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STUDIES IN FIBER-OPTIC COUPLERS AND RESONATORS

by

Robert P. Dahlgren

ABSTRACT

Polished fiber optic couplers are widely used as building blocks for interferometric sensors. The development of fiber optic sensors was facilitated by the availability of adjustable couplers in the early 1980s, which were widely used in prototypes and experiments. This document outlines the theoretical and practical aspects of polished coupler and resonator technology.

First, a matrix technique is presented which permits calculation of the characteristics of non ideal birefringent coupler devices, including asymmetry effects. A method for modeling complex fiber optic resonant topologies is also introduced, which allows for backscattering effects. Together, these permit computer simulation of most of the linear errors in resonant cavities, and permit comparison of different topologies.

Fiber optic coupler polishing is described, for the first time in such detail. A survey of substrate preparation techniques is made, and an advanced groove-generation machine is presented, which produces very consistent substrate grooves. Specific materials and polishing compounds are listed, providing the reader a "recipe" for processing of PM fiber couplers. Design and operation of a state-of-the-art principal axis alignment system is discussed, with approximately ±2-deg accuracy. Assembly techniques for adjustable couplers, and for the first time, optical contact bonded couplers. An attempt is made to remove some of the mystery from this ancient "black art."

Resonators were constructed from the couplers, and data is presented for four devices made. Polarization-maintaining fiber couplers were spliced onto fiber coils, both polarization-maintaining and single-polarization types. A spliceless all-polarization-maintaining ring was also assembled, having a finesse in excess of 300. All of the resonators had very low polarization cross-coupling and lineshape asymmetry.

Thesis Supervisor: Dr. Shaoul Ezekiel, Professor of Aeronautics and Astronautics
Draper Supervisor: Dr. Raymond Carroll, Principal Member of the Technical Staff
Biographical Note

Mr. Dahlgren was born in 1960 and raised in Austin, Minnesota. He later attended the University of Minnesota and received the Bachelor's Degree in Electrical Engineering in 1983. It was while a sophomore that he was introduced to optical technology at Honeywell Inc. as a part-time photolithography worker. He subsequently started working on ring laser gyroscopes as a U. of M. intern in 1980, and he was subsequently assigned to the System and Research Center, under Dr. Gordon Mitchell. It was there he started working on fiber-optic couplers, components, and sensors.

Upon graduation in 1983, Bob was employed at Magnetic Peripherals Inc., where he developed optical characterization techniques for read/write heads and magnetic media. In 1985 he returned to GN&C, working for Sperry Aerospace in Phoenix, which is now owned by Honeywell. There he worked on fiber optic gyro and component projects with Drs. Chin Chang and Loren Stokes. Polished and fused fiber optic couplers were investigated in support of IFOG activities, and also devices such as resonators and polarizers. A number of high-quality components were made and integrated into brassboard FOGs, and Bob was promoted to Sr. Scientist for his efforts.

Subsequently, he accepted a technical staff position at The Charles Stark Draper Laboratory Inc., in Byong Ahn's RFOG group. There he further refined polished coupler technology and built a number of devices for different RFOG projects. He also performed radiation hardness experiments on birefringent fibers and couplers, and worked on other fiber optic sensors. During that time, Bob took a number of optics courses at MIT, and was eventually awarded a Draper Fellowship in the Aeronautics and Astronautics Department. Most recently, he has accepted a position of Manager of Fiber Optic R&D for Fujikura Technology America Corp. in California. He is the author of a number of technical publications, and holds eight patents in fiber optic and optical component technology.
Acknowledgments

I have had the pleasure of being assisted by a number of people during my stay at MIT and Draper as I wrote this dissertation. First, I gratefully acknowledge the early encouragement from, and discussions with, Prof. Shaoul Ezekiel, Dr. Raymond Carroll, Byong Ahn, and Dr. Robert Smith. Two master polishers, Robert Sutherland and Paul Marchi, performed much of the hands-on work, and were willing to share their vast experience with me throughout this project, and were responsible for much if its success. Other technicians at Draper Lab who contributed were Kevin Champagne, Nick Katis, and Steve Root. Other Draper employees that have earned my thanks include Ken Spratlin, who made a Sun workstation available at no charge for the computationally-intensive analysis of the coupler, and Greg Cappiello who made the ray-tracing diagram of the PANDA fiber used in chapter IV. I would also like to thank Dr. John Sweeney, Dr. John Deyst, Bob Gauthier, Dr. David Burke, and Dr. Leonard Wilk. From the Photonics department, the support of Dr. Jack Haavisto, Todd Kaiser, and Dr. John Hopps is also gratefully acknowledged. Work on fiber optic couplers, and other optical components at Draper, would not have been possible without the context provided by various FOG contracts from Delco Inc., NASA, US Army Strategic Defense Command, and US Air Force. While this work was largely supported by Draper IR&D, is acknowledged that these contracts did provide the driving force behind it.

A special thanks goes to Bev Tuzzalino who performed essential proofreading. The support of The Charles Stark Draper Laboratory Inc., MIT Aeronautics/Department, and MIT Center for Advanced Engineering Studies is gratefully acknowledged. Honeywell Inc., Fujikura Ltd., Alcoa Fujikura Inc., and Struehers Inc. are also appreciated for providing technical information, figures, and hardware. Special thanks to Daniel Lichtblau and K.J. Paradise from Wolfram Inc. for assisting with Mathematica problems. Finally, I would like to thank all of my friends, professors, and colleagues I have met in Boston, including Alan Yarbrough, Daniel Laurent, John Hess, Loretta Wiltse, Bim, Dr. Paul Brown, and Profs. Humblet, Smith, Haus, and Warde. Dr. Alan Shiota and my colleagues at FTAC are also thanked for letting me finish up my writing in California, and tolerating my bleary-eyed countenance in the morning. Finally, I would like to express my gratitude and love for my parents Beverly and Vernon Dahlgren who have been most supportive during this time. This document is dedicated to my uncle Ernest Guhnus, who passed away during the last months of this writing.
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Partial List of Abbreviations

- $\equiv$Si–O–Si$\equiv$ amorphous bulk silica showing siloxane bonds
- $\equiv$Si–OH hydrated silica at surface of glass
- $a,b,c,d$ general-purpose matrix coefficients
- $A, B$ fiber designator
- $A$ solution matrix describing the coupler
- $ac$ alternating current
- AU arbitrary units
- $B_{AC}$ AC birefringence modulation
- $B_E$ extrinsic birefringence due to squeezing
- $B_I$ intrinsic birefringence due to stress-applying parts
- $B_T$ total birefringence due to stress-applying parts and squeezing
- $C$ coupling coefficient between two fiber cores
- $C_{ab}, C_{ba}$ coupling coefficient between fiber A and B
- $CC$ cross-coupling, polarization cross-coupling
- CCD charge-coupled device
- CFR cross-coupled fiber resonator ($k \sim 1$)
- $C_p$ Preston coefficient
- CTE coefficient of thermal expansion
- $C_{xy}, C_{yx}$ coupling coefficient associated with polarization
- $c_0$ the speed of light in vacuum
- $D$ coilform diameter
- $D$ a diagonal matrix
- $\text{det}[ \ ]$ determinant of a matrix
- $d_c$ fiber cladding diameter
- $dc$ direct current
- $DFR$ direct-coupled fiber resonator ($k \ll 1$)
- $d_j$ fiber jacket diameter
- $d(y_o)$ equivalent core separation for y-offset
- $d(y_o,z_o)$ equivalent core separation for y- and z-offset
- DI $H_2O$ deionized water
- $d_o$ minimum core separation in a coupler
- $d(z)$ centerline separation as a function of z
- $E_A(z), E_B(z)$ scalar describing propagation in each fiber
\[ E_0 \] field at \((x,y) = (0,0)\)

\[ E_{in} \] input scalar field

\[ E_{LED} \] edge-emitting LED

\[ E_{circ} \] circulating scalar field

\[ E_{out} \] output scalar field

\[ E_j \] field entering or leaving port \(j\)

\[ E_{j+1} \] field entering or leaving port \(j+1\)

\[ E_x, E_y, E_z \] field components in \((x,y,z)\) direction

\[ E(z) \] vector describing field propagation in all paths

\[ E_1, E_2, E_3, E_4 \] amplitude at coupler input/output ports

\[ E_I \] fields entering or leaving port \(I\) of embedded network

\[ E_{II} \] fields entering or leaving port \(II\) of embedded network

\[ ESOP \] eigenstate-of-polarization

\[ f \] fast axis designation

\[ f_o \] modulation frequency

\[ F \] force, finesse

\[ FSR \] free spectral range

\[ FWHM \] full width half maximum

\[ G \] geometry matrix

\[ Ge \] the element Germanium

\[ GN&C \] guidance, navigation and control

\[ h \] h-parameter

\[ HE_{11}^x \] lowest-order mode \(x\)-polarized

\[ HE_{11}^y \] lowest-order mode \(y\)-polarized

\[ Hz \] Hertz, cycles per second

\[ I \] identity matrix

\[ IFOG \] interferometric fiber optic gyroscope

\[ IMO \] index-matching oil

\[ IMU \] inertial measurement unit

\[ IO \] integrated optic

\[ IR&D \] internal research and development funding

\[ I_1, I_2, I_3, I_4 \] intensity at coupler input/output ports

\[ J_0, J_1 \] Bessel functions

\[ k \] intensity coupling ratio

\[ K \] matrix describing coupler differential equation

\[ k_o \] free-space propagation constant

15
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_0$, $K_1$</td>
<td>Modified Bessel functions</td>
</tr>
<tr>
<td>$L$, $L_1$, $L_2$</td>
<td>length of a piece of fiber</td>
</tr>
<tr>
<td>$L_B$</td>
<td>beat length</td>
</tr>
<tr>
<td>$L_c$</td>
<td>coupling length</td>
</tr>
<tr>
<td>$L_d$</td>
<td>depolarization length</td>
</tr>
<tr>
<td>$L_e$</td>
<td>effective length</td>
</tr>
<tr>
<td>$L_f$</td>
<td>length of applied force</td>
</tr>
<tr>
<td>$L_v$</td>
<td>coherence length</td>
</tr>
<tr>
<td>$M$</td>
<td>number of layers in a fiber coil</td>
</tr>
<tr>
<td>$M$</td>
<td>general Jones matrix</td>
</tr>
<tr>
<td>MEK</td>
<td>methyl ethyl ketone</td>
</tr>
<tr>
<td>MGD&amp;T</td>
<td>miniature gyro development and test</td>
</tr>
<tr>
<td>MFD</td>
<td>mode field diameter</td>
</tr>
<tr>
<td>$M_P$</td>
<td>Jones matrix of polarizer</td>
</tr>
<tr>
<td>$M_{PM}$</td>
<td>Jones matrix of PM fiber</td>
</tr>
<tr>
<td>$M_{PZ}$</td>
<td>Jones matrix of PZ fiber</td>
</tr>
<tr>
<td>$M_R$</td>
<td>Jones matrix of linear retarder</td>
</tr>
<tr>
<td>$M_S$</td>
<td>Jones matrix of PM fiber splice</td>
</tr>
<tr>
<td>$M_{\Omega}$</td>
<td>Jones matrix of circular retarder</td>
</tr>
<tr>
<td>$n$</td>
<td>refractive index</td>
</tr>
<tr>
<td>$N$</td>
<td>number of turns per layer in a fiber coil</td>
</tr>
<tr>
<td>NA</td>
<td>numerical aperture</td>
</tr>
<tr>
<td>$n_{\text{clad}}$</td>
<td>fiber cladding refractive index</td>
</tr>
<tr>
<td>$n_{\text{core}}$</td>
<td>fiber core refractive index</td>
</tr>
<tr>
<td>Nd</td>
<td>elemental Neodymium</td>
</tr>
<tr>
<td>$n_{\text{eff}}$</td>
<td>effective index</td>
</tr>
<tr>
<td>$n_o$</td>
<td>external medium refractive index</td>
</tr>
<tr>
<td>nm</td>
<td>nanometer unit</td>
</tr>
<tr>
<td>$n_{\text{SAP}}$</td>
<td>stress-applying-part refractive index</td>
</tr>
<tr>
<td>$n_{25}$</td>
<td>matching oil refractive index at standard conditions</td>
</tr>
<tr>
<td>$n(x,y)$</td>
<td>refractive index distribution</td>
</tr>
<tr>
<td>$n_x$, $n_y$</td>
<td>refractive index in birefringent media</td>
</tr>
<tr>
<td>$n_1$, $n_2$, $n_3$, $n_4$, $n_5$, $n_6$</td>
<td>refractive index perturbations in ellipsoid</td>
</tr>
<tr>
<td>$P_{11}$, $P_{12}$</td>
<td>elasto-optic coefficients</td>
</tr>
<tr>
<td>OCB</td>
<td>optical contact bond, optical contact bonded</td>
</tr>
<tr>
<td>$P$</td>
<td>polishing pressure, optical power</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$P_{\text{core}}$</td>
<td>optical power guided in the core</td>
</tr>
<tr>
<td>$P_{\text{clad}}$</td>
<td>optical power guided in the clad</td>
</tr>
<tr>
<td>PM</td>
<td>polarization maintaining, polarization-preserving</td>
</tr>
<tr>
<td>PMF</td>
<td>polarization-maintaining fiber</td>
</tr>
<tr>
<td>$P_{\text{r}}$</td>
<td>the element praseodymium</td>
</tr>
<tr>
<td>$P_{\text{Z}}$, $P_{\text{ZF}}$</td>
<td>polarizing fiber, single-polarization fiber</td>
</tr>
<tr>
<td>$Q$</td>
<td>matrix containing the eigenvectors</td>
</tr>
<tr>
<td>$r$</td>
<td>radius, as in cylindrical coordinates, reflectivity</td>
</tr>
<tr>
<td>$R$</td>
<td>radius, reflectivity</td>
</tr>
<tr>
<td>$r_b$</td>
<td>spherometer ball bearing diameter</td>
</tr>
<tr>
<td>$r_c$</td>
<td>core radius for step-index fiber</td>
</tr>
<tr>
<td>RFOG</td>
<td>resonant fiber optic gyroscope</td>
</tr>
<tr>
<td>RIP</td>
<td>refractive index profile</td>
</tr>
<tr>
<td>RLG</td>
<td>ring laser gyroscope</td>
</tr>
<tr>
<td>RMS</td>
<td>root-mean-square</td>
</tr>
<tr>
<td>$R(\theta)$</td>
<td>rotation matrix</td>
</tr>
<tr>
<td>$s$</td>
<td>slow axis designation</td>
</tr>
<tr>
<td>$S$</td>
<td>general scattering matrix</td>
</tr>
<tr>
<td>$\bar{S}$</td>
<td>global scattering matrix</td>
</tr>
<tr>
<td>SAP</td>
<td>stress applying part</td>
</tr>
<tr>
<td>SOP</td>
<td>state of polarization</td>
</tr>
<tr>
<td>SM</td>
<td>single-mode</td>
</tr>
<tr>
<td>SR</td>
<td>splitting ratio, coupling ratio</td>
</tr>
<tr>
<td>SSD</td>
<td>subsurface damage</td>
</tr>
<tr>
<td>STC</td>
<td>standard test conditions</td>
</tr>
<tr>
<td>STM</td>
<td>scanning tunneling microscopy</td>
</tr>
<tr>
<td>$\text{SiO}_2$</td>
<td>silicon dioxide - glass in this context</td>
</tr>
<tr>
<td>$t$</td>
<td>time, transmission coefficient</td>
</tr>
<tr>
<td>$T$</td>
<td>transformation matrix</td>
</tr>
<tr>
<td>$u$</td>
<td>transverse propagation constant in core</td>
</tr>
<tr>
<td>USAF</td>
<td>United States Air Force</td>
</tr>
<tr>
<td>USASDC</td>
<td>United States Army Strategic Defense Command</td>
</tr>
<tr>
<td>UV</td>
<td>ultraviolet</td>
</tr>
<tr>
<td>$V$</td>
<td>fiber V-number</td>
</tr>
<tr>
<td>$V_L$</td>
<td>average polishing velocity</td>
</tr>
<tr>
<td>$V_3$</td>
<td>equivalent v-number for oil drop test</td>
</tr>
</tbody>
</table>
\( v \) transverse propagation constant outside core
\( w \) width of fiber coil

**WDM** wavelength division multiplexing

**X** \(_1\), **X** \(_2\) eigenvectors

**x**, **y**, **z** Cartesian coordinates

\( x_0 \) distance from fiber surface to core centerline
\( x_s \) spherometer x-offset
\( y_e \) width of polished fiber ellipse
\( y_o \) half-coupler transverse offset
\( y_s \) spherometer ball bearing to centerline distance
\( y_c \) half-coupler longitudinal offset

Å Angstrom unit

\( \alpha \) amplitude attenuation coefficient
\( \alpha_{dB} \) amplitude attenuation coefficient in decibels
\( \alpha_x, \alpha_y \) amplitude attenuation coefficient in s and f mode

\( \beta \) propagation constant along fiber

\( \beta_A, \beta_B \) propagation constant of fiber A and B

\( \beta_x, \beta_y \) propagation constant in slow and fast axes

\( \delta \) normalized intensity dip depth \( I_{\text{min}}/I_{\text{max}} \)
\( \delta\phi \) circular retardance

\( \Delta \) normalized core/cladding index difference

\( \Delta\beta \) propagation constant difference between f and s

\( \Delta\lambda \) source spectral bandwidth

\( \Delta\phi \) linear retardance

\( \Delta\nu \) source frequency bandwidth

\( \gamma \) intensity fractional loss

\( \gamma_{dB} \) decibel fractional loss

\( \Gamma(\zeta) \) power spectrum of birefringence perturbations

\( \lambda_o \) free-space optical wavelength

\( \lambda_c \) fiber cutoff wavelength

\( \Lambda \) diagonal matrix containing eigenvalues

**\( \Lambda_1, \Lambda_2 \)** individual eigenvalues

\( \mu m \) micrometer

\( \eta \) polarizer transmission, y-axis

\( \nu \) optical frequency

\( \omega \) angular optical frequency
$\Omega_L$ polishing wheel rotation rate
$\phi$ phase shift
$\Phi$ principal axes rotation due to perturbations
$\pi$ pi constant $3.14159...$
$\varphi$ phase shift
$\rho$ amplitude transmission coefficient
$\rho_a$ transmission coefficient for antisymmetric mode
$\rho_s$ transmission coefficient for symmetric mode
$\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6$ stresses in $x,y,z$
$\theta$ general rotation angle
$\theta_a$ angle of analyzing polarizer transmissive axis to fiber slow axis
$\theta_A$ angle of fiber A slow axis to polished surface
$\theta_B$ angle of fiber B slow axis to polished surface
$\theta_p$ angle of launch polarizer transmissive axis to fiber slow axis
$\theta_s$ angle of squeezer line of compressive force to fiber slow axis
$\epsilon$ differential symmetric/antisymmetric normal mode loss
$\xi$ fractional polarization cross-coupling or extinction ratio
$\xi_{dB}$ decibel polarization cross-coupling or extinction ratio
$\zeta$ spatial frequency (of perturbations)

Several brand names are mentioned in this writing, and the author is mentioning them for informative purposes, not as an endorsement of a particular product or vendor. Cab-o-Sil is a trademark of the Cabot Corp., Epo-Tek is the trademark of Epoxy Technology Inc., Syton is the trademark of Monsanto Corp., and Pyrex is the trademark of Corning Inc. My apologies for any trademark that has not been acknowledged.
Fiber-optic directional couplers are one of the fundamental subcomponents for guided-wave sensors and communications. The function of a coupler is to divide or combine two or more optical guided waves at a given ratio, called the splitting ratio, which is denoted by $k$. Communications uses for couplers include local area networks, long-haul communication, cable television, optical amplifiers, and wavelength-division multiplexed (WDM) systems. The measurands sensed with fiber-optic sensors include rotation; acceleration; chemical composition; strain; and acoustic, magnetic, and electric fields. Other important applications for coupler technology include medical, signal processing, metrology, and are the basis for devices such as polarizers. In this dissertation, the fabrication of polished fiber-optic couplers and resonant cavities will be described. Matrix-based coupler theory and a formalism will be introduced to facilitate the modeling of complex fiber optic assemblies.

1.1 Fiber Optic Couplers

Several approaches have been demonstrated to realize single-mode (SM) fiber coupling [1] with varying degrees of success. Types of couplers include micro-optic, integrated optic, etched, spliced, mixer-rod, fused biconical taper, and polish-and-fuse. Of these techniques, polished, SiO$_2$ integrated optic (IO), and fused biconical taper have matured to commercialization, and are shown in Figure 1-1.

![Figure 1-1. Fiber-Optic Coupler Technologies.](image)

The polished variety of coupler, which is the subject of this dissertation, is fabricated by side polishing the fiber, which has been embedded in a substrate. This
polishing removes somewhat less than one-half of the cladding, stopping just short of the core. Two such polished fibers are assembled such that the core regions are overlapping, which enables the light to couple from one fiber to another.

Fused and IO couplers can be mass produced at low cost, which is important for communications applications. One technique used to make fused couplers is by twisting a pair of stripped fibers and applying heat while pulling the pair into a biconic taper shape. Properly packaged, fused couplers have demonstrated environmental ruggedness with moderate losses [2, 3]. Other advantages of fused and IO couplers include simple implementation of multi port topology, WDM, and polarization-selective couplers. Some recent improvements to the fused coupler are the hybrid polished-fused approach [4] and the multiple-index coupler design [5]. Further evidence of the maturity of coupler technology is the numerous commercial vendors, annual conferences, and the introduction of industry-wide standards.

IO devices are fabricated out of planar materials, for example, glass for passive devices or lithium niobate to make use of the electro-optic effect. Planar waveguides can be fabricated by several techniques, dependent on the material and device: oxidation of silicon, titanium indiffusion, and silver ion-exchange are used to raise the refractive index of the surface slightly. Standard microlithographic techniques are used to delineate patterns on photoresist, and the waveguides are prepared by lift-off or etching techniques. IO technology has the advantage of enabling higher-level integration, but requires the joining of fibers to the wafer for a practical coupler. Considerable work has been expended to develop a rugged, low-loss interface between the IO chip and fiber pigtales.

Since the polished coupler was first demonstrated in 1980 at Stanford [6], it has been the technology of choice for experimenters. Polishing is the only coupler technology that can realize variable coupling, which is often important in breadboard and prototype development. Beyond that, fused couplers are typically selected for later stages of development and for fielded systems. The polished fiber/substrate, also called half-coupler, is also the basis for a family of devices based upon coupling to thin films, such as polarizers, phase modulators, and spectrometers. The advent of polarization-maintaining (PM) optical fibers for interferometric sensors necessitated the development of a PM coupler, a device for which the polishing process is ideally suited.
While polished couplers have an edge in performance, they have been less successful in meeting the environment of military and commercial applications than its fused and integrated optical counterparts. It is also difficult to realize anything more than a 2 × 2 coupler with polished technology. It is worth noting that the cost is roughly the same for PM couplers of all three types, on the order of $2000. It is the author's opinion that this is largely due to the low volume of PM couplers needed by the market. Fused single-mode couplers are more common, and their costs are roughly a factor of ten less expensive than their PM counterparts.

1.2 Polished 2 × 2 Fiber Couplers

The basics of the process for fabricating polished coupler devices have changed little since the first devices were demonstrated. Figure 1-2 outlines the process steps for the polished coupler.

Figure 1-2. Polished Coupler Process.

The processing starts by preparing a glass block having a curved groove cut in it, having a large radius of curvature of about 30 centimeters. A short section of the fiber is
stripped of its protective jacket, and is carefully bonded into the groove with an adhesive. In the case of PM fiber, an alignment step precedes the adhesive application to orient the fiber axially with respect to the polished surface.

Once the adhesive is fully cured, a rough grind removes the bulk of the material in a rapid manner. By changing to finer and finer polishing compounds, a highly polished finish is achieved, which permits optical propagation through the fiber with low losses. Some of the guided optical energy will propagate in free space near the polished surface, if polished to within a few microns of the core boundary. Two such half-couplers are cleaned and assembled such that the optical field overlaps the second fiber, which permits coupling to occur. Various techniques are used to hold a desired splitting ratio, either to a fixed or adjustable value. Several review articles discuss polished coupler technology [7-10]. The interested reader is also referred to a series of conference proceedings that are also available [11-18].

1.3 Polished Fiber Ring Resonators

The first ring resonator for rotation sensing was fashioned out of a precisely-aligned set of bulk-optical mirrors [19]. Subsequently, guided-wave resonators were fashioned out of a spliceless piece of optical fiber [20] with an integral polished coupler as shown in Figure 1-3. These were developed shortly after the first polished fiber coupler was demonstrated [6].

![Figure 1-3. Fiber-Optic Reflection-Mode Resonator.](image-url)
These initial resonant cavities were made from SM coupler, and required low-loss couplers with nearly 100 percent splitting ratio. In this type of coupler, nearly all of the light launched into port 1 is coupled to port 4 and is observed as a large background intensity. Any light in the loop was coupled efficiently from port 2 to port 3 and circulated with low loss. When the optical path length of the loop becomes an integer multiple of the wavelength of the light, the circulating intensity builds up to a large steady-state value. Simultaneously, the output intensity is seen to be reduced and a resonant "dip" is observed in the output intensity. If γ is the round-trip loss, conservation of energy requires that the output signal be zero at resonance if $k + \gamma = 1$.

The resonator has a transfer function similar to a classical Fabry-Perot interferometer operated in the reflection mode, hence, the "reflection" designation. Addition of a second coupler in the loop constitutes a transmission resonator, whose output is inverted from that in Figure 1-3. The original impetus for developing resonant cavities toward a resonant fiber-optic gyroscope (RFOG). There was little commercial activity throughout the 1980s, as the RFOG was surpassed by ring laser gyroscope (RLG) and interferometric fiber-optic gyroscope (IFOG) technology.

Generally speaking, less government funding has been available for RFOG than IFOG research. More recently, there has been a resurgence of interest in RFOG technology, with several avionics and photonics firms performing studies under government contract or internal funding. Table 1-1 summarizes the past and present commercial activities the author is aware of to date.

Table 1-1. Industrial Fiber-Optic Resonator Gyroscope Research.

<table>
<thead>
<tr>
<th>Company</th>
<th>Project Description</th>
<th>Funding</th>
</tr>
</thead>
<tbody>
<tr>
<td>C. S. Draper Lab Inc.</td>
<td>MGD&amp;T [21]</td>
<td>USASDC</td>
</tr>
<tr>
<td>C. S. Draper Lab Inc.</td>
<td>Solid-State IMU [22]</td>
<td>USASDC</td>
</tr>
<tr>
<td>C. S. Draper Lab Inc.</td>
<td>Cryogenic RFOG [23]</td>
<td>USASDC</td>
</tr>
<tr>
<td>Honeywell Inc.</td>
<td>Dual-Pol. RFOG [25]</td>
<td>USASDC</td>
</tr>
<tr>
<td>Litton Guidance Systems</td>
<td>[26]</td>
<td>IR&amp;D</td>
</tr>
<tr>
<td>British Aerospace Ltd.</td>
<td>[27]</td>
<td>IR&amp;D</td>
</tr>
<tr>
<td>Tokyo Aircraft Ltd.</td>
<td>[28]</td>
<td>IR&amp;D</td>
</tr>
<tr>
<td>Mitsubishi Ltd.</td>
<td>[29]</td>
<td>IR&amp;D</td>
</tr>
<tr>
<td>Canadian Inst. Ltd.</td>
<td>[30]</td>
<td>Commercial</td>
</tr>
</tbody>
</table>
University research is summarized in Table 1-2; the numerous publications could not all be listed, but a representative paper has been cited. There has been several comparisons made of RFOG to IFOG technology [31-34] for different applications. This is still the subject of considerable debate [35], and the introduction of low-cost micromechanical gyroscopes adds a new factor in low-performance rotation sensing. Non gyro ring resonator applications have been investigated by a number of researchers, and are cataloged in Table 1-3.

Table 1-2. University Fiber-Optic Resonator Gyroscope Research.

<table>
<thead>
<tr>
<th>M.I.T.</th>
<th>RFOG, Brillouin [36]</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>U. of Tokyo</td>
<td>RFOG [37]</td>
<td>Japan</td>
</tr>
<tr>
<td>Strathclyde</td>
<td>RFOG [38]</td>
<td>Scotland</td>
</tr>
<tr>
<td>Tokyo Inst. of Technology</td>
<td>RFOG, Laser [39]</td>
<td>Japan</td>
</tr>
<tr>
<td>Stanford</td>
<td>IFOG, RFOG [40]</td>
<td>USA</td>
</tr>
<tr>
<td>University College London</td>
<td>RFOG [41]</td>
<td>England</td>
</tr>
<tr>
<td>Wilfrid Laurer Institute</td>
<td>RFOG [42]</td>
<td>Canada</td>
</tr>
<tr>
<td>Max Planck Institute</td>
<td>[43]</td>
<td>Germany</td>
</tr>
</tbody>
</table>

Table 1-3 Non Gyro Applications for Fiber-Optic Resonators.

<table>
<thead>
<tr>
<th>Accelerometer [44]</th>
<th>Laser Narrowing [45, 46]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrophone [47]</td>
<td>Diode Ring Laser [48, 49]</td>
</tr>
<tr>
<td>Spectroscopy [50]</td>
<td>Er ring Laser [51, 52]</td>
</tr>
<tr>
<td>Filtering [53]</td>
<td>Nd ring Laser [54]</td>
</tr>
<tr>
<td>Multiplexed Sensors [55]</td>
<td>Pr ring Laser [56]</td>
</tr>
<tr>
<td>FDM Systems [57]</td>
<td>Brillouin Laser [58]</td>
</tr>
<tr>
<td>Delay Line [59, 60]</td>
<td>Raman Laser [61]</td>
</tr>
<tr>
<td>Polarizer [66]</td>
<td>Soliton Ring Laser [67, 68]</td>
</tr>
<tr>
<td>Coupler Testing [69]</td>
<td>Discriminator [70]</td>
</tr>
<tr>
<td>Photon STM [71]</td>
<td>Dye Laser [72, 73]</td>
</tr>
<tr>
<td>Seismic Sensor [74]</td>
<td>Pulse Generator [75]</td>
</tr>
<tr>
<td>Spectrum Analyzer [53]</td>
<td>Frequency Shifter [76]</td>
</tr>
</tbody>
</table>
1.4 References


In this chapter, a matrix formalism will be developed to understand the mathematics of coupled waveguides. Coupled differential equations will be used to model the optical propagation through the coupler. Starting with scalar equations, the basic behavior of coupled waveguides will be examined; the polarization properties of the fiber and coupler will be ignored for now. This will then be generalized to include nonidealities, including a novel approach to incorporating differential symmetric/antisymmetric mode losses.

2.1 Coupled Waveguides

Before starting the coupler matrix analysis, we will briefly discuss some of the parameters which are used to describe fiber and couplers. It is assumed that the reader has a knowledge of single-mode (SM) fiber waveguides. There are numerous monographs on this subject [1-3], and there is no reason to repeat this analysis, except for some simple asymptotic expressions for modal parameters. It should be noted that this approach involves several assumptions, but is quite accurate over the usual range of operating conditions for SM fiber.

2.1.1 Assumptions and Conventions Used in This Chapter

Several common assumptions will be invoked: for example we will limit the discussion to circularly symmetric, step-index, weakly guiding, isotropic waveguides. For a small refractive index difference between the core and cladding, a great simplification of the calculations can be realized. The \( -i\omega t \) convention is used throughout this analysis, where \( \omega \) is the optical angular frequency, having units of radians per second. In this manner, increasing fiber length results in increasing phase and delay for positive propagation constant \( \beta \). The field will have \( z \) and \( t \) dependence of the form
\[ E(z,t) = E_0 e^{i(\beta z - \omega t - \phi)} \] (2.1)

where \( \phi \) is a constant phase factor, and \( E_0 \) is a constant amplitude factor. The angular frequency can be defined in terms of the free-space wavelength \( \lambda_0 \) as

\[ \omega = 2\pi \nu = \frac{2\pi c_0}{\lambda_0} \] (2.2)

where \( \nu \) is the optical frequency (on the order of \( 2 \times 10^{14} \) Hz for \( \lambda_0 = 1.3 \) \( \mu \)m), and \( c_0 \) is the speed of light in vacuum, which is approximately \( 3 \times 10^8 \) m/s. The propagation constant \( \beta \) can also be defined in terms of the free-space wavelength of the light

\[ \beta = n_{\text{eff}} \frac{2\pi}{\lambda_0} = n_{\text{eff}} k_0 \] (2.3)

where we have defined the free-space propagation constant \( k_0 \) in terms of \( \lambda_0 \). The effective index \( n_{\text{eff}} \) of the guided mode lies somewhere between \( n_{\text{clad}} \) and \( n_{\text{core}} \), whose exact value needs to be computed numerically.

Alternatively, the \( E_0 \) term in equation (2.1) can be a complex phasor to eliminate the need for the phase variable \( \phi \) as an exponent. \( E_0 \) implicitly contains the time- and \( z \)-independent transverse (x,y) dependence of the field. For this model, the exact spatial distribution information is not needed or desired for understanding the basic physics of the coupler.

2.1.2 Description of the Fiber

The fiber is made of pure SiO\(_2\), and is drawn to an outside diameter, or cladding diameter, of 125 \( \mu \)m. The central region of the fiber is typically doped with a small concentration of germanium, which raises the refractive index of this core region by a fraction of a percent. Typical core radius \( r_c \) is on the order of 3.5 \( \mu \)m; the fiber geometry for a circularly symmetric, step-index optical fiber can be summarized by the normalized frequency, often called the fiber V-number [4]:

\[ V \equiv \sqrt{u^2 + v^2} = k_0 r_c \sqrt{n_{\text{core}}^2 - n_{\text{clad}}^2} = k_0 r_c n_{\text{clad}} \sqrt{2\Delta} \] (2.4)
where \( \Delta = (n_{\text{core}} - n_{\text{clad}})/n_{\text{clad}} \), and \( k_0 \) was defined earlier. For the case of \( \Delta n/n \) being small, the V-number may be approximated as shown on the right-hand side of the equation. For the fiber to propagate only the lowest-order HE\(_{11}\) mode, the V-number must be below 2.405, which is the first zero of the \( J_0 \) bessel function.

The transverse modal parameter \( v \) can be estimated from the fiber V-number with

\[
v = 1.1428 V - 0.9960 \tag{2.5}
\]

This approximation has less than 1% error over the range of \( 1 < V < 3 \) [5]. \( v \) describes the extent of the field which propagates outside of the core region, the so-called evanescent field of the waveguide.

The term "single-mode" is somewhat of a misnomer, because there are two possible solutions to the differential equations that describe the waveguide. These orthogonal modes \( HE_{11}^x \) and \( HE_{11}^y \) are degenerate, i.e., will have equal propagation constants [4]. It can be shown that the effective index, and the propagation constants can be estimated with the help of equation (2.5) for the weakly-guiding fiber

\[
\beta_x = \beta_y = k_0 n_{\text{eff}} = \sqrt{(k_0 n_{\text{clad}})^2 + (v/r_c)^2} \tag{2.6}
\]

Other commonly used approximations that are useful in the SM regime, are the Gaussian mode approximation [6], and mode-field-diameter dependence on V-number [7].

2.1.3 Physical Description of Coupling

Consider a pair of optical fibers, capable of SM propagation with a portion of the field propagating outside of the core. If core of one optical fiber is brought into close proximity to the other fiber core, evanescent optical energy may be transferred from one core to another. Qualitatively speaking, it is the result of the overlapping of the two core wavefunctions, integrated over some volume. Figure 2-1 illustrates an ideal coupler which consists of two fibers, labeled A and B, having overlap region from \( z = 0 \) to \( z = L \). There are two input fields and two output fields, which will each look like equation (2.1),
and it is assumed that the coupling is zero outside of the overlap region. Consider the case with initial condition \( E_2 = 0 \):

![Single-Mode Coupler Block Diagram](image)

**Figure 2-1. Single-Mode Coupler Block Diagram.**

It is intuitively obvious that it is possible for the light propagating in fiber A to couple to fiber B, through evanescent coupling. Since the coupling is symmetric, one might expect that as coupling length increases, the coupled power to B saturates, reaching a condition of \( E_3 = E_4 \). However as \( z \) increases, more light gets coupled into fiber B, until almost 100% of the light is in B, which is not intuitively obvious. If the length \( L \) is increased further, the light that is coupled to B is found to be periodic in a sinusoidal fashion.

The physical mechanism for this phenomenon is the field in fiber A induces motion in the atomic dipoles in the dielectric waveguide B. These dipoles, in turn, radiate optical energy with a small phase shift added with respect to the forcing function. As this propagation continues down the fiber, the summation of these small phase shifts produces a differential propagation for the two "supermodes" of the waveguide structure. These two modes will interfere, producing a periodic coupling of the optical energy from A to B and back, as \( L \) increases.

### 2.1.4 Coupling Coefficient for Parallel Waveguides

The strength of the coupling between two waveguides is called the coupling coefficient \( C \), which is a real number and has units of inverse length. \( C \) is a function of the difference of the square of the core and cladding indices and the \((x,y)\) overlap integral of the eigenfunctions [8-10].
\[ C_{AB} = \pm \frac{\omega \varepsilon_0}{2} (n_{\text{core}}^2 - n_{\text{clad}}^2) \int_x \int_y E_A(x,y)^* E_B(x,y) \, dx \, dy \]  

(2.7)

where the integration is over the cross section surrounding guides A and B, and * denotes the complex conjugate. The fields are assumed to be described by scalar amplitudes, still neglecting polarization in this SMF analysis. The leading + is for propagation in the +z direction and the - is for the -z direction, which cancels the sign of the unit normal for the integration making C positive. Since C is a real number, A and B can be interchanged

\[ \int \int E_A^* E_B \, dx \, dy = \int \int E_B^* E_A \, dx \, dy \]  

(2.8)

which implies that \( C_{AB} = C_{BA} \) [8] and the 2 x 2 matrix describing the coupler transfer function is symmetrical. The subscripts to C are thus unnecessary and will be dropped for the rest of this analysis. Assuming weak guidance, weak coupling \( C \ll \beta \), and matched fibers, a simple expression for the coupling coefficient may be derived [11, 12]. The geometry is assumed to be two parallel fiber cores, whose centerlines are separated by a distance \( x_o \).

\[ C(x_o) = \frac{\lambda}{2\pi n_{\text{core}}} \frac{u^2}{r_c^2 v^2} \frac{K_0(v x_o/r_c)}{K_1^2(v)} \]  

(2.9)

\( K_0 \) and \( K_1 \) are modified Bessel functions of the second kind of order 0 and 1, respectively. We will use this definition for the C coefficient for all of the calculations in the following analysis. Figure 2-2 plots the theoretical coupling coefficient as a function of core-to-core spacing \( x_o \), as predicted in equation (2.9) for several core diameters with \( \Delta = 1\% \). Note that for smaller cores and closer cores, the coupling coefficient increases in an exponential manner.
For SM fiber the coupling coefficient is to first order independent of polarization, i.e., $C_{xx} = C_{xy}$ [13]. The wavelength dependence in the coupling coefficient can be exploited to fabricate wavelength-dependent couplers [14].

2.1.5 Curved Fibers and the Effective Length

Figure 2-3 illustrates a common polished coupler configuration, where the fibers are cemented into the bottom of a curved groove. If two identical fibers both have a circular fiber arc of radius $R$, and minimum spacing $d_0$ the separation $d(z)$ can be approximated by the sum of two parabolic equations for the case of large $R >> d(z)$.
Since the fibers are no longer parallel, the core separation, and hence $C$, is a function of $z$. For fibers that are perfectly aligned atop one another, it was shown [13] that an effective coupling length can be defined. This is defined as the length of equivalent parallel fibers with spacing $d_o$ that produces an equal splitting ratio to the curved coupler. This definition is written mathematically as an integral equation which is

$$C L_e = \int_{-\infty}^{\infty} C(z) \partial z$$

(2.11)

that may be solved for $L_e$. The coupling coefficient $C$ is for fibers with constant spacing $x_o = d_o$, and is written in equation (2.9). Combining this expression and (2.10), and then integrating along $z$ results in a very simple expression for effective interaction length

$$L_e = \sqrt{\frac{\pi r_e R}{2v}}$$

(2.12)

Note that $L_e$ is independent of $d_o$. Some values for $L_e$ for different bend radii are tabulated for two types of Fujikura birefringent fiber are listed in Table 5-1.

<table>
<thead>
<tr>
<th>TYPE</th>
<th>$r_e$</th>
<th>$\Delta n/n$</th>
<th>V</th>
<th>$v$</th>
<th>R</th>
<th>$L_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>High NA</td>
<td>3.0 (\mu)m</td>
<td>1 %</td>
<td>2.0</td>
<td>1.3</td>
<td>30 cm</td>
<td>1.0 mm</td>
</tr>
<tr>
<td>PM</td>
<td>3.5 (\mu)m</td>
<td>0.4 %</td>
<td>2.3</td>
<td>1.6</td>
<td>10 cm</td>
<td>0.6 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>30 cm</td>
<td>1.0 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>100 cm</td>
<td>1.8 mm</td>
</tr>
</tbody>
</table>

The calculated difference between the high NA fiber and the standard PM fiber is less than the accuracy of the approximations used. By adjusting the radius $R$, one can control the effective interaction length.
2.2 **Matrix Derivation of Coupling in Parallel Fibers**

An optical fiber coupler can be modeled with coupled differential equations, using the $C_{AB}$ coefficient derived in the previous section. The formalism for finding solutions to coupled differential equations will now be introduced, ignoring polarization. A "generating function" will be used to set up the differential equations which govern the coupling [15, 16]. The fields in the coupled waveguides are represented by a pair of complex numbers, and a 2-dimensional vector space is sufficient to model the amplitudes in the fibers. Refer to Appendix II, which is the companion Mathematica code for this analysis.

2.2.1 **Coupled Fiber Scalar Differential equation**

Refer to Figure 2-2, showing the idealized coupler, with both waveguides having equal propagation constants $\beta$. At any point $z$ along the coupler, the total field can be expressed as a $2 \times 1$ column vector containing complex coefficients that are of the form of equation (2.1). The vector will describe the phase and magnitude of the fields in fiber A and B as a function of $z$. Ignoring coupling effects for the moment the vector is

$$
\begin{bmatrix}
E_A(z) \\
E_B(z)
\end{bmatrix} =
\begin{bmatrix}
E_1 e^{i(\beta z - \omega t)} \\
E_2 e^{i(\beta z - \omega t)}
\end{bmatrix}
$$

where $E_1$ and $E_2$ are the initial conditions at $z = 0$ which is the field amplitude and phase entering ports 1 and 2 of Figure 2-2. As usual, the time-dependent terms will be dropped, and the differential equation describing the propagation and coupling between waveguide A and waveguide B is written as the sum of two matrices that operate on the fields [17]

$$
\frac{i \partial}{\partial z} \begin{bmatrix} E_A(z) \\ E_B(z) \end{bmatrix} = \begin{bmatrix} \beta_A & 0 \\ 0 & \beta_B \end{bmatrix} \begin{bmatrix} E_A(z) \\ E_B(z) \end{bmatrix} + C \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} E_A(z) \\ E_B(z) \end{bmatrix}
$$

(2.14)

The first term on the right-hand side of the differential equation is for the propagation in waveguides A and B. The $\beta$'s are the propagation constants for the two guides, as defined in equation (2.3). The rightmost term accounts for the coupling between the guides, with the coupling coefficient $C$ computed with equation (2.9). It is convenient to
write the matrix equation in shorthand notation, where bold characters represent matrices

\[
\frac{\partial}{\partial z} E(z) = (\beta I + K)E(z)
\]  \hspace{1cm} (2.15)

where \( \beta \) is the average propagation constant, and the matrix \( K \) is defined as

\[
K \equiv \begin{bmatrix} b & c \\ c & -b \end{bmatrix}
\]  \hspace{1cm} (2.16)

where \( b \) is half of the difference of the propagation constants of the two fibers.

\[
b \equiv (\beta_A - \beta_B)/2 \quad \text{and} \quad \beta \equiv (\beta_A + \beta_B)/2
\]  \hspace{1cm} (2.17)

Lower-case letters will be used as matrix coefficients, so the off-diagonal elements are rewritten as \( c = C \). Note the symmetry in the matrix, i.e., \( K = K^T \) (\( K \) transposed), since \( C_{AB} = C_{BA} \), as stated earlier. The differential equation (2.15) has an exponential solution

\[
E(z) = e^{i\beta z} e^{iKz} E(0)
\]  \hspace{1cm} (2.18)

The average phase shift \( e^{i\beta z} \) is dropped and will be added later as needed. The solution will be solved explicitly when we have an explicit expression of \( e^{iKz} \), which is generally a complex matrix. The exponential matrix is computed by diagonalizing \( K \) by computing

\[
\det (K - \Lambda I) = 0
\]  \hspace{1cm} (2.19)

For the \( 2 \times 2 \) case, the characteristic equation is quadratic and will have 2 roots

\[
(b - \Lambda)(-b - \Lambda) - c^2 = \Lambda^2 - c^2 - b^2 = 0
\]  \hspace{1cm} (2.20)

therefore there will be two eigenvalues found, that will be opposite in sign

\[
\Lambda_{1,2} = \pm \sqrt{c^2 + b^2}
\]  \hspace{1cm} (2.21)
Using the methods described in [18], the matrix $K$ can be factored into a form that contains a diagonal matrix $D$. The matrix $K$ is then said to be diagonalized

$$K = QDQ^{-1}$$  \hspace{1cm} (2.22)

The matrix $D$ contains the eigenvalues $\Lambda_1$ and $\Lambda_2$ along the diagonal

$$D = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix}$$  \hspace{1cm} (2.23)

and $Q$ is a matrix whose columns are the corresponding eigenvectors of $K$. The eigenvectors are solutions to the eigenvalue equations

$$KX_1 = \Lambda_1 X_1 \quad \text{and} \quad KX_2 = \Lambda_2 X_2$$  \hspace{1cm} (2.24)

By calculating the inverse of $Q$ and multiplying out $QDQ^{-1}$ one can calculate the $K$ matrix again. It will always be possible to compute $Q^{-1}$ because the eigenvectors $X_1$ and $X_2$ are orthogonal linearly independent vectors and nontrivial, which is written as

$$X_1 X_2^* = 0 \quad \text{and} \quad X_1^* X_2 = 0$$  \hspace{1cm} (2.25)

$$X_1 X_1^* \neq 0 \quad \text{and} \quad X_2 X_2^* \neq 0$$

If the eigenvectors are normalized such that their magnitude is unity, they are called orthonormal eigenvectors. In this case, the inverse of the normalized $Q$ has the identity that the inverse of an orthonormal matrix is the matrix transposed

$$Q^{-1} = Q^T$$  \hspace{1cm} (2.26)

In this case, equation (2.18) can be modified slightly and called the coupling matrix $A$

$$e^{Kz} \equiv A = e^{i(QDQ^T)z} = Q e^{Dz} Q^T$$  \hspace{1cm} (2.27)

since $D$ is diagonal, its exponential can therefore be written simply as
\[ e^{\lambda z} = \begin{bmatrix} e^{i\lambda_1 z} & 0 \\ 0 & e^{i\lambda_2 z} \end{bmatrix} \] (2.28)

Dropping the constant phase term, the resultant solution will then have the form

\[ E(z) = A E(0) \] (2.29)

where \( E(0) \) is the vector containing the input fields \[ \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \] and explicitly the solution is

\[ A = Q \begin{bmatrix} e^{i\lambda_1 z} & 0 \\ 0 & e^{i\lambda_2 z} \end{bmatrix} Q^T \] (2.30)

Explicit examples of this formalism are given in the following sections, and are computed using symbolic algebra in the Appendix II.

2.2.2 Amplitude Solution with Identical Fibers

To find the solution for the coupling matrix \( A \), starting from \( K \) with \( b = 0 \) for the case of identical fiber, the characteristic polynomial is calculated from equation (2.19)

\[ \text{det} (K - \Lambda I) = \Lambda^2 - c^2 = 0 \] (2.31)

the eigenvalues are immediately found to be \( \Lambda_1 = -c \) and \( \Lambda_2 = +c \), and the matrix \( D \) can be written with the two eigenvalues along the diagonal and zero elsewhere.

\[ D = \begin{bmatrix} c & 0 \\ 0 & -c \end{bmatrix} \] (2.32)

To find the eigenvectors, both of the eigenvalues are plugged into equation (2.24), and the two orthonormal eigenvectors are solved for and in this case are found to be

\[ X_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix} \] (2.33a)
\[ X_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \]  

Longhand calculation eigenvalues and eigenvectors is time consuming for matrices more complicated than this. Computer algebra packages are available that perform this computation in practice, and are used extensively in the Appendices. The \( Q \) matrix is assembled from the eigenvectors as column vectors, and its inverse is then derived

\[
Q = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \quad (2.34)
\]

\[
Q^{-1} = \frac{1}{\sqrt{2}} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \quad (2.35)
\]

Since \( Q \) is symmetric, orthonormality will then have the result \( Q^{-1} = Q^T = Q \).

The \( A \) matrix is computed by taking the product \( Qe^{Dz}Q^T \), which is shown to be

\[
A = e^{ikz} = \frac{1}{2} \begin{bmatrix} e^{icz} + e^{-icz} & e^{icz} - e^{-icz} \\ e^{icz} - e^{-icz} & e^{icz} + e^{-icz} \end{bmatrix} = \begin{bmatrix} \cos cz & i \sin cz \\ i \sin cz & \cos cz \end{bmatrix} \quad (2.36)
\]

This clearly illustrates the periodic nature of the coupling process. Note that 100% coupling occurs when \( z = \pi/2C \equiv L_k \) or a multiple thereof; this characteristic length is called the coupling length.

The imaginary off-diagonal coefficient implies that coupling from fiber A to fiber B has a relative +\( \pi/2 \) phase shift relative to fiber A. This is seen by recalling that \( i \equiv \exp(i\pi/2) \). In other words, the coupled wave lags the throughput wave by 90 deg, i.e., a quarter of a wave. This is consistent with our original field assumption, and is qualitatively understood as follows: the evanescent energy guided by fiber A perturbs the dipoles in fiber B in phase. The perturbation becomes the forcing function in the wave equation for guide B, and the resultant field in B lags the driving field by 90 deg.
Alternately, it is often desirable to define $A$ in terms of coupling ratio $k$. For a fixed, uniform coupling region of length $z = L_c$, we define the intensity splitting ratio $k \equiv \sin^2(CL)$.

Substituting this relation yields the following equivalent matrix for $A$:

$$
A = \begin{bmatrix}
\sqrt{1-k} & i \sqrt{k} \\
i \sqrt{k} & \sqrt{1-k}
\end{bmatrix}
$$

(2.38)

which describes the transfer function of the 4-port coupler in terms of a single parameter. Not only is the matrix $A$ symmetric because $C_{AB} = C_{BA}$, but for the lossless case, $A$ is also a unitary matrix.

2.2.3 Intensity Solution with Identical Fibers

The standard test conditions (STC) for a coupler as discussed in this text describes a certain set of initial conditions for equation (2.29). In this case, STC is defined as unity optical power is launched into port #1 (fiber A) of the coupler, and port #2 (fiber B) has zero illumination, the output fields can then be found by calculating the matrix algebra

$$
\begin{bmatrix}
E_3 \\
E_4
\end{bmatrix} = \begin{bmatrix}
\cos cz & i \sin cz \\
i \sin cz & \cos cz
\end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix}
\cos cz \\
i \sin cz
\end{bmatrix}
$$

(2.39)

The optical power exiting ports #3 and #4 from the coupler is the field times its conjugate

$$
P_3 = E_3E_3^* = \cos^2(cz) \quad \text{and} \quad P_4 = \sin^2(cz)
$$

(2.40)

Figure 2-4 plots the normalized intensity of the light in fiber A and fiber B as a function of $z$, for a coupling coefficient $C = 0.001 \, \mu m^{-1}$.

The splitting ratio $k$ of a coupler can be characterized from the measured power exiting fiber A and fiber B when operating the coupler under STC. Under these test conditions, the splitting ratio is calculated from the measured power values

$$
k = \frac{P_4}{P_3 + P_4}
$$

(2.41)
Any common factors, such as losses in the device, are cancelled out in the above expression. There is now agreed-upon procedures for measurement of fiber optic couplers [19], which describe in great details the full STC for testing.

![Graph showing I/I₀ vs z in um](image)

**Figure 2-4. Coupling vs Length.**

Periodic coupling is observed for increasing z, with maximum power coupled to fiber B at odd multiples of the coupling length. $L_k$ is given by $\pi/2C = 1570 \mu m$; note that power is conserved for all z.

2.2.4 Coupling Curves for Transverse and Longitudinal Offset

While the amount of cladding separating the fiber cores is fixed for a given coupler assembly, there are two degrees of freedom which may be adjusted. To obtain the desired splitting ratio, the transverse and longitudinal offset of the waveguides is adjusted. A cross section of the side-polished fibers as they are assembled in a variable coupler is shown in Figure 2-5. The vertical separation $x_o$ is set by the polishing depth, and is rarely changed after the device has been finished. The interface thickness is neglected in this figure, and that boundary condition is assumed to be perfectly matched, and the fibers are free to slide horizontally past one another. In a typical variable coupler, this $y_o$ motion is adjusted by a differential micrometer with submicron resolution. The distances $x_o$ and $y_o$ are both $<< R$. 

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The distance between the two cores $d(y_o)$ is calculated with the Pythagorean theorem:

$$d(y_o) = \sqrt{x_o^2 + y_o^2} \quad (2.42)$$

This result is plugged into equation (2.9) to give the coupling coefficient versus the offset

$$C(y_o) = \frac{\lambda}{2\pi n_{core}} \frac{u^2}{r_c^2} \frac{v^2 K_1(v)}{K_0} \left( \frac{v \sqrt{x_o^2 + y_o^2}}{r_c} \right) \quad (2.43)$$

and the splitting ratio is given by substituting into our original definition of $k \equiv \sin^2(C y_o)$:

$$k = \sin^2[C(y_o) L_c] \quad (2.44)$$

This produces the familiar coupling versus displacement curves as illustrated in Figure 2-6. This plot is for a device with a 1 mm interaction length using the standard PM fiber. The intensity starts with 100% of the light in the throughput leg (Port 3) when the fibers are widely spaced, i.e., zero coupling ratio. As the fiber cores come nearer, the coupling ratio periodically cycles between zero and unity.
As polishing reduces $x_0$, more and more cycles of the coupling curve as $y_0$ is scanned from $y_0 = 0$. Refer to Appendix II for coupling curves as a function of increasing polishing depth.

For comparison, the case of longitudinal adjustment in the $z$ direction is now examined. Figure 2-7 depicts the geometry of the two half-couplers in the centered and offset positions. The effect of longitudinal offset $z_0$ is to increase the affective separation of two cores over the original $x_0$ value.

The dashed line is an equivalent parting line with respect to the new axis of symmetry. The longitudinal offset is $z_0$, and the equivalent separation for $z_0 \neq 0$ is denoted...
as \( x_0 \). By constructing a right-angle triangle, it is seen with the Pythagorean theorem that

\[
\left( \frac{R + x_0}{2} \right)^2 + \left( \frac{z_0}{2} \right)^2 = \left( \frac{R + x_0}{2} \right)^2
\]

(2.45)

which can be expanded out and multiplied through by \( 1/R^2 \) which has, in the limit of \( x_0 < R \) and \( z_0 < R \), the following approximate result for the equivalent separation is

\[
x_0^\prime = x_0 + \frac{z_0^2}{4R}
\]

(2.46)

which is the equivalent separation of the fibers as a function of longitudinal offset. Combining this result with equation (2.42) permits computing the new separation \( d \) which is a function of \( y \)- and \( z \)-displacement of the two half-couplers. For the small offsets, it may be expressed as a correction factor to equation (2.42), shown on the right-hand side

\[
d(y_0, z_0) = \sqrt{x_0^2 + y_0^2} + \frac{z_0^2}{4R \sqrt{1 + y_0^2 / x_0^2}}
\]

(2.47)

By using the density plotting routines in Mathematica, it is possible to plot the coupling ratio as a function of longitudinal and transverse offset. In Figure 2-8, black corresponds to zero coupling and white is equivalent to a 100% splitting ratio value. The interested reader is also referred to [17, 20-25] for more details on coupled-mode theory.

2.3 Loss and Asymmetric Loss

Up until this point, the coupler has always been assumed to be lossless. We will now generalize the analysis to include effects of different kinds of loss mechanisms. Coupler losses impact the design of communications links, and distribution networks because this inefficiency reduces the SNR of the system proportional to \( (1 - \text{Loss}) \). In resonators, the effects of loss are more profound - the quality of the resonant cavity goes as \( 1/\text{Loss} \), so it is crucial to get low losses in these devices.
2.3.1 Coupler with Loss

If the coupler is made from fiber with distributed losses, there will be an exponential decay in the coupling curves. Due to the short length of fiber used in the coupler and the low intrinsic losses of SMF, this effect can be discounted. There is always some finite losses in the device due to scattering or absorption, for example, which make the transmission slightly less than unity.

The loss in couplers is often lumped as an amplitude transmission coefficient, and the Greek symbol \( \rho \) is again adopted to permit devices with less than unity transmission.

\[
\begin{bmatrix}
E_3 \\
E_4
\end{bmatrix} = \rho \begin{bmatrix}
\cos cz & i \sin cz \\
i \sin cz & \cos cz
\end{bmatrix} \begin{bmatrix}
E_1 \\
E_2
\end{bmatrix}
\]

(2.48)

The intensity loss in decibels is converted from the amplitude transmission coefficient by \( \text{loss} = 20 \log(\rho) \). To calculate the intensity loss as a fractional number, one computes \( \text{loss} = (1 - \rho^2) \), or \( \text{loss} = (1 - \rho^2) \times 100\% \) for percentage units. For STC in measuring the output power levels, the sum of the output intensities reduces to the form

\[
P_3 + P_4 = [\rho^2(1 - k) + \rho^2k]P_1
\]

(2.49)
and normalizing out the input power $P_1$ cancels $k$ and yields the transmission coefficient

$$\frac{P_3 + P_4}{P_1} = \rho^2$$

(2.50)

which can be converted to decibels by the conversion stated earlier. The power entering port $P_1$ is measured by the so-called cutback technique [26], which must be performed carefully for low-loss devices. Three power measurements, along with the above equation and equation (2.41) are sufficient to characterize $\rho$ and $k$ for a SM coupler under STC. Some sources for loss which would make $\rho$ less than unity would include:

(i) Grinding-induced defects and subsurface damage.
(ii) Scattering of light from imperfect polishing.
(iii) Contamination/outgassing of the index-matching oil.
(iv) Interface refractive index mismatch.
(v) Microbending of the fiber in the groove.
(vi) Excess offset in $y$ or $z$ from center position.

The loss is roughly periodic with coupling [27], but is approximately constant in the regions of zero and 100% coupling where resonators are made. For large $z$-offset, there can be increases of loss from the typically small values observed [28]. Champion polished couplers can be made with losses well below 0.1 dB.

2.3.2 Symmetric and Antisymmetric Supermodes

In coupled waveguides, the transverse distribution of the propagating field can be decomposed into two orthogonal "supermodes." These supermodes, or "normal modes" correspond to the two unique eigenvectors in equation (2.33) and have even and odd symmetry. The consequences of this will now be examined for couplers and resonators.

Figure 2-9a illustrates the field distribution in waveguide A ignoring the presence of the other. The opposite condition is depicted in Figure 2-9b for light propagating in B.
For the $2 \times 2$ case, the eigenvectors correspond to the two possible orthogonal combinations of the two waveguide modes. The first eigenvalue/eigenvector pair corresponds to the field $X_1 e^{i\Lambda_1 z}$, which is a solution of the differential equation (2.15). If this is multiplied by a scalar or a complex number, it will still be a valid solution to equation (2.15). So examining the first eigenvector having with its positive eigenvalue

$$\Lambda_1 = +c, \quad X_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$  \hspace{1cm} (2.51)

where the eigenvector has been normalized, the analogous supermode is described as the sum of the two individual waveguide modes and has a symmetric distribution, which is shown in Figure 2-10. This is called the symmetric supermode, and has even symmetry [29]. The graph is not an exact representation of the symmetric normal mode but is meant to illustrate the $E(x,0)$ distribution [24, 30].
The second solution is for the negative eigenvalue and its associated eigenvector

\[ \lambda_2 = -c, \quad X_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \]  

and corresponds to \( E_A - E_B \) as shown in Figure 2-11. This supermode has odd symmetry and is often called the antisymmetric mode.

The field in the coupler at any point can be thought of as the superposition of the two normal modes with the appropriate phase shifts and amplitudes [31, 32]. The coupling phenomenon can be thought of as the interference and beating of the two normal
modes, whose propagation constants are \( \beta - c \) and \( \beta + c \), and differ by the amount \( 2c \)

\[
E_s X_1 e^{iA_1 z} + E_a X_2 e^{iA_2 z}
\]  

(2.53)

\( E_s \) and \( E_a \) are the complex weighting factors, which are functions of the two basis waveguide fields. For identical fibers the fields can be decomposed as a linear sum

\[
\begin{bmatrix}
E_3 \\
E_4
\end{bmatrix} = \frac{E_s}{\sqrt{2}} \begin{bmatrix}
1 & 1 \\
1 & -1
\end{bmatrix} e^{i(\beta + c)z} + \frac{E_a}{\sqrt{2}} \begin{bmatrix}
1 & 1 \\
1 & -1
\end{bmatrix} e^{i(\beta - c)z}
\]

(2.54)

\[
= \frac{1}{\sqrt{2}} \begin{bmatrix}
e^{icz} & e^{-icz} \\
e^{icz} & -e^{-icz}
\end{bmatrix} \begin{bmatrix}
E_s \\
E_a
\end{bmatrix}
\]

(2.55)

The inverse normalized relation for the symmetric and antisymmetric supermodes is

\[
E_s \frac{1}{\sqrt{2}} (E_A + E_B) \quad \text{and} \quad E_s \frac{1}{\sqrt{2}} (E_A - E_B)
\]

(2.56)

\[
\begin{bmatrix}
E_3 \\
E_4
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
e^{icz} & e^{-icz} \\
e^{icz} & -e^{-icz}
\end{bmatrix} \begin{bmatrix}
1 & 1 \\
1 & -1
\end{bmatrix} \begin{bmatrix}
E_A \\
E_B
\end{bmatrix}
\]

(2.57)

which reduces to our result in equation (2.66). Some of the implications for polished couplers are summarized in Table 2-2, assuming positive \( C \) values.

