MICROWAVE BEAM-SHAPING ANTENNAS

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by

L. J. Chu

Abstract

A simple procedure for designing microwave beam-shaping antennas is described. The power from a line or point source is reflected by a reflector of a special design so that it is distributed in a prescribed fashion in space. Simple laws of geometric optics are used. Some typical experimental results are given. This work was started in the Radiation Laboratory, Massachusetts Institute of Technology, in 1943.

MICROWAVE BEAM-SHAPING ANTENNAS

1. Introduction

The range of a radio communication system varies angularly with the field intensity distribution of the radiation pattern of the antenna. It is often desirable to distribute the radiated power in a prescribed fashion such that there will be enough intensity available in all desired directions for detection at the appropriate range. This paper describes a scheme for designing this type of antenna at microwave frequencies where the wavelength is short compared to the physical dimensions of the antenna. Since the fundamental scheme was conceived in the early part of 1943, many modifications and extensions have been worked out by members of the Radiation Laboratory, M.I.T., and others. For completeness, the latter will also be discussed in this paper.

The method is based upon the principles in geometric optics concerning the reflection and transmission of waves by curved surfaces. The application of these principles to microwave antennas is justified provided we are dealing with large antennas as well as divergent waves. Otherwise the diffraction effect must be taken into consideration.

2. Line Sources and Cylindrical Reflectors

In this section we shall discuss the design procedure of cylindrical reflectors which will focus the power from a line source into a shaped fan beam. A broadside linear array or other types of line sources can be used provided the amplitude and phase are fairly constant over the length of the aperture. This type of radiator gives a cylindrical wavefront in its neighborhood. Over the main portion of the wavefront, it appears that the power originates from a geometric line. This allows us to treat the problem as a two-dimensional one.

Let $I(\Psi)$ be the power per unit angle radiated from the line source and $I(\theta)$ be the power per unit angle desired from the two-dimensional antenna system. The angles Ψ and θ are measured from a reference axis as in Fig. 1.
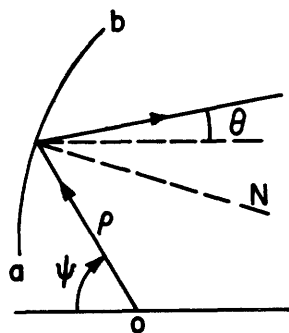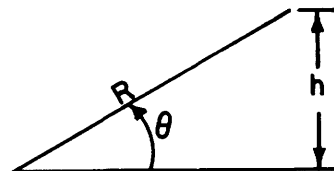

Fig. 1   Fig. 2

The line source is located at the origin O. The problem is to design a cylindrical reflector which will reflect the power from the line source in a prescribed angular distribution in the plane normal to the line source.

Let curve ab be the reflector desired and (ρ, Ψ) be the polar coordinates of a point. Let N be the normal to the surface at that point. From the theorem in geometric optics that the angle of incidence is equal to the angle of reflection, and from the relation

between the normal and the slope of a curve, we have

$$\frac{d\rho}{\rho d\Psi} = \tan \tfrac{1}{2}(\Psi+\Theta).$$ (1)

The functional relationship between Ψ and Θ is supplied by the power relationship

$$I(\Psi)d\Psi = I(\Theta)d\Theta .$$ (2)

It says that the power within a small angle $d\Psi$ from the line source must be equal to the power within the corresponding angle $d\Theta$ in space. The last equation is subjected to the condition that the total power reflected by the reflector must be equal to the power intercepted by the reflector.

$$\int_{\Psi_1}^{\Psi_2} I(\Psi)d\Psi = \int_{\Theta_1}^{\Theta_2} I(\Theta)d\Theta$$ (3)

where Ψ_1, Ψ_2, Θ_1, and Θ_2 are the limiting angles of Ψ and Θ, respectively. By integrating Eq. (2) and subjecting the integral to the condition of Eq. (3), we have

$$\frac{\int_{\Psi_1}^{\Psi} I(\Psi)d\Psi}{\int_{\Psi_1}^{\Psi_2} I(\Psi)d\Psi} = \frac{\int_{\Theta_1}^{\Theta} I(\Theta)d\Theta}{\int_{\Theta_1}^{\Theta_2} I(\Theta)d\Theta} .$$ (4)

The angle Θ can be solved in terms of Ψ either graphically or analytically. The coordinates of the reflector can then be obtained by integrating Eq. (1)

$$\frac{\rho}{\rho_1} = \exp \int_{\Psi_1}^{\Psi} \tan \frac{\Psi+\Theta}{2} d\Psi$$ (5)

where ρ_1 is the distance from the line source to the end of the reflector (ρ,Ψ_1).

