1x}^2(\sigma^2+\tau^2-2) - \alpha^2 + \tau^2 k_y^2\right] + k_{1x}(\sigma^2 k_{1x}^2 + \alpha^2)}{4\sigma^2\tau^2 k_{1x}^2 k_y^2 + (\sigma^2 k_{1x}^2 + \alpha^2 - \tau^2 k_y^2)^2}\,. \tag{12}$$

It has been known that, for the lossless symmetric configuration, the lateral shifts of both the reflected and transmitted beams are the same [13, 23-25]. Nevertheless, in gyrotropic case, it is not true. According to Eqs. (10) and (12), a difference ΔS_{rt} ($\Delta S'_{rt}$) between S_r (S'_r) and S_t (S'_t) arises directly from the external magnetic field $\bar{B}_0$ and the magnitude of the difference is independence of d, thickness of the slab.

3. Result and discussion

Here, we consider an indium antimony (InSb) slab in vacuum. The material parameters used in the computation are: μ_1=μ_2=μ_0, ε_1=ε_0, ε_∞=15ε_0, N=10^{22}m^{-3}, and m_{eff}=0.015m_0 [26, 27]. Hence, ω_p=1.19×10^{13}rad/s and ω_c/ω_p=0.98B_0. For the lossless symmetric gyrotropic slab, in Fig. 2, we plot the lateral displacements in the (ω, θ_i) plane with a color scale proportional to the magnitude of the displacement, normalized to the incident wavelength in vacuum λ_0=2$\pi c/\omega$. The horizontal axis is frequency, normalized to the plasma frequency ω_p, and the vertical axis is incident angle, from -90 degree to +90 degree. We define the angle to be positive if the wave comes from the lower left and consider it to be negative if the wave comes from the upper left. (For example, the incident angle shown in Fig. 1 is positive.)

From Fig. 2(a), we can see that, corresponding to the variation of equivalent permittivity ε_V with frequency, there are two regions (A and B) in the (ω, θ_i) plane. In region A, total reflection occurs on the boundary at x=0; while region B is where total reflection does not occur. As shown in Figs. 2(b) and 2(c), the existence of the applied magnetic field splits each region up into two, marked with subscript 1 and 2.

Fig. 2(a) shows magnitude of displacement of both transmitted and reflected waves when there is no applied magnetic field. In this case, since ε_g=0 and τ=0, ΔS_{rt} ($\Delta S'_{rt}$) is zero [see Eqs. (10) and (12)]. Hence S_t = S_r ($S'_t = S'_r$), i.e., the lateral displacements of waves transmitted and reflected from the lossless symmetric gyrotropic slab are always the same in the absence of external magnetic field. However, it is not true when an external magnetic field is applied. The presence of an applied magnetic field $\bar{B}_0$ results in a nonzero ε_g and τ, and gives rise to ΔS_{rt} ($\Delta S'_{rt}$), a difference between S_t (S'_t) and S_r (S'_r), which means that if there is an applied magnetic field $\bar{B}_0$, the lateral shifts of TM waves transmitted and reflected from the gyrotropic slab are not the same even for the lossless symmetric configuration. We show them in Fig. 2(b) and Fig. 2(c) respectively.

It is important to note that the shifts in Fig. 2(a) and 2(b) are both symmetric with respect to the incident angle θ_i while asymmetric in Fig. 2(c). This is due to the nonreciprocal nature of the gyrotropic medium. If we keep the magnitude of incident angle θ_i but change the sign from positive to negative, i.e., the wave is incident from the upper left instead of the lower left, the field in gyrotropic medium changes and causes the modification of reflection and transmission coefficients. In fact, we can obtain the formulas for negative θ_i case by the replacement $\tau \rightarrow -\tau$ in all equations that include τ. From Eqs. (8) and (11), we can see that lateral shift of transmitted wave S_t (S_t') remains the same because it is the function of τ^2; while Eqs. (10) and (12) show that ΔS_{rt} ($\Delta S_{rt}'$) will change sign. That is why shifts of transmitted wave in Fig. 2(b) are symmetric however those of the reflected wave in Fig. 2(c) are not. Without an applied magnetic field, τ=0, the sign of the incident angle does not affect the lateral shift at all, resulting in lateral shifts of both the transmitted and reflected wave in Fig. 2(a) are symmetric too.

