CHARGE EXCHANGE SCATTERING OF
NEGATIVE PI-MESONS NEAR 1000 MEV

by

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CHARGE EXCHANGE SCATTERING OF NEGATIVE PI-MESONS NEAR 1000 MEV

by

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ABSTRACT

The differential and total cross-section for the charge exchange reaction: \[ \pi^- + p \rightarrow \pi^0 + n \] (I)

and the total cross-sections for the reactions:

\[ \pi^- + p \rightarrow \pi^0 + \pi^0 + n \] (II)

\[ \pi^- + p \rightarrow \pi^0 + \pi^0 + \pi^0 + n \] (III)

have been measured at an incident \( \pi^- \) laboratory kinetic energy of 910±5 Mev. The instrument used was a 15" heavy liquid bubble chamber equipped with a 7¼" x 1 3/8" dia. internal liquid hydrogen target. The reactions were produced in the hydrogen target and gamma rays from \( \pi^0 \) decays were observed in the heavy liquid. The differential cross-section for the charge-exchange reaction (I) was found to be:

\[
\frac{d\sigma}{d\Omega} = (0.02\pm 0.03) + (0.67\pm 0.47)x + (0.30\pm 0.32)x^2 - (5.03\pm 0.72)x^3 - (6.6\pm 0.55)x^4 + (5.5\pm 1.20)x^5 \quad x = \cos \theta
\]

The total cross-sections for reactions (I), (II), (III) were (4.21±0.42), (3.93±0.44), and (0.36±0.27) m.b. respectively.
The differential cross-section for reaction (I) indicates a $D \frac{5}{2}, F \frac{5}{2}$ interference in the $T=\frac{1}{2}$ iso-spin state at the 900 Mev $\pi^-$-$p$ resonance. The total cross-section of reaction (I) together with the known cross-sections for $\pi^-$-$p$ charged elastic, and $\pi^+$-$p$ cross sections are found to be consistent with the charge-independence hypothesis.

Thesis supervisor: Irwin A Pless
Title: Associate Professor of Physics
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I would also like to acknowledge assistance in the early phases of this experiment by Professor M. Chretien and Dr. D. Firth.

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I. INTRODUCTION

In recent years, it has been established that the total cross-section as a function of energy displays two distinct peaks at around 600 and 900 Mev incident pion kinetic energy for the reactions

\[ \pi^- + p \rightarrow \pi^+ + p \]  
\[ \pi^- + p \rightarrow \pi^0 + n \]  \hspace{1cm} (V) \hspace{1cm} (I)  

On the other hand, the cross-section for the reaction

\[ \pi^+ + p \rightarrow \pi^+ + p \]  \hspace{1cm} (VI)  

does not peak at these energies (3,4). These results seem to indicate a resonant behavior at 600 and 900 Mev of the pion-nucleon system in the T=\frac{1}{2} iso-spin state.

The 600 Mev peak is believed to be a resonance in the J=3/2, T=\frac{1}{2} state 7-9 whereas the 900 Mev peak may possibly be in a state with J=5/2, T=\frac{1}{2} 10-11. However, it has been recently suggested by Kycia, et al 12 that the 900 Mev peak may be due to an interference between the D=5/2, T=3/2, and F=5/2, T=\frac{1}{2} wave amplitudes. The purpose of this paper is to report on the study of reaction (I) which may contribute toward the establishment of states involved at the 900 Mev resonance.

The charge-exchange differential cross-section (I) was measured by Weinberg, et al 13 in a 15" heavy liquid bubble chamber, with the chamber liquid itself supplying the pro-tonic target. In order to be certain that there would be no
contamination from quasi-elastic interactions, we have chosen to carry out our measurements in a 15" heavy liquid chamber, identical to that used by Weinberg, et al, but equipped with an internal liquid hydrogen target.

This experiment was carried out at the Cosmotron, Brookhaven National Laboratory, in the fall of 1961.
I. APPARATUS AND EXPERIMENTAL SET-UP

As mentioned before, the $\pi^0$-decay gamma ray detector used in this experiment was a heavy liquid bubble chamber, with an internal liquid hydrogen target which provided the target protons. The target is cylindrical (7½" length by 1 3/8" diameter) and is encased in a 2" diameter outer cylinder which extends into the chamber. (See Fig.I-1). A detailed description of this internal hydrogen target will be given elsewhere 14-15. The 15" diameter by 14" depth (51 liter capacity) heavy liquid chamber used in this experiment was identical to that discussed by L. Rosenson and J. Szymanski except for provisions to accept the hydrogen target. All equipment associated with the operation of the chamber itself was also identical to those already described 16.

Photographic exposures were made on 35mm. non-perforated Linograph-Shellburst film under bright field illumination. Three cameras were situated symmetrically around the perimeter of the bubble chamber, allowing for 150° degree stereo photography with any two cameras. The cameras are discussed in detail by Averell 18.

A. BUBBLE CHAMBER LIQUID

The bubble chamber liquid used in this experiment consisted of three components: methyl iodide (CH$_3$I), freon B 13 (CF$_3$BR), and ethane (C$_2$H$_6$). The volumetric fractions used, of each component were 68%, 18%, and 14% respectively.
The proportions of each component were determined by weight, and the volumetric fractions were then computed using known densities. The radiation length was then computed to be 9.949 gms/cm². The density of this mixture was computed using the known densities of each component at room temperature. During the course of heating the chamber from \(27^\circ C\) to \(110^\circ C\), 6 liters of the liquid were taken out of the chamber to allow for thermal expansion. Taking into account this volumetric expansion, we find a liquid density of \(1.64\pm5\%\) gm/cc at the operating temperature. The 5\% error on the density is an estimate of the error in measuring the amount of liquid removed during the heating process.

The gamma ray conversion length, as a function of gamma ray energy, is related to the radiation length in the following way:

\[
\lambda(E) = \frac{x_0}{\lambda(E)}
\]

\[
\lambda(E) = -0.0330 + 0.2850 \log(E) - 0.0183(\log(E))^2
\]

where \(x_0\) is the radiation length of the bubble chamber liquid in cm., and \(E\) is the energy of the gamma ray, in Mev.

The energy dependent coefficient \(\lambda(E)\) was determined for this mixture by the use of Averell's computer program which carries out a least-squares fit to the Bethe-Heitler formula in terms of \(\log(E)\). The conversion length as a function of gamma ray energy is plotted in Fig. I-2.