As a simple example, we shall specify that all the power from the line source be reflected in the direction Θ. Then Θ in Eq. (1) is equal to Θ_1. By integrating Eq. (5) we obtain

$$\rho = \frac{2f}{1 + \cos(\Psi+\Theta_1)} ,$$ (6)

the equation of a parabola of focal length f. Obviously, this is a limiting case where the wave is no longer divergent. For a parabolic cylinder of finite size, the field distribution in the neighborhood of the reflector will approximately satisfy the required condition. Away from the reflector, we must expect to have a diffraction pattern consistent with the size and shape of the reflector, the wavelength, as well as the primary angular distribution $I(\Psi)$ of the line source.

The most common type of beam shaping is used in connection with aircraft communication. It is often desired for the aircraft receiver to get a constant power level from a ground transmitting station when the aircraft is flying at a constant height. Let h be the height of the aircraft above a flat earth and R the distance from the transmitting station to the aircraft as shown in Fig. 2. Then $R = h \csc \Theta$. In order to satisfy the above requirement we must have

$$I(\Theta) = A \csc^2\Theta$$ (7)

where A is a constant. The angle Θ varies from a value Θ_1, greater than zero, to Θ_2.

From Eq. (4) we obtain

$$\frac{\cot \Theta_1 - \cot \Theta}{\cot \Theta_1 - \cot \Theta_2} = \frac{\int_{\Psi_1}^{\Psi} I(\Psi) d\Psi}{\int_{\Psi_1}^{\Psi_2} I(\Psi) d\Psi}$$

or

$$\Theta = \cot^{-1}\left[(\cot \Theta_2 - \cot \Theta_1)\frac{\int_{\Psi_1}^{\Psi} I(\Psi) d\Psi}{\int_{\Psi_1}^{\Psi_2} I(\Psi) d\Psi} + \cot \Theta_1\right]. \tag{8}$$

The reflector surface can be computed from Eqs. (5) and (8).
If $I(\Psi)$ is constant, we have

$$\Theta = \cot^{-1}[(\cot \Theta_2 - \cot \Theta_1)\frac{\Psi - \Psi_1}{\Psi_2 - \Psi_1} + \cot \Theta_1]; \tag{9}$$

and from Eq. (5) the equation of the surface is given by

$$\rho = \rho_1 \exp \int_{\Psi_1}^{\Psi} \tan \tfrac{1}{2}\left[\Psi + \cot^{-1}\left\{(\cot \Theta_2 - \cot \Theta_1)\frac{\Psi - \Psi_1}{\Psi_2 - \Psi_1} + \cot \Theta_1\right\}\right] d\Psi \tag{10}$$

where ρ_1 is the distance from the line source to one end of the reflector corresponding to Θ_1.

Equation (10) is given only as an example to illustrate this procedure. We observe that for a given primary pattern $I(\Psi)$ and a given secondary pattern $I(\Theta)$, Eq. (10) represents a family of curves all identical in shape. The portion of the reflector contributing to the peak of $I(\Theta)$ is essentially a parabola.

In practice, the function $I(\Psi)$ is specified by the design of the line source. It is not possible to have a steep cutoff of $I(\Theta)$ at the peak as specified in the example, because of the diffraction. The function $I(\Theta)$ must be modified to a form more consistent with the characteristics of a conventional narrow beam antenna. The peak must be rounded off and the half-power beam width of $I(\Theta)$ must be consistent with the wavelength and the dimension of that portion of the aperture responsible for the peak.

3. Design Considerations

The beam characteristics in the plane parallel to the line source are primarily governed by the phase and amplitude distributions along the line source. We expect to have a sharp beam with reasonable side lobes, in that plane.