It is also of interest to note that in regions B_1 and B_2 of Fig. 2(c), there is a positively or negatively large displacement for the reflected wave but no such phenomenon in Fig. 2(b) for the transmitted wave. This is related to the Brewster angle, shown in white dashed line. For TM waves, there is an abrupt phase change of the reflection coefficient near the Brewster angle [9, 10, 17] which causes a large lateral shift for the reflected beam. Nevertheless, there is no abrupt change for the phase of the transmission coefficient at Brewster angle. Therefore, we expect a large lateral shift of the reflected beam near the Brewster angle but not for the transmitted beam. To compare the lateral shifts of the reflected waves S_r and those of the transmitted waves S_t at the Brewster angle, we choose two frequencies in region B_2, and plot S_r and S_t versus the incident angle θ_i in Fig. 3(a). (In region B_1, since the equivalent permittivity ε_V changes greatly with frequency, the Brewster angle is sensitive to the frequency. Furthermore, the frequency band of region B_1 is narrower, so we focus on the region B_2 instead.) From it we can see that, for a fixed frequency, near the Brewster angle, the lateral displacement of the reflected wave S_r can be positively or negatively large while no such phenomenon for the transmitted beam. Furthermore, near the Brewster angle, the sign of the incident angle θ_i affects the sign of S_r greatly. That is because ΔS_{rt}, which will change the sign when the sign of incident angle θ_i changes [see Eq. (10)], plays an important role in the total lateral shift for the reflected beam under this situation.

Without an applied magnetic field, the lateral shift of the reflected wave is of no interest near the Brewster angle because the reflection coefficient vanishes. However, due to the presence of an applied magnetic field $\bar{B}_0$, the existence of τ makes it reasonable to expect a large finite slope of the phase change with a nonzero reflection. Fig. 3(b) shows the corresponding reflectance $|R|^2$ of cases in Fig. 3(a) versus incident angle. It is shown that, for a fixed frequency, the reflectance changes with different incident angle, and it will be the smallest when the incident angle equals to the Brewster angle. Although the reflectance is small at the Brewster angle, it can be nonzero. For instance, when ω=1.34ω_p, the smallest reflectance is 0.036. In addition, we focus on the Brewster angle cases and show the corresponding lateral shifts for reflected waves S_r and the reflectance $|R|^2$ (multiplied by 1000) versus frequency in Fig. 4. From it we can see that, when the condition $k_{2x}d=m\pi$ is satisfied, magnitude of the reflection coefficient vanishes. For instance, when the working frequency is near 1.237ω_p, 1.287ω_p, 1.383ω_p, and 1.528ω_p, $|R|$=0. But within the chosen frequency band (for example, 1.22ω_p to 1.50ω_p) we can obtain a relatively large reflectance (for instance, ω=1.32ω_p, $|R|^2$=54.45×10^{-3}). It can be larger than that in weakly absorbing media [9], making it easier to detect. However, when the frequency is very high, the effect of the magnetic field vanishes, causing the zero reflection at the Brewster angle.

Furthermore, we want to mention that, due to nonreciprocal characteristics of the gyrotropic medium, for reflection near the Brewster angle within the chosen frequency band, there are several methods to realize the negative displacement. As shown in Fig. 5, when the applied magnetic field $\bar{B}_0$ is in +z direction, if both the incident and reflected waves are at the left side of the slab and the incident wave comes from the lower left (case 1), the lateral shift for reflected wave is positive; while it becomes negative when the incident wave comes from the upper left (case 2). If the incident and reflected waves move to the right side of the slab and the incident wave comes from the lower right (case 3), the lateral shift is negative, the same as case 2. Moreover, we can change the signs of all the lateral shifts by adjusting $\bar{B}_0$ from +z direction to –z direction.