Table 2-2. Supermode Characteristics.

<table>
<thead>
<tr>
<th>SUPERMODE</th>
<th>VELOCITY</th>
<th>CUTOFF</th>
<th>LOSSES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetric</td>
<td>Slower</td>
<td>Lower Cutoff</td>
<td>at Interface</td>
</tr>
<tr>
<td>Antisymmetric</td>
<td>Faster</td>
<td>Higher Cutoff</td>
<td>at Perturbations</td>
</tr>
</tbody>
</table>

For \( c > 0 \), the antisymmetric wave is faster than the symmetric wave. This is discovered by taking \( (\beta \pm c)z - \omega t = \) constant, and solving for velocity of the phasefronts. The symmetric wave has no cutoff for exactly matching waveguides. This property makes it less susceptible to waveguide perturbations than the antisymmetric case [29]. However, if there is absorption in the inter core region, there will be more losses.
experienced by the symmetric supermode. In a polished coupler, a physical explanation of differential mode losses would arise from scattering in the interfacial region.

2.3.3 Differential Symmetric/Antisymmetric Mode Losses

Consider the case where there is an absorbing layer in between the cores in Figure 2-10 at \( x = 0 \). Clearly, the symmetric mode will be attenuated more than the antisymmetric mode. For the amplitude transmittances \( \rho_s \) and \( \rho_a \) associated with the two modes, rewrite equation (2.54) with two transmission coefficients

\[
\begin{bmatrix}
E_3 \\
E_4
\end{bmatrix} = \rho_s \frac{E_s}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{i(\beta + c)z} + \rho_a \frac{E_a}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{i(\beta - c)z}
\] (2.58)

This can be rewritten in the form of an \( A \) matrix like (2.36) with different losses on the \( e^{+icz} \) and \( e^{-icz} \) components [29, 33]. The \( e^{\beta z} \) has been factored out.

\[
A = \frac{1}{2} \begin{bmatrix}
\rho_s e^{icz} + \rho_a e^{-icz} & \rho_s e^{icz} - \rho_a e^{-icz} \\
\rho_s e^{-icz} - \rho_a e^{icz} & \rho_s e^{icz} + \rho_a e^{-icz}
\end{bmatrix}
\] (2.59)

We define average transmission coefficient \( \rho \) and differential transmission coefficient \( \varepsilon \) in terms of the transmission coefficients

\[
\rho \equiv \frac{\rho_s + \rho_a}{2} \quad \text{and} \quad \varepsilon \equiv \frac{\rho_s - \rho_a}{2}
\] (2.60)

which permits recalculating \( A \) in terms of \( \rho \) and \( \varepsilon \) parameters. For \( \varepsilon = 0 \), the equation reduces to equation (2.36) with a single factor \( \rho \) in front. To make sure that power conservation holds in the system, \( \rho + |\varepsilon| \leq 1 \). For nonzero \( \varepsilon \), calculating \( A' \) we observe

\[
A' = \begin{bmatrix}
\rho \cos cz + i\varepsilon \sin cz & i\varepsilon \sin cz + \varepsilon \cos cz \\
\varepsilon \sin cz + \varepsilon \cos cz & \rho \cos cz + i\varepsilon \sin cz
\end{bmatrix}
\] (2.61)

or alternately, as the sum of the original coupling matrix plus an asymmetry matrix.
which can be compacted into matrix form by defining new matrix \( \mathbf{N} \) which contains \( \varepsilon \) equation (2.63) permits a particularly convenient technique for calculating asymmetry effects for arbitrary \( \mathbf{A} \) matrices for matched fibers. Qualitative effects of finite \( \varepsilon \) are (i) deviation from the ideal 90 deg phase shift in coupling, (ii) decrease of maximum coupling, and (iii) increase of minimum coupling. It is the author's experience that couplers with low loss exhibit a very small \( \varepsilon \) value because there is no scattering or perturbations to induce differential symmetric/antisymmetric normal mode losses.

2.3.4 Effects in 50/50 Splitting Ratio Couplers

The most common fiber coupler has a 50/50 splitting ratio, i.e., \( k = 0.5 \), and incorporating differential symmetric/antisymmetric effects, equation (2.62) reduces to

\[
\mathbf{A}' = \rho \left[ \begin{array}{cc} 1 + i\varepsilon/\rho & i(1 + i\varepsilon/\rho) \\ i(1 + i\varepsilon/\rho) & 1 + i\varepsilon/\rho \end{array} \right] \quad (2.64)
\]

In the limit of \( \varepsilon/\rho << 1 \), the magnitude of the matrix coefficients and the splitting ratio does not change. There is a small phase shift induced to the coefficients, approximately

\[
\phi = \tan^{-1} \left( \frac{\varepsilon}{\rho} \right) = \frac{\varepsilon}{\rho} \quad (2.65)
\]

These effects are not observable in typical coupler applications, and are usually not specified by manufacturers. In interferometric sensors, particularly resonators, the asymmetry affect becomes more important and is observed as an asymmetric interference effect.
2.3.5 Effects in Low- and High-Coupling Ratio Couplers

By representing the coefficients of the $A'$ matrix as phasors on the complex plane, the effects of differential supermode loss can be visualized. These phasors are shown in Figure 2-12, which are have a finite angle with the real and imaginary axes as shown.

\[
\begin{align*}
\text{Im} & \quad \varepsilon \sqrt{1 - k} \\
\rho \sqrt{k} & \quad \phi \\
\rho \sqrt{1 - k} & \quad \text{Re} \\
\end{align*}
\]

Figure 2-12. Phasor Representation of $A'$ Matrix Coefficient.

For the diagonal matrix elements of $A'$ the phasor will be rotated by an angle $\phi$, which is

\[
\phi = \tan^{-1} \left( \frac{\varepsilon \sqrt{k}}{\rho \sqrt{1 - k}} \right) \tag{2.66}
\]

Similarly, for the off-diagonal coefficients of the matrix, there is also a rotation

\[
\phi = \tan^{-1} \left( \frac{\varepsilon \sqrt{1 - k}}{\rho \sqrt{k}} \right) \tag{2.67}
\]

and corresponding magnitude change. The general matrix for the coupling matrix is

\[
A' \approx \rho \begin{bmatrix}
\frac{\varepsilon^{\phi}}{\cos \phi} \sqrt{1 - k} & \frac{i \varepsilon^{-\phi}}{\cos \phi} \sqrt{k} \\
\frac{i \varepsilon^{-\phi}}{\cos \phi} \sqrt{k} & \frac{\varepsilon^{\phi}}{\cos \phi} \sqrt{1 - k}
\end{bmatrix} \tag{2.68}
\]
In the limit of small $k$ and $\varepsilon/\rho << 1$, the factor $\phi = 0$, and the diagonal components are unaffected. However, since $\tan(\phi) \approx k^{-1/2}$, that coefficient may not be negligible for small coupling ratio devices. Still, for small $\phi$, the change in magnitude of the off-diagonal coefficient is negligible. However, a small rotation angle is introduced

$$A' = \rho \begin{bmatrix} \sqrt{1 - k} & i e^{-ik} \sqrt{k} \\ i e^{-\phi} \sqrt{k} & \sqrt{1 - k} \end{bmatrix} \quad k << 1 \quad (2.69)$$

In the case of high coupling ratio couplers, the situation is reversed with $\phi$ vanishing:

$$\phi = \frac{\varepsilon}{\rho \sqrt{1 - k}} \quad \text{and} \quad \phi = 0 \quad (2.70)$$

Multiplying the matrix through by $e^{-ik}$ results in a new matrix of the same form as (2.69)

$$A' = \rho \begin{bmatrix} \sqrt{1 - k} & i e^{-ik} \sqrt{k} \\ i e^{-ik} \sqrt{k} & \sqrt{1 - k} \end{bmatrix} \quad k = 1 \quad (2.71)$$

where the common phase factor has been dropped without any loss of generalization for $k$ near unity. Figure 2-13 illustrates the imperfect coupling for a device with differential symmetric/antisymmetric losses. The effect has been exaggerated for clarity.

![Figure 2-13. Intensity Behavior with Differential Normal Mode Losses.](image-url)
Figure 2-14 depicts the phase behavior of the device. The plot is of the deviation from the ideal π/2 phase shift experienced when coupling from fiber A to fiber B. The phase error for high and low splitting ratio devices is magnified as shown, resulting in asymmetric dips in resonant cavities.

![Phase Behavior Plot](chart.png)

Figure 2-15. Phase Behavior with Differential Normal Mode losses.

This phase error will be evidenced as an asymmetric resonance dip in a resonant ring assembly [29]. Dip asymmetry is a major error source in RFOGs that use these devices in a resonant cavity [34, 35]. As mentioned in Section 1.3, high-quality resonant cavities requires the splitting ratio to equal the losses. When using low-loss couplers and fiber in a resonator, it requires the splitting ratio to either be very near unity, or slightly above zero. For all else remaining constant, there does not seem to be an advantage of low or high splitting ratio on the loss asymmetry, for a given amount of loss. The differential normal mode losses can be measured by launching light of known phase into both ports 1 and 2 at different phases [36].
2.5 References


Chapter III

Polarization-Preserving Coupler Theory

In this chapter, the theoretical methods introduced in Chapter 2 for calculating the coupler will be expanded to include the effects of fiber birefringence. As a prelude to this, we will examine optical propagation in birefringent optical fiber, and also dichroic (polarizing) fiber. The Jones formalism will be introduced, and the Poincaré sphere will be used as a visual tool for understanding the state-of-polarization in birefringent media.

3.1 Wave Propagation in Birefringent Fiber Waveguides

Recall the discussion in Chapter II about perfectly isotropic SMF, with its two degenerate lowest-order modes. In the ideal case, since there is cylindrical symmetry, guided light with a certain state-of-polarization (SOP) will maintain that SOP as it propagates down the fiber. This would be analogous to optical transmission in free space or in strain-free isotropic bulk media. Unfortunately, in optical fiber there are numerous randomly distributed perturbations will cause the SOP to drift in an undeterministic manner [1]. This phenomena will cause contrast reduction in interferometric sensors when the polarization in the two arms of the interferometer drift to orthogonal states. Signal fading will be periodically observed in this situation for all but the most quiescent environments. Highly birefringent fibers have been most successful in eliminating this fundamental limitation for sensors based on interference, and have made the development of practical fiber-optic sensors possible.

3.1.1 Construction of Birefringent Optical Fibers

In order to reduce the polarization cross-coupling, it is necessary to break the degeneracy between the $HE_{11}^x$ and $HE_{11}^y$ modes by making $\beta_x \neq \beta_y$. One way to do this is by making $n_x \neq n_y$, and the fiber is said to be birefringent, polarization-preserving, or polarization-maintaining (PM). Birefringent fiber incorporates a permanent anisotropy, such that the random external perturbations are small in comparison. The birefringence $B$...
is defined as the difference of the index experienced by the two polarization modes

\[ B = n_x - n_y \]  

(3.1)

There have been several techniques demonstrated to generate anisotropy in optical fiber, most successfully with stress birefringence and geometric birefringence. For a broader introduction and bibliography, see the survey papers [2, 3]. The fibers used in this study are of the stress-birefringent variety; PM fiber based on geometric and material birefringence will not be discussed here, as they are not in wide use in sensors.

To generate a large intrinsic birefringence B via the elasto-optic effect, highly doped stress-applying-parts (SAPs) are incorporated in the fiber cladding. One particularly effective embodiment of a stress birefringence fiber is the PANDA design and shown in Figure 3-1. The SAPs are on the order of 20 μm and are doped sufficiently with Boron has a coefficient of linear expansion that is \( \approx 2 \times 10^{-6} \) per °C. This is roughly four times the expansion of the surrounding cladding material, which has a CTE \( \approx 5 \times 10^{-7} / °C \).

![Figure 3-1. PM Fiber Cross Section and Refractive Index Profile.](image)

The differential expansion coefficients between cladding and SAP will create stress when the fiber cools below the freezing point during the drawing process. SAPs want to shrink at a greater rate than the pure silica cladding, so there is tension created between the
SAPs. Symmetry demands that compressive stress be on the line bisecting the stress rods. There is a focusing effect in the stress distribution between the SAPs, and the stress field is approximately linear in the core region [4, 5].

The stress field, and the birefringence it introduces, has a symmetry that parallels the SAP geometry. What was the x-axis in Figure 2-1 is relabeled the s-axis, and the y-axis is now the f-axis. The slow and fast labels refer to relative wave velocity and their meaning will become clear shortly. Since the fiber, and the SAPs, can twist in general usage the (s,f) coordinate system can be at arbitrary angles with respect to the laboratory (x,y) coordinate system. Even mechanically twist-free fiber has been observed to have internal rotation of the SAPs [6]. This has been attributed to the fiber drawing process, and occurs as a result of the fiber "walking off" the drawing capstan.

3.1.2 Stress-Induced Birefringence in Fiber Waveguides

General anisotropic media having direction-dependent refractive index can be described by the index ellipsoid, alternately called the optical indicatrix [7]. The indicatrix for isotropic media is represented by a sphere, because the refractive index is independent of the direction of propagation, and in Cartesian coordinates is written as

$$\frac{x^2}{n^2} + \frac{y^2}{n^2} + \frac{z^2}{n^2} = 1$$  \hspace{1cm} (3.2)

where n is the refractive index of the material. Light propagating in the material will see the same n independent of the direction of the k vector. Of course, such a perfectly isotropic fiber does not exist, with many factors contributing to anisotropy and asymmetry [8]. Core noncircularity, coating imperfections, and fiber core undulations are some of the intrinsic sources of anisotropy, and tend to be randomly distributed along z. Extrinsic sources include bending, pinching, twist, tension, lateral pressure, the Faraday, and electro-optic Kerr effects. These effects tend to be time-varying as well, and are the cause of SOP drift and scrambling.

To account for sources of anisotropic index of refraction, the isotropic indicatrix in equation (3.2) is rewritten with n perturbed. The perturbations are denoted as $n_1...n_6$ and
the perturbed 3-dimensional ellipsoid is rewritten in the following general form

\[
\left( \frac{1}{n_0^2} + \frac{1}{n_1^2} \right) x^2 + \left( \frac{1}{n_0^2} + \frac{1}{n_2^2} \right) y^2 + \left( \frac{1}{n_0^2} + \frac{1}{n_3^2} \right) z^2 + \frac{2yz}{n_4^2} + \frac{2xz}{n_5^2} + \frac{2xy}{n_6^2} = 1
\] (3.3)

where the \((x,y,z)\) coordinate system has been retained, and the unperturbed index \(n\) has been relabeled \(n_0\). The \(n_1, n_2, n_3\) terms correspond to \(x, y, z\) perturbations, and \(n_4, n_5, n_6\) refer to perturbations that couple \(y\leftrightarrow z, x\leftrightarrow z,\) and \(x\leftrightarrow y\), respectively. The ordering is consistent to the Einstein convention, used for crystallographic, elasto-optic and electro-optic calculations[9]. The relation of the perturbations to stress is given by [9]

\[
\begin{bmatrix}
\frac{1}{n_1^2} & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{n_2^2} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{n_3^2} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{n_4^2} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{n_5^2} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{n_6^2}
\end{bmatrix}
\begin{bmatrix}
P_{11} \\
P_{12} \\
P_{12} \\
P_{12} \\
P_{12} \\
P_{12}
\end{bmatrix}
= \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\sigma_4 \\
\sigma_5 \\
\sigma_6
\end{bmatrix}
\]  (3.4)

The sign convention chosen for stress is that \(\sigma\) is positive for compressive stress, and negative for tensile stress. Equation (3.4) is multiplied out to yield the \(6 \times 1\) column vector

\[
\begin{bmatrix}
P_{11} \sigma_1 + P_{12} \sigma_2 + P_{12} \sigma_3 \\
P_{12} \sigma_1 + P_{11} \sigma_2 + P_{12} \sigma_3 \\
P_{12} \sigma_1 + P_{12} \sigma_2 + P_{11} \sigma_3 \\
\frac{1}{2} (P_{11} - P_{12}) \sigma_4 \\
\frac{1}{2} (P_{11} - P_{12}) \sigma_5 \\
\frac{1}{2} (P_{11} - P_{12}) \sigma_6
\end{bmatrix}
\]  (3.5)
where \( p_{11} \) and \( p_{12} \) are the optoelastic constants of silica. In the 1.3-\( \mu \)m wavelength region, 
\[
p_{11} - p_{12} = 3.17 \times 10^{-5} \text{ mm}^2/\text{kg} \quad [10].
\]

It will be assumed that the guided wave propagates in the +z-direction. The indicatrix is projected onto the x-y plane, and reduces to a 2-dimensional problem, where the \( \sigma_3 \) stress and the shear stresses \( \sigma_4 \) and \( \sigma_5 \) are all dropped from the calculation

\[
\left( \frac{1}{n_0^2} + p_{11} \sigma_1 + p_{12} \sigma_2 \right) x^2 + \left( \frac{1}{n_0^2} + p_{12} \sigma_1 + p_{11} \sigma_2 \right) y^2 + (p_{11} - p_{12}) \sigma_6 x y = 1
\]

The isotropic circle in the x-y plane will become an ellipse whose major and minor axes will be rotated by an angle \( \Phi \) with respect to the unperturbed coordinate axes. Figure 3-2 illustrates the ellipse and the unperturbed circle, which is drawn in a lighter shade.

![Figure 3-2. Index Ellipsoid Projected onto the x-y Plane.](image)

To solve for the indices, the above equation is transformed to a new coordinate system having symmetry with respect to the ellipse. The principal axes are relabeled \( s \) and \( f \)

\[
x = s \cos \Phi - f \sin \Phi \quad \text{and} \quad y = s \sin \Phi + f \cos \Phi
\]

(3.7)
The angle $\Phi$ is revealed by expanding out (3.6) with respect to the (s,f) coordinate system, and requiring that the s-f cross-terms vanish. That condition is satisfied when

$$(p_{12}\sigma_1 + p_{11}\sigma_2 - p_{11}\sigma_1 - p_{12}\sigma_2)\sin 2\Phi + (p_{11} - p_{12})\sigma_6 \cos 2\Phi = 0 \quad (3.8)$$

which can be reduced with some algebraic manipulation to yield the equation

$$\tan 2\Phi = \frac{\sigma_6}{\sigma_1 - \sigma_2} \quad (3.9)$$

When constraining the indicatrix with (3.9), essentially another eigenvalue problem is being solved for the symmetric coordinate system. The new orthogonal axes of symmetry (sometimes called principal axes) correspond to the eigenvectors. The major and minor axes of the ellipse correspond to the perturbed index of refraction values and are analogous to the eigenvalues. This graphical technique of eliminating cross-terms is analogous to the matrix diagonalization problem.

The ordering convention adopted means that $\sigma_6$ is associated with shear stress in xy, which is zero for the symmetric geometry of the SAPs. This has the important result

$$\Phi = 0^\circ \quad (3.10)$$

i.e., the (s,f) principal axes are not rotated from the original (x,y) system. The perturbed 2-dimensional ellipse subject to the above conditions is now written as

$$\left(\frac{1}{n_0^2} + p_{11}\sigma_1 + p_{12}\sigma_2\right)s^2 + \left(\frac{1}{n_0^2} + p_{12}\sigma_1 + p_{11}\sigma_2\right)f^2 = 1 \quad (3.11)$$

and can be redefined with respect to the new coordinate system as

$$\frac{s^2}{n_s^2} + \frac{f^2}{n_f^2} = 1 \quad (3.12)$$

Thus the orthogonal linear eigenmodes in the (s,f) coordinate system are called the slow and fast principal axes, and are parallel to the (x,y) system [11]. The slow and fast eigenmode propagates at velocity $c_0/n_s$ and $c_0/n_f$, respectively. Now it is written
\[
\frac{1}{n_s^2} - \frac{1}{n_o^2} = p_{11} \sigma_1 + p_{12} \sigma_2 = \frac{n_o^2 - n_s^2}{n_o^2 n_s^2} \quad (3.13)
\]

\[
= \frac{(n_o + n_s)(n_o - n_s)}{n_o^2 n_s^2} \approx \frac{2(n_o - n_s)}{n_o^3} \quad (3.14)
\]

where the approximation is for the condition that \(n_o \approx n_s\). Rearranging to solve for \(n_o - n_s\)

\[
n_o - n_s = \frac{n_o^3}{2} (p_{11} \sigma_1 + p_{12} \sigma_2) \quad (3.15)
\]

and similarly for the fast axis, the index perturbation is found to be

\[
n_o - n_f = \frac{n_o^3}{2} (p_{12} \sigma_1 + p_{11} \sigma_2) \quad (3.16)
\]

and solving for the birefringence which is the index difference

\[
B \equiv n_s - n_f = \frac{-n_o^3}{2} (p_{11} - p_{12})(\sigma_1 - \sigma_2) \quad (3.17)
\]

for small perturbations. Equation (3.17) and Figure 3-3 illustrate the functional relationship between birefringence and internal fiber stress.

Figure 3-3. PM Fiber Stress Detail and Refractive Index Profile.
Checking signs, the leading minus sign in equation (3.17) indicates that the compressive stress reduces the index of refraction. Since $\sigma_1 < 0$ and $\sigma_2 > 0$ as we have defined earlier, $\sigma_1 - \sigma_2 < 0$ and therefore $n_s > n_r$. From now on, we will choose the coordinate system so that it is lined up with the SAPs, such that $n_x = n_s$ and $n_y = n_r$. The $(x,y)$ subscripts will be used interchangeably with $(s,f)$ when referring to the fiber coordinate system.

3.1.3 Jones Matrix Representation of PM Fiber

As stated earlier, two coherent orthogonal linear polarization states, aligned with the x- and y-axes, correspond to the eigenvectors and propagate unchanged. They are called eigenstates of polarization (ESOP); the eigenvectors and their physical interpretation are

\[
\begin{bmatrix}
1 \\
0
\end{bmatrix} \leftrightarrow x\text{-pol} \quad \text{and} \quad \begin{bmatrix}
0 \\
1
\end{bmatrix} \leftrightarrow x\text{-pol}
\]  

(3.18)

which is defined as a SOP that propagates through a medium unaltered, except for a overall change in amplitude and phase. Any coherent SOP can be decomposed into a linear combination of the two ESOPs [11, 12].

Representation of arbitrary SOP is accomplished with Jones vectors, of which the two vectors on (3.18) are specific cases [13]. As implied in the above paragraph, general SOP is represented by a $2 \times 1$ vector with complex coefficients. Propagation through birefringent optical media is represented by a Jones matrix, which operates on the Jones vectors [14]. This formalism permits calculating the polarization behavior in complex optical systems, by taking the product of the individual Jones matrices of sub components. For the case of an ideal birefringent fiber, the Jones matrix has vanishing off-diagonal coefficients

\[
\begin{bmatrix}
E_x(z) \\
E_y(z)
\end{bmatrix} = \begin{bmatrix}
e^{i\beta_x z} & 0 \\
0 & e^{i\beta_y z}
\end{bmatrix}\begin{bmatrix}
E_x(0) \\
E_y(0)
\end{bmatrix}
\]  

(3.19)

Either ESOP will be transmitted unaffected through the system, i.e., linear SOP is preserved if it is aligned to either of the fiber's principal axes. A misaligned linear polarization will be transformed into a generally elliptical SOP. This is best visualized with
the Poincaré sphere, shown in Figure 3-4, which is a conformal mapping technique of a general SOP onto a spherical surface [15]. The equator of the sphere corresponds to linear SOP, and the North and South poles refer to left- and right-hand circular polarization, respectively. In this manner, the latitude is proportional to the ellipticity, and the longitude is the azimuth or angle of the major axis of the general SOP. x- and y-polarization states lie opposite one another on the equator, as do -45° and +45° linear polarization. Appendix III describes the Poincaré sphere and techniques for calculating and plotting it.

Figure 3-4. State-of-Polarization in PM Fiber with Linear Polarization Input.

The above figure is an example of a Poincaré sphere representation of optical propagation in an ideal birefringent fiber, which displays the SOP for different initial conditions. Linear polarization is assumed to be launched into the fiber at various angles $\theta_p = 0°, 5°, 10°, 20°, 30°,$ and $45°$ with respect to the to the x-axis. The SOP is calculated
with equation (3.19), using a rotation matrix to calculate the initial x- and y-polarization field components.

\[
\begin{bmatrix}
E_x(0) \\
E_y(0)
\end{bmatrix} = \begin{bmatrix}
\cos \theta_p & -\sin \theta_p \\
\sin \theta_p & \cos \theta_p
\end{bmatrix} \begin{bmatrix}
1 \\
0
\end{bmatrix} = \begin{bmatrix}
\cos \theta_i \\
\sin \theta_i
\end{bmatrix}
\]

(3.20)

Note the single point at the horizontal (x) state, which is linear polarization with zero angular misalignment; this SOP does not change with increasing z. As the misalignment is increased, the loci of the SOP traces out progressively larger circles; the SOP varies in a periodic fashion. At the extreme case of 45° misalignment, equal power is launched in the x- and y-ESOP. The resulting output SOP varies from 45° linear, to left circular, to -45° linear, then to right circular polarization, and the cycle is repeated with increasing fiber length. The SOP is generally elliptical in varying degrees with an azimuth of ±45°. The SOP varies in a periodic fashion; Figure 3-5 illustrates snapshots of the SOP at different parts of the beat cycle.

![Figure 3-5. Polarization State Evolution Along the Fiber with \( \theta_p = 45° \).](image)

To put the Jones matrix in a more symmetric form, the common phase shift in the diagonal coefficients is often factored out. The \( n_o k_o z \) term then comes out front.
\[ M = e^{i\beta z} \begin{bmatrix} e^{ibz} & 0 \\ 0 & e^{-ibz} \end{bmatrix} \quad (3.21) \]

where \[ \beta = \frac{2\pi}{\lambda} n_0 \equiv \frac{\beta_x + \beta_y}{2} \quad (3.22) \]

and \[ b = \frac{2\pi B}{\lambda} \equiv \frac{\beta_x - \beta_y}{2} \quad (3.23) \]

It is convenient to introduce the term beat length, which is the length of PM fiber for the two linear polarization eigenmodes to have 2\(\pi\) phase slippage relative to one another. \(L_B\) is commonly specified by vendors, and is inversely proportional to the birefringence. The various parameters in terms of beat length are given by

\[ B = \frac{\lambda}{L_B} \quad \Delta\beta = \frac{2\pi}{L_B} \quad b = \frac{\pi}{L_B} \quad (3.24) \]

Typical high-birefringence fibers have a beat length of 2 mm, which corresponds to a birefringence value of \(B = 6.5 \times 10^{-4}\) at a wavelength of 1.3 \(\mu\)m.

In summary, fiber anisotropy permits decoupling of the two eigenmodes, by breaking their degeneracy. This is accomplished by raising the intrinsic birefringence such that external perturbations are small by comparison. In this manner, an eigenstate of polarization is relatively unaffected by changes in the environment, if it is aligned to the fibers principal axes. In other words, a linear SOP launched parallel to the fast or slow axis of the PM fiber will be preserved, even in the case of fiber perturbations.

3.1.4 Extrinsic Birefringence Sources

Rearranging equation (3.17), one can express the internal stress as a function of \(B\)

\[ \sigma_x - \sigma_y = \frac{-2 B}{n_0^3 (p_{11} - p_{12})} \quad (3.25) \]
which has the correct units of force per unit area. In terms of beat length, the internal stress can be calculated for a 2-mm beat length birefringent fiber to be a tensile stress of

\[
\sigma_x - \sigma_y = \frac{-2 \lambda / L_B}{n^3(p_{11} - p_{12})} = -13.5 \text{ kg/mm}^2
\]  

(3.26)

which should be numerically negative for SAPs with a larger CTE than cladding. The upper limit of birefringence in this type of fiber is held by the yield stress of silica, which is on the order of 25 kg/mm\(^2\). Because of the risk of breakage, fibers with \(L_B < 1\) mm are seldom made.

If a circular isotropic fiber with diameter \(D\) is squeezed with a uniform force \(F\) in the \(y\)-direction over length \(L_F\), the induced stress near the core is given by [16]

\[
\sigma_x - \sigma_y = -2F/\pi DL_F \quad (3.27)
\]

equating to equation (3.22) and rearranging to solve for force per unit length results in

\[
\left( \frac{F}{L_F} \right) = \frac{-B \pi D}{n^3(p_{11} - p_{12})} = \frac{-\pi \lambda D}{n^3(p_{11} - p_{12})L_B}
\]  

(3.28)

which is on the order of 3.6 kg/mm compressive force that would have to be applied along the \(y\)-axis to equal the birefringence induced by the SAPs for 2-mm beat length PM fiber.

Small amounts of birefringence can also be induced in fiber by bending, which can be significant in single-mode fiber. In the core region of a fiber bent in a loop, compression is in the plane of bending, which creates a local fast axis in the bending plane. The birefringence generated in a fiber loop with a bending radius \(R\) and cladding diameter \(D\) is quite small, and is approximately given by the expression [8]

\[
B = 0.135 \left( \frac{D}{2R} \right)^2
\]  

(3.29)

The fiber loop acts as a retarder having \(\Delta \beta L\) retardance, where \(L\) is the length of the loop(s). To make a quarter-wave retarder, there must be 90° retardance incurred.
\[ \Delta \beta L = \frac{2\pi}{\lambda} B (2\pi m R) = \frac{\pi}{2} \text{ radians} \quad (3.30) \]

The previous equation can be rearranged to solve for the desired radius for 90° retardance:

\[ \Rightarrow R_{90°} = 0.135 \frac{2\pi D^2}{\lambda} \quad (3.31) \]

which for \( m = 1 \) turn, works out to be roughly 1 cm bending radius for 1.3-μm wavelength. At this bending radius, the bend-induced birefringence is roughly \( 5.3 \times 10^{-6} \), or about 100\( \times \) less than typical PM fiber. It can easily be seen that this type of bend will have little effect upon a PM fiber, because of the relatively small perturbation to the SAP-induced stress.

Quarter-wave and half-wave (made with \( m = 2 \) turns) retarders fabricated from non-PM fiber can be used to make polarization adjusters [17]. A series of two rotatable quarter-wave retarders can be used to transform arbitrary SOPs in an optical system. Several variants on this approach have been developed, including real-time polarization control with servo electronics.

It is also possible, by winding longer coils of SM fiber, to produce birefringent coils that preserve polarization with standard SM fiber. The random perturbations must be minimized, which requires extremely high-quality winding techniques and careful encapsulation. If loss can be avoided, the birefringence can be further increased by adding tension during the wind. Unfortunately, the SM fiber pigtails will tend to scramble the SOP, so PM fiber pigtails must be spliced onto the coil [18]. The principal axes of the PM fiber and the now-birefringent SM fiber are aligned, and the SM-to-PM splices are encapsulated within the coilform to reduce the sensitivity to the environment.

3.1.5 PM Fiber Cross-Coupling

In nonideal PM fiber, random \( \Delta \beta \) perturbations distributed along a birefringent fiber will cause a finite cross-coupling of the two orthogonally polarized modes. The statistical nature of the random fluctuations will determine the ultimate polarization-holding capability of the fiber [19]. For linear polarization launched on the x-axis, the amount of
light cross-coupled into the y-polarization will be an exponentially growing function. The notation for \(<P>\) denotes an ensemble average of the optical power for this measurement. The detailed analysis for PM fiber will not be presented here, but may be found in References [20] and [21]. For launching pure linear polarization in the x-axis of the fiber, the power coupled to the y-axis is a hyperbolic function of length

\[
\frac{<P_y>}{<P_x>} = \xi = \tanh(hL) \approx hL
\]  

(3.32)

where the extinction ratio \(\xi\) is defined as the ratio of the y-polarization to the x-polarization power. The approximation on the right-hand side of equation (3.32) is for small \(hL\) values.

The variable \(h\) is called the h-parameter and is a number often provided by PM fiber vendors. The h-parameter is measured by the fiber vendor under the most benign perturbations possible, and therefore represents the ultimate polarization-holding capability of a fiber. As an example, a 1-km fiber with \(h=10^{-6}/m\), one would expect an extinction ratio 0.001 or -30 dB. Typical fibers have an h-parameter in the range of \(10^{-5}\) to \(10^{-6}/m\). The h-parameter is convenient to estimate the ultimate polarization-holding capability of a fiber coil; however in actual use, the polarization performance is often dominated by the external perturbations [22].

If the birefringence perturbations are known, h-parameter may be calculated [21]

\[
h = \frac{k_0^2}{4} <|\Gamma(\zeta)|^2>
\]

(3.33)

where \( <|\Gamma(\zeta)|^2> \) denotes the power spectrum of the birefringent perturbations evaluated at spatial frequency \(\zeta\). It is difficult to quantify \( <|\Gamma(\zeta)|^2>\) for practical fibers, but qualitatively, it exhibits a low-pass type of dependence [23]. The cross-coupling can become large if the perturbations have the right periodicity, due to resonant coupling effects. This is only of concern if there is significant spectral content at \(\zeta = 1/L_B\), but can be exploited to make special devices [24].

For simplicity, the cross-coupling is often handled as a lumped coefficient, an approach commonly used in a number of models. A matrix is used to incorporate the mixing of the two eigenmodes that is equivalent to a small angle rotation matrix like that
used in equation (3.20), with the small angle approximation being $\theta = \sqrt{hL} \ll 1$

$$M = e^{ibz} \begin{bmatrix} 1 & -\sqrt{hL} \\ \sqrt{hL} & 1 \end{bmatrix} \begin{bmatrix} e^{ibz} & 0 \\ 0 & e^{-ibz} \end{bmatrix}$$

(3.34)

This is similar to the effect of a PM fiber splice with a small misalignment angle present.

3.1.6 Characterization Limitations and Depolarization Length

In typical characterization situations, the measured $\xi$ is much greater than would be suggested by the $h$-parameter alone, especially for short fiber lengths. This is due to limitations in lens and polarizer quality used in the characterization setup. In addition, unavoidable perturbations to the fiber and mode-field curvature place fundamental limitations on characterization of extinction ratio [25]. It is necessary to procure the highest-quality lenses and polarizers to measure the extinction ratio to a $-40$-dB level. For a detailed description of PM fiber characterization test equipment, calibration, and techniques, refer to [26].

When a highly coherent source is used to characterize PM fiber, there is considerable drift when the analyzer is rotated to the minimum (crossed) polarization state. This effect is due to interference of the cross-coupled light with the primary linear polarization. The measurement of PM fiber and components is greatly simplified by use of a broadband source [27, 28]. In this case, the $< >$ in equation (3.32) now has the meaning of a spectral average instead of a time average.

The so-called depolarization length is the length of fiber where the x- and y-polarization modes are phase shifted beyond the coherence length. For $L > L_d$, the two polarizations are mutually incoherent and will not interfere with each other. The length is

$$L_d = \frac{L_a L_c}{\lambda_0}$$

(3.35)

For an optical source having no multilongitudinal mode structure, and FWHM frequency width $\Delta\nu$ Hz, or FWHM spectral bandwidth $\Delta\lambda$, the coherence length is [29]
\[ L_c = \frac{c_0}{\Delta \nu} = \frac{\lambda_0^2}{\Delta \lambda} \]  

(3.36)

If one attempts to characterize a resonator with a poorly coherent source, the finesse of the resonator will be underestimated because of the insufficient coherence [30].

3.2 Wave Propagation in Single-Polarization Fibers

In an effort to improve fiber h-parameter, birefringent fibers have been developed that have a high loss in one axis [31]. Figure 3-6 plots attenuation as a function of bending radius for Fujikura polarizing (PZ) fiber.

![Figure 3-6. Fujikura PANDA PZ Fiber Bending Loss Curves (Reprinted with permission of Fujikura Ltd.).](image)

An important application for PZ fiber is the fabrication of fiber-optic polarizers, which reject a given linear SOP. High-quality polarizers are an important component of the IFOG, as it enforces reciprocity for the counterpropagating waves [32]. The RFOG uses polarizers outside of the cavity to reduce polarization cross-coupling induced errors,
and polarizers have been proposed to be incorporated inside the cavity [33]. A ring resonator with a polarizing cavity will operate with only one resonance, greatly reducing polarization errors.

3.2.1 PZ Fiber Loss Curves

These fibers work by trimming the spectral loss curves such that one mode is lossy and the orthogonal mode is guided with low attenuation. The fiber design can be optimized to yield a high degree of polarization for moderate lengths of fiber. Losses are strong functions of both bending diameter and wavelength, which allows flexibility in designing polarizers. In the case of PANDA fiber, the transmitting mode is the slow (x) eigenmode. From Figure 3-6, the attenuation coefficients can be estimated for this fiber to be

\[
\begin{align*}
\alpha_x &= 8.2 \times 10^{-3} \text{ dB/m} \\
\alpha_y &= 1.8 \text{ dB/m}
\end{align*}
\]

(\(\alpha_x = 1.9 \times 10^{-3} \text{ m}^{-1}\))

at a bending diameter of 75 mm. At this bending diameter, any light that is cross-coupled into the y-axis will be attenuated at a rate of 0.66/m. This effect is shown in the Poincaré sphere in Figure 3-7, for initial launching of a 45° oriented linear polarization state. Note that the SOP starts at the +45° linear polarized state on the equator, then spirals toward the x-polarization state.

3.2.2 Operation of PZ Fibers

A differential equation is used to describe the operation of the PZ fiber [34].

\[
\frac{\partial}{\partial z} \begin{bmatrix} E_x(z) \\ E_y(z) \end{bmatrix} = \begin{bmatrix} -\alpha_x + i\beta_x - ic & ic \\ -ic & -\alpha_y + i\beta_y - ic \end{bmatrix} \begin{bmatrix} E_x(z) \\ E_y(z) \end{bmatrix}
\]

(3.37)

and assuming that the random variable \(c < \beta_x - \beta_y << \alpha_x - \alpha_y\), permits writing the solution for the measured extinction ratio as a function of fiber length that looks like
Figure 3-7. SOP in Polarizing Fiber with 45° Linear Polarization Input.

\[
\xi = \frac{a^2 \cosh(2cbz) - 1 + ab \sinh(2cbz)}{a^2 \cosh(2cbz) - 1 - ab \sinh(2cbz)}
\]  \hspace{1cm} (3.38)

where the intermediate variables have been defined as:

\[
a \equiv \frac{\alpha_x - \alpha_y}{2c} \quad \text{and} \quad b \equiv \sqrt{a^2 - 1}
\]  \hspace{1cm} (3.39)
The theoretical behavior of the extinction ratio, in dB units, is plotted in Figure 3-8. The extinction ratio is seen to logarithmically rise with increasing fiber length, then saturates at a value of $20 \log(a)$, due to the finite polarization cross-coupling $c$.

![Graph](https://example.com/graph.png)

Figure 3-8. Predicted Extinction Ratio vs Length for PZ Fiber.

PZ fibers are by nature bend-sensitive and wavelength sensitive, i.e., $\alpha_x$ and $\alpha_y$ are sensitive to the fiber axial alignment $\theta$ to the bending plane [35]. Development of flat PZ fiber has attempted to improve the repeatability of coil winding [36]. PZ fiber is very sensitive to sharp bends and pressure, so a high degree of workmanship is required when making components out of, or characterizing this type of fiber. Experience has shown that it is very difficult to make polished couplers out of PZ fiber and that they have poor performance and reliability [37]. This rules out the possibility of fabricating a spliceless resonator, as shown in Figure 1-3, out of PZ fiber. To make a resonator out of this type of fiber requires a spliced assembly of a PZ fiber coil to a PM fiber coupler.

### 3.3 Matrix Derivation for Coupled Birefringent Fibers

The analysis in Chapter II is now expanded to include birefringent fiber by combining it with the Jones formalism. Scalars $E_A$ and $E_B$ are replaced with Jones vectors as described in the section on PM fibers. There are several differing approaches to this
type of analysis [38-40], but this analysis will continue to use the generating matrix approach that was so successful in the previous chapter.

3.3.1 PM Coupler Analysis

In the following analysis the fiber is assumed to be lossless, and the polarization cross-coupling in the fiber is assumed to be negligible. Figure 3-9 denotes a coupler similar to Figure 2-1, except that the scalar field inputs and outputs are replaced by vectors.

![PM Coupler Block Diagram](image)

Figure 3-9. PM Coupler Block Diagram.

The two coupled differential equations for the PM fibers are conveniently described by a set of coupled $2 \times 2$ matrices [41, 42].

$$\frac{i\hbar}{\partial z} \begin{bmatrix} E_{Ax}(z) \\ E_{Ay}(z) \end{bmatrix} = \begin{bmatrix} \beta_x & 0 \\ 0 & \beta_y \end{bmatrix} \begin{bmatrix} E_{Ax}(z) \\ E_{Ay}(z) \end{bmatrix} + \begin{bmatrix} c & d \\ -d & c \end{bmatrix} \begin{bmatrix} E_{Bx}(z) \\ E_{By}(z) \end{bmatrix}$$ (3.40)

The subscripts $x, y$ refer to the orthogonal linear polarization components of the in the two fibers $A$ and $B$. Assumptions implied include no loss or cross-coupling in the fiber, matching fiber, and no twist in the fibers. It is also assumed that the coupling is independent of SOP. There is a similar differential equation for fiber $B$, which is

$$\frac{i\hbar}{\partial z} \begin{bmatrix} E_{Bx}(z) \\ E_{By}(z) \end{bmatrix} = \begin{bmatrix} c & -d \\ d & c \end{bmatrix} \begin{bmatrix} E_{Ax}(z) \\ E_{Ay}(z) \end{bmatrix} + \begin{bmatrix} \beta_x & 0 \\ 0 & \beta_y \end{bmatrix} \begin{bmatrix} E_{Bx}(z) \\ E_{By}(z) \end{bmatrix}$$ (3.41)

The diagonal matrices with the $\beta$ coefficients are for the propagation of the light in the waveguides, as before. The coefficients $c$ and $d$ are the amplitude coupling coefficients. From examination of Figure 3-10 and applying trigonometry, one can derive $c$ and $d$: 82
Figure 3-10. Cross Section of Misaligned Fibers.

The convention used in this drawing is that $\theta_B$ is negative as measured from the datum:

$$\theta = \theta_A - \theta_B$$  \hspace{1cm} (3.42)

where $\theta$ is the relative angle between the principal axes of the two birefringent fibers. Thus

$$c = C \cos \theta$$  \hspace{1cm} (3.43)

$$d = C \sin \theta$$  \hspace{1cm} (3.44)

may be defined where $C$ is the coupling coefficient as calculated in equation (2.9). These expressions provide the rotation transformation as in equation (3.7); note that the lower left-hand $2 \times 2$ submatrix in the matrix below resembles the matrix in equation (3.20) [43]. The upper right-hand submatrix is an antirotation matrix, because the coupling is from fiber B to A. The matrix describing the coupled-mode equations can be rewritten as

$$\frac{i\partial}{\partial z} \begin{bmatrix} E_{Ax}(z) \\ E_{Ay}(z) \\ E_{Bx}(z) \\ E_{By}(z) \end{bmatrix} = \begin{bmatrix} \beta_s & 0 & c & d \\ 0 & \beta_f & -d & c \\ c & -d & \beta_s & 0 \\ d & c & 0 & \beta_f \end{bmatrix} \begin{bmatrix} E_{Ax}(z) \\ E_{Ay}(z) \\ E_{Bx}(z) \\ E_{By}(z) \end{bmatrix}$$  \hspace{1cm} (3.45)
where the basis has been expanded to a $1 \times 4$ vector space. The same approach is now followed with the $2 \times 2$ matrix for scalar couplers in Chapter II. If the average and $\Delta/2$ of the propagation constants for the fast and slow axes are defined to be equal to

$$
\beta \equiv (\beta_s + \beta_f)/2 \quad \text{and} \quad b \equiv (\beta_s - \beta_f)/2
$$

permits writing the coupled differential equations in an even more compact form

$$
\frac{dE}{dz} = (\beta I + K)E(z)
$$

where the $4 \times 4$ matrix $K$ is written in a symmetric form that looks like

$$
K = \begin{bmatrix}
    b & 0 & c & d \\
    0 & -b & -d & c \\
    c & -d & b & 0 \\
    d & c & 0 & -b
\end{bmatrix}
$$

where the vector $E$ contains the four field components

$$
E(z) = \begin{bmatrix}
    E_{Ax}(z) \\
    E_{Ay}(z) \\
    E_{Bx}(z) \\
    E_{By}(z)
\end{bmatrix}
$$

which has an exponential solution with the same form as during the scalar case

$$
E(z) = e^{i\beta z} e^{iKz} E(0)
$$

The same diagonalization procedure for $K$ is used as for the $2 \times 2$ case, with the aid of Mathematica software package [44]. Using the Mathematica routines in Appendix IV, two of the four eigenvalues are found to be opposite in sign, with the result that

$$
\Lambda_1 = -\Lambda_2 = -\sqrt{(b-c)^2 + d^2}
$$

$$
\Lambda_3 = -\Lambda_4 = -\sqrt{(b+c)^2 + d^2}
$$

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The eigenvalues go into the diagonal matrix $D$ which is written as

$$D = \begin{bmatrix}
 e^{i\Lambda_1 z} & 0 & 0 & 0 \\
 0 & e^{i\Lambda_2 z} & 0 & 0 \\
 0 & 0 & e^{i\Lambda_3 z} & 0 \\
 0 & 0 & 0 & e^{i\Lambda_4 z}
\end{bmatrix}$$  \hspace{1cm} (3.53)

and the eigenvectors are placed in column form in the $Q$ matrix

$$Q = \begin{bmatrix}
 1 & 1 & 1 & 1 \\
 \frac{-b+c+\Lambda_1}{d} & \frac{-b+c+\Lambda_2}{d} & \frac{b+c-\Lambda_3}{d} & \frac{b+c-\Lambda_4}{d} \\
 -1 & -1 & 1 & 1 \\
 \frac{-b+c+\Lambda_1}{d} & \frac{-b+c+\Lambda_2}{d} & \frac{-b-c+\Lambda_3}{d} & \frac{-b-c+\Lambda_4}{d}
\end{bmatrix}$$  \hspace{1cm} (3.54)

The matrix $K$ is then factoried into the symmetric form as in equation (2.22):

$$K = QDQ^{-1}$$  \hspace{1cm} (3.55)

A solution for the four coupled differential equations, is then found by

$$A \equiv e^{-iKz} = Q e^{-iDz}Q^{-1}$$  \hspace{1cm} (3.56)

The resulting $4 \times 4$ matrix with 16 coefficients is computed in the Appendix IV, section 2. There are now four normal orthonormal modes: symmetric and antisymmetric for the $x$- and $y$-polarization [45]. It can be completely specified by a set of only four coefficients by defining

$$A \equiv \begin{bmatrix}
 k_t & c_t & k_r & c_r \\
 c_t & k_t^* & -c_r & -k_r^* \\
 k_r & -c_r & k_t & -c_t \\
 c_r & -k_r^* & -c_t & k_t^*
\end{bmatrix}$$  \hspace{1cm} (3.57)
Comparing this with the single-mode waveguide case in equation (2.38), we find that

\[ k_t \] corresponds to \( \sqrt{1-k} \) in the SM case, for the \( x \)-eigenmode.

\[ k_r \] corresponds to \( i\sqrt{k} \) in the SM case, for the \( x \)-eigenmode.

\( k_t^* \) and \( -k_r^* \) are similar, but describe the \( y \)-eigenmodes.

\( c_t \) and \( c_r \) relate the cross-coupling between the \( x \)- and \( y \)-eigenmodes.

where \( c_t \) is the amplitude polarization cross-coupling (\( f \to s \)) for the throughput path and \( c_r \) is the polarization cross-coupling for the coupling path. The coefficients are found to be

\[
2k_t = \cos\lambda_1 z + \cos\lambda_3 z + \frac{i}{\Lambda_1} (b-c)\sin\lambda_1 z + \frac{i}{\Lambda_3} (b+c)\sin\lambda_3 z \quad (3.58)
\]

\[
2c_t = \frac{i}{\Lambda_1} d \sin\lambda_1 z - \frac{i}{\Lambda_3} d \sin\lambda_3 z \quad (3.59)
\]

\[
2k_r = -\cos\lambda_1 z + \cos\lambda_3 z + \frac{i}{\Lambda_1} (-b+c)\sin\lambda_1 z + \frac{i}{\Lambda_3} (b+c)\sin\lambda_3 z \quad (3.60)
\]

\[
2c_r = \frac{i}{\Lambda_1} d \sin\lambda_1 z + \frac{i}{\Lambda_3} d \sin\lambda_3 z \quad (3.61)
\]

The interested reader is also referred to [46-49] for more in-depth analysis of birefringent fiber couplers.

3.3.2 PM Coupler Examples - Optical Splitting

It is useful for the \( A \) matrix to be grouped into a set of four \( 2 \times 2 \) submatrices. Neglecting the birefringence for now, the result for the \( A \) matrix for \( b = 0 \) is simplified
This can be rearranged into a 2 x 2 system with 2 x 2 matrix "coefficients" to look like

\[
A = \begin{pmatrix}
\cos(Cz) & i \sin(Cz) \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
\cos \theta & \sin \theta \\
-sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\cos(Cz) \\
0 & 1
\end{pmatrix}
\] (3.62)

where \( I \) is the identity matrix, \( R(\theta) \) is the rotation matrix, and \( R(-\theta) = R(\theta) \) transposed. For the general case as in equation (3.57), a similar matrix can be defined to (3.34):

\[
A = \begin{pmatrix}
A_T & A_R \\
A_R^T & A_S
\end{pmatrix}
\] (3.64)

where the \( A \) submatrices are defined in terms of the coefficients in equations (3.58-61):

\[
A_T = \begin{pmatrix}
k_t & c_t \\
c_t & k_t^*
\end{pmatrix}
\] (3.65)

\[
A_R = \begin{pmatrix}
k_r & c_r \\
-c_r & -k_r^*
\end{pmatrix}
\] (3.66)

\[
A_S = \begin{pmatrix}
k_l & -c_l \\
-c_l & k_l^*
\end{pmatrix}
\] (3.67)

For well-aligned couplers, the rotation matrix coefficients \( c \) and \( d \) can be simplified via

\[
\sin \theta \approx \theta \quad \text{and} \quad \cos \theta \approx 1 \quad \text{where } \theta \ll 1 \text{ radian}
\] (3.68)
\[ c = C \]  \hspace{1cm} (3.69)

\[ d = C \theta \]  \hspace{1cm} (3.70)

and the calculations are greatly simplified, as shown in Appendix IV. The power in the two waveguides is plotted as a function of \( z \) in Figures 3-11 and 3-12.

![Graph 1](image1)

**Figure 3-11.** Optical Power in Fiber A and Fiber B for x-Polarization.

![Graph 2](image2)

**Figure 3-12.** Optical Power in Fiber A and Fiber B for y-Polarization.

In these Figures, \( C = 0.001/\mu\text{m} \), \( b = 0.001 \) (roughly 3-mm beat length), and the misalignment angle is about 5.7 deg. Starting with 100% of the light in fiber A x-polarization, one can easily see the periodic coupling from fiber to fiber, as in Figure 2-4. However, the misalignment couples a small amount per unit length over to the orthogonal polarization, which is seen to increase as integration along \( z \) increases.
3.3.3 PM Coupler Examples - State of Polarization

Figure 3-13 shows the Poincaré sphere, with that SOP mapped out for fiber A and fiber B, for 3 mm of propagation down the fiber. In this example, the principal axes are misaligned by 5 deg.

![Figure 3-13. SOP Evolution in Coupled PM Fibers.](image)

The loci of the SOP can be traced for fiber A and fiber B on the Poincaré sphere, with each dot representing 10 μm of propagation. When the path crosses the equator of the sphere, the light becomes linear, and the measured extinction ratio becomes very high.
Plotting the extinction ratio from the Poincaré sphere with

$$\xi_{\text{dB}} = 20 \log(\sin(\text{latitude}))$$

(3.71)

yields the interesting Figure 3-14, which should be studied carefully with Figure 3-13 and Table 3-1. One easily sees the resonant type of effect where $\xi$ is maximized, which occur near zero and 100% splitting ratio, plus at two other ratios, denoted as $k_1$ and $k_2$.

Near the 50/50 splitting ratio point, the coupled fiber has approximately -20 dB of extinction ratio, which can be compared to the case of a misaligned fiber splice [50]

$$\xi_{50\%} = 10 \log(\tan^2 \theta)$$

(3.72)

which gives an estimate of -21 dB extinction ratio for 5-deg misalignment. The light that stays in fiber A also has reduced extinction ratio, even though there was no assumed crosstalk in the fiber itself. At the second 50% splitting ratio point, the two $\xi$ values are
equal because the light has coupled twice between the fibers. This type of extinction ratio behavior as been reported by others [51, 52].

Table 3-1. State-of-Polarization as Light Propagates in PM Coupler.

<table>
<thead>
<tr>
<th>SR Region</th>
<th>SOP - Fiber A</th>
<th>SOP - Fiber B</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k \approx 0$</td>
<td>linear $0^\circ$</td>
<td>$\sim \infty$</td>
<td>linear $+5^\circ$</td>
</tr>
<tr>
<td>$k \approx 50%$</td>
<td>$\sim$ linear $+2^\circ$</td>
<td>$-35$ dB</td>
<td>$\sim$ ellipt. $+2^\circ$</td>
</tr>
<tr>
<td>$k \approx k_1$</td>
<td>linear $-5^\circ$</td>
<td>$\sim \infty$</td>
<td>elliptical $0^\circ$</td>
</tr>
<tr>
<td>$k \approx 100%$</td>
<td>linear $-90^\circ$</td>
<td>$\sim \infty$</td>
<td>elliptical $-2^\circ$</td>
</tr>
<tr>
<td>$k \approx k_2$</td>
<td>elliptical $-10^\circ$</td>
<td>$-10$ dB</td>
<td>linear $-5^\circ$</td>
</tr>
<tr>
<td>$k \approx 2nd 50%$</td>
<td>elliptical $-8^\circ$</td>
<td>$-20$ dB</td>
<td>elliptical $-8^\circ$</td>
</tr>
</tbody>
</table>

One comment resulting from this analysis: It appears that low splitting ratio couplers would have better polarization performance than for 50% or high splitting ratio couplers, all other things being equal. Given an fixed uncertainty of locating the principal axes, the likelihood for achieving high performance is much greater for the small splitting ratio devices than for high splitting ratio devices. The ramifications of this on resonator design will be discussed in Chapter 6.
3.4 References


Chapter IV

Coupler Substrate Preparation

The processing of polished fiber-optic couplers, which was summarized in Chapter I, has been arbitrarily divided into two chapters. This chapter will describe the fabrication of the substrates and the placement of the fiber into the substrate and bonding it into place. The steps that will be done in this chapter include:

(i) Substrate fabrication.
(ii) Principal axis alignment.
(iii) Fiber jacket stripping and cleaning.
(iv) Fiber-to-groove bonding and curing.

There is a natural break in the process after processing of the substrates has been completed and once the adhesive has been cured, the parts can be set aside and stockpiled for later polishing. Chapter V will discuss the polishing and assembly aspects of different types of polished couplers, including one of the few detailed discussions on optical contact bonding. An attempt has been made to survey techniques that have been reported to provide a background and alternate approaches. The specific processes used in this study will then be described in detail, for the benefit of the reader.

4.1 Materials Considerations

The selection of materials used for polished coupler fabrication is of great importance. There are four materials that go into making a half-coupler, which are shown in Figure 4-1. Those materials are (i) substrate, (ii) the fiber itself, (iii) fiber adhesive, and (iv) lower-modulus strain-relief adhesive. Variable couplers also have a metal housing and adjustment mechanism, and index-matching oil in between the half-couplers. These materials will be examined in detail in this chapter, and the rationale and data will be presented to support the selection of specific materials.
4.1.1 Substrate Materials

The selection of the material for supporting the optical fiber during the polishing needs to be given careful consideration. Most of the published literature describes the use of silicon, quartz, or glass for substrate materials. Some of their salient mechanical properties are listed in Table 4-1, which is compiled from the various vendors' product literature. The first row is the properties of the bulk glass used in the fabrication of Fujikura fiber preforms, which are assumed to be representative of the properties of the undoped cladding.

Table 4-1. Fiber and Substrate Mechanical Properties.

<table>
<thead>
<tr>
<th>Material</th>
<th>Young's Mod. $\times 10^3$ kg/mm$^2$</th>
<th>CTE $\times 10^{-6}$/°C</th>
<th>Poisson's Ratio</th>
<th>Hardness, Knoop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sinetsu-Sekiei Synth. Silica</td>
<td>7.3</td>
<td>0.51</td>
<td>0.17</td>
<td>590-620 †</td>
</tr>
<tr>
<td>Corning 7740 Fused Silica</td>
<td>6.1</td>
<td>3.2</td>
<td>0.20</td>
<td>418</td>
</tr>
<tr>
<td>Corning 7940 Pyrex™</td>
<td>7.4</td>
<td>0.52</td>
<td>0.17</td>
<td>500-560 †</td>
</tr>
<tr>
<td>Schott B-270 Crown Glass</td>
<td>7.3</td>
<td>9.5</td>
<td>0.22</td>
<td>542</td>
</tr>
<tr>
<td>Schott BK-7 Crown Glass</td>
<td>8.3</td>
<td>7.1</td>
<td>0.21</td>
<td>520</td>
</tr>
<tr>
<td>Quartz            Crystalline</td>
<td>8.0/10.0 *</td>
<td>7.4</td>
<td>0.17</td>
<td>710/790 *</td>
</tr>
<tr>
<td>Silicon           Crystalline</td>
<td>17.2</td>
<td>2.3</td>
<td>0.28/.36 *</td>
<td>850</td>
</tr>
</tbody>
</table>

† range of values given by vendor.  * values for different crystal faces.

The material chosen to be a substrate should be selected to match the fiber as much as possible in terms of coefficient of thermal expansion (CTE). The optical contact bonding process necessitates a transparent substrate in order to see the Newton's rings.
during assembly, and moderate UV transmission is helpful for fiber-to-groove bonding. Silicon has the advantage of being able to be anisotropically etched to form v-grooves and such, but lacks both of the above properties.

The polishing rate of the substrate is \( \propto 1/\) hardness, which permits the control of the differential polishing rate of the fiber with respect to the substrate. Both silicon and quartz have a much higher hardness than the synthetic silica, and the fiber surface would be below flush with the substrate surface after polishing. This would result in an unavoidable gap between the fibers when the substrates are assembled, and make it impossible to achieve an OCB coupler.

The 7740 was chosen for our fabrication process as a compromise between low hardness with respect to the fiber, and low CTE. Pyrex™ seems to work well with this process, producing fibers with the correct height above the polished surface.

Table 4-2 lists the specifications for substrate blanks; note that there are two sizes of substrates, one for variable couplers, and a thicker block for OCB coupler devices.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Specification</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Corning 7740 Pyrex™</td>
<td></td>
</tr>
<tr>
<td>Size: variable device</td>
<td>6.00 ( \times ) 25.00 ( \times ) 3.00 mm</td>
<td>+0.025/-0.0 mm</td>
</tr>
<tr>
<td>Size: OCB device</td>
<td>12.00 ( \times ) 20.00 ( \times ) 6.00 mm</td>
<td>+0.025/-0.0 mm</td>
</tr>
<tr>
<td>Parallelism</td>
<td>&lt; 1 arc minute</td>
<td></td>
</tr>
<tr>
<td>Flatness</td>
<td>1 wave, ( \lambda = 589 ) nm</td>
<td>both sides</td>
</tr>
<tr>
<td>Surface finish</td>
<td>80-50 scratch-dig</td>
<td>per MIL-O-13830</td>
</tr>
<tr>
<td>Edge finish</td>
<td>fine grind</td>
<td>do not bevel vertices</td>
</tr>
</tbody>
</table>

The flatness and parallelism specification has been relaxed to lower the cost because the blocks will be polished very flat later in the process. Blank substrate blocks can be procured to the above specifications for \( \approx $10 \) in quantity [1]. These glass blocks serve as a good starting point for fabrication of quality substrates and coupler devices.
4.1.2 Fiber-to-Groove Adhesive

The dominant material in the composite coupler structure is the adhesive used for bonding the optical fiber into the substrate groove. Most of the literature reports the use of UV-curing adhesives or epoxies. Notable exceptions are using a low melting-point glass [2], and electrostatic bonding [3] to an oxide layer grown on Silicon v-grooves. A suitable adhesive must have the following properties:

(i) Low shrinkage during cure.
(ii) Good adhesion to glass surfaces.
(iii) Low-temperature cure schedule.
(iv) Similar polishing characteristics to glass.
(v) Minimal water absorption and outgassing.
(vi) Compatibility with index matching oil.
(vii) Compatibility with Syton™ polishing solution.