B. INCIDENT BEAM

The "pencil" beam used in this experiment was designed to give from 5-15 particles per pulse with a beam cross-section of \(\frac{1}{8}\) " and with incident momentum of \(1038\pm\frac{1}{2}\) Mev/c.
The details of this beam are discussed elsewhere.

The pion beam (See Fig. I-3) was produced by collision of protons from the Cosmotron's external beam $P_2$ onto a 3/" x 3/8" x 7" carbon target situated in the bending magnet 204. The negative pions produced at $0^\circ$ with respect to the target axis were then deflected $29^\circ$ by magnet 204. Quadrupoles 205 and 206 were used to focus the beam at the Heavy-met acceptance slit. The momentum selection was made by bending magnets 207 and 208. Magnet 209 deflected the beam, allowing photons from $\pi^0$ decays to travel in a direction $16^\circ$ from the beam, reducing the photon background to a negligible level. The final focus, in the center of the beam finger target, was carried out by quadrupole magnets 210 and 211.

The beam trajectory was designed by R. Lanou using the computer programs of J. Stanford and J. Sandweiss and those of R. Pinon and R. Lanou. The beam itself was then simulated by wire-orbit techniques and properties of the beam measured by ray tracing.

The muon and electron contamination in the beam have been estimated to be 6.7% and 0.2% respectively, by Lanou. The photon contamination is negligible due to bending of the beam at Magnet 209 as stated above.

During the set-up of the beam, a 2½" freon bubble chamber was used to locate and study the cross-sectional profile of the beam. This was done by placing the window plane of the study chamber at right angles to the beam -- thus directly
observing the cross-sectional profile of the beam. This technique is reported in detail by Bulos, et al.

The resulting beam was found to be tightly collimated with a maximum cross-sectional diameter of \( .67^\circ \). A study of the beam profile in the plane parallel to the bubble chamber windows has been carried out by the Brown University group by observing directly on scanning projectors the track distributions in pictures. The distribution of the tracks along the axis perpendicular to the window plane has been obtained by measuring centers of nuclear stars that occur in the liquid of the chamber due to interactions by the beam emerging from the hydrogen target. Measurements were carried out in three views and the coordinates of the stars were reconstructed in space. The distributions obtained are shown in Fig. 1-4a,b.

Because the distributions were narrow and the events fell well within the target, it was not necessary to consider the unfolding of resolution spreads due to measurement errors and multiple scattering of the beam in the liquid. It is clear from these distributions that the beam was well centered within the target. As another check on the position of the beam, 1300 pictures were taken without any hydrogen in the target, and two pictures contained possible 27 events. These events were measured and found to have come from interactions in the chamber liquid, and not the beam finger wall.
II. SCANNING AND MEASURING

A. SCANNING PROCEDURE -

A total of 98 rolls of film, each containing an average of 620 exposed frames, were divided into two groups and were scanned independently by the M.I.T. team and the scanning team at Brown University. The M.I.T. team was responsible for the scanning of 48 rolls and the Brown University team for 50 rolls. Scanning by both teams was carried out in the same fashion. Three views of each picture were looked at simultaneously, and all events containing two or more gamma rays ("gamma rays" will be used to denote the tracks due to the converted positron-electron pairs) pointing towards the beam finger (hydrogen target) were recorded as possible good events. The total number of such gamma rays and the number of charged tracks scattered out of the beam finger by more than $\sim 10^6$ were recorded along with the picture numbers of each selected frame.

B. TRACK COUNT -

Throughout the scanning procedure the number of tracks in every tenth picture frame was reported. The total number of negative pions encountered in this experiment was found to be $3.872 \times 10^5$.

C. SCANNING MACHINES -

The scanning machines used at M.I.T. consisted of two sets of three (Model) MPC-1 Recordak 35 mm. film projectors.
The Brown University team used one set of Recordak projectors and one "Yale" projector. These latter are capable of driving three rolls of film and because it is possible to switch from one view to another, they may be operated by a single scanner to scan the three views at the same time.

D. CHECKING -

Every event selected by the scanners was rechecked and decided upon by the physicists to determine its category. The quality of the gamma rays themselves was divided into two types—"Clean" gamma rays and "CIS" gamma rays. A "clean gamma ray is defined as one whose point of conversion corresponding to the origin of the seemingly single track ("nose of gamma ray) formed by the narrow angle (45°) vertex of positive-negative electron pair, is not obscured in more than one view by the beam finger or is not in the fiducial beam region in more than one view—the fiducial region being defined by a band equal in width to one third the diameter of the beam finger and with axis corresponding to that of the beam. A "CIS" (Converts in Steel /Shadow) gamma ray, on the other hand, is one that is obscured by the finger or falls in the fiducial beam region in two or three views or one that converts in the steel wall of the beam finger.

A fiducial chamber volume was defined as the truncated conical volume formed by the projection, onto the front window (as viewed by a camera), of a circle of radius lcm. smaller than that of the rear window and concentric with the rear window, with major and minor base planes parallel to and
5.8 cm. from the windows, (Fig.II-1). Any gamma ray that does not convert in the fiducial volume in at least two views is considered as non-existent. Thus, for example, an event containing three "clean" gamma rays, two of which convert in at least two fiducial volumes and the third in only one is considered as a "clean" two gamma ray event.

The intersection points of the "clean" gamma rays were determined by extending their "noses" with a straight edge on the projector screen. The intersection points were then checked for consistency in the three views to determine whether they were from interactions in the beam finger or accidental intersection of two or more unrelated gamma rays. All events with two or more "clean" gamma rays and with charged tracks scattered from the beam direction by more than 10° were checked to determine whether the charged track(s) were associated with the gamma rays. This was accomplished by extending the charged tracks back into the beam finger to see if they formed a common interaction point with the gamma rays.

E. SCAN-CHECK EFFICIENCY

Thirty rolls of the total ninety eight were scanned twice, and forty eight rolls were checked twice. The scanning efficiency of a single scan is defined as the ratio of the number of identical events found by both scans to the number of events found by the other scan only. The checking efficiency is similarly defined -- the ratio of the number of events selected in common to that selected by the other check only.
The overall scan-check efficiency was then found to be 0.91 ± 0.02. The error is due to statistics only.