4. Conclusion

This paper investigates the lateral displacements of TM waves transmitted and reflected from a symmetric gyrotropic slab. For TM waves, due to the external magnetic field and the nonreciprocal nature of the gyrotropic medium, the displacements for transmitted beam are different from those for reflected, even for the lossless symmetric configuration. Furthermore, within the chosen frequency band, when incident angle is near the Brewster angle, the shift of reflected wave can be positively or negatively large with relatively large reflectance, making it easier to detect. Meanwhile, several cases for negative lateral shift are shown. The results here may have potential applications in magnetic modulations. For instance, switching the direction of the external magnetic field from +z to –z can result in a significant variation of the difference between the lateral displacements of the transmitted wave and the reflected wave.

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List of Figures

FIG. 1. Symmetric gyrotropic slab with thickness d in an isotropic medium. Region 1 and Region 3 are the same isotropic media, with permittivity ε_1 and permeability μ_1. Region 2 is a gyrotropic medium with $\overline{\overline{\varepsilon_2}}$ and μ_2. An applied magnetic field $\overline{B}_0$ is in $+z$ direction, parallel to the interfaces and perpendicular to the direction of the wave propagation (Voigt configuration). S_r and S_t refer to the lateral displacements of reflected and transmitted waves respectively.

FIG. 2. Lateral displacements of TM waves transmitted and reflected from a lossless symmetric gyrotropic slab, the Brewster angle is also shown (in white dashed line). The incident and reflected waves are both on the left side of the slab. We consider the angle as positive when the incident wave comes from the lower left. The frequencies are normalized to ω_p, and the displacements are normalized to $\lambda_0=2\pi c/\omega$, the incident wavelength. The thickness of the slab $d=0.5\lambda_p=\pi c/\omega_p$. Without an applied magnetic field, the lateral displacement of the transmitted wave is equal to that of reflected: $S_t=S_r$; while the presence of an applied magnetic field results in a difference between S_t and S_r. (a) Both S_t and S_r with no applied magnetic field; (b) S_t with an applied magnetic field $\overline{B}_0 = +\hat{z}0.4\mathrm{T}$; (c) S_r with an applied magnetic field $\overline{B}_0 = +\hat{z}0.4\mathrm{T}$.

FIG. 3. (a)Dependence of S_t and S_r on the incident angle θ_i. S_t is lateral displacement for transmitted wave while S_r is that for reflected. Both are normalized to $\lambda_0=2\pi c/\omega$. (b)Dependence of magnitude of the reflectance $|R|^2$ on the incident angle θ_i. The thickness $d=0.5\lambda_p=\pi c/\omega_p$ and the applied magnetic field $\overline{B}_0 = +\hat{z}0.4\mathrm{T}$. The red and blue lines are for cases of $\omega=1.34\omega_p$ and $\omega=1.40\omega_p$ respectively.

FIG. 4. When the incident angle equals to the Brewster angle θ_B, the lateral displacements of reflected wave S_r (normalized to λ_0), and the reflectance $|R|^2$ (multiplied by 1000) versus frequency. The thickness $d=0.5\lambda_p=\pi c/\omega_p$ and the applied magnetic field $\overline{B}_0 = +\hat{z}0.4\mathrm{T}$.

FIG. 5. Comparison of several incident wave cases. Case 1: incident wave and reflected wave are both on the left side of the slab, and the incident wave comes from the lower left. $S_{r1}>0$. Case 2: incident wave and reflected wave are also on the left side, but the wave is incident from the upper left. Due to the nonreciprocal nature of the gyrotropic medium, the shift in case 1 is different from that in case 2. $S_{r2}<0$. Case 3: incident wave also comes from below, but on the right side of the slab. $S_{r3}<0$. Case 2 and case 3 are equivalent, and lateral shifts are the same.