Norland 61 is an excellent general-purpose optical adhesive, meeting military specifications for optical adhesives and environment. Unfortunately, this compound was unsuitable for polished couplers because of its low modulus and low hardness compared to glass. Rounding of the fiber and groove edges at the bond line was excessive when using this type of relatively soft adhesive. These caused a difficulty in holding a flat surface, particularly in the neighborhood of the fiber.

A new adhesive that is a 2-part epoxy was formulated to eliminate the problems associated with the Norland cement. Initially, an adhesive was selected that was known to be hard, resistant to polishing, and unaffected by oil. The Epo-tek 353 series has been widely used in the fiber-optics field for bonding optical fibers into connector ferrules. Unfortunately, this adhesive lacked the hardness required to prevent edge rounding. One way to modify the mechanical properties of the adhesive to better match both the glass substrate and the fiber is to fill the epoxy with various materials. Glass filling the epoxy resulted in a high degree of scratching, since the polishing action pulled out small grains of glass from the matrix, which became embedded in the polishing pad. Table 4-3 lists some of the mechanical properties of the different adhesives used in this study. The final entry is for Norland 68, which is a low-modulus UV-curing adhesive, used as a stress-relief material as shown in Figure 4-1.
A major improvement was realized by replacing the glass filler with fumed silica, which is produced by the flame hydrolysis of silicon tetrachloride [4]. The average primary particle diameter of the amorphous silica is 0.014 μm, and the aggregates are on the order of 5 μm, having a high surface area =200 m$^2$/gm. A 2-3% amount of Cab-o-sil™ M-5, which is a treated version of this silica "soot," is mixed with the 353 adhesive, which increases the viscosity and hardness. Table 4-3 lists the mechanical properties of the different adhesives with and without the compound.

**4.1.3 Strain-Relief Adhesive**

As stated earlier, a low-modulus adhesive is applied at the ends of the substrate where the fiber exits the block. This is to provide some flexibility and stress-relief at the fiber pigtail, and to seal the stripped region of the fiber jacket. Several such adhesives were evaluated for suitability with the polishing process and are listed in Table 4-4. In this experiment, the materials were prepared in grooves that were cut into scrap substrate blocks. These samples were processed according to our polishing procedure, and the adhesives were observed for any indications of breakdown or increase in modulus.

**Table 4-4. Results of Stress-Relief Adhesive Experiment.**

<table>
<thead>
<tr>
<th>Adhesive</th>
<th>Viscosity, Centipoise</th>
<th>Young's Mod. x10$^3$ kg/mm$^2$</th>
<th>Hardness, Shore D</th>
<th>CTE × 10$^{-6}$/°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Norland 61</td>
<td>UV-cure</td>
<td>350</td>
<td>105</td>
<td>85</td>
</tr>
<tr>
<td>Epo-tek 353</td>
<td>2-part</td>
<td>2000</td>
<td>612</td>
<td>87</td>
</tr>
<tr>
<td>Epo-tek 353NDT</td>
<td>2-part</td>
<td>43000</td>
<td>680</td>
<td>88</td>
</tr>
<tr>
<td>Norland 68</td>
<td>UV-cure</td>
<td>5000</td>
<td>14</td>
<td>60</td>
</tr>
</tbody>
</table>

Norland 68 UV-Cure No effect
Urelane 5750 2-part urethane Swelling, peeling
Dow Corning Q-6662 UV-cure silicone Swelling, peeling, breakdown
Dow Corning 93-500 2-part silicone Complete breakdown (missing)
Dow Corning 6-1104 1-part silicone Peeling, picked up contamination
Only the Norland adhesive was found to be suitable for a low-modulus strain relieve adhesive; all of the others either broke down, had unacceptable flatness, or had poor adhesion to the groove walls. The Norland 68 had excellent adhesive qualities, polished flat with respect to the substrate surface, and resisted Syton™. Details of the application and curing of the adhesives are given in Section 4.4, which describes fiber stripping, cleaning, and fiber-to-groove bonding.

4.1.4 Fiber

Typically, the fiber choice is driven by system or customer needs, and is fixed. Unfortunately, the mechanical properties of the fiber have a strong influence on the polishing of the half-coupler surface. Judicious selection of the other half-coupler materials with respect to a given fiber is important to get a successful polishing process. While the fiber is essentially SiO2, in the case of birefringent fiber, many of the polishing characteristics are dominated by the stress-applying parts (SAPs). The incorporation of boron or other dopants to create the SAPs changes the refractive index, hardness, CTE, and other properties of the glass. While the initial grinding effectively planarizes the surface, a differential polishing rate is observed for the SAPs during subsequent processing. The SAPs will tend to polish at a faster rate than the fiber or substrate, which has some important ramifications in coupler design and processing.

Polished couplers have been successfully fabricated out of all-birefringent fiber designs, including PANDA [5, 6], bow-tie [7], elliptical SAP fiber [8], and others. Single-polarization fibers have been more difficult to polish than polarization-preserving fibers due to their increased stress levels [9].

4.1.5 Index-Matching Oil

In variable polished couplers, an optical matching compound is used to match the boundaries between the waveguides. This index-matching 'oil' (IMO) is a clear mixture of alicyclic and aliphatic hydrocarbons, the precise formulation of which is adjusted to achieve a given refractive index. Low viscosity permits it to be wicked into the fiber interfacial area by capillary action; unfortunately, IMO is unsuitable for anything but laboratory environments. IMOs, in general, evaporate, attract contaminants, can change...
in prolonged sunlight, and can damage some plastics. Because of this, the long-term stability of adjustable couplers is poor, and the oil needs to be replaced periodically.

IMOIs can be purchased in a refractive index ranging from 1.400 to 1.640 in 0.002 increments, with an accuracy of ±0.0002. These numbers are quoted for \( \lambda = 589.3 \) nm (the Sodium D-line), and \( T = 25^\circ C \). A difficulty with practical use of these oils is that they have high dispersion and temperature coefficients. For \( n_{25^\circ C} = 1.456 \) oil, the data given by the manufacturer for temperature compensation of the index is [10]

\[
n(T) = n_{25^\circ C} - 0.000389 (T - 25)
\]

(4.1)

where \( T \) is in \( ^\circ C \). The wavelength dependence is given by the Sellmeyer equation

\[
n(\lambda) = 1.4442 + \frac{400289}{\lambda^2} + \frac{3.36220 \times 10^{11}}{\lambda^4}
\]

(4.2)

where wavelength \( \lambda \) is in Angstroms. Figure 4-2 illustrates the wavelength dependence of this specific oil according to Eq. (4.2) at a 25°C temperature.

![Figure 4-2. Index-Matching Oil Refractive Index vs Wavelength.](image)

According to Eq. (4.2) at our operating wavelength of 1.3 mm, the refractive index of the 1.456-labeled oil is 1.447, which matches the index of refraction of the fiber cladding as desired. The variation of IMO refractive index will induce loss and change the splitting ratio over temperature [11, 12]. Typical values for the splitting ratio coefficients are on the order of 0.2% per \( ^\circ C \) and 0.3% per nanometer for adjustable couplers with this type of oil.
4.2 Substrate Preparation

The substrate is a critical element of the polished fiber-optic coupler because it must rigidly support the fiber without perturbing it. The substrate design has a large effect on the fiber profile, polishing characteristics, process yield, and assembly options of the device. Several substrate fabrication techniques have been reported, and these will be surveyed in this section. The most common substrate design consists of a slightly curved groove that has been cut in a glass block, which is shown in Figure 4-3.

![Figure 4-3. Top and Side View of Coupler Substrate.](image)

The substrate, which is made from a glass block, contains a centrally located groove with a large radius $R$. Two wider, short radius grooves receive the fiber jacket, which is held by the flexible adhesive mentioned earlier. The objective of a good substrate is to provide a repeatable, stable support for polishing and optically joining the two waveguides. The substrate also serves to protect the fiber and acts as a reference for aligning birefringent fiber principal axes. Major criteria to meet this goal are:
As suggested by Eq. (2.42), the radius of curvature, $R$, controls the interaction length of the device. The groove must have minimal deviation from the ideal curvature.

(ii) The absolute depth of the groove needs to be controlled to $\pm 2\ \mu m$ to obtain repeatable polishing time. Note that the block thickness values up to $25\ \mu m$ in Table 4-2.

(iii) It is desirable to minimize the amount of epoxy used to bond the optical fiber into the groove, to improve polishing flatness and coupler environmental behavior. This can be accomplished by controlling the groove width to a value slightly wider than the fiber.

(iv) The bottom groove surface, upon which the fiber rests, must be smooth to avoid inducing microbending losses or inducing damage to the fiber.

(v) A wider groove is needed at the extremities of the substrate to receive the fiber jacket, which is typically from 250 - 500 $\mu m$ in diameter.

(vi) The process should be repeatable and operator insensitive, and permit rapid processing with minimal maintenance and set-up. Consumable items such as cutting blades should be long lasting.

(vii) The groove should be able to be cut with arbitrary radii and shapes. This is because different interaction lengths are needed for WDM couplers and other devices.

Several techniques have been developed to meet the different criteria listed above, with varying degrees of success, which are now described. A novel approach for the fabrication of substrates, the numerically-controlled groove generator, will be described in detail.

4.2.1 Wire Saw

The first method that was used for generating curved grooves in glass blocks employed a wire saw, commonly used in the fabrication of crystalline optics [2, 13]. These saws are designed to maintain high tension on the cutting wire to cut a straight line. To reduce tension on the endless wire, some modification was required to permit cutting
a curved groove. Figure 4-4 shows such a wire saw with a spring-loaded pulley that places a light tension on the wire.

![Figure 4-4. Modified Wire Saw.](image)

The wire is made of steel and requires a grinding agent, or can be bought with a diamond-impregnated surface. With low tension, the natural tendency is to cut near the edges of the block first, which generates a curved shape. In practice, it is difficult to predict what groove shape or radius results, but empirical adjustment can produce blocks of good quality. The cross section of the groove has a rounded bottom, and it can have quite a smooth surface. To ensure consistent groove depth and shape, care must be taken to monitor such variables as wire speed, tension, slurry flow, and grit size and concentration. This method has been amenable to batch processing, fabricating several blocks in parallel [14] with ganged wire saw arrangements.

In some processes, a constant-depth slot was cut into the glass blocks prior to generating the curved groove with the wire saw. The straight cut was performed with a thin cutting wheel in a low-speed cutting saw [15]. This is the same saw used to cut the raw material into the proper dimensions for substrates and the reliefs for the fiber jacket.

### 4.2.2 Curved Substrate

Another early method consisted of mounting an optical fiber onto a spherical lens with wax or adhesive, as depicted in Figure 4-5. This provided a well-defined radius, and a very smooth surface [16, 17]. This technique was later modified by sectioning a cylinder lens into wafers and attaching the fiber to the curved face [18]
The radius for a plano-convex lens is calculated from the lensmaker's formula is

\[ R = (n - 1)f \]  

(4.3)

where \( f \) is the focal length and \( n \) is the refractive index of the lens. This technique has seen several different embodiments, such as molded-plastic blocks with integral reference surfaces [19].

These have the drawback that there is no planar reference surface surrounding the optical fiber, which makes the alignment and affixment of the polished fibers difficult. One method around this is to build up a supporting layer of epoxy over the fiber, which is then polished flat [20]. A molded plastic block with a curved groove, not unlike the glass blocks described in the previous subsection, has also been used [21, 22]. These techniques suffer from flatness problems, due to the large amounts of soft material adjacent to the fiber.

4.2.3 Silica Sandwich

A major improvement to the lens-bonding technique that eliminates the flatness problem is to make up fused silica sandwiches, as shown in Figure 4-6. The technique developed by British Telecom consists of a pair of support blocks bonded on either side of the curved section [23]. The curved section is a thin wafer sliced out of a polished cylinder of known radius, which has been polished down to a controlled thickness, slightly wider than the fiber.
This design produces substrates having good reproducibility and control of groove dimensions, which is evidenced by the high-quality PM couplers reported with this technique. Polished fiber couplers fabricated from elliptical-core birefringent fiber have exhibited polarization cross-coupling levels below $-35$ dB with this method [24].

Champion device performance up to the $-42$-dB measurement limit is largely due to the smooth groove surface. The process is complex and labor-intensive, requiring several additional polishing steps, but the substrate has close to ideal groove properties.

4.2.4 Swing-Arm Cutting

A simple technique of fabricating polished coupler substrates employs a modified low-speed cutting saw, already mentioned in Section 4.2.1. Low-speed diamond cutting has been shown to produce low levels of damage to brittle and difficult materials [25]. These blades consist of a thin metal disk with abrasives embedded around the periphery. A swing arm and clamp assembly is supplied with the saw for holding the samples against the saw to section. To enable cutting curved grooves, a slight modification to the swing-arm is required. It is lengthened and relocated such that the glass block sweeps an arc past the cutting blade, as illustrated in Figure 4-7. The radius of the groove is determined by the length of the arm, which can be set up quite accurately.
The specifications for groove cutting blades for cutting are listed in Table 4-5. If the cutting wheel becomes glazed or loaded up with material, a dressing block is applied to the spinning wheel. The dressing block, made of a resin-bonded block of fine carbide powder, removes any soft material clogging the diamond-impregnated surface. This procedure needs only to be done on a new blade, but is rarely needed when cutting brittle materials such as we are. There are several sources for off-the-shelf stock item [26] and custom blades [27].

Table 4-5. Low-Speed Cutting Wheel Specifications.

<table>
<thead>
<tr>
<th>Abrasive</th>
<th>Diamond</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grit size</td>
<td>600</td>
</tr>
<tr>
<td>Concentration</td>
<td>low</td>
</tr>
<tr>
<td>Diameter</td>
<td>3 in</td>
</tr>
<tr>
<td>Thickness</td>
<td>0.006 in and 0.020 in</td>
</tr>
<tr>
<td>Lubricant</td>
<td>Isocut or kerosene</td>
</tr>
<tr>
<td>Speed</td>
<td>100 rpm</td>
</tr>
</tbody>
</table>
The swing-arm technique produces quality blocks, with well-defined groove radius profiles, but is limited to circular profiles. It is difficult to extend this to very long radius grooves, because vibration in the arm and pivot bearing will degrade the groove quality. It also requires quite a bit of effort to consistently control the groove width and depth to μm levels.

4.2.5 Cam-Follower

A slightly modified approach is to use a cam-follower arrangement to move the block in an arc past the blade. This requires the arm to pivot and also move the block toward and away from the wheel as it moves along the blade shown in Figure 4-8. A master cam controls the block's vertical position with respect to the rotating wheel, and is machined to have a magnified displacement for a given radius of curvature at the substrate. This method was used to fabricate resonators and couplers with $<0.1$-dB loss [28].

Figure 4-8. Cam-Follower Groove Cutting Technique.
(with permission of Honeywell Inc.)
4.2.6 Silicon V-Grooves

Anisotropic etching of $<100>$ Silicon has been known for some time to produce 54.74 deg angled v-grooves [29]. A mixture of KOH and isopropyl alcohol will produce a 400:1 etch ratio for the $<100> : <111>$ crystal planes. A SiO$_2$ mask is required, and with suitable photolithography, a curved groove can be delineated as shown in Figure 4-9 [30, 31]. This produces a substrate that is well-suited for polished couplers and other silicon-based optical sensors [32].

![Figure 4-9. Silicon V-Groove Substrate.](image)

The arbitrary pattern shapes available are useful in the inclusion of other features, such as fiber jacket grooves and fiducial alignment features [33]. This method also has potential for mass production of substrates, and is used in a commercially-available adjustable coupler [34]. However, it would be difficult to fabricate an adhesive-bonded coupler using this method, due to the UV-opacity of Si. Optical contact bonding (OCB) would also be difficult because the fringes would be unobservable and the existence of dissimilar materials.
Promise of adhesiveless substrate pair assembly does exist, with several techniques for producing Si-Si bonds having been demonstrated. One method is to thermally oxidize a thin layer on the silicon surfaces [35] and essentially making a glass-glass OCB; another method is by sputtering SiO₂ onto the Si [36]; and also by producing a hydrophylic layer with H₂SO₄ etching [37]. To hold the fiber into the v-groove without adhesive arequires a Si-glass bond, has also been demonstrated using high voltage fields [38], and special glass formulations [39]. All of these techniques require various degrees of heating, in some cases up to 1100°C; lower temperature processes need to be developed for this to become a reality.

4.2.7 Numerically-controlled groove generator

A significant improvement in the low-volume fabrication of precision grooved substrates is a computer-controlled cutter [40]. Substrates made in this manner overcomes the limitations of the methods described in Sections 4.2.1-6. It consists of a low-speed motor, precision bearing assembly, and various arbors to hold the saw blades. A pair of Klinger motorized linear stages is combined to give x and z motion, and are controlled by a personal computer, via the IEEE-488 bus. The z stage, which controls horizontal motion in the direction of the groove, has a resolution of 1 micron and a range of 75 mm. The x stage, which controls the depth of cut, has 0.1 micron resolution and a range of slightly over 4 mm. A manually-adjusted y-axis stage permits offsetting the groove position in the block to compensate for blade position and thickness. The fixture and associated electronics is illustrated in Figure 4-10.

Limit switches, high-accuracy origin search, and other functions are connected to the drive electronics, which can inform the computer of the motion status. The position of the block surface relative to the blade is controlled by software that has been written in Pascal and the National IEEE-488 driver. An IBM AT-series personal computer with the National GPIB interface card is the hardware of choice for this application. The program initially asks the operator for some information, then iterates through a series of straight and then curved cuts to create a groove of a given radius.
The x and z motion of the stages are controlled by a simple algorithm, where z is stepped by a constant amount and x movement is determined by the equation for a circle

\[ x(z) = x_0 + R - \sqrt{z^2 + R^2} \]  

(4.4)

which can be approximated for the region \( |z| \ll R \) with a parabolic expression for x

\[ x(z) = x_0 + \frac{z^2}{2R} \]  

(4.5)

where R is groove radius and \( x_0 \) is groove depth at \( z = 0 \) in the center of the block. The 0.1 \( \mu \text{m} \) resolution of the x translation stage ensures that there are no 'steps' in the curve. By changing a variable in the software, the radius may be adjusted without reconfiguring the mechanical system. Rearranging Eq. (2.12) for a given effective length yields
\[ R = \frac{2vL_e^2}{\pi r_e} \]  

(4.6)

which is entered in the software. 30 cm is the radius used for these devices, which results in approximately a 1-mm effective length, as tabulated in Table 2-1.

4.2.8 Groove Generator Processing

In an effort to understand the relationship between groove width and resultant flatness, an experiment was performed with optical fiber bonded into various width grooves. The substrates were made of B-270 glass, and the 353-NDT epoxy was used for fiber-to-groove bonding. Figure 4-11 illustrates the fiber slightly above flush with respect to the polished block surface and the faster polishing of the epoxy regions.

![Figure 4-11. Fiber and Groove Dimensions.](image)

The fiber offset is \( \Delta x_{fiber} \) and the depth of the epoxy trench is denoted as \( \Delta x_{epoxy} \) as measured with a Dektak stylus profilometer having a 10-\( \mu \)m stylus radius. Measurements for the above figure are tabulated in Table 4-6, which shows the improvement with narrower grooves; the dimensions are with respect to the substrate surface.

<table>
<thead>
<tr>
<th>Condition</th>
<th>( \Delta w )</th>
<th>( \Delta x_{fiber} )</th>
<th>( \Delta x_{epoxy} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wide groove</td>
<td>100</td>
<td>+0.5</td>
<td>-2.2</td>
</tr>
<tr>
<td>Narrow groove</td>
<td>20</td>
<td>+0.1</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

Table 4-6. Effects of Groove Width.
The narrower grooves produce a superior flatness across the fiber, and produces a fiber which protrudes slightly above the surface. This condition, along with control of the overall substrate surface figure and roughness will produce a substrate suitable for optical contact bond. Thus, one would like to have a groove perhaps $20 \sim 40 \mu m$ wider than the fiber to avoid these problems and to allow an adequate bond line for the epoxy. Pyrex has an even lower $\Delta x_{\text{fiber}}$ than B-270, on the order of 50 nm.

The initial testing of the groove generator used a coarse 200-grit blade, which produced substrates with unacceptably high roughness levels in the groove and excessive edge chipping. Some improvement was observed with a finer 600-grit blade, also supplied by Buehler Ltd. Both of these blades were 3-in diameter and had approximately 0.006-in width, as listed in Table 4-5. These blades produced a rounded groove cross section, and had unacceptable groove roughness. Figure 4-12 shows photographs of typical grooves in Pyrex blocks, all taken with a Zeiss Universal microscope with bright-field illumination at 100x magnification. There is a poor thickness tolerance and large kerf losses associated with these blades, that is shown in Figure 4-12a and 4-12b. Another problem with these blades was that they wore thinner with successive cuts, so the grooves were too narrow after roughly ten substrates. In this case, the fiber gets wedged in the groove with epoxy under the fiber, with a high probability of fiber breakage or polishing through the core. If the substrate survives polishing, pinching stresses will degrade cross-coupling and losses, and the epoxy under the fiber will introduce environmental sensitivity.

To improve the quality of the substrate grooves, alternative saw blade vendors were sought, but were unsuitable because of the poor thickness tolerance and coarse grits used. Diamond-coated nickel dicing blades, commonly used by the IC industry, were applied with great success. The blades are bonded onto a thick backing plate, which provides support for the thin blade, and also has precision diametrical and side-mounting surfaces. These blades produced smoother grooves with less damage because of the increased stability afforded [41].

This and the smaller blade diameter vastly reduced blade flex and wobble, resulting in about a 3X reduction in groove width. Figure 4-12c illustrates the vast improvement in chipping and kerf loss when using the dicing wheel. In addition, these dicing blades are available in 0.0005 inch increments down to 0.003 inch thickness,
which allows control of the groove width with a high degree of precision. Table 4-7 lists the specifications for the dicing wheels for fabrication of substrates in this study.

Table 4-7. Dicing Wheel Specifications.

<table>
<thead>
<tr>
<th>Abrasive</th>
<th>Diamond</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grit size</td>
<td>5 μm</td>
</tr>
<tr>
<td>Concentration</td>
<td>medium</td>
</tr>
<tr>
<td>Diameter</td>
<td>2.188 in</td>
</tr>
<tr>
<td>Thickness</td>
<td>0.0045 in</td>
</tr>
<tr>
<td>Lubricant</td>
<td>Isocut</td>
</tr>
<tr>
<td>Speed</td>
<td>300 rpm</td>
</tr>
</tbody>
</table>

A cross-section of a substrate made with the new process with a bonded fiber is shown in Figure 4-13. Note the smooth groove surface, flat groove bottom and sides, and tight gap between fiber and groove walls. The steps in block processing are now described: The blank substrate is held with paraffin onto a precision fixture, which accurately registers the block with respect to the motorized translation stages.

(i) Wax glass block onto substrate holder at 50°C and cool.
(ii) Align fixture to registration pins and tighten screws.
(iii) Mount dicing blade on mandrel, and tighten clamp screw.
(iv) Invoke program on PC, turn on spindle and set to 300 rpm.
(v) When blade is centered on block, jog blade (+x) toward block.
(vi) Observe gap, shim stock, or accelerometer for contact.
(vii) Instruct computer to cut groove, apply Isocut fluid liberally.
(viii) When curved groove is finished, switch to 0.020-in blade.
(ix) Adjust y-micrometer, if necessary, to recenter thicker blade.
(x) Follow computer prompts to cut strain-relief grooves.
(xi) Remove holder, and melt wax to remove finished substrate.
(xii) Inscribe serial number on the side of block with scribe.
(xiii) Ultrasonic in toluene, Freon, and alcohol 5 minutes each.
(xiv) Inspect at high magnification, and record groove width.
(xv) Sort by size, and place in glassine envelope in storage bin.
Figure 4-12. Substrate Groove, 100x bright field illumination. (a) 200-grit saw blade: groove width = 375 μm; (b) 600-grit saw blade: groove width = 225 μm; (c) dicing blade: groove width = 165 μm; all ±3 μm.

Figure 4-13. Groove Cross-Section.
The last cut of the curved groove is made at an especially slow feed rate, taking only a 1-μm cut. This is an attempt to cut in the so-called ductile-mode grinding regime [42], which produces a smooth surface.

Figure 4-14 shows a histogram of a run of 24 consecutive substrates on a 0.0040-in thick dicing blade. The average width is 148 μm, and standard deviation is ≈14 μm; note the sharp cutoff at the lower limit. For a fiber nominal diameter of 125 μm, the desired groove width is in the neighborhood of 160 ~ 170 μm.

![Groove Width Histogram](image)

The absolute depth of the groove is an important parameter to control, and the operators have developed various techniques to compensate for the varying block thicknesses. By careful observation of the gap between the rotating blade and approaching block surface, a technician with good eyes can judge contact. Alternatively, a thin piece of plastic shim stock can be used as a gauge of the remaining gap. It will be forced out of the gap at a repeatable point, which is a known distance from the surface. The good control of groove depth makes polishing time more repeatable, and removes the requirement of tight control of blade diameter.

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An accelerometer may be mounted on the block-holding fixture and used to pick up vibrations of blade contact, which are displayed on an oscilloscope. When the blade is being manually jogged in the vertical (x) direction, the bearing and motor noise is observed as a low-frequency signal. Periodic high-frequency vibrations indicate blade contact and is repeatable to within 10 μm. When using old coarse grit blades, one merely has to listen for initial blade contact.

4.3 Principal Axis Alignment

As shown in Chapter III, it is important to obtain a high degree of alignment between the principal axes of the two fibers in a PM coupler. A good rule of thumb for a 50/50 coupler is that the polarization cross coupling \( \xi \) in dB is given by

\[
\xi = 10 \log(\tan^2 \theta) = 20 \log \theta \quad \text{for small } \theta
\]  

(4.7)

for fiber A to fiber B. For example, a 1-mrad misalignment (~ 0.5 deg) will induce roughly −30 dB (0.1%) polarization cross coupling. It should be reminded that poor workmanship or polishing-induced stress relief could limit the cross coupling to a higher level. There are two basic techniques that are in use for identification and orientation of the birefringent principal axes, based on direct observation of the SAPs and acousto-optics. After a survey of these approaches is made, a system for the latter method will be described in detail.

4.3.1 Transverse Optical Techniques

The first attempts at aligning the principal axes of PM fiber as described in the literature were with a microscope, perhaps with Nomarski phase-contrast enhancement techniques. This method produced alignment to ±10 deg and is highly operator-sensitive, so several techniques have been developed that lend themselves to improved accuracy and automation.

A simple and elegant method to align the principal axes of birefringent optical fiber is to observe the fiber and stress rods with transverse illumination. The fiber is illuminated with a collimated beam of 830-nm light, and is imaged onto a CCD array with a simple lens arrangement [43] as shown in Figure 4-15. The 1-dimensional image
captured by the array at the focal plane is represented by the shaded line in the figure. Note the uniform illumination beyond the fiber and two shadows just inside the cladding region.

Figure 4-15. Transverse Illumination Alignment Technique.

This technique is used in a commercial splicer [44] to make optical fiber splices in the "passive" mode, i.e., without launching light in the core and measuring the output SOP. Due the refraction of light at the SAP/cladding interface, there will be some variation in the observed pattern at the focal plane as the fiber is rotated, as long as \( n_{\text{SAP}} \neq n_{\text{clad}} \). Figure 4-16 is a ray-trace diagram for a PANDA fiber with depressed-index SAPs, made with the Oslo optical design program.

The widely diverging optical rays produce the shadow regions in the focal plane, and the dark concentration of rays at the extreme right produce the more complex central structure. There will be an asymmetry associated with the peak intensity regions of the central image, which varies as a function of misalignment. This technique has the best sensitivity to misalignment for the optical axis nearly parallel to the slow axis of the fiber as shown in the schematic, rather than in the ray-tracing diagram. In that mode, the slow axis can be aligned sufficiently parallel to the optic axis to produce \(-25\) to \(-35\)-dB splices. Calculating the angle that produces this performance level allows estimation of the misalignment in the range of \( 1 \sim 3 \) deg.

If coherent transverse illumination is used in lieu of a broadband source, a diffraction pattern is generated, which may be read directly with a CCD array to yield alignment information [45]. In this case, the lens in Figure 4-15 may be eliminated, and an operator interprets the complex diffraction pattern. The authors reported \( \xi < -15 \) dB, which implies alignment accuracies of better than 10 deg, if their device performance is limited by alignment.
Although fast and simple, these methods require that the SAPs have a different refractive index than the cladding, which is not true for all fibers. Passive techniques align the SAP geometry that may not have exact symmetry with the principal axes in nonideal cases. The big advantage of transverse illumination is that light does not need to be launched in the fiber; the disadvantage is that the images generated require some interpretation either by an operator or by a simple image processor.

### 4.3.2 Acousto-Optic Technique

The technique used to make the couplers in this project employs acousto-optical interaction of the fiber and an external squeezer device [46]. The $3 \times 3$ stress tensor used to handle 3-dimensional problems can be truncated to a $2 \times 2$ matrix for our purposes. Consider a short length of birefringent fiber, which is squeezed between a pair of parallel jaws, as shown in Figure 4-17. The angle of the squeezer (line of compressive force) to the slow axis, which is under tensile stress ($\sigma < 0$) is given by $\theta_s$. Since the stresses are opposite in sign, at the alignment condition $\theta_s = 0$, the squeezer will tend to partially

---

**Table 4-1**: Optical System Layout

<table>
<thead>
<tr>
<th>PANDA Fiber</th>
<th>SCALE</th>
</tr>
</thead>
<tbody>
<tr>
<td>OPTICAL SYSTEM LAYOUT</td>
<td>900</td>
</tr>
</tbody>
</table>

---

Figure 4-16. Ray-Trace Diagram of Side-Illuminated PANDA Fiber.

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cancel the fiber birefringence. For fast axis alignment condition, $\theta_s = 90$ deg and the squeezer will add to the fiber's birefringence.

Figure 4-17. Squeezed PANDA Fiber Showing Slow Axis Rotation $\Phi$.

Qualitatively, the effect of squeezing a PM fiber is to rotate the principal axes by an angle $\Phi$. The total stress in the fiber is given by the sum of the two tensors

\[
\begin{bmatrix}
\sigma_1 & \frac{1}{2}\sigma_6 \\
\frac{1}{2}\sigma_6 & \sigma_2
\end{bmatrix}_{\text{total}} = \begin{bmatrix}
\sigma_1 & 0 \\
0 & \sigma_2
\end{bmatrix}_{\text{int}} + \begin{bmatrix}
\sigma_1(\theta_s) & \sigma_6(\theta_s) \\
\sigma_6(\theta_s) & \sigma_2(\theta_s)
\end{bmatrix}_{\text{ext}}
\]  

(4.8)

where the internal stresses are due to the SAPs, and are fixed with respect to the fiber coordinate system. As assumed earlier, there are no shear stresses $\sigma_6$ from the SAPs in the unsqueezed fiber. There will be shear stress components imparted by the squeezer, which are a function of the angle of the applied force. Applying the coordinate transformation from the squeezer to the fiber coordinate system results in the expression

\[
\begin{bmatrix}
\sigma_1 & \sigma_6 \\
\sigma_6 & \sigma_2
\end{bmatrix}_{\text{ext}} = \begin{bmatrix}
\cos\theta_s & \sin\theta_s \\
-\sin\theta_s & \cos\theta_s
\end{bmatrix}\begin{bmatrix}
\sigma_{1\text{E}} & 0 \\
0 & \sigma_{2\text{E}}
\end{bmatrix}\begin{bmatrix}
\cos\theta_s & -\sin\theta_s \\
\sin\theta_s & \cos\theta_s
\end{bmatrix}
\]  

(4.9)

\[
= R(-\theta_S) \begin{bmatrix}
\sigma_{1\text{E}} & 0 \\
0 & \sigma_{2\text{E}}
\end{bmatrix} R(\theta_S)
\]  

(4.10)
where the squeezer is designed to also induce no shear stresses in the fiber. The resultant total stress components can then be shown to be

\[
\sigma_1 = \sigma_{1I} + \frac{1}{2} (\sigma_{1E} + \sigma_{2E}) + \frac{1}{2} (\sigma_{1E} - \sigma_{2E}) \cos(2\theta_S)
\]  
\[
\sigma_2 = \sigma_{2I} + \frac{1}{2} (\sigma_{1E} + \sigma_{2E}) - \frac{1}{2} (\sigma_{1E} - \sigma_{2E}) \cos(2\theta_S)
\]

\[
\sigma_6 = - (\sigma_{1E} - \sigma_{2E}) \sin(2\theta_S)
\]

\[
\sigma_1 - \sigma_2 = (\sigma_{1I} - \sigma_{2I}) + (\sigma_{1E} - \sigma_{2E}) \cos(2\theta_S)
\]

There is now the possibility for a nonzero shear stress component that is a function of the misalignment angle. From Eq. (4.11c), it can be seen that the net shear stress vanishes for \(\sin(2\theta_S) = 0\), i.e., \(\theta_S = 0\) or 90 deg. The stresses \(\sigma_6\) and \(\sigma_1 - \sigma_2\) are substituted into Eq. (3.9), and it can be seen that the principal axes rotate for nonvanishing shear stresses

\[
\tan(2\Phi) = \frac{\sigma_6}{\sigma_1 - \sigma_2} = \frac{-\sigma_E \sin(2\theta_S)}{\sigma_I + \sigma_E \cos(2\theta_S)}
\]

The shorthand \(\sigma_I \equiv (\sigma_{1I} - \sigma_{2I})\) and \(\sigma_E \equiv (\sigma_{1E} - \sigma_{2E})\) has been adopted since there are no shear components in the internal and external stress fields. Equation (4.12) is the key to this type of alignment technique. The new total birefringence is

\[
B_T = \frac{-n_0^3}{2} (p_{11} - p_{12}) \sqrt{(\sigma_1 - \sigma_2)^2 + \sigma_6^2}
\]

which is the general case for Eq. (3.17). Substituting in the perturbed stress values

\[
B_T = \frac{-n_0^3}{2} (p_{11} - p_{12}) \sqrt{\sigma_I^2 + \sigma_E^2 + 2\sigma_I \sigma_E \cos(2\theta_S)}
\]

Thus, the fiber in the squeezed region can have a rotated principal axes, and a different birefringence from the unsqueezed fiber, since \(\sigma_I < 0\) and \(\sigma_E > 0\). The birefringence cycles periodically, with minima at 0 and 180 deg and maxima at \(\theta_S = 90\) and 270 deg. At these four angles the principal axis rotation is also zero and are aligned
with respect to the line of force defined by the squeezer. The effect of Eq. (4.12) is plotted in Figure 4-18, for a 2-mm beat length fiber and constant \( B_E \).

![Figure 4-18](image)

Figure 4-18. \( \Phi \) as a function of \( \theta_S \) with \( B_E = 0 \) and \( B_E = 0.0005 \).

By substitution and cancellation of terms, the above expressions can be rewritten in terms of numerically positive birefringence values, by introducing a sign change as

\[
\tan(2\Phi) = \frac{B_E \sin(2\theta_S)}{B_I - B_E \cos(2\theta_S)}
\]

(4.15)

\[
B_T = \sqrt{B_I^2 + B_E^2 - 2B_I B_E \cos 2\theta_S}
\]

(4.16)

where \( B_I \) is the intrinsic birefringence, and \( B_E \) is the externally-induced birefringence due to squeezing [47]. There is also a small phase delay induced by the change of the average refractive index of the fiber in the squeezed region, which is typically neglected. The effect of nonzero \( \Phi \) is to induce polarization cross-coupling in the fiber, and is observed as a reduction in measured extinction ratio. The amount of cross-coupling is dependent on the amount of \( \Phi \) and change in birefringence in the squeezed region.
4.3.3 Squeezer Alignment System

The optical system for acousto-optic alignment is depicted in Figure 4-19, which is modeled as three segments of fiber. Collimated light from an unpolarized source passes through a polarizer and is launched into the fiber with a lens, where the polarizer's transmission axis is defined to be at angle $\theta_p$ with the slow axis of the fiber. In this manner, a linear polarized beam may be launched into the fiber at an arbitrary angle with respect to the principal axes. The output of the fiber is recollimated and is passed through a second analyzing polarizer at angle $\theta_a$ before illuminating a photodetector. The lengths of the three fiber segments are denoted $L_1$, $L_2$, and $L_3$, where $L_2$ is the squeezed section length that is shaded in the drawing.

The fiber is squeezed with force $F$ in the $L_2$ region, at an angle $\theta_s$ with the slow axis, by virtue of a precision stepper motor arrangement. For the propagation of the first length of fiber, the Jones vector for the linear polarization exiting the polarizer is written down on the right. The rotation matrix performs the coordinate transformation with respect to the principal axes, consistent with our definitions. These two matrices define a linear polarization at an arbitrary fiber with respect to the slow axis.

\[
\begin{bmatrix}
E_x \\
E_y
\end{bmatrix}
= e^{i\beta_1} \begin{bmatrix}
e^{i\phi_1} & 0 \\
0 & e^{-i\phi_1}
\end{bmatrix}
\begin{bmatrix}
\cos \theta_p & -\sin \theta_p \\
\sin \theta_p & \cos \theta_p
\end{bmatrix}
\begin{bmatrix}
1 \\
0
\end{bmatrix}
\] (4.17)

The differential phase shift, or retardance, in fiber segment 1 is accounted for by the left-hand matrix, and the $\phi_1$ term is the common phase shift that has been factored out.

\[
\phi_1 = \frac{(\beta_x - \beta_y)L_1}{2} \ll \beta L_1
\] (4.18)
The product of the right-hand side of Eq. (4.13) is a vector representing the SOP of the field exiting the fiber segment. Multiplying this out and assuming \( \theta_p = 0 \) results in

\[
\begin{bmatrix}
E_x \\
E_y
\end{bmatrix}
= e^{i\beta L_1}
\begin{bmatrix}
e^{i\phi_1 \cos \theta_p} \\
e^{-i\phi_1 \sin \theta_p}
\end{bmatrix}
\approx e^{i\beta L_1}
\begin{bmatrix}1 \\
0
\end{bmatrix}
\tag{4.19}
\]

showing that the linear SOP is preserved in this special case, and undergoes a phase shift of \( \beta L_1 \). The small additional retardance term can be neglected and has been dropped.

The squeezed segment of the fiber is rotated by \( \Phi \), retarded by \( k_0B_1L_2 \) radians, and then rotated back to the original principal axes by an angle \( -\Phi \)

\[
\begin{bmatrix}
E_x \\
E_y
\end{bmatrix}
= \begin{bmatrix}
\cos \Phi & \sin \Phi \\
-\sin \Phi & \cos \Phi
\end{bmatrix}
e^{i\beta L_2}
\begin{bmatrix}
e^{i\phi_1} & 0 \\
0 & e^{-i\phi_1}
\end{bmatrix}
\begin{bmatrix}
\cos \Phi & -\sin \Phi \\
\sin \Phi & \cos \Phi
\end{bmatrix}
\begin{bmatrix}
E_x \\
E_y
\end{bmatrix}
\tag{4.20}
\]

where the differential retardance in the fiber matrix is given by

\[
\phi_1 = \frac{k_0B_1L_2}{2} = \frac{\pi L_2}{\lambda} \sqrt{B_1^2 + B_E^2 - 2B_1B_E \cos 2\theta_S}
\tag{4.21}
\]

Now, multiplying out Eq. (4.20) results in a Jones matrix describing the second segment

\[
\begin{bmatrix}
E_x \\
E_y
\end{bmatrix}
= e^{i\beta L_1 + L_2}
\begin{bmatrix}
\cos \phi_2 + i \sin \phi_2 \cos (2\Phi) & -i \sin \phi_2 \sin (2\Phi) \\
-i \sin \phi_2 \sin (2\Phi) & \cos \phi_2 + i \sin \phi_2 \cos (2\Phi)
\end{bmatrix}
\begin{bmatrix}
E_x \\
E_y
\end{bmatrix}
\tag{4.22}
\]

\[
= e^{i\beta (L_1 + L_2)}
\begin{bmatrix}
\cos \phi_2 + i \sin \phi_2 \cos (2\Phi) \\
-\sin \phi_2 \sin (2\Phi)
\end{bmatrix}
\tag{4.23}
\]

where the final result is the Jones vector assuming out linear, aligned input SOP. Assuming the detector is polarization independent, the third fiber segment is written as

\[
\begin{bmatrix}
E_x \\
E_\perp
\end{bmatrix}
= \begin{bmatrix}1 & 0 \\
0 & 0
\end{bmatrix}
e^{i\beta L_3}
\begin{bmatrix}
\cos \theta_a & \sin \theta_a \\
-\sin \theta_a & \cos \theta_a
\end{bmatrix}
e^{i\phi_3}
\begin{bmatrix}0 & 0 \\
e^{-i\phi_3} & 0
\end{bmatrix}
\begin{bmatrix}
E_x \\
E_y
\end{bmatrix}
\tag{4.24}
\]
\[ E_\parallel = e^{i\Phi} [ \cos \theta_a (\cos \phi_2 + i \sin \phi_2 \cos 2\Phi) - i \sin \phi_2 \sin 2\Phi ] \]  

(4.26)

where the total length of the fiber is \( L = L_1 + L_2 + L_3 \). We will now examine two cases of polarizer/analyzer combinations: (i) crossed polarizers and (ii) parallel polarizers.

Case (i): \( \theta_a = 90\text{deg} \) crossed polarizers

\[ E_\parallel = - i e^{i\Phi} \sin \phi_2 \sin 2\Phi = - i e^{i\Phi} \sin(k_o B T L_2/2) \sin 2\Phi \]  

(4.27)

The intensity at the detector is proportional to \( E_\parallel^* E_\parallel \), which is calculated to be

\[ I = \sin^2 \phi_2 \sin^2(2\Phi) \]  

(4.28)

Eq. (4.11) is rearranged to solve for the angle of axis rotation during squeezing

\[ \Phi = \frac{1}{2} \tan^{-1} \left( \frac{B_E \sin(2\theta_S)}{B_I - B_E \cos(2\theta_S)} \right) \]  

(4.29)

using the trigonometric identity \( \sin^2(\tan^{-1}(x)) = x^2/(1+x^2) \) permits eliminating the intermediate variable \( \Phi \), which has the simple result

\[ \sin^2(2\Phi) = \frac{B_E^2 \sin^2(2\theta_S)}{B_I^2 + B_E^2 - 2B_I B_E \cos(2\theta_S)} = \frac{B_E^2}{B_T^2} \sin^2(2\theta_S) \]  

(4.30)

which permits writing a simplified expression for the intensity output from the analyzer:

\[ I(B_I, B_E, \theta_S) = \frac{B_E^2}{B_T^2} \sin^2(2\theta_S) \sin^2(k_o B T L_2/2) \]  

(4.31)
We now have an expression for the intensity as a function of fiber birefringence, squeezer-induced birefringence, and the angle of the applied force. Note that there is zero intensity for the case of the squeezer being aligned to the fast or the slow axes.

Case (ii): $\theta_a = 0$-deg aligned polarizers

In the previous case, the polarizers were crossed, which means that little light gets through the system for unperturbed fiber. In this situation, the polarizers are aligned, and a large amount of light is passed by the analyzer. From (4.26), this is written as

$$E_\parallel = e^{jB_1L} (\cos\phi_2 + i \sin\phi_2 \cos2\Phi)$$

(4.32)

And the intensity detected now includes an additional term that is independent of $\Phi$

$$I = \cos^2\phi_2 + \sin^2\phi_2 \cos^2(2\Phi)$$

(4.33)

and a similar trigonometric expansion as in case (i) results in the following identity

$$\cos^2(2\Phi) = \frac{(B_1 - B_E \cos(2\theta_S))^2}{B_1^2 + B_E^2 - 2B_1B_E \cos(2\theta_S)} = 1 - \frac{B_E^2}{B_T^2} \sin^2(2\theta_S)$$

(4.34)

The intensity can now be expressed for aligned polarizer and analyzer transmissive axes

$$I(B_1, B_E, \theta_S) = \cos^2\phi_2 + \sin^2\phi_2 - \frac{B_E^2}{B_T^2} \sin^2(2\theta_S) \sin^2\phi_2$$

(4.35)

$$= 1 - \frac{B_E^2}{B_T^2} \sin^2(2\theta_S) \sin^2\phi_2$$

(4.36)

Compare this result to the case (i) result for crossed polarizers, Eq. (4.31). The above result includes a large steady-state signal that is not a function of squeezer angle. Thus, case (ii) will have an increased shot noise at the detector compared to crossed polarizer situation.
4.3.4 Sensitivity Maximization

To maximize the sensitivity to $\theta_s$, the $\sin^2 \phi_2$ term must be maximized for either Eq. (4.26) or Eq. (4.31). This will occur only for the condition that $\phi_2$ equals

$$\phi_2 = \frac{\pi L_2}{\lambda} \sqrt{B_1^2 + B_E^2 - 2B_1 B_E \cos \theta_s} = \frac{(2m-1)\pi}{2}$$  \hspace{1cm} (4.37)

where $m$ is an integer. Rearranging the above equation and squaring both sides yields

$$B_1^2 + B_O^2 - 2B_1 B_O \cos \theta_s = \left( \frac{(2m-1)\frac{\lambda}{2}}{\pi L_2} \right)^2$$  \hspace{1cm} (4.38)

Completing the square permits calculating an expression for optimal external birefringence $B_O$ that will maximize the sensitivity to $\sin^2(\theta_s)$. The normalized $B_O$ is

$$\frac{B_O}{B_1} = \cos \theta_s + \sqrt{1 + \cos^2 \theta_s + \left( \frac{(2m-1)\frac{L_B}{2}}{L_2} \right)^2}$$  \hspace{1cm} (4.39)

Thus if $B_E = B_O$, then the sensitivity is maximized. This can be accomplished if one knows a priori the squeezer angle by computing and applying the correct force. In practice this is difficult; the technician will typically adjust the squeezing force manually to maximize the signal. An easier method is to make the $\theta_s$-dependent terms small such that $B_O$ is approximately constant. This is done by making $L_2 \ll L_B$, or choosing a large $m$; this will cause it to dominate the equation and now the optimum $B_E$ is found to be

$$B_O = \frac{(2m-1)\frac{L_B}{2}}{L_2} B_1$$  \hspace{1cm} (4.40)

which is independent of the squeezer angle. This is an important constraint on the design.

Recalling the expression of birefringence induced by a squeezer as a function of external force per unit length from Chapter III, and rearranging Eq. (3.28), we get

$$B_O = \frac{n^3(p_{11} - p_{12})}{\pi D} \frac{F_O}{L_2}$$  \hspace{1cm} (4.41)
setting this equal to Eq. (4.36) and canceling terms and solving for optimum force

$$F_0 = \frac{(2m-1)}{2} \frac{\lambda_o \pi D}{n^3(p_{11} - p_{12})}$$  \hspace{1cm} (4.42)

Note that the force required is independent of the length of the squeezed region. Calculating $F_0$ for $m = 1, \lambda_o = 1.3 \mu m, D = 125 \mu m$ results in about $\approx 2.6$ kg of force for the condition that $L_2 \ll L_B$. The next maximum occurs at $m = 2$, which is at $F_0 = 7.9$ kg; at higher forces, there are problems with reaching the yield stress ($\sigma_{\text{max}} = 25$ kg/mm$^2$), especially for short $L_2$. The maximum force the fiber can endure is computed from Eq. (3.27)

$$F_{\text{max}} = \frac{\pi D L_2}{2} \sigma_{\text{max}} = 4.9 L_2 \text{ kg} \quad \text{where } L_2 \text{ is in mm} \quad (4.43)$$

For $F > F_{\text{max}}$, there is risk of damaging or breaking the fiber; the squeezer requires a large amount of energy to generate higher forces with increased thermal dissipation. For the $m = 2$ condition, Eq. (4.38) cannot be satisfied simultaneously with the requirement that $L_2 \ll L_B$. At $m = 1$ however, to be able to reach a 3-kg loading, it is possible to meet both conditions, and a minimum squeezed length would be $\approx 0.6$ mm.

Neglecting the breakage problem for now, assuming that we are squeezing the fiber over a short length with the optimum force results in the $\sin^2 \phi_2$ term being unity

$$I_{\text{opt}} = \frac{B_E^2}{B_T^2} \sin^2(2\theta_S) \quad (4.44)$$

Now the idealized behavior will be examined for the two possible near-alignment situations. In reality, the deviation from optimum force will result in a reduction of the signal from the optimized value. The denominator $B_T^2$ has the following extrema for the two possible alignment conditions:

$\theta_S = 0^\circ \hspace{1cm} B_T^2 = (B_I - B_E)^2 \quad (4.45)$

$\theta_S = 90^\circ \hspace{1cm} B_T^2 = (B_I + B_E)^2 \quad (4.46)$
which intuitively makes sense, because at 0 deg, the two stresses tend to cancel. For a small misalignment, $B_T$ is assumed constant and the intensity can be approximated by a parabolic function of the misalignment angle, so that in these near-alignment regions

$$\theta_S = 0^\circ \quad I = \frac{4B_E^2}{(B_1 - B_E)^2} \theta_S^2$$

$$\theta_S = 90^\circ \quad I = \frac{4B_E^2}{(B_1 + B_E)^2} (\theta_S - 90^\circ)^2$$

Note that the two parabolic relationships have differing scale factors, with the 0-deg case intensity magnified by $[(B_T + B_E)/(B_T - B_E)]^2$. This is how one discerns the slow from the fast axis as one varies the $\theta_S$ angle. Figure 4-20 is a plot of data in 1-deg increments near the slow and fast axes, demonstrating the wider parabola observed for the fast axis.

![Figure 4-20. Typical Data for Slow and Fast Axis Near Alignment.](image)

In this experiment, the force is reoptimized for the two regions by adjusting the current to the squeezer solenoid. The exact alignment condition for the slow axis can be estimated within ±0.5 deg without much trouble directly from the plot. Increasing the gain of the lock-in permits ±0.1 deg resolution around the null; curve-fitting the null region to a parabola should permit even better precision. The resolution limit of the setup...
is in the stepper motors that rotate the fiber, which have a 50,000 steps/revolution, which corresponds to a ±0.007-deg angle for ±1 step.

4.3.4 Squeezer and System Design

Fiber-optic squeezers have been employed for numerous applications besides principal axis alignment for coupler fabrication. Measurement of opto-elastic constants [48], birefringence [49, 50], and distributed cross-coupling points [51] have been reported. The characterization [52] and control [53] of SOP at arbitrary points along the fiber with negligible loss, have also been demonstrated. Squeezers have also been used for phase modulators [54, 55] and birefringence modulators [56] for IFOG applications.

The squeezers in the above list often consisted of piezoelectric elements with manual or spring preloading; alternatively, the fiber could be placed between the pole pieces of a suitable electric relay. To achieve milliradian errors, the squeezer must apply the force in a known direction with respect to the block surface, and the fiber principal axes. The force applied needs to have a fine dc adjustment with a small ac dither, and must be distributed over a known length of fiber. After the alignment is accomplished, the fiber must be transferred to the substrate groove without disturbing the alignment.

The squeezer assembly consists of a set of four flexible pivots that are highly accurate and have zero hysteresis and minimal friction over a limited range. Figure 21 is a diagram of a squeezer similar to that used in this study, which has a highly polished flat jaw and a curved jaw to pinch the fiber. This design is made to squeeze the fiber over a well-defined length in a purely vertical direction. The flat upper jaw of the squeezer is free to move up and down as shown by the large arrow, and is attached to a voice coil at the top of the upper jaw.

There is an L-shaped frame that supports the lower curved jaw, two of the flexible pivots, and a strong permanent magnet. When a current passes through the voice coil, the jaws close and remain highly parallel to each other and the mounting surface of the frame. A pair of banana jacks facilitate applying current to the squeezer; not shown is a thermocouple that monitors the temperature of the voice coil. The whole assembly rests upon a precision xyz stage, which combined have < 300 microradians angular error over
their range. The aligned fiber may then be dropped into the substrate groove by translation of the squeezer assembly without disturbing the alignment.

Figure 4-21. Squeezer Mechanical Assembly.

ac modulation is used to improve sensitivity by use of synchronous detection. Assuming that there is a small ac component superimposed the squeezer force

\[ B_E \rightarrow B_E + B_{AC} \quad \text{where} \quad |B_{AC}| \ll B_E \]  

where \( B_{AC} \approx \cos(2\pi f_m t) \) and \( f_m \) is the modulation frequency. This has been done in the past by cementing a piezoelectric element under the lower jaw of the squeezer and applying a high voltage, supplied by a RF amplifier. This requires hazardous RF voltages, and the power amplifier must drive a nearly pure capacitive load, which reflects nearly all of the energy back to the amplifier where it must be dissipated. An expensive (and heavy) RF power amplifier is required, or a high-frequency piezo driver is needed to drive the squeezer, and electrical interference can be a problem.

A much simpler way is to add a small ripple current to the voice coil driving current -- only a 10-mA sinusoidal current is required to produce a large output signal, comparable to the piezo technique. Since the force-current relationship of the voice coil is known, it is possible to calibrate the ac force in this squeezer design. Taking Eq.
(4.26), and plugging in Eq. (4.44) the ac modulation, which becomes approximately

\[ I(B_1, B_E, B_{AC}, \theta_S) = \frac{B_E^2 + 2B_EB_{AC}}{B_T^2} \sin^2(2\theta_S) \sin^2(k_0B_TL_2/2) \]  

(4.50)

for small ac modulation. Expanding the above result and throwing away the dc component, the ac component, which is selected by a high-pass filter or a lock-in amp is

\[ I_{AC} = \frac{2B_EB_{AC} \sin^2(2\theta_S)}{B_T^2} \sin^2\left(\frac{k_0B_TL_2}{2}\right) \]  

(4.51)

This expression is plotted in Figure 4-22 for a 2-mm beat length \( B_1 = 0.00065 \) and a constant force, which incidentally is not the optimum force. The length of the squeezed region is assumed to be 0.5 mm, and the external birefringence \( = 0.0006 \) which is slightly less than \( B_1 \).

![Figure 4-22. ac Signal vs Squeezer Angle.](image)

Because the SAPs polish at a faster rate than the cladding, the orientation of the fiber will affect the surface flatness. If the slow axis is aligned such that \( \theta_S = 0^\circ \), the resulting substrate will have better flatness because one of the SAPs will be completely polished away. If the slow axis is aligned parallel to the block surface, i.e., \( \theta_S = 90^\circ \), both SAPs will be polished less than halfway through, which would relieve compressive stress normal to the surface [57]. The result after polishing is a shallow trench along either side of the core where the SAPs meet the polished surface, which would make OCB difficult.
Thus, the fast axis is aligned parallel to the substrate surface, which, with this squeezer configuration, is near $\theta_s = 0^\circ$, the more sensitive of the two nulls.

Figure 4-23 is a schematic of the setup with its associated electronics and optics. The fiber is held between a pair of 50,000 steps per revolution stepper motors, which are mounted in a fixed position, and are ganged electrically. Motion of the steppers toward and away from each other permits application of a slight tension on the fiber. Dashpots damp out any motion in the tensioning system that could break the fiber, and a pneumatic loading system compresses the tensioning springs for loading.

Figure 4-22. Principal Axis Alignment System and Electronics.

To align the fiber, the operator unwinds a 2-m length of fiber to make a coupler with 1-m pigtails, and strips 2 cm of fiber jacket at the center point, and the stripped region is cleaned. The stripped region is clamped into the stepper motor subassembly and the loading subsystem is released, applying tension to the fiber. The ends of the fiber are cleaved and are placed into the launching apparatus and the laser output is launched into the fiber. The polarizer and analyzer are alternately rotated until minimum light is observed, i.e., they are aligned to orthogonal principal axes. The squeezer dc voltage is adjusted while monitoring the ac output on an oscilloscope, such that the first of the
maxima is reached. After waiting a few time constants, the output of the lock-in amplifier is stable and can be recorded. The fiber is unclamped, rotated 10 deg, then the force is reapplied and data is taken every 10 deg until a pair of nulls are recorded. The fiber is rotated to the narrower of the two nulls, and now 1-deg increment data is taken, which permits its location of the slow axis. The fiber is moved out of the jaws and placed into the block groove, being careful not to disturb the alignment established. The fiber is ready for adhesive application to attach it to the substrate.

The effect of the squeezing upon the fiber strength was measured with an Instron tester for stripped and cleaned fibers, and fibers that were squeezed with the above procedure, but not mounted in substrates (See Table 4-8). For five samples each, there was a slight degradation of the strength, and no discernible degradation of the polarization cross-coupling of the fiber. The fiber is proof tested at 100 kpsi, and the strength reduction is minor compared to that introduced by polishing.

Table 4-8. Effect of Squeezing upon Fiber.

<table>
<thead>
<tr>
<th>Effect</th>
<th>Unsqueezed</th>
<th>Squeezed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average strength</td>
<td>124 kpsi</td>
<td>74 kpsi</td>
</tr>
<tr>
<td>Avg. cross coupling</td>
<td>&lt; -35 dB</td>
<td>&lt; -35 dB</td>
</tr>
</tbody>
</table>

In summary, several techniques have been discussed that permit alignment to <1 deg which would produce couplers having -35 dB of cross-coupling. The acousto-optic technique has been outlined that incorporates several major improvements over the state-of-the-art in this type of alignment method.

4.4 Fiber-to-Groove Bonding

The preparation, handling, and attachment of the fiber to the substrate are described in this section.
4.4.1 Fiber Stripping and Cleaning

Each fiber is stripped mechanically, using a new razor blade and mirror for backing or carefully trained fingers. The length of the stripped region must be controlled to within 2 mm to fit within the strain relief areas of the two block designs. Results of a strength test for five stripped fibers are an average of 124 kpsi, with a minimum of 91 kpsi. Since the virgin fiber strength is specified to be 100 kpsi, it can be concluded that there is no degradation induced by the stripping operation. Once stripped, the fibers must promptly be processed to minimize water absorption from the air, which would reduce strength levels significantly.

If the fiber surface is not completely free from organic contaminants, delamination with the bonding adhesive can occur. The fiber is cleaned with a minimal amount methyl ethyl ketone (MEK) using a Q-tip, being careful to avoid wicking the solvent under the fiber jacket. MEK will remove any residual primary coating material from the fiber; next, Freon is next used with a clean-room cloth to remove any grease from fingerprints and the like.

4.4.2 Fiber-to-Groove Bonding

The adhesive itself is described in Section 4.1, and cleaning of the substrates is described in Section 4.2 on groove generation. After the principal axes are identified and the aligned, the fiber must be bonded into the substrate groove with epoxy. It is crucial to minimize bubbles in the adhesive, and to have a clean fiber and substrate prior to bonding. Steps must be taken to ensure consistently mixed, qualified, bubble-free epoxy samples. To maintain quality control, premixed, degassed, and dated samples of epoxy are made up, and are stored in miniature canisters in the freezer. Samples are taken from each lot prepared, and are cured and tested for their mechanical properties, to ensure proper mixing and quality conformance. The process that is followed to perform the bonding procedure is as follows:

(i) Thaw adhesive sample before opening to ambient humidity.
(ii) Press adhesive into groove with spatula while avoiding air bubbles.
(iii) Carefully lower fiber into the groove with the alignment system.
(iv) Liquify and set epoxy with heat gun at 8-in distance.

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(v) Adhesive is cured when color changes from straw to dark brown.
(vi) Remove substrate from the squeezer and place on hot plate.
(vii) Precure epoxy at 50°C for 30 minutes.

4.4.3 Fiber Stress-Relief Application

The Norland 68 adhesive is applied to the wide groove areas, forming a low-modulus stress-relief area. This material is cured with the following cure schedule:

(i) Wear protective UV goggles for high-intensity cure.
(ii) Ultracure™ UV flux was >500 mW/cm^2 at 365 nm.
(iii) Precure for 60 seconds at 2-3 inches.
(iv) Block can be safely removed from fixture.
(v) Blak-Ray™ UV flux was >1 mW/cm^2 at 366 nm.
(vi) Continue cure for 15 minutes at 8-12 inches.
(vii) Post-cure at 50°C overnight.

A witch oiler works well to apply controlled amounts of this adhesive. The post-cure overnight bake step also serves to fully cure the epoxy in the central groove. Now the substrate is ready for the polishing process.
4.5 References

1. Vogel Optical Co., POB 95, Norwood, MN 55368.


27. Mark V Laboratories, POB 310, 18 Kripes Rd., East Granby CT 06026.


34. Centre Suisse d'Electronique et de Microtechnique S.A., Maladière 71, CH-2007 Neuchâtel Switzerland.


Once the fiber substrates are prepared as described in Chapter IV, the fiber substrates are polished to remove roughly half of the cladding and to expose the evanescent field. The substrate, when polished to the desired depth, is referred to as a half-coupler, and is then assembled with a second half-coupler into a finished device. It is these process steps that will be discussed in this chapter.

Half-couplers can be used for a number of applications besides fiber coupling. For example, interaction of the evanescent field with different materials placed upon the half-coupler surface has permitted the demonstration of chemical sensors [1]. By placing gain media on the surface, lasing has been demonstrated [2]. Phase modulation has also been demonstrated using an electro-optic film [3], and intensity modulation with liquid crystal overlays [4] has also been reported. Several techniques for the fabrication of polarizers have also been mentioned using birefringent crystals [5], nemantic liquid crystals [6], thick metal films [7, 8], thin films [9, 10], and multilayer metal/dielectric films [11] to induce differential attenuation of the two polarization modes. The interested reader is referred to the above references for further details, as these applications will not be discussed further.