F. MEASUREMENTS -

The gamma rays in each selected event were measured on Hydel measuring machines. Two points, one on the vertex of the gamma ray "nose" and the other 2.5 cm. back along the straight edge extension of the gamma "nose" were measured on each gamma ray. The output coordinates in each view consisting of coordinates for the points on each gamma ray and coordinates of fiducial marks on the bubble chamber windows were automatically punched on IBM cards for data reduction. Each event was measured twice by a different person.

G. GEOMETRY RECONSTRUCTION -

The measurement data cards were used in conjunction with the geometry reconstruction program written by M. Chretien. The results of the geometry program were then used with the Least-Squares Fitting program, TRKEXT, of C. Bordner, and A. Brenner. This program will construct lines passing through the origin of the "noses" of each gamma ray and will constrain these directional lines to intersect at a common point. Then by iterative procedure, the intersection point may be allowed to move throughout all of space, or, as an alternative mode, along a prescribed line, until a minimum $\chi^2$ value is obtained between the originally measured gamma ray directions (determined by the two points measured on each extended gamma ray "nose") and the directions resulting from this adjustment.

In order to select the final sample of events, the two measurements of each event have been treated by allowing the intersection points to move throughout all of space. Any event having its intersection points of both measurements falling
inside the hydrogen target (within their errors) was accepted for the final sample. 673 original events were treated in this fashion and 627 were accepted. Of the 46 events that were rejected, 34 were found to be consistent with interactions of stray pions in the bubble chamber liquid and 12 were found to have inconsistent results between the two measurements due to very badly defined gamma ray "noses".

Both measurements of these selected events were then treated according to the second mode of fitting, allowing the intersection point to move along the axis of the beam. The measurement displaying the better $\chi^2$ value was selected for use and their gamma rays were Lorentz transformed back into the $\pi^-$-p.C.M. System. The opening angle between each combinatorial pair of gamma rays, and the cosine of their bisectors, were then determined in this frame of reference.

H. SAMPLE

The length of the beam finger was divided into three adjacent sections -- each 5 cm. in length, from 0 to 5 cm., 5 to 10 cm., and 10 to 15 cm. along the x axis (See Fig.II-2). 0 corresponds to the geometric center of the bubble chamber. The events were then divided into three sub-groups according to which of the three sections of the tube the interaction occurred. The number of events of each type in the three sections of the tube is tabulated in Table II-1. The entry called 2 + "CIS" represents the events found in 45 rolls with at least two "clean" gamma rays plus any number of associated "CIS" gamma rays. This was recorded for the purpose of internal checking and will be spoken of later in the text.
III. ANALYSIS

A. SIMULATION PROGRAM -

Because of the highly complicated geometry of the bubble chamber-target arrangement, it was necessary to determine the various efficiencies for observing gamma rays by means of a Monte-Carlo type event simulation program. This program, by taking into account the energy dependence of the gamma ray conversion probability, will simulate an event and check to see if the gamma rays lie within a prescribed fiducial volume, and also checks to see if the gamma rays are obscured by the beam finger or fiducial beam region. In particular, it uses a phase-space generating sub-program \textsuperscript{27} (RDECAY) written by Brenner and Ronat. Thus, the sequential operation of the simulation program for a single simulated event is as follows:

1) An interaction point is picked randomly in the beam finger according to a prescribed beam distribution. In this problem, a square beam cross-section was used, with the side of the square equal to 1.70 cm. The distribution used along the length of the target was uniform.

2) In the case of multiple pion production the momentum and direction (in the $\pi^-p$ center of mass system) of the neutral pions produced are determined by RDECAY, according to phase-space distributions. In the case of elastic scattering, the momentum of the $\pi$ is constant and the angular...
distribution is determined by any distribution that is fed into RDECAy.

3) The neutral pion(s) are then allowed to decay.

4) The energy and direction of the pion decay gamma rays are then Lorentz-transformed into the laboratory system.

5) Each gamma ray is then allowed to convert according to the exponential law:

\[
\frac{dP}{dR} = e^{-\frac{R}{\lambda(E)}}
\]  

(VIII)

where \( \lambda \) is a function of the gamma ray energy \( E \) and is given by equation (VII) of page 8. \( R \) is the distance of the gamma ray conversion point from the \( \pi^0 \) decay point.

6) The gamma rays are then viewed by three cameras to see if they are within the prescribed fiducial volume.

7) The gamma rays that are within the fiducial volume are then tested to see if they are of the "clean" or "CIS" category, and are identified as such.

8) The center of mass values of the opening angle and cosine of the bisector of the opening angle of the combinatorially paired gamma rays are then stored for future read out.

9) The event is categorized according to the number of gamma rays that fall outside the fiducial region, the number that are of the "clean" type and the number of the "CIS" type.

After a predetermined number of simulated events have been treated in this way, the probabilities for observing the various categories of gamma ray conversion modes are
given. In the case of elastic scattering, the efficiency for observing events, as a function of their bisector directions in the C.M. System, is also given. Histograms of the various opening angle and bisector directions are also given by the program.

The probabilities for observing events of the various conversion modes are shown in Table III-1. The criteria for accepting events were the same as those used in accepting events for the actual samples.

The opening angle distributions of the combined 3 and 4 gamma ray observed sample is shown along with the combined 3 and 4 gamma ray sample produced by the simulation program in Fig. III-1 assuming a phase-space distribution for $\pi^0$ production. The 3 and 4 gamma ray contributions from $3\pi^0$ productions have been neglected. The distributions for the X, Y, and Z (See Fig. II-2) coordinates of the gamma conversion points for the combined $2\gamma$, $3\gamma$, and $4\gamma$ observed samples are compared with proportional combinations of the $2\gamma$, $3\gamma$, and $4\gamma$ samples from the simulated events. These distributions are shown in Figs.III -la, b and c. It should be pointed out that since the simulation program approximates the hemispherical end of the beam finger as a straight end, we have excluded this region from the samples before normalizing.

All comparisons are in substantial agreement and indications are that the phase-space model accounts for the multiple $\pi^0$ momentum distributions.
B. 2\( \pi \) AND 3\( \pi \) PRODUCTIONS

Using the probabilities for the various conversion modes (Table III-1), and the number of 3 gamma, 4 gamma, and 5 gamma events in the observed sample, we are able to determine the number of 2\( \pi \) and 3\( \pi \) productions. We shall use the entire sample, from 0 to 15 cm. of the beam finger in order to increase the statistics.