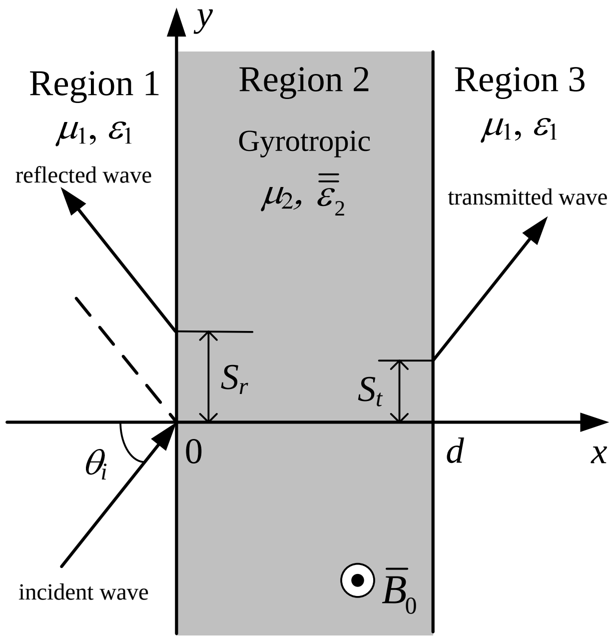
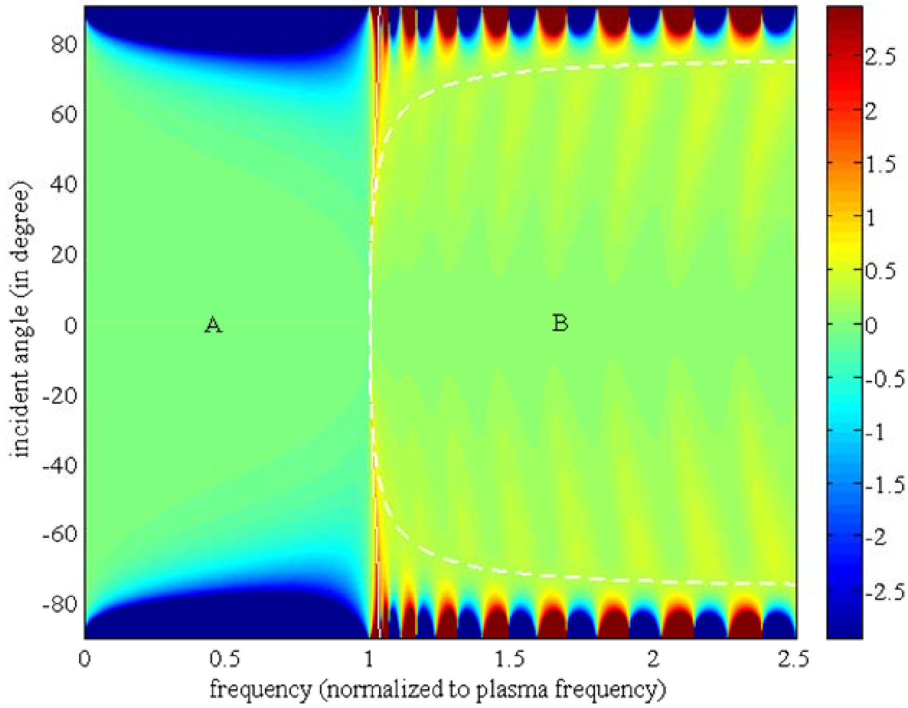
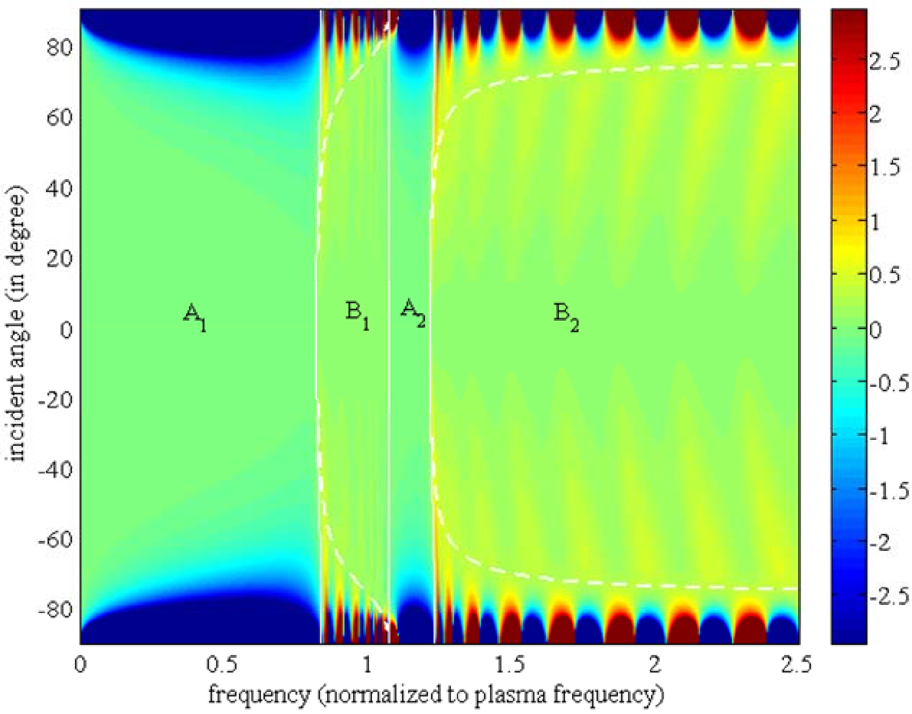