5.1 Substrate Polishing

The steps taken during polishing the fiber and substrate largely determine the performance and assembly yield of polished couplers. A technique will be outlined that has been successful in polishing high-quality half-couplers for OCB assembly, but is by no means fully optimized. The process is carried out in four steps: (i) grinding, (ii) lapping, (iii) polishing, and (iv) superpolishing. They are sequenced to remove the maximum material with the coarsest grit initially, switching to finer grades of polishing compounds. The polishing times are such that the residual surface roughness and subsurface damage from the previous step is completely removed, while allowing reasonable throughput.
5.1.1 Polishing Fixturization and Blocking

Figure 5-1 is an illustration of the polishing jig, which is made in Scotland [12], that is used for the polishing of the substrates. The jig consists of a precision piston assembly that has a steel plate attached to the central shaft, which moves up and down with respect to the jig. The piston plate is the middle disk in the drawing, and is tapped with various mounting holes for sample mounting tooling.

![Figure 5-1. PP5 Polishing Fixture (Courtesy Logitech Ltd.)](image)

At the bottom of the fixture is a cast iron conditioning ring with radial grooves on the bottom surface that support the jig while it sits on the polishing wheel. The conditioning ring helps to stabilize the fixture and ensures that the polishing wheel maintains flatness, even though there is nonuniform material removal from the polishing
wheel due to the annular path of the substrate. At the top of the jig is a ring that serves as a handle and a dial gauge with 2-μm resolution that measures piston motion, indicating material removal from the sample relative to the conditioning ring.

This type of polishing jig is well-suited for polishing noncurved samples [13]. A number of optional accessories such as vacuum chucking, sample tilt adjustment, and automatic monitoring are available but are not used in our polished coupler fabrication process. Not shown in the figure is an adjustment to control the polishing pressure from 0 to 2800 grams, and a mechanical depth stop.

The fiber and substrate are mounted on sample holder, which is bolted to the underside of the piston plate. After an early investigation of vacuum-mounting and clamp-mounting, it was decided to use the traditional wax-mounting techniques to reduce the problems with stress and movement. Low-stress mounting is one of the keys to obtaining a high degree of flatness in parts with moderate to high aspect ratios, which required the development of an advanced sample holder. If stress is induced into the part during the mounting operation, the flat surface obtained by polishing will warp when the part is released from the mounting fixture. The sample holder featured three-point mounting, a thin annular diaphragm, and is machined from Invar, which is a low-expansion metal.

The glass blocks are held to the sample holder with paraffin wax, which is heated with a programmable hot plate. The wax is precoated upon the surface of the sample holders in a thin layer beforehand to facilitate the mounting operation. The temperature cycle has been chosen to gradually warm and cool the sample holder at a rate of roughly 0.5 °C per minute, as illustrated in Figure 5-2.

Similar precautions must be used when curing adhesives to avoid "freezing" stress into the part. The programmable hot plate is an ideal way to ensure the consistency of these steps and to avoid the unintentional thermal shocking of the parts.
Blocking material is used to surround the parts to ensure flatness across the half-coupler and to eliminate rounding at the edges of the half-coupler [14]. Figure 5-3 illustrates different blocking techniques (shaded regions) used when setting up adjustable coupler substrates and OCB substrates, which require higher flatness. The blocking material is made from ungrooved substrate blocks, or scrap substrates that have been flipped over, and are later discarded after processing.

Figure 5-3. Blocking Techniques for Adjustable and OCB Substrates.

Figure 5-4 is a photograph of the underside of the polishing jig, showing the conditioning ring, fiber sample holder, and a pair of substrates with blocking material. There is a lip surrounding the periphery of the holder that retains the fiber pigtails of the substrate during the polishing operation. This particular sample holder incorporates the mounting of a coil of fiber for spliceless resonator fabrication. Figure 5-5 is a high-magnification photograph of a partially polished fiber in the groove, showing the epoxy regions on either side of the fiber.
Figure 5-4. Spliceless Resonator and Blocking on Polishing Jig.

Figure 5-5. Scanning Electron Micrograph of Partially Polished Half-coupler.
5.1.2 Grinding and Lapping

The objects of the grinding and lapping steps are to remove the bulk of the material and to reduce the surface roughness to a level that is suitable for polishing. Grinding is performed on a 30-cm diameter ungrooved cast iron polishing wheel as indicated in row 1 of Table 5-1. Aluminum oxide polishing compound with a 3-µm grit size is mixed with deionized water to make a grinding slurry mixture that permits rapid material removal. Initial attempts at grinding with diamond and other hard materials, such as silicon carbide or boron carbide, resulted in a high degree of scratching and fiber breakage. It is desirable to use the softest grinding compound possible when dealing with highly stressed samples such as PM fiber. For general reviews of polishing techniques and mechanisms, refer to References [15-17].

<table>
<thead>
<tr>
<th>Process Step</th>
<th>Polish Pad</th>
<th>Polish Compound</th>
<th>Conc:DI Water</th>
<th>Speed rpm</th>
<th>Pressure g/mm²</th>
<th>Time min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grind</td>
<td>Cast Iron</td>
<td>3 µm Al₂O₃</td>
<td>1:6</td>
<td>30-35</td>
<td>3</td>
<td>15-30</td>
</tr>
<tr>
<td>Lap</td>
<td>Polytron™</td>
<td>3 µm CeO₂</td>
<td>1:8</td>
<td>30-35</td>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>Polish</td>
<td>Polytron™</td>
<td>Syton™</td>
<td>pH ≈ 11</td>
<td>30-35</td>
<td>3</td>
<td>AR</td>
</tr>
<tr>
<td>Superpolish</td>
<td>Polytron™</td>
<td>Syton™</td>
<td>pH ≈ 11</td>
<td>5-10</td>
<td>0.2</td>
<td>4 h</td>
</tr>
</tbody>
</table>

To prepare the cast iron grinding wheel, a smaller conditioning wheel, also made of cast iron, is run on the grinding wheel to true the flatness of the polishing wheel. This is typically done for 10 minutes at the start of the day, and the flatness of the conditioning wheel, which mirrors the flatness of the grinding wheel, is checked with a spherometer. Flatness measurement and control will be discussed in the next section.

There is always damage to the amorphous molecular structure that is induced by the grinding, which is known as subsurface damage (SSD). In the region just below the surface, glass density, refractive index, and stress variations can exist even for microscopically smooth polished surfaces. SSD phenomena will be evidenced as scattering, absorption, and altered mechanical properties of the finished part [18]. It is particularly troublesome in RLG mirrors and fiber resonators, because of the induced scattering that tends to couple the counterpropagating optical waves. In couplers, it also induces losses and polarization cross-coupling, and has the potential to reduce the...
reliability of the device. The mechanism of SSD formation is not well understood, but has been empirically correlated with high polishing rates, large abrasive grit sizes, and hard abrasives and polishing pads. One difficulty in detecting SSD is that there is no practical technique for nondestructive measurement; SSD can be made visible by a short etch in dilute hydrofluoric acid, which ruins the part. Other suspected culprits for SSD include poor particle sizing, machine vibration and chatter, and contamination. A poor choice of initial processing steps will often guarantee a large amount of SSD; the rule of thumb is that SSD layer is approximately 4x the surface roughness [19].

Polishing pressure is controlled by the adjustment facility on the polishing jig mentioned earlier, and the pressure is calibrated with a load-cell prior to grinding. The load cell fixture rigidly supports the conditioning ring and measures the downward force on the central piston assembly of the polishing jig. This step is important to achieve consistent polishing rates. The grinding operation initially removes the excess epoxy on the surface of the substrate, and is continued until the fiber cladding is reached. The fixture is then removed from the polishing machine, and carefully cleaned with water to remove all traces of the grinding compound before proceeding to the finer abrasives. It is also important to wash any remaining grit from the grinding wheel to avoid scratching when switching to the next finer grit. The polishing and superpolishing steps are performed on a separate machine to further reduce the probability of cross-contamination. If the processing is temporarily stopped, the water must never be allowed to dry upon the glass surface, because it will result in the staining of the glass.

The lapping is an intermediate step that uses a cerium oxide compound, producing a smoother surface finish. The CeO2 has the advantage of breaking down into finer and finer particles as the glass is worked [20, 21], producing a superior ultimate finish at the expense of polishing rate. This avoids the need to switch to intermediate grit sizes, and removes subsurface damage as it is created. Both the grinding and lapping steps are characterized as free-abrasive grinding, and the material removal rate during these operations is described by the Preston wear equation, which is given as

\[ \Delta x = C_P P V_L \Delta t \]  

(5.1)

where the material removed from the sample is \( \Delta x \) during time \( \Delta t \) [22]. The variable \( P \) is the polishing pressure, and \( V_L \) is the average relative velocity between the polishing wheel and the part. All of the other parameters are lumped in the Preston coefficient \( C_P \),
which has units of area/force and is fixed for a given material and polishing technique. The velocity of the part with respect to the polishing wheel is simply given by

\[ V_L = \pi D_{\text{avg}} \Omega_L \]  

(5.2)

where \( D_{\text{avg}} \) is the average diameter that the sample traces around the polishing wheel, and \( \Omega_L \) is the rotation rate of the lap. Hydroplaning is avoided by holding \( \Omega_L < 40 \text{ rpm} \) with our grinding equipment.

5.1.3 Polishing

Several techniques have been reported in the literature for polishing fiber optic couplers. Methods have included diamond paste [23], carborundum [24], cerium oxide [25], diamond-impregnated wheels (i.e., bound-abrasive) [26], and other polishing compounds. Selecting a polishing compound whose hardness is comparable to the sample produces a very smooth surface with low stress and subsurface damage [27, 28]. We have chosen a colloidal silica polishing solution, which is well-known to produce a superior finish on a number of materials [29-32]. The colloidal silica used in our process is manufactured under the trade name Syton\textsuperscript{TM}, and consists of a dispersion of nearly spherical, \(~11-\mu m\) particles of SiO\textsubscript{2}. Syton\textsuperscript{TM} is a water-based suspension with a high pH, and has very uniform particle size distribution and high surface area. The high pH stabilizes the dispersion, and prevents agglomeration [30]. Two precautions when using colloidal silica are to avoid freezing and to not allow the solution to evaporate. If these occur, crystallization can result, causing a high degree of scratching if the dispersion is used.

The traditional technique of polishing with a pitch polishing pad has been replaced by a Polytron\textsuperscript{TM} pad material [33] with a high degree of success. When properly conditioned, synthetic polishing pad materials produce excellent surface quality and flatness without the flow, mess, and toxicity problems associated with pitch [22]. The conditioning procedure consists of running the wheel with a diamond-impregnated conditioning wheel for 10 minutes at 20 rpm. This step removes any glazing and crystallized silica from the polishing wheel. Figure 5-6 illustrates the microscopic operation of the Polytron\textsuperscript{TM} polishing pad, which consists of a napped polyurethane material, with a closed-cell porous microstructure. The figure illustrates the porous
microstructure of the polishing pad, and the colloidal silica particles, which are represented by the small triangles.

Figure 5-6. Polishing Mechanism of Polytron Pad with Colloidal Silica.

Polishing with aqueous abrasive carriers produces a hydrated silica layer at the surface, which is denoted by the dashed lines. The chemical reaction at the surface is

\[ \equiv\text{Si-O-Si}\equiv + \text{H}_2\text{O} \leftrightarrow 2(\equiv\text{Si-OH}) \]

where the siloxane bonds of the amorphous bulk SiO\textsubscript{2} are broken to form the hydrated surface layer \([29, 34]\). The Syton\textsuperscript{TM} particles readily adhere to the hydrated silica layer, and are "wiped off" as the pores on the raised nap sidewalls are dragged across the substrate surface. This combination of chemical and mechanical processes effectively removes the hydrated layer from the high spots without inducing damage that would be incurred by a purely mechanical polishing compound. The motion of the textured surface over the sample produces a pumping action that circulates fresh Syton\textsuperscript{TM} into the pores.

As polishing continues, the hydration-deposition-wiping process is repeated until the surface is planar on nearly a molecular level. An ultra-smooth surface is obtained by reducing the polishing pressure and speed for the last fraction of a micron, as described in Table 5-1. With this process, a scratch-free surface is obtained with roughness measured to be 20Å rms over several square millimeters. Figure 5-7 shows a pair of finished substrates, one for adjustable coupler use and one for OCB coupler assembly. The good results are due to the combination of the polishing fixture, Polytron\textsuperscript{TM} polishing pad, Syton\textsuperscript{TM} polishing solution, and the skill of the polishing technician. There are no features visible after a short etching step, which indicates no subsurface damage.
Figure 5-7. Polished Half-Couplers for Adjustable and OCB Devices.

Figure 5-8. Surface Damage Effect at 1 μm and 5 μm after SAP Boundary.
An interesting phenomena, first pointed out by R. Sutherland that we have not seen reported in the literature, is a surface fracture effect when the first SAP boundary is reached during the grinding process. Figure 5-8 shows photomicrographs of the fiber in the groove where polishing has been started early to show the damage effect. The left-hand figure is approximately 1 μm beyond the SAP boundary, which shows a damaged fiber surface; note the SAP outline and that the substrate is not rough like the fiber. The right-hand photo was taken roughly 5 μm beyond the SAP boundary, and shows that the effect is truly at the surface and is removed by subsequent polishing. It is worth commenting that no sudden change in the optical properties is observed when this condition occurs. The cause of this phenomenon is not well understood; it is hypothesized that it is a result of a shock wave produced when the last of the surrounding cladding is removed from the SAP. The removal of the constraining material suddenly releases pent-up stress in the highly-doped SAP that is observed as the surface damage. If this were a contamination effect, there would have been damage on both the substrate and the fiber surface. Similar odd effects have been related to the author for polishing PM fiber preforms and other highly-stressed materials [35]. This curious phenomena does not affect the device, and normally passes unnoticed due to the high roughness during the grinding stage of the process.

The polarization properties of PM fiber half-couplers are observed to change as grinding and polishing proceeds [36, 37]. Typically, the polarization cross-coupling of a good unpolished fiber/substrate is on the order of 40 dB. This value is seen to reduce to a worse value between 20 and 30 dB [38, 39] as the fiber is polished toward the core. This is due to the increasing stress relief [40] and associated perturbation of the birefringence. It has long been known that polishing induces undesired surface changes in the glass [41]. A layer of increased density and refractive index, which is 10 to 20Å thick, has been observed [42, 43]. For Pyrex, the index increase has been measured to be on the order of 1 to 2 percent [44]. As mentioned earlier, this layer could in theory affect the operation of the coupler [45], but has not been investigated.

Other advanced polishing techniques reported for general use have included Teflon pad polishing [46] and float polishing [47], which is done while hydroplaning. These techniques have demonstrated roughness levels in the 1-2Å rms range, but to the author's knowledge have not been tried with polished couplers.
5.2 Process Monitoring

There are a number of metrology techniques that can be used during polished coupler fabrication to monitor the progress of the polishing and condition of the half-coupler. It was decided to summarize all of them in a separate chapter in their rough order of use rather than to include them in the polishing chapter.

5.2.1 Ellipse Measurement

The polishing depth can be initially estimated by the so-called ellipse measurement [36, 48], which is performed with a microscope. The surface of the half-coupler is depicted in Figure 5-9, and the feature that is measured is the minor axis of the ellipse, denoted as $y_e$.

$$x_o = \frac{1}{2} \sqrt{d_c^2 - y_e^2}$$

Unfortunately, the uncertainty in the fiber cladding diameter, $d_c$, makes this test less reliable as the core is approached. The fiber manufacturers typically specify a ±1 to ±3-μm tolerance on cladding diameter, which makes an accurate estimation of $x_o$ difficult. Care must be taken to calibrate the filar eyepiece with a microscopically-ruled scale, and the microscope must be set up to minimize parallax errors.
5.2.2 Flatness Control and Measurement

The control of the overall surface flatness is most critical during the initial grinding and lapping stages of the processing. If this is not achieved early on, gross deviations from the flat condition cannot be corrected during the polishing step, due to the small amount of material removed. Flatness of the cast iron grinding wheel is controlled during the conditioning step, prior to grinding any components. After the conditioning step has been performed, the conditioning wheel will have the opposite curvature of the grinding plate. This curvature may then be measured by a spherometer, consisting of a three-point stand and a dial gauge, which is also equipped with a spherical ball of diameter $2r_b$, as illustrated in Figure 5-10.

![Figure 5-10. Spherometer upon Inverted Conditioning Wheel Surface.](image)

To calibrate the spherometer, it is placed upon a flat surface plate, and the gauge is zeroed. The conditioning wheel is removed from the polishing wheel and is inverted, and the spherometer is placed on it, and the dimension $x_s$ is read from the gauge. The expression can be calculated, if the bearings are spaced a distance $y_s$ apart, to be

$$R = \frac{y_s^2}{2x_s} + \frac{x_s}{2} - r_b \approx \frac{y_s^2}{2x_s} \quad (5.4)$$

where the approximation is for the near-flat condition $x_s \ll y_s$ and $r_b \ll R$. This simple device is useful for measuring gross out-of-flat conditions with simple curvature. The flatness can be corrected by adjusting $D_{avg}$ inward or outward on the polishing plate.
5.2.3 Stress-Rod Observation

The boron-doped stress rods polish at a slightly higher rate than the cladding, which results in a sub-µm depression, which is shown in Figure 5-5 for a partially polished coupler. The polishing step was started deliberately early to reduce the roughness and to enhance this feature for that photograph. The progress of the polishing is monitored by observing the stress-rod depression under a microscope with a Nomarski phase contrast attachment [49]. After the upper stress rod is observed to be polished away, there is some distance remaining to the core boundary (See Appendix I). At that point, superpolishing is initiated by reducing polishing speed and pressure as described in Table 5-1, and the oil drop test is commenced.

5.2.4 Surface Flatness

Microscope objective interferometers are a valuable tools for examining the flatness, edge rounding, and fiber standoff near the fiber. Michaelson, Linnik, and Mireau interferometers can be procured for most microscopes, and are invaluable tools in this type of work. Figure 5-11 is an example of a Mireau interferogram of a polished fiber surface; our system has a green filter, which produces fringes having roughly 250-nm spacing. To remove the 2π ambiguity, the filter is removed, and white-light fringes are observed instead of the fringes produced by the narrow-band green light.

Another tool occasionally used is a stylus profilometer, which can measure the roughness statistics and provide a trace of the surface profile in one dimension. Figure 5-12 is an example of a polished fiber region in a Pyrex half-coupler. Note the two depressions that correspond to the epoxy regions beside the fiber and the slight height difference between fiber and substrate.

To look at the surface flatness over the whole substrate, a large phase-stepping interferometer is employed. This system is made by Zygo Inc., and can also yield quantitative data in the form of 3D computer plots of the surface height [50]. Figure 5-13a is an interferogram of the substrate and blocking material, and 5-13b shows the same half-coupler after it has been dewaxed from the polishing fixture.
Figure 5-11. Mireau Interferogram of Polished Fiber in Substrate.

Figure 5-12. Dektak Stylus Profilometer Plot of Polished Fiber in Substrate.
Figure 5-13a. Zygo Interferogram of Mounted Half-Coupler and Blocking.

Figure 5-13b. Zygo Interferogram of Unmounted Half-Coupler.
Note that the flatness has been retained upon removal from the polishing jig. This is an expensive tool, but less quantitative techniques such as optical flats and various interferometers can be used to measure overall flatness.

5.2.5 Oil Drop Test

Once the polished surface is in close proximity to the fiber core, a more exact gauging technique is required to measure the core-to-surface distance. The oil drop test was recognized early on as an effective method for estimating the remaining polishing time [51]. It consists of measuring the adsorption induced by a drop of index-matching oil (IMO) that is placed onto the half-coupler. If the refractive index of the oil is higher than $n_{\text{eff}}$, the oil will act as a mode sink, and power will be removed from the core. The advantage of the oil drop test over geometrical methods is that it is independent of concentricity and core and cladding diameter variations. The experimental setup for this test is shown in Figure 5-14, which consists of a laser source, launching optics, and a photodetector on the right.

To perform this test, light is launched into one of the fiber pigtails of a clean half-coupler, and the power exiting the other pigtail is recorded with an optical power meter. A drop of IMO is then placed upon the polished fiber surface, and the power is then remeasured. The difference of the measurements is the loss, $\gamma_{\text{dB}}$, induced by the oil, which is a strong function of the distance to the polished surface. The rather complex expression for the induced fractional intensity loss has been calculated to be [52]

\[
\gamma \propto \left(1 - \frac{v^2}{V_3^2}\right) \int_0^1 \frac{\sqrt{1-x^2}}{4(V_3^2-v^2)x^2 + v^2} \exp\left(-\frac{2x_0}{r_c} \sqrt{(V_3^2-v^2)x^2 + v^2}\right) \, dx \quad (5.5)
\]

where the step-index, weak guidance assumptions have been made. The transverse propagation constant, $v$, can be gotten from equation (2.5) and the fiber design, and $V_3$ is...
the equivalent V-number for the refractive index oil layer, which is given by

$$V_3 = k_{o}r_0 \sqrt{n_{oil}^2 - n_{clad}^2}$$  \hspace{1cm} (5.6)

Figure 5-15 lists some data for various polishing times, including a theoretical plot using the parameters for PANDA fiber, which was fitted to the data in the vertical direction. Both curves show the cutoff effect that occurs when the oil has a refractive index below the cladding index.

![Figure 5-15. Oil Drop Test Data and Theory.](image)

For a constant index, loss is a linear function of polishing time, i.e., proportional to material removed. We have chosen the Cargille "1.464" oil, which has an index of 1.454 at the 1.3-μm wavelength region. The target losses for polishing that are used for our process are given in Table 5-2 for 50/50 splitting couplers and resonators. In this manner, it is not necessary to know the absolute polishing depth, which is never measured in practice. The only method for absolute distance measurement is to section and polish the half-coupler, and to etch the surface to highlight the core. A variation of the oil drop test allows the fiber profile to be characterized by measuring loss as the oil flows across the surface [53]. Yet another method uses a specially-prepared MCVD preform whose refractive index varies linearly as a function of position, that is placed upon the half-coupler [54]. The silica block has an index variation from −0.4% to +1.3% from the intrinsic index of refraction of glass, and permits continuous adjustment in a deterministic manner.
Table 5-2. Target Oil Drop Test Loss.

<table>
<thead>
<tr>
<th>Device</th>
<th>Target Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coupler</td>
<td>10-15 dB</td>
</tr>
<tr>
<td>Resonator</td>
<td>15-20 dB</td>
</tr>
</tbody>
</table>

Several attempts have been made to automate the polishing process by measuring the induced loss by polishing in situ [24, 55]. Commercial devices are becoming available to permit the control of polishing depth [12] and flatness [12, 56].

5.3 Cleaning and Assembly

The steps for assembling the completed half-couplers will now be described for adjustable and OCB couplers. The cleaning and mating steps are virtually the same for both adjustable and optically contacted couplers.

5.3.1 Half-Coupler Cleaning

After repeating the heating cycle of Figure 5-2 to release it from the lapping jig, the finished substrate is ready to be cleaned in preparation for mating. Small amounts of wax can be removed with a Q-tip and toluene, being careful to not damage the surface or the fiber jacket. Toluene is harmful, so it should be used under a fume hood while wearing chemically-resistant gloves. Cotton balls or Webri™ cotton pads are useful for gross contamination removal. It is very important to clean the half-coupler immediately after polishing is completed because water and Syton™ will leave persistent stains upon the block surface. Other contaminants, such as fingerprints and spittle, are known to etch glass in 48 hours [57] and should be avoided. Petri dishes were found to be suitable for storage of half-couplers, and were stored in a nitrogen-purged drybox to further reduce the chances of surface contamination.

High-quality solvents need to be used to avoid leaving particulates and other contaminants. Spectral-grade solvents that have <2 ppm nonvolatile residue are recommended for this purpose. Small Teflon squeeze bottles are used to store the solvents to further reduce the risk of contamination. Commercially-available cotton
swabs contain oils that must be extracted to avoid contaminating the half-coupler at later stages of the cleaning process. The cleaning operation is done in a class 100 laminar flow bench (except for toluene), and a air ionizer is useful to reduce static charge buildup on the glass. Of course, talc-free rubber gloves or finger cots are worn during this operation.

The cleaning sequence has been chosen such that the solvent used is itself soluble in the subsequent solvent. This type of procedure is commonly used in optics and microelectronics [58]. The block is held sideways, and the solvents are flushed down the substrate surface from top to bottom. The total sequence is as follows:

(i) Toluene.
(ii) Acetone.
(iii) Freon.
(iv) Ethyl alcohol.
(v) DI water.

After rinsing with DI water and drying with filtered nitrogen, the water-break-free method is used to determine the state of cleanliness of the surface [59]. DI water is flooded onto the surface, and there should be no sheeting observed, and surface tension should hold the water with a high contact angle (much like on a freshly waxed car). If the surface is not water-break-free to all four corners, the blocks are recleaned and reinspected. Stubborn organic contaminants can be removed with a microdetergent and DI water solution and carefully scrubbed with a cotton swab, if necessary.

5.3.2 Mating of the Half-Couplers

Two half-couplers are mated in the class 100 laminar flow bench after cleaning, and the resultant fringe pattern is observed under white light. Diffuse white light will produce the familiar colored fringes, which have a specific color sequence [60]. If a centrally symmetric zero-order fringe is observed (black to dark brown) the blocks are sufficiently close for the next step. If higher-order (color) fringes are found, the blocks must be recleaned and reassembled until a zero-order fringe is observed in the central region of the blocks. If the gap is sufficiently small, the two half-couplers are placed into an adjustable coupler fixture, which is shown in Figure 5-16.
The adjustable coupler fixture is machined out of stainless steel, with a differential micrometer head and a regular micrometer head attached to control the transverse offset of the fiber. Six plastic-tipped spring-loaded plungers are employed to preload the two half-couplers toward one another and against the micrometer heads. This type of design using stainless steel can be highly ground and polished, and is more stable than the aluminum designs, which have a higher CTE. One of the fiber pigtails is connected to a laser source as shown in Figure 5-17, and a dual-channel power meter is used to monitor the splitting ratio by connecting to the output pigtails of fiber A and B.

The two fibers are aligned roughly with the aid of a microscope, and Cargille "1.458" index-matching oil is wicked in between the coupler halves and is allowed to penetrate the gap. In this manner, the splitting ratio can be monitored when setting up the coarse adjustments of the micrometer. When the device is tuned in (usually to 50 percent), the two coarse micrometer adjustments are fixed with a drop of adhesive, and the fine tuning is accomplished with the differential micrometer.
5.3.3 Optical Contact Bonding

Optical contact bonding (OCB) has been known for some time to produce an adhesiveless bond between a pair of properly prepared glass parts [61]. In OCB, Van der Waals forces are employed to attach the parts, which must be smooth, flat, and clean on a molecular level. It is the preferred technique for the assembly of prism polarizers [62] and RLG mirrors [63], because it produces a hermetic, stable seal. By eliminating any material in the gap between the fibers, a great improvement in loss and differential normal mode losses is observed. This process picks up from the previous subsection just before the index matching oil is introduced into the device.

References [64, 65] should acquaint the reader with the general techniques of OCB, and [66, 67] are applied specifically to fiber optic coupler manufacture. Table 5-3 lists some of the specifications for substrates that are to be used as OCB couplers, using the $6 \times 12 \times 20$ mm substrate blocks [68, 69]. The high degree of flatness is made possible by the narrow groove and blocking technique used in our process.

Table 5-3. Requirements for OCB.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>flatness</td>
<td>&lt; $\lambda/15$</td>
</tr>
<tr>
<td>roughness</td>
<td>20 Å RMS</td>
</tr>
<tr>
<td>cleanliness</td>
<td>particulate-free</td>
</tr>
</tbody>
</table>

The cleaned half-couplers are assembled as before in the adjustable coupler fixture, and the fibers are carefully lined up over one another with the aid of a microscope. The two halves, if in pristine condition and the preloading force is light, will not scratch one another when they are adjusted. The attractive force per unit area $P$ between a pair of ideal planar glass surfaces is approximately given by the relation

\[ P = \frac{\chi}{d^4} \] (5.5)
where \( d \) is the gap separation and \( X \) is a proportionality constant [70]. The strength of the attraction is a function of flatness, roughness, and dielectric permittivities of the two substrate materials. There is also a strong role for adsorbates, which is not well understood at this time. The attractive force is a very strong function of gap, and \( X \) has been experimentally measured to be on the order of \( X \approx 10^{-19} \text{ erg-cm, or } 10^{-30} \text{ kg-m}^2 \), for BK-7 type glass. For gaps much less than 100 nm, the force becomes significant and it is sometimes very difficult to keep the half-couplers from spontaneously bonding. For our hypothetical perfectly smooth 6 \( \times \) 12 mm surface, a 10 nm uniform gap will produce a 24 gram force. For this reason, adjustable couplers have a slight crown polished onto them to keep them from sticking, but they can occasionally OCB as well. Figure 5-18 is a photograph of such a coupler made from the thinner substrates; note the surrounding Newton's rings. Note the central region where the OCB has occurred, and where the substrates have become transparent because there is no interference in the fused region.

To get more control of the process, more complex fixturization has been designed. Figure 5-19 is an advanced OCB fixture which has piezoelectric control and 6 degrees of freedom. This mechanism is designed such that all six motions are orthogonal and do not interact with one another. With either fixture the procedure is the same: a gentle "wringing" motion is used to slowly bring the two surfaces into parallelism and then reduce the gap. By watching the fringe pattern, it can be centered over the fiber ellipses by adjusting the preload screws on the adjustable coupler, or the tilt adjustment on the larger fixture. If the zero-order fringe cannot be attained, they must be disassembled and recleaned until it is possible to get a symmetric, black fringe. The splitting ratio is actively monitored during the assembly process with the same setup as shown in Figure 5-17. For a 50% splitting ratio device, the transverse offset is adjusted to achieve equal powers in both outputs of the power meter.

To test whether the coupler is ready to bond, a slight increase in the preloading pressure is applied with a finger, for example. If the gap is sufficiently small, this should produce an enlargement of the zero-order fringe area, but only a small (< 10%) fluctuation in the splitting ratio. A further increase in pressure will initiate the OCB, which proceeds rapidly and is evidenced by the black fringe turning transparent. At that point, the blocks are bonded by the attractive forces present between the half-couplers. In our process, there is typically a 5% shift in splitting ratio, which may be precompensated for. If the bonded device has the wrong splitting ratio, they may be separated by applying a heat gun to gently heat one of the half-couplers. Alternatively, a few seconds on a hot
plate will also induce a differential expansion that will separate the two halves. The pair is then inspected and recleaned and the process is repeated until the desired splitting ratio is achieved. This type of "Kentucky windage" requires some practice for the technician to master, but is quite successful in getting good devices. Devices which are stubborn to bond are recleaned and/or hand-polished, and as a last resort, isopropyl alcohol can be used to promote the OCB.

The debonding step can only be performed for relatively young devices, because the strength of the OCB is low during the first few hours [71]. After a few days has passed, the strength is seen to increase and then saturate. This is probably due to molecular rearrangement to reduce the dangling bonds and associated energy of the surface. There is a water layer adsorbed from ambient humidity when the glass surface is exposed to air. The characteristics of the trapped H₂O layer has been measured with multiple frustrated total internal reflection spectroscopy to be ≈ 10Å thick [72], and ≈ 95Å thick in [73]; Akhmatov, 1975 #595 using transmission spectroscopy. Hydrocarbon compounds have also been detected in the first set of experiments in the OCB layer; adsorbates are claimed to increase the tensile strength of the OCB. Another set of experiments with quartz samples used a vacuum bake step to remove nearly all of the water layer, and showed little difference in the OCB properties [74]. The physics of OCB phenomena are still not well understood, and the author is not aware of a full theoretical treatment in English.

A problem with poor or incomplete OCBs is the generation of large amounts of stress that induce index variations via the elasto-optic effect [64]. These stresses and the resultant index inhomogeneities are negligible for highly flat surfaces [75]. This would tend to cause poor polarization performance if this occurred near the fiber core region. The strength of OCB assemblies has been the subject of considerable research in the former Soviet Union [64, 71, 76-81]. It is generally agreed that heating the contacted part to 400°C to 800°C, which is below the annealing point of the glass, will produce an increase in the strength of the OCB [64, 79, 82-84]. Heating will also promote bonding in less-than-flat surfaces in some cases; clearly, optical contacting is a subject that needs further investigation and the application of modern solid-state and surface physics approaches.
Figure 5-18. Assembled Half-Couplers Showing Newton's Rings.

Figure 5-19. Optical Contact Bonded Coupler Assembly Fixture.
Figure 5-20. OCB Strength vs Time [71]. (a) K8 glass, (b) BK10 glass, (c) TF1 glass, (e) SO115 pyroceramic, (f) K8 glass.
5.4 References


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In this chapter, a formalism will be introduced that permits the calculation of arbitrarily complex fiber-optic assemblies, including interferometers and resonators. Some of the important components, and their Jones matrix representation will be summarized. The scattering matrix will then be introduced for 2- and 4-port devices. Graphical techniques used to visualize the results of the model will be discussed.

6.1 Matrix Description of 2-Port Devices

Two-port devices, such as fibers, splices, and polarizers are used as subcomponents in coupler-based fiber optic sensors. Figure 6-1 illustrates a generic 2-port device, that has unidirectional propagation and no reflection. As shown, the device is polarization-independent, and scalars are used to represent the input and output fields, as in equation (2.1). In the case of polarization-preserving devices, the scalars $E_j$ and $E_{j+1}$ are replaced with the appropriate Jones vectors.

![2-Port Device Representation](image)

Figure 6-1. 2-Port Device Representation.

6.1.1 Single-Mode Fiber and Attenuation

Real fibers absorb and scatter a small amount of the light during propagation, and behave according to Beer's law. This states that the intensity in uniform lossy media will have an exponential decay as a function of $z$. A common parameter used to describe absorption effects is called loss $\gamma$ and is usually small and is often expressed in percent.
The attenuation coefficient \( \alpha \) sets the amplitude decay rate and has units of inverse meters. \( I_0 \) is the intensity entering the medium, and \( L \) is the physical length of the uniformly lossy medium. The amplitude transmission coefficient \( \rho \) is also defined in equation (6.1). In the case of lossy media, the \( z \)-dependence in the field solution that was originally proposed in equation (2.1) has a real part that is an exponential decay:

\[
E(z,t) = E_0 e^{(-\alpha + i\beta)z} e^{-i\omega t}
\]  

(6.2)

For the scalar representation, i.e., single-mode fiber, there is no polarization information carried in the equations. A single complex number is sufficient to define the field, and the transfer function that describes the box in Figure 6-1 is expressed as

\[
E_{j+1} = e^{-\alpha L} e^{i\beta L} E_j
\]  

(6.3)

Loss is often given in units of decibels (dB), and is denoted by the \( \gamma_{dB} \) designation

\[
\gamma_{dB} = 10 \log_{10}(1-\gamma) = 20 \log_{10}(\rho)
\]  

(6.4)

The attenuation coefficient can also be written in terms of dB, by defining \( \gamma_{dB} = \alpha_{dB}L \) where it is in dB/m. Equating (6.4) and (6.1), and eliminating \( L \) permits the conversion factor from a dB coefficient to an exponential coefficient to be written approximately as

\[
\alpha = 0.116 \alpha_{dB}
\]  

(6.5)

As an example, consider a typical PM fiber having an attenuation specification of 2 dB/km at 1.3-\( \mu \)m wavelength. After 1 km of propagation, 2-dB loss has been induced, or roughly 37% of the optical power has been absorbed as measured at the end of the fiber. Fiber components often use =1-m lengths of fiber, which would have on the order of 0.05% loss, which is negligible. A typical ring resonator with a 20-m cavity length would see coil losses of 0.04 dB for this fiber. Calculating the intensity loss

\[
\gamma = 1 - 10^{-0.004} = 1\%
\]  

(6.6)
note that for small $\alpha L$, the above equation reduces to $\gamma = 2\alpha L$. In many calculations, a lumped transmission coefficient $\rho$, hopefully only slightly less than unity, is used in lieu of exponential attenuation. Rearranging equation (6.1) and for the same approximation

$$\rho \equiv \sqrt{1 - \gamma} = 1 - \alpha L$$

(6.7)

An alternative to equation (6.3) is where the attenuation could be included as a lumped multiplicative coefficient as with the coupler. Equation (6.3) now becomes

$$E_{j+1} = \rho e^{i\gamma L} E_j$$

(6.8)

Again, for small $\gamma$, equation (6.7) can be written in terms of coefficients in dB units

$$\rho = 1 - 0.116 \alpha_{\text{dB}} L = 1 - 0.116 \gamma_{\text{dB}}$$

(6.9)

which is a convenient conversion to use when calculating the transmission coefficient for low-loss devices and fibers.

6.1.2 PM and PZ Fiber Representation

To include polarization effects in birefringent fiber, the optical field is represented by $1 \times 2$ column vectors, whose coefficients describe the field in the x (slow) and y (fast) eigenmodes of the fiber. This situation is illustrated in Figure 6-2, which is a generalization of Figure 6-1 to include both orthogonal polarization states. The device under discussion, which has two physical ports, looks like a 4-port device functionally.

![Figure 6-2. 2-Port Birefringent Device.](image)

For an E field whose SOP is described by the vector at port $j$, the output at the $(j+1)$ port is described by a $2 \times 2$ matrix. For an ideal fiber with no cross-coupling,
would describe the light as it propagates through the fiber. This matrix is similar to the matrix which describes an ideal bulk-optical retarder [1]. Non coupler devices will be denoted by a $M$ matrix

$$M_{PM} = \begin{bmatrix} e^{j\phi_x L} & 0 \\ 0 & e^{j\phi_y L} \end{bmatrix}$$ \hspace{1cm} (6.11)$$

This is the so-called Jones matrix for PM fiber; neglecting cross-coupling, a compact notation can be used, assuming the $E$'s are vectors as done in Section 3:

$$E_{j+1} = M_{PM} E_j \hspace{1cm} (6.12)$$

For a more realistic piece of fiber that exhibits a small but finite loss and polarization cross-coupling, $M$ takes on a more complex form given by (3.34)

$$M_{PM} = \rho \begin{bmatrix} 1 & -\sqrt{hL} \\ \sqrt{hL} & 1 \end{bmatrix} \begin{bmatrix} e^{j\phi_x L} & 0 \\ 0 & e^{j\phi_y L} \end{bmatrix} \hspace{1cm} (6.13)$$

where loss is equal for both $x$- and $y$-polarization, and is a lumped multiplicative factor, and $hL \ll 1$. For PZ optical fiber, the losses are different for the two polarization modes:

$$M_{PZ} = \begin{bmatrix} e^{-\alpha_x L + j\beta_x L} & 0 \\ 0 & e^{-\alpha_y L + j\beta_y L} \end{bmatrix} \hspace{1cm} (6.14)$$

To avoid the exponentials, the same matrix may be written with transmission coefficients

$$M_{PZ} = \begin{bmatrix} \rho_x e^{j\beta_x L} & 0 \\ 0 & \rho_y e^{j\beta_y L} \end{bmatrix} \hspace{1cm} (6.15)$$

in the matrix, where $\alpha_x$ and $\alpha_y$ ($\rho_x$ and $\rho_y$) are the amplitude attenuation (transmission) coefficients for the $x$- and $y$-polarization modes. Typically, the average transmission and propagation constants are factored out and pulled out front of the matrix.
6.1.3 Fiber Polarizer

The fiber polarizer has low attenuation in one axis and high attenuation in the other. The Jones matrix describing a polarizer is similar to that for PZ fiber, and is also represented by the two-port device in Figure 6-2. One difference between a polarizer and a PZ fiber is that a polarizer has negligible length. In this model, the x-polarization is always the transmitting channel, having amplitude transmission coefficient \( p \), which is near unity. The loss of the device \( \gamma_{dB} \) specified in decibels for the x-polarization

\[
p = 1 - 0.116 \gamma_{dB(x)}
\]  

(6.16)

where again it is assumed that the loss is low. The same approximation cannot be taken for the y-polarization, which is heavily attenuated. When converting from decibels, use

\[
\eta = \sqrt{10^{-0.116 \gamma_{dB(y)} / 20}} = 10^{-0.05 \gamma_{dB(y)}}
\]  

(6.17)

which is the exact conversion from dB loss to fractional transmission coefficients. Obviously, \( \eta \ll p \), and the Jones matrix for an imperfect polarizer is written [1]

\[
M_p = \begin{bmatrix}
\rho & 0 \\
0 & \eta
\end{bmatrix}
\]  

(6.18)

For a high-quality fiber polarizer, \( \eta^2 < 10^{-4} \), and bulk optic polarizers can attain \( 10^{-7} \) extinction ratios. Small polarization cross-coupling contributions can be handled as in equation (6.13). Occasionally, the device will be specified in terms of dB extinction ratio \( \xi_{dB} \), which is the difference of the dB loss in the x- and y-polarization.

\[
\xi_{dB} = \gamma_{dB(y)} - \gamma_{dB(x)}
\]  

(6.19)

For low-loss polarizers, the approximation \( \gamma_{dB(y)} = \xi_{dB} \) is often used. In these discussions, the loss and extinction ratio values are numerically positive.
6.1.4 PM Fiber Splice

An ideal splice between two pieces of birefringent fiber has no loss or cross-coupling, and its Jones matrix would be the identity matrix. However, real splices have loss, polarization cross-coupling, and backreflection. Not including the latter effect, the $2 \times 2$ Jones matrix for principal axes with misalignment angle $\theta$ is as shown in Figure 6-3. The effect of the misalignment angle is to mix the orthogonal polarization modes. The Jones matrix for an arbitrary splice angle $\theta$ is given by the equation

$$M_S = \rho \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} = \rho \, R(\theta)$$  \hspace{1cm} (6.20)

![Figure 6-3. PM Fiber Splice Misalignment.](image)

The matrix $R(\theta)$ is a unitary matrix and is sometimes called a rotation matrix. For an inverse rotation, $R(-\theta) = R(\theta)^T$. Typical performance of PM-to-PM and PM-to-PZ splices is listed in Table 6-1.

<table>
<thead>
<tr>
<th>Splice</th>
<th>Loss</th>
<th>Polarization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dB</td>
<td>% CC dB</td>
</tr>
<tr>
<td>PM/PM</td>
<td>0.03</td>
<td>0.7 &lt; -30</td>
</tr>
<tr>
<td>PM/PZ</td>
<td>0.1</td>
<td>2.3 &lt; -30</td>
</tr>
</tbody>
</table>

The splicing machine used permits the principal axes of two splices to be aligned to a high degree. This permits rewriting equation (6.20) for small $\Phi$ in radian units:
There is occasion to make a splice which has a 90° offset to interchange the two polarization modes [3]. In that case, the matrix becomes

\[ M_S = \rho \begin{bmatrix} 1 & -\theta \\ \theta & 1 \end{bmatrix} \]  \hspace{1cm} (6.21)

Again, this is assuming that \( \theta \), that is now the deviation from 90° offset, is small.

6.1.5 Cascading of Jones Matrices

It is desirous to be able to concatenate several optical fiber components together, and this is done by cascading the representative Jones Matrices. Figure 6-4 is such an example, a section of fiber with a 20-m length of PZ fiber that is spliced to a 1-m length of PM fiber.

The individual Jones matrices are assembled from right to left, then they are multiplied.

\[ M = M_{PM} \times M_S \times M_{PZ} \]  \hspace{1cm} (6.23)
Assuming that PM and PZ fiber have the same propagation constants, the splice is lossless, and the PM fiber is lossless with zero cross-coupling results in the matrix

\[
M = \begin{bmatrix}
\rho_x e^{i(\beta_L x L_1 + \beta_x L_2)} & -\rho_x \theta e^{i(\beta_L x L_1 + \beta_x L_2)} \\
\rho_y \theta e^{i(\beta_L y L_1 + \beta_y L_2)} & \rho_y e^{i(\beta_L y L_1 + \beta_y L_2)}
\end{bmatrix}
\] (6.24)

It is worth noting that any arbitrary Jones matrix cascade can be multiplied out into a 2 \times 2 matrix. This, in turn, can be factored into a canonical form that consists of a linear retarder represented by \(M_R\), a rotation matrix, and a circular retarder \([4]\). Therefore, it will resemble equations (6.11), (6.20), and the matrix equivalent of the circular retarder which is denoted by \(M_\Omega\).

\[
M = \rho e^{i\theta L} M_\Omega M_R R(\theta)
\] (6.25)

where the matrices for the linear and circular retarders are in Appendix 2 of [1]

\[
M_R = \begin{bmatrix}
e^{i\Delta \phi/2} & 0 \\
0 & e^{-i\Delta \phi/2}
\end{bmatrix}
\] (6.26)

\[
M_\Omega = \begin{bmatrix}
\cos \frac{\delta \phi}{2} & \sin \frac{\delta \phi}{2} \\
-\sin \frac{\delta \phi}{2} & \cos \frac{\delta \phi}{2}
\end{bmatrix}
\] (6.27)

and \(\Delta \phi\) is the linear retardance, and \(\delta \phi\) is the amount of circular retardance for the two orthogonal modes, and \(\theta\) is the rotation angle.

6.2 Bidirectional Propagation and the Scattering Matrix

The model will now be expanded to include propagation in both directions in the device. This will require matrices with twice the rows and columns as the devices modeled for unidirectional propagation. Figure 6-5 shows a revised block diagram for a nonbirefringent 2-port device that accommodates bidirectional propagation.
6.2.1 Reflection at a Mirror and Reciprocity

Consider the case of a partially transmitting mirror as a 2-port device. The mirror has reflectivity \( r^2 \) and transmission \( t^2 \) and is lossless such that \( r^2 + t^2 = 1 \). The mirror is a reciprocal device, and its scattering matrix will have special characteristics [7]

\[
\begin{bmatrix}
E_j \\
E_{j+1}
\end{bmatrix}_{\text{out}} =
\begin{bmatrix}
ir & t \\
t & ir
\end{bmatrix}
\begin{bmatrix}
E_j \\
E_{j+1}
\end{bmatrix}_{\text{in}}
\] (6.28)

Reciprocity means that the device under question is completely independent of the direction of propagation of the incident field. This is true for most materials, not including certain nonreciprocal effects due to the environment. It is known that the scattering matrix \( S \) of a lossless device is unitary, which is mathematically expressed as

\[(S^T)^* \ S = I \] (6.29)

where \((S^T)^*\) is the Hermitian (transpose conjugate) of the \( S \) matrix. Rearranging yields

\[(S^T)^* = S^{-1} \] (6.30)

In addition, Maxwell's equations are invariant in dielectric materials for time reversal, so
combining equations (6.30) and (6.31) yields the result for a reciprocal device, which is

\[
S^{-1} = S^* \tag{6.31}
\]

which implies that the matrix is symmetric for reciprocal media. If the system is linear, and the loss in the system removes energy independent of direction, the system is also reciprocal and the S matrix is symmetric.

As mentioned, there are several sources for nonreciprocal behavior. The Faraday effect [8], Sagnac effect [9, 10], time-varying thermal gradients [11], and other effects such as Kerr effect [12, 13] have a nonreciprocal effect. Other problems such as polarization-mode dispersion and scattering effects can produce nonreciprocal-type effects [14].

### 6.2.1 Splice with Backreflection

Splices have some inherent backreflection, on the order of a ppm. The scattering matrix description of a splice can be carried out like any 2-port device, as in microwaves. If the splice is modeled as in Figure 6-2, the fields at the jth and j+1 ports for the scalar case, the 2 x 2 matrix describing an imperfect splice with backreflection r

\[
\begin{bmatrix}
E_j \\
E_{j+1}
\end{bmatrix}_{\text{out}} = \begin{bmatrix}
ir & \rho \\
\rho & ir
\end{bmatrix}\begin{bmatrix}
E_j \\
E_{j+1}
\end{bmatrix}_{\text{in}}
\tag{6.33}
\]

where the i accounts for the 90° phase shift upon reflection, and \( \rho = 1 \). The scattering matrix S formalism is defined for this arrangement of vectors. The dependent variables are vectors describing the fields leaving the device, port by port. The S matrix for a single-mode fiber splice with backreflection would therefore be equal to the S matrix

\[
S_S = \begin{bmatrix}
ir & \rho \\
\rho & ir
\end{bmatrix}
\tag{6.34}
\]
where \( r \) is a small reflection coefficient. This is an example to illustrate scattering matrices, and will not be used in this analysis, but will be generalized for PM fiber.

### 6.2.2 PM Fiber Splice with Backreflection

The case for a PM splice with backreflection is handled with \( 1 \times 4 \) column vector basis, and the vector space is operated on by a \( 4 \times 4 \) matrix. This scattering matrix has on its diagonal four equal coefficients, whose value \( r \) is assumed to be independent of polarization. The off-diagonal matrix coefficients are mapped from the non reflecting splice Jones matrix. Four off-diagonal zeros mean that there is polarization preservation upon reflection, which is also a reasonable assumption for normal reflection. The four coefficients in the upper right-hand corner submatrix is the same as equation (6.21) for a unidirectional PM splice.

\[
S_s = \begin{bmatrix}
ir & 0 & \rho & \theta \\
0 & ir & -\theta & \rho \\
\rho & -\theta & ir & 0 \\
\theta & \rho & 0 & ir
\end{bmatrix}
\]  
(6.35)

Note that the matrix is symmetrical, which implies reciprocity. What this means is that the effect of the splice is independent of the propagation direction. The lower left-hand submatrix is the same as the splice matrix transposed \( M_s^T \), which is because the coordinate system rotates \(-\theta\) for \(-z\) propagation.

### 6.2.3 PM Fiber 2-port Device with Bidirectional Propagation

Consider the arbitrary Jones matrix \( M \) for a series of PM fiber components, for example, equation (6.23). For the case of \( m \) nonreflecting elements that are concatenated,

\[
M = M_m \times M_{m-1} \times M_{m-2} \times \ldots \times M_2 \times M_1 = \prod_{n=1}^{m} M_n = \begin{bmatrix} a & b \\ c & d \end{bmatrix}
\]  
(6.36)
the corresponding $4 \times 4$ scattering matrix $S$ would be symmetric and will be given by [15]

$$S = \begin{bmatrix}
0 & 0 & a & b \\
0 & 0 & c & d \\
a & c & 0 & 0 \\
b & d & 0 & 0
\end{bmatrix} \quad (6.37)$$

The zeros indicate there is no coupling between the counterpropagating fields as we have assumed. To further illustrate this point, the appropriate vectors are written down:

$$\begin{bmatrix}
E_{1x} \\
E_{1y} \\
E_{2x} \\
E_{2y} \text{out}
\end{bmatrix} = \begin{bmatrix}
0 & 0 & a & b \\
0 & 0 & c & d \\
a & c & 0 & 0 \\
b & d & 0 & 0
\end{bmatrix} \begin{bmatrix}
E_{1x} \\
E_{1y} \\
E_{2x} \\
E_{2y} \text{in}
\end{bmatrix} \quad (6.38)$$

Equation (6.38) permits transforming from unidirectional Jones matrices to scattering matrices for nonreflecting constituent devices. The lower left-hand submatrix for $-z$ propagation has to be equation (6.36) transposed [16]. Birefringent fiber is by nature reciprocal and its associated matrices are symmetric [8]. To incorporate backreflection, for example at splices in the optical train, an operation must be performed upon the $S$ matrix, which will be described in Section 6.3; in shorthand notation:

$$E_{\text{out}} = S \ E_{\text{in}} \quad (6.39)$$

This notation will be used to compute the case for backreflections, and to calculate resonator transfer functions.

### 6.2.4 SM Coupler with Backreflection

Figure 6-6 illustrates the model for a single-mode coupler that is also described by a $4 \times 4$ matrix for the scalar field case. Recall that the matrix for the unidirectional
The scattering matrix can be shown for the SM device, with no backreflection, to be

\[
\begin{bmatrix}
 E_1 \\
 E_2 \\
 E_3 \\
 E_4
\end{bmatrix}_{\text{out}} =
\begin{bmatrix}
 0 & 0 & \sqrt{1-k} & i\sqrt{k} \\
 0 & 0 & i\sqrt{k} & \sqrt{1-k} \\
 \sqrt{1-k} & i\sqrt{k} & 0 & 0 \\
 i\sqrt{k} & \sqrt{1-k} & 0 & 0
\end{bmatrix}_{\text{in}}
\begin{bmatrix}
 E_1 \\
 E_2 \\
 E_3 \\
 E_4
\end{bmatrix}_{\text{in}}
\]  

(6.41)

The scattering matrix can be further generalized by including nonzero coefficients. As defined, \( r_{jj} \) is the backreflected light that would be seen in the jth port.

\[
\begin{bmatrix}
 E_1 \\
 E_2 \\
 E_3 \\
 E_4
\end{bmatrix}_{\text{out}} =
\begin{bmatrix}
 r_{11} & r_{12} & \sqrt{1-k} & i\sqrt{k} \\
 r_{21} & r_{22} & i\sqrt{k} & \sqrt{1-k} \\
 \sqrt{1-k} & i\sqrt{k} & r_{33} & r_{34} \\
 i\sqrt{k} & \sqrt{1-k} & r_{43} & r_{44}
\end{bmatrix}_{\text{in}}
\begin{bmatrix}
 E_1 \\
 E_2 \\
 E_3 \\
 E_4
\end{bmatrix}_{\text{in}}
\]  

(6.42)

The off-diagonal terms describe the directivity of the coupler, i.e., the amount of light backreflected to an adjacent port. \( r_{12} \) is the coefficient for output in port 2, which was scattered from light entering port 1. Due to reciprocity, \( r_{12} = r_{21} \) and \( r_{34} = r_{43} \).
An important 2-port device that is often neglected is the fiber pigtails on a coupler. In Chapter II, the coupler was modeled as a point coupling; phase shifts of the coupler leads was neglected. This can be factored in by multiplying the matrix by a matrix $P$ which is a diagonal matrix containing the phase shifts on the appropriate pigtails. One simply substitutes $PSP$ for the original scattering matrix and continue as before, as done in Appendix VI.

6.2.5 PM Coupler Scattering Matrix Description

For the PM fiber case, the vectors are now $8 \times 1$ column vectors of the form

$$E_{\text{in}} = \begin{bmatrix} E_{1x} \\ E_{1y} \\ E_{2x} \\ E_{2y} \\ E_{3x} \\ E_{3y} \\ E_{4x} \\ E_{4y} \end{bmatrix} \quad \text{and} \quad E_{\text{out}} = \begin{bmatrix} E_{1x} \\ E_{1y} \\ E_{2x} \\ E_{2y} \\ E_{3x} \\ E_{3y} \\ E_{4x} \\ E_{4y} \end{bmatrix} \quad \text{(6.43)}$$

Consider Eq. (3.57) that is the $4 \times 4$ matrix that describes the unidirectional PM coupler propagation matrix $A$. The scattering matrix $S$ now becomes an $8 \times 8$ matrix which is found to be

$$S = \begin{bmatrix}
  r_{11} & r_{12} & r_{13} & r_{14} & k_i & c_i & k_r & c_r \\
  r_{21} & r_{22} & r_{23} & r_{24} & c_i & k_i^* & -c_r & -k_r^* \\
  r_{31} & r_{32} & r_{33} & r_{34} & k_r & -c_r & k_i & -c_i \\
  r_{41} & r_{42} & r_{43} & r_{44} & c_r & -k_r^* & -c_i & k_i^* \\
  k_i & c_i & k_i & c_i & r_{11} & r_{12} & r_{13} & r_{14} \\
  k_i^* & -c_r & -k_r^* & r_{21} & r_{22} & r_{23} & r_{24} \\
  c_r & -c_r & k_i & -c_i & r_{31} & r_{32} & r_{33} & r_{34} \\
  c_r^* & -k_r^* & -c_i & k_i^* & r_{41} & r_{42} & r_{43} & r_{44} 
\end{bmatrix} \quad \text{(6.44)}$$

Again, the above matrices have to be symmetric, for reciprocal devices. A SM 6-port coupler can also be modeled with scattering matrices [17], and 3-port couplers can be handled with the addition of a "virtual" 4th port [18, 19].

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6.3 Resonator Analytical Modeling

In this section, a technique will be introduced for theoretical modeling of ring resonators, which can have arbitrary complexity and backreflecting points. The techniques are generalized by an approach described by [20] and [21].

6.3.1 The Global Scattering Matrix

A resonator can consist of one or more couplers and optical fiber, which forms a closed path to form a resonant cavity. More complex designs incorporate cascaded cavities, polarizing components, cavities within cavities; these will not be discussed in this section, but the methods presented are completely general. The complexity of the system is limited by the size of the matrices with which one is willing to deal.

Figure 6-7 depicts the components for the construction of a reflection-mode resonator. It consists of a single-mode fiber coupler and an optical fiber, whose ports are numbered sequentially. The convention used is for the coupler to be listed first, with its numbering port sequence as defined in earlier chapters, and then the fiber device's ports are numbered.

A global scattering matrix can now be assembled from the constituent components, which in this case, will map the input fields to the output fields:

\[
E_{out} = S \ E_{in}
\]

(6.45)

where \(S\) is the 6 \(\times\) 6 global scattering matrix, and the E's are now 6 \(\times\) 1 column vectors.
where an ideal coupler has been assumed. The $\rho e^{iBL}$ terms account for the loss and propagation down the length of fiber L, which relates $E_5$ to $E_6$, and vice versa. This matrix is also symmetrical, because of reciprocity. This matrix accounts for all fields entering and leaving the components in a consistent manner. One could easily add parasitic backreflections in the coupler or fiber at this point. We will now provide the connections between the different components in the following section.

### 6.3.2 The Connectivity Matrix

Once the global scattering matrix has been established, the assembly can be realized by connecting the ports together. Figure 6-8 shows one possible topology where port 1 and 3 of the coupler are the input/output channels and ports 2, 4, 5, and 6 are connected as shown. Using the nomenclature of [22], this is called a direct-coupled fiber resonator (DFR), and requires a coupler with $k << 1$.

$$S = \begin{bmatrix} 0 & 0 & \sqrt{1-k} & i\sqrt{k} & 0 & 0 \\ 0 & 0 & i\sqrt{k} & \sqrt{1-k} & 0 & 0 \\ \sqrt{1-k} & i\sqrt{k} & 0 & 0 & 0 & 0 \\ i\sqrt{k} & \sqrt{1-k} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho e^{iBL} \\ 0 & 0 & 0 & 0 & \rho e^{iBL} & 0 \end{bmatrix}$$ (6.46)

It is desirous to construct the connectivity or geometry matrix $G$ such that

$$E_{in} = G E_{out} + E_o$$ (6.47)
which links the output fields to the input fields, which are dependent upon the particular topology of the assembly. The initial conditions are provided by $E_0$, which represents the laser source for the ring. For Figure 6-8, it can be seen that the following are true:

$$E_5 = E_2 \quad E_6 = E_4 \quad \text{and} \quad E_2 = E_5 \quad E_4 = E_6 \quad (6.48)$$

which can now be assembled into the matrix form of the geometry matrix which is thus

$$G = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix} \quad (6.49)$$

This is a convenient way to separate the effects of topology from the components themselves. The alternate connection topology for Figure 6-7 is to connect ports 2 to 5, and connect ports 3 to 6, where port 4 now becomes the output. This is called a cross-coupled fiber resonator (CFR), and requires a high splitting ratio near unity. In this case, the $G$ matrix would be modified to link up the correct ports.

6.3.3 The $(S^{-1} - G)^{-1}$ Formalism

The geometry and global scattering matrices are now combined to produce the transfer function matrix. This will relate the output fields and circulating fields to the input fields into port 1 (or port 3) of the assembly. Taking equation (6.45), and dropping the overbar on the $\bar{S}$ shows the dependence of the component output fields on input fields

$$E_{\text{out}} = S \ E_{\text{in}} \quad (6.50)$$

and since $S$ is not singular, the inverse can be computed, which relates the input fields as a function of the fields leaving the component's ports. This is expressed as

$$E_{\text{in}} = S^{-1} \ E_{\text{out}} \quad (6.51)$$
which can now be substituted into equation (6.40), and the input fields can be eliminated

\[ E_{in} = G E_{out} + E_0 = S^{-1} E_{out} \]  \hfill (6.52)

\[ \Rightarrow E_0 = (S^{-1} - G) E_{out} \]  \hfill (6.53)

To compute the fields leaving the components, the expression in the brackets is inverted, which is no problem for modern PCs and symbolic algebra software such as Mathematica

\[ E_{out} = (S^{-1} - G)^{-1} E_0 \]  \hfill (6.54)

The circulating and output fields are computed in Appendix VI, and for the STC of launching unit amplitude into port #1, the output from port 3 is found to be equal to

\[ E_3 = \frac{2pe^{i\phi} - \sqrt{1-k} \left(1 + \rho^2 e^{2i\phi}\right) - kpe^{i\phi}}{-1 - \rho^2 e^{2i\phi} + 2pe^{i\phi} \sqrt{1-k} + k\rho^2 e^{2i\phi}} \]  \hfill (6.55)

and the magnitude squared is plotted in Figure 6-9 for a low-loss single-mode resonator.

![Figure 6-9. Intensity Leaving Port 3 of Coupler (Output)](image)

The definitions shown are the free spectral range (FSR) which is the frequency difference between adjacent resonance dips. The FSR is a well-known quantity, and is given by
The full width half maximum (FWHM) of the resonance is called the linewidth, and is determined by the loss. The finesse $F$, is defined as the ratio of the FSR to the FWHM:

$$F = \frac{\text{FSR}}{\text{FWHM}}$$

(6.57)

which is a measure of the $q$ of the cavity. The lower the losses in the resonator, the higher the finesse of the resonator. The dip depth $\delta$ is defined as a ratio also, and is

$$\delta = \frac{I_{\text{min}}}{I_{\text{max}}}$$

(6.58)

Finesse has been defined in relative terms, and for Figure 6-9, may be done with phase (or length) in lieu of frequency. The circulating intensity leaving port #4 is found to be

$$E_4 = \frac{i \sqrt{k} \sqrt{1-k} (-1 + \rho e^{i\phi})}{-1 - \rho^2 e^{2i\phi} + 2\rho e^{i\phi} \sqrt{1-k} + \rho^2 e^{2i\phi}}$$

(6.59)

And is plotted below in Figure 6-10 for the same STC, showing the circulating intensity maxima that correspond to the resonance dips in Figure 6-9.