Taking the number of 5 gamma events observed and the appropriate gamma ray conversion probability, we find the number of 3\( P^- \) to be:

\[ \eta(3\pi) = 78 \pm 5.5 \]

The number of events appearing as 4\( \pi \), 3\( \pi \), 2\( + \) CIS, and 2\( \pi \) events due to 3\( \pi \) decays are then found to be:

\[ N_{4\pi}(3\pi) = 3.4 \pm 2.4 \]
\[ N_{3\pi}(3\pi) = 3.0 \pm 2.1 \]
\[ N_{2\pi+\text{CIS}}(3\pi) = 4.7 \pm 3.0 \]
\[ N_{2\pi}(3\pi) = 1.5 \pm 1.1 \]

The number of 2\( \pi \) events can be determined from the number of 3\( \pi \) events and independently by the number of 4\( \pi \) events. Using a weighted average of the numbers obtained both ways, we find the number of 2\( \pi \) events to be:

\[ \eta(2\pi) = 806 \pm 86 \]

Using the number of 2\( \pi \) and 3\( \pi \) events thus obtained, the number of 2\( + \) CIS events is predicted to be 329 \( \pm 45 \). However, only 44 of the 96 rolls have been scanned for 2\( + \) CIS events, and by taking this fraction into account, 147 \( \pm 22 \) 2\( + \) CIS events should be observed in 45 rolls of film. The number actually observed was 125 \( \pm 11 \) which is in good agreement with the predicted number.
C. OPENING ANGLE DISTRIBUTION -

The theoretical opening angle (angle between the decay gamma rays) distribution of the gamma rays from $\pi^0$ decays is given by
\[
\frac{dN}{d\theta} = \frac{\cos(\theta)}{2\sqrt{\pi^0 mass}} \sin^2(\theta) \sin^2(\theta - \theta_0) - 1
\]
where $\theta$ is the opening angle and $\pi^0$ is the total energy of the $\pi^0$, in units of the $\pi^0$ mass. The distribution (Fig.III 2) is sharply peaked at the minimum opening angle given by:
\[
\theta_{min} = 2 \sin^{-1}(\frac{1}{\sqrt{2}})
\]

The actual distribution which should be observed in the bubble chamber was estimated with the use of the simulation program. This opening angle distribution was obtained using a second guess to the charge exchange angular distribution (this second guess will be spoken of in detail below) and allowing for a Gaussian resolution function of $2.0^\circ$ width to be folded into the distribution. This resolution width was obtained by plotting the difference of the opening angle values taken from the results of two measurements. If the error in the opening angle is Gaussian distributed, with width $\theta^\circ$ then it can be shown that this difference distribution is also Gaussian distributed with its width being equal to $2\theta^\circ$. The opening angle difference distribution is shown in Fig.III-3. It cannot be fitted by a single Gaussian curve but a close approximation would be a Gaussian of $\sim 2.8^\circ$ width.

The observed $2 \pi^0$ opening angle distribution is shown in Fig.III-4. This sample is contaminated by the $2 \pi^0$ events due
to 2 \( \eta^0 \) decays, and by a negligible amount due to 3 \( \eta^0 \) decays. The number of events that appear as 2 converted \( \eta^0 \) rays, due to 2 \( \eta^0 \) decays, is given by:

\[
N_{2\eta}(2\eta) = \eta(2\eta) \rho_{2\eta}(2\eta) = (806 \pm 86 \times 0.0882) = 71 \pm 8
\]

By using the shape of the combined 3 and 4 \( \eta^0 \) opening angle distribution and normalizing this distribution to 71 \( \pm 8 \) events we are then able to subtract the background distribution from the observed 2 \( \eta^0 \) opening angle distribution. The resulting distribution is shown in Fig. III-5. The dotted curve represents the opening angle distribution from the simulated program, normalized to the number of events in the subtracted distribution between 0° and 85°. This limited opening angle range for normalization was chosen since any neutral decays of \( \eta^0 \) and \( \eta^0 \) should show up with minimum opening angle between the decay \( \eta^0 \) rays to be about 90°. The existence of neutral \( \eta^0 \) mesons due to charge exchange in the observed sample is evident from this comparison. The 29 \( \pm 8 \) residual 2 \( \eta^0 \) ray events beyond 90° opening angle are due chiefly to the 2 \( \eta^0 \) decay mode of the \( \eta^0 \) and to a lesser extent to decays from the neutral decay mode of \( \eta^0 \), and to an almost negligible amount due to decays of \( \eta^0 \) in the tail of the \( \eta^0 \) distribution.

D. METHOD TO OBTAIN THE ANGULAR DISTRIBUTION –

When a \( \eta^0 \) in motion decays, the actual \( \eta^0 \) direction lies between the decay gamma rays and for opening angles near the minimum opening angle \( \theta_{\text{min}} \), lies very close to the bisector of the opening angle. The angle between the \( \eta^0 \) and
the bisector, $\theta_{b-\pi}$, can be represented as a function of the opening angle by the following relation:

$$\cos(\theta_{b-\pi}) = \frac{1}{\rho} \cos \left( \frac{\theta}{\rho} \right)$$

This function is plotted in Fig. III - 6.

One can see that most of the time the bisector direction is a very good approximation to the $\pi^0$ direction since the opening angle distribution is highly peaked at the minimum opening angle (at the minimum opening angle, the bisector and $\pi^0$ directions coincide). Thus, one would expect the opening angle bisector angular distribution in the $\pi^–p$ C.M. System (where the $\pi^+$s from charge exchange events are mono-energetic) to be very nearly the same as the $\pi^0$ angular distribution. If the $\pi^0$ angular distribution is expanded in a Legendre polynomial series

$$\frac{dN}{d\Omega} = \sum_{L=0}^{L_{max}} A_L P_L(cos \theta)$$

it can be shown that the opening angle bisector distribution is given by:

$$\frac{dN}{d\Omega_{bis}} = \sum_{L=0}^{L_{max}} A_L B_L P_L(cos \theta) = \sum_{L=0}^{L_{max}} C_L P_L(cos \theta)$$

where $B_L$ is given by:

$$B_L = (1-\beta^2) \int_{\theta_{\pi-\pi_{max}}} P_L(cos \theta \pi \pi) \cot(\theta_{\pi-\pi}) \frac{d(cos \theta \pi \pi)}{[1-\beta^2 \cos^2(\theta_{\pi-\pi})]^{3/2}}$$

The values for $B_L$ for an opening angle limit of 50° and various $\pi^0$ velocities have been numerically integrated and are presented in Table III 2.
The general procedure to obtain the $\pi^+$ angular distribution is then as follows:

1) Obtain the opening angle bisector distribution in the $\pi^-$-p C.M. System for observed $2\pi^-$ events.