FIG. 1. Symmetric gyrotropic slab with thickness d in an isotropic medium. Region 1 and Region 3 are the same isotropic media, with permittivity ε_1 and permeability μ_1. Region 2 is a gyrotropic medium with $\overline{\overline{\varepsilon}}_2$ and μ_2. An applied magnetic field $\overline{B}_0$ is in $+z$ direction, parallel to the interfaces and perpendicular to the direction of the wave propagation (Voigt configuration). S_r and S_t refer to the lateral displacements of reflected and transmitted waves respectively.



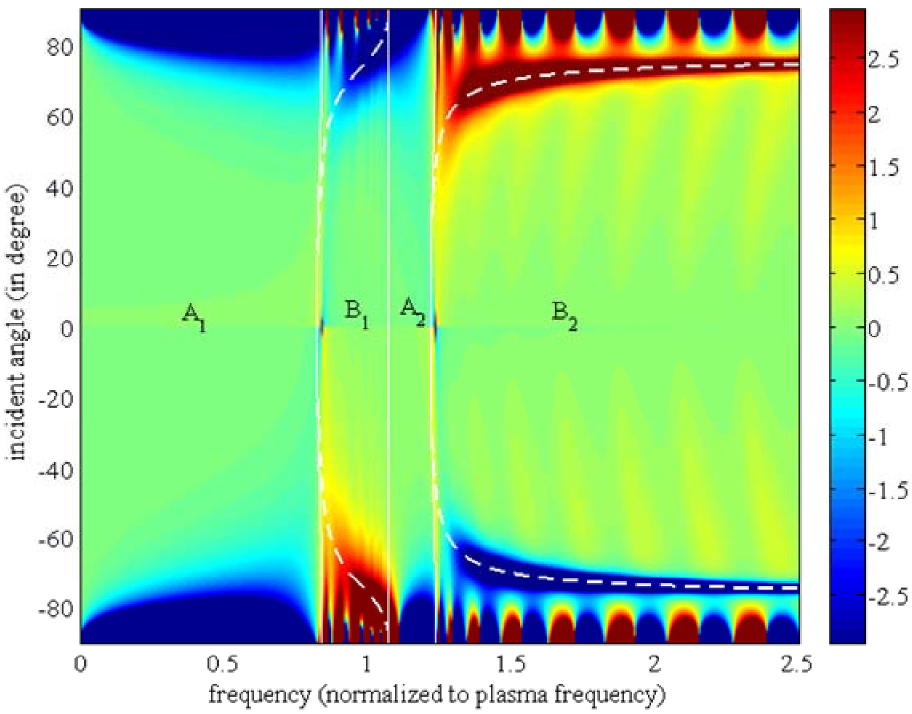
incident angle (in degree)
frequency (normalized to plasma frequency)
A
B

(a)