![Figure 6-10. Intensity Leaving Port 4 of Coupler (Circulating)](image-url)
As mentioned earlier, the dip depth is a strong function of the degree of matching between the total losses and the splitting ratio [23]. Fiber dips are shown in Figure 6-11 for the case of a system with approximately 1 dB loss ($\rho^2 = 0.8$), for a splitting ratio of 1%, 20%, and 90%. Note that when the two parameters sum to equal unity, $\delta = 0$ and there is complete destructive interference at resonance.

![Figure 6-11. Effect of Splitting Ratio on Resonator Transfer Function.](image)

The minimum dip intensity as a function of loss and splitting ratio is shown in Figure 6-12, and finesse is noted at different operating points on the various loss curves.

![Figure 6-12. Minimum Dip Intensity as a Function of Splitting Ratio.](image)
For resonators that have no asymmetry, simple expressions for finesse and dip depth can be derived from [24], relating these parameters to the \( \rho \) and \( k \) values.

\[
F = \frac{-\pi}{\ln(\rho \kappa)} \quad \text{where} \quad \kappa \equiv \sqrt{1 - k} \quad (6.60)
\]

\[
\delta = \left( \frac{\ln(\rho \kappa)}{\ln(\rho^2)} \right)^2 \quad (6.61)
\]

For the case of a high splitting ratio type resonator, use \( \kappa \equiv \sqrt{k} \) instead of the definition above. These can also be simplified for the case of matched SR, i.e., \( \delta = 0 \):

\[
F = \frac{-\pi}{\ln(\rho^2)} \approx \frac{4.34 \pi}{\gamma_{DB}} \quad (6.62)
\]

where \( \gamma_{DB} \) is the total decibel loss in the coupler, fiber loop, and splices, if any.

6.3.4 Observable Fields

The technique of "embedding" is now introduced, which permits higher levels of complexity to be handled, by eliminating the unobservable variables in the system [25].

![Figure 6-13. Resonator as Component: Hidden Variables.](image)

Often, it is not necessary or desirable to know the fields that are within the ring, and the resonator is only used as a "black box." Figure 6-13 shows a resonator, with external fields denoted with roman numerals \( E_I \) and \( E_{II} \). These observable fields \( E_I \) and \( E_{II} \) are related to the six fields in the matrix solution by the following identities:

\[
E_I = E_1 \quad E_{II} = E_3 \quad \text{and vice-versa} \quad (6.63)
\]
A transformation matrix $T$ can be written which can be used to relate the fields

$$E_{\text{ext}} \equiv \begin{bmatrix} E_I \\ E_{\text{II}} \end{bmatrix} = TE_{\text{out}}$$

(6.64)

and $T$ will have a rectangular $2 \times 6$ matrix, that eliminates the unobservable variables

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

(6.65)

Substituting this transformation into equation (6.54), the transformed equation will be

$$E_{\text{ext}} = T (S^{-1} - G)^{-1} E_o$$

(6.66)

and the input field can be transformed with the $T$ matrix transposed, which is written

$$E_{\text{ext}} = T (S^{-1} - G)^{-1} T^T E_{\text{launch}}$$

(6.67)

Which produces a $2 \times 2$ scattering matrix that describes the resonator as a 2-port device. This may, in turn, be used as a building block for further modeling using these techniques. An example where this technique could be used is for analysis of a ring resonator with a in-cavity Fabry-Perot resonator element [26].

6.4 Polarization and Other Topics

The techniques presented in section have been generalized for arbitrarily complex resonant assemblies. As the system becomes more complex than the simple resonators presented here, the algebra rapidly becomes cumbersome. For example, the transmission-type, or "peak," resonator that was modeled in Appendix VI has two couplers and one splice uses a $10 \times 10$ matrices. The availability of computer algebra packages such as Mathematica and Matlab promise to simplify the modeling process, given sufficient computing power. Unfortunately, the author did not have sufficient time to fully investigate the topics in this Section, and only summaries are presented.
Some of the more difficult topologies which could be analyzed with these techniques would include cascaded transmission-type ring resonators [27], coupled Fabry-Perot cavities [28], fiber Fox-Smith resonators [29], and "figure eight" dual peak rings [30]. Even more complex types of topologies employ couplers with 3 input ports and 3 output ports [31, 32] can be handled with suitable modification of the coupler scattering matrix which would be a $6 \times 6$ size. A combination of a Mach-Zehnder interferometer and a ring resonator [33], and even more complex topologies [34-36] could be handled, depending on the computing platform.

6.4.1 Polarization

The techniques introduced in Section 6.3 can also be expanded for polarization-maintaining fiber components, which doubles the matrix dimensions. For example, the birefringent peak ring with two splices would grow to $24 \times 24$ matrix computations! Several resonators have been demonstrated with birefringent fiber [37-40] over the past 10 years, and these methods deliver a consistent method to model and compare such devices.

It has been known for some time that the two resonances are supported in birefringent fiber resonators, which can interact with one another [41-47]. Figure 6-14 illustrates this phenomenon, with trace (a) showing the two resonances strongly coupling with resultant double-dip structure and reduced contrast. When the two resonances move farther apart, a trace like (b) is observed, and when they are not coincident, a single dip is observed (c).

When the overlap condition occurs, the distortion in the resonance produces large errors in RFOG output [48-50]. There have been several proposals to removing this error, including placing a polarizer within the cavity [51], and polarizers outside of the cavity [40]. Single-polarization fiber has been used successfully in lieu of the in-cavity polarizer, to suppress the orthogonal polarization resonance [52, 53]. Such a resonator was fabricated, and it's operation is described in the next chapter.
Figure 6-14. Effect of PM Resonator Orthogonal Mode Overlap. 
(a) Near Total Overlap; (b) Partial Overlap; (c) Desired Dip with No Overlap.

Another approach is to incorporate a 90-deg splice into the cavity, which exchanges the fast and slow axes for every round trip [3, 54-57]. These approaches have been modeled in Appendix VII. Alternately, a polarization-rotating coupler has been demonstrated [58] to achieve the same effect as the 90-deg splice. Several theoretical treatments for birefringent resonant cavities exist in the literature [43, 48, 59-61]. A number of authors are now examining the relative merits of these types of resonator solutions to the polarization problem [62-64].

6.4.2 Backscattering

There are two types of backscattering seen in fiber gyroscopes: distributed Rayleigh scattering [65], and point-like scatterers. The latter could come from a splice, coupler, a local "hot spot" in the fiber with larger than average scattering, or fiber endfaces [66]. This type of imperfection produces scale factor errors in the gyro operation, and increases gyro noise at the detector [67-70]. Backreflections are also well-known to produce the lock-in effect near zero rotation rate [71], and doubly-backscattered light is also a problem.

Several methods have been investigated to mitigate the effects of backreflections, which use various sinusoidal phase modulations applied to the light entering the resonator
These techniques show promise of reducing backscatter-induced gyro errors to levels low enough to permit satisfactory gyro performance [77].

6.4.3 Graphical Techniques

Several graphical techniques are valuable for visualizing the physics of the resonator, and are briefly mentioned here. The plotting on the complex plane shows the behavior of the phasors which describe the fields [78, 79], and has been done as part of Appendix VI. Figure 6-15 qualitatively illustrates the complex plane behavior for ideal resonators and resonators with differential normal mode losses.

Figure 6-15. Complex Plane Mapping of Output and Circulating Fields.

Another graphical technique that is useful for understanding the operation of PM fiber resonators is the Poincaré sphere [80]. It is with great regret that time did not permit the combining of Appendix III and Appendix VII to plot the resonator ESOP as the fiber is perturbed. Summarizing, for the case of overlap of the two orthogonal resonances (Figure 6-12a), the ring ESOPs are located near the two poles of the Poincaré sphere. One ESOP is right-hand circular polarization, and the other is left-handed circular polarization. This permits excitation of both eigenmodes for even small amounts of polarization cross-coupling. Another approach which has been suggested [79] is to use
state-space techniques [81] to model the resonator. Systems analysis and graphical techniques are powerful tools for modeling resonators, and can be readily applied with the symbolic algebra programs such as in the Appendices.

6.4.4 Low-SR vs High-SR Resonators

There are some interesting comparisons between the DFR, and the CFR which uses a high-SR coupler [22]. Table 6-2 summarizes in a qualitative manner some of the salient characteristics, for the coupler modeled in Appendix V.

Table 6-2. Comparison Between DFR and CFR Resonators.

<table>
<thead>
<tr>
<th>Type</th>
<th>Config.</th>
<th>Loss</th>
<th>Fiber</th>
<th>Loop</th>
<th>In/Out</th>
<th>Finesse</th>
<th>Biref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k &lt; 1$</td>
<td>spliced</td>
<td>splice</td>
<td>PM, PZ</td>
<td>AA $\rightarrow$</td>
<td>AB $-40$</td>
<td>medium</td>
<td>NC</td>
</tr>
<tr>
<td>$k = 1$</td>
<td>spliced</td>
<td>splice</td>
<td>PM, PZ</td>
<td>AB $-15$</td>
<td>AA $-25$</td>
<td>medium</td>
<td>CC gets</td>
</tr>
<tr>
<td>spliceless</td>
<td>coupler</td>
<td>PM</td>
<td></td>
<td></td>
<td></td>
<td>high</td>
<td>worse</td>
</tr>
</tbody>
</table>

Obviously, it is not possible to make a spliceless DFR; the losses are assumed to be in the order of coil $<<$ coupler $<<$ splice for PM fiber. The low-splitting ratio DFR topology has the advantages of less polarization crosstalk in the coupler between the cavity ports, where it counts. The RFOG electronics has difficulty in acquiring and locking to extremely sharp dips, with an upper limit on F of about 100 [82]. In light of this, there is no advantage in going to a spliceless design, if the splices can be made with good polarization and loss characteristics.
6.5 References


Chapter VII

Device Performance

In Chapters IV and V, fabrication techniques were described that permitted the fabrication of fiber couplers. Some device data are now presented for typical couplers and resonator assemblies. Although nearly a dozen resonators were assembled, there was not a lot of experimental activity toward the end of this project. It was decided to present only four resonator examples, which were some of the better resonators that were made during this project.

7.1 Couplers

Slightly over one hundred half-couplers were made with this study, including the initial process development when there were very high scrap rates experienced. In this section, the operation and performance of birefringent fiber couplers will be examined, both OCB and adjustable. All of these devices used the Pyrex substrates with a 30-cm groove radius, Fujikura PM fiber, and were prepared as described in Chapters IV and V.

7.1.1 Adjustable Couplers

The characterization setup used for the couplers is the same as shown in Figure 5-17, with the addition of an input polarizer that has been added to the launching optical train. A broadband ELED source is used to get accurate splitting ratio and polarization cross-coupling measurements. The standard test condition (STC) is for linear polarized light launched in fiber A, with the azimuth of the linear polarization aligned parallel to one of the fiber principal axes. Power measurements of fiber A and fiber B are made with the 2-channel large-area optical power meter, and splitting ratio is calculated by equation (2.41). By measuring the launched power with the cutback technique [1], loss is characterized using equations (2.50), and (6.4). The polarization cross-coupling for a given output port is measured by collimating the output of the fiber and placing a second polarizer in the path of the detector. As the output polarizer is rotated, the difference of
the minimum and maximum readings of the power meter is the polarization cross-coupling $\xi_{db}$. Reference [2] describes in detail the type of optical setup and calibration procedures needed to make reliable polarization extinction ratio measurements.

Table 7-1 summarizes the data for some of the better adjustable devices made in this study. The substrates were numbered sequentially, and the serial number is the combination of the half-coupler numbers. The two columns for extinction ratio are for fiber A and fiber B outputs, and the measurement system limitation is better than $-40$ dB.

Table 7-1. Performance of Adjustable Couplers.

<table>
<thead>
<tr>
<th>Serial Number</th>
<th>Loss, dB</th>
<th>Max. Splitting Ratio, %</th>
<th>$\xi_{A}$, dB at 50% SR</th>
<th>$\xi_{B}$, dB at 50% SR</th>
</tr>
</thead>
<tbody>
<tr>
<td>41/42</td>
<td>0.13</td>
<td>70%</td>
<td>-27</td>
<td>-20</td>
</tr>
<tr>
<td>45/51</td>
<td>0.22</td>
<td>90%</td>
<td>-22</td>
<td>-24</td>
</tr>
<tr>
<td>47/48</td>
<td>0.25</td>
<td>99 - 25%</td>
<td>-11</td>
<td>-17</td>
</tr>
<tr>
<td>54/55</td>
<td>0.03</td>
<td>40%</td>
<td>-21</td>
<td>-25</td>
</tr>
<tr>
<td>61/67</td>
<td>0.05</td>
<td>&gt; 50 %</td>
<td>-30</td>
<td>-19</td>
</tr>
<tr>
<td>73/75</td>
<td>0.20</td>
<td>90%</td>
<td>-32</td>
<td>-22</td>
</tr>
<tr>
<td>94/96</td>
<td>0.40</td>
<td>99.5%</td>
<td>-22</td>
<td>-26</td>
</tr>
<tr>
<td>102/103</td>
<td>0.03</td>
<td>98%</td>
<td>-26</td>
<td>-21</td>
</tr>
<tr>
<td>111/112</td>
<td>0.04</td>
<td>96%</td>
<td>-27</td>
<td>-27</td>
</tr>
</tbody>
</table>

The loss for the devices was very low, in several cases below 1 percent, and large coupling of nearly 100 percent could be attained. Coupler #47/48 was overpolished such that it coupled up to 100% and then started back down as shown in Figure 2-6. The rest of the devices had only one splitting ratio maxima as the transverse offset was adjusted.

Figure 8-1 presents some data for an adjustable coupler, serial no. 54/55, where the optical power in mW is plotted for fiber A and fiber B. The differential micrometer was adjusted in 0.05-μm increments, and the power levels were read from the digital readouts. This type of data is called a coupling curve, and is typical and compares to the plots found in Appendix II.
This coupler could be adjusted to a splitting ratio of no more than 40%, and had a loss of 0.03 dB. It was subsequently spliced into a ring resonator, which had a finesse of over 130 and \( \delta = 0.01 \) dip depth. A photograph of an adjustable coupler in its stainless steel fixture is shown in Figure 7-2, showing the differential micrometer in the background. Coupler 41/42 was also used to construct a spliced PM resonator assembly, which exhibited a finesse of 65 and a dip depth of 1.3 percent.

![Figure 7-1. Optical Power as a Function of Transverse Offset.](image)

For the nine devices in Table 7-1, the extinction ratio of fiber A averaged to be \(-24 \) dB, with the B output averaging \(-22 \) dB, with the champion \( \xi_{db} \) value implying \( \pm 2.5 \)-deg alignment. These are reasonable numbers, but in several of the devices, both A and B had nearly the same ER, which implies that something other than misalignment was contributing to the crosstalk. It is believed that nonuniform stress-relief during polishing [3] was to blame for the increase of the cross-coupling between the polarization modes.

7.1.2 OCB Couplers

Once the process was well established for the smaller, adjustable coupler halves, the thicker blocks were polished to produce OCB couplers. About a dozen OCB couplers were fabricated, starting with a new serial number sequence, with some device data listed in Table 7-2. These couplers were considerably more difficult to polish and assemble.
than their adjustable counterparts; Figure 7-3 shows a typical OCB device with its thicker substrates.

Table 7-2. Performance of OCB Couplers.

<table>
<thead>
<tr>
<th>Serial Number</th>
<th>Loss, dB</th>
<th>Splitting Ratio</th>
<th>$\xi_A$, dB at 50% SR</th>
<th>$\xi_B$, dB at 50% SR</th>
</tr>
</thead>
<tbody>
<tr>
<td>2/7</td>
<td>&lt; 0.02</td>
<td>91%</td>
<td>-17</td>
<td>-16</td>
</tr>
<tr>
<td>8/9</td>
<td>&lt; 0.02</td>
<td>77%</td>
<td>-22</td>
<td>-22</td>
</tr>
<tr>
<td>10/11</td>
<td>0.30</td>
<td>55%</td>
<td>-18</td>
<td>-15</td>
</tr>
<tr>
<td>12/13</td>
<td>0.20</td>
<td>52%</td>
<td>-10</td>
<td>-20</td>
</tr>
</tbody>
</table>

OCB devices 10/11 and 12/13 were used for the construction of an RFOG brassboard for the photodetector output couplers. The splitting ratio could be held ±5 percent to desired values once the "Kentucky windage" technique was mastered. Device 2/7 was made into a ring resonator that was used in the same RFOG project [4], and will be described in the next section.

These devices were stable in the laboratory, but did not perform well under environmental stress. The splitting ratio is not stable over temperature, and they are susceptible to thermal shock. OCB devices have remained functional in the lab for over a year with no degradation in performance. There is a need for laboratory devices with a very high isolation between the polarization modes for gyro experiments. It is hoped that further advances in alignment technology, processing, and design optimization will make polished couplers feasible.
Figure 7-2. Adjustable Polished PM Fiber Optic Coupler.

Figure 7-3. Optical Contact Bonded PM Fiber Optic Coupler.
7.2 Resonator Assembly

Coil winding, splicing, and resonator characterization will now be discussed, before the resonators themselves are presented. Spliceless resonators are made by affixing the fiber substrates on to the coil pigails, and mounting the whole assembly on the polishing machine (see Figure 5-4). The cleaning, mating, and alignment steps are the same as described in Section 5.3, only with different test equipment.

7.2.1 Coil Winding

The coils for this study were wound on a custom-made coil winding machine, which is shown in Figure 7-4. The winding machine has active tension control [5], and permits winding to <10-gm tension, which is important to reduce losses [5, 6], and to maintain long lifetime [7]. To produce acceptable results at small winding diameters and low temperatures, a special high numerical aperture fiber is usually used [8], such as that listed in Appendix I.

The machine has a programmable capability to handle different fiber jacket diameters, and closed-circuit video monitoring of the fiber winding at high magnification. Figure 7-5 shows a 400-m coil of PM fiber, which was wound with this machine, that has been spliced to a coupler to form a resonator.

To design the fiber coils, a simple calculation is required, given a target length, fiber jacket diameter \(d_j\) and coilform diameter \(D\). Consider the first two layers, with \(N\) turns per layer wound over diameter \(D\):

\[
L_1 = 2 \pi (D + d_j) N \tag{7.1}
\]

\[
L_2 = 2 \pi (D + 2d_j) N \tag{7.2}
\]

and the length of the \(m\)th layer, which is denoted by \(L_m\), is given by the general expression

\[
L_m = 2 \pi (D + m d_j) N \tag{7.3}
\]
If there are a total of \( M \) layers, with each layer having \( N \) turns per layer, the total length is

\[
L_t = 2\pi ND + 2\pi ND \sum_{m=1}^{M} m d_j = \pi N \left[ 2D + d_j M (M + 1) \right] \quad (7.4)
\]

Now defining \( N = w/d_j \), where \( w \) is the "ideal" width of the coilform, permits solving for \( N \) as a function of the number of layers. \( N \) has to be an integer, so it is rounded up

\[
w = \frac{L/\pi}{M^2 + M + 2D/d} = \frac{L d_j}{2\pi D} \quad \text{if} \quad D/d_j >> M^2 \quad (7.5)
\]

\[
N = \text{Round} \left( \frac{w}{d_j} \right) \quad (7.6)
\]

and the actual coilform width can be computed by \( W = N d_j \). Typically, a tabulation is made to see which combinations of \( M \) and \( N \) meet the mechanical requirements of the resonator or gyro design.

The winding machine has been equipped to wind the anti-Shupe configuration, in a semi-manual mode where the operator changes the supply reels. The fiber jacket was assumed to be 400-μm diameter, and the coilform diameter is 76.96 mm in all cases. The coilforms were machined out of 6061 aluminum, and had a highly-polished surface. Table 7-3 lists the winding parameters and performance of typical coils wound for resonators made from PM and PZ fiber.

<table>
<thead>
<tr>
<th>Fiber</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>W</th>
<th>Loss</th>
<th>CC or ER</th>
</tr>
</thead>
<tbody>
<tr>
<td>PM</td>
<td>400 m</td>
<td>28</td>
<td>54</td>
<td>21.6 mm</td>
<td>0.1 dB</td>
<td>-19.2 dB</td>
</tr>
<tr>
<td>PZ</td>
<td>20 m</td>
<td>2</td>
<td>42</td>
<td>16.8 mm</td>
<td>0.24 dB</td>
<td>&gt; 40 dB</td>
</tr>
<tr>
<td>PM</td>
<td>20 m</td>
<td>2</td>
<td>42</td>
<td>16.8 mm</td>
<td>&lt; 0.02 dB</td>
<td>&lt; -40 dB</td>
</tr>
</tbody>
</table>

Anti-Shupe winding is required to reduce nonreciprocal phase shifts due to time-varying thermal gradients [9, 10]. This type of winding is done in a symmetrical manner with respect to the center of the fiber loop [11-13], such that perturbations are symmetrized and tend to cancel out. This technique is essential to reduce sensitivity to thermal gradients for long IFOG coils to improve gyro drift [14, 15].
Figure 7-4. Coil Winding Machine.

Figure 7-5. 400-m PM Fiber Coil Spliced to Form Resonator.
7.2.2 Splicing

Three of the resonators are spliced assemblies that were fusion spliced with a Fujikura FSM-20PM automated splicer [16]. Table 6-1 lists the expected parameters for different kinds of splices made with this machine. This machine was chosen because it permitted the splicing of the fibers passively, i.e., without light in the cores for position feedback. That feature is crucial for the reliable assembly of closed-fiber loop systems, which would otherwise be difficult to set up and optimize while splicing. Once the splice is made, it is recoated with silicone, on a commercial splice recoating machine.

7.2.3 Characterization

The test setup for the characterization of resonator transfer functions is schematically shown in Figure 7-6. A narrow-line laser output is linearly polarized, and launched into the fiber pigtail of the resonator. A tunable Nd:YAG non planar ring laser was used for a source, which had a wavelength of 1319 nm. Its optical frequency was swept by a triangle wave, which was produced by a function generator that also triggered the oscilloscope. A digitizing oscilloscope was used because of the many features available, such as built-in mathematical functions and trouble-free plotting using the IEEE-488 interface.

![Figure 7-6. Resonator Characterization Setup.](image-url)
The resonator was assembled and tuned in while watching the output on an analog oscilloscope to see where the dips commenced. Then, the detector output was connected to the digital scope to fine tune of the splitting ratio until the dip depth becomes minimum at the optimum splitting ratio. The transfer function is recorded by the digital oscilloscope, and the cursors are used to measure the finesse and dip depth according to Figure 6-9 and equations (6.57) and (6.58).

Dip asymmetry is measured by invoking the differentiation subroutine in the oscilloscope, which produces a display as shown Figure 7-7. By using the cursors and comparing the peak slope of the lineshape, a rough estimate of the asymmetry can be obtained. Unfortunately, the resolution of the scope prohibited a very precise measurement of this parameter. The backscattering was measured with the aid of a 50/50 coupler to observe the resonant peak [17], which is proportional to the circulating intensity in the cavity (See Appendix VI).

![Figure 7-7. Typical Resonator Dip Symmetry Measurement.](image)

The free spectral range was measured using the technique described in [18], that uses an IOC phase modulator which is sinusoidal-driven near the FSR frequency.

7.3 **Resonators**

Three types of resonators will be presented in this section, including spliced PM, spliceless PM, and spliced PZ rings. All of the resonators are the reflection-mode type, which have a transfer function described by resonant dips.
7.3.1 Spliced 400-m PM Fiber Resonator

The 400-m PM fiber coil was spliced to a pair of half-couplers in the CFR configuration of Figure 1-3. The half-couplers were polished according to Table 5-2 for resonators, but loss of the coupler was unknown at the time of assembly. The performance of the resonator is listed in Table 7-4 for two different adjustments of the PM fiber coupler transverse offset.

Table 7-4. 400-m Resonator Performance at Two Splitting Ratios.

<table>
<thead>
<tr>
<th>FSR</th>
<th>Finesse</th>
<th>Linewidth</th>
<th>Dip Depth</th>
<th>Crosstalk</th>
<th>Backscatter</th>
</tr>
</thead>
<tbody>
<tr>
<td>459 kHz</td>
<td>31</td>
<td>14.8 kHz</td>
<td>~ 0 %</td>
<td>&lt; -20 dB</td>
<td>not measured</td>
</tr>
<tr>
<td>459 kHz</td>
<td>65</td>
<td>7.0 kHz</td>
<td>15 %</td>
<td>&lt; -20 dB</td>
<td>-46 dB</td>
</tr>
</tbody>
</table>

The device was measured for two different splitting ratio adjustments to show the effect of k on the lineshape. This is a very narrow linewidth device, by virtue of the long cavity length and low free spectral range. The transfer function is shown in Figure 7-8, for the higher-finesse (overcoupled) adjustment in Table 7-4. The lower trace is when beam is temporarily blocked to get a zero-light reference; the sweep is also plotted.

![Figure 7-8. 400-m Spliced Resonator Transfer Function.](image)

The dip symmetry was not measured for this device, but is estimated to be less than 5 percent. The 7 kHz FWHM value is one of the more narrow linewidths reported to
date, thanks to the long cavity length which produces a low FSR. No orthogonal polarization dip could be observed in the resonator, and the polarization cross-coupling is estimated to be below -20 dB.

7.3.2 Spliced PM / PZ Resonator

It was an early suggestion that single-polarization fiber be used in the cavity to eliminate the orthogonal polarization resonance [19]. A spliceless SM fiber ring resonator was fabricated with an integral crystal-type polarizer in 1984 [20]. It had a cavity length of approximately 2 m, and exhibited a finesse of 60 at a 632-nm wavelength. A polarizing resonator made from single-polarization fiber was not demonstrated for some time [21]. This is partly due to the difficulty of obtaining good PZ fiber, and the problems making spliceless resonator assemblies when the fiber was available.

In this subsection, an adjustable coupler (45/51) and an OCB coupler (2/7) were selected for ring fabrication. Since the OCB device has a splitting ratio of 91 percent, the CFR topology of Figure 3-1 is demanded. To get high contrast, the optimum losses would have to equal the splitting ratio, which is calculated to be ~0.41 dB. For a spliced all-PM fiber assembly, this is more than the sum of the coupler, coil, and expected splice loss; the resonator would be extremely undercoupled. If the splice loss could be deliberately increased to about 0.2 dB each without introducing backreflections, decent contrast could be obtained.

Table 7-5. Possible Resonator Loss Budget for OCB Coupler 2/7.

<table>
<thead>
<tr>
<th>Coil</th>
<th>Coupler Loss</th>
<th>Fiber Loss</th>
<th>Splice Loss × 2</th>
<th>Total Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>PM Coil</td>
<td>0.02 dB</td>
<td>0.02 dB</td>
<td>0.06 dB est</td>
<td>0.1 dB</td>
</tr>
<tr>
<td>PZ Coil</td>
<td>0.02 dB</td>
<td>0.24 dB (x)</td>
<td>0.2 dB est</td>
<td>0.46 dB</td>
</tr>
</tbody>
</table>

Rather than following that approach, it was decided to make the resonator out of PZ fiber, which had enough loss in the x-mode to match the OCB coupler. The data for the OCB resonator is listed in Table 7-6. A similar decision-making process went into the adjustable ring device, which was made in the CFR topology (Figure 6-8) which used a low splitting ratio.
Table 7-6. 24-m PM/PZ OCB Resonator Performance.

<table>
<thead>
<tr>
<th>FSR</th>
<th>Finesse</th>
<th>Linewidth</th>
<th>Contrast</th>
<th>Crosstalk</th>
<th>Backscatter</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.6 MHz</td>
<td>25</td>
<td>350 kHz</td>
<td>2 %</td>
<td>&lt; -20 dB</td>
<td>not measured</td>
</tr>
</tbody>
</table>

Figure 7-9 is a representative plot of the resonant dips for the OCB device, which illustrates the quality of the resonator. No orthogonal resonance could be observed, even for deliberate perturbations to the fiber and for misaligned input launching conditions. Figure 7-10 is a photograph of the device, showing the coil and the coupler.

Figure 7-9. Spliced PZ Fiber Coil OCB Resonator Transfer Function.

The situation of Figure 6-14 never occurred, even for moderate heating of the coilform. This demonstrates the practicality of this approach for producing single-polarization resonators with PZ fiber. Table 7-7 summarizes the performance of the adjustable device when the splitting ratio has been optimized to get minimum dip depth.

Table 7-7. 27-m PM/PZ Adjustable Resonator Performance.

<table>
<thead>
<tr>
<th>FSR</th>
<th>Finesse</th>
<th>Linewidth</th>
<th>Contrast</th>
<th>Crosstalk</th>
<th>Backscatter</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0 MHz</td>
<td>27</td>
<td>260 kHz</td>
<td>2 %</td>
<td>&lt; -20 dB</td>
<td>-37.5 dB</td>
</tr>
</tbody>
</table>

229
Rotation of the input polarizer only reduced the overall intensity of the output for either device, as predicted by theory. Both assemblies have nearly 100-percent dip depth and very symmetrical lineshapes below the measurement threshold.

7.3.3 Spliceless PM Resonator

To fabricate spliceless resonator, the coupler substrates are bonded directly to the fiber pigtails of a coil and processed. The coil and one of the substrates must be mounted onto the polishing jig during the polishing of the other half-coupler, which increases the likelihood of breakage. Table 7-8 summarizes the performance of this device, using the STC described at the beginning of this Section.

Table 7-8. Spliceless PM Resonator Performance.

<table>
<thead>
<tr>
<th>FSR</th>
<th>Finesse</th>
<th>Linewidth</th>
<th>Contrast</th>
<th>Crosstalk</th>
<th>Backscatter</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.9 MHz</td>
<td>320</td>
<td>31 kHz</td>
<td>7 %</td>
<td>&lt; -20 dB</td>
<td>not measured</td>
</tr>
</tbody>
</table>

Figure 7-11 is a photograph of a spliceless resonator assembly, showing the short pigtails going from the coil to the adjustable coupler fixture. The data for this device is plotted in Figure 7-12, showing the very narrow lineshape. There is a high degree of symmetry, which cannot be measured with our differentiation technique. Note the presence of a small orthogonal polarization dip, and parasitic Fabry-Perot effect from fiber endface reflections. It is estimated that the splitting ratio is 91 percent, and that the total losses are approximately 0.02 dB.

Table 7-9 lists some other high-finesse devices found in the literature for both SM and PM fiber spliceless designs. Such high finesse values are not so good for practical applications, because of the large amount of backscattering and low Brillouin threshold. For RFOG applications, a very high finesse makes it difficult for servo electronics to acquire and lock on to the resonance. Temporal effects include "ringing" [22] and characterization difficulty because of the problems with sampling a narrow dip with the digital oscilloscope.
Figure 7-10. Spliced PM Coupler and PZ Coil Resonator.

Figure 7-11. Adjustable Spliceless PM Fiber Resonator.
blank page
Table 7-9. High-Finesse Reflection Resonators.

<table>
<thead>
<tr>
<th>Coil</th>
<th>Finesse</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-m SMF</td>
<td>500</td>
<td>[23, 24]</td>
</tr>
<tr>
<td>20-m PMF</td>
<td>1045</td>
<td>[18]</td>
</tr>
<tr>
<td>60-cm SMF</td>
<td>1260</td>
<td>[25]</td>
</tr>
</tbody>
</table>

The spliceless resonator requires the CFR topology, and a coupler with near-unity splitting ratio. For a finesse of 1000, total loss must be below roughly 0.015 dB, and the splitting ratio would be 99.7% to get zero dip depth.

Figure 7-12. Spliceless Resonator Transfer Function.
7.4 References


In this final chapter, conclusions and topics for possible future work will now be suggested. There are several more topics that the author had wished to explore than time permitted. The results of the PM coupler analysis suggest an optimum resonator topology with respect to resonance asymmetry and polarization cross-coupling. Unfortunately, at the time this was realized, no further processing was being done, and it was not possible to verify this information.

8.1 Conclusions

To summarize and conclude, a matrix-based technique for coupled-mode analysis was introduced in Chapter II to permit the modeling of polished fiber couplers. The relationship between the coupled-mode and normal-mode models was discussed, and a new and simple technique was introduced to incorporate differential normal mode losses. The addition of the asymmetry matrix equation (2.63) to the solution of the differential equations permits modeling of resonators with asymmetric resonance lineshape.

In Chapter III, the matrix techniques were broadened to include polarization effects that generalized the model for identical twist-free birefringent fiber. The Poincaré sphere was introduced, and the relationship of polarization cross-coupling to coupler design was explored. The theory predicts a lower sensitivity to axial misalignment for low splitting ratio couplers than for high splitting ratio couplers, all other things being equal. Jones calculus can readily be applied to the modeling of birefringent fiber-optic assemblies. The general PM coupler matrix can be fully described by four unique coefficients of equations (3.57) and (3.58).

The first resonators demonstrated were spliceless in design [1, 2] because low-loss splicing technology was not generally available. These were, by necessity, fabricated out of high splitting ratio couplers. The commercialization of low-loss PM fiber splicers now makes spliced rings of either topology producible with low or high splitting ratio
couplers. Figure 3-14 implies an optimum topology for resonant rings, where only couplers with $k << 1$ would be used. For applications where a small amount of splice loss can be tolerated, such as RFOGs, it is concluded that polarization-induced errors can be reduced by using the DFR topology with $k << 1$.

One of the main conclusions from this study is that coupler making is not a "black art," but can be routinely done. Fiber-optic substrate preparation and fiber-to-groove bonding are critical steps to obtaining high-performance coupler and resonator devices. Equipment and processing is described in detail to obtain PM couplers with losses < 0.1 dB and polarization crosstalk below −20 dB. The principal axis alignment of birefringent fiber was analyzed, and an acousto-optic system was devised to permit precise axial alignment. The advances to the state-of-the-art in principal axis alignment and substrate groove generation in Chapter IV have enabled repeatable, low-volume preparation of two prepared coupler substrates per day.

A "recipe" was presented for polishing fiber couplers that takes advantage of new materials such as synthetic polishing pads and polishing suspensions in Chapter V. The use of special polishing jigs, and the avoidance of pitch polishing lessens the amount of training for the polishing person. Often, one coupler was completed in a day, and an operator can be trained in a few months to produce acceptable results. Optical contact bonding was examined for the first time in several years, and it too can be performed reputably and reversibly. This most mysterious craft can be done by starting with a high-quality half-coupler, proper cleaning techniques, and common sense tooling. The same common-sense applies to coil winding for resonators (use low tension, avoid twist and kinks, etc.)

If one has access to a splicing machine, he does not need to make couplers to fabricate resonators. Rings may be spliced out of commercially-available coils and fiber couplers. Chapter VI introduces a technique that will permit modeling of spliced assemblies, which has capability to include nonidealities in splices, coils and couplers. Multiple backscattering points can be handled by the technique, as can arbitrarily complex topologies. Appendices II through VII contain software that execute the algorithms developed for this analysis.

Several resonators were fabricated under this study, and the device performance was presented in Chapter VII for three different types of resonators. Reflection-type PM
rings were made having (i) a 7-kHz FWHM linewidth, (ii) single-polarization fiber, and (iii) finesse of >300. All of these resonators demonstrated a high degree of dip symmetry, and low polarization crosstalk. These are currently being used in RFOG and other experiments.

8.2 Future Work

As mentioned earlier, it would be desirable to experimentally verify the predictions of the modeling. The different topologies need to be examined and compared, with a systematic set of experiments using the matching parts, if possible. Another experiment that comes to mind is the monitoring of SOP for couplers during adjustment, an easy thing now that real-time SOP analyzers are available.

The performance of computer experiments with the software would be simplified by the addition of a CAD-type program for setting up the topology and global scattering matrices. Any number of computer simulations come to mind, such as the trade-offs of effective coupler length with respect to splitting ratio, asymmetry, and polarization performance. Perhaps the optimization of such factors can make a coupler with a large "sweet spot" where it can meet splitting and polarization cross coupling over a wide adjustment range. The effects of different polishing techniques on backscatter and asymmetry have not been investigated, but similar studies for RLG mirrors have produced extremely low scattering and loss levels.

The current limitation of coupler performance is by the alignment and polishing-induced polarization cross-coupling. Methods for reducing the polarization degradation observed when polishing half-couplers need to be investigated. More accurate alignment methods (and alignment transfer methods) need to be developed as well. For example, curve fitting to the nulls of the squeezer system could greatly improve the accuracy and repeatability of the system.

Finally, the optical contact bond phenomena has not been reexamined in light of new surface physics theories. It would be a good thing to see a rigorous study from a modern materials and surface science viewpoint to see if any further understanding of this interesting effect can be made.
8.3 **Future Technology Directions**

Incorporation of rare-earth doped optical fibers has enabled loss compensation [3-5] to produce ultrahigh-finesse rings. This development has produced finesses much higher than has been currently possible with passive devices, and permits cascading of rings for spectroscopic applications.

The work on silica-based integrated optics has shown much promise for fabricating low-loss ring resonators in planar waveguides. This, along with the higher levels of integration available, holds the promise of a truly low-cost optical gyroscope. Recently a transmission-type resonator has been fabricated on a planar silicon with a finesse of 110, a respectable number [6]. By doping planar silica waveguides with rare-earths, lasers have been demonstrated [7]. Silica-based integrated-optics technology could be a new vehicle for miniaturized resonant gyro designs [8-10].

The recently demonstrated 2nd-order electro-optic effect in silica surfaces could be a major breakthrough in this technology area [11]. This would permit the fabrication of electro-optic modulators and devices based on nonlinear optical effects [12]. Nonlinear effects, which have been traditionally been avoided in RFOGs, may be exploited in some schemes [13, 14] for rotation sensing. Another advanced approach uses nonlinear effects to enhance the Sagnac effect [15, 16], which could theoretically increase the sensitivity of RFOGs.
8.3 References


Appendix I

FIBER MANUFACTURER DATA

Fujikura polarization-maintaining PANDA, single-polarization, and high numerical aperture fibers. The latter was not used in this study, but was included for comparative purposes.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SM13P PMF</th>
<th>SM13P-P PZF</th>
<th>SM13P high-NA</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>core diameter</td>
<td>7.0 ± 1</td>
<td>5.0 ± 1</td>
<td>4.5 ± 1</td>
<td>μm</td>
</tr>
<tr>
<td>clad diameter</td>
<td>125 (1)</td>
<td>124.5</td>
<td>125 (1)</td>
<td>μm</td>
</tr>
<tr>
<td>coating diameter</td>
<td>410</td>
<td>400</td>
<td>250 (1)</td>
<td>μm</td>
</tr>
<tr>
<td>MFD</td>
<td>9.5</td>
<td>8.8</td>
<td>6.0</td>
<td>μm</td>
</tr>
<tr>
<td>cutoff λ</td>
<td>1.0 ~ 1.29</td>
<td>0.8 ~ 1.1</td>
<td>1.1 ~ 1.2</td>
<td>μm</td>
</tr>
<tr>
<td>attenuation</td>
<td>0.5</td>
<td>1.8 (2)</td>
<td>&lt; 2 (1)</td>
<td>dB/km</td>
</tr>
<tr>
<td>beat length</td>
<td>3.1</td>
<td>1.7</td>
<td>&lt; 2 (1)</td>
<td>mm</td>
</tr>
<tr>
<td>h-parameter</td>
<td>&lt; 3 × 10⁻⁵</td>
<td>10⁻⁵</td>
<td>&lt; 3 × 10⁻⁶</td>
<td>–</td>
</tr>
<tr>
<td>nominal Δ</td>
<td>0.3</td>
<td>0.3</td>
<td>0.8</td>
<td>%</td>
</tr>
<tr>
<td>Core-to-SAP distance</td>
<td>~ 6.0</td>
<td>~ 2.5</td>
<td>~ 2.8</td>
<td>μm</td>
</tr>
</tbody>
</table>

(1) design specification supplied by vendor
(2) loss is for the x-axis for PZ fiber
(3) estimated
Appendix II

SCALAR COUPLER MODEL

For Parallel and Curved Fibers

Version 6 10/20/92
1.1 MByte Mathematica Version 2.0
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This Mathematica notebook contains the coupled-mode calculations for coupled fiber-optic waveguides. The polarization properties of the light are neglected for now, and the individual fiber fields are described by a pair of complex scalars. A 2x2 matrix representation of the coupled differential equations will be introduced and explored.

2.1 Parallel Fiber Coupling Coefficient

IMPORTANT VERSION 2.1 INFORMATION: Mathematica Version 2.1 contains a bug which results in difficulty in plotting of arrays of functions. If this notebook is evaluated with Version 2.1, the following string must be evaluated before attempting to plot any graphics. This cell has been deactivated for Version 2.0 running.

Unprotect[Plot]
ClearAttributes[Plot, HoldAll]

This set of calculations evaluates the coupling coefficient c for a pair of parallel, identical step-index optical fibers. A fixed, monochromatic light source is assumed with 1.3-micron wavelength, and polarization is neglected in this analysis. Assumptions include weak guidance and weak coupling, for propagation in the +z direction.
\[ c \ll \beta \quad \text{and} \quad \Delta \ll 1 \]
The core radius is given by \( a \); \( d \) is the core-to-core separation; \( \lambda \) is the wavelength; \( n \) is
the refractive index of the cladding; and \( \Delta n/n \) is the relative index \( \Delta n/n \) between the core and
cladding. The fiber and coupler are assumed to be lossless, and all dimensions are in
microns in this notebook.

First, the propagation constant \( k_0 \) and the V-number \( V \) is determined from the
waveguide parameters and operating wavelength. Then the transverse modal parameters
\( u \) and \( v \) are estimated - valid to within \( 1\% \) in the range \( 1 < V < 3 \) [1]. Finally, the
coupling coefficient is computed using the result of [2] for parallel fibers.

\[
\begin{align*}
\pi &= N[\Pi]; \\
k_0 &= 2 \pi / 1.300; \\
n &= 1.45; \\
d \Delta &= 0.01; \\
V &= k_0 a n \sqrt{2.0 \Delta n/n} \\
v &= 1.1428 V - 0.9960; \\
u &= \sqrt{V^2 - v^2}; \\
K_1 &= BesselK[1, v] \\
K_0 &= BesselK[0, (v \Delta n/n)/a] \\
c &= (u^2 K_0) / (k_0 n a^2 V^2 K_1^2) \\
3.46887 \\
0.0417169 \\
BesselK[0, 0.848063 \Delta] \\
1.79259 BesselK[0, 0.848063 \Delta]
\end{align*}
\]

Now, the coupling coefficient is plotted as a function of the normalized core-to-core
spacing, given by the ratio \( d/2a \) which is defined as the \( r \) variable. In this plot, \( \lambda = 1.3 \mu m; \Delta n/n = 1\% \); and \( n = 1.45 \) for Silica. The core diameters are 6, 7 and 8 microns, and
minimum separation occurs at \( d/2a = 1 \), where the cores are just touching. In the strong
coupling region, corrections are required to the simple equation used here which are beyond the scope of this model.

\[ a = \ldots \]

\[ \text{Cr} = c /. \ d \to 2 \ r \ a \]

\[ a = (3, 3.5, 4); \]

\[ \text{theLabels} = \{\text{Text}[^{2a = 6\text{um}}], \{1.1, .003\}], \text{Text}[^{7\text{um}}], \{1.06, .002\}], \text{Text}[^{8\text{um}}], \{1.06, .0008\}]\}; \]

\[ \text{Plot}[\text{Cr}, \{r, 1, 1.5\}, \text{AxesLabel} \to \{" d/2r", "\}, \text{Prolog} \to \text{theLabels}]; \]

\[ 1.79259 \ BesselK[0, 1.69613 \ a \ r] \]

where the horizontal scale, the core separation, is normalized by the core diameters. This clearly illustrates the dependence of coupling coefficient upon core spacing.
2.2 Parallel Matched Fiber Coupler

In this section we introduce the formalism for calculating the coupler transfer function matrix $\mathbf{A}$, which describes the input/output relationship of the coupler. Firstly, the matrix $\mathbf{K}$ describing the coupled differential equations is generated, which has the form

$$i \frac{\partial \mathbf{E}}{\partial z} = (\beta \mathbf{I} + \mathbf{K}) \mathbf{E}$$

This system of coupled differential equations will have an exponential solution like

$$\mathbf{E}(z) = e^{I\beta z} e^{iKz} \mathbf{E}_0$$

where $\beta$ is the average propagation constant of the four waveguide modes and $\mathbf{I}$ is the identity matrix. In this case, we neglect any common phase terms, and are assuming perfectly matched fiber such that the diagonal components of $\mathbf{K}$ are zero. Firstly, the eigenvalues and eigenvectors of $\mathbf{K}$ are determined, and the last two lines check the validity of the eigenvalues and eigenvectors for that matrix. By diagonalization, we can compute the exponential matrix

$$e^{i\mathbf{K}z}$$

where $\mathbf{K}$ is an arbitrary matrix.

The exponential of the matrix $\mathbf{K}$ yields the explicit solution for the differential equations. The trigonometry package is loaded by the first command below.

```
Needs["Algebra`Trigonometry`"]

c = .
K = {{0, c}, {c, 0}};
MatrixForm[K]
{eVal, eVec} = Eigensystem[K];

K . eVec[[1]] == eVal[[1]] eVec[[1]]
K . eVec[[2]] == eVal[[2]] eVec[[2]]

eVal
eVec
```
0  c

True

True

{-c, c}

{{-1, 1}, {1, 1}}

Now, the eigenvectors become the column vectors in the matrix \( QQ \), and the diagonal matrix \( EE \) contains the eigenvalues. The orthogonal matrix to \( QQ \) is generated, and is denoted as \( QI \); this permits calculating the exponential matrix to be calculated as [4]

\[
e^{iKz} = QQ e^{iEEz} QI = QQ DD QI
\]

and is computed in the section below. The matrix \( QQ \) is constructed and then the eigenvectors are normalized such that their magnitude is one. This orthonormal set of eigenvectors becomes the matrix \( QN \).

\[
QQ = \text{Transpose}[\text{eVec}]
\]

\[
QN = \frac{QQ}{\text{Abs}[QQ . \{1, i\}]} \]

\[
QI = \text{Inverse}[QN]
\]

\[
\{\{-1, 1\}, \{1, 1\}\}
\]

\[
\{\{-0.707107, 0.707107\}, \{0.707107, 0.707107\}\}
\]

\[
\{\{-0.707107, 0.707107\}, \{0.707107, 0.707107\}\}
\]

Note that the inverse of the normalized matrix is the normalized matrix transposed, which is the property of matrices having orthonormal column vectors.
EE = DiagonalMatrix[eVal];
DD = DiagonalMatrix[E^(I eVal z)];
MatrixForm[QN]
MatrixForm[DD]
MatrixForm[QI]

M = QN . DD . QI;
MatrixForm[M]

Some simplifications are applied to the matrix M, which clearly show the periodic nature of the coupling. The ComplexToTrig function which can be found in the package Algebra`Trigonometry` which was loaded in the beginning of this section. The command Rationalize removes the floating-point 1.000 in the matrix coefficients.

M1 = Together[ComplexToTrig[M]];
MatrixForm[M1]

M2 = Rationalize[M1];
MatrixForm[M2]

1. Cos[c z] 1. I Sin[c z]
1. I Sin[c z] 1. Cos[c z]
Cos[c z]    I Sin[c z]    
I Sin[c z]    Cos[c z]    

To plot the evolution of the coupling with z, assume that there is zero field in port 2, and port 1 is illuminated with unity amplitude. z is plotted from zero to 10 millimeters, and the backslash-n is a newline character so the printer prints the leading space characters.

```
fields = (M2 /. c -> 0.001) . Transpose[{1, 0}]
intensity = fields Conjugate[fields];
Plot[intensity, {z, 1, 10^4},
AxesLabel -> {"\n z in um", "I/Io"}]
{Cos[0.001 z], I Sin[0.001 z]}
```

The maximum coupling occurs at \( c \ z = \pi/2 \) and we can define the coupling length = \( \pi/2c \), and 50% coupling occurs at \( L = L_c/2 \).

An alternate expression for the coupler transfer function matrix uses the substitution of intensity coupling ratio \( k = \sin^2(2cz) \), and results in

```
rule1 = {Sin[c z] -> Sqrt[k], Cos[c z] -> Sqrt[1 - k]};
M3 = Rationalize[M1 /. rule1];
MatrixForm[M3]
```
\[ \sqrt{1 - k} \quad \text{I} \quad \sqrt{k} \\]
\[ \text{I} \quad \sqrt{k} \quad \sqrt{1 - k} \]

Which is the same result as computed in the text of this dissertation in Chapter II.

This model is for an ideal device; this will now be generalized for imperfect couplers.

### 2.3 Parallel Fiber Coupler with Loss

To include loss effects, for matched fiber the \( \beta \) coefficient is replaced by a complex value \( \beta \rightarrow \beta + i\alpha \) where if \( \alpha \) is positive, when the complex propagation coefficient is multiplied later by \( i \), the attenuation will have the correct negative sign which results in exponential decay. The eigenvalues will be of the form \( \beta + i\alpha \pm c \) therefore there will be an \( e^{i(\beta + i\alpha)z} \) factor out in front of the matrix, whose imaginary part is again dropped. This leaves only the the exponential decay factor common to all of the matrix coefficients.

\[ a a = 2.0 \times 10^{-5}; \]
\[ \text{fields} = E^{(-aa \ z)} * \]
\[ (M2 /. \ c \rightarrow 0.001) \ . \ Transpose[{1, 0}] \]

\[ \text{intensity} = \text{fields} \ \text{Conjugate}[\text{fields}]; \]
\[ \text{Plot}[\text{intensity}, \{z, 1, 10^4\}, \]
\[ \text{AxesLabel} \rightarrow \{"z in \text{um}", \text{"I/Io"}\}] \]
Which decays as expected. This effect is highly exaggerated in this plot; it would take 5 kilometers to reach this attenuation level with modern fiber. In practice, loss induced due to polishing, etcetera is included as a lumped factor.

**Coupling Behaviour in Asymmetrically Lossy Couplers**

For the case of differential symmetric/antisymmetric supermode loss [5], the matrix is modified by multiplying by the \( \mathbf{N} \) matrix which mixes the coefficients of the \( \mathbf{A} \) matrix to yield \( \mathbf{A}\mathbf{A} \).

\[
\begin{align*}
\rho &= 0.8; \\
\varepsilon &= 0.2; \\
\mathbf{N} &= \{\{\rho, \varepsilon\}, \{\varepsilon, \rho\}\}; \\
\text{MatrixForm}[\mathbf{N}] \\
\mathbf{A}\mathbf{A} &= \mathbf{N} \cdot \mathbf{M}_2 \\
\text{fields} &= (\mathbf{A}\mathbf{A}/.c\to 0.001) \cdot \text{Transpose}[\{1, 0\}] \\
\text{intensity} &= \text{fields} \text{Conjugate}[\text{fields}]; \\
\text{Plot}[\text{intensity}, \{z, 1, 10^4\}, \\
\text{AxesLabel} \to \{"n z in um", "I/I_0"}, \\
\text{PlotRange} \to \{0, 1\}]
\end{align*}
\]
The mismatch of the two transmission coefficients will keep the coupler from having a splitting ratio below a minimum value, in this case \((0.2)^2\).

Phase Behaviour in Asymmetrically Lossy Couplers

In Section 2.2, it was shown that normally there is a \(\pi/2\) phase lag for the coupled leg with respect to the throughput leg of the device. In the case of differential
symmetric/antisymmetric supermode loss, the phase change associated with the preceding result is

\[
\text{phDiff} = \text{Arg}[\text{fields}[[2]]] - \text{Arg}[\text{fields}[[1]]];
\]

\[
\text{Plot}[\text{phDiff}, \{z, 0, 10^4\},
\text{AxesLabel} \to \{"\text{\textbackslash n \ z in um}"", "\text{phase}"\}]
\]

\[
\begin{aligned}
\text{phase} & \\
1 & 0.5 \ 0 \ -0.5 \ -1 \\
2000 & 4000 & 6000 & 8000 & 10000 \ z \ \text{in um}
\end{aligned}
\]

The above plot is simplified by making the calculation modulo \(\pi\), and redisplaying for 3 cases of increasing differential losses. The normal \(\pi/2\) phase difference has been subtracted out. Note the large deviation of the phase from the ideal case near the high- and low-coupling ratio regions. Plugging in more realistic values of epsilon yields

\[
rho = 0.99;
\]
\[
\text{eps} = 0.0001;
\]
\[
\text{NN} = \{\{\text{rho}, \text{eps}\}, \{\text{eps}, \text{rho}\}\};
\]
\[
\text{AA} = \text{NN} \cdot \text{M2}
\]
\[
\text{fields} = (\text{AA} /\ c \to 0.001) \cdot \text{Transpose}[\{1, 0\}]
\]
\[
\text{phase} = \text{Arg}[\text{fields}[[2]]] - \text{Arg}[\text{fields}[[1]]];
\]
\[
\text{phRel} = \text{Mod}[\text{phase}, \pi] - \pi/2;
\]
\[
\text{Plot}[\text{phRel}, \{z, 0, 10^4\},
\text{AxesLabel} \to \{"\text{\textbackslash n \ z in um}"", "\text{phase - Pi/2}"\}]
\]
\{(0.99 \cos[c \ z] + 0.0001 \ I \ \sin[c \ z], \\
0.0001 \ \cos[c \ z] + 0.99 \ I \ \sin[c \ z]), \\
(0.0001 \ \cos[c \ z] + 0.99 \ I \ \sin[c \ z], \\
0.99 \ \cos[c \ z] + 0.0001 \ I \ \sin[c \ z]\}\} \\

\{(0.99 \ \cos[0.001 \ z] + 0.0001 \ I \ \sin[0.001 \ z], \\
0.0001 \ \cos[0.001 \ z] + 0.99 \ I \ \sin[0.001 \ z]\}\}

\text{phase} - \frac{\pi}{2}

\text{which illustrates the increase of the phase error for couplers near zero or 100\% coupling.}

\section{2.4 Parallel Non-Matching Fiber Coupler}

Now we generalize for the case of non matched fiber, to see the effect upon the maximum coupling ratio achievable. In this subsection, $b$ is the difference of the propagation constants of fiber $A$ and fiber $B$, divided by two. Some of the assumptions are

\[ \beta_A - \beta_B \ll c \ll \beta \quad \text{and} \quad \Delta \ll 1 \]

The first command deletes all prior variable definitions in this Mathematica session.
Remove["Global`*"]

K = {{b, c}, {c, -b}};
MatrixForm[K]
{eVal, eVec21} = Eigensystem[K];
eVal
eVec21
b  c
c  -b

{-Sqrt[b^2 + c^2], Sqrt[b^2 + c^2]}

{{b - Sqrt[b^2 + c^2], 1}, {b + Sqrt[b^2 + c^2], 1}}

VERSION 2.0.4 derived a different result for the eigenvectors than the v2.1 above. Until this discrepancy is resolved, plug in the 2.0.4 values to avoid messing up the intermediate algebra steps. Both eigenvectors are equivalent and produce the same result.

eVec = {{(b - Sqrt[b^2 + c^2])/c, 1}, {(b + Sqrt[b^2 + c^2])/c, 1}}
Together[eVec - eVec21]

{{b - Sqrt[b^2 + c^2], 1}, {b + Sqrt[b^2 + c^2], 1}}

{{0, 0}, {0, 0}}

which shows that they are identical. Now, the matrix is multiplied out.
Which is the matrix describing the coupler. Two new rules are defined to permit better viewing of the matrix, which substitute e\(1\) and -e\(1\) for the two eigenvalues. The matrix is redisplayed with the simplified variables (the characters "/." can be interpreted as meaning "such that." rule4 is for back-substitution purposes later in the calculations.
rule2 = {{Sqrt[b^2 + c^2] -> e1,
            1/(Sqrt[b^2 + c^2]) -> 1/e1}
rule4 = {e1 -> Sqrt[b^2 + c^2]}

eVal /. rule2
eVec /. rule2
Expand[M /. rule2]
Together[ComplexToTrig[%]]

{Sqrt[b^2 + c^2] -> e1, \(\frac{1}{\sqrt{b^2 + c^2}}\) -> \(\frac{1}{e1}\)}

{e1 -> Sqrt[b^2 + c^2]}

{-e1, e1}

\[\left\{\frac{b - e1}{c}, 1\right\}, \left\{\frac{b + e1}{c}, 1\right\}\]

\[\left\{\frac{-I e1 z}{2} + \frac{I e1 z}{2} - \frac{b E}{2 e1} + \frac{b E}{2 e1},\right\}
\[\left\{\frac{b e1}{2 c} - \frac{I e1 z}{2 c e1} - \frac{I e1 z}{2 c e1} + \frac{I e1 z}{2 c e1},\right\},\]
\[\left\{\frac{-(c E - I e1 z)}{2 e1} + \frac{c E}{2 e1},\right\}
\[\left\{\frac{-I e1 z}{2} + \frac{I e1 z}{2} + \frac{b E}{2 e1} - \frac{I e1 z}{2 e1} - \frac{b E}{2 e1}\right\}\]

\[\left\{\frac{e1 \text{Cos}[e1 z]}{e1},\right\}
\[\left\{\frac{-I (b^2 \text{Sin}[e1 z] - e1^2 \text{Sin}[e1 z])}{c e1}\right\},\]
\[\left\{\frac{I c \text{Sin}[e1 z]}{e1}, \frac{e1 \text{Cos}[e1 z] - I b \text{Sin}[e1 z]}{e1}\right\}\]

We can back-substitute the eigenvalue in terms of b and c if desired to eliminate e1
\[ \text{M} = \text{Apart[\%]} /\text{. rule4} \]

\[ \left\{ \frac{\cos(\sqrt{b^2 + c^2}) \ z}{\sqrt{b^2 + c^2}}, \right. \]
\[ \left. \frac{\frac{I \ b \ \sin(\sqrt{b^2 + c^2}) \ z}{\sqrt{b^2 + c^2}}, \right\} \]
\[ \frac{I \ c \ \sin(\sqrt{b^2 + c^2}) \ z}{\sqrt{b^2 + c^2}}, \]
\[ \frac{\frac{I \ c \ \sin(\sqrt{b^2 + c^2}) \ z}{\sqrt{b^2 + c^2}}, \right\} \}
\[ \cos(\sqrt{b^2 + c^2}) \ z - \frac{I \ b \ \sin(\sqrt{b^2 + c^2}) \ z}{\sqrt{b^2 + c^2}} \} \]

Note that this reduces to the ideal matrix for the case of \( b = 0 \). Now we will plot the coupler matrix transfer function, subject to the same initial conditions as in the section 2.2. First, a small mismatch is introduced and the intensity is plotted.

\[
(* \text{PROCEDURE \hspace{1em} } \text{Launch1} *)
\]

\[
\text{Launch1[bb_] := (}
\[
\text{fields = (M1 /\text{. b->bb}) . Transpose[{1, 0}];}
\[
\text{fields Conjugate[fields]}
\]
\]

\[
c = 0.001;
\]

\[
\text{Plot[Launch1[0.0002], \{z, 0, 10^4\},}
\]
\[
\text{AxesLabel -> \{"\{n \hspace{1em} z \hspace{1em} in \hspace{1em} um\}, \"I/Io\}\}}
\]

\[
\text{Plot::plnr: CompiledFunction[\{z\}, <<1>>, -Co<<8>>de-][z]}
\]
\[
is \text{not a machine-size real number at } z = 0.. \]
Note the reduction in maximum coupling ratio achievable from 100 percent to somewhat less than 100 percent coupling. Now, try a larger $b$ value, and change the initial $z$ value to 1 um to eliminate the error message.

```
Plot[Launch1[0.0005], {z, 1, 10^4},
AxesLabel -> {
"\n z in um",
"I/Io"}]
```

The coupling is reduced to a significant fraction of 100%, and this device would be unsuitable for a spliceless resonator which requires on the order of 95% coupling. Highly mismatched fibers will further degrade the coupling performance of the device:
Performing a three-dimensional plot of this phenomena illustrates the dependence better.
To plot the dependence of maximum attainable coupling ratio on fiber matching, set $z$ equal to the coupling length, and evaluating the normalized the intensity leaving port four

$$L_c = \frac{\pi}{2} \div \sqrt{b^2 + c^2}$$

$$z = L_c;$$

```
PLOT[Launchl[b][[2]], {b, 0, 2 c},
  PlotRange -> {0, 1},
  AxesLabel -> {"\n b", "c max"}]
```

$$\frac{\pi}{2 \sqrt{1 \times 10^{-6} + b^2}}$$
This would limit the finesse of a spliceless reflection-type ring resonator, which needs a coupler which has a splitting ratio near unity.

### 2.5 Curved Fiber Coupler & Coupling Curves

In this excercise, the coupling curves are calculated for fiber lateral displacement, as defined by the polished coupler geometry. First, the V-number and coupling coefficient is calculated for Fujikura PM fiber. The effective interaction length is estimated for a 30 cm bending radius, using the relation described by [2] for curved fiber geometry.

```
Remove["Global`*""]

a = 3.5;
d = .;
w = 1.3;
pi = N[Pi];
ko = 2 pi / w;
n = 1.45;
dd = 0.003;

V = ko a n Sqrt[2.0 dd]

v = 1.1428 V - 0.9960;
```
\[ u = \sqrt{v^2 - v^2}; \]
\[
K_1 = \text{BesselK}[1,v];
\]
\[
K_0 = \text{BesselK}[0, (v \, d / a)];
\]
\[
c = \frac{(u^2 \, K_0)}{(\kappa \, n \, a^2 \, v^2 \, K_1^2)}
\]
\[
R = 300 \times 10^3;
\]
\[
\text{Le} = \sqrt{\pi \, a \, R / (2 \, v)}
\]

1.89998

0.0352267 \text{BesselK}[0, 0.335798 \, d]

1184.63

Thus the interaction is on the order of 1 millimeter. The core-to-core separation is now a function of the sum of the polishing depths \(x_0\), and the transverse offset \(y_0\), which is adjusted by micrometer. The ideal coupler is assumed, with \(I_1 = 1\) and \(I_2 = 0\) initial conditions. All dimensions are in microns. It should be noted that there is a factor of 0.5 with respect to the indication on the micrometer, which is not included in this calculation. The function `couplingCurve` is defined to calculate and display the coupling curves for various \(x_0\) values.

