# 1 Reputation

## 1.1 A Repeated Buyer-Supplier Model

- A Buyer who wants one of two types of goods: Colors and Whites.

- If he orders colors this period and the color is good his utility is $H$ but if the color is bad his utility is $D$.

- If he orders whites, and they are good his utility is $h$ and if they are bad, his utility is $d : H > h > d > D$.

- Assume that the quality is not-contractable.

- Assume that the buyer proposes the equilibrium.

- Three types of suppliers: honest (fraction $\alpha$), dishonest ($\beta$) and strategic ($\gamma$). Honest suppliers always produce high quality, dishonest ones always produce low quality and strategic ones do what is in their best interest.

- Suppliers have a cost $G$ per period of supplying the good quality and a cost of 0 of supplying the bad quality.

- Assume the efficient outcome is to produce high quality colors:$H - G > d$.

- The supplier's outside option is getting zero for ever. The supplier also cannot be paid a negative price.

- The relation goes on till the supplier dies, which happens with probability $\lambda$ each period. There is no discounting.

### 1.1.1 Equilibrium

- In a one-shot game this is not an equilibrium– the dishonest and strategic types will choose low quality.

- In a repeated game this can be avoided:

  The seller orders high quality colors from all new sellers and offers a price of zero.

  He buys only low quality whites from all those who ever gave him low quality colors and pays them zero.

  He orders high quality colors from all buyers who have given him high quality colors for the last $n$ periods at a price $P(n)$.

  Dishonest sellers will not sell.

Strategic sellers will deliver high quality in the $n$th period if $\sum_{s=n}(1-\lambda)^{s-n}[P(s)-G] \geq P(n)$ for all $n$

What should the optimal price sequence look like?

### 1.1.2 Implications

- Older sellers will be paid more.

- New sellers have to take a loss.

- If buyers can take on a new seller, $P(n)$ cannot go up very much.

- A buyer who has an established seller may refuse a new supplier even if the price is zero. Why?

- Always an equilibrium where the seller only orders low quality whites and strategic sellers always cheat them.

## 1.2 A Random Matching Model

- Imagine that buyers and sellers are randomly matched each period and that there is a large population of buyers and sellers.

- Assume that people do not know the histories of the buyers they meet but if a buyer has supplied low quality at least once in the past, they will find out with probability $x$.

- Finally assume that the price of colors is fixed at $B$ and that of whites is fixed at $b < B$.(in Tirole these are private benefits.)

### 1.2.1 Equilibrium

- Once a strategic seller supplies low quality he will always do so in the future.

- Suppose sellers are expected to always demand whites in the future. Then all strategic agents will produce low quality today if the seller orders colors. But then seller are better off asking for whites today. Bad equilibrium.

- Suppose the seller orders colors from any buyer with whom he is matched and who is not known to have delivered low quality in the past. Then a hitherto untainted strategic seller may produce high quality colors if $x$ is high enough and the gap $B-b$ is large enough. Knowing this the buyer will order colors as long as there are enough untainted strategic sellers. Good equilibrium.

### 1.2.2 Implications

- History matters. Once enough strategic sellers are tainted, it is clear that the buyer will not order colors for the foreseeable future (till enough of these tainted people 'die'). But then a seller will only benefit from being honest today in the very distant future. Unless he is very patient he will produce low quality.

- Good and bad equilibria need not be symmetric.