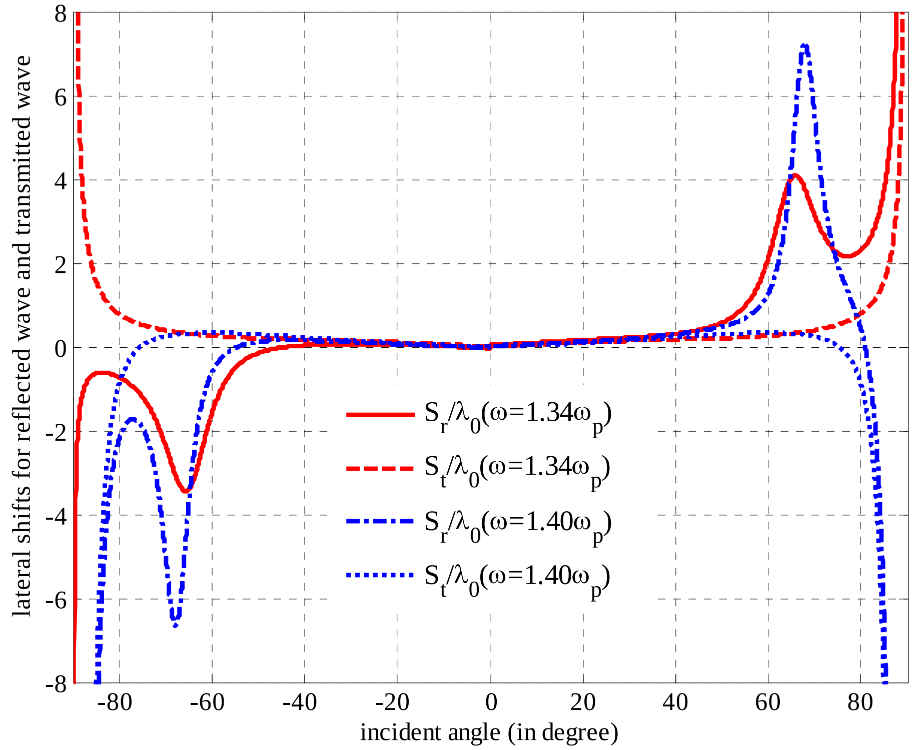
incident angle (in degree)
frequency (normalized to plasma frequency)
A_1
B_1
A_2
B_2

(b)



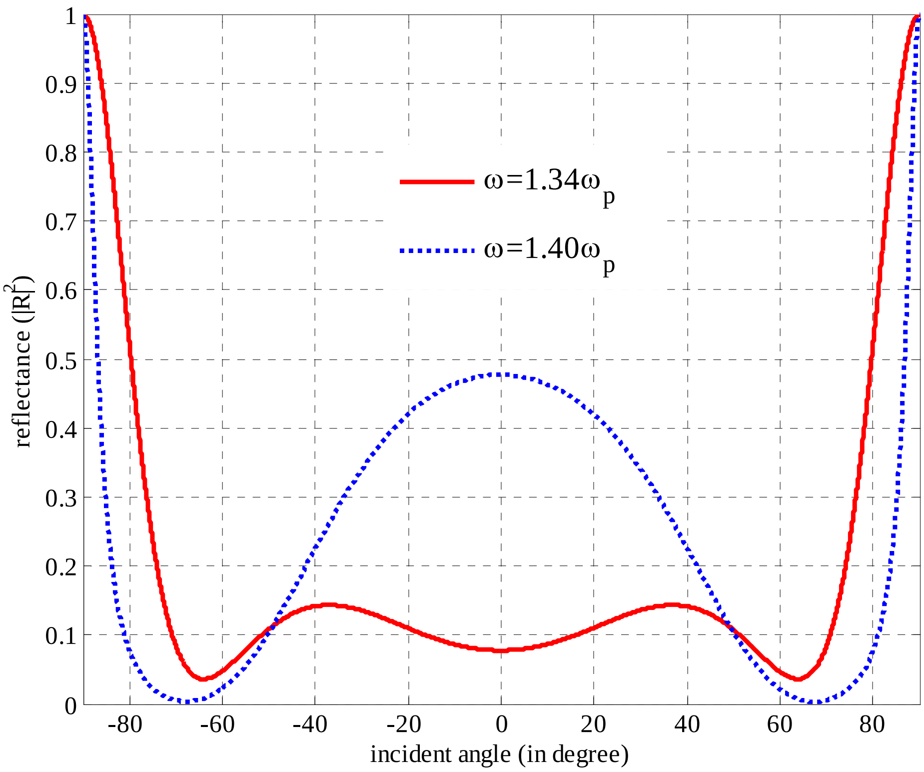
(c)