(* FUNCTION    couplingCurve *)

couplingCurve[x_] := (  
d = \sqrt{x^2 + y_0^2};  
i3 = (\cos[c \, \text{Le}])^2;  
i4 = (\sin[c \, \text{Le}])^2;  
\text{do} = x/2 - a;  
\text{Print}[:Core/clad boundary to polished surface = "},  
\text{do, " um"};  
\text{Plot}[(i3,i4), \{y_0, -20,20\}, \text{PlotPoints} \to 100]  
)

\(x_0 = 11;\)
couplingCurve[x0]

Core/clad boundary to polished surface = 2. um
Polishing further, such that the minimum core spacing is only 1 micron, permits nearly 100% coupling to occur for zero offset.

\[ x_0 = 9; \]
\[ \text{couplingCurve}[x_0] \]

Core/clad boundary to polished surface = 1. um

Polishing still further, such that there is 1/2 micron minimum core spacing, results in 100% coupling to occur, as shown below. The coupling ratio starts to decrease near the zero offset position, as expected.
Increasing the groove radius to 1 meter, and therefore the interaction length to about 2 millimeters has the same result: more cycles as the device is adjusted about zero offset.

```plaintext
R = 10^6;
Le = Sqrt[pi a R / (2 v)]
```

```
xo = 8;
couplingCurve[xo]
```

```
2162.82
```

Core/clad boundary to polished surface = 0.5 um
A three-dimensional plot will now be attempted, to illustrate the effect of longitudinal fiber offset which was calculated in Chapter II. This is plotted as a density plot, where the grey level is proportional to coupling ratio; white is normalized 100% level.

(* FUNCTION couplingSurface *)

couplingSurface[x_] := (
  d = Sqrt[x^2 + yo^2] + zo^2/(4 R Sqrt[1 + (yo/x)^2]);
  i3 = (Cos[c Le])^2;
  i4 = (Sin[c Le])^2;
  do = x/2 - a;
  Print["Core/clad boundary to polished surface = ",
    do, " um"];
  DensityPlot[i4, {zo, -3000, 3000}, {yo, -20, 20},
    PlotPoints -> 100, AspectRatio -> 0.5, FrameLabel -> {
      "z-displacement, um", "y-displacement, um"}]
)

couplingSurface[9]

Core/clad boundary to polished surface = 1. um
Which illustrates the intensity in the coupled fiber as a function of displacement in \( y \) and \( z \) from the center position.
2.6 References


Appendix III

POINCARÉ SPHERE

Black & White Graphics Version

Version 6 10/18/92
1.0 MBytes Mathematica Version 2.0
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This Mathematica notebook contains the procedure greyPoincare which performs
the conformal mapping of optical state-of-polarization to what is known as the Poincaré
sphere. Black & white 3-dimensional graphics are used to conserve memory usage to
about 150KBytes per image, and a bug in the color version was corrected.

3.1 Poincaré Sphere

An important graphical tool is introduced, which enables mapping optical
state-of-polarization onto a spherical surface known as the Poincaré sphere [1-4]. The
packages Shapes and Graphics3D are requires for this model and need to be loaded,
and the sphere graphic is generated in the initialization cell below. The cartesian xyz axes
are defined, which correspond to Horizontal linear polarization, $+45^\circ$ linear polarization,
and right-handed circular polarization, respectively. The sphere is set up to have
10-degree spacing in longitude and latitude.
Needs["Graphics`Shapes`"]
Needs["Graphics`Graphics3D`"]
niceSphere = Evaluate[Graphics3D[Sphere[1, 36, 18]]]

xyzAxes = 

{Line[{{0, 0, 1.2}, {0, 0, -1.2}}],
  Line[{{1.2, 0, 0}, {-1.2, 0, 0}}],
  Line[{{0, 1.2, 0}, {0, -1.2, 0}}]},

{Text["L", {0, 0, 1.3}], Text["R", {0, 0, -1.3}],
  Text["H", {1.3, 0, 0}], Text["V", {-1.3, 0, 0}],
  Text["+45", {0, 1.3, 0}], Text["-45", {0, -1.3, 0}]};

The initialization takes some time to run so be patient, especially on a slower CPU. No
graphics is produced at this point; this will be performed by the procedure entitled
plotPoincare, which takes input in the form of Ein = \{Ex, Ey\} or an array of
\{\{Ex1, Ey1\}, \{Ex2, Ez2\}, . . .\}. The state-of-polarization (azimuth and
ellipticity) will be calculated converted to longitude and latitude and plotted on the
Poincaré Sphere.

The variable R is the ratio of the absolute values of the x and y components, and d is the
phase difference between the two field components.

a1 = \sin(2 \text{ ellipticity}) = \sin(\text{latitude}) \quad \text{and} \quad a2 = \tan(2 \text{ azimuth}) = \tan(\text{longitude})

the side variable corrects the bug that is in the color version.
(* Procedure grayPoincare *)

grayPoincare[xyfield_] := (
{shouldbe2, Elength} = Dimensions[xyfield];

R = Evaluate[Abs[xyfield[[2]]] / Abs[xyfield[[1]]]];  
side = Sign[1 - R^2];  

dd = Evaluate[Arg[xyfield[[2]]] - Arg[xyfield[[1]]]];  
a1 = 2 R Sin[dd] / (1 + R^2);  
a2 = 2 R Cos[dd] / (1 - R^2);  
xx = side Sqrt[(1 - a1^2)/(1 + a2^2)];  
yy = side a2 Sqrt[(1 - a1^2)/(1 + a2^2)];  
zz = a1;

pplot = Table[Point[{xx[[i]], yy[[i]], zz[[i]] }],
{i, Elength}];  
poincare = {niceSphere, Graphics3D[{pplot, xyzAxes}]};  

Show[poincare, ColorOutput -> GrayLevel]
)

where the color information has been suppressed by the last statement. Note that there
are singularities when R approaches 1 - this occurs at the "poles" of the sphere, which
must be avoided.

3.2 PM Fiber Propagation

In this subsection, the behaviour of the state-of-polarization is analyzed for the case of
ideal lossless birefringent fiber. To more fully illustrate the evolution of the
state-of-polarization, linear polarization is launched at 45° angle ot the x-axis of the fiber.
The +ikz convention is used throughout these examples; first, the propagation constants
are evaluated as a function of fiber beat length (birefringence). An array of x- and y-field
decomposition values are computed as the light steps down the fiber at fixed increments.
\[ n = 1.5; \]
\[ w = 1.3; \quad \text{(* 1.3 \text{ um wavelength } *)} \]
\[ Lb = 2000; \quad \text{(* 2\text{mm beat length } *)} \]
\[ \pi = \pi \text{N} \]
\[ ko = 2 \pi n / w; \]
\[ kx = ko + \pi / Lb; \]
\[ ky = ko - \pi / Lb; \]

\[ imax = 500; \]
\[ Zstep = 25.1; \]
\[ Z = Zstep \text{Range}[imax]; \]

\[ \text{Exo} = 1; \]
\[ \text{Eyo} = 0.99; \]
\[ Ein = \{\text{Exo} \, E^{(I \, kx \, Z)}, \, \text{Eyo} \, E^{(I \, ky \, Z)}\}; \]

In the color mode, the plotPoincare function takes roughly 12 seconds of CPU time on a Sun workstation, with approximately 1 minute to ship 380 KBytes of data over the Ethernet and rendering of the sphere, which is done on the MacIntosh IIcx front end. The new procedure reduces total time to 38 seconds, and memory to approximately 140 KBytes per sphere rendered, a major improvement.

\[ \text{grayPoincare}[Ein] \]
The state-of-polarization traces out a great circle which crosses both poles, traversing the circumference a number of times. We can see that the state-of-polarization is linear where it crosses the equator at the +45° and -45° azimuth (longitude/2) and becomes circular near the two poles. Right-handedness is defined on the upper hemisphere. The evolution is illustrated below for increasing z.
and repeats in a periodic fashion. In this analysis we neglect polarization cross-coupling between the two principal axes. For random length changes, such as with thermal perturbations, the state-of-polarization varies in an undeterministic manner around that great circle.

Let's now assume a better launch condition, having a linear polarization which is tilted slightly off of the slow-axis of the fiber. 3-degrees offset corresponds to only 5% of the light launched in the y-axis.
n = 1.5;
w = 1.3; (* 1.3 um wavelength *)
Lb = 2000; (* 2mm beat length *)
pi = N[Pi];

ko = 2 pi n / w;
kx = ko + pi / Lb;
ky = ko - pi / Lb;

imax = 50;
Zstep = 50;
Z = Zstep Range[imax];

Exo = 1;
Eyo = 0.05;
Epm = {Exo E^(I kx Z), Eyo E^(I ky Z)};

The procedure is modified slightly, to permit passing of plotting options through the procedure to the final Show command. This will permit rotating the sphere and incorporating other plotting options, like any Mathematica graphics command.

(* Procedure greyPoincare *)

greyPoincare[xyfield_, opts__] := (
{shouldbe2, Elength} = Dimensions[xyfield];

R = Evaluate[Abs[xyfield[[2]]] / Abs[xyfield[[1]]]];  
side = Sign[1 - R^2];

dd = Evaluate[Arg[xyfield[[2]]] - Arg[xyfield[[1]]]]; 

a1 = 2 R Sin[dd] / (1 + R^2);
a2 = 2 R Cos[dd] / (1 - R^2);

xx = side Sqrt[(1 - a1^2)/(1 + a2^2)];

yy = side a2 Sqrt[(1 - a1^2)/(1 + a2^2)];

zz = a1;

pplot = Table[Point[{ xx[[i]], yy[[i]], zz[[i]] }],
{i, Elength}];
poincare = {niceSphere, Graphics3D[{pplot, xyzAxes}]};

Show[poincare, ColorOutput -> GrayLevel, opts]
)

General::spell1:
Possible spelling error: new symbol name "greyPoincare"
is similar to existing symbol "grayPoincare".

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And the evolution of the state-of-polarization is plotted and the sphere is rotated to a better viewing angle of the horizontal or \( x \)-polarization.

\[
grey\text{Poincare}[Epm, \text{ViewPoint} \rightarrow \{4, -2, 2\}];
\]

Comment: If the user wishes to restore the color graphics capability, simply pass \text{ColorOutput} \rightarrow \text{RGBColor} to the \text{greyPoincare} procedure as an option.

The polarization stays near the H-polarized state, but is generally slightly elliptical. The state-of-polarization of the light is linear polarized at a \( 3^\circ \) angle, then becomes a slightly elliptical right-handed polarization, at a \( 0^\circ \) angle. The ellipse flattens out and tilts until it
becomes linear polarization at a $-3^\circ$ angle, then becomes a left-handed ellipse, and back to
linear polarization at $+3^\circ$ angle. This cycle is repeated every beat length, as before.

The procedure doesn’t care how many curves are in its input data file, as long as the data
is in the right format. Now, the polarization state will be plotted for various
misalignment angles, with the help of a new procedure which calculates the propagation
down a length of PM fiber.

(* Procedure generateArray *)

generateArray[angle_, Zstep_, Nsteps_] := {
Exo = N[Cos[angle]];  
Eyo = N[Sin[angle]];  
Z = Zstep Range[Nsteps];

Earray = {Exo E^(I kx Z), Eyoe^(I ky Z)}
}

list45 = generateArray[Pi (44.9/180), 12.0, 200];
list30 = generateArray[Pi (30/180), 20.0, 150];
list20 = generateArray[Pi (20/180), 25.0, 100];
list10 = generateArray[Pi (10/180), 50.0, 50];
list05 = generateArray[Pi (5/180), 80.0, 30];
list00 = generateArray[0, 1, 1];

fiveCircles = Transpose[Join[
Transpose[list45], Transpose[list30],
Transpose[list20], Transpose[list10],
Transpose[list05], Transpose[list00]]];

{shouldbe2, Elength} = Dimensions[fiveCircles]
{2, 531}

greyPoincare[fiveCircles, ViewPoint -> {4, -2, 2}]
To modify the above graphics, a Postscript-interpreting graphics software must be used, such as Adobe Illustrator. This permits adding stuff to the sphere which can be later converted to an encapsulated Postscript format, that may then be inserted into most word processors. This procedure is for MacIntosh, and probably varies for other platforms.

The procedure for transfer is:

1) click mouse on graphic and copy
2) convert clipboard to Adobe 88 file type (QMS option for color graphics)
3) name file and save file  
4) launch Adobe Illustrator and open the saved file  
5) modify graphic and save as Adobe Illustrator 3.0 file (color or B&W preview)  
6) EPS file may now be used by other software

3.3 PZ Fiber Propagation

This example describes the propagation in a polarizing birefringent optical fiber, having high loss in the y-axis. Due to the limited number of steps used, the loss (5 dB per meter) is exaggerated by a factor of 10.

\[ n = 1.5; \]
\[ w = 1.3; \]
\[ Lb = 2000; \]
\[ \pi = \text{N}[\pi]; \]

\[ ko = 2\pi n / w; \]
\[ kx = ko + \pi / Lb; \]
\[ ky = ko - \pi / Lb; \]

\[ imax = 500; \]
\[ Zstep = 25; \]
\[ Z = Zstep \text{Range}[imax]; \]

\[ Exo = 1; \]
\[ Eyo = 1; \]
\[ Epz = \{Exo \, \text{E}^{(I \, kx \, Z)}, \, Eyo \, \text{E}^{(-0.00023 \, Z + I \, ky \, Z)}\}; \]

The field is plotted on the sphere, which shows the periodic cycling of the state-of-polarization. Even though equal amounts of x and y-polarized light is initially launched, the attenuation of the y-component is such that the fiber is eventually propagating purely x-polarized light.

\[ \text{greyPoincare}[Epz, \text{ViewPoint} \rightarrow \{4, -2, 2\}] \]
Which illustrates the spiraling of the SOP on the sphere toward the x-polarization due to the attenuation of the light in the y-axis of the fiber, and any cross-coupled light as well.
3.4 Example of Aliasing

To demonstrate the effects of aliasing which can lead to incorrect conclusions if misinterpreted, the z step is purposely chosen large, and funny effects are displayed. Note that the y-loss has been reduced also in this example.

```plaintext
n = 1.5;
w = 1.3;
Lb = 2000;
pi = N[Pi];

ko = 2 pi n / w;
px = ko + pi / Lb;
py = ko - pi / Lb;

imax = 500;
Zstep = 210;
Z = Zstep Range[imax];

Exo = 1;
Eyo = 1;
Eal = {Exo E^(I px Z), Eyo E^(-0.000023 Z + I py Z)};

greyPoincare[Eal, ViewPoint -> {4, -2, 2}]
```
This produces the optical illusion which could lead to incorrect interpretation of the SOP,
3.5 References


Two out-of-print books also contain lots of information on polarization phenomena:


This Mathematica notebook contains the model for birefringent fiber couplers, which is an extension of the analysis performed in Appendix II. Different cases will be examined, for zero, small, and general misalignment conditions, which will serve as the foundation for subsequent models on resonators.

4.1 PM Coupler Calculations

The differential equations describing the polarization characteristics of the coupler are now solved. The author wishes to thank Dan Lichtblau and K. J. Paradise at Wolfram Research for assistance. Assumptions include identical, ideal (no loss or polarization cross-coupling) birefringent fibers, and weak coupling. It is also assumed that the coupling coefficient is independent of birefringence and axial alignment; the misalignment angle $\theta$ is the only mechanism coupling the polarization states. For the 4x1 column vector describing the $(E_x, E_y)$ fields in fiber A and fiber, the optical propagation through the two coupled waveguides is described by the following differential equation [1,2]

$$i \frac{\partial \mathbf{E}}{\partial z} = (\beta \mathbf{I} + \mathbf{K}) \mathbf{E}$$
where $\beta$ is the average propagation constant of the four waveguide modes and $I$ is the identity matrix. The technique we will use is to evaluate the exponential of the matrix $K$, to yield a solution of the form

$$E = e^{i\beta z} e^{iKz} E_0$$

which requires diagonalization of $K$. Typically, the constant phase-factor $e^{i\beta z}$ is dropped. We begin by setting up the 4x4 matrix that describes the $K$ matrix.

$$K = \{\{b, 0, c, d\}, \{0, -b, -d, c\}, \{c, -d, b, 0\}, \{d, c, 0, -b\}\};$$

Note the symmetry for the matrix. The coefficients are defined as follows, where $C$ is the coupling coefficient as calculated in Appendix II:

$$\beta = (\beta_S + \beta_F)/2 \quad \text{average propagation constant} = \frac{2\pi n}{\lambda}$$

$$b = (\beta_S - \beta_F)/2 \quad \text{proportional to the fiber birefringence} = \frac{\pi}{L_b}$$

$$c = C \cos(t) \quad \text{and} \quad d = C \sin(t) \quad \text{relate to misalignment}$$
The eigenvalues and eigenvectors are calculated, using the EigenSystem command, and the eigenvalues are displayed. The corresponding eigenvectors can be found in eigen[[2]].

\[
\text{eigen} \equiv \text{Eigensystem}[K];
\]
\[
\text{Print}["\text{Eigenvalues of matrix } K \text{ are calculated to be}\""]
\]
\[
\text{eVal} \equiv \text{eigen[[1]]}
\]
\[
\text{eVec} \equiv \text{eigen[[2]]};
\]

Eigenvalues of matrix $K$ are calculated to be

\[
\{-\sqrt{b^2 - 2bc + c^2 + d^2}, \sqrt{b^2 - 2bc + c^2 + d^2},
\-\sqrt{b^2 + 2bc + c^2 + d^2}, \sqrt{b^2 + 2bc + c^2 + d^2}\}
\]

The eigenvectors $E_i$ and the eigenvalues $\Lambda_i$ are plugged into the equation $K E_i = \Lambda_i E_i$ and simplifying results in zero result for all cases

\[
\text{shouldBeZero} \equiv K . \text{eVec[[1]]} - \text{eVal[[1]]} \times \text{eVec[[1]]};
\text{Together[\%]}
\]
\[
\text{shouldBeZero} \equiv K . \text{eVec[[2]]} - \text{eVal[[2]]} \times \text{eVec[[2]]};
\text{Together[\%]}
\]
\[
\text{shouldBeZero} \equiv K . \text{eVec[[3]]} - \text{eVal[[3]]} \times \text{eVec[[3]]};
\text{Together[\%]}
\]
\[
\text{shouldBeZero} \equiv K . \text{eVec[[4]]} - \text{eVal[[4]]} \times \text{eVec[[4]]};
\text{Together[\%]}
\]
\[
\{0, 0, 0, 0\}
\]
Thus the eigenvalues and eigenvectors are correct. For now we ignore birefringence by setting $\beta_S = \beta_F$ and define the resulting eigenvalues $\text{eVal0}$ and the variable $\text{eVec0}$ equal to the eigenvectors. This is not a valid approximation, but is useful for some qualitative understanding.

\[
\begin{align*}
\text{rule1} &= b \rightarrow 0 \\
\text{eVal0} &= \text{eigen[[1]]} /\text{. rule1} \\
\text{eVec0} &= \text{eigen[[2]]} /\text{. rule1}
\end{align*}
\]

\[
b \rightarrow 0
\]

\[
\{-\text{Sqrt}[c^2 + d^2], \text{Sqrt}[c^2 + d^2], -\text{Sqrt}[c^2 + d^2], \\
\text{Sqrt}[c^2 + d^2]\}
\]

\[
\{\{1, \frac{c - \text{Sqrt}[c^2 + d^2]}{d}, -1, \frac{c - \text{Sqrt}[c^2 + d^2]}{d}\},
\{1, \frac{c + \text{Sqrt}[c^2 + d^2]}{d}, -1, \frac{c + \text{Sqrt}[c^2 + d^2]}{d}\},
\{1, \frac{c + \text{Sqrt}[c^2 + d^2]}{d}, 1, \frac{-c - \text{Sqrt}[c^2 + d^2]}{d}\},
\{1, \frac{c - \text{Sqrt}[c^2 + d^2]}{d}, 1, \frac{-c + \text{Sqrt}[c^2 + d^2]}{d}\}\}
\]

To calculate $\exp(K)$ the matrix is diagonalized by factoring into the form $K = Q D Q^{-1}$ where $D$ is a diagonal matrix containing the eigenvalues [3]. $Q$ is the matrix whose column vectors contains the eigenvectors, and is always invertable since the column
vectors are linearly independent for non-repeated eigenvalues. This permits writing

$$\exp(i \mathbf{K} z) = \exp(i \mathbf{Q} \mathbf{D} \mathbf{Q}^{-1} z) = \mathbf{Q} \exp(i \mathbf{D} z) \mathbf{Q}^{-1}$$

The orthogonal eigenvectors are set up as column vectors in a matrix $\mathbf{QQ}$, and its inverse $\mathbf{QI}$ is computed. The exponential of the diagonal matrix is calculated as Mathematica variable $\mathbf{DD}$.

$$\mathbf{QQ} = \text{Transpose}[\text{eVec0}]$$

$$\mathbf{QI} = \text{Together}[\text{Inverse}[\mathbf{QQ}]]$$

$$z = .$$

$$\mathbf{DD} = \text{DiagonalMatrix}[\mathbf{E}^\mathbf{I} (\mathbf{eVal0} z)]$$

\[
\begin{bmatrix}
1, 1, 1, 1, & \frac{c - \sqrt{c^2 + d^2}}{d}, & \frac{c + \sqrt{c^2 + d^2}}{d}, \\
\frac{c + \sqrt{c^2 + d^2}}{d}, & \frac{c - \sqrt{c^2 + d^2}}{d}, & \{1, -1, 1, 1\}, \\
\frac{c - \sqrt{c^2 + d^2}}{d}, & \frac{c + \sqrt{c^2 + d^2}}{d}, & \{1, -1, 1, 1\}, \\
\frac{-c - \sqrt{c^2 + d^2}}{d}, & \frac{-c + \sqrt{c^2 + d^2}}{d}
\end{bmatrix}
\]
This rather cumbersome result can be displayed in a more compact form by replacing the radical by it's simplified value $C$ by virtue of the trigonometric identity $\sin^2(t) + \sin^2(t) = 1$.

The eigenvalues are also redisplayed.
rule2 = {Sqrt[c^2 + d^2] -> C}
eVal0 /. rule2

QQ = Transpose[eVec0] /. rule2;
MatrixForm[QQ]

QI = Together[Inverse[QQ]];
MatrixForm[QI]

z = .
DD = DiagonalMatrix[E^(I eVal0 z)] /. rule2;
MatrixForm[DD]

{Sqrt[c^2 + d^2] -> C}

{-C, C, -C, C}

\[
\begin{array}{cccc}
1 & 1 & 1 & 1 \\
\frac{c - C}{d} & \frac{c + C}{d} & \frac{c + C}{d} & \frac{c - C}{d} \\
-1 & -1 & 1 & 1 \\
\frac{c - C}{d} & \frac{c + C}{d} & \frac{-c - C}{d} & \frac{-c + C}{d}
\end{array}
\]

\[
\begin{array}{cccc}
c + C & -d & -(c + C) & -d \\
\frac{4 C}{4 C} & \frac{4 C}{4 C} & \frac{4 C}{4 C} & \frac{4 C}{4 C} \\
-c + C & d & c - C & d \\
\frac{4 C}{4 C} & \frac{4 C}{4 C} & \frac{4 C}{4 C} & \frac{4 C}{4 C} \\
-c + C & d & -c + C & -d \\
\frac{4 C}{4 C} & \frac{4 C}{4 C} & \frac{4 C}{4 C} & \frac{4 C}{4 C} \\
c + C & -d & c + C & d \\
\frac{4 C}{4 C} & \frac{4 C}{4 C} & \frac{4 C}{4 C} & \frac{4 C}{4 C}
\end{array}
\]
The matrix $A$ which describes the coupler is now computed. The Trigonometry
package is now loaded and various Mathematica algebraic manipulation and
simplification commands are invoked to gather and cancel terms.

\begin{verbatim}
Needs["Algebra`Trigonometry`"]
A0 = QQ . DD . QI;

Expand[%]
Apart[%]
\end{verbatim}

\[
\begin{pmatrix}
-\frac{I C z}{2} & \frac{I C z}{2} & 0 & 0 \\
0 & \frac{I C z}{2} & 0 & 0 \\
0 & 0 & -\frac{I C z}{2} & 0 \\
0 & 0 & 0 & \frac{I C z}{2}
\end{pmatrix}
\]
The matrix is now in a form to convert from exponential to trigonometric functions, and cancel terms.

**ComplexToTrig[%]**

\[ A2 = \text{Apart}[\%] \]

\[ \{ \frac{-\text{Cos}[C \, z]}{2} + \frac{i \, \text{Sin}[C \, z]}{2}, 0, \frac{\text{Cos}[C \, z] - i \, \text{Sin}[C \, z]}{2}, 0, \frac{\text{Cos}[C \, z] + i \, \text{Sin}[C \, z]}{2}, 0, \frac{\text{Cos}[C \, z] - i \, \text{Sin}[C \, z]}{2}, 0, \frac{\text{Cos}[C \, z] + i \, \text{Sin}[C \, z]}{2} \} \]
\[
\begin{align*}
\frac{(-c^+ + C^-) \cos(C \, z) - I \sin(C \, z)}{2 \, C \, d} + \\
\frac{(c^2 - C^2) \cos(C \, z) + I \sin(C \, z)}{2 \, C \, d}, \\
\frac{-(c \cos(C \, z) - I \sin(C \, z))}{2 \, C} + \\
\frac{c \cos(C \, z) + I \sin(C \, z)}{2 \, C}, \\
\frac{-(c \cos(C \, z) - I \sin(C \, z))}{2 \, C} + \\
\frac{c \cos(C \, z) + I \sin(C \, z)}{2 \, C}, \\
\frac{d \cos(C \, z) - I \sin(C \, z)}{2 \, C} - d \cos(C \, z) + I \sin(C \, z)}{2 \, C} + \\
\frac{\cos(C \, z) + I \sin(C \, z)}{2}, 0}, \\
\{\frac{c^2 - C^2 \cos(C \, z) - I \sin(C \, z)}{2 \, C \, d} + \\
\frac{(-c^2 + C^2) \cos(C \, z) + I \sin(C \, z)}{2 \, C \, d}, \\
\frac{-(c \cos(C \, z) - I \sin(C \, z))}{2 \, C} + \\
\frac{c \cos(C \, z) + I \sin(C \, z)}{2 \, C}, 0} \\
\{\frac{\cos(C \, z) - I \sin(C \, z)}{2} + \frac{\cos(C \, z) + I \sin(C \, z)}{2} \}
\end{align*}
\]
To get rid of the $C^2$ terms, a new substitution rule is written, yielding the result for the coupler matrix. The intermediate variables are then eliminated to show the functional dependence.

\[
\text{rule3} = C^2 \rightarrow c^2 + d^2
\]
\[
A1 = A2 /. \text{rule3}
\]

\[
\text{rule4} = \{c \rightarrow C \cos[t], \ d \rightarrow C \sin[t]\}
\]
\[
A0 = A1 /. \text{rule4}
\]

\[
c^2 \rightarrow c^2 + d^2
\]

\[
\begin{align*}
\{\{\cos[C \ z], 0, \frac{I \ c \ \sin[C \ z]}{C}, \frac{I \ d \ \sin[C \ z]}{C}\}, \\
\{0, \cos[C \ z], -\frac{I \ d \ \sin[C \ z]}{C}, \frac{I \ c \ \sin[C \ z]}{C}\}, \\
\{\frac{I \ c \ \sin[C \ z]}{C}, -\frac{I \ d \ \sin[C \ z]}{C}, \cos[C \ z], 0\}, \\
\{\frac{I \ d \ \sin[C \ z]}{C}, \frac{I \ c \ \sin[C \ z]}{C}, 0, \cos[C \ z]\}\}
\end{align*}
\]

\[
\{c \rightarrow C \cos[t], \ d \rightarrow C \sin[t]\}
\]

\[
\begin{align*}
\{\{\cos[C \ z], 0, I \ \cos[t] \ \sin[C \ z], I \ \sin[t] \ \sin[C \ z]\}, \\
\{0, \cos[C \ z], -I \ \sin[t] \ \sin[C \ z], I \ \cos[t] \ \sin[C \ z]\}, \\
\{I \ \cos[t] \ \sin[C \ z], -I \ \sin[t] \ \sin[C \ z], \cos[C \ z], 0\}, \\
\{I \ \sin[t] \ \sin[C \ z], I \ \cos[t] \ \sin[C \ z], 0, \cos[C \ z]\}\}
\end{align*}
\]

As in the 2x2 case, the coupling ratio is defined, with an additional shorthand

\[
\cos(cz) = (1 - k)^{1/2} \equiv k_t \\
\sin(cz) = k^{1/2} \equiv k_t
\]

\[
\text{rule5} = \{\sin[C \ z] \rightarrow kr, \ \cos[C \ z] \rightarrow kt\}
\]
\[
\text{MatrixForm}[A0 /. \text{rule5}]
\]

\[
\{\sin[C \ z] \rightarrow kr, \ \cos[C \ z] \rightarrow kt\}
\]
Note the matrix is symmetric, like our original matrix $K$. The $A$ matrix can be divided into four submatrices consisting of identity, rotation, and antirotation 2×2 matrices.

Compare the coupling coefficient (C $z$) terms to the result for the scalar device in Appendix II.

\[
A = \begin{bmatrix}
\begin{bmatrix}
\cos(C z) \\
i \sin(C z)
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\cos(\theta) & \sin(\theta) \\
-sin(\theta) & \cos(\theta)
\end{bmatrix}
\begin{bmatrix}
\cos(C z) \\
i \sin(C z)
\end{bmatrix}
\begin{bmatrix}
\cos(\theta) & -\sin(\theta) \\
\sin(\theta) & \cos(\theta)
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\end{bmatrix}
\]

Note that when $\theta = 0$, there are four 2×2 identity submatrices, and four independent channels in the coupler. Using the shorthand for the submatrices:

\[
I = \text{Identity 2×2 matrix} \quad \text{and} \quad R(\theta) = \text{Rotation 2×2 matrix by } \theta
\]

permits rewriting the above 4×4 matrix in a more compact form, which emphasizes the similarity to the single-mode coupler result calculated in Appendix II.

\[
A = \begin{bmatrix}
I \cos(C z) & i R(-\theta) \sin(C z) \\
i R(\theta) \sin(C z) & I \cos(C z)
\end{bmatrix}
\]
IMPORTANT VERSION 2.1 INFORMATION: The Eigenvector routines comes up with a different (but equivalent) result for the eigenvectors than Version 2.0. Subsequent algebraic manipulation techniques that assumed a certain form of the eigenvectors will not work with the new version, especially in the following sections.

### 4.2 PM Coupler - General Case

Now we will include the birefringence effects by allowing the b coefficient to be nonvanishing. The eigenvalues and eigenvectors are calculated as in the previous section, using the `EigenSystem` command

```
b = .
c = .
d = .
K = {{b, 0, c, d), {0,-b,-d,c), {c,-d,b,0), {d,c,0,-b));
MatrixForm[K]

eigen = Eigensystem[K];
Print["Eigenvalues of matrix K are calculated to be"]
eigen[[1]]
b 0 c d
0 -b -d c
c -d b 0
d c 0 -b
```

Eigenvalues of matrix K are calculated to be

\{ -\sqrt{b^2 - 2 b c + c^2 + d^2}, \sqrt{b^2 - 2 b c + c^2 + d^2}, \\ -\sqrt{b^2 + 2 b c + c^2 + d^2}, \sqrt{b^2 + 2 b c + c^2 + d^2} \}\n
Note that the first two eigenvalues \( \Lambda_1 \) and \( \Lambda_2 \) are opposite in sign, as are the 3rd and 4th eigenvalues. We now define a substitution rule `rule6` which permits defining the eigenvalues in terms of \( e_1 \) and \( e_3 \). The eigenvalues are redisplayed using /. which
should be interpreted as meaning "such that" and \( \rightarrow \) means "goes to" or "is replaced by." In addition, rule7 is written for back-substitution purposes.

\[
e_1 = .
\]
\[
e_3 = .
\]
\[
\text{rule6} = \{\sqrt{b^2 - 2 b c + c^2 + d^2} \rightarrow -e_1,
\sqrt{b^2 + 2 b c + c^2 + d^2} \rightarrow -e_3\};
\]
\[
\text{rule7} = \{e_1 \rightarrow -\sqrt{b^2 - 2 b c + c^2 + d^2},
\quad e_3 \rightarrow -\sqrt{b^2 + 2 b c + c^2 + d^2}\};
\]
\[
e\text{Vals} = \text{eigen}[[1]]/.\text{rule6}
\quad e\text{Vecs} = \text{eigen}[[2]]/.\text{rule6};
\]
\[
\{e_1, -e_1, e_3, -e_3\}
\]

The matrices are assembled and multiplied to yield the initial result for the coupler matrix \( A_2 \), in its unsimplified form.

\[
\text{QQ} = \text{Transpose}[e\text{Vecs}];
\text{MatrixForm}[\text{QQ}]
\]
\[
\text{QI} = \text{Together}[\text{Inverse}[\text{QQ}]];
\text{MatrixForm}[\text{QI}]
\]
\[
z = .
\]
\[
\text{DD} = \text{DiagonalMatrix}[\text{E}^{i e\text{Vals}\cdot z}];
\text{MatrixForm}[\text{DD}]
\]
\[
\text{A}_2 = \text{QQ}\cdot\text{DD}\cdot\text{QI}
\]

\[
\begin{array}{cccc}
1 & 1 & 1 & 1 \\
-\frac{b + c + e_1}{d} & -\frac{b + c - e_1}{d} & \frac{b + c - e_3}{d} & \frac{b + c + e_3}{d} \\
-1 & -1 & 1 & 1 \\
-\frac{b + c + e_1}{d} & -\frac{b + c - e_1}{d} & -\frac{b - c + e_3}{d} & -\frac{b - c - e_3}{d}
\end{array}
\]
\[
\begin{align*}
\frac{b - c + e_1}{4 e_1} & \quad \frac{d}{4 e_1} & \quad \frac{-b + c - e_1}{4 e_1} & \quad \frac{d}{4 e_1} \\
\frac{-b + c + e_1}{4 e_1} & \quad \frac{-d}{4 e_1} & \quad \frac{b - c - e_1}{4 e_1} & \quad \frac{-d}{4 e_1} \\
\frac{b + c + e_3}{4 e_3} & \quad \frac{-d}{4 e_3} & \quad \frac{b + c + e_3}{4 e_3} & \quad \frac{d}{4 e_3} \\
\frac{-b - c + e_3}{4 e_3} & \quad \frac{d}{4 e_3} & \quad \frac{-b - c + e_3}{4 e_3} & \quad \frac{-d}{4 e_3}
\end{align*}
\]

\[
\begin{align*}
E & \quad 0 & \quad 0 & \quad 0 & \quad 0 \\
0 & \quad -I e_1 z & \quad 0 & \quad 0 \\
0 & \quad 0 & \quad I e_3 z & \quad 0 \\
0 & \quad 0 & \quad 0 & \quad E -I e_3 z
\end{align*}
\]

\[
\begin{align*}
\frac{I e_1 z}{4 e_1} (b - c + e_1) & + \frac{E -I e_1 z}{4 e_1} (-b + c + e_1) + \frac{E -I e_3 z}{4 e_3} (b + c + e_3) \\
\frac{-d E}{4 e_1} & + \frac{d E I e_1 z}{4 e_1} + \frac{d E -I e_3 z}{4 e_3} + \frac{d E}{4 e_3} - \frac{d E}{4 e_3}
\end{align*}
\]

\[
\begin{align*}
\frac{-I e_1 z}{4 e_1} (b - c - e_1) & + \frac{I e_1 z}{4 e_1} (-b + c - e_1) + \frac{E -I e_3 z}{4 e_3} (b + c + e_3) \\
\frac{-d E}{4 e_1} & + \frac{d E I e_1 z}{4 e_1} - \frac{d E -I e_3 z}{4 e_3} + \frac{d E}{4 e_3} - \frac{d E}{4 e_3}
\end{align*}
\]

\[
\begin{align*}
\frac{I e_1 z}{4 d e_1} (-b + c - e_1) & + \frac{I e_1 z}{4 d e_1} (-b + c - e_1) + \frac{E}{4 d e_3} (b + c - e_3) & + \frac{E}{4 d e_3} (b + c - e_3)
\end{align*}
\]
\[- \left( \frac{4d e3}{e3} \right) \left( -b + c + e3 \right) \],

\[- \left( \frac{4d e3}{e3} \right) \left( b - c + e3 \right) \left( -b + c - e3 \right) \].
Next, the matrix is converted from exponential to trigonometric functions, and the coefficients are expanded out in an intermediate matrix A1.

```
Needs["Algebra`Trigonometry`"]
ComplexToTrig[A2];
A1 = Expand[%]
{{Cos[e1 z] 2 + Cos[e3 z] 2 + I 2 b Sin[e1 z] e1 I 2 c Sin[e1 z] e1 +
  I 2 b Sin[e3 z] e3 + I 2 c Sin[e3 z] e3,
}}
```
\[
\begin{align*}
&\frac{I}{2} d \sin(e_1 z) - \frac{I}{2} d \sin(e_3 z) \\
&\frac{\cos(e_1 z)}{2} + \frac{\cos(e_3 z)}{2} - \frac{I}{2} b \sin(e_1 z) + \frac{I}{2} c \sin(e_1 z) \\
&\frac{I}{2} b \sin(e_3 z) + \frac{I}{2} c \sin(e_3 z) \\
&\frac{I}{2} d \sin(e_1 z) + \frac{I}{2} d \sin(e_3 z) \\
&\frac{-I}{2} b^2 \sin(e_1 z) + \frac{-I}{2} b^2 \sin(e_3 z) \\
&\frac{-I}{2} e_1 \sin(e_1 z) - \frac{I}{2} e_1 \sin(e_3 z) \\
&\frac{I}{2} c^2 \sin(e_3 z) - \frac{I}{2} c^2 \sin(e_3 z) \\
&\cos(e_1 z) + \cos(e_3 z) - \frac{I}{2} b \sin(e_1 z) + \frac{I}{2} c \sin(e_1 z) \\
&\frac{I}{2} b \sin(e_3 z) - \frac{I}{2} c \sin(e_3 z) \\
&\frac{I}{2} b^2 \sin(e_1 z) - \frac{I}{2} b^2 \sin(e_3 z) \\
&\frac{I}{2} e_1 \sin(e_1 z) - \frac{I}{2} e_1 \sin(e_3 z) \\
&\frac{I}{2} c^2 \sin(e_3 z) - \frac{I}{2} c^2 \sin(e_3 z) \\
&\cos(e_1 z) - \cos(e_3 z) - \frac{I}{2} b \sin(e_1 z) + \frac{I}{2} c \sin(e_1 z)
\end{align*}
\]
\[
\begin{align*}
&\text{Sin}\left[\frac{1}{2} b \text{Sin}[e3 z] \right] \cdot \text{e3} + \frac{1}{2} \text{c Sin}[e3 z] \cdot \text{e3} \\
&\text{-Cos}[\text{el z}] + \text{Cos}[\text{e3 z}] \cdot \frac{1}{2} b \text{Sin}[\text{el z}] \cdot \text{el} + \frac{1}{2} \text{c Sin}[\text{el z}] \cdot \text{el} \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
&\text{Cos}[\text{el z}] + \text{Cos}[\text{e3 z}] + \frac{1}{2} b \text{Sin}[\text{el z}] \cdot \text{el} + \frac{1}{2} \text{c Sin}[\text{el z}] \cdot \text{el} \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
&\text{Cos}[\text{el z}] \cdot \text{Cos}[\text{e3 z}] \cdot \frac{1}{2} b \text{Sin}[\text{el z}] \cdot \text{el} + \frac{1}{2} \text{c Sin}[\text{el z}] \cdot \text{el} \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
&\text{Cos}[\text{el z}] \cdot \text{Cos}[\text{e3 z}] + \frac{1}{2} b \text{Sin}[\text{el z}] \cdot \text{el} + \frac{1}{2} \text{c Sin}[\text{el z}] \cdot \text{el} \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
&\frac{1}{2} b \text{Sin}[e3 z] \cdot \frac{1}{2} \text{c Sin}[e3 z] \\
\end{align*}
\]
Careful inspection of the coefficients of the above matrix show some redundancy. In fact, the 4x4 matrix can be fully described by four unique coefficients. Compare row one and row three, and it can be seen that

\[
a_{31} = a_{13} \quad a_{32} = -a_{14} \quad a_{33} = a_{11} \quad \text{and} \quad a_{34} = -a_{12}
\]

looking at row two of the A1 matrix to see if it can be defined in terms of row 1 coefficients.

\[
a_{22} = a_{11}^* \quad a_{24} = -a_{13}^* \quad \text{also} \quad a_{42} = -a_{13}^* \quad a_{44} = a_{11}^*
\]

but it is not clear of the relationship of the remaining coefficients \(a_{21}, a_{23}, a_{41} \) and \(a_{43} \) but some further algebra will illustrate the dependence on the row 1 coefficients. The problem is terms like

\[
\frac{-I}{2} \left( \frac{b}{d} \right)^2 + \frac{I}{2} \left( \frac{b}{d} \right) \left( \frac{c}{e_3} \right) \quad \frac{I}{2} \left( \frac{c}{d} \right)^2 \quad \frac{I}{2} \left( \frac{e_1}{d} \right) \quad \frac{I}{2} \left( \frac{e_3}{d} \right)
\]

\[
\text{Sin}(e_1 z) + \text{Sin}(e_3 z)
\]
To simplify these expressions, we begin by multiplying through by $e_1$ and $e_3$, which makes the last term in the parenthesis become $\Lambda_i^2$, then plug in the expressions for the eigenvalues.

Only a partial listing of the matrix is shown in the output notebook cell. On a MacIntosh, click and drag on the box at the lower right-hand corner of the output cell to observe more of the result.

\[
\text{Expand}[(e_1 e_3) A_1] /. \text{rule7}
\]

\[
\left\{\left(\sqrt{b^2 - 2 b c + c^2 + d^2}\right) \sqrt{b^2 + 2 b c + c^2 + d^2} \right. \\
\left. \cos\left(\sqrt{b^2 - 2 b c + c^2 + d^2} z\right)\right) / 2 + \\
\left(\sqrt{b^2 - 2 b c + c^2 + d^2}\right) \sqrt{b^2 + 2 b c + c^2 + d^2} \\
\left. \cos\left(\sqrt{b^2 + 2 b c + c^2 + d^2} z\right)\right) / 2 + \\
\frac{i}{2} b \sqrt{b^2 + 2 b c + c^2 + d^2}
\]

↑↑↑↑ NOTE: The above results are only a partial listing. ↑↑↑↑

The multiplicative terms are cancelled, and then the $e_1$ and $e_3$ are substituted back in again.

\[
\text{Together}[\%] /. \text{rule6}
\]

\[
\left\{\left(e_1 e_3 \cos(e_1 z) + e_1 e_3 \cos(e_3 z) + I b e_3 \sin(e_1 z) - I c e_3 \sin(e_1 z) + I b e_1 \sin(e_3 z) + I c e_1 \sin(e_3 z)\right) / 2, \\
\frac{I}{2} \left(d e_3 \sin(e_1 z) - d e_1 \sin(e_3 z)\right), \\
\left(-e_1 e_3 \cos(e_1 z) + e_1 e_3 \cos(e_3 z) - I b e_3 \sin(e_1 z) + I c e_3 \sin(e_1 z) + I b e_1 \sin(e_3 z) + I c e_1 \sin(e_3 z)\right) / 2\right\}
\]

↑↑↑↑ NOTE: The above results are only a partial listing. ↑↑↑↑
The matrix is then divided by the two eigenvalues, resulting in the simplified version of the matrix \( A_1 \):

\[
A_0 = \text{Expand} \left[ \frac{A}{(e_1 e_3)} \right]
\]

\[
\begin{align*}
&\left\{ \begin{array}{c}
\frac{\cos e_1 z}{2} + \frac{\cos e_3 z}{2} + \frac{\frac{i}{2} b \sin e_1 z}{e_1} - \frac{\frac{i}{2} c \sin e_1 z}{e_1} + \\
\frac{\frac{i}{2} b \sin e_3 z}{e_3} + \frac{\frac{i}{2} c \sin e_3 z}{e_3}, \\
\frac{\frac{i}{2} d \sin e_1 z}{e_1} - \frac{\frac{i}{2} d \sin e_3 z}{e_3}, \\
\frac{\frac{i}{2} d \sin e_1 z}{e_1} - \frac{\frac{i}{2} d \sin e_3 z}{e_3}, \\
\frac{\cos e_1 z}{2} + \frac{\cos e_3 z}{2} - \frac{\frac{i}{2} b \sin e_1 z}{e_1} + \frac{\frac{i}{2} c \sin e_1 z}{e_1} + \\
\frac{\frac{i}{2} b \sin e_3 z}{e_3} + \frac{\frac{i}{2} c \sin e_3 z}{e_3}, \\
\frac{-\frac{i}{2} d \sin e_1 z}{e_1} + \frac{\frac{i}{2} d \sin e_3 z}{e_3}, \\
\frac{\cos e_1 z}{2} - \frac{\cos e_3 z}{2} - \frac{\frac{i}{2} b \sin e_1 z}{e_1} + \frac{\frac{i}{2} c \sin e_1 z}{e_1} - \\
\frac{\frac{i}{2} b \sin e_3 z}{e_3} - \frac{\frac{i}{2} c \sin e_3 z}{e_3}, \\
\frac{-\frac{i}{2} d \sin e_1 z}{e_1} - \frac{\frac{i}{2} d \sin e_3 z}{e_3}, \\
\end{array} \right. \\
\end{align*}
\]
\[-\frac{\cos(e_1 z)}{2} + \frac{\cos(e_3 z)}{2} - \frac{I}{2} \frac{\sin(e_1 z)}{e_1} + \frac{I}{2} \frac{\sin(e_3 z)}{e_3},\]

\[-\frac{I}{2} \frac{\sin(e_1 z)}{e_1} - \frac{I}{2} \frac{\sin(e_3 z)}{e_3},\]

\[\frac{\cos(e_1 z)}{2} + \frac{\cos(e_3 z)}{2} + \frac{I}{2} \frac{\sin(e_1 z)}{e_1} - \frac{I}{2} \frac{\sin(e_3 z)}{e_3};\]

\[-\frac{I}{2} \frac{\sin(e_1 z)}{e_1} + \frac{I}{2} \frac{\sin(e_3 z)}{e_3},\]

\[\frac{I}{2} \frac{\sin(e_1 z)}{e_1} + \frac{I}{2} \frac{\sin(e_3 z)}{e_3};\]

\[\cos(e_1 z) - \cos(e_3 z) - \frac{I}{2} \frac{\sin(e_1 z)}{e_1} + \frac{I}{2} \frac{\sin(e_3 z)}{e_3},\]

\[-\frac{I}{2} \frac{\sin(e_1 z)}{e_1} + \frac{I}{2} \frac{\sin(e_3 z)}{e_3},\]

\[\cos(e_1 z) + \cos(e_3 z) - \frac{I}{2} \frac{\sin(e_1 z)}{e_1} + \frac{I}{2} \frac{\sin(e_3 z)}{e_3} - \frac{I}{2} \frac{\sin(e_1 z)}{e_1} - \frac{I}{2} \frac{\sin(e_3 z)}{e_3};\]
The remaining coefficients can now be expressed in terms of the four coefficient of interest

\[ a_{21} = a_{12} \quad a_{23} = a_{14}^* \quad \text{also} \quad a_{41} = a_{14} \quad a_{43} = a_{12}^* \]

The assembled coefficients in the matrix form is like

\[
\begin{bmatrix}
  a_{11} & a_{12} & a_{13} & a_{14} \\
  a_{12} & a_{11}^* & -a_{14} & -a_{13}^* \\
  a_{13} & -a_{14} & a_{11} & -a_{12} \\
  a_{14} & -a_{13}^* & -a_{12} & a_{11}^* \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
  k_t & c_t & k_r & c_r \\
  c_t & k_t^* & -c_r & -k_r^* \\
  k_r & -c_r & k_t & -c_t \\
  c_r & -k_r^* & -c_t & k_t^* \\
\end{bmatrix}
\]

Now a new matrix \( \mathbf{A} \) is defined in terms of the coefficients \( k_t, c_t, k_r, \) and \( c_r \).
Unfortunately the Conjugate command does not work on symbolic expressions, so a new function \( \text{Conj} \) will be defined to eliminate that problem.

\[
\text{test} = a_r + I a_i \\
\text{nogood} = \text{Conjugate}[\text{test}] \\
\text{Conj}[x_] := \text{ComplexExpand}[\text{Conjugate}[x]] \\
\text{good} = \text{Conj}[\text{test}] \\
I a_i + a_r \\
\text{Conjugate}[I a_i + a_r] \\
-I a_i + a_r
\]
\{kt, ct, kr, cr\} = A0[[1]]; 

\text{OutputForm["kt matrix coefficient a"Subscript[11]]} 
\text{kt} 
\text{OutputForm["ct matrix coefficient a"Subscript[12]]} 
\text{ct} 
\text{OutputForm["kr matrix coefficient a"Subscript[13]]} 
\text{kr} 
\text{OutputForm["cr matrix coefficient a"Subscript[14]]} 
\text{cr} 

\text{kt matrix coefficient a}_{11} 
\begin{align*} 
\frac{\cos(e_1 z)}{2} + \frac{\cos(e_3 z)}{2} &= \frac{i}{2} \frac{b \sin(e_1 z)}{e_1} - \frac{i}{2} \frac{c \sin(e_1 z)}{e_1} + \\
&\frac{i}{2} \frac{b \sin(e_3 z)}{e_3} + \frac{i}{2} \frac{c \sin(e_3 z)}{e_3} 
\end{align*} 

\text{ct matrix coefficient a}_{12} 
\begin{align*} 
\frac{i}{2} \frac{d \sin(e_1 z)}{e_1} - \frac{i}{2} \frac{d \sin(e_3 z)}{e_3} 
\end{align*} 

\text{kr matrix coefficient a}_{13} 
\begin{align*} 
-\frac{\cos(e_1 z)}{2} + \frac{\cos(e_3 z)}{2} &= \frac{i}{2} \frac{b \sin(e_1 z)}{e_1} + \frac{i}{2} \frac{c \sin(e_1 z)}{e_1} + \\
&\frac{i}{2} \frac{b \sin(e_3 z)}{e_3} + \frac{i}{2} \frac{c \sin(e_3 z)}{e_3} 
\end{align*} 

\text{cr matrix coefficient a}_{14}
\[
\frac{1}{2} e_1 \sin(e_1 z) + \frac{1}{2} e_3 \sin(e_3 z)
\]

\[A = \{\{kt, ct, kr, cr\}, \{ct, \text{Conj}[kt], -cr, -\text{Conj}[kr]\}, \{kr, -cr, kt, -ct\}, \{cr, -\text{Conj}[kr], \text{Conj}[ct], \text{Conj}[kt]\}\};
\]

\[\text{ShouldBeZer0} = \text{Expand}[A_0 - A]
\]
\[
\{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}
\]

The zeroes indicate that the matrix can be assembled from the four coefficients. The last thing to do is to reserve these expressions by protecting them, and to place them in an array called the basis.

\[
\text{Protect}[\{\{kt, ct, kr, cr\}\}]
\]

\[\text{basis} = \{\{kt, ct, kr, cr\}\};
\]

\[\{\{kt, ct, kr, cr\}\}
\]

### 4.3 PM Coupler Calculations - Perfect Alignment

In the last section, it was demonstrated that the PM coupler could be modeled with a 4×4 matrix with 4 unique coefficients without loss of generality. The four basis coefficients calculated at the end of section 4.3 will be used in the rest of the calculations. In this section, we will include birefringence effects but ignore angular misalignment for now. The basis has been defined in the previous section in terms of the coefficients \(kt, ct, kr, \) and \(cr\). The eigenvalues are redisplayed, and are now simplified, assuming perfect alignment, i.e. \(t = 0\).
\texttt{rule8} = \{c \rightarrow C, \, d \rightarrow 0\}
\texttt{eigen[[1]]} /. \texttt{rule8}
\texttt{Simplify[\%]}
\{c \rightarrow C, \, d \rightarrow 0\}

\{-\text{Sqrt}[b^2 - 2 b C + C^2], \, \text{Sqrt}[b^2 - 2 b C + C^2], \}
\{-\text{Sqrt}[b^2 + 2 b C + C^2], \, \text{Sqrt}[b^2 + 2 b C + C^2]\}

\{-\text{Sqrt}[(b - C)^2], \, \text{Sqrt}[-(b + C)^2], \, -\text{Sqrt}[(b + C)^2], \}
\text{Sqrt}[(b + C)^2]\}

\textit{Mathematica} does not evaluate the above result fully to its simplest form. The matrix coefficients are thus redefined with respect to the simplified eigenvalues

\texttt{e1 = .}
\texttt{e3 = .}
\texttt{rule9} = \{\texttt{e1} \rightarrow -(b - C), \, \texttt{e3} \rightarrow -(b + C)\}
\texttt{basis} /. \texttt{rule8}
\texttt{\%} /. \texttt{t} \rightarrow 0
\texttt{newbasis} = \texttt{Simplify[\%} /. \texttt{rule9}\]
\texttt{\{e1} \rightarrow -b + C, \, \texttt{e3} \rightarrow -b - C\}

\texttt{\{Cos[e1 z]} + \texttt{Cos[e3 z]} + \frac{\texttt{I} \, b \, \texttt{Sin[e1 z]}}{2 \, \texttt{e1}} - \frac{\texttt{I} \, C \, \texttt{Sin[e1 z]}}{2 \, \texttt{e1}} + \frac{\texttt{I} \, b \, \texttt{Sin[e3 z]}}{2 \, \texttt{e3}} + \frac{\texttt{I} \, C \, \texttt{Sin[e3 z]}}{2 \, \texttt{e3}}, \texttt{0},

\texttt{-Cos[e1 z]} + \texttt{Cos[e3 z]} - \frac{\texttt{I} \, b \, \texttt{Sin[e1 z]}}{2 \, \texttt{e1}} + \frac{\texttt{I} \, C \, \texttt{Sin[e1 z]}}{2 \, \texttt{e1}} + \frac{\texttt{I} \, b \, \texttt{Sin[e3 z]}}{2 \, \texttt{e3}} + \frac{\texttt{I} \, C \, \texttt{Sin[e3 z]}}{2 \, \texttt{e3}}, \texttt{0}\}
\[
\begin{align*}
\frac{\cos(e_1 z)}{2} + \frac{\cos(e_3 z)}{2} &+ \frac{i}{2} \frac{b \sin(e_1 z)}{e_1} - \frac{i}{2} \frac{c \sin(e_1 z)}{e_1} + \frac{i}{2} \frac{b \sin(e_3 z)}{e_3} - \frac{i}{2} \frac{c \sin(e_3 z)}{e_3}, 0, \\
-\cos(e_1 z) &+ \cos(e_3 z) - \frac{i}{2} \frac{b \sin(e_1 z)}{e_1} + \frac{i}{2} \frac{c \sin(e_1 z)}{e_1} + \frac{i}{2} \frac{b \sin(e_3 z)}{e_3} + \frac{i}{2} \frac{c \sin(e_3 z)}{e_3}, 0)
\end{align*}
\]

\{\cos(C z) \ (\cos(b z) + i \sin(b z)), 0, \\
I \ (\cos(b z) + i \sin(b z)) \ \sin(C z), 0\}

The \(\cos(b z) + i \sin(b z)\) will cancel out when the optical power is calculated because it has unit magnitude for all possible \(b\) values.

\{kt3, ct3, kr3, cr3\} = newbasis;

\[A = \{\{kt3, ct3, kr3, cr3\},
\{ct3, \text{Conj}[kt3], -cr3, -\text{Conj}[kr3]\},
\{kr3, -cr3, kt3, -ct3\},
\{cr3, -\text{Conj}[kr3], -ct3, \text{Conj}[kt3]\}\}\]

\{\cos(C z) \ (\cos(b z) + i \sin(b z)), 0, \\
I \ (\cos(b z) + i \sin(b z)) \ \sin(C z), 0\},
\{0, \cos(b z) \ \cos(C z) - I \ \cos(C z) \ \sin(b z), 0, \\
I \ \cos(b z) \ \sin(C z) + \sin(b z) \ \sin(C z)\},
\{I \ (\cos(b z) + i \sin(b z)) \ \sin(C z), 0, \\
\cos(C z) \ (\cos(b z) + I \ \sin(b z)), 0\},
\{0, I \ \cos(b z) \ \sin(C z) + \sin(b z) \ \sin(C z), 0, \\
\cos(b z) \ \cos(C z) - I \ \cos(C z) \ \sin(b z)\}\}

The matrix is illustrated below, showing zero terms that would tend to coupler the \(x\) to the \(y\) polarization in both waveguides. It is almost the same as the final result for Section 4.1, for the case of \(\theta = 0\), but with the phase factors in front of the diagonal terms.
To illustrate this further, the intensity transfer function is computed, and the fiber is tested with the standard test conditions described in the text.

\[ b = 0.001; \]
\[ \text{Unprotect}[C]; \]
\[ C = 0.001; \]
\[ \text{eOut} = A \cdot \text{Transpose}[[1, 0, 0, 0]] \]
\[ \text{iOut} = \text{eOut Conjugate}[	ext{eOut}]; \]
\[ \text{Plot}[[\text{iOut[[1]], iOut[[3]], \{z, 1, 10^4\}], \text{AxesLabel} \to \{"\text{\text{n z in um}}, "\text{I/Io}"\}] \]

\( e^{i b z} \cos(C z) \quad 0 \quad e^{i b z} i \sin(C z) \quad 0 \)

\[ \begin{bmatrix}
    0 & e^{-i b z} \cos(C z) & 0 & e^{-i b z} i \sin(C z) \\
    e^{i b z} i \sin(C z) & 0 & e^{i b z} \cos(C z) & 0 \\
    0 & e^{-i b z} i \sin(C z) & 0 & e^{-i b z} \cos(C z)
\end{bmatrix} \]
This plot shows the same response as for the single-mode coupler derived in Section 2. Of course, it is not possible to obtain perfect alignment usually, so this effect needs to be included to see how birefringence and misalignment interact.

