IV. COMMUNICATIONS AND RELATED PROJECTS

A. Modulation Studies: Spectrum Utilization, Efficiency of Power Utilization, Signal to Noise Ratios

Staff: Mr. W. Tuller
Mr. E. Kretzmer
Mr. T. Cheatham

A survey has been made of most of the various known systems of impressing intelligence on an electromagnetic wave in an attempt to determine the relative advantages of these systems with regard to efficiency of spectrum utilization, efficiency of power utilization, and obtainable signal-to-noise ratios.

A. Systems considered so far:
1. Frequency modulation
2. Amplitude modulation
3. Pulse phase modulation
4. Pulse frequency modulation
5. Pulse amplitude modulation
6. Pulse width modulation
7. Amplitude modulated d-c pulses used as a subcarrier to frequency modulate the main carrier.

B. Factors considered in evaluating the performance of each system:
1. Ratio of the bandwidth required to transmit the signal to the bandwidth of intelligence capable of being transmitted. This ratio may be symbolically represented by $BWS/BWI$.
2. Signal-to-noise ratio at receiver output as compared to signal-to-noise ratio at the worst point in the receiving system. This factor is called $SNO/SNM$. Only thermal or quasithermal noise has so far been considered.
3. Ratio of the "interference bandwidth", i.e., the bandwidth not required to transmit intelligence but used by the system and therefore not available to other services, to the actual bandwidth required to transmit the signal. This ratio is called $BWN/BWS$.
4. Ratio of the product of the number of channels transmitted and the bandwidth of intelligence capable of being transmitted in each channel to the total bandwidth available for the transmission of intelligence. This ratio has meaning only in a system using spectrum division methods for multiplexing channels. It may be represented by $NBWI_1/ NBW$. The corresponding factor in a system employing time division for multiplexing is the ratio of the product of number of channels transmitted by the time taken to transmit each channel to the total time during which the system is available. This latter factor may be called $FT_1/T$. Both of these factors are dependent on the amount of crosstalk between channels considered tolerable.

A system performance coefficient may be made up from these factors.

It is:

$$SFC = \frac{BWI \times SNO \times BWS \times NBWI_1}{BWS \times SNM \times BWN \times NBW}$$

for spectrum division multiplex, or

$$SFC = \frac{BWI \times SNO \times BWS \times FT_1}{BWS \times SNM \times BWN \times T}$$
for time division multiplex. In evaluating systems by means of this coefficient, average power and receiver noise figure are assumed identical for all systems. Systems employing limiters are assumed to be operating in such a manner that the limiters are effective.

Although quantitative investigations are not complete on all systems the following tentative table may be made of modulation systems in order of decreasing system performance coefficients, i.e., the best first.

1. Amplitude modulation, single side band, suppressed carrier
2. Frequency modulation
3. Single channel pulse phase modulation
   Amplitude modulation, carrier and both sidebands transmitted
   Pulse amplitude modulation
4. Pulse phase modulation - multichannel
5. Pulse frequency modulation
6. Pulse width modulation

The exact position in the table of the system using amplitude modulated d-c pulses to frequency modulate the transmitted carrier has not been established, but it is known to be high.

It is realized that this system performance coefficient does not give complete evaluation of the worth of the system considered, particularly from an engineering viewpoint, but it is believed that the coefficient will be useful in indicating profitable directions for endeavor.

This survey has been based on the Hartley law, relating amount of intelligence transmitted, bandwidth occupied by the intelligence, and time required to transmit the intelligence. The work is being continued, particularly with a view toward finding out the elements which make up an efficient transmission system so as to give the possibility of synthesizing new and better modulation systems on the basis of these elements.

2. The analysis of frequency modulation discriminators reported on previously has been further extended to cover discriminators operating with relatively high distortion. Two results have come as a by-product of this investigation. The first is an analysis of the conditions necessary to produce minimum distortion in microwave discriminators. As a result of this analysis microwave discriminators may be designed with six decibels less distortion than is found in types now used.