In the plane where the beam shaping is intended, the pattern will be distorted from the desired smooth one by the following reasons:

1. Direct radiation from the line source.
2. Scattering of the reflected wave by the structure of the line source.
3. Diffraction effect.

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Because of the linear construction, the power radiated directly into space from the line source as well as that scattered by it will be concentrated in the form of a fan beam, coinciding with the reflected beam. The former will be added to the reflected beam with a varying phase to form a series of maxima and minima in the radiation pattern. It is essential to confine nearly all the power radiated from the line source within the angles Ψ_1 and Ψ_2. The reflector should be arranged in such a way so that the reflected rays clear the reflector by an appropriate distance.

Were the radius of curvature of the reflector almost infinite compared to the wavelength, there would be no diffraction except for the knife-edge effect at the edge. For an antenna of finite size, geometrical optics cannot be used to calculate the radiation pattern precisely. We must first calculate, as accurately as possible, the current induced on the reflector and then figure out the radiation pattern by the standard method.[1] The distortion due to diffraction shows up again as a series of minor maxima and minima in the radiation pattern. This distortion may be reduced by using a larger reflector and tapering the intensity towards the ends of the reflector.

There are two types of reflectors we can use. If the ratio $d\Theta/d\Psi$ is positive, the reflector will have less curvature than the corresponding parabola. In this case the rays reflected from the surface will not cross each other and will be strictly divergent in nature. If the ratio $d\Theta/d\Psi$ is negative, the bottom portion of the reflector will reflect power upward, while the top portion will reflect power downward. This arrangement is usually more economical in space and allows for more angular coverage in Θ.

The gain of this type of antenna is approximately given by the following formula:

$$G(\Theta) \cong g \frac{1.1 \times 4\pi \, I(\Theta)}{\phi_0 \int_0^{2\pi} I(\Theta) d\Theta} \tag{11}$$

where $I(\Theta)$ is the power intensity measured in the plane normal to the line source, ϕ_0 is the half-power width in the plane parallel to the line source, and g is a numerical factor (about $\frac{1}{2}$) to take care of incidental losses. The gain at the peak is radically lower than that of an antenna of equal aperture designed to produce a pencil beam. Part of the radiated power is diverted into the shaped part of the beam. Also, only a fraction of the aperture is used for the peaked portion of the beam; as a consequennce a broader beam is produced at the peak.

Figure 3 illustrates an antenna designed with the above procedure and Fig. 4. shows the experimental result. A pillbox is used as the line source. It is essentially a cylindrical parabolic reflector and a waveguide feed confined between two parallel metallic plates spaced less than half wavelength apart. The power emerges from the flared aperture with a constant phase across. The pillbox is pointed toward the center of the reflector and $I(\Psi)$ at the edges is about 10 db down from the peak. The value

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1. T. J. Keary has gone through the procedure outlined here and the calculated results checked surprisingly well with the experimental curves.

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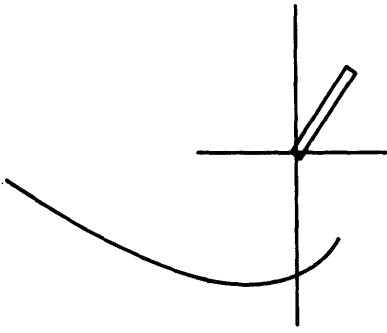
Fig. 3