4.4 PM Coupler Calculations - with Misalignment

We now include misalignments, and birefringence effects in the PM coupler. For the condition of small misalignments, the coefficients and eigenvalues are approximated by

\[ c = C \cos(t) = C \quad \text{and} \quad d = C \sin(t) = C \times t \]

This is appropriate for angles less than 15 degrees or approximately 0.25 radians.

\[
\begin{align*}
\text{rule10} & \equiv \{c \rightarrow C, \ d \rightarrow C \times t\} \\
\text{rule11} & \equiv \{e_1 \rightarrow -\sqrt{(b - C)^2 + (C \times t)^2}, \ e_3 \rightarrow -\sqrt{(b + C)^2 + (C \times t)^2}\}
\end{align*}
\]

basis /. rule11; 
newbasis = % /. rule10;
{c \rightarrow C, \ d \rightarrow C \times t}

\[
\begin{align*}
e_1 & \rightarrow -\sqrt{(b - C)^2 + C^2 \times t^2}, \\
e_3 & \rightarrow -\sqrt{(b + C)^2 + C^2 \times t^2}
\end{align*}
\]

{kt4, ct4, kr4, cr4} = newbasis

\[
A_4 = \{(kt4, ct4, kr4, cr4), \\
\{ct4, \text{Conj}[kt4], -cr4, -\text{Conj}[kr4]\}, \\
\{kr4, -cr4, kt4, -ct4\}, \\
\{cr4, -\text{Conj}[kr4], -ct4, \text{Conj}[kt4]\}\};
\]

\[
\begin{align*}
\text{Cos}[\text{Sqrt}[(b - C)^2 + C^2 \times t^2] z] + \\
\end{align*}
\]
\begin{align*}
\cos[\sqrt{(b + C)^2 + C^2 t^2} z] + \\
\frac{1}{2} b \sin[\sqrt{(b - C)^2 + C^2 t^2} z] \\
\frac{1}{2} C \sin[\sqrt{(b - C)^2 + C^2 t^2} z] \\
\frac{1}{2} b \sin[\sqrt{(b + C)^2 + C^2 t^2} z] \\
\frac{1}{2} C \sin[\sqrt{(b + C)^2 + C^2 t^2} z] \\
\frac{1}{2} C t \sin[\sqrt{(b - C)^2 + C^2 t^2} z] \\
\frac{1}{2} C t \sin[\sqrt{(b + C)^2 + C^2 t^2} z] \\
\cos[\sqrt{(b - C)^2 + C^2 t^2} z] + \\
\frac{1}{2} b \sin[\sqrt{(b - C)^2 + C^2 t^2} z] \\
\frac{1}{2} C \sin[\sqrt{(b - C)^2 + C^2 t^2} z] \\
\frac{1}{2} b \sin[\sqrt{(b + C)^2 + C^2 t^2} z] \\
\frac{1}{2} C \sin[\sqrt{(b + C)^2 + C^2 t^2} z]
\end{align*}
\[ \sqrt{(b + C)^2 + C^2 t^2} \]
\[ \frac{1}{2} Ct \sin \left[ \sqrt{(b - C)^2 + C^2 t^2} z \right] \]
\[ \frac{\sqrt{(b - C)^2 + C^2 t^2}}{2} \]
\[ \frac{1}{2} Ct \sin \left[ \sqrt{(b + C)^2 + C^2 t^2} z \right] \]
\[ \frac{\sqrt{(b + C)^2 + C^2 t^2}}{2} \]

The above result is now plotted for the standard test conditions, launching x-polarization in waveguide A only. The power for the x-polarization is seen to couple back and forth as before for the single-mode case. However, the finite cross-coupling to the y-polarization shows up as an exponential-like decay in the optical power as shown. In this experiment, the fiber misalignment \( t \) is 0.1 radian, or roughly 5.7 degrees.

\[ b = 0.001; \]
\[ \text{Unprotect}[C]; \]
\[ C = 0.001; \]
\[ t = 0.1 \]

\[ eOut = A4 \cdot \text{Transpose}[\{1, 0, 0, 0\}] \]
\[ iOut = \text{Conjugate}[eOut]; \]
\[ \text{Plot}[\{iOut[[1]], iOut[[3]]\}, \{z, 1, 10^4\}, \]
\[ \text{AxesLabel} \rightarrow \{"\text{z in um}, "I/Io"\}] \]

0.1

\[ \frac{\text{Cos}[0.0001 z]}{2} + 0. I \text{Sin}[0.0001 z] + \]
\[ 0.499376 I \text{Sin}[0.0020025 z], \]
\[ 0.5 I \text{Sin}[0.0001 z] - 0.0249688 I \text{Sin}[0.0020025 z], \]
\[ -\frac{\text{Cos}[0.0001 z]}{2} + \frac{\text{Cos}[0.0020025 z]}{2} + 0. I \text{Sin}[0.0001 z] + \]
\[ 0.499376 I \text{Sin}[0.0020025 z], \]
\[ 0.5 I \text{Sin}[0.0001 z] + 0.0249688 I \text{Sin}[0.0020025 z]\]

General::spell:
Possible spelling error: new symbol name "iOut"
is similar to existing symbols \{eOut, Out\}. 318
Now, examining the y-polarization output for ports #3 and #4 requires plotting the 2nd and 4th coefficient of the output intensity vector.

\[
\text{Plot}\left[\{\text{iOut}[2], \text{iOut}[4]\}, \{z, 1, 10^4\}, \text{AxesLabel} \to \{"\text{z in um"}, "I/I_o"\}\right]
\]

This illustrates the increasing light coupler over to the y-polarized modes of the coupler outputs. The periodic nature of the cross-coupled light stems from the periodic variations of the optical power guided by the two waveguides.
4.5 References


In this notebook, the Poincaré sphere will be further utilized to examine the evolution of the state-of-polarization in birefringent fiber couplers. The effects of coupling length, birefringence relaxation due to polishing, principal axis misalignment, and coupling coefficient will be investigated.

### 5.1 PM Coupler Calculations Summary

For the K matrix describing the coupled differential equations of the form

\[
\begin{pmatrix}
\begin{array}{cccc}
 b & 0 & c & d \\
 0 & -b & -d & c \\
 c & -d & b & 0 \\
 d & c & 0 & -b \\
\end{array}
\end{pmatrix}
\]

where \( b = \pi / L_B \), \( c = C \cos(t) \), \( d = C \sin(t) \), and \( C = \) coupling coefficient.

The eigenvalues for the K matrix were calculated to be \( \{e_1, e_2, e_3, e_4\} = \)
\{-\sqrt{b^2 - 2bc + c^2 + d^2}, \sqrt{b^2 - 2bc + c^2 + d^2},
-\sqrt{b^2 + 2bc + c^2 + d^2}, \sqrt{b^2 + 2bc + c^2 + d^2}\}

where it can be seen that \(e_2 = -e_1\) and \(e_4 = -e_3\). The exact eigenvalues will be simplified for the small-angle approximation later. The eigenvectors that were also calculated in Appendix IV are listed below in column form, and are in terms of the two eigenvalues \(e_1\) and \(e_3\) in the above equation.

\[
\begin{array}{cccc}
1 & 1 & 1 & 1 \\
-b + c + e_1 & -b + c - e_1 & b + c - e_3 & b + c + e_3 \\
\frac{d}{d} & \frac{d}{d} & \frac{d}{d} & \frac{d}{d} \\
-1 & -1 & 1 & 1 \\
-b + c + e_1 & -b + c - e_1 & -b - c + e_3 & -b - c - e_3 \\
\frac{d}{d} & \frac{d}{d} & \frac{d}{d} & \frac{d}{d} \\
\end{array}
\]

After diagonalization, the A matrix solution is generated which describes the propagation of the fields in the PM coupler, in terms of four fundamental coefficients:

\[
\begin{array}{cccc}
kt & ct & kr & cr \\
ct & Conjugate[kt] & -cr & -Conjugate[kr] \\
kr & -cr & kt & -ct \\
kr & -Conjugate[kr] & -ct & Conjugate[kt] \\
\end{array}
\]

We will now examine these fundamental coefficients which constitute the A matrix elements in more detail. The fundamental coefficients that were computed in Appendix IV are now defined:
kt = \cos[el z]/2 + \cos[e3 z]/2 + (I/2 b \sin[el z])/el - (I/2 c \sin[e1 z])/el + (I/2 b \sin[e3 z])/e3 + (I/2 c \sin[e3 z])/e3

ct = (I/2 d \sin[e1 z])/el - (I/2 d \sin[e3 z])/e3

kr = -\cos[el z]/2 + \cos[e3 z]/2 - (I/2 b \sin[e1 z])/el + (I/2 c \sin[e1 z])/el + (I/2 b \sin[e3 z])/e3 - (I/2 c \sin[e3 z])/e3

cr = (I/2 d \sin[e1 z])/el + (I/2 d \sin[e3 z])/e3

\text{fundCoeff} = \{kt, ct, kr, cr\};

\frac{\cos[el z]}{2} + \frac{\cos[e3 z]}{2} + \frac{I}{2} \frac{b \sin[el z]}{el} - \frac{I}{2} \frac{c \sin[e1 z]}{el} + \frac{I}{2} \frac{b \sin[e3 z]}{e3} + \frac{I}{2} \frac{c \sin[e3 z]}{e3}

\frac{I}{2} \frac{d \sin[e1 z]}{el} - \frac{I}{2} \frac{d \sin[e3 z]}{e3}

-\frac{\cos[el z]}{2} + \frac{\cos[e3 z]}{2} - \frac{I}{2} \frac{b \sin[el z]}{el} + \frac{I}{2} \frac{c \sin[e1 z]}{el} - \frac{I}{2} \frac{b \sin[e3 z]}{e3} + \frac{I}{2} \frac{c \sin[e3 z]}{e3}

\frac{I}{2} \frac{d \sin[e1 z]}{el} + \frac{I}{2} \frac{d \sin[e3 z]}{e3}

The c and d coefficients are now defined, then simplified assuming good alignment, i.e. the angle |t| << 1. The approximations applied are \cos(t) = 1 and \sin(t) = t

The coupler matrix \textbf{A0} is then defined in terms of the coefficients \textbf{kt}, \textbf{ct}, \textbf{kr}, and \textbf{cr}.
The matrix $\mathbf{A}_0$ will be the starting point for several "experiments" to ascertain the polarization behavior of birefringent-fiber couplers. The simplified coefficients are

$$
\begin{align*}
\mathbf{e}_1 &= \sqrt{(b - C)^2 + (C t)^2} \\
\mathbf{e}_3 &= \sqrt{(b + C)^2 + (C t)^2} \\
\mathbf{A}_0 &= \{(\mathbf{k}_t, \mathbf{c}_t, \mathbf{k}_r, \mathbf{c}_r), \{\mathbf{c}_t, \text{Conjugate}[\mathbf{k}_t], -\mathbf{c}_r, -\text{Conjugate}[\mathbf{k}_r]\}, \{\mathbf{k}_r, -\mathbf{c}_r, \mathbf{k}_t, -\mathbf{c}_t\}, \{\mathbf{c}_r, -\text{Conjugate}[\mathbf{k}_r], -\mathbf{c}_t, \text{Conjugate}[\mathbf{k}_t]\}\};
\end{align*}
$$

The matrix $\mathbf{A}_0$ will be the starting point for several "experiments" to ascertain the polarization behavior of birefringent-fiber couplers. The simplified coefficients are

$$
\begin{align*}
\mathbf{e}_1 &= \mathbf{e}_3 = \mathbf{C} \\
\text{rule1} &= \{(\sqrt{(b - C)^2 + (C t)^2} \rightarrow -\mathbf{e}_1, \sqrt{(b + C)^2 + (C t)^2} \rightarrow -\mathbf{e}_3)\}
\end{align*}
$$

$$
\begin{align*}
\text{kt} /. \text{rule1} \\
\text{ct} /. \text{rule1} \\
\text{kr} /. \text{rule1} \\
\text{cr} /. \text{rule1}
\end{align*}
$$

$$
\begin{align*}
\frac{\cos(\mathbf{e}_1 z)}{2} + \frac{\cos(\mathbf{e}_3 z)}{2} + \frac{\frac{I}{2} \mathbf{b} \sin(\mathbf{e}_1 z)}{\mathbf{e}_1} - \frac{\frac{I}{2} \mathbf{c} \sin(\mathbf{e}_1 z)}{\mathbf{e}_1} + \\
\frac{\frac{I}{2} \mathbf{b} \sin(\mathbf{e}_3 z)}{\mathbf{e}_3} + \frac{\frac{I}{2} \mathbf{c} \sin(\mathbf{e}_3 z)}{\mathbf{e}_3}
\end{align*}
$$

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\[
\begin{align*}
\frac{1}{2} \sin(e_1 z) & - \frac{1}{2} \sin(e_3 z) \\
\cos(e_1 z) & + \cos(e_3 z) & - \frac{1}{2} b \sin(e_1 z) & + \frac{1}{2} c \sin(e_3 z) \\
\frac{1}{2} b \sin(e_3 z) & + \frac{1}{2} c \sin(e_1 z) \\
\end{align*}
\]

5.2 Procedure Definitions for Graphics

This section lists all of the procedures used in this Mathematica notebook. It consists of a set of initialization cells which must be executed before proceeding in the model.

IMPORTANT VERSION 2 INFORMATION: First, an initialization cell which contains a bug fix must be run to permit plotting of arrays of functions:

```
Unprotect[Plot]
ClearAttributes[Plot, HoldAll]
{Plot}
```

The Mathematica command Conjugate does not work on symbolic expressions, so a new function is defined that does:

```
Conj[xinput_] := ComplexExpand[Conjugate[xinput]]
```
The next few procedures are for mapping the optical state-of-polarization onto the Poincaré sphere. For details and examples, see Appendix III. This builds the sphere:

```math
Needs["Graphics`Shapes`"]
Needs["Graphics`Graphics3D`"]
niceSphere = Evaluate[Graphics3D[Sphere[1,36,18]]]

xyzAxes = {{Line[{{0,0,1.2},{0,0,-1.2}}]},
Line[{{1.2,0,0},{-1.2,0,0}}],
Line[{{0,1.2,0},{0,-1.2,0}}]},

{Text["L", {0,0,1.3}], Text["R", {0,0,-1.3}],
Text["H",{1.3,0,0}], Text["V", {-1.3,0,0}],
Text["+45", {0,1.3,0}], Text["-45", {0,-1.3,0}]);
```

z::shdw: Warning: Symbol z appears in multiple contexts
{Calculus`VectorAnalysis`, Global`}
; definitions in context Calculus`VectorAnalysis`
may shadow other definitions.

This procedure converts SOP to longitude and latitude and plots on a colored sphere.

The input file must be of the form \{(EX1, EX2,...EXN), (EY1, EY2,...EYN)\}
(* Procedure plotPoincare *)

plotPoincare[xyfield_, opts__] := (
{
shouldbe2, Elength} = Dimensions[xyfield];

R = Evaluate[Abs[xyfield[[2]]] / Abs[xyfield[[1]]]];  
side = Sign[1 - R^2];
dd = Evaluate[Arg[xyfield[[2]]] - Arg[xyfield[[1]]]];  
a1 = 2 R Sin[dd] / (1 + R^2);  
a2 = 2 R Cos[dd] / (1 - R^2);  
xx = side Sqrt[(1 - a1^2)/(1 + a2^2)];  
yy = side a2 Sqrt[(1 - a1^2)/(1 + a2^2)];  
zz = a1;

pplot = Table[Point[{{xx[[i]], yy[[i]], zz[[i]]}],  
{i, Elength}}];  
poincare = {niceSphere, Graphics3D[{pplot, xyzAxes}]};

Show[poincare, opts]
)

By passing the option GolorOutput -> GrayLevel, the colorization of the sphere is 
suppressed, with a large savings in memory and rendering speed. An additional routine has been 
written to plot a second set of data, plotted in red, on a Poincaré sphere.
(* Procedure redAddOn *)

redAddOn[xyfield_, opts__] := (
{shouldbe2, Elength} = Dimensions[xyfield];

R = Evaluate[Abs[xyfield[[2]]] / Abs[xyfield[[1]]]];  
side = Sign[1 - R^2];

dd = Evaluate[Arg[xyfield[[2]]] - Arg[xyfield[[1]]]];  

a1 = 2 R Sin[dd] / (1 + R^2);

a2 = 2 R Cos[dd] / (1 - R^2);

xx = side Sqrt[(1 - a1^2)/(1 + a2^2)];

yy = side a2 Sqrt[(1 - a1^2)/(1 + a2^2)];

zz = a1;

redplot = Table[Point[{xx[[i]], yy[[i]], zz[[i]] }],
{i, Elength}];
poincare = {niceSphere, Graphics3D[{pplot, xyzAxes}],
Graphics3D[{Hue[0], redplot}]};

Show[poincare, opts]
)

The above procedure cannot be run until after first calling plotPoincare on the same data array, which generates the pplot file. If desired, the plotting of the fist sphere can be avoided by calling out DisplayFunction -> Identity as an option. A slightly different technique is required to get a black and white sphere and still get the red plot. The illumination LightSources -> {{x, y, z}, GrayLevel[1]} of the sphere is set to be a white light rather than the three primary colors in the default.

To compute the extinction ratio of the optical intensity, as measured with an ideal analyzer and detector setup, the following procedure returns a plot in decibel units.

(* Procedure extRatio *)

extRatio[xyfield_] := (  
{shouldbe2, Elength} = Dimensions[xyfield];

R = Evaluate[Abs[xyfield[[2]]] / Abs[xyfield[[1]]]];  

dd = Evaluate[Arg[xyfield[[2]]] - Arg[xyfield[[1]]]];  

a1 = 2 R Sin[dd] / (1 + R^2);

20 Log[10, Abs[a1]]
)
5.3 Cross-Coupling as a Function of Z

Setting birefringence, coupling coefficient, and misalignment. In this case, $t = 5\degree$

$$b = \pi / L_b$$ corresponds to a 2mm beat length

$$C = \text{coupling coefficient per unit length}$$

The first experimental conditions are set up, and the $A$ matrix is generated as a function of the propagation distance in the $z$-direction

```math
\text{couplerLength} = 3000
\text{Unprotect}[C];
\text{couplerExpl} = \{ 
  b \to 0.0015, 
  C \to 0.001, 
  t \to 0.0872665 \}

A = N[A0 /. \text{couplerExpl}]

3000

\{b \to 0.0015, C \to 0.001, t \to 0.0872665\}

\{0.5 \cos[0.000507558 z] + 0.5 \cos[0.00250152 z] +
  0.492554 i \sin[0.000507558 z] +
  0.499696 i \sin[0.00250152 z],
  0.0859669 i \sin[0.000507558 z] -
  0.0174427 i \sin[0.00250152 z],
-0.5 \cos[0.000507558 z] + 0.5 \cos[0.00250152 z] -
  0.492554 i \sin[0.000507558 z] +
  0.499696 i \sin[0.00250152 z],
  0.0859669 i \sin[0.000507558 z] +
  0.0174427 i \sin[0.00250152 z]},

\{0.0859669 i \sin[0.000507558 z] -
  0.0174427 i \sin[0.00250152 z],
\text{Conjugate}[0.5 \cos[0.000507558 z] +
  0.5 \cos[0.00250152 z] +
  0.492554 i \sin[0.000507558 z] +
  0.499696 i \sin[0.00250152 z],
-0.5 \cos[0.000507558 z] + 0.5 \cos[0.00250152 z] -
  0.492554 i \sin[0.000507558 z] +
  0.499696 i \sin[0.00250152 z],
  0.0859669 i \sin[0.000507558 z] -
  0.0174427 i \sin[0.00250152 z] +
  0.0174427 i \sin[0.00250152 z]]\}
```

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For the standard test conditions (STC) of linear polarization of unity amplitude launched into the slow axis of the fiber, the output fields are computed and plotted for the slow axis intensity for 3 mm of z-propagation.

\[
\begin{align*}
0.499696 \sin(0.00250152 z) + 0.5 \cos(0.00250152 z) + 0.492554 \sin(0.000507558 z) + 0.499696 \sin(0.00250152 z), \\
-0.0859669 \sin(0.000507558 z) - 0.0174427 \sin(0.00250152 z), \\
\end{align*}
\]

\[
\begin{align*}
-0.5 \cos(0.000507558 z) + 0.5 \cos(0.00250152 z) - 0.492554 \sin(0.000507558 z) + 0.499696 \sin(0.00250152 z), \\
-0.0859669 \sin(0.000507558 z) - 0.0174427 \sin(0.00250152 z), \\
0.5 \cos(0.000507558 z) + 0.5 \cos(0.00250152 z) + 0.492554 \sin(0.000507558 z) + 0.499696 \sin(0.00250152 z), \\
-0.0859669 \sin(0.000507558 z) + 0.0174427 \sin(0.00250152 z), \\
\end{align*}
\]

\[
\begin{align*}
-1. \text{Conjugate}[-0.5 \cos(0.000507558 z) + 0.5 \cos(0.00250152 z) - 0.492554 \sin(0.000507558 z) + 0.499696 \sin(0.00250152 z)], \\
\end{align*}
\]

\[
\begin{align*}
\text{Conjugate}[0.5 \cos(0.000507558 z) + 0.5 \cos(0.00250152 z) + 0.492554 \sin(0.000507558 z) + 0.499696 \sin(0.00250152 z)],
\end{align*}
\]

\[
eOut = A . \text{Transpose}[[1, 0, 0, 0]]
\]
\[
iOut = eOut \text{Conjugate} [eOut];
\]
\[
\text{Plot}[[iOut[[1]], iOut[[3]]], \{z, 1, \text{couplerLength}\},
\text{AxesLabel} \rightarrow \{"\text{n z in um}, \text{"I/Io"}\},
\text{Epilog} \rightarrow \{\text{Text} ["A", \{2800, 0.75\}],
\text{Text} ["B", \{2800, 0.25\}]\}]
\]
\[
\{0.5 \cos(0.000507558 z) + 0.5 \cos(0.00250152 z) + \\
0.492554 i \sin(0.000507558 z) + \\
0.499696 i \sin(0.00250152 z), \\
0.0859669 i \sin(0.000507558 z) - \\
0.0174427 i \sin(0.00250152 z), \\
-0.5 \cos(0.000507558 z) + 0.5 \cos(0.00250152 z) - \\
0.492554 i \sin(0.000507558 z) + \\
0.499696 i \sin(0.00250152 z), \\
0.0859669 i \sin(0.000507558 z) + \\
0.0174427 i \sin(0.00250152 z)\}
\]

which looks like the scalar coupling curves plotted in Appendix II. Now examining the light that is cross-coupled over to the fast axes of the two output fibers is done by

\[
\text{Plot}\{\{\text{iOut}[2], \text{iOut}[4]\}, \{z, 1, \text{couplerLength}\}, \\
\text{AxesLabel} \rightarrow \{"\text{n} \ z \ \text{in um}, \ "\text{I/Io}\}, \\
\text{Epilog} \rightarrow \{\text{Text}["A", \{2800, 0.0045\}], \\
\text{Text}["B", \{2700, 0.01\}]\}}
\]
One can see the increase of the undesirable orthogonal polarization component. Two data files with the x- and y-polarized components are created from the matrix calculations, one for each fiber. The state-of-polarization is now plotted on the Poincaré sphere [1].

\[
\begin{align*}
\text{EinA} &= \text{Transpose[Table[{eOut[[1]], eOut[[2]]},
\{z, 1, \text{couplerLength}, 10\}]}]; \\
\text{EinB} &= \text{Transpose[Table[{eOut[[3]], eOut[[4]]},
\{z, 1, \text{couplerLength}, 10\}]}];
\end{align*}
\]

General::spell1:
Possible spelling error: new symbol name "EinB" is similar to existing symbol "EinA".

\[
\text{plotPoincare[EinA, ViewPoint -> \{4, -2, 2\}, DisplayFunction -> Identity]}
\]

\[
\text{redAddOn[EinB, ViewPoint -> \{4, -2, 2\]}
\]
```
ListPlot[Table[(iOut[[3]] / (iOut[[3]] + iOut[[1]])),
{z, 1, couplerLength, 10}]]
```
The reader is reminded that the horizontal scale is for 10 micron increments. Now plot ER:

\[
\text{Show[ListPlot[extRatio[EinA]], ListPlot[extRatio[EinB]]]}
\]
The periodic nature of the unwanted polarization can be fully appreciated over a longer length, in this case 2 centimeters.

\[ \text{couplerLength} = 20000 \]

\begin{verbatim}
Plot[{iOut[[2]], iOut[[4]]}, {z, 1, couplerLength},
AxesLabel -> {"z in um", "I/Io"},
AspectRatio -> 0.4]
\end{verbatim}

One can see a longer-periodicity in the coupled light, as predicted by theory [2,3].
EinA = Transpose[Table[{eOut[[1]], eOut[[2]]}, {z, 1, couplerLength, 10}]];
EinB = Transpose[Table[{eOut[[3]], eOut[[4]]}, {z, 1, couplerLength, 10}]];

plotPoincare[EinA, ViewPoint -> {4, -2, 2}, DisplayFunction -> Identity]
redAddOn[EinB, ViewPoint -> {4, -2, 2}]

-Graphics3D-
The complex polarization for multiple coupling lengths is illustrated on the sphere. Obviously, a coupler is not going to be made with a 2-cm coupling length, but the SOP has regions of very good extinction ratio and very poor extinction ratio, even circular SOP.

5.4 Cross-Coupling vs. Birefringence

Because polishing tends to relieve the stress in the fiber [4], the birefringence is probably less than the value for the unpolished fiber. The coupler has 5° misalignment, and 0.001 coupling coefficient, and is plotted up to 3 mm propagation distance. Normal birefringence corresponding to a 2-mm beat length, 67% reduction, and 33% of the original birefringence is plotted.

```
couplerExp3 = {
    b -> {0.0015, 0.001, 0.0005},
    C -> {0.001, 0.001, 0.001},
    t -> {N[5 Degree], N[5 Degree], N[5 Degree]}}

birel = {Text[".0015", {2700, 0.003}],
          Text[".001", {2000, 0.017}],
          Text[".0005", {2600, 0.014}];

eOut = A0 . Transpose[{1, 0, 0, 0}];
iOut = eOut Conjugate[eOut];
data2 = iOut[[2]] /. couplerExp3;
data4 = iOut[[4]] /. couplerExp3;

Plot[data2, {z, 1, 3000},
     AxesLabel -> {"z in um", "IA/Io"}, Epilog -> birel]
     {b -> {0.0015, 0.001, 0.0005}, C -> {0.001, 0.001, 0.001},
      t -> {0.0872665, 0.0872665, 0.0872665})
```
Several observations are made from these plots, for launching of linear polarization into the slow-axis of fiber A:

(i) There is very little change in fiber A or fiber B polarization predicted for $k \ll 1$ couplers.

(ii) For $z \approx 1.5$ mm, $k = 1$ and fiber A sees less cross-coupled power as birefringence relaxes.
(iii) For $z = 1.5$ mm and $k = 1$, fiber B sees more cross-coupled power as birefringence relaxes.

(iv) Situation (iii) suggests more sensitivity to stress relief for high-$k$ resonators than for $k << 1$, because the A-B coupling path is part of the resonant cavity.

### 5.5 Cross-Coupling vs. Misalignment

Of course, as misalignment gets worse, there will be more polarization cross-coupling induced. This is now examined for the case of $1^\circ$, $5^\circ$, and $10^\circ$ principal axis misalignment.

```math
\text{couplerExp4} = \{
  b \rightarrow \{0.0015, 0.0015, 0.0015\},
  C \rightarrow \{0.001, 0.001, 0.001\},
  t \rightarrow \{N[1 \text{ Degree}], N[5 \text{ Degree}], N[10 \text{ Degree}]\}
```

```math
\text{eOut} = \text{A0} . \text{Transpose}[[1, 0, 0, 0]]; \\
\text{iOut} = \text{eOut} \text{Conjugate}[\text{eOut}]; \\
\text{data1} = \text{iOut}[[1]] /\text{. couplerExp4}; \\
\text{data2} = \text{iOut}[[2]] /\text{. couplerExp4}; \\
\text{data3} = \text{iOut}[[3]] /\text{. couplerExp4}; \\
\text{data4} = \text{iOut}[[4]] /\text{. couplerExp4};
```

```math
\text{Plot}[\text{Flatten}[[\text{data1}, \text{data3}]], \{z, 1, 3000\}, \\
  \text{AxesLabel} \rightarrow \{"\text{z in um", } \text{"Iz/I0"}\}]
```

```math
\text{Plot}[[\text{data2}, \{z, 1, 3000\}, \\
  \text{AxesLabel} \rightarrow \{"\text{z in um", } \text{"IAy/I0"}\}, \\
  \text{PlotRange} \rightarrow \text{All}]
```

```math
\text{Plot}[[\text{data4}, \{z, 1, 3000\}, \\
  \text{AxesLabel} \rightarrow \{"\text{z in um, } \text{"IBy/I0"}\}]
```

```
{b \rightarrow \{0.0015, 0.0015, 0.0015\}, C \rightarrow \{0.001, 0.001, 0.001\}, \\
t \rightarrow \{0.0174533, 0.0872665, 0.174533\}}
```
One can see that as the misalignment is increased, the cross-coupled component increases by a large amount, as would be expected. The light comes from the x- (slow) axis light, which is reduced accordingly.

5.6 Cross-Coupling vs. Coupling Coefficient

In polished couplers, z is not adjusted, but is fixed by the radius of curvature of the substrate grooves. In this experiment, z is held constant at 1 mm, and the coupling coefficient is varied for the 5° misalignment case and 2-mm beat length. Coupling is varied continuously from zero to a coefficient of 0.003 in these plots.

```
Unprotect[C]

C = .
couplerExp4 = {
  b -> 0.0015,
  z -> 1000,
  t -> N[5 Degree]}

A = N[A0 /. couplerExp4];
eOut = A . Transpose[{1, 0, 0, 0}];
iOut = eOut Conjugate[eOut];

Plot[{iOut[[1]], iOut[[3]]}, {C, 0, 0.003},
AxesLabel -> {"\n C", "Ix/Io"}]
{}

{b -> 0.0015, z -> 1000, t -> 0.0872665}

Plot::plnr: CompiledFunction[{{C}, <<1>>, -Co<<8>>de-}][C]
is not a machine-size real number at C = 0. .
```
This is indistinguishable from the plot made in Section 5.3, for the x-polarization. This is because \( \sin(c z) \) and \( \cos(c z) \) are the dominant terms in the expressions. Now examine y-polarization in both fibers, starting with \( C \) slightly above zero to eliminate the error message.

\[
\text{fibID} = \{\text{Text["A", \{0.0025, 0.011\}],} \\
\quad \text{Text["B", \{0.0025, 0.004\}]};
\]

\[
\text{Plot}\left[\{\text{iOut[[2]], iOut[[4]]}, \{C, 10^{-6}, 0.003\}, \right. \\
\quad \text{AxesLabel} \to \{"\n C", "Iy/Io"}, \text{Epilog} \to \text{fibID}\right]
\]

Which looks quite different than for the situation plotted in Section 5.3 for the y-polarization.

Now, plot the response of the 1-mm coupler on the Poincaré sphere for \( 0.0001 < C < 0.01 \):
C = 0.0001 CC / 2;
EinA = Transpose[Table[{eOut[[1]], eOut[[2]]},
{CC, 1, 200}]];

EinB = Transpose[Table[{eOut[[3]], eOut[[4]]},
{CC, 1, 200}]];

plotPoincare[EinA, ViewPoint -> {4, -2, 2},
DisplayFunction -> Identity]

edAddOn[EinB, ViewPoint -> {4, -2, 2}]

General::spell1:
Possible spelling error: new symbol name "EinB"
is similar to existing symbol "EinA".

-Graphics3D-
Which has a completely different look than the case of C being fixed. In this situation, the SOP of fiber A stays nearer the equator, and there is no "loop" in fiber B path. Now plot SR and ER.
ListPlot[Table[(iOut[[3]] / (iOut[[3]] + iOut[[1]])),
{CC, 1, 200}]]

-Graphics-

Where it can be seen that several coupling cycles are being traversed as \( C \) is increased.

Show[ListPlot[extRatio[EinA]], ListPlot[extRatio[EinB]]]
Both fibers have good ER for $C < 0.002$, which includes the $k \ll 1$ and the first $k = 1$ regions. Again, the extinction ratio of light leaving fiber B is seen to become very poor periodically for subsequent $k = 0$ regions. There are still funny resonance regions where the ER gets very high for fiber B, but fiber A’s resonances are diminished in comparison to Section 5.3.

This section is meant to be an example of how Mathematica and the Poincare sphere can be used to examine coupler physics for various types of nonidealities. A more thorough study of the tradeoffs for coupler design parameters, and experimental verification, is hoped to someday produce high-performance devices which are insensitive to process variations.
5.7 References


blank page
The scattering matrix approach will be introduced, and resonators with different types of imperfections will be simulated in this Notebook.

### 6.1 Scattering Matrix Analysis (6x6) System

An ideal single-mode fiber optic resonator will now be examined, as described in the text. For the assembly below, consisting of a 4-port coupler and a length of optical fiber, the global scattering matrix S and geometry matrix G can be written down readily [1,2].

```
SM COUPLER
```

```
S[x_] := 
{ {0, 0, Sqrt[1-x], I Sqrt[x], 0, 0}, 
{0, 0, I Sqrt[x], Sqrt[1-x], 0, 0}, 
{Sqrt[1-x], I Sqrt[x], 0, 0, 0, 0}, 
{I Sqrt[x], Sqrt[1-x], 0, 0, 0, 0}, 
{0, 0, 0, 0, a E^b}, 
{0, 0, 0, 0, a E^b, 0}}

S[k] = Inverse[S[k]]
```
In the above output, $k$ is the intensity splitting ratio of the coupler, and $a$ is the amplitude transmission coefficient for the fiber. The coupler and splices are assumed to be lossless.

\[
G = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
\end{pmatrix}
\]

The calculation for the transfer function is derived as follows:

Given scattering matrix $S$ such that $E_{\text{out}} = S E_{\text{in}}$

and geometry matrix $G$ such that $E_{\text{in}} = G E_{\text{out}} + E_{0}$

where $E_{0}$ is the initial conditions, by substitution one can find
\[ E_{\text{out}} = S (G E_{\text{out}} + E_0) \]
\[ E_{\text{out}} = (S^{-1} - G)^{-1} E_0 \]
which is computed in the following cells and plotted.

\[ \text{SIG} = S I - G \]

\[ \text{SIGI} = \text{Simplify}[\text{Inverse}[\text{SIG}]] \]
\[
\{\{0, 0, \text{Sqrt}[1 - k], -\text{I Sqrt}[k], 0, 0\},
\{0, 0, -\text{I Sqrt}[k], \text{Sqrt}[1 - k], -1, 0\},
\{\text{Sqrt}[1 - k], -\text{I Sqrt}[k], 0, 0, 0, 0\},
\{-\text{I Sqrt}[k], \text{Sqrt}[1 - k], 0, 0, 0, -1\},
\{0, -1, 0, 0, 0, 0\}, \{0, 0, 0, -1, \frac{\text{E}^{-\text{I b}}}{a}, 0\}\}
\]

General::spell1:
Possible spelling error: new symbol name "SIGI"
is similar to existing symbol "SIG".
\begin{align*}
\{ & (I (-1 + a E I b) Sqrt[1 - k]) Sqrt[k]) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& (I (-1 + a E I b) Sqrt[1 - k]) Sqrt[k]) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& 0, 0, 0, (I a E I b (-1 + a E I b) Sqrt[1 - k]) Sqrt[k]) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& (I (-1 + a E I b) Sqrt[1 - k]) Sqrt[k]) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& (a E - Sqrt[1 - k] - a E I b) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& 0, 0, 0, (a E^2 I b (1 - \frac{E I b Sqrt[1 - k]}{a} - k)) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& (I a E I b (-1 + a E I b) Sqrt[1 - k]) Sqrt[k]) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& 2 E^2 I b (1 - \frac{E I b Sqrt[1 - k]}{a} - k)) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& 0, 0, 0, (a E I b (-1 + a E I b) Sqrt[1 - k]) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& 0, 0, (I a E I b (-1 + a E I b) Sqrt[1 - k]) Sqrt[k]) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& 2 E^2 I b (1 - \frac{E I b Sqrt[1 - k]}{a} - k)) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& (a E (-1 + a E I b) Sqrt[1 - k])) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& (a E^2 I b (1 - \frac{E I b Sqrt[1 - k]}{a} - k)) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& 0, 0, 0, (I a E I b (-1 + a E I b) Sqrt[1 - k]) Sqrt[k]) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& 2 E^2 I b (1 - \frac{E I b Sqrt[1 - k]}{a} - k)) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& (a E (-1 + a E I b) Sqrt[1 - k])) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& (a E^2 I b (1 - \frac{E I b Sqrt[1 - k]}{a} - k)) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& 0, 0, 0, (I a E I b (-1 + a E I b) Sqrt[1 - k]) Sqrt[k]) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& 2 E^2 I b (1 - \frac{E I b Sqrt[1 - k]}{a} - k)) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& (a E (-1 + a E I b) Sqrt[1 - k])) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& (a E^2 I b (1 - \frac{E I b Sqrt[1 - k]}{a} - k)) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& 0, 0, 0, (I a E I b (-1 + a E I b) Sqrt[1 - k]) Sqrt[k]) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& 2 E^2 I b (1 - \frac{E I b Sqrt[1 - k]}{a} - k)) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& (a E (-1 + a E I b) Sqrt[1 - k])) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& (a E^2 I b (1 - \frac{E I b Sqrt[1 - k]}{a} - k)) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& 0, 0, 0, (I a E I b (-1 + a E I b) Sqrt[1 - k]) Sqrt[k]) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& 2 E^2 I b (1 - \frac{E I b Sqrt[1 - k]}{a} - k)) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& (a E (-1 + a E I b) Sqrt[1 - k])) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& (a E^2 I b (1 - \frac{E I b Sqrt[1 - k]}{a} - k)) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& 0, 0, 0, (I a E I b (-1 + a E I b) Sqrt[1 - k]) Sqrt[k]) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& 2 E^2 I b (1 - \frac{E I b Sqrt[1 - k]}{a} - k)) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& (a E (-1 + a E I b) Sqrt[1 - k])) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& (a E^2 I b (1 - \frac{E I b Sqrt[1 - k]}{a} - k)) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& 0, 0, 0, (I a E I b (-1 + a E I b) Sqrt[1 - k]) Sqrt[k]) / \\
& (-1 - a E^2 I b + 2 a E I b Sqrt[1 - k] + a E^2 I b k), \\
& 2 E^2 I b (1 - \frac{E I b Sqrt[1 - k]}{a} - k)) / 
\end{align*}
Launching light of unit amplitude into port 1 of the coupler is represented by $E_0$ and the output fields are then computed.

\[
\text{Launch} = \{1,0,0,0,0,0\};
\]

\[
\text{MatrixForm[ fields = SIGI . Launch]}
\]

\[
\begin{align*}
\text{fields} &= \{0, 0, (2 a E^2 I b - \sqrt{1 - k} - a E^2 I b \sqrt{1 - k}) - a E^2 I b / (-1 - a E^2 I b + 2 a E I b \sqrt{1 - k} + a E^2 I b k), \\
&\quad (I (-1 + a E I b \sqrt{1 - k}) \sqrt{k}) / (-1 - a E^2 I b + 2 a E I b \sqrt{1 - k} + a E^2 I b k), \\
&\quad (I a E I b (-1 + a E I b \sqrt{1 - k}) \sqrt{k}) / (-1 - a E^2 I b + 2 a E I b \sqrt{1 - k} + a E^2 I b k), 0\}
\end{align*}
\]

The output intensity (port 3) and the circulating intensity (e.g. port 4) are then calculated for a specific splitting ration and loss value of 0.46 dB.

\[
\text{ratio} = 0.1 \\
\text{trans} = \text{Sqrt[0.9]} \\
\text{outlight} = \text{Abs[fields[[3]]]^2 /. \{k -> ratio, a -> trans\}} \\
\text{cirlight} = \text{Abs[fields[[4]]]^2 /. \{k -> ratio, a -> trans\}}
\]

\[
\begin{align*}
0.1 \\
0.948683
\end{align*}
\]

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Abs\left[\frac{-0.948683 + 1.8025 \times 10^{-1} \times I_b - 0.853815 \times 10^{-1} \times I_b^2}{-1 + 1.8 \times 10^{-1} \times I_b - 0.81 \times 10^{-1} \times I_b^2}\right]^2

Abs\left[\frac{0.316228 \times I_b (-1 + 0.9 \times 10^{-1} \times I_b)}{-1 + 1.8 \times 10^{-1} \times I_b - 0.81 \times 10^{-1} \times I_b^2}\right]^2

\text{Plot}\left[\text{outlight, \{b, 0, 20\}}, \text{PlotRange} \to \{0, 1\}, \text{PlotPoints} \to 500\right]

\text{Plot}\left[\text{cirlight, \{b, 0, 20\}}\right]
Now, examine the resonance obtained as the splitting ratio of the coupler is adjusted. The splitting ratio is set to a very small value, then adjusted optimally, then overcoupled by a large amount. The optimum condition occurs when the splitting ratio plus the intensity transmission coefficient add up to equal 1.

```
Unprotect[Plot]
ClearAttributes[Plot, HoldAll]
{Plot}

ratio = {0.01, 0.2, 0.9}
trans = Table[Sqrt[0.8], {3}]
outlight3 = Abs[fields[[3]]]^2 /. {k -> ratio, a -> trans};

Plot[outlight3, {b, 0, 20}, PlotRange -> {0, 1},
     PlotPoints -> 500]
{0.01, 0.2, 0.9}

{0.894427, 0.894427, 0.894427}

-One can see the strong effect that splitting ratio has upon the resonator transfer function. -Graphics-

1) Undercoupled: shallow, high-finesse dips at k = 1%
2) Optimally coupled: deep, ≈100% contrast dips with moderate finesse at k = 20%.
3) Overcoupled: shallow, nearly sinusoidal interference at k = 90%
The above was for 80% intensity transmission, i.e., almost 1 dB of loss. Now to plot the minimum dip depth as a function of splitting ratio for various losses in the system.

\[
\begin{align*}
    k &= . \\
    \text{Loss} &= \{0.05, 0.1, 0.2, 0.4\} \quad (* \text{dBs} *) \\
    \text{trans} &= \sqrt{10^{-0.1 \text{Loss}}} \\
    \text{minlight} &= \frac{\text{Abs}[\text{fields}[[3]]]^2}{. (b \rightarrow 0, \ a \rightarrow \text{trans})} \\
    \text{pl} &= \text{Plot}[\text{minlight}, \{\text{k}, 0, 0.5\}] \\
\end{align*}
\]

\[
\begin{align*}
    \{0.05, 0.1, 0.2, 0.4\} \\
    \{0.99426, 0.988553, 0.977237, 0.954993\}
\end{align*}
\]

Plotting this on a logarithmic horizontal scale where the units are powers of ten.

\[
\begin{align*}
    \text{logminlight} &= \text{minlight} \/. \ k \rightarrow 10^x; \\
    \text{Plot}[\text{logminlight}, \{x, -3, -0.3\}]
\end{align*}
\]
Now plot the real and imaginary components of the output amplitude on the complex plane, as a function of the fiber phase shift.

\[
X = \text{Re}[\text{fields[[3]]} /. \{k \rightarrow \text{ratio}, a \rightarrow \text{trans}\}]
\]

\[
Y = \text{Im}[\text{fields[[3]]} /. \{k \rightarrow \text{ratio}, a \rightarrow \text{trans}\}]
\]

\[
\text{ParametricPlot[\{X,Y\}, \{b, 0, 20\}, \text{AspectRatio} \rightarrow \text{Automatic}]}\]

\[
\text{Re}[\frac{-0.948683 + 1.8025 E^I b - 0.853815 E^2 I b}{-1 + 1.8 E^I b - 0.81 E^2 I b}]
\]

\[
\text{Im}[\frac{-0.948683 + 1.8025 E^I b - 0.853815 E^2 I b}{-1 + 1.8 E^I b - 0.81 E^2 I b}]
\]
Repeating for the circulating light amplitude.

\[ X = \text{Re}[\text{fields}[[4]]] /. \{k \to \text{ratio}, a \to \text{trans}\} \]
\[ Y = \text{Im}[\text{fields}[[4]]] /. \{k \to \text{ratio}, a \to \text{trans}\} \]

\[
\text{ParametricPlot[}\{X,Y\}, \{b, 0, 20\}, \text{AspectRatio} \to \text{Automatic}] \\
\text{Re}\left[\frac{0.316228 I (-1 + 0.9 E^I b)}{-1 + 1.8 E^I b - 0.81 E^2 I b}\right] \\
\text{Im}\left[\frac{0.316228 I (-1 + 0.9 E^I b)}{-1 + 1.8 E^I b - 0.81 E^2 I b}\right]
\]
## 6.2 Ring Resonator with Coupler Asymmetry

To do this, a slightly different approach is used to generate the $S$ matrix from the coupler matrix which has been modified to include effects of differential symmetric and antisymmetric normal mode losses. Now, the fiber is assumed to be lossless, the amplitude transmission coefficient being lumped in the coupler matrix, with the asymmetry parameter $a$ which accounts for differential losses in the two normal modes.

$$
S = \begin{cases}
\{0, 0, \text{k}t, \text{k}r, 0, 0\}, \\
\{0, 0, \text{k}r, \text{k}t, 0, 0\}, \\
\{\text{k}t, \text{k}r, 0, 0, 0, 0\}, \\
\{\text{k}r, \text{k}t, 0, 0, 0, 0\}, \\
\{0, 0, 0, 0, 0, \text{E}^{(I b)}\}, \\
\{0, 0, 0, 0, \text{E}^{(I b)}, 0\}\).
\end{cases}
$$

MatrixForm[S]

coupler = Flatten[\{\{a, d\}, \{d, a\}\} . \\
\{\text{Sqrt}[1-k], \text{I Sqrt}[k]\}, \{\text{I Sqrt}[k], \text{Sqrt}[1-k]\}\}]

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\[\begin{align*}
0 & \quad 0 & \quad k_t & \quad k_r & \quad 0 & \quad 0 \\
0 & \quad 0 & \quad k_r & \quad k_t & \quad 0 & \quad 0 \\
k_t & \quad k_r & \quad 0 & \quad 0 & \quad 0 & \quad 0 \\
k_r & \quad k_t & \quad 0 & \quad 0 & \quad 0 & \quad 0 \\
0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad I b \\
0 & \quad 0 & \quad 0 & \quad 0 & \quad E & \quad I b \\
\end{align*}\]

\[\{a \text{ Sqrt}[1 - k] + I d \text{ Sqrt}[k], \quad d \text{ Sqrt}[1 - k] + I a \text{ Sqrt}[k], \quad a \text{ Sqrt}[1 - k] + I d \text{ Sqrt}[k], \quad a \text{ Sqrt}[1 - k] + I d \text{ Sqrt}[k]\}\]

\[\text{SA} = S / . \{k_t \rightarrow \text{coupler}[[1]], \quad k_r \rightarrow \text{coupler}[[2]]\}\]

\[\{\{0, 0, a \text{ Sqrt}[1 - k] + I d \text{ Sqrt}[k], \quad d \text{ Sqrt}[1 - k] + I a \text{ Sqrt}[k], \quad 0, 0\},\]
\[\{0, 0, d \text{ Sqrt}[1 - k] + I a \text{ Sqrt}[k], \quad a \text{ Sqrt}[1 - k] + I d \text{ Sqrt}[k], \quad 0, 0\},\]
\[\{a \text{ Sqrt}[1 - k] + I d \text{ Sqrt}[k], \quad d \text{ Sqrt}[1 - k] + I a \text{ Sqrt}[k], \quad 0, 0, 0, 0, 0\},\]
\[\{d \text{ Sqrt}[1 - k] + I a \text{ Sqrt}[k], \quad a \text{ Sqrt}[1 - k] + I d \text{ Sqrt}[k], \quad 0, 0, 0, 0, 0\},\]
\[\{0, 0, 0, 0, 0, 0, 0, 0, E, E, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, E, I b, 0\}\]

\[\text{G} = \{\{0, 0, 0, 0, 0, 0, 0\},\]
\[\{0, 0, 0, 0, 1, 0\},\]
\[\{0, 0, 0, 0, 0, 0\},\]
\[\{0, 0, 0, 0, 0, 0\},\]
\[\{0, 0, 0, 0, 0, 0\},\]
\[\{0, 1, 0, 0, 0, 0\},\]
\[\{0, 0, 0, 1, 0, 0\}\};\]

\[\text{MatrixForm}[G]\]

\[\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0
\end{pmatrix}\]
The inverse matrix operations are much more complex and take significantly longer.

\[
\begin{align*}
SI &= \text{Inverse}[SA]; \\
SIG &= SI - G; \\
SIGI &= \text{Inverse}[SIG]; \\
\text{Launch} &= \{1, 0, 0, 0, 0, 0\}; \\
\text{fields} &= SIGI . \text{Launch}; \\
\end{align*}
\]

\[
\begin{align*}
0, 0, (a - 2a^2 &+ d^2)E^2ib \\
-2a &+ 10\ E^5ib \\
(-a &+ 2a^2dE^2ib - 4E^2ib^3) \\
(-a&+ 2a^2dE^2ib - 4E^2ib^3) \\
(-a &+ 2a^2dE^2ib - 4E^2ib^3) \\
(-a&+ 2a^2dE^2ib - 4E^2ib^3) \\
9a &+ 8\ E^5ib \\
16a &+ 4\ E^5ib \\
9a &+ 8\ E^5ib \\
\end{align*}
\]

↑↑↑↑ NOTE: The above results are only a partial listing. ↑↑↑↑

The model is run with the same losses and splitting ratios as before, and with asymmetry factors of 0.02 and 0.05 and plotted.

\[
\begin{align*}
asym &= 0.02; \\
\text{ratio} &= 0.1; \\
\text{trans} &= \text{Sqrt}[0.9]; \\
\text{specs} &= \{k \rightarrow \text{ratio}, \ a \rightarrow \text{trans}, \ d \rightarrow \text{asym}\}; \\
\text{outlight} &= \text{Abs}[\text{fields}[[3]]]^2 \/. \text{specs} \\
\text{cirlight} &= \text{Abs}[\text{fields}[[4]]]^2 \/. \text{specs} \\
\text{maxlight} &= \text{cirlight} \/. \text{b} \rightarrow 0 \\
\text{Plot}[\text{outlight}, \{b, 0, 20\}, \text{PlotRange} \rightarrow \{0, 1\}, \text{PlotPoints} \rightarrow 500] \\
\text{Plot}[\text{cirlight}, \{b, 0, 20\}, \text{PlotRange} \rightarrow \{0, \text{maxlight}\}]
\end{align*}
\]
\{k \rightarrow 0.1, a \rightarrow 0.948683, d \rightarrow 0.02\}

\text{Power[Abs[(0.80928 \ E^{2 \ I \ b} \ (-1.00044 - 0.00703041 \ I + (-1.1121 - 0.00781504 \ I) \ E^{-2 \ I \ b} + (2.11245 + 0.0140671 \ I) \ E^{-I \ b}) / (-1 + (1.8 + 0.0126491 \ I) \ E^{I \ b} + (-0.80996 - 0.0113842 \ I) \ E^{2 \ I \ b})], 2]}

\text{Power[Abs[(0.80928 \ E^{2 \ I \ b} \ ((-0.0234451 - 0.3707 \ I) \ E^{-2 \ I \ b} + (0.0187561 + 0.333778 \ I) \ E^{-I \ b}) / (-1 + (1.8 + 0.0126491 \ I) \ E^{I \ b} + (-0.80996 - 0.0113842 \ I) \ E^{2 \ I \ b})], 2]}

9.

-Graphics-
asym = 0.05;
ratio = 0.1;
trans = Sqrt[0.9];
specs = {k -> ratio, a -> trans, d -> asym}
outlight = Abs[fields[[3]]]^2 /. specs
Plot[outlight, {b, 0, 20}, PlotRange -> {0, 1},
     PlotPoints -> 500]
{k -> 0.1, a -> 0.948683, d -> 0.05}

Power[Abs[(0.805506 E^(-1.00279 - 0.0176171 I) + (-1.11731 - 0.0196291 I) E^{-2 I b} + (2.11947 + 0.0353324 I) E^{-I b})) / (-1 + (1.8 + 0.0316228 I) E^{I b} + (-0.80975 - 0.0284605 I) E^{2 I b})], 2]
Now, try plotting the output and circulating light on the complex plane to see if asymmetry has an effect.

```math
X = \text{Re}[\text{fields}[3]] /. \text{specs};
Y = \text{Im}[\text{fields}[3]] /. \text{specs};
\text{ParametricPlot}[\{X, Y\}, \{b, 0, 20\}, \text{AspectRatio} -> \text{Automatic}]
```

```math
X = \text{Re}[\text{fields}[4]] /. \text{specs};
Y = \text{Im}[\text{fields}[4]] /. \text{specs};
\text{ParametricPlot}[\{X, Y\}, \{b, 0, 20\}, \text{AspectRatio} -> \text{Automatic}]
```
-Graphics-
Note how the circles have shifted in the phase plane from the ideal case which was plotted in Section 6.1

### 6.3 Ring Resonator with Coupler Backreflection

The S matrix is now modified to include a small symmetric backreflection in the coupler.

![Diagram of SM COUPLER and SM FIBER with labels 1, 2, 3, 4, 5, 6]

\[ R = I r \]

\[ S = \begin{cases} [R, 0, Sqrt[1-k], I Sqrt[k], 0, 0], \\ [0, R, I Sqrt[k], Sqrt[1-k], 0, 0], \\ [Sqrt[1-k], I Sqrt[k], R, 0, 0, 0], \\ [I Sqrt[k], Sqrt[1-k], 0, R, 0, 0], \\ [0, 0, 0, 0, 0, a E^I b], \\ [0, 0, 0, 0, a E^I b, 0] \end{cases} \]

\[ SI = \text{Inverse}[S]; \]

\[ I r \]

\[ \begin{cases} [I r, 0, Sqrt[1 - k], I Sqrt[k], 0, 0], \\ [0, I r, I Sqrt[k], Sqrt[1 - k], 0, 0], \\ [Sqrt[1 - k], I Sqrt[k], I r, 0, 0, 0], \\ [I Sqrt[k], Sqrt[1 - k], 0, I r, 0, 0], \\ [0, 0, 0, 0, a E^I b], \\ [0, 0, 0, 0, a E^I b, 0] \end{cases} \]
\[ G = \{\{0,0,0,0,0,0\}, \{0,0,0,1,0\}, \{0,0,0,0,0,0\}, \{0,0,0,0,0,1\}, \{0,1,0,0,0,0\}, \{0,0,0,1,0,0\}\}; \]

```
MatrixForm[G]
0 0 0 0 0 0
0 0 0 0 1 0
0 0 0 0 0 0
0 0 0 0 0 1
0 1 0 0 0 0
0 0 0 1 0 0
```

The inverse matrix operations are much more complex and take significantly longer.

\[ \text{SIG} = \text{SI} - G; \]
\[ \text{SIGI} = \text{Inverse}[	ext{SIG}]; \]
\[ \text{Launch} = \{1,0,0,0,0,0\}; \]
\[ \text{fields} = \text{SIGI} . \text{Launch} \]

\[ \begin{align*}
6 & 6 I b & 2 & 2 & 4 & 2 \\
8 & 8 I b & \Phi(a E) & (1 + 2Ib - 4 Kr + r) / \\
((I a E) & 2 & 2 & 2 I b & 2 + \\
Power[\Phi(-a E I b) & 2 & 2 & 2 I b & 4] + \\
4 a E I b & 2 & 2 & 2 I b & 4, 5] + \\
(8 I a E I b) & 2 & 2 & 2 I b & 4] / \\
Power[\Phi(-a E I b) & 2 & 2 & 2 I b & 4] - \\
4 a E I b & 2 & 2 & 2 I b & 4, 5] - \\
\end{align*} \]

↑↑↑↑ NOTE: The above results are only a partial listing. ↑↑↑↑

Now calculate for a -60 dB backscatter at the coupler:
refl = N[Sqrt[10^-6]];
ratio = 0.1;
trans = Sqrt[0.9];
specs = {k -> ratio, a -> trans, r -> refl}

outlight = Abs[fields[[3]]]^2 /. specs;
cirlight = Abs[fields[[4]]]^2 /. specs;
backlight = Abs[fields[[1]]]^2 /. specs;
{k -> 0.1, a -> 0.948683, r -> 0.001}

The first term of the output fields[[1]] was zero in the ideal analysis, but the presence of
backscatter makes this term nonzero. This can be plotted as well.

maxlight = cirlight /. b -> 0
maxback = backlight /. b -> 0
Plot[outlight, {b, 0, 20}, PlotRange -> {0, 1},
    PlotPoints -> 500]
Plot[cirlight, {b, 0, 20}, PlotRange -> {0, maxlight}]
Plot[backlight, {b, 0, 20}, PlotRange -> {0, maxback}]

9.9982

0.000063987

-Graphics-
Which illustrates the backreflected resonance peak, as seen in experiments [3].

6.4 Ring Resonator with Fabry-Perot Effect

A resonator is now simulated which has a finite backreflection on the fiber endfaces, which is known as $r << 1$. 
In this situation, the splice has a small reflection \( r \ll 1 \), and is lossless. The geometry matrix is modified to include the reflection from the fiber endfaces, and the phase shift in the pigtails.

\[
R = I r E^{2 I \text{pp}}
\]

\[
G = \{\{R, 0, 0, 0, 0, 0\},
\{0, 0, 0, 0, 1, 0\},
\{0, 0, R, 0, 0, 0\},
\{0, 0, 0, 0, 0, 1\},
\{0, 1, 0, 0, 0, 0\},
\{0, 0, 0, 1, 0, 0\}\}
\]

\[I E^{2 I \text{pp} r}\]
The fields are then computed, and displayed as before, for an endface reflection of 1%.

The pigtails are assumed to be roughly 90% of the cavity length.

\[\text{ratio} = 0.1;\]
\[\text{trans} = \sqrt{0.9};\]
\[\text{refl} = N[\sqrt{0.01}];\]
\[\text{specs} = \{k \rightarrow \text{ratio}, \ a \rightarrow \text{trans}, \ r \rightarrow \text{refl}, \ pp \rightarrow 0.9\ \text{b}\}\]

\[
\text{SIG} = (\text{SI} / . \ \text{specs}) - (\text{G} / . \ \text{specs});
\]
\[
\text{SIGI} = \text{Inverse}[\text{SIG}];
\]
\[
\text{Launch} = \{1,0,0,0,0,0\};
\]
\[
\text{fields} = \text{SIGI} \cdot \text{Launch}
\]

\[
\{k \rightarrow 0.1, \ a \rightarrow 0.948683, \ r \rightarrow 0.1, \ pp \rightarrow 0.9\ \text{b}\}
\]

\[
\{\{\text{E}^2 \ I \ pp \ r, \ 0, \ 0, \ 0, \ 0, \ 0\}, \ \{0, \ 0, \ 0, \ 0, \ 1, \ 0\}, \ \{0, \ 0, \ I \ pp \ r, \ 0, \ 0, \ 0\}, \ \{0, \ 0, \ 0, \ 0, \ 0, \ 1\}, \ \{0, \ 1, \ 0, \ 0, \ 0, \ 0\}, \ \{0, \ 0, \ 0, \ 1, \ 0, \ 0\}\}
\]
20 dB backscatter level at the splice and coupler are extremely high values, only to illustrate the effects of backreflections. The output can be plotted for a 20 meter cavity length and a stretching of 20 radians of phase shift:

```
outlight = Abs[fields[[3]]]^2;
cirlight = Abs[fields[[4]]]^2;
phlen = 20 (7 10^6);

Plot[outlight, {b, phlen, phlen + 20},
   PlotRange -> All, PlotPoints -> 100]
```

The parasitic reflections produce the familiar sinusoidal interference on the output.
6.5 Ring Resonator with Splice Backreflection

The $S$ matrix is now modified to include a short fiber pigtail on port 2 of the coupler, which has phase shift $b_1$, which is approximately $7 \times 10^6$ for a 1-meter pigtail.