2) Using the results from the simulated program, determine the amount of background events expected from $2\pi^-$ (the background contribution due to $3\pi^+$ is negligible).

3) Normalize the opening angle bisector distributions of the observed $3\pi^-$ and $4\pi^+$ samples to the determined number expected and subtract from the observed $2\pi^-$ bisector distribution.

4) Correct the number of resulting events as a function of bisector angle by the approximate efficiencies (Table III-3) for observing the gamma rays giving rise to the bisector.

5) Determine the $C_L$'s by means of a least-squares fit between the corrected data and the Legendre polynomial series.

6) Divide each $C_L$ by the approximate $B_L$ to obtain the expansion coefficients $A_L$ for the $\pi^+$ distribution.

E. THE BISECTOR ANGULAR DISTRIBUTION -

For the determination of the bisector angular distribution we have limited our sample to events having opening angles between 25 and 50 degrees only. The reason for this limitation is that, in taking events with wide opening angles, the number of single $\pi^+$ events gained is not much
more than the number of background events gained. This is so since the $\pi^*$ distribution peaks at 25 degrees and drops off very rapidly as the opening angle increases, whereas the background distribution is very broad.

This limited opening angle sample was then sub-divided into three groups in accordance with the target region (0-5, 5-10, 10-15 cm.) in which the interaction occurred. This was done since the efficiency corrections for the bisectors are slightly different for these three regions (See Table III-3). Each regional sample was then treated independently.

The background distribution was determined by scaling the bisector distribution of the combined 3 and 4$\gamma$ sample to the number of background events predicted by the simulation program results. This distribution was then subtracted from the observed 2$\gamma$ distribution. The resulting distributions were then corrected by the bisector efficiencies obtained in the manner below.

F. BISECTOR EFFICIENCY -

In reality the bisector efficiency (the efficiency for observing an event giving rise to a opening angle bisector lying along a given direction) is not independent of the $\pi^*$ angular distribution, since a given bisector polar angle may correspond to bisectors from decays of $\pi^*$'s with different polar angles and necessarily different opening angles. However, the gamma rays may be going in various directions and possess various energies, and it is their conversion pro-
babilities which determine the bisector efficiencies. It is clear that if only events with the minimum opening angle are considered, the bisector efficiencies would be independent of the distribution since, in this case, the $\pi^*$ direction and bisector directions coincide.

In our case the opening angle range (25°-50°) is sufficiently small that large deviations in the bisector efficiencies due to different $\pi^*$ angular distributions are not expected. Table III-3 contains the bisector efficiencies obtained by using isotropic $\pi^*$ distribution, (ISO), using the bisector distribution obtained from the observed $2\gamma$ sample, without subtractions or corrections, as a first guess to the $\pi^*$ distribution (Bl 1), and finally, using the bisector distribution obtained from the background subtracted sample which was corrected by efficiencies from (Bl 1) as the second guess to the $\pi^*$ distribution (Bl 2). In principle, one could carry out an iterative procedure of this kind, but it is clear from these results that after two iterations the bisector efficiencies are identical, within statistics.

G. FINAL BISECTOR DISTRIBUTIONS -

The final bisector distributions in the three regions of the target are shown in Fig.III-7. These distributions were obtained using the final bisector efficiencies (Bl 2) given in Table III-3. Any bisector direction with efficiency less than 0.10 was not considered since the correction necessary was thought to be too large (approximately four times the smallest correction made which was 1/0.40). After
correcting the distributions for each region, an average of the three independent distributions was taken. This final average is shown in Fig.III-8. The dotted curve in Fig.III-8 represents the corrected bisector distribution obtained by using the efficiency corrections resulting from the first guess (BI 1). (See TableIII-3).

One can see that both distributions are in good agreement as expected. It should be noted that the last cosine interval bin (-1.0 to 0.90) in the extreme backward direction has a bisector efficiency of < .10 in all three regions of the target, and according to our criterion, should not be used. However, in order to make an estimate of the number of events expected in this interval, we make the following observation: no true events in the sample, in any region of the target, were found to fall in this interval - thus, the best estimate of the number in this bin is 0. If we found at most 1 event to fall in this interval, and using the highest efficiency (0.049), we would expect an error of ±(9.7). We therefore estimate the number in this interval to be 0.0 ±(9.7).

As an internal check, the sum of the events in the useful cosine interval bins (bins with θ > .10) was compared with the sum of events in the corresponding bins of the averaged distribution, for each region of the target. The comparisons are:
H. ANGULAR DISTRIBUTION

A least-squares fit for the function

$$\frac{dN}{dS_{\text{bis}}} = \sum_{L=0}^{L_{\text{max}}} C_L P_L(\cos \theta)$$

(1)

to the final averaged bisector distribution was carried out for \( L \) ranging from 0 to 6. The \( X \) probabilities are plotted as a function of \( L \) in Fig. III-9. We see that at least the \( L=5 \) term is necessary in order to obtain a good fit to the distribution. The solid smooth curve in Fig. III-10 represents the fit with \( L=5 \).

The true angular distribution for the various fits are obtained by correcting each polynomial coefficient \( (C_L) \) by the appropriate correction factor \( (B_L) \) given in Table III-2.

The true angular distribution coefficients are shown in Table III-4. Also shown in Table III-4 are the coefficients for the cosine expansions

$$\frac{dN}{dS_{\text{bis}}} = \sum_{L=0}^{L_{\text{max}}} A_L \cos^L \theta$$

for the 5th, and 6th order fits to the final angular distribution. The dotted smooth curve in Fig. III-10 represents the angular distribution for \( L=5 \).