ELEVATION ANGLE IN DEGREES
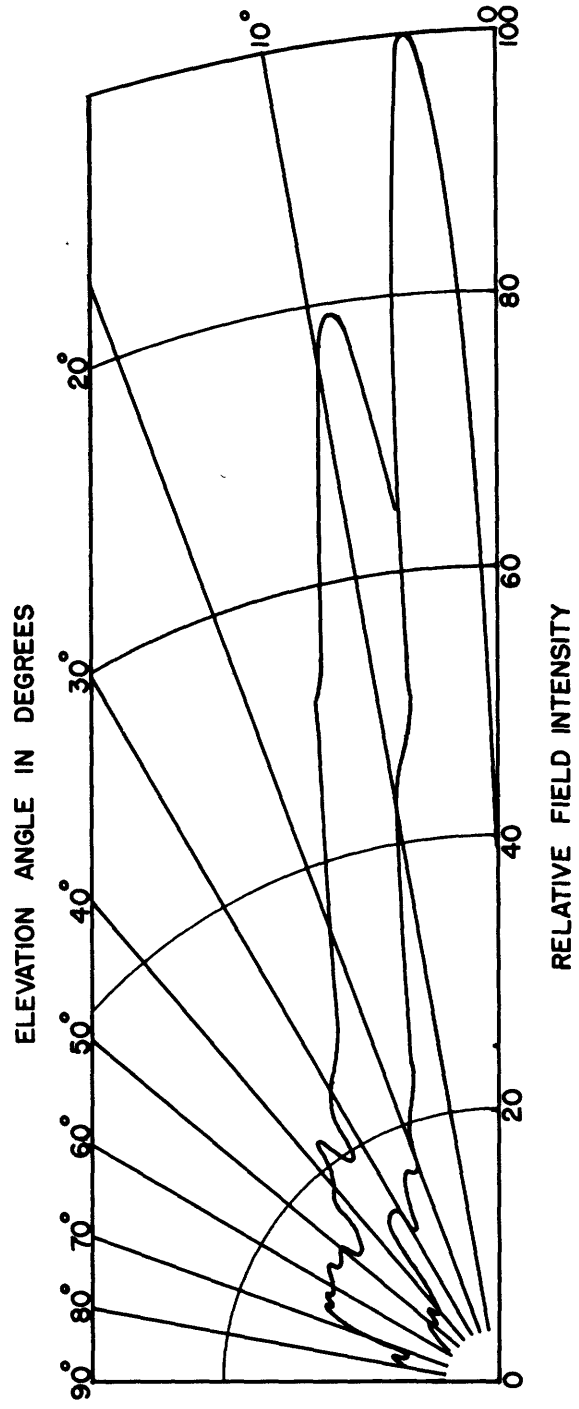
RELATIVE FIELD INTENSITY

Fig. 4

of $d\theta/d\Psi$ is negative. The bottom of the reflector reflects power skyward over a wide angle. Since $I(\Psi)$ here is a slowly varying function of Ψ, the cosecant pattern can be adjusted slightly for different altitudes by rotating the pillbox about its aperture as shown in Fig. 4. The deviation of the pattern from a true cosecant pattern is less than 1 db. The antenna was designed by W. Sichak.

4. Line Source and Lens

The principle outlined above was applied by A. S. Dunbar to a line source and lens combination to obtain a shaped beam. For simplicity, we shall consider the case where the source is imbedded in the dielectric. The ray from the source making an angle Ψ with the reference axis will emerge from the dielectric in the direction θ (Fig. 5). By Snell's law, we have,

$$\frac{\sin i}{\sin r} = \frac{1}{n} \tag{12}$$

where i = angle of incidence

r = angle of refraction

and n = index of refraction of the dielectric medium. The angle between the normal and the reference axis is given by

$$\Psi - i = \theta - r. \tag{13}$$

By eliminating r from Eqs. (12) and (13), we have

$$\tan i = \frac{\sin(\theta - \Psi)}{n - \cos(\theta - \Psi)} . \tag{14}$$

Let (ρ, Ψ) be the polar coordinates of the lens surface. It is obvious that

$$\frac{d\rho}{\rho d\Psi} = \tan i = \frac{\sin(\theta - \Psi)}{n - \cos(\theta - \Psi)} \tag{15}$$

and

$$\frac{\rho}{\rho_o} = \exp \int_{\Psi_o}^{\Psi} \frac{\sin(\theta - \Psi)}{n - \cos(\theta - \Psi)} \, d\Psi \tag{16}$$

The functional relationship between θ and Ψ is supplied by the energy condition (Eq. 4). If the primary pattern of the feed and the desired secondary pattern are known, the lens surface can be calculated graphically from Eq. (16).