```
Remove["Global\"\"]

pig = E^(I b1);

S = {{0, 0, Sqrt[1-k], I Sqrt[k], 0, 0},
    {0, 0, I pig Sqrt[k], pig Sqrt[1-k], 0, 0},
    {Sqrt[1-k], I pig Sqrt[k], R, 0, 0, 0},
    {I Sqrt[k], pig Sqrt[1-k], 0, R, 0, 0},
    {0, 0, 0, 0, 0, a E^(I b)},
    {0, 0, 0, 0, a E^(I b), 0}}

SI = Inverse[S];

{{0, 0, Sqrt[1 - k], I Sqrt[k], 0, 0},
    {0, 0, I E^I b1 Sqrt[k], E^I b1 Sqrt[1 - k], 0, 0},
    {Sqrt[1 - k], I E^I b1 Sqrt[k], R, 0, 0, 0},
    {I Sqrt[k], E^I b1 Sqrt[1 - k], 0, R, 0, 0},
    {0, 0, 0, 0, a E^I b}, {0, 0, 0, 0, a E^I b, 0}}
```

In this situation, the splice has a small reflection $r << 1$, and is lossless. The geometry
matrix is modified to include the reflection from the splice.

\[
\begin{align*}
R &= \mathbf{I} \, r; \\
G &= \{ \{0,0,0,0,0,0\}, \\
&\{0,R,0,0,1,0\}, \\
&\{0,0,0,0,0,0\}, \\
&\{0,0,0,0,1,0\}, \\
&\{0,1,0,0,R,0\}, \\
&\{0,0,0,1,0,0\}\} \\
&\{\{0,0,0,0,0,0\}, \{0, I \, r, 0, 0, 0, 1\}, \{0,0,0,0,0,0\}, \{0,0,0,0,0,1\}, \{0,1,0,0,1,0\}, \{0,0,0,1,0,0\}\}
\end{align*}
\]

The fields are then computed and displayed as before, for a splice reflection of -20 dB. The loss has been increased, and the splitting ratio adjusted accordingly to reduce the finesse of the resonator.

Yo better have a lot of RAM in order to compute the following (> 10 MByte)

\[
\begin{align*}
\text{ratio} &= 0.2; \\
\text{trans} &= \text{Sqrt}[0.8]; \\
\text{refl} &= \text{N[Sqrt[10^{-2}]]}; \\
\text{specs} &= \{k \rightarrow \text{ratio}, \text{a} \rightarrow \text{trans}, \text{r} \rightarrow \text{refl}, \text{bl} \rightarrow 0.2\}
\end{align*}
\]

\[
\begin{align*}
\text{SIG} &= (\text{SI} \, . \, \text{specs}) - (\text{G} \, . \, \text{specs}); \\
\text{SIGI} &= \text{Inverse}[(\text{SIG})]; \\
\text{Launch} &= \{1,0,0,0,0,0\}; \\
\text{fields} &= \text{SIGI} \, . \, \text{Launch}; \\
&\{k \rightarrow 0.2, \text{a} \rightarrow 0.894427, \text{r} \rightarrow 0.1, \text{bl} \rightarrow 0.2\}
\end{align*}
\]

20 dB backscatter level is excessive, but is used only to illustrate the effects of backreflections. The output can be plotted for a 20 meter cavity length and a stretching of 40 radians of phase shift.
The effects of in-loop splice scattering centers produces a periodic shift of the minima.

### 6.6 3 Random Scatterers and Embedding

Now, an attempt at modelling random scatterers in optical fiber will be attempted. In this model, it will be assumed that there is $N=3$ scatterers, each having small backreflection $r$. 

\[
\begin{align*}
\text{outlight} & = |\text{Abs[fields[[3]]]|}^2; \\
\text{phlen} & = 20 \ (7 \ 10^6); \\
\text{Plot}(&\text{outlight, \{b, phlen, phlen + 40\},} \\
&\text{PlotRange } \rightarrow \{0,1\}, \text{PlotPoints } \rightarrow 100]
\end{align*}
\]
Remove["Global`*"]

\[ S = \{\{i, 1, 0, 0, 0, 0\}, \{1, i, 0, 0, 0, 0\}, \{0, 0, i, 1, 0, 0\}, \{0, 0, 1, i, 0, 0\}, \{0, 0, 0, 0, i, 1\}, \{0, 0, 0, 0, 1, i\}\}; \]

\textbf{MatrixForm}[S]

\[ S^{-1} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
i & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & i & 1 & 0 & 0 \\
0 & 0 & 1 & i & 0 & 0 \\
0 & 0 & 0 & 0 & i & 1 \\
0 & 0 & 0 & 0 & 1 & i
\end{pmatrix} \]

\textbf{Inverse}[S]

\[ \left\{ \begin{array}{c}
\frac{i r + 2 i r^3 + i r^5}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6}
\end{array} \right\} \]

\[ \left\{ \begin{array}{c}
\frac{i r + 2 i r^3 + i r^5}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6}
\end{array} \right\} \]

\[ \left\{ \begin{array}{c}
\frac{i r + 2 i r^3 + i r^5}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6}
\end{array} \right\} \]

\[ \left\{ \begin{array}{c}
\frac{i r + 2 i r^3 + i r^5}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6}
\end{array} \right\} \]

\[ \left\{ \begin{array}{c}
\frac{i r + 2 i r^3 + i r^5}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6}
\end{array} \right\} \]

\[ \left\{ \begin{array}{c}
\frac{i r + 2 i r^3 + i r^5}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6} \\
\frac{-1 - 2 r^2 - r^4}{-1 - 3 r^2 - 3 r^4 - r^6}
\end{array} \right\} \]

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\[
\begin{align*}
-1 - 3r^2 - 3r^4 - r^6 \\
\frac{I^2 + 2I^3}{r + 2I^3 + I^5} \\
-1 - 3r^2 - 3r^4 - r^6
\end{align*}
\]

Single-mode fiber has been measured to be on the order of -70 dB backscatter for 1 meter of fiber [5]. The mean expected backscatter then would be \(2 \times 10^{-6}\) for 20 meters (-57 dB). Assuming that \(N\) scatterers have identical reflection coefficients (a dubious assumption), one can estimate the reflection coefficient from

\[
(Nr)^2/2 = L 10^{-7} \quad \Rightarrow \quad r \sim (4.5 \times 10^{-4} L^{1/2})/N = 6.7 \times 10^{-4}
\]

\(\text{SI}_{\text{exact}} = \text{SI} / r \rightarrow 6.7 \times 10^{-4}\)

\(\text{SI}_{\text{approx}} = \{(-I r, 1, 0, 0, 0, 0), (1, -I r, 0, 0, 0, 0), (0, 0, -I r, 1, 0, 0), (0, 0, 1, -I r, 0, 0), (0, 0, 0, 0, -I r, 1), (0, 0, 0, 1, -I r)\}\)

\(\{(-0.00067 I, 1., 0, 0, 0, 0), (1., 0.00067 I, 0, 0, 0, 0), (0, 0, -0.00067 I, 1., 0, 0), (0, 0, 1., -0.00067 I, 0, 0), (0, 0, 0, 0, -0.00067 I, 1.), (0, 0, 0, 1., -0.00067 I)\}\)

\(\{(1, -I r, 0, 0, 0, 0), (0, 0, -I r, 1, 0, 0), (0, 0, 1, -I r, 0, 0), (0, 0, 0, 0, -I r, 1), (0, 0, 0, 1, -I r)\}\)

where the above approximation has been made for the inverse S matrix for small reflections. Note that S inverse has only the diagonal element's sign changed.

The geometry matrix is now constructed, to connect ports 2 to 3, and ports 4 to 5:
\[
G = \{(0, 0, 0, 0, 0),
\{0, 0, E^{(I \ p_1)}, 0, 0, 0\},
\{0, E^{(I \ p_1)}, 0, 0, 0, 0\},
\{0, 0, 0, 0, E^{(I \ p_2)}, 0\},
\{0, 0, 0, E^{(I \ p_2)}, 0, 0\},
\{0, 0, 0, 0, 0, 0\}\};
\]

\[
SG = SIapprox - G;
\]

\[
SG = SIapprox - G;
\]

\[
\begin{array}{cccccc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & E^{I \ p_1} & 0 & 0 & 0 \\
0 & E^{I \ p_1} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & E^{I \ p_2} & 0 \\
0 & 0 & 0 & E^{I \ p_2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

Now the inverse matrix is computed, forcing all high-order terms in r to vanish:

\[
SGI = Inverse[SG] \/. \{r^2 \rightarrow 0, \ r^3 \rightarrow 0, \ r^4 \rightarrow 0, \ r^5 \rightarrow 0, \ r^6 \rightarrow 0\}
\]
The circulating fields 2 through 5 are of no interest, as only the fields entering and leaving the fiber are observable. To eliminate the extraneous information, a transformation is done with a rectangular matrix $T$:

$$
T = \{(1,0), (0,0), (0,0), (0,0), (0,0), (0,1)\};
$$

$$
TT = \text{Transpose}[T];
$$

MatrixForm[T]

MatrixForm[TT]

solution = Together[TT . SGI . T]

1 0
0 0
0 0
0 0
0 0
0 1

1 0 0 0 0 0
0 0 0 0 0 1
\[
\{(r + E^2 I p_1 + E^2 I p_1 + 2 I p_2 r, E I p_1 + I p_2),
\{E I p_1 + I p_2, I (r + E^2 I p_1 + 2 I p_2 r + E^2 I p_2 r)\}\}
\]

which is a 2×2 matrix describing the scattering matrix for the fiber exclusive of what goes on in between the fiber endfaces. The phase shifts between the first and last scatterer and the endfaces has been dropped, because it has no effect.

\[
\begin{align*}
\text{launch} &= \{1, 0\} \\
\text{backref} &= \text{Transpose[launch]} . \text{solution} . \text{launch} \\
\text{backint} &= \text{Abs[backref]}^2 \\
&= (1, 0)
\end{align*}
\]

\[
I (r + E^2 I p_1 + E^2 I p_1 + 2 I p_2 r)
\]

\[
\text{Abs}[I (r + E^2 I p_1 + E^2 I p_1 + 2 I p_2 r)]^2
\]

The scattering points are modeled as being roughly 10 meters apart, with a random phase distribution over the interval 0...2\pi

\[
\begin{align*}
n &= 100 \\
p1\text{random} &= 7 \times 10^7 + \text{Table}[2\ \text{N[Pi]} \ \text{Random[]}, \{n\}] \\
p2\text{random} &= 7 \times 10^7 + \text{Table}[2\ \text{N[Pi]} \ \text{Random[]}, \{n\}] \\
\text{ListPlot}[p1\text{random}, \text{PlotJoined} \to \text{True}] \\
&= 100
\end{align*}
\]
refl = 6.7 \times 10^{-4}

randombackint = backint /. \{p1 -> p1random, p2 -> p2random\};
ListPlot[randombackint /. r -> refl, PlotJoined -> True]

0.00067

which shows the random backscattered intensity observed. Now try again and calculate the mean:
randombackint = backint /. {p1 -> p1random, p2 -> p2random};
ListPlot[randombackint /. r -> refl, PlotJoined -> True]

Sum[%%[[i]]/. r -> refl, {i, 1, 100}] / 100

The := in the random number generator recomputes a new set each time that variable is invoked. Note that the mean level for scattering is near what is predicted -58 dB.

6.7 Coupled Fabry-Perot Resonator

A modified Fabry-Perot resonator will now be examined, which has a coupler in the cavity [6,7]. Two mirrors are added onto the fiber ends, which have negligible losses. The fiber and coupler are assumed to be lossless in this design.
Remove["Global`*"]

\[ S = \{ \{0, 0, \sqrt{1-k}, I \sqrt{k}, 0, 0\}, \]
\[ \{0, 0, I \sqrt{k}, \sqrt{1-k}, 0, 0\}, \]
\[ \{\sqrt{1-k}, I \sqrt{k}, 0, 0, 0\}, \]
\[ \{I \sqrt{k}, \sqrt{1-k}, 0, 0, 0\}, \]
\[ \{0, 0, 0, 0, E^{(I b)}\}, \]
\[ \{0, 0, 0, 0, E^{(I b)}, 0\}\} \]

\[ S^{-1} = \text{Inverse}[S] \]
\[ \{ \{0, 0, \sqrt{1-k}, I \sqrt{k}, 0, 0\}, \]
\[ \{0, 0, I \sqrt{k}, \sqrt{1-k}, 0, 0\}, \]
\[ \{\sqrt{1-k}, I \sqrt{k}, 0, 0, 0\}, \]
\[ \{I \sqrt{k}, \sqrt{1-k}, 0, 0, 0\}, \]
\[ \{0, 0, 0, 0, E^{I b}\}, \{0, 0, 0, 0, E^{I b}, 0\}\} \]

The geometry matrix is different for this topology, and recalling a 90° phase shift at the mirrors is written as

\[ r \cdot \]
\[ G = \{ \{0, 0, 0, 0, 0, 0\}, \]
\[ \{0, 0, 0, 0, 1, 0\}, \]
\[ \{0, 0, 0, 0, 0, 0\}, \]
\[ \{0, 0, 0, 0, 0, 0\}, \]
\[ \{0, 1, 0, 0, 0, 0\}, \]
\[ \{0, 0, 0, 0, I r\}\}; \]

\text{MatrixForm}[G]
The mirror attached to port 4 of the coupler has unity reflection, and the distance between the mirror and the coupler is neglected in this simple analysis.

\[
\begin{align*}
\text{SIG} &= \text{SI} - G \\
\text{SIGI} &= \text{Simplify}[\text{Inverse}[\text{SIG}]]; \\
\text{Launch} &= \{1,0,0,0,0,0\}; \\
\text{fields} &= \text{SIGI} \cdot \text{Launch}
\end{align*}
\]

\[
\left\{ \begin{array}{c}
I k \\
Sqrt[1 - k], -I Sqrt[k], 0, 0, 0, 0
\end{array} \right\}, \left\{ \begin{array}{c}
Sqrt[1 - k], -I Sqrt[k], Sqrt[1 - k], -1, 0, 0
\end{array} \right\}, \\
\left\{ \begin{array}{c}
Sqrt[1 - k], -I Sqrt[k], 0, 0, 0, 0
\end{array} \right\}, \\
\left\{ \begin{array}{c}
-I Sqrt[k], Sqrt[1 - k], 0, -I, 0, 0
\end{array} \right\}, \\
\left\{ \begin{array}{c}
0, -1, 0, 0, 0, E^{-Ib}, 0, 0, 0, E^{-Ib}, -I r
\end{array} \right\}
\]

\[
\left\{ \begin{array}{c}
\frac{I k}{-1 - E^2 I b r + E^2 I b k r}, \\
\frac{Sqrt[1 - k] Sqrt[k]}{-1 - E^2 I b r + E^2 I b k r}, \\
\frac{Sqrt[1 - k] (1 + E^2 I b r)}{-1 - E^2 I b r + E^2 I b k r}, \\
\frac{-I Sqrt[k]}{-1 - E^2 I b r + E^2 I b k r}, \\
\frac{I E^2 I b Sqrt[1 - k] Sqrt[k] r}{-1 - E^2 I b r + E^2 I b k r}, \\
\frac{E^2 I b Sqrt[1 - k] Sqrt[k]}{-1 - E^2 I b r + E^2 I b k r}
\end{array} \right\}
\]

The output intensity (port 3) and the cavity intensity (port 6) are then calculated for a specific splitting ratio and 90% mirror reflection. The fiber length is chosen to be nominally 1.00 meters, and the phase length in radians is calculated, including \(\Delta \phi\).
ratio = 0.1;
mirror = Sqrt[0.9];
outlight = Abs[fields[[3]]]^2 /. {k -> ratio, r -> mirror}
cirlight = Abs[fields[[6]]]^2 /. {k -> ratio, r -> mirror}

b = 1.00 (2 Pi 1.5)/(1.3 10^-6) + phase

Plot[outlight, {phase, 0, 10}, PlotRange -> {0, 1},
PlotPoints -> 100]

Plot[cirlight, {phase, 0, 10}, PlotPoints -> 100]

Abs[0.948683 (1 + 0.948683 E^2 I b)^2 /
1 + 0.853815 E^2 I b]^2

Abs[0.3 E^I b / -1 - 0.853815 E^2 I b]^2

phase + 2.30769 10^6 Pi

-Graphics-
Since the mirror attached to port 6 was assumed to be lossless, the transmission out that fiber endface would be $I_6(1-r^2)$.

### 6.8 Peak Resonator (10x10) System

A larger system will be examined, having two couplers and a length of fiber. This is set up with 10×10 matrices, and the computation is done as before.
Remove["Global`*"]

\[ S = \{ \{0, 0, \text{Sqrt}[1-k], I \text{Sqrt}[k], 0, 0, 0, 0, 0\}, \{0, 0, I \text{Sqrt}[k], \text{Sqrt}[1-k], 0, 0, 0, 0, 0\}, \{\text{Sqrt}[1-k], I \text{Sqrt}[k], 0, 0, 0, 0, 0, 0, 0\}, \{I \text{Sqrt}[k], \text{Sqrt}[1-k], 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, \text{Sqrt}[1-k], I \text{Sqrt}[k], 0, 0\}, \{0, 0, 0, 0, 0, I \text{Sqrt}[k], \text{Sqrt}[1-k], 0, 0\}, \{0, 0, 0, 0, 0, \text{Sqrt}[1-k], I \text{Sqrt}[k], 0, 0\}, \{0, 0, 0, 0, 0, I \text{Sqrt}[k], \text{Sqrt}[1-k], 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, a \text{E}^{I b}\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, a \text{E}^{I b}\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \}\]

\[ S_I = \text{Inverse}[S] \]

\[ \{ \{0, 0, \text{Sqrt}[1-k], I \text{Sqrt}[k], 0, 0, 0, 0, 0\}, \{0, 0, I \text{Sqrt}[k], \text{Sqrt}[1-k], 0, 0, 0, 0, 0\}, \{\text{Sqrt}[1-k], I \text{Sqrt}[k], 0, 0, 0, 0, 0, 0, 0\}, \{I \text{Sqrt}[k], \text{Sqrt}[1-k], 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, \text{Sqrt}[1-k], I \text{Sqrt}[k], 0, 0\}, \{0, 0, 0, 0, 0, I \text{Sqrt}[k], \text{Sqrt}[1-k], 0, 0\}, \{0, 0, 0, 0, 0, \text{Sqrt}[1-k], I \text{Sqrt}[k], 0, 0\}, \{0, 0, 0, 0, 0, I \text{Sqrt}[k], \text{Sqrt}[1-k], 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, a \text{E}^{I b}\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, a \text{E}^{I b}\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0\} \} \]
\[
\begin{align*}
&\{0, 0, \text{Sqrt}[1-k], -\text{I} \text{Sqrt}[k], 0, 0, 0, 0, 0, 0, 0\}, \\
&\{0, 0, -\text{I} \text{Sqrt}[k], \text{Sqrt}[1-k], 0, 0, 0, 0, 0, 0, 0\}, \\
&\{\text{Sqrt}[1-k], -\text{I} \text{Sqrt}[k], 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
&\{-\text{I} \text{Sqrt}[k], \text{Sqrt}[1-k], 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
&\{0, 0, 0, 0, 0, 0, \text{Sqrt}[1-k], -\text{I} \text{Sqrt}[k], 0, 0, 0, 0\}, \\
&\{-\text{I} \text{Sqrt}[k], \text{Sqrt}[1-k], 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \\
&\{0, 0, 0, 0, 0, 0, \text{Sqrt}[1-k], -\text{I} \text{Sqrt}[k], 0, 0, 0, 0\}, \\
&\{0, 0, 0, 0, -\text{I} \text{Sqrt}[k], \text{Sqrt}[1-k], 0, 0, 0, 0, 0\}, \\
&\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{-\text{I} \text{b}}{\text{a}}, \frac{-\text{I} \text{b}}{\text{a}}, 0\}, \\
&\{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \frac{-\text{I} \text{b}}{\text{a}}, \frac{-\text{I} \text{b}}{\text{a}}, 0\} \\
\end{align*}
\]

The geometry matrix is written down from the figure, and matrix algebra performed.

\[
G = \{0,0,0,0,0,0,0,0,0,0,0\}, \\
\{0,0,0,0,1,0,0,0,0,0,0\}, \\
\{0,0,0,0,0,0,0,0,0,0,0\}, \\
\{0,0,0,0,0,0,0,0,0,1,0\}, \\
\{0,0,0,0,0,0,0,0,0,0,0\}, \\
\{0,0,0,0,0,0,0,0,0,0,0\}, \\
\{0,0,0,0,0,1,0,0,0,0,0\}, \\
\{0,0,0,0,0,0,0,0,0,0,0\};
\]

\[
\text{MatrixForm}[G] \\
\text{SIG} = \text{SI} - G; \\
\text{SIGI} = \text{Simplify}[\text{Inverse}[\text{SIG}]];
\]

\[
\begin{align*}
&0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
&0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
&0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
&0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
&0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
&0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
&0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
&0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
&0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
&0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{align*}
\]

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The light is launched into port #1, and the 10 fields are computed. Note that 5 of the fields are zero because no backreflections have been assumed.

\[ \text{Launch} = \{1, 0, 0, 0, 0, 0, 0, 0, 0, 0\}; \]
\[ \text{fields} = \text{SIGI} \cdot \text{Launch} \]

\[
\begin{align*}
&0, \quad 0, \quad \frac{(1 - a E^b) \sqrt{1 - k}}{1 - a E^b + a E^b k}, \quad \frac{I \sqrt{k}}{1 - a E^b + a E^b k}, \quad 0, \\
&I a E^b \sqrt{1 - k} \sqrt{k} \quad 1 - a E^b + a E^b k, \quad \frac{I a E^b \sqrt{k}}{1 - a E^b + a E^b k}, \quad 0, \\
&0, \quad \frac{I a E^b \sqrt{k}}{1 - a E^b + a E^b k}, \quad 0
\end{align*}
\]

The output intensity from the first coupler, the circulating intensity in the cavity, and the output field from the second coupler (which has a "peak" response) are computed.

\[
\begin{align*}
\text{ratio} & = 0.1 \\
\text{trans} & = \sqrt{0.99} \\
\text{outlight} & = \text{Abs}[\text{fields[[3]]}]^2 / \{k \rightarrow \text{ratio}, a \rightarrow \text{trans}\} \\
\text{cirlight} & = \text{Abs}[\text{fields[[4]]}]^2 / \{k \rightarrow \text{ratio}, a \rightarrow \text{trans}\} \\
\text{peaklight} & = \text{Abs}[\text{fields[[6]]}]^2 / \{k \rightarrow \text{ratio}, a \rightarrow \text{trans}\} \\
& 0.1 \\
& 0.994987 \\
& \text{Abs}[\frac{0.948683 (1 - 0.994987 E^b)}{1 - 0.895489 E^b}]^2 \\
& \text{Abs}[\frac{0.316228 E^b}{1 - 0.895489 E^b}]^2
\end{align*}
\]
These are in turn plotted, showing the output, circulating, and "peak" output intensities.

\[
\text{maxcirc} = \text{cirlight} / . \ b \to 0 \\
\text{maxpeak} = \text{peaklight} / . \ b \to 0
\]

Plot[outputlight, \{b, 0, 20\}, PlotRange \to \{0, 1\},
\text{PlotPoints} \to 500]
Plot[cirlight, \{b, 0, 20\}, PlotRange \to \{0, \text{maxcirc}\}]
Plot[peaklight, \{b, 0, 20\}, PlotRange \to \{0, \text{maxpeak}\}]

9.15532

0.906377
6.9 References


The single-mode analytical techniques introduced in Appendix VI will now be further generalized to include polarization effects. Specific resonator types will be modeled with the software.

### 7.1 Ring Resonator with PM Coupler (12x12) System

The scattering matrix for the PM fiber coupler is an 8x8 symmetric matrix, and the PM fiber is also expanded by a factor of 2 and is a 4x4 matrix. The exact coefficients for the PM coupler are defined first, as in Appendix IV. They are then approximated for small misalignment between the birefringent fiber's principal axes $\theta\ll 1$ radian.
ktl = \frac{\cos(e_1 z)}{2} + \frac{\cos(e_3 z)}{2} + \frac{i}{2} \frac{b \sin(e_1 z)}{e_1} - \frac{i}{2} \frac{c \sin(e_1 z)}{e_1} + \frac{i}{2} \frac{d \sin(e_3 z)}{e_3} + \frac{i}{2} \frac{c \sin(e_3 z)}{e_3}

ctl = \frac{(i/2 \ d \ \sin(e_1 z))}{e_1} - \frac{(i/2 \ d \ \sin(e_3 z))}{e_3}

krl = -\cos(e_1 z)/2 + \cos(e_3 z)/2 - \frac{(i/2 \ b \ \sin(e_1 z))}{e_1} + \frac{(i/2 \ c \ \sin(e_1 z))}{e_1} + \frac{(i/2 \ b \ \sin(e_3 z))}{e_3} + \frac{(i/2 \ c \ \sin(e_3 z))}{e_3}

crl = \frac{(i/2 \ d \ \sin(e_1 z))}{e_1} + \frac{(i/2 \ d \ \sin(e_3 z))}{e_3}

c = C;

d = C \ t;

e_1 = -\sqrt{(b - C)^2 + (C \ t)^2};

e_3 = -\sqrt{(b + C)^2 + (C \ t)^2};

The characteristics for the coupler are then defined: \(\Delta \beta / 2 = \pi / L_B\), \(\Phi\), and coupling coefficient \(C\).
The units for length in this analysis are all in meters, not in microns as in the other Appendices.

```math
\text{coupler} = \{b \rightarrow \text{N}[\pi/0.002], t \rightarrow \text{N}[5 \text{ Degree}], C \rightarrow 1000\}
\text{kt} = \text{kt1} /\text{coupler}
\text{ct} = \text{ct1} /\text{coupler}
\text{kr} = \text{kr1} /\text{coupler}
\text{cr} = \text{cr1} /\text{coupler}
\{b \rightarrow 1570.8, t \rightarrow 0.0872665, C \rightarrow 1000\}

\frac{\cos[577.429 z]}{2} + \frac{\cos[2572.28 z]}{2} +
\quad 0.494257 \text{I sin}[577.429 z] + 0.499712 \text{I sin}[2572.28 z]

0.0755647 \text{I sin}[577.429 z] - 0.0169629 \text{I sin}[2572.28 z]

- \frac{\cos[577.429 z]}{2} + \frac{\cos[2572.28 z]}{2} -
\quad 0.494257 \text{I sin}[577.429 z] + 0.499712 \text{I sin}[2572.28 z]

0.0755647 \text{I sin}[577.429 z] + 0.0169629 \text{I sin}[2572.28 z]
```

The numerical values are inserted as early as possible, to reduce the time for inversion of the large 12x12 global matrix and subsequent operations. The S and G matrices are now constructed:
Since most of the entries in the 12x12 scattering matrix are zeroes, the Table function is used to generate a template. "Copy Output From Above" is used to create the S matrix with editing.

```math
\text{X[xx]} := \text{Conjugate[xx]}
\text{Table[Table[0, \{12\}], \{12\}]}
```
\{
  \{0, 0, 0, 0, 0, 0, 0, 0\},
  \{0, 0, 0, 0, 0, 0, 0, 0\},
  \{0, 0, 0, 0, 0, 0, 0, 0\},
  \{0, 0, 0, 0, 0, 0, 0, 0\},
  \{0, 0, 0, 0, 0, 0, 0, 0\},
  \{0, 0, 0, 0, 0, 0, 0, 0\},
  \{0, 0, 0, 0, 0, 0, 0, 0\},
  \{0, 0, 0, 0, 0, 0, 0, 0\},
\}\n
S = \{
  \{0, 0, 0, 0, kt, ct, kr, cr, 0, 0, 0, 0\},
  \{0, 0, 0, 0, ct, X[kt], -cr, -X[kr], 0, 0, 0, 0\},
  \{0, 0, 0, 0, kr, -cr, kt, ct, 0, 0, 0, 0\},
  \{0, 0, 0, 0, -ct, -X[kr], -ct, X[kt], 0, 0, 0, 0\},
\}

\{kt, ct, kr, cr, 0, 0, 0, 0, 0, 0, 0, 0\},
\{ct, X[kt], -cr, -X[kr], 0, 0, 0, 0, 0, 0, 0, 0\},
\{kr, -cr, kt, -ct, 0, 0, 0, 0, 0, 0, 0, 0\},
\{cr, -X[kr], -ct, X[kt], 0, 0, 0, 0, 0, 0, 0, 0\},
\}

\{0, 0, 0, 0, 0, 0, 0, 0, a E^(I bx), 0, 0, 0, 0\},
\{0, 0, 0, 0, 0, 0, 0, 0, a E^(I by), 0, 0, 0, 0\},
\{0, 0, 0, 0, 0, 0, 0, 0, a E^(I bx), 0, 0, 0, 0\},
\{0, 0, 0, 0, 0, 0, 0, 0, a E^(I by), 0, 0, 0, 0\}\}

\{0, 0, 0, 0, \frac{\cos[577.429 z]}{2} + \frac{\cos[2572.28 z]}{2} + \\
0.494257 I \sin[577.429 z] + 0.499712 I \sin[2572.28 z], \\
0.0755647 I \sin[577.429 z] - 0.0169629 I \sin[2572.28 z], \\
-\frac{-\cos[577.429 z]}{2} + \frac{\cos[2572.28 z]}{2} - \\
0.494257 I \sin[577.429 z] + 0.499712 I \sin[2572.28 z], \\
0.0755647 I \sin[577.429 z] + 0.0169629 I \sin[2572.28 z], \\
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\},
\{0, 0, 0, 0, 0, 0, 0, 0, \cos[577.429 z] + \cos[2572.28 z]\}

\uparrow \uparrow \uparrow \uparrow \text{NOTE: The above results are only a partial listing.} \uparrow \uparrow \uparrow \uparrow
The same technique is used to build the geometry matrix from the port-to-port connections:

\[ G = \{ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \]
\[ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \]
\[ \{ 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0 \}, \]
\[ \{ 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0 \}, \]
\[ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \]
\[ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \]
\[ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \]
\[ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \]
\[ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \}, \]
\[ \{ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \} \}

The splitting ratio is set by adjusting the coupling length \( L_c \). The relationship is plotted, and a small splitting ratio is set by estimating the length desired to achieve about 10% coupling, which occurs at \( L \sim 3.3 \) mm.

```math
Plot[Abs[kr]^2, \{z, 0, 0.002\}]
Lc = 0.00033
SR = Abs[kr]^2 /. z -> Lc
```
The parameters for the ring are now defined, namely splitting ratio and loss. The only undefined terms in the matrices are the phase shifts in the x- and y-modes of the fiber.

```mathematica
ring = {z -> Lc, a -> Sqrt[0.9]};
SI = Inverse[S /. ring];
SIG = SI - G;

SIGI = Inverse[SIG];
Launch = {1, 0, 0, 0, 0, 0, 0, 0, 0, 0};
fields = SIGI . Launch
{z -> 0.00033, a -> 0.948683}
```

General::spell1:
Possible spelling error: new symbol name "SIGI" is similar to existing symbol "SIG".
\[ \{0, 0, 0, 0, (E^{2 I \text{bx} + 2 I \text{by}}) + (0.735126 + 0.419431 I + (0.81739 - 0.466385 I) E^{-2 I \text{bx}} + (-1.78769 + 0.0000146381 I) E^{-I \text{bx}} + (1.01561 + 0.579486 I) E^{-2 I \text{bx} - 2 I \text{by}} + (-1.13017 - 1.91218 I) E^{-I \text{bx} - 2 I \text{by}} + (-2.09801 - 6.69664 \times 10^{-20} I) E^{-2 I \text{bx} - I \text{by}} + (3.46157 + 1.97504 I) E^{-I \text{bx} - I \text{by}} + (0.0161006 + 1.05148 I) F^{-2 I \text{by}} + \} \]

\[ \text{\uparrow\uparrow\uparrow \text{NOTE: The above results are only a partial listing. \uparrow\uparrow\uparrow} \]

The x-polarization and y-polarization out of the coupler pigtail is extracted from the result.

\[ \text{xout} = \text{Chop[Abs[fields[[5]]]]}^2 \]
\[ \text{yout} = \text{Chop[Abs[fields[[6]]]]}^2 \]
\[\text{Power[Abs[(E}^{(0.735126 + 0.419431 I) + (0.81739 - 0.466385 I) E^{-2 I bx} + (-1.78769 + 0.0000146381 I) E^{-I bx} + (1.01561 + 0.579486 I) E^{-2 I bx - 2 I by} + (-1.13017 - 1.91218 I) E^{-I bx - 2 I by} - 2.09801 E^{-2 I bx - I by} + (3.46157 + 1.97504 I) E^{-I bx - I by} + (0.0161006 + 1.05148 I) E^{-2 I by} + (-0.960053 - 1.62433 I) E^{-I by}) / (1.23648 + (-1.92699 - 1.0995 I) E^{I bx} + (0.506354 + 0.856753 I) E^{2 I bx} + 3.9808 E^{I bx + I by} + (-1.55097 - 0.884948 I) E^{2 I bx + I by} + (-1.55097 + 0.884948 I) E^{I bx + 2 I by} + 0.801004 E^{2 I bx + 2 I by} + (-1.92699 + 1.0995 I) E^{I by} + (0.506354 - 0.856753 I) E^{2 I by}]], 2]\]

General::spell1:
Possible spelling error: new symbol name "yout" is similar to existing symbol "xout".
\[ \text{Power[Abs[} \left( E^2 I \text{bx} + 2 I \text{by} \right) \left( 0.00141605 I + (0.00604708 - 0.00713679 I) \right) E^2 I \text{bx} + \left( -0.00866734 + 0.00150165 I \right) E^{-I \text{bx}} + \left( 0.00195484 \right) E^{-2 I \text{bx} - 2 I \text{by}} + \left( 0.0106659 + 0.00204576 I \right) E^{-I \text{bx} - 2 I \text{by}} + \left( -0.0106659 + 0.00204576 I \right) E^{-2 I \text{bx} - I \text{by}} + \left( 0.00648209 \right) E^{-I \text{bx} - I \text{by}} + \left( -0.00604708 - 0.00713679 I \right) E^{-2 I \text{by}} + \left( 0.00866734 + 0.00150165 I \right) E^{-I \text{by}} \right) / \left( 1.23648 + (-1.92699 - 1.0995 I) E^2 I \text{bx} + \left( 0.506354 + 0.856753 I \right) E^2 I \text{bx} + 3.9808 E^2 I \text{bx} + I \text{by} + \left( -1.55097 - 0.884948 I \right) E^2 I \text{bx} + I \text{by} + \left( -1.55097 + 0.884948 I \right) E^2 I \text{bx} + 2 I \text{by} + 0.801004 E^2 I \text{bx} + 2 I \text{by} + \left( -1.92699 + 1.0995 I \right) E^2 I \text{by} + \left( 0.506354 - 0.856753 I \right) E^2 I \text{by} \right), 2] \]

The Chop function removes very small constants which can be neglected, and the intensity is calculated with the magnitude squared of the field. 20 meter fiber length, the propagation constant \(2\pi n/\lambda\) at \(\lambda = 1300\) nm, and birefringence effects are added:

\[
L = 20 + \text{deltaL};
B = 7.3 \times 10^6;
bx = (B + N[\pi/0.002]) \times L
by = (B - N[\pi/0.002]) \times L
\]

\[
\text{Plot[xout, \{deltaL, 0, 2 \times 10^{-6}\}, PlotRange -> All]}
\text{Plot[yout, \{deltaL, 0, 2 \times 10^{-6}\}, PlotRange -> All]}
\]

7.30157 \times 10^6 \times (20 + \text{deltaL})
If this was observed with a photodetector without polarization selection, both plots are added incoherently. The circulating intensities are also plotted for the x- and y-polarizations.

\[ x_{cir} = \text{Abs}[\text{fields}[[7]]]^2; \]
\[ y_{cir} = \text{Abs}[\text{fields}[[8]]]^2; \]
\[ \text{Plot}[x_{cir}, \{\text{deltaL, 0, 2 \times 10^{-6}}, \text{PlotRange} \to \text{All}\}] \]
\[ \text{Plot}[y_{cir}, \{\text{deltaL, 0, 2 \times 10^{-6}}, \text{PlotRange} \to \text{All}\}] \]
Note that there are two separate resonances, one for each polarization eigenmode of the resonator. The x-polarization resonates as in a single-mode resonator, and gives rise to the large dips in the x-polarization output from the coupler. However, the small misalignment in the fiber optic coupler permits a finite coupling to the orthogonal eigenmode which can also resonate. Due to polarization dispersion, i.e., $n_x > n_y$ the two resonances are separate and usually independent; however under the right conditions they can strongly couple to one another [1].
7.2 Examples of Polarization Error

Now examine the case for a misaligned input polarization with the same resonator used in section 7.1. In this example, linear polarization is launched at a $20^\circ$ misalignment with the slow (x) axis of the input fiber, with a unit intensity. One would not normally encounter such a large misalignment in practice, but it is useful for emphasizing the errors produced.

```math
Launch = \{0.940, 0.342, 0, 0, 0, 0, 0, 0, 0, 0, 0\};

fields = SIGI . Launch;
xout = Chop[Abs[fields[[5]]]]^2;
yout = Chop[Abs[fields[[6]]]]^2;
Plot[xout, \{deltaL, 0, 2 \times 10^{-6}\}, PlotRange -> \{0,1\}]
Plot[yout, \{deltaL, 0, 2 \times 10^{-6}\}, PlotRange -> \{0,0.2\}]
```

-Graphics-
If there is no output polarizer, both x- and y-polarizations are viewed simultaneously by the detector, which produces the familiar output plot of one large dip and smaller dip due to the orthogonal polarization resonance. They are added incoherently, producing the 2nd dip and raising the minimum of the larger resonance dip.

\[
\text{combo} = \text{xout} + \text{yout};
\]

\[
\text{Plot}[\text{combo}, \{\text{deltaL}, 0, 2 \times 10^{-6}\}, \text{PlotRange} \rightarrow \{0,1\}]
\]

As the fiber is thermally perturbed further, the two resonances can drift nearer to one another. To see this effect, the cavity length is shortened by 200 microns, near where both x- and x-modes have an integral number of standing waves in the cavity.
offset = -200 N[10^-6]
Plot[combo, {deltaL, offset, offset + 2 10^-6),
    PlotRange -> {0,1}]

-0.0002

offset = -200 N[10^-6] + 0.4 10^-6
Plot[combo, {deltaL, offset, offset + 0.4 10^-6},
    PlotRange -> {0,1}]

-0.000196
This illustrates the distortion of the dip when the two polarization resonances are coincident. This "asymmetry" of the dip results in large gyro errors when this effect is seen.

### 7.3 Ring Resonator with PM Coupler and PZ Fiber

The same matrix is used as for section 7.1 except that different transmission coefficients are applied to the x- and y-polarized modes of the optical fiber. In this model, the x-polarization is the guided polarization, and has transmission coefficient \( a \) which is near unity. The scattering matrix is modified such that y-polarization is severely attenuated and is included by the factor \( r \).
a = .
bx = .
by = .

\[ S = \{\{0,0,0,0, k_t, c_t, k_r, c_r, 0,0,0,0\}, \]
\[ \{0,0,0,0, c_t, X[k_t], -c_r, -X[k_r], 0,0,0,0\}, \]
\[ \{0,0,0,0, k_r, -c_r, k_t, -c_t, 0,0,0,0\}, \]
\[ \{0,0,0,0, -c_t, -X[k_r], -c_t, X[k_t], 0,0,0,0\}, \]
\[ \{k_t, c_t, k_r, -c_t, 0,0,0,0, 0,0,0,0\}, \]
\[ \{c_t, X[k_t], -c_r, -X[k_r], 0,0,0,0, 0,0,0,0\}, \]
\[ \{k_r, -c_r, k_t, -c_t, 0,0,0,0, 0,0,0,0\}, \]
\[ \{c_t, -X[k_t], -c_t, X[k_t], 0,0,0,0, 0,0,0,0\}, \]
\[ \{0,0,0,0,0,0,0,0,0,0\}, 0, 0, a E^{i b x}, 0 \} \}
\[ \{0,0,0,0,0,0,0,0,0,0\}, 0, 0, 0, r E^{i b y} \} \}
\[ \{0,0,0,0,0,0,0,0,0,0\}, a E^{i b x}, 0, 0 \} \}
\[ \{0,0,0,0,0,0,0,0,0,0\}, 0, r E^{i b y}, 0, 0 \} \}; \]

The same parameters are used in the device, except for the 30 dB attenuation for the y-mode.

\[ \text{ring} = \{z \rightarrow Lc, \ a \rightarrow \text{Sqrt}[0.9], \ r \rightarrow 0.032\} \]
\[ \text{SI} = \text{Inverse}[S \/. \text{ring}] ; \]
\[ \text{SIG} = \text{SI} - G ; \]
\[ \text{SIGI} = \text{Inverse}[\text{SIG}] ; \]
\[ \text{Launch} = \{1,0,0,0,0,0,0,0,0,0,0\} ; \]
\[ \text{fields} = \text{SIGI} \cdot \text{Launch} \]
\[ \{z \rightarrow 0.00033, \ a \rightarrow 0.948683, \ r \rightarrow 0.032\} \]
\{0, 0, 0, 0, (E^{2} I bx + 2 I by
(0.735126 + 0.419431 I +
(0.81739 - 0.466385 I) E^{-2} I bx +
(-1.78769 + 0.0000146381 I) E^{-I} bx +
(892.628 + 509.314 I) E^{-2} I bx - 2 I by +
(-993.313 - 1680.63 I) E^{-I} bx - 2 I by +
(-62.1985 - 2.33861 \times 10^{-18} I) E^{-2} I bx - I by +
(102.623 + 58.5528 I) E^{-I} bx - I by +
(14.1509 + 924.151 I) E^{-2} I by +

\uparrow\uparrow\uparrow\uparrow \text{NOTE: The above results are only a partial listing.} \uparrow\uparrow\uparrow\uparrow

The x-polarization and y-polarization out of the coupler pigtail is now plotted.

\begin{verbatim}
xout = Chop[Abs[fields[[5]]]]^2;
yout = Chop[Abs[fields[[6]]]]^2;
L = 20 + deltaL;
B = 7.3 \times 10^6;
bx = (B + N[Pi/0.002]) L
by = (B - N[Pi/0.002]) L
Plot[xout, {deltaL, 0, 2 \times 10^{-6}}, PlotRange -> All]
Plot[yout, {deltaL, 0, 2 \times 10^{-6}}, PlotRange -> All]
\end{verbatim}

7.30157 \times 10^6 (20 + deltaL)

7.29843 \times 10^6 (20 + deltaL)
Compare this with the result for section 7.1, where there were two peaks in the y-output. The 2nd resonance in the y-eigenmode has been effectively squelched by the attenuation in that mode [2]. The only component in the y-output is simply the x-polarization circulating field cross-coupled to the output fiber by the coupler. This is not a problem because it can be removed with a polarizer outside of the ring.

Now, look at the circulating intensity for both polarizations. Indeed, the circulating x-intensity looks almost exactly like the y-output intensity as hypothesized.
The y-resonant mode has been suppressed on the order of 30 dB and does not show up in the x-polarized output.

Now consider the same 20° input polarization misalignment with this type of ring.
Launch = {0.940, 0.342, 0, 0, 0, 0, 0, 0, 0, 0, 0};
fields = SIGI . Launch;

xout = Chop[Abs[fields[[5]]]]^2;
yout = Chop[Abs[fields[[6]]]]^2;

Plot[xout, {deltaL, 0, 2 10^-6}, PlotPoints -> 500,
PlotRange -> All]
Plot[yout, {deltaL, 0, 2 10^-6}, PlotRange -> All]

Even with deliberate launching of light into the y-polarized mode, the undesired resonance is still
suppressed, except for a DC component.
\section*{7.4 90° Spliced Resonators}

Now examine the case for a resonator made from PM fiber with an intentional 90° splice misalignment [3]. The splice is assumed to be ideal - note the changes in the S matrix.

\begin{verbatim}
a = .
bx = .
by = .
S = 
  {0,0,0,0, kt, ct, kr, cr, 0,0,0,0},
  {0,0,0,0, ct, X[kt], -cr, -X[kr], 0,0,0,0},
  {0,0,0,0, kr, -cr, kt, -ct, 0,0,0,0},
  {0,0,0,0, -ct, -X[kr], -ct, X[kt], 0,0,0,0},
  {kt, ct, kr, -ct, 0,0,0,0, 0,0,0,0},
  {ct, X[kt], -cr, -X[kr], 0,0,0,0, 0,0,0,0},
  {kr, -cr, kt, -ct, 0,0,0,0, 0,0,0,0},
  {cr, -X[kr], -ct, X[kt], 0,0,0,0, 0,0,0,0},
  {0,0,0,0,0,0,0,0, a E^(I bx)},
  {0,0,0,0,0,0,0,0, a E^(I by), 0},
  {0,0,0,0,0,0,0,0, a E^(I bx), 0, 0},
  {0,0,0,0,0,0,0,0, a E^(I by), 0, 0};
\end{verbatim}

The light propagates through the cavity, then changes from x- to y-polarization mode, then traverses the cavity a second time before interfering at the coupler. Calculating and plotting:
bx = .
by = .
xring = \{z \rightarrow Lc, a \rightarrow \text{Sqrt}[0.9]\}
SI = \text{Inverse}[S / . \text{xring}];
SIG = SI - G;

SIGI = \text{Inverse}[SIG];
Launch = \{1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\};
fields = SIGI . Launch;

xout = \text{Chop}[\text{Abs}[\text{fields[[5]]}]]^2;
yout = \text{Chop}[\text{Abs}[\text{fields[[6]]}]]^2;

L = 20 + \Delta L;
B = 7.3 \times 10^6;
bx = (B + N[\frac{\pi}{0.002}]) L;
by = (B - N[\frac{\pi}{0.002}]) L;
Plot[xout, \{\Delta L, 0, 2 \times 10^{-6}\}, \text{PlotPoints} \rightarrow 500,
         \text{PlotRange} \rightarrow \{0,1\}]
Plot[yout, \{\Delta L, 0, 2 \times 10^{-6}\}, \text{PlotRange} \rightarrow \{0,1\}]
\{z \rightarrow 0.00033, a \rightarrow 0.948683\}
Note that there are twice as many dips for 2 microns of fiber length change, because the ring is effectively twice as long. There is also twice the loss, all other things being equal. If no polarizer is present at the output, both x- and y-polarizations are viewed simultaneously by the detector.

\[
\text{combo} = xout + yout;
\]
\[
\text{Plot}[\text{combo}, \{\text{deltaL}, 0, 2\ 10^{-6}\}, \text{PlotPoints} \to 100, \text{PlotRange} \to \{0,1\}]
\]

And the dip depth is seen to be reduced to about 50%, as seen experimentally.
7.5 References


In this model, we will examine the operation of the acousto-optic principal axis alignment system, as originally described by Carrara et. al. The results from the derivation in Chapter 4 for an ideal squeezer system model will be plotted and discussed.

### 8.1 Principal Axis Rotation During Squeezing

These calculations are for a squeezer which compresses the fiber along a line at an angle $\theta_s$ with respect to slow axis, which imparts a birefringence $B_E$ into the fiber. The stress rods impart an internal birefringence $B_I$, and the sum of the two is called the total birefringence $B_T$ which was shown to be given by the expressions [1]

$$ B_T = \sqrt{B_I^2 + B_E^2 - 2B_I B_E \cos 2\theta_s} $$

$$ \Phi = \frac{1}{2} \tan^{-1} \left( \frac{B_E \sin (2\theta_s)}{B_I - B_E \cos (2\theta_s)} \right) $$

where $\Phi$ is the amount of rotation that the principal axes experience during squeezing. The intrinsic birefringence is set to 0.00065 which is equivalent to 2mm beat length, at a 1.3-micron wavelength. The intrinsic and extrinsic birefringence values are calculated [2]

$$ B_I = \frac{\lambda}{L_B} $$
\[ B_E = 0.00025 \text{ F/L}_2 \]

where the coefficients have been lumped for the second equation. In this case, \( F \) is the squeezer force in kg, the opto-elastic constants are measured at 1.3 microns, and the fiber is 125 microns in diameter. \( L_2 \) should be in millimeters to get the units to work out in the lower equation.

\[
DA = 0.5 \arctan\left(\frac{B_E \sin[2 \ t \ \text{Degree}]}{B_I - B_E \cos[2 \ t \ \text{Degree}]})\right)
\]

\( \text{expl} = \{B_I \to 0.00065, \ B_E \to 5 \times 10^{-4}\} \)

\[
\text{newAngle} = \frac{(t \ \text{Degree} + DA)}{\text{Degree}};
\]

\[ \text{Plot\{\{newAngle /. \text{expl}, t\}, \{t, -10, 370\}, \text{Frame} \to \text{True}, \text{AspectRatio} \to 0.95\} \]

\[
0.5 \arctan\left(\frac{B_E \sin[2 \ \text{Degree} \ t]}{B_I - B_E \cos[2 \ \text{Degree} \ t]}\right)
\]

\( \{B_I \to 0.00065, \ B_E \to 0.0005\} \)
In the above plot of principal axis angle as a function of squeezer angle, the straight line is the case for no squeezing and should have slope $= 1$. Note that with a squeezer-induced birefringence of $0.0005$, the optical principal axes rotate away about $\pm 15^\circ$ from their unperturbed state. Now plot the perturbed birefringence as a function of squeezer angle.

$$\text{BT} = \sqrt{B_{11}^2 + B_{22}^2 - 2 B_{11} B_{22} \cos[2 \theta \text{ Degree}]}$$

\[
\text{BT} /. \ \text{expl}; \\
\text{Plot}[%, \{t, -10, 370\}, \text{Frame} \rightarrow \text{True}, \\
\text{PlotRange} \rightarrow \{0, 0.0014\}]
\]

The above plot of birefringence as a function of squeezer angle for the same conditions.

Increasing the squeezing force so it almost matches the birefringence of the fiber results in cancellation of the birefringence when squeezing force is parallel to the slow axis.

$$\text{exp2} = \{\text{BI} \rightarrow 0.00065, \ B_{11} \rightarrow 0.00064\}$$

\[
\text{Plot}[\{\text{newAngle} /. \ \text{exp2}, \ t\}, \{t, -10, 370\}, \text{Frame} \rightarrow \text{True}, \\
\text{AspectRatio} \rightarrow 0.95]
\]

\[
\text{Plot}[\text{BT} /. \ \text{exp2}, \{t, -10, 370\}, \text{Frame} \rightarrow \text{True}, \\
\text{PlotRange} \rightarrow \{0, 0.0014\}]
\]
\{BI \rightarrow 0.00065, BE \rightarrow 0.00064\}

- Graphics -

- Graphics -

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In the above plot of $B_T$ as a function of $\Phi$ when the squeezer-induced birefringence is near the fiber's birefringence value, cancellation is observed every 180° or rotation. At those points, the angle of the principal axes are seen to rotate rapidly with respect to $\Phi$.

### 8.2 AC Output of Alignment System

For linear polarization launched into the slow axis, and for the analyzer aligned to the fast axis, the DC and AC intensity as a function of birefringence and squeezer angle are given

\[
I_{DC} = \frac{B_E^2}{B_T^2} \sin^2(2\theta_S) \sin^2(k_oB_T L/2)
\]

\[
I_{AC} = \frac{2B_E B_{AC}}{B_T^2} \sin^2(2\theta_S) \sin^2(k_oB_T L/2)
\]

The optical intensity output of the squeezer system will be now plotted as a function of constant force upon the squeezer, for a fixed squeezer length, at 1.3 micron wavelength.

\[
IDC = (B_E^2 \sin[2 \text{ t Degree}]^2 / B_T^2) \sin[ko L BT]^2;
\]

\[
IAC = (2 B_E B_{AC} \sin[2 \text{ t Degree}]^2 / B_T^2) \sin[ko L BT]^2;
\]

\[
constant1 = \{ko -> 4883, L -> 0.5, BAC -> 0.0001, BI -> 0.00065\}
\]

\[
ACsignal = IAC /. constant1
\]

\[
Plot[ACsignal /. BE -> 0.0002, \{t, -10, 370\}, PlotPoints -> 100, Frame -> True]
\]

\[
\{ko -> 4883, L -> 0.5, BAC -> 0.0001, BI -> 0.00065\}
\]
\[
(0.0002 \ \text{BE} \ \sin(2 \ \text{Degree} \ t))^2 \\
\text{Power}[(\sin(2441.5 \ \sqrt{4.225 \times 10^{-7} + \text{BE}^2 - 0.0013 \ \text{BE} \ \cos(2 \ \text{Degree} \ t)})], \ 2)] / \\
(4.225 \times 10^{-7} + \text{BE}^2 - 0.0013 \ \text{BE} \ \cos(2 \ \text{Degree} \ t))
\]

The above plot is the AC intensity output for the squeezer system modeled in Chapter 4, for 2 mm beat length, and AC birefringence modulation of 100 ppm. As the fiber is rotated in the squeezer, this is the characteristic signal for the squeezer system, as demodulated with a lock-in amplifier. The maxima are near \(\pm 45^\circ\) from alignment; by seeking the narrower of the two nulls, one can align the squeezer to the slow axis of the fiber. If the force is increased to the higher level the wider of the nulls flattens out:

\[
\text{Plot[ACsignal /. BE -> 0.00064, \{t, -10, 370\},}
\text{PlotPoints -> 100, Frame -> True]}
\]
Both of the above plots are simulations for a 0.5 mm squeezed length of fiber, which is shorter than the beat length. Now we will plot for a longer squeezer, having a 2.5 mm interaction length. Note the spurious nulls which would make it difficult to locate the correct null which would indicate an alignment condition.

\[
\text{constant2} = \{\text{ko} \to 4883, \text{L} \to 2.5, \\
\hspace{1cm} \text{BAC} \to 0.0001, \text{BI} \to 0.00065\}
\]

\[
\text{ACsignal2} = \text{IAC} / . \text{constant2}
\]

\[
\text{Plot}[\text{ACsignal2} / . \text{BE} \to 0.00064, \{t, -10, 370\}, \\
\hspace{1cm} \text{PlotPoints} \to 100, \text{Frame} \to \text{True}]
\]

\[
\{\text{ko} \to 4883, \text{L} \to 2.5, \text{BAC} \to 0.0001, \text{BI} \to 0.00065\}
\]

\[
(0.0002 \text{ BE Sin}[2 \text{ Degree } t]^2 \\
\text{Power}[\text{Sin}[12207.5 \text{ Sqrt}[4.225 \times 10^{-7} + \text{BE}^2 - \hspace{1cm} 0.0013 \text{ BE Cos}[2 \text{ Degree } t]]], 2)] / \\
(4.225 \times 10^{-7} + \text{BE}^2 - 0.0013 \text{ BE Cos}[2 \text{ Degree } t])
\]
This is an important situation to avoid when designing principal axis alignment systems. The simplest way is to design the squeezer such that it perturbs only a short length of fiber, less than the shortest beat length fiber that one is likely to encounter.

### 8.3 Sensitivity as a Function of Force

The optical intensity output of the squeezer system will be now plotted as a function of constant squeezer angle, with various forces on the squeezer.

\[
\text{Plot}[\text{ACsignal} /. \text{t} \to 10, \{\text{BE}, 0, 0.003\}]
\]

\[
\text{Plot}[\text{ACsignal} /. \text{t} \to 30, \{\text{BE}, 0, 0.003\}]
\]

\[
\text{Plot}[\text{ACsignal} /. \text{t} \to 45, \{\text{BE}, 0, 0.003\}]
\]
It can bee seen that the best sensitivity occurs at the lower force settings. These graphs are plotted in 3D to give a better visualization of the intensity/force/squeezer angle relationship.

\[
\text{Plot3D}[\text{ACsignal, \{t, -10, 370\}, \{\text{BE}, 0, 0.003\}, \\
\text{PlotPoints \to 50}]}
\]

The vertical (z) scale is intensity, and the lefthand (x) axis is squeezer angle, and the y-axis is the extrinsic birefringence from the squeezer. This data can be replotted using a density plot, which better illustrates the force-angle dependence. The brightness is proportional to the output AC signal, with \(\Phi\) horizontal and \(B_E\) on the vertical axis.
DensityPlot[ACsignal, {t, -10, 370}, {BE, 0, 0.003}, PlotPoints -> 100]

Now, returning to the 2.5 mm long squeezer, and replotting the AC response as a function of BE at $\Phi = 10^\circ$ and $30^\circ$ from alignment, shows a more complex dependence:

Plot[ACsignal2 /. t -> 10, {BE, 0, 0.003}]
Plot[ACsignal2 /. t -> 30, {BE, 0, 0.003}]

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Again, do the density plot (this takes quite a while to calculate and render).

\[
\text{DensityPlot}[\text{ACsignal2}, \{t, -10, 370\}, \{\text{BE}, 0, 0.003\}, \\
\text{PlotPoints} \to 200]
\]
Which illustrates where the additional nulls come from. The optimum squeezing force for maximum sensitivity is a strong function of the squeezer angle for long squeezer.

\section{8.4 Optimum Force Operation}

The optimum external birefringence for a given set of curves was calculated to be

\[
\frac{B_0}{B_1} = \cos^2 \theta_s + \sqrt{1 + \cos^2 2 \theta_s + \left( \frac{(2m-1)L_B}{L_2} \right)^2}
\]

this is now plotted as a function of m, beat length, and squeezer length for the short squeezer. In this case m=1 and m=2 will be examined for the 0.5 mm long squeezer.

\[
R = \left(2m - 1\right) L_B / (2L)
\]

\[
B_0 = B_1 (\cos[2 \text{Degree} t] + \sqrt{1 + \cos[2 \text{Degree} t]^2 + R^2})
\]

\[
LB \frac{(-1 + 2m)}{2L}
\]

\[
B_1 (\cos[2 \text{Degree} t] + \sqrt{1 + \frac{L_B^2 (-1 + 2m)^2}{4L^2}} + \cos[2 \text{Degree} t]^2))
\]

\[
\text{curvem1} = \{B_1 \to 0.00065, L_B \to 2, L \to 0.5, m \to 1\}
\]

\[
\text{curvem2} = \{B_1 \to 0.00065, L_B \to 2, L \to 0.5, m \to 2\}
\]

\[
\text{Plot}\{\{B_0 /. \text{curvem1}, B_0 /. \text{curvem2}\}, \{t, -10, 370\}, \text{PlotRange} \to \{0, 0.005\}\}
\]

\[
\{B_1 \to 0.00065, L_B \to 2, L \to 0.5, m \to 1\}
\]
In the above plot of optimum birefringence as a function of squeezer angle, it is seen to periodically vary, being higher at the slow-axis alignment and lower at the fast axis case.

Now, do the same plot for the longer squeezer case, showing the vide variation $B_{opt}$.

```math
\text{curveM1} = \{\text{BI} \to 0.00065, \text{LB} \to 2, \text{L} \to 2.5, \text{m} \to 1\}
\text{curveM2} = \{\text{BI} \to 0.00065, \text{LB} \to 2, \text{L} \to 2.5, \text{m} \to 2\}
```

```math
\text{Plot}[\{\text{BO} /. \text{curveM1}, \text{BO} /. \text{curveM2}\}, \{t, -10, 370\},
\text{PlotRange} \to \{0, 0.002\}]
\{\text{BI} \to 0.00065, \text{LB} \to 2, \text{L} \to 2.5, \text{m} \to 1\}
\{\text{BI} \to 0.00065, \text{LB} \to 2, \text{L} \to 2.5, \text{m} \to 2\}
```
We were able to get useful characteristic curves for a short squeezer with constant force, because there were no overlaps in the different m-ordered $B_{opt}$. Plotting with $B_E = B_{opt}$

constant3 = {ko -> 4883, L -> 2.5, BAC -> 0.0001, 
             BI -> 0.00065, LB -> 2, L -> 2.5, m -> 1}

BT = Sqrt[BI^2 + BE^2 - 2 BI BE Cos[2 t Degree]]; 
IAC = (2 BE BAC Sin[2 t Degree]^2 / BT^2) Sin[ko L BT]^2; 
IACopt = IAC /. BE -> BO; 
ACsignal3 = IACo = constant3; 

Plot[ACsignal3, {t, -10, 370}, 
     PlotPoints -> 100, Frame -> True]

{ko -> 4883, L -> 2.5, BAC -> 0.0001, BI -> 0.00065, 
   LB -> 2, L -> 2.5, m -> 1}
By optimizing the applied force, a useful characteristic curve can be obtained with a longer squeezer, which reduces stress in the fiber and lowers the chance of breakage. One must either a priori know the $\Phi$, or iteratively (or with a servo) lock onto the same m-order optimum force curve during the measurements.

### 8.5 Squeezer Design

In this section, the design of the squeezer jaws will be explored, as it has been established that it is important to control the length of the squeezed region. The empirical formula for calculation of the elliptical Hertzian contact region between two cylinders which meet at right angles will be used, to estimate the squeezed length of fiber.
The expression for the major and minor axis lengths of the elliptical contact region as given by [3] for the above situation, where the fiber diameter is 2R₁, the jaw radius is R₂:

\[ A = \alpha \left( \frac{3 FR_e}{2 E_e} \right)^{1/3} \]

and the expression for the minor axis is

\[ B = \beta \left( \frac{3 FR_e}{2 E_e} \right)^{1/3} \]

where the variables are defined as

\[ \frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} \]

and the effective modulus is given by

\[ E_e = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \]

Poisson's ratio and Young's modulus

We will now examine the functional dependence of the \( \alpha \) coefficient to estimate the squeezed length of fiber during the principal axis alignment. First, define the effective radius:

\[
\text{RE} = 1 / \left( \frac{1}{R_1} + \frac{1}{R_2} \right)
\]

\[
\text{jaws} = \{ R_1 \to 0.0625, R_2 \to 100 \}
\]

\[
\frac{1}{\text{RE} \text{/ . jaws}} \quad \text{bigratio} = \frac{R_2}{R_1 \text{/ . jaws}}
\]

\[
\frac{1}{R_1} + \frac{1}{R_2}
\]

\[
\{ R_1 \to 0.0625, R_2 \to 100 \}
\]

0.062461

1600.

Using the 5th-order polynomial fit to obtain \( \alpha \) and \( \beta \), as described on page 232 in [3]:

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\[
a = -4522790.6 + 24146275.9 f - 51557740 f^2 + 55036391 f^3 - 29371139 f^4 + 6269014.53 f^5
\]

\[
b = 51254.01 - 273306.27 f + 582926 f^2 - 621625 f^3 + 331436.01 f^4 - 70684.52 f^5
\]

\[
\cos f = \text{RE} \sqrt{R1^{-2} + R2^{-2} - \frac{2}{R1 R2}}
\]

\[
\alpha = a /\cdot f \to \%
\]

\[
\beta = b /\cdot f \to \%
\]

\[
\cos f /\cdot \text{jaws}
\]

\[
\alpha /\cdot \text{jaws}
\]

\[
\beta /\cdot \text{jaws}
\]

\[
-4.52279 10^6 + 2.41463 10^7 f - 51557740 f^2 + 55036391 f^3 - 29371139 f^4 + 6.26901 10^6 f^5
\]

\[
51254. - 273306. f + 582926 f^2 - 621625 f^3 + 331436. f^4 - 70684.5 f^5
\]

\[
\sqrt{R1^{-2} + R2^{-2} - \frac{2}{R1 R2}} = \frac{1}{R1} + \frac{1}{R2}
\]

General::spell1:
Possible spelling error: new symbol name "beta"
is similar to existing symbol "Beta".

0.998751

11.246

0.239509

It should be noted that there is approximately a 8% overestimate for \( \alpha \) for large ratios, when comparing to tabulated data. Whether this approach is valid in extrapolating out to such high diameter ratios is valid question, as the table only goes to ratios of 100:1.
alpha = 0.92 a /. f -> cosf;

fib = R1 -> 0.125 / 2
Plot[cosf /. fib, {R2, 1, 100}, PlotRange -> All]
Plot[alpha /. fib, {R2, 1, 100}, PlotRange -> All]
Plot[beta /. fib, {R2, 1, 100}, PlotRange -> All]
R1 -> 0.0625
which agrees reasonably well with the tabulated values. For now we will assume that the function can be extrapolated for larger ratios in the 100 to 1000 range. The effective Young's modulus is computed from the mechanical properties of the materials:

\[
\begin{align*}
glass: & \quad E_1 = 7.3 \times 10^6 \text{ kg/mm}^2 \quad v_1 = 0.17 \\
stainless: & \quad E_2 = 2.0 \times 10^4 \text{ kg/mm}^2 \quad v_2 = 0.12 \\
\ee & = \frac{(1 - 0.17^2)}{(7.3 \times 10^6)} + \frac{(1 - 0.12^2)}{(2 \times 10^4)} \\
A & = (\alpha \left(3 \frac{F \cdot RE}{(2 \ee)^{0.333}} \right))^{0.333} / \text{fib}; \\
B & = (\beta \left(3 \frac{F \cdot RE}{(2 \ee)^{0.333}} \right))^{0.333} / \text{fib}; \\
& 0.000049413 \\
\end{align*}
\]

Finally, the squeezed length as a function of squeezer jaw radius is examined for a 1 kg applied force to the jaws. This needs to be verified by FEM or experiment.

\[
\begin{align*}
\text{force} & = F \to 1 \\
\text{Plot}[2 \ A \ / \ . \ \text{force}, \ \{R2, \ 1, \ 100\}, \ \text{PlotRange} \to \text{All}] \\
F & \to 1
\end{align*}
\]
The vertical scale in the above two plots are in microns, and the horizontal scale is mm.

Our squeezer has a jaw radius of 100 mm, and this situation is modeled as a function of applied force, where the cube root dependence can be observed.
For forces up to 10 kg, the squeezed length is predicted to be less than 2 mm, which it a typical value for high-performance birefringent fiber. A constant DC force can be used.

8.6 References


"A thesis is never finished; it is abandoned."

- Prof. Charles Kittel