0 - 5 Target region cosine range -1.0 to -0.9

\( N_{0-5\text{AV}} = 298 \pm 28 \quad N_{0-5\text{notAV}} = 347 \pm 43 \)

5 - 10 Target region cosine range -1.0 to -0.6

\( N_{5-10\text{AV}} = 268 \pm 24 \quad N_{5-10\text{notAV}} = 254 \pm 37 \)

10 - 15 Target region cosine range -1.0 to -0.5

\( N_{10-15\text{AV}} = 176 \pm 17 \quad N_{10-15\text{notAV}} = 141 \pm 29 \)

We see that the comparisons are in substantial agreement.
I. TOTAL CROSS-SECTION -

The total cross-section corresponding to \( N \) events is given by the following expression:

\[
\sigma = \frac{N \times 10^{61}}{P \lambda o \times L \times \varepsilon_s (1-\lambda)(1-\varepsilon)(1-s)} \cdot \lambda N \text{ mb.}
\]

\( N \) - Total number of events
\( P \) - 0.07 gms/cc - density of liquid hydrogen
\( \lambda o \) - 6.02 \times 10^{23} -- Avagadro's number
\( L \) - 15.00 - 0.42 cm. - Length of Target (the uncertainty is due to the uncertainty in the reconstruction of the gamma ray intersection; 0.42 cm uncertainty)

\( N_\pi \) - (3.872 \times 10^{5} - Total number of particles
\( \varepsilon_s \) - 0.91 \pm 0.02 - Scan-Check efficiency
\( \lambda \) - 0.067 - Muon contamination of the beam
\( e \) - 0.002 - Electron contamination of the beam
\( s \) - 0.0119 - Fraction of \( \pi^0 \)'s decaying by a single Dalitz pair.

\( \lambda \) is then found to be:

\[
\lambda = (0.488 \pm 0.017) \times 10^{-2}
\]

Using the number of \( 2 \pi^0 \) and \( 3 \pi^0 \) found above, we find the corresponding total cross-sections to be:

\[
\sigma_{2\pi^0} = (3.93 \pm 4.4) \text{ mb}
\]

\[
\sigma_{3\pi^0} = (0.38 \pm 0.27) \text{ mb}
\]

The total cross-section for \( 3\pi^0 \) production is presented on the basis of only 2 five-gamma-ray events.

The differential cross-section for charge-exchange using
the results of the 5th order fit is then:

\[
\frac{d\sigma}{d\Omega} = (0.03 \pm 0.03) + (0.69 \pm 0.47)X + (0.30 \pm 0.13)X^2 - (5.03 \pm 7.2)X^3 + (41 \pm 55)X^4 + (5.56 \pm 1.12)X^5 \text{mb}
\]

and the total cross-section is:

\[
\sigma_{\gamma\gamma} = 4.21 \pm 0.42 \text{ mb}
\]

The 29+8 excess $\gamma$ ray events observed in the opening angle distribution beyond 90° amount to a cross-section of

\[
\sigma_{>90^\circ} = 0.6 \pm 0.2 \text{ mb}
\]
IV. DISCUSSION OF RESULTS

A. TOTAL CROSS-SECTIONS -

The ratio of the total cross-sections for the reactions:

\[ \pi^- + p \rightarrow \pi^0 + n \quad (I) \]

and

\[ \pi^- + p \rightarrow \pi^0 + n^0 + n \quad (II) \]

found in this experiment is \( \frac{37}{37} = (0.71 \pm 0.16) \) whereas the ratio observed by the group at Saclay\(^6\) is \( \frac{37}{37} = (2.51 \pm 0.54) \). There seems to be a clear discrepancy between the results. However, the sum of the cross-sections for the above reactions seem to be in substantial agreement -- \((8.1 \pm 0.6)\) mb. from this experiment as compared to \((10.9 \pm 0.9)\) mb. from the Saclay experiment. The reason for this slight difference in the sums of the cross-sections may easily be due to the fact that the cross-section is rapidly changing (falling) with energy in this region. The discrepancy in the ratio of the cross-sections, however, cannot be accounted for by this fact. Even if we assume our measurement of the background due to \(3\pi^0\)'s to be low by as much as a factor of two, we obtain a ratio of \((1.2 \pm 0.2)\), which is still in disagreement with the Saclay results. It therefore seems that this discrepancy cannot be accounted for by possible uncertainties in the \(3\pi^0\) contamination.

Another possibility for this discrepancy may simply be due to errors in separating out the various \(\pi^0\) productions, in either or both of the experimental methods used. In reference
to this alternate possibility for discrepancy, we can speak only of advantages in our method. They are as follows:

1) The single $\eta^\circ$ cross-section does not depend strongly on $2\eta^\circ$ contamination since our limited opening angle range (25° to 50°) automatically excludes most of this background and all of that due to $\eta^\circ$ and $\Lambda^\circ$'s.

2) The $2\eta^\circ$ production is determined directly from the observed number of events having more than two gamma rays—thus, this sample cannot be contaminated by the $1\eta^\circ$ events.

3) The conversion efficiencies used are results from a simulation program which allows for several internal checks. (These internal checks were previously mentioned throughout the text.)

4) Since we "observe" the gamma rays, we have further checks between various observed distributions (opening angles; conversion points etc.), and corresponding distributions predicted by the simulation program.

5) Because of the strongly peaked characteristic distribution of the opening angle due to single charge exchange $\eta^\circ$'s, we are certain we are dealing with charge exchange events. Weinberg, et al., have also measured the cross-sections for the reactions (I), and (II), at 960 kev, and their results agree with those of the Saclay group. However, in the report of their results, there is no indication of any clear separation of hydrogen and quasi-elastically produced events. Furthermore, they did not use any known distributions in order to estimate their background and to obtain a reliable sample.
of charge exchange events. Hence, their experiment does not necessarily support the Saclay result.

B. \( \eta^0 \) PRODUCTION -

Chretien et al., give the following estimate to the ratio of the \( \eta^0 \) production cross-section to that of charge exchange:

\[
\frac{\sigma_{\eta^0}}{\sigma_{\pi^+}} = 0.17 \pm 0.03
\]

Using this value and our charge exchange cross-section of \((4.2 \pm 0.4)\) mb, we expect an \( \eta^0 \) production cross-section of \((0.1 \pm 0.1)\) mb.

If we assume our residual events with opening angles greater than 90° to be due to \( \eta^0 \) decays (the effective cross-section due to neutral decays of \( \eta^0 \)'s is 0.1 mb.) then our value of \((0.6 \pm 0.2)\) mb is consistent with the value deduced.

C. FORWARD SCATTERING -

Using the values obtained by Cronin for the determination of the forward scattering differential cross-section for charge exchange, we find the predicted value for \( \frac{d\sigma}{d\Omega} \) to be:

\[
\frac{d\sigma}{d\Omega} \text{ pred.} = (2.5 \pm 0.1) \text{ mb/st}
\]

We find from our angular distribution the value:

\[
\frac{d\sigma}{d\Omega} \text{ exp.} = (1.9 \pm 0.3) \text{ mb/st}
\]

These values are in substantial agreement.