   Another, and possibly more important by-product of this analysis is the design of an adjustable bandwidth discriminator for use at any radio-frequency at which lumped circuit elements are employed. The bandwidth of this discriminator is readily adjustable by one easily obtained control. An experimental check of this unit has been begun.

3. Harmonic analyses of pulse width modulation have been carried through in a general form. It is thereby possible to predict the relative magnitudes of the various undesired components (harmonics and intermodulation products) which may be

---

1. Fuller, W. G., Distortion in F.M. Discriminators, NDRC, Division 14, Research Laboratory of Electronics, Massachusetts Institute of Technology, Technical Report No. 1, March 8, 1946

---
present at the output of the demodulator discussed below, (4-2) and how these magnitudes depend on the degree of modulation and other variables. Such considerations are of prime importance in high fidelity systems using pulse-time (or pulse phase) modulation, and also in those using pulse width modulation directly.

The results of the analyses show the important influence of the law of modulation, a concept which has been virtually neglected in the published literature. By the term "law" as applied to pulse width modulation, is meant the exact relationship between the time of occurrence of a pulse edge and the modulating voltage. Analyses of the type described above, but based on a different law of modulation recently appeared in the literature. A comparison of the results of these two sets of analyses shows certain fundamental differences, resulting in a wide variation in the amplitude of the various distortion products with change of modulation law. Letters pointing out the fact have been sent to the editors of the Proceedings of the Institute of Radio Engineers and The Wireless Engineer, respectively.

4. The following pieces of equipment have been designed, built and tested for use in the study of pulse modulated transmissions:
   (a) Low power gas tube pulser
   (b) Ten kilocycle repetition frequency pulse generator comprising:
       (1) Synchronizing pulse and timing wave generator
       (2) Phase modulated pulse generator
       (3) Width modulated pulse generator
   (c) High power video amplifier
   (d) Phase modulated pulse decoder of the type which gives an output proportional to the center of gravity of the received pulse.
   (e) Pulse amplitude modulator
   (f) Phase modulated pulse decoder of the type which produces width modulated pulses as an intermediate step in the demodulation process.
   (g) Pulse repetition frequency filter for use with decoders.

The first three of these units have been described in the Interim Progress Report for March 15, 1946 of this Laboratory, pp. 39-40.

The pulse amplitude modulator is of simple and conventional design. It produces a train of pulses of about 20 volts average amplitude. This amplitude can be modulated by an external signal. The pulses so produced may be used to amplitude modulate a klystron amplifier integral with the modulator or to frequency modulate an external transmitter.

The new decoder (f) is based on the conventional method whereby an incoming pair of synchronizing and channel pulses are "bridged" to form a relatively wide width-modulated pulse. While the generation of this pulse offered no obstacles the problem of subsequent reduction of the 10 KC pulse repetition frequency component was found to be more difficult. The signal of the pulses is of the order of 40 db below the 10 KC component with the degree of modulation practical in a ten-channel system. Hence at least 80 db attenuation of the p.r.f. must be attained to give a 40 db signal-to-whistle ratio. A satisfactory filter (g) was found to consist of a constant-k two-section \pi filter and a bridge-T rejection network in cascade.

Components of the 10 KC pulse generator are to be subjected to some research. The modulator has been investigated in detail and modified considerably, since it was found to produce excessive intermodulation distortion. The modified circuit is being tested; it is expected that, among other improvements, this circuit will exhibit better linearity of modulation and greater freedom from "jitter" modulation.

IV. B. Stabilized Oscillator Problems
Staff: Prof. T. S. Gray
Mr. F. P. Zaffarano
Mr. W. Galloway
Mr. P. Richman

Numerous requests for data to assist in the construction of a number of i-f type 723A/B stabilizers for general laboratory usage have led to an investigation of the conditions under which the waveguide discriminator should operate.