For the ordinary dielectric medium, the index of refraction is greater than unity. If we use a metallic lens[1] made of equally spaced thin metallic sheets perpendicular to the line source, with polarization parallel to the sheets, the medium will have an effective index of refraction

$$n = \sqrt{1 - \left(\frac{\lambda}{2a}\right)^2} \tag{17}$$

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1. W. E. Kock, "Metal Lens Antennas", Proc. I.R.E., _34_, 828, (1946).

where $\underline{a}$ is the spacing of the parallel sheets, and is subject to the condition $\frac{\lambda}{2} < a < \lambda$. Figure 6 shows the vertical pattern of a mettalic-lens antenna with a pillbox as the line source. The lens has an effective index of refraction equal to 0.5 and is shaped as shown in Fig. 5. The pattern deviates from a cosecant one by less than $\pm$ 1.5 db from 7° to 80°.
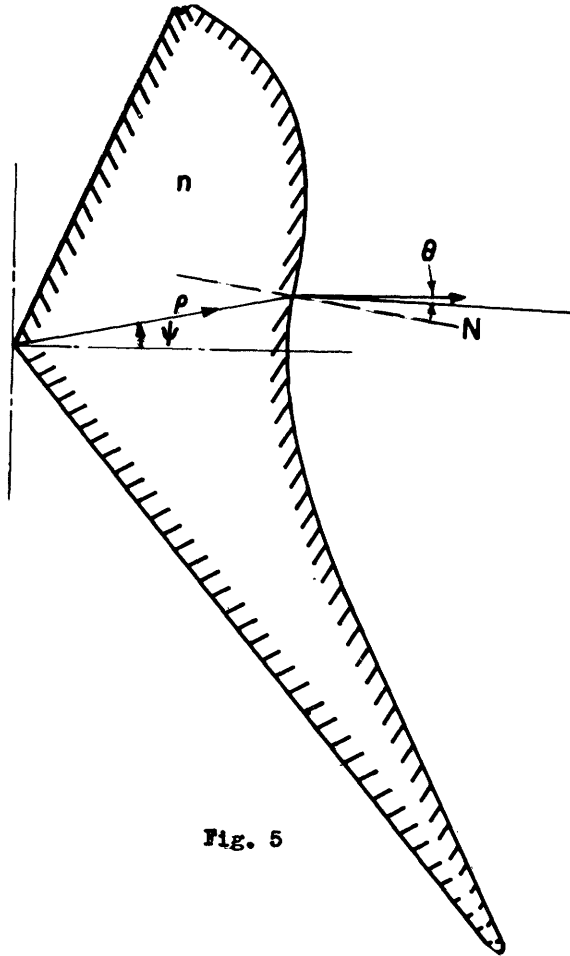

Fig. 5

5. Point Sources and Double-Curvature Reflectors

To design a reflector for converting a broad spherical wave from a point source into a beam, narrow in one plane but shaped in the other plane, we have to investigate first some of the characteristics of a paraboloid. By construction, a paraboloid has the property of focusing the divergent rays of a point source into rays all parallel to the axis of the paraboloid. If we choose arbitrarily a narrow strip on the paraboloid, this strip will have the same property as that of a straight strip parallel to a line source in the sense that the reflected rays are parallel. We shall impose the further restriction that the reflected rays must be in one plane. Then as shown in Fig. 7, the strip is a parabola of focal length f. The distance from the focus of the paraboloid to the center of the strip is
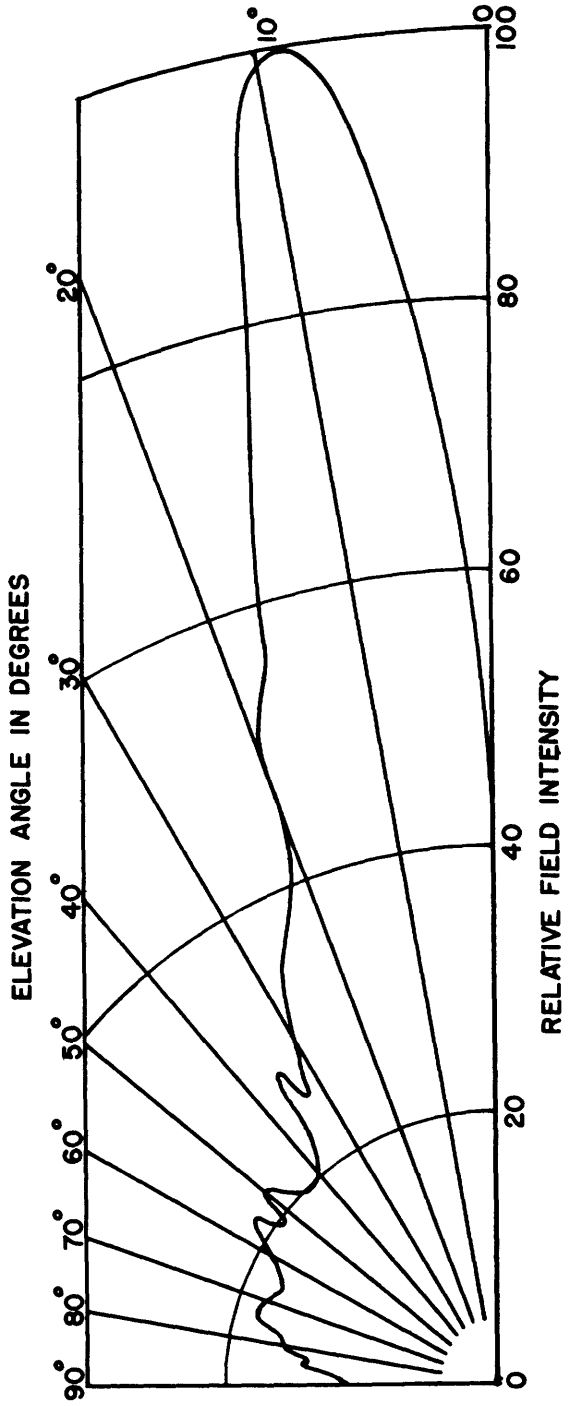
$$\rho = \frac{2f}{1 - \cos \alpha} \tag{18}$$