D. D 5/2 - F 5/2 INTERFERENCE -

With the results of our differential cross-section, we are able to determine the partial waves involved at 900 kev.

If we use the 5th order fit to the angular distribution, and take \( a_6 \) to be 0, then the following is true:

\[
F_{5/2} = 0
\]

\[
a_6 \propto \text{Re} D_{5/2} F_{5/2}^*
\]
and if we take the largest term in $a_3$, assuming all other terms negligible, we get:

$$a_3 \propto -11.7 \text{ Re } D_{5/2} \pi F_{5/2}^*$$

$a_3$ is then found to be equal to $a a_5$ in magnitude, but negative in sign. This is indeed true of our results – we find a large value for $a_5 (5.6 \pm 1.2) \text{ mb/st}$ and an equally large but negative value for $a_3 (-5.0 \pm 0.9) \text{ mb/st}$. Thus, indications are that the $F 7/2$ amplitude is 0 or very small and that the $D 5/2 - F 5/2$ interference predominates.

There may be a question as to why the 6th order fit is not used. By including the 6th order term the $x^2$ probability does not change appreciably and this additional 6th order term enters with a very large error. We therefore assume that it is not needed or is at most very small.

E. CHARGE INDEPENDENCE

If charge independence is assumed to hold true in pion-nucleon interactions, the following relationship between the various elastic cross-sections ($\sigma^- \equiv \sigma_{\pi^- p} \rightarrow \pi^- p$,$\sigma^+ \equiv \sigma_{\pi^+ p} \rightarrow \pi^+ p$,$\sigma^0 \equiv \sigma_{\pi^+ p} \rightarrow \pi^0 N$,$\sigma^+ \equiv \sigma_{\pi^+ p} \rightarrow \pi^+ p$) must be satisfied:

$$R \equiv \left| \frac{3\sigma^- - 6\sigma^0 + \sigma^+}{4\sqrt{3}(\sigma^0 + \sigma^- - \sigma^+)} \right| \leq 1$$

Using the following values for the various cross-sections,

$$\sigma^+ = (12.0 \pm 0.4) \text{ mb}$$
$$\sigma^- = (25.0 \pm 3.0) \text{ mb}$$
$$\sigma^0 = (4.2 \pm 0.4) \text{ mb}$$

We find $R = (0.73 \pm 0.12)$ which satisfies the inequality required by the charge independence hypothesis.
F. ISOTOPIC-SPIN ASSIGNMENT

Using our preliminary results, it was suggested by Dalitz \(^{33}\) that the isotopic-spin associated with the D \(5/2\) and F \(5/2\) wave amplitudes are both in the \(T=\frac{1}{2}\) state. The argument for this iso-spin assignment which depends on our final results is reproduced below.

Assuming charge in dependence, it can be shown that:

\[
\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}(\pi^+\pi^-\pi^0\pi^-) + \frac{d\sigma}{d\Omega}(\pi^+\pi^-\pi^0\pi^+) - \frac{1}{2} \frac{d\sigma}{d\Omega}(\pi^+\pi^-\pi^+\pi^-)
\]

The experimental values for the various cross-sections are:

\[
\frac{d\sigma}{d\Omega}(\pi^+\pi^-\pi^0\pi^-) = (0.28 \pm 0.08) + (0.1 \pm 0.3)x + (2.1 \pm 0.7)x^2 - (9.1 \pm 1.9)x^3
\]

+(5.8 \pm 1.1)x^4 + (17.0 \pm 2.0)x^5 \text{ mb/sr}

from F. Grard, et al

\[
\frac{d\sigma}{d\Omega}(\pi^+\pi^-\pi^0\pi^+) = (0.07 \pm 0.09) + (0.9 \pm 0.17)x + (1.30 \pm 0.43)x^2 - (5.03 \pm 0.92)x^3
\]

+(6.6 \pm 5.5)x^4 + (5.56 \pm 1.20)x^5 \text{ mb/sr}

from this experiment

\[
\frac{d\sigma}{d\Omega}(\pi^+\pi^-\pi^0\pi^-) = (0.06 \pm 0.05) + (5.0 \pm 1.8)x + (1.71 \pm 0.6)x^2
\]

+(9.6 \pm 3.2)x^3

from F. Grard, et al

Therefore:

\[
\frac{d\sigma}{d\Omega}(T=\frac{1}{2}) = (0.28 \pm 0.08) + (0.1 \pm 0.3)x + (2.1 \pm 0.7)x^2 - (9.1 \pm 1.9)x^3
\]

+(5.8 \pm 1.1)x^4 + (17.0 \pm 2.0)x^5 \text{ X = cos } \Theta

If we assume a pure D \(5/2\) - F \(5/2\) interference (all other amplitudes equal to 0) then:

\[
\frac{d\sigma}{d\Omega}(T=\frac{1}{2}) \rightarrow \left[\frac{d\sigma}{d\Omega}(\pi^+\pi^-\pi^0\pi^-) + \frac{d\sigma}{d\Omega}(\pi^+\pi^-\pi^0\pi^+)\right] + 2 \Re(\pi^+\pi^-\pi^0\pi^-) \text{ (25 \text{ cos} \Theta \text{ - 26 \text{ cos} }^3 \Theta \text{ + 3 \text{ cos} } \Theta)}
\]

Keeping in mind the assumption made (pure D \(5/2\) - F \(5/2\) interference ), this seems to fit the data quite well. The experimental(M\(\pi^+\pi^-\)) differential cross-section does not even
require a 4th order $\cos \theta$ term. This indicates that there is no strong $D \frac{5}{2}, T=\frac{3}{2}$ contribution. Hence, we can conclude that the interfering $D \frac{5}{2} - F \frac{5}{2}$ amplitudes are both in the $T=\frac{3}{2}$ isotopic-spin state.
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<th>5-10</th>
<th>10-15</th>
<th>0-15</th>
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### TABLE III-1

**CONVERSION PROBABILITIES OF GAMMA RAYS FROM $\pi^0$, $2\pi^0$ AND $3\pi^0$ DECAYS**

**TARGET REGION**

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<th>5-10</th>
<th>10-15</th>
<th>0-15</th>
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57
TABLE III-2  
CORRECTION FACTORS  
FOR  
LEGENDRE-POLYNOMIAL COEFFICIENTS

