Homework Assignment 2

Development Economics 14.771 / 2390b

Problem 1 *Regression Discontinuity (Final exam, Fall 1999)*

You are asked by the government of Transylvania to evaluate the effects of a scholarship program on school attendance. The program covers the cost of education and gives a small stipend to the beneficiaries. To be a beneficiary, a household needs to satisfy certain conditions such as: have at least one school-aged child, be eligible and apply.

Eligibility is decided by the government of Transylvania based upon a socio-economic index (SEI) which takes continuous values between 0 and 1 and is increasing with household wealth. Only those with SEI<0.25 are eligible. This rule is observed pretty strictly. You have access to a household survey which includes the SEI, child level variables (school attendance) and whether the household is a beneficiary.

1. What are the sources of bias is you regress school attendance of a child on a dummy of whether the child is a beneficiary?

2. Assume that the households do not know the exact value of their SEI before they apply. Plot the probability of applying against the SEI.

3. Plot the probability of receiving the grant against the SEI.

4. Plot school attendance against SEI in the following cases:

   (a) counterfactual in which this program doesn't exist

   (b) this program exists, and the scholarship is effective in increasing attendance

   (c) this program exists, but the scholarship is ineffective.

5. Assume that you have access to a very large sample (so large that even if you restrict the sample to a fairly narrow range of SEI, you have many observations). How would you estimate the impact of the grant on attendance? Provide a formula and an explanation.

Notation: $y_i = 1$ if child i attends school
$p_i = 1$ if the household of child i is a beneficiary
$s_i = SEI$ of the household of child i

Problem 2 *Land Distribution and Productivity.*

Corn is produced from land (A) and labor (L) using the aggregate technology

$$F(A, L) = A^{\alpha} L^{1-\alpha}.$$

There are H consumers who each allocate one unit of time across labor and leisure. Preferences are identical across consumers and take the form

$$U(c, l) = c^{\beta} l^{1-\beta}.$$

Normalize the prices of corn to unity and let the wage rate be w and the rental rate be q. Both of these prices are in terms of corn due to the normalization. Consumers vary in their holdings of land with consumer $h$owning quantity of land a_h.

1. Show that profit maximization implies $\frac{q}{w} = \frac{\alpha}{1-\alpha}\left(\frac{L}{A}\right)$
2. Determine consumer $h's$ labor supply as a function of q, w and a_h.
3. Assume all consumers supply a strictly positive amount of labor, using your results from (a) and (b) find the Walrasian equilibrium (i.e., solve for the aggregate supply of labor, and the equilibrium price vector (q^*, w^*)).
4. Using your previous results, find a condition for all consumers to supply a strictly positive amount of labor in equilibrium. Interpret this condition.
5. Suppose that land is equally distributed across M landowners with the other $N = H - M$ consumers owning no land at all. Find a condition for the value of $\frac{M}{N}$ so that the landowners do not work. Find the Walrasian equilibrium in this case.
6. Suppose that we observe an agricultural economy characterized large numbers of landless peasants and a few rich landlords who do not work. This corresponds to the Walrasian equilibrium discussed in (e) above. Now imagine a land reform program that is progressive enough to ensure that everyone works. What happens to the wages (in terms of corn) and rent (also in terms of corn)? Discuss.

Problem 3 *Tenancy and Adverse Selection. (Final Exam, Fall 1998)*

Consider a model of tenancy where the tenant can be of two types- lazy or hard-working. A lazy tenant will generate an output of y on the land while a hard-working tenant generates an output of Y > y. The fraction of hard-working tenants is p, 0<p<1. Assume that the outside option of the lazy tenant is u while that of the hard-working tenants is U.

Suppose now that only one tenant chosen at random from the population will show up at the landlords door and the landlord has to offer him a contract on the spot. Moreover, assume that the landlord does not know his type (i.e. whether he is lazy or not). Assume that only linear contracts can be offered and they take the form of the tenant paying the landlord R+s[output] (where

output is of course either y and Y, and s is a fraction between 0 and 1). Assume further that the tenant will stay and work if the contract makes him better off than his outside option and not otherwise. If the tenant leaves the land remains fallow and generates an income of zero.

1. First assume that the landlord can offer only one contract to the tenant. Find the optimal contract the landlord should offer the tenant as a function of the various parameters of the model (p,y,Y,u,U)

2. Next let the landlord be able to offer two contracts to the tenant and let the tenant choose. What contracts will the landlord choose to offer as a function of the various parameters? Does being able to offer two contracts make the landlord better off than when he could only offer one? Would it be even better if he could offer three contracts?

3. Suppose the landlord chooses the optimal pair of contracts. What correlation would one observe between choice of contract and productivity? What do you think of this as an explanation of the evidence on productivity differences described in class?

Problem 4 *A Model of Sharecropping*

Assume a simple production function: $y = e + \theta$, where y is output, e is effort put in by the cultivator, and θ is a normally distributed random shock with zero mean and variance σ^2. Also assume that all agents are risk averse with a mean-variance expected utility a function of income (y):

$E[U(y)] = E(y) - \frac{r(w)}{2} Var(y)$

where r(w) is the coefficient of risk-aversion with $r'(w) < 0$. (This formulation of expected utility (U) is equivalent to exponential instantaneous utility (u) in the presence of normally distributed income shocks: $u(y) = -exp(-r(w)y)$, y˜Normal.)

Consider two agents: Agent L is a landlord who owns a piece of land, has monetary wealth w_L and has no labor, and Agent T is the tenant who owns no land, has monetary wealth w_T and one unit of labor. Also assume that land, labor and goods markets are perfectly competitive, but insurance markets are nonexistent. This means that if the landowner wants, he could buy the tenants labor services at the market wage rate m, and if the labor-owner wanted, he could buy the landlords land at the market rental rate p.

Note that there is a difference between labor and effort. The cost of exerting effort is $\frac{1}{2}ce^2$ and the competitive wage rate in the labor market is net of the effort cost. Assume that monitoring is costless, so e is observable and contractible. Also assume that contracts are linear, so the tenants income y_T is written: $y_T = sy - R$, where s is the share of total output going to the tenant (with $0 \leq s \leq 1$) and R is the fixed rent. (In the following analysis, you can ignore the restrictions on R imposed by the wealth levels of the landlord and tenant, i.e. assume that $-w_L \leq R \leq w_T$ is satisfied).

1. Write equations for the expected utility of the tenant and the landlord (both as a function of e, s and R). Then solve for the socially optimal level of effort (e) and the optimal share of output that goes to the tenant (s). What are the maximized value of the social surplus S, the tenants utility and the landlords utility? When will there be gains from trade between the landowning and labor-owning agents?

2. Next, assume that the landowner can make a take-it-or-leave-it offer to the laborowner. What would be the resulting levels of e, s, R and S? How does the riskiness of production affect the fixed rent component, the level of effort and the share of output that goes to the tenant? Why? How does the productivity of labor and the wage rate (m) affect the same three variables? Finally, how does the risk aversion of the tenant and landlord affect these results? What if the landlord is so rich that he is risk-neutral (i.e. $r_L = 0$)? Or if the tenant is risk-neutral (i.e. $r_T = 0$) but the landlord is risk averse?

3. Now assume that the Tenant owns the land. What level of effort would he exert? Would the social surplus be higher or lower than in part b)? (Assume the utility of the "landowner" is now zero.

4. Finally some interpretation of the above results. Could share contracts, pure rent contracts and pure wage contracts all coexist? What determines which form is taken?