Since these units are to be as reliable as is practical, a reasonable requisite is that the stabilizer should be incapable of "locking" on any but the desired frequency. Moreover, the locking action should be automatic and not require a particular sequence of manipulation of the controls. A good approximation to the ideal discriminator characteristic is therefore mandatory.

A change in the detector crystal match (due to r-f power level shift) of 4:1 in power S.W.R. was found to be the cause of undesired loops in the discriminator characteristic. D-c biasing of 1 to 5 ma (depending on the conditions under which the crystal is matched) permits the ideal discriminator curve to be realized. A pre-plumbed version of the discriminator with only one adjustment (a phase shifter allowing a few degrees compensation of the reference cavity position) has exhibited an ideal phase shift versus frequency characteristic in the 100 mc range over which it was tested.

A simplified i-f strip - to conserve the diminishing supply of AN/APS-10 units - is under consideration. It is quite certain that the phase response of the APS-10 strips leave something to be desired especially if long-time stability is required without resorting to stabilization of the intermediate frequency source.

Modulation of the stabilized oscillators at frequencies higher than 20 KC shows the phase detector to be the most restrictive limitation. The i-f amplifier, phase detector, i-f frequency, and reference cavity Q all limit the high frequency stabilization obtainable at modulation frequencies greater than 1 mc.
Study of the stability problem in d-c amplifiers for use in measurement and control applications and for improvement of microwave discriminators is proposed. In particular, the relative merits of the series-balanced, parallel-balanced, cathode-compensated, and diode-compensated circuits for use in measurement problems and in the discriminator work described above will be made.

IV. C. Multipath Transmission

Staff: Prof. L. B. Arguinbau
Mr. J. Granlund

The chief limitation on long-distance radio communication with speech and music is the interference caused by transmission over two or more paths. The generally poor results on transatlantic broadcasts using amplitude modulation are well known.

A few papers have been published concerning the effect of multipath transmission on frequency-modulated signals. No similar data have been published on pulse modulation.

The present project has been started with the object of determining the relative advantages of the various types of modulation with two parallel paths of variable strength. At first it was proposed to use a microwave model but further consideration showed that this would not be desirable as the relative time delays on two microwave paths could not be made comparable to the long delays (0.5-1 millisecond) met in practice. For this reason it is planned to carry out the experimental work at a frequency of around 30 megacycles. At this frequency acoustic delay lines (mercury columns with piezoelectric electrodes) are available. These particular lines have delays of about a half millisecond.

Fig. 1

---

The arrangement is shown in the functional diagram of Figure 1. A modulated signal is transmitted through two parallel paths to a receiver. One of the paths includes the half-millisecond delay line. The two paths are provided with volume controls to permit the relative magnitudes of the signals to be varied at will, either statically or dynamically.

The previous work has shown that with frequency-modulation and two almost equal paths serious waveform distortion occurs. When one path is much stronger than the other it predominates and there is little trouble. Loran experience shows that when two or more paths are present the relative strengths of the two paths may switch over from time to time. Most trouble can be expected during this transition.

In the present project it is proposed to compare the different systems of modulation with respect to their operation during this transition. One would expect trouble from the AM system as long as the two signals are spaced by less than 20 decibels while comparable trouble would be expected with FM when the signal strengths differed by 6 decibels. In an idealized pulse system one would expect the transitional region to be much smaller. As long as one signal is slightly larger than the other that signal should blank out the other. This would indicate that the trouble occurring during a switch in level between the two paths should last longest for amplitude modulation and should be of very short duration for an idealized pulse system. It is planned to determine the effect of such transition on speech and to find out something about the effect of the time taken by the change over between the two paths.

Experimental equipment corresponding to Figure 1 has been built up. Unfortunately the delay line has an attenuation of something like 60 decibels. This fact and the necessity for shutting off one path and letting the other through imposes severe shielding requirements. At the moment of writing, parasitic paths are being traced down so that the two paths can be kept separate. So far work has been limited to getting the equipment in good working order and no overall tests have been made.