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ELEVATION ANGLE IN DEGREES
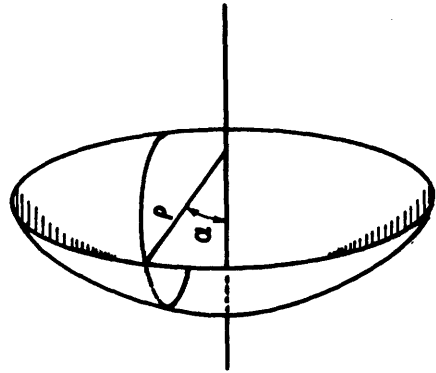
RELATIVE FIELD INTENSITY

Fig. 6

Fig. 7

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where α is the angle between the axis of the paraboloid and the line joining the focus to the center of the strip. The complete reflector can be constructed of such strips with f and α as parameters. Each strip is oriented to coincide with the corresponding Θ direction.

Figure 1 can be used to represent a cut through the center plane of this double-curvature reflector. Here (ρ, Ψ) is a point on the center section. All the outgoing parallel rays in the Θ direction lie in a plane perpendicular to the paper. The intersection of this plane and the complete reflector is a parabola of focal length f which is given by the following formula

$$f(\rho, \Psi) = \rho \sin^2 \frac{\Psi + \Theta}{2} \ . \tag{19}$$

From geometric optics, again, we have for the center section

$$\frac{d\rho}{\rho} = \tan \frac{\Psi + \Theta}{2} d\Psi \ . \tag{20}$$

These two equations completely determine the geometry of the reflector except for the functional relationship between Θ and Ψ.

Many factors come into the picture when a point source is used, so that the power relationship between the incident and reflected waves is no longer a clean-cut proposition. The distribution of intensity on each strip is not easily determinable without knowing first the position and other constants of the strip. The width of each strip is no longer constant along the strip. All that affects the secondary pattern and its peak intensity in the plane where the narrow beam is desired.

In order to devise a simple procedure, we shall assume that the primary and secondary patterns are simply related by the geometry of the center section.

Let $P(\Psi)$ be the power per unit solid angle radiated from the point source to the point (ρ, Ψ) on the center section. Let $I(\Theta)$ be the power per unit angle of the desired secondary pattern in the Θ plane. We have

$$\frac{1}{\rho} P(\Psi)d\Psi = k \ I(\Theta)d\Theta \tag{21}$$

where k is a normalization constant. The factor $1/\rho$ is introduced to convert the spherical wavefront into the corresponding cylindrical wavefront.