FIG. 2. Lateral displacements of TM waves transmitted and reflected from a lossless symmetric gyrotropic slab, the Brewster angle is also shown (in white dashed line). The incident and reflected waves are both on the left side of the slab. We consider the angle as positive when the incident wave comes from the lower left. The frequencies are normalized to ω_p, and the displacements are normalized to $\lambda_0=2\pi c/\omega$, the incident wavelength. The thickness of the slab $d=0.5\lambda_p=\pi c/\omega_p$. Without an applied magnetic field, the lateral displacement of the transmitted wave is equal to that of reflected: $S_t=S_r$; while the presence of an applied magnetic field results in a difference between S_t and S_r. (a) Both S_t and S_r with no applied magnetic field; (b) S_t with an applied magnetic field $\bar{B}_0=+\hat{z}0.4\mathrm{T}$; (c) S_r with an applied magnetic field $\bar{B}_0=+\hat{z}0.4\mathrm{T}$.



lateral shifts for reflected wave and transmitted wave
incident angle (in degree)
S_r/λ_0(ω=1.34ω_p)
S_t/λ_0(ω=1.34ω_p)
S_r/λ_0(ω=1.40ω_p)
S_t/λ_0(ω=1.40ω_p)

(a)



reflectance (|R|²)
incident angle (in degree)
ω=1.34ω_p
ω=1.40ω_p

(b)

FIG. 3. (a)Dependence of S_t and S_r on the incident angle θ_i. S_t is lateral displacement for transmitted wave while S_r is that for reflected. Both are normalized to $\lambda_0=2\pi c/\omega$. (b)Dependence of magnitude of the reflectance $|R|^2$ on the incident angle θ_i. The thickness $d=0.5\lambda_p=\pi c/\omega_p$ and the applied magnetic field $\bar{B}_0=+\hat{z}0.4\text{T}$. The red and blue lines are for cases of $\omega=1.34\omega_p$ and $\omega=1.40\omega_p$ respectively.