IV. D. Impulse Interference

Staff: Mr. G. E. Stannard

An investigation of "Impulse Interference on Frequency Modulated Receivers" is being studied with equipment similar to that described by Landon.¹ The equipment has been carefully shielded and provided with attenuators so that quantitative results can be expected. The work is being done to get an experimental check on recent (unpublished) work by Smith and Bradley of Philco and to study limiter circuits developed since Landon wrote his paper.

IV. E. Circuit Problems

1. Study of Waveguide with Dissipative Walls

Staff: Mr. R. M. Fano

Waveguides with highly dissipative walls have been used successfully in the design of broadband high-power loads. It was found in this connection

that if a rectangular guide with highly dissipative walls is jointed to a non-
dissipative guide of the same internal dimensions there is always a wavelength
at which a perfect match is obtained. For instance, in a rectangular guide of
internal dimensions 0.400" x 0.900" this wavelength has been found to be equal
approximately to 3.3 cms.

A theoretical study of the propagation of the dominant mode in
a lossy guide shows that, to a first approximation, the wave impedance is given by:

\[
Z_{TB} = \sqrt{\frac{\mu}{\epsilon}} \frac{1}{\sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}} \left[ 1 - j \frac{\eta_2}{\omega \mu b} \frac{1 - 2(1+j)\left(\frac{\lambda}{\lambda_c}\right)^2}{1 - \left(\frac{\lambda}{\lambda_c}\right)^2} \right]
\]

\(\mu, \epsilon\) are the permeability and the dielectric constant of the medium filling the
guide, \(\lambda\) is the free-space wavelength, \(\lambda_c\) is the cut-off wavelength, \(b\) and \(a\) are
the narrow and wide dimensions of the guide respectively, \(\omega\) is the angular fre-
quency, \(\eta_2 = \sqrt{\frac{1}{\omega \mu c}}\) is the intrinsic impedance of the walls. The equation is
valid for small values of \(\eta_2\), that is, for large values of the conductivity \(\sigma_2\).

It has been found, at the same time, that the propagation function
is given by:

\[
\gamma = \frac{j2\pi}{\lambda} \sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2} + \eta_2 \frac{\lambda}{\mu b} \sqrt{\frac{1}{1 - \left(\frac{\lambda}{\lambda_c}\right)^2}}
\]

The real part of the term proportional to \(\eta_2\) is the well-known expression for the
attenuation. Note that the angle of \(\eta_2\) is very closely equal to 45° because \(j\omega \epsilon_2\)
is negligible for large values of \(\sigma_2\).

In the expression for the wave impedance, the term proportional
to \(\eta_2\) vanishes when:

\[2(1+j)\left(\frac{\lambda}{\lambda_c}\right)^2 = 1\]

It can be shown that when this condition is satisfied, the power dissipated by the
longitudinal currents is equal to the power dissipated by the transverse currents.
Note that in the case of conventional transmission lines, that is of TEM modes, the
wave impedance is not affected by dissipation when the power dissipated in the con-
ductors is equal to the power dissipated in the dielectric. The analogy between
these two cases is evident.

In order to determine the condition of match between a non-
dissipative guide and a dissipative guide one must consider the junction effect as
well as the change of wave impedance. The problem of determining the junction
effect has not yet been solved. However, a formula based on physical reasoning
has been derived which seems to check the few experimental results available at
present.