By taking logarithmic derivatives of Eq. (21) with respect to Ψ and substituting from Eq. (20), S. Silver obtained the differential equation

$$\frac{d^2\Theta}{d\Psi^2} + \left[\tan\left(\frac{\Psi+\Theta}{2}\right) - \frac{1}{P}\frac{dP}{d\Psi} \right] \frac{d\Theta}{d\Psi} + \frac{1}{I}\frac{dI}{d\Theta}\left(\frac{d\Theta}{d\Psi}\right)^2 = 0 \tag{22}$$

which can be integrated numerically, although it is somewhat troublesome.

If we consider $\rho(\Psi)$ to be a constant as a first approximation, Eq. (21) reduces to the form of Eq. (2), whence a direct integration can be performed immediately to obtain an approximate relation between Θ and Ψ. From Eq. (20) $\rho(\Psi)$ can be computed as a first approximation which can now be used in Eq. (21) to obtain the second approximation of $\Theta(\Psi)$. The second approximation is usually accurate enough for design purposes.

The design considerations are about the same as that of the cylindrical reflectors with one exception. The direct radiation from, and scattering by, the point

source are not directive. The deterioration of the pattern caused by that will not be serious.

One outstanding design of this type is shown in Fig. 8. A waveguide feed is used to illuminate the reflector. The top half of the reflector is a regular paraboloid. The bottom half is also a paraboloid of revolution generated by revolving a parabola of the same focal length about a line passing through the focus but perpendicular to the axis. By choosing a feed of proper P(Ψ), it is possible to obtain a fairly decent secondary pattern as shown in Fig. 9. This antenna was designed by S. J. Mason, E. A. Martinelli, and A. S. Dunbar.
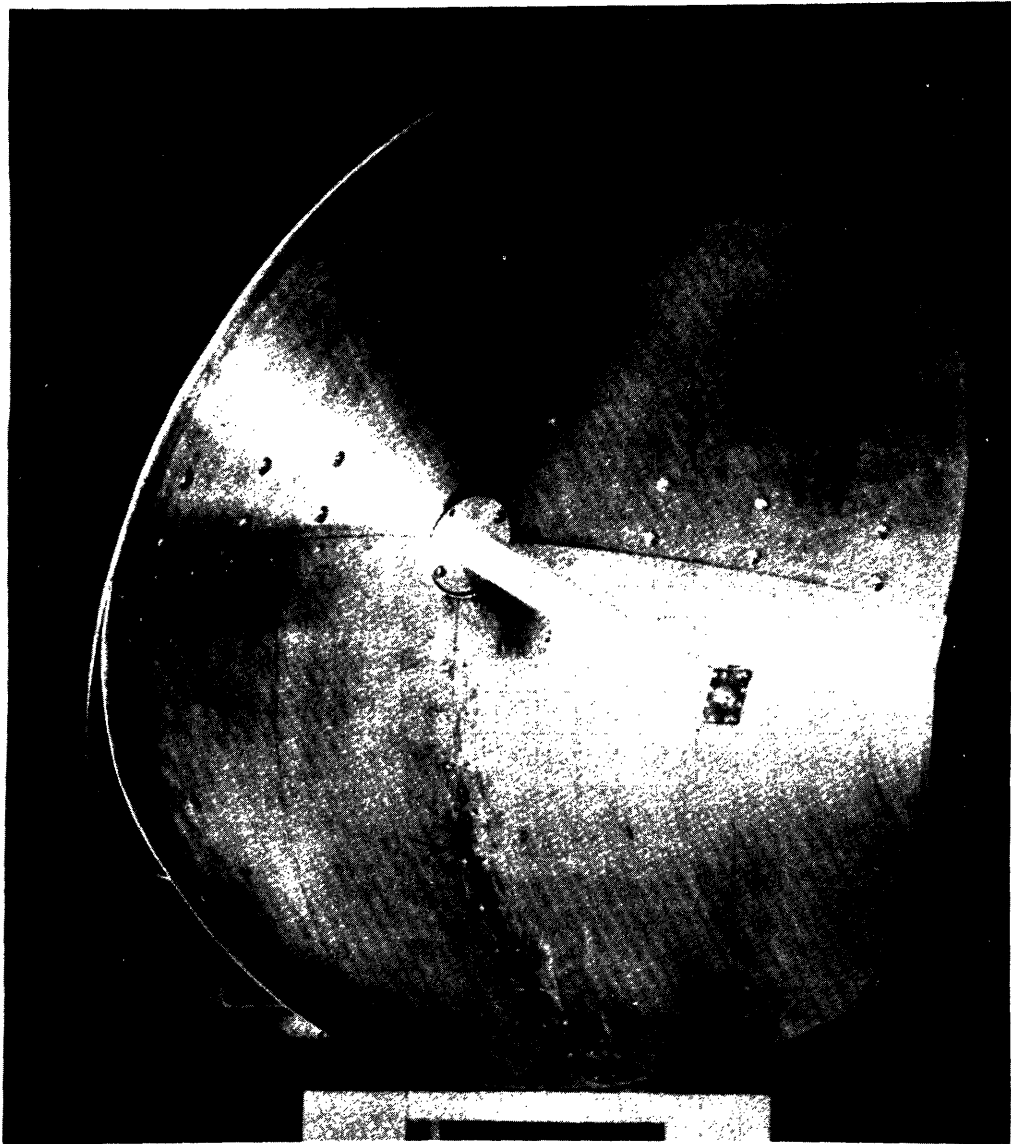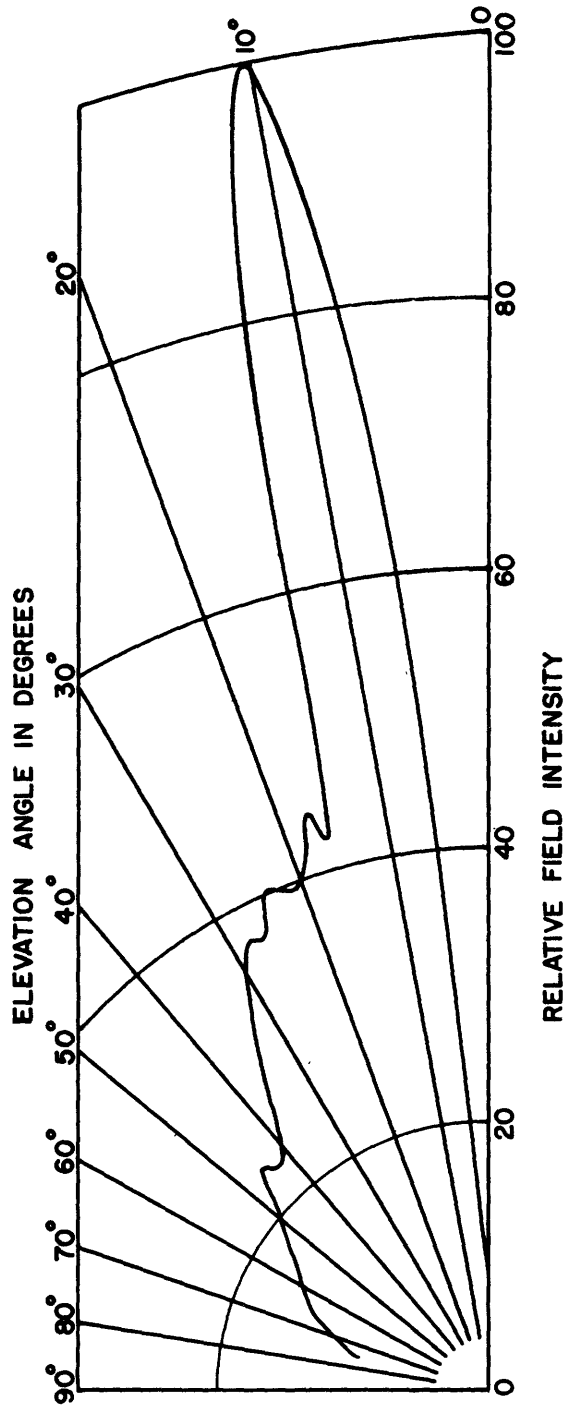

Fig. 8

ELEVATION ANGLE IN DEGREES

RELATIVE FIELD INTENSITY

Fig. 9

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