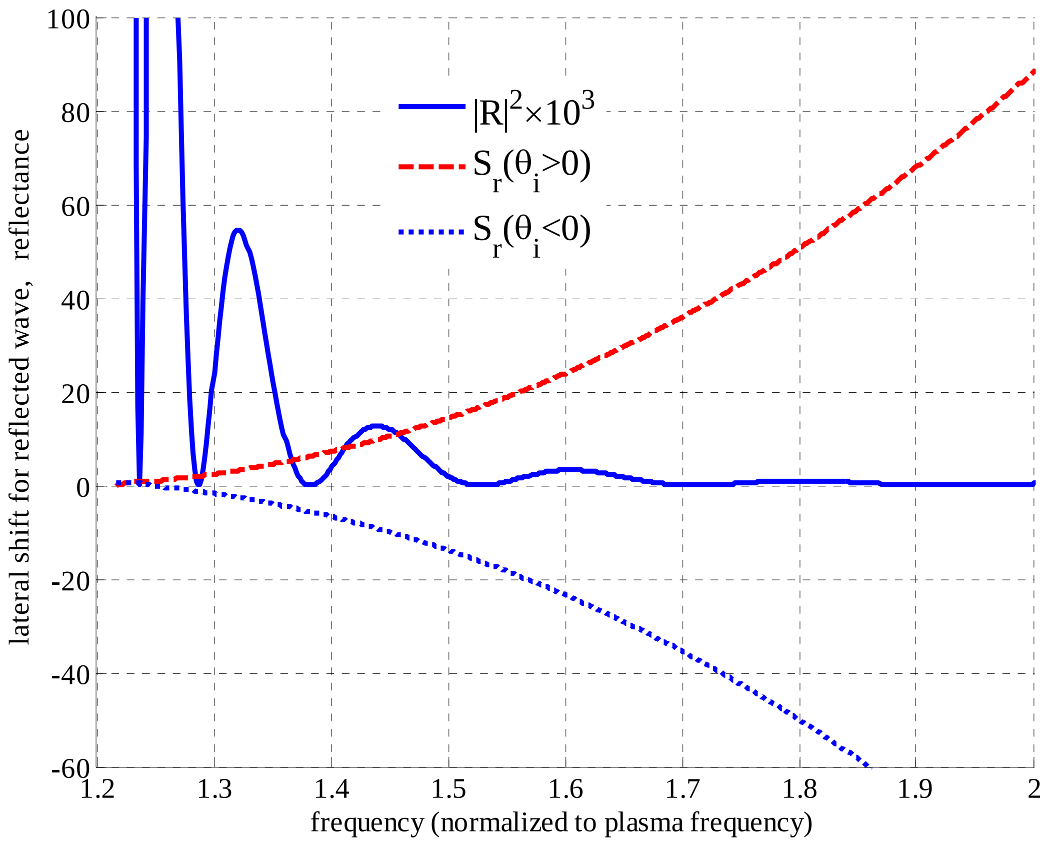


FIG. 4. When the incident angle equals to the Brewster angle θ_B, the lateral displacements of reflected wave S_r (normalized to λ_0), and the reflectance $|R|^2$ (multiplied by 1000) versus frequency. The thickness d=0.5λ_p=$\pi c/\omega_p$ and the applied magnetic field $\bar{B}_0 = +\hat{z}0.4\mathrm{T}$.

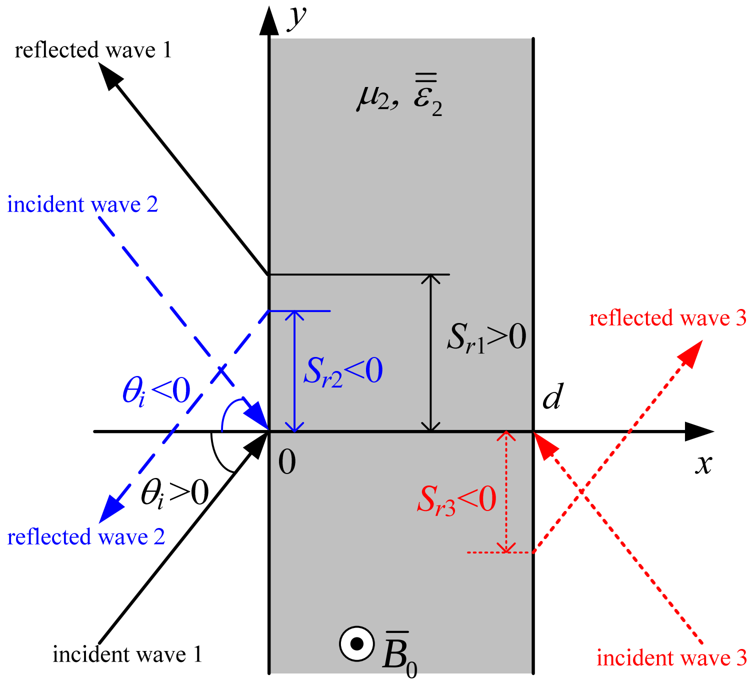


FIG. 5. Comparison of several incident wave cases. Case 1: incident wave and reflected wave are both on the left side of the slab, and the incident wave comes from the lower left. $S_{r1}>0$. Case 2: incident wave and reflected wave are also on the left side, but the wave is incident from the upper left. Due to the nonreciprocal nature of the gyrotropic medium, the shift in case 1 is different from that in case 2. $S_{r2}<0$. Case 3: incident wave also comes from below, but on the right side of the slab. $S_{r3}<0$. Case 2 and case 3 are equivalent, and lateral shifts are the same.