Marcuvitz has shown that the reflection taking place at the
junction of two guides of slightly different dimensions can be computed by means
of conventional transmission line theory if one defines a characteristic impedance
equal to the product of the wave impedance by the ratio \(\frac{b}{a}\). It seems reasonable
to extrapolate this result to our case by taking as ratio \(\frac{b}{a}\) for the lossy guide
the ratio of the inside dimensions increased by twice the complex skin depth:

\[ t_s = \frac{\eta_2}{j\omega \mu} \]

One has then for the characteristic impedance of the lossy guide:

\[
\begin{align*}
Z_o^2 &= \frac{1}{\sqrt{1 - (\frac{\lambda}{\lambda_c})^2}} \left[ 1 - j \frac{\eta_2}{\omega \mu \mu} \frac{1 - 2(1 + b)(\frac{\lambda}{\lambda_c})^2}{1 - (\frac{\lambda}{\lambda_c})^2} \right] \frac{b(1 + 2\eta_2)}{a(1 + 2\eta_2)} \\
&= \frac{1}{\sqrt{1 - (\frac{\lambda}{\lambda_c})^2}} \left[ 1 - j \frac{\eta_2}{\omega \mu \mu} \frac{4 - (\frac{\lambda}{\lambda_c})^2}{1 - (\frac{\lambda}{\lambda_c})^2} \right]
\end{align*}
\]

This expression becomes equal to the corresponding expression for a non-dissipative guide of the same internal dimensions when:

\[ 1 - (\frac{\lambda}{\lambda_c})^2 = \frac{1}{4}(1 + \frac{b^2}{a^2}) \]

One would then expect to obtain a perfect match when this equation is satisfied. For a 0.400" x 0.900" guide this equation yields \( \lambda = 3.32 \), a value very close to the wavelength determined experimentally. A similar check is obtained in the case of a 1 \( \frac{\lambda}{2} \)" x 3" guide.

2. Magic-T Problems

Staff: G. W. Zabel

Wave Guide Ring-circuit Analysis.

Two types of ring circuit magic-T's have been considered: a magic-T employing a ring whose mean circumference is (1) equal to one guide wavelength, (2) equal to one and one-half guide wavelengths. Expressions have been found for the elements of the scattering matrices of both types in terms of their dimensions and the parameters of the equivalent circuit of the \( \eta \)-plane T. The relations were applied to the magic-T used at a wavelength of 3.23 cm. for balanced duplexing. The agreement with the experimentally determined values of J. Reed and with the simple theory indicates almost complete canceling of junction effects. Since the junction effects are not small, it is suspected that another choice in guide dimensions would not show this cancellation. Although no further calculations were made, the equivalent circuit parameters are available for a large range of dimensions of the \( \eta \)-plane T.

A Waveamplitude Multiplier.

Dr. R. H. Dicke has suggested an application of the magic-T in which a circulating traveling wave is formed whose amplitude is much greater than the amplitude of the incident wave. The device consists of a standard magic-T (combination of an \( \eta \)-plane and an \( \eta \)-plane T) whose \( \eta \)-plane arm is returned to one of the symmetrical arms, forming a loop. The power circulating in the loop depends upon the length and attenuation of the loop and varies from 5.28 to .414 times the incident power. The device is always matched if a matched load is placed on the remaining arm. The power delivered
to the load varies from 0, when there is an attenuation of 3 db in the loop and the loop is a wavelength long, to 1, when there is no attenuation in the loop. Additional magic T's may be inserted in the loop in which case the power in the traveling wave in the last loop is \((5.28)^n\) times the incident power, where \(n\) is the number of T's added.

3. Miscellaneous Waveguide Problems

Staff: R. M. Redheffer

As an immediate corollary of the general reciprocity theorem, it has been shown that radome transmission, as measured in the usual way, will be the same whether the antenna is transmitting or receiving. That the (normalized) pattern will be the same is evident. This result also indicates that error due to the spherical nature of the wave front, when dielectric constants are measured by the transmission method, will be the same whether the sample is near the receiver or transmitter.

In the bridge method of measuring dielectric constants error due to frequency drift has been briefly investigated, as has error in \(\tan \delta\) from the variable transmission of the line stretcher. A simple method of compensating this latter has been obtained.