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<th>$B_3$</th>
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$\beta_{\pi} = 0.9735$ for this experiment.
### TABLE III-3

**BISECTOR EFFICIENCIES**

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**ALL EFFICIENCIES WITH TWO DIGITS SHOULD BE MULTIPLIED BY 10^-2**

**ALL EFFICIENCIES WITH THREE DIGITS SHOULD BE MULTIPLIED BY 10^-3**

**ALL COSINE VALUES SHOULD BE MULTIPLIED BY 10^-2**
### TABLE III

#### $\pi^0$ ANGULAR DISTRIBUTION

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<td>$0.05\pm0.09$</td>
<td>$0.43\pm0.11$</td>
<td>$0.09\pm0.10$</td>
<td>$-0.08\pm0.12$</td>
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<td>5</td>
<td>$0.38\pm0.04$</td>
<td>$0.05\pm0.09$</td>
<td>$0.49\pm0.11$</td>
<td>$0.46\pm0.13$</td>
<td>$-0.15\pm0.13$</td>
<td>$0.71\pm0.15$</td>
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<td>6</td>
<td>$0.39\pm0.04$</td>
<td>$0.02\pm0.09$</td>
<td>$0.48\pm0.11$</td>
<td>$0.45\pm0.13$</td>
<td>$-0.26\pm0.16$</td>
<td>$0.75\pm0.15$</td>
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#### COSINE SERIES COEFFICIENTS

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<td>$-5.03\pm0.92$</td>
<td>$-0.66\pm0.55$</td>
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<tr>
<td>6</td>
<td>$1.11\pm0.03$</td>
<td>$0.74\pm0.21$</td>
<td>$0.44\pm0.90$</td>
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<td>$5.88\pm1.23$</td>
<td>$2.79\pm2.63$</td>
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REFERENCES


References con't.


16. Lawrence Rosenson, 1960 Int. Conf. on Instrumentation for High-Energy Physics, Berkeley, 135 (1960)


21. LGP-30 program for particle trajectories, written by J. Sanford, and J. Sandweiss, Yale U.


24. These projectors were built by the Brown University group, but originally designed at Yale U.

25. IBM 704 program written by M. Chretien, Brandeis U., modified for IBM 7090 use by A. Brenner, Harvard U.

26. IBM 7090 program written by C. Bordner, Harvard U.

27. IBM 7090 program written by A. Brenner, and E. Ronat, Harvard University.


33. R.H. Dalitz, private communication.

Expansion Head

Target Reservoir

Beam Finger

15" Chamber

TT Beam

BUBBLE CHAMBER WITH INTERNAL HYDROGEN TARGET (BEAM FINGER)

Figure I-1
Fig.I-2

MEAN FREE-PATH FOR GAMMA RAY CONVERSION.
Figure I - 3
Fig. I-4a

BEAM DISTRIBUTION ALONG Y-AXIS
Figure I-4b

BEAM DISTRIBUTION
ALONG Z-AXIS

Target Wall

Target Wall

N
44
36
28
20
12

Z (cm)
3
2
1
0
1
2
3

Figure I-4b
FIDUCIAL VOLUME FOR BUBBLE CHAMBER

Figure II-1
COORDINATE SYSTEM

Figure II-2
COMBINED (3+4) Γ
OPENING ANGLE DISTRIBUTION

PROB. > X^2 = 15%

Figure III - 1
DISTRIBUTION IN X-COORD. OF $\gamma$ CONVERSION
POINTS COMBINED (2+3+4) $\gamma$ SAMPLES

PROB. $> x^2 = 44\%$

Figure III-1a
DISTRIBUTION IN Y-COORD. OF $\gamma$ CONVERSION POINTS COMBINED (2+3+4) $\gamma$ SAMPLES

PROB. $>X^2 = 45\%$

Figure III-1b
DISTRIBUTION IN Z-COORD. OF $\gamma$ CONVERSION POINTS COMBINED (2+3+4) $\gamma$ SAMPLES

- Data
- Simulation Program

PROB. $> X^2 = 31\%$

Figure III.10
For Isotropic Decay in $X_0$ Rest Frame

\[ N(\Theta) \, d\Theta = 0 \text{ for } \Theta < \Theta_{\text{min}} = 2\sin^{-1} \frac{1}{\gamma} \]

\[ = \frac{1}{2\beta\gamma} \frac{\cos \Theta/2 \, d\Theta}{\sin^2 \Theta/2 \sqrt{\gamma^2 \sin^2 \Theta/2 - 1}} \]

\[ \gamma = \frac{1}{\sqrt{1 - \beta^2}} \]

THEORETICAL OPENING ANGLE DISTRIBUTION

Figure III-2
Fig. III-3

OPENING ANGLE DIFFERENCE DISTRIBUTION

$|\Theta_{op1} - \Theta_{op2}|$
Fig. III-4

RAW (OBSERVED DATA) 2 \gamma
OPENING ANGLE DISTRIBUTION.
BACKGROUND SUBTRACTED
2Y OPENING ANGLE
DISTRIBUTION

PROB. > $\chi^2$ = 70%
Normalized Between 0°-85°

--- Background Subtracted Data.
---- Simulation Program.

Fig. III-5
Fig. III - 6

$\Theta_{\beta-\pi} \text{ vs } \Theta_{op}$

$\cos(\Theta_{\beta-\pi}) = \frac{1}{\beta} \cos(\Theta_{op})$
CORRECTED OPENING ANGLE
BISECTOR DISTRIBUTION (0-5)
Fig. III-7b

CORRECTED OPENING ANGLE
BISECTOR DISTRIBUTION (5-10).

Region of Exclusion $\epsilon < 0.10$
FIG. III-7C
CORRECTED OPENING ANGLE
BISECTOR DISTRIBUTION
(10-15)
Fig. III-8
FINAL AVERAGED OPENING
ANGLE BISECTOR DISTRIBUTION.

--- Using Efficiency Corrections From (BI 2)
--- Using Efficiency Corrections From (BI 1)
FIG. III-2

PROB. X VS MAX
Fig. III—10

5th ORDER FIT TO OPENING ANGLE BISECTOR DISTRIBUTION.

- Fit To Bisector Distribution.
- Final $\pi^0$ Angular Distribution.