In plotting graphs for computation of dielectric constants from the original data one requires knowledge of the physically possible range of values; and thus none of the work of computation will be wasted. With this idea in mind the dependence of the phase shift \(\varphi_1\) associated with interface reflection \(r_1\) has been briefly considered. If the loss tangent of the material, \(\tan \delta\), is zero, then this phase shift is \(\pi\), as is well known; and it is likewise \(\pi\) when \(\tan \delta\) is infinite. At some intermediate value of \(\tan \delta\), then, the deviation from \(\pi\) will be a maximum.

The elementary result established is that this deviation is maximum when \(\cos \delta = K/(2K-1)\) and its value is given by \(\cot^2(\pi - \varphi_1)_{\text{max}} = 2(K-1)\). The interface reflection \(r_1\) is now equal to \(\sqrt{(K-1)(2K-1)}/[K/2 + \sqrt{3K-1}]\) and from these relations the desired limits for the range of graphical computation have been readily obtained.

Earlier investigations of the effect of detector non-linearity have been systematized and elaborated. It has been shown that the error introduced in measurements of transmission is the same as that introduced in measurements of standing wave ratios by the maximum/minimum technique, the fractional error in both cases being given by \(1 - (\text{true value})^{\alpha/2}\) if \(\alpha\) is the law of the detector. A similar relation has been found when the standing wave ratio is computed from the width at \(K\) times the minimum, namely, fractional error in \((\text{SWR})^2 = [K - K^2/\alpha]/[K-1]\). This result, which like the others was derived for the dielectric chapter, had been previously obtained for \(K = 2\).
4. Transient Phenomena in Waveguides  
Staff: Mr. M. V. Cerrillo  

1) By successive Laplace transformations of Maxwell’s equations, for the general cylinder, a set of simultaneous algebraic and differential equations were obtained. Solutions of this system were already made up to the last Laplace inversion with respect to time. These solutions contain all the initial conditions which can be specified in a problem of this type.

2) The set of these last Laplace transforms were carefully studied and it was found that all of them are so related that only the inversion of one prototype is needed.

3) To find the inversion of this prototype, two methods were used:

   a) The convolution theorem and the use of the well-known inverse transform

\[
\mathcal{L}^{-1}\left(\frac{-a\sqrt{s^2 + b^2}}{s^2 + b^2}\right) = \begin{cases} 
0 & \text{for } t < a \\
J_0\left(b\sqrt{t^2 - a^2}\right) & \text{for } a < t
\end{cases}
\]

Both lead to an integral of the type

\[
\mathcal{\Lambda}(x, \xi, \xi_o) = \int_x^\infty Z_0(\xi, y)e^{-j\xi_0 y} dy
\]

which might be considered as the fundamental generating function for the propagation in hollow cylinders.

b) By the integration, along a proper contour of

\[
\int \frac{e^{-st-a\sqrt{s^2 + b^2}}}{s-j\omega_o} ds
\]

4) The function \(\mathcal{\Lambda}(x, \xi, \xi_o)\) was carefully studied and solutions in series, for small, medium and large values of \(t\), were obtained.

The integration was performed in terms of Bessel functions and in power series expansion. These solutions are rather complicated and very hard to handle, for sometimes they appear as double series.

5) The direct inversion of the fundamental transform, following the method in 3) (b), is very advanced, more suitable and some definite results are now available. The existence of secondary waves in any cross section was found, so that there exist direct, reflected and transmitted waves. The last two waves disappear when the transient is over.

The mathematical expression for the secondary waves is simple. The part of the frequency spectrum below the cut-off is responsible for its existence. The direct wave is a sort of step function whose numerical expression is the same as the permanent state but properly delayed.

Numerical integration and expression for the transient terms were made by two methods of summation. Unfortunately, these results cannot be applied at very large distance from the origin of the perturbation. The mechanism of the limiting process is not well understood.
and one effort is now made to solve this part. The next step in this connection is the integration around the saddle points of the integrand.