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ABSTRACT

This paper presents a practical method for project evaluation using techniques of financial economics which were developed originally for valuing stock options and other financial assets. It is based on the formulation and estimation of an "information model" which represents the resolution over time of uncertainties underlying a project. Cash flows can then be valued using techniques of derivative asset valuation. The method overcomes shortcomings of conventional methods which are either imprecise about the relation between economic value and uncertainty, or are rigid and unrealistic in the assumptions that must be made about this relation.

For ease of implementation, the method has been designed to be as close as possible to approaches familiar in industry today. The formulation of decision alternatives, the selection of underlying uncertainties, and the design of a cash-flow model are essentially the same as in conventional discounted cash-flow methods, as are the simulation and valuation results. The information model also can be estimated using analysis and judgment similar to that applied in conventional evaluation. The approach is illustrated in application to an oil development project under a complex tax system, where oil prices are the underlying source of uncertainty.
1. INTRODUCTION

Uncertainty analysis is a critical part of project evaluation in most industries. Moreover, it is not just uncertainty at the start of a project that matters. The information that managers have will change over time as events occur: things unknown before a project is begun become known as it progresses toward its end. An evaluation should take account of the process by which managers might use this new information over time to resolve the uncertainties about the project itself. Until recently, however, valuation methods were based on only a weak and inflexible link between aspects of project uncertainty and economic notions of value, resulting in a potential for bias in project analysis.\(^1\) Fortunately, techniques of financial economics, developed originally for valuing stock options and other financial assets, now provide an improved basis for representing this aspect of investment in real assets. In this paper we use the results of this work to formulate a practical method for project evaluation.

For ease of implementation, the procedure is designed to be as close as possible to analyses familiar in industry today. Several steps, including the formulation of the decision alternatives, the selection of key uncertainties, and the layout of a project cash-flow model, are essentially the same as in conventional discounted cash-flow (DCF) evaluation. The results also come in the same form: simulations of project cash-flows and present value numbers for these cash flow sequences. Two key distinctions lie in the way the uncertain inputs into the cash-flow model are represented, and in the manner that value is calculated. Our approach is

\(^1\) For reviews of the problem, see Myers (1984) and Barwise et al. (1989).
more formal and quantitative than is typically the case today, which forces more careful attention to uncertainty and its effects on the project and allows an economically more rigorous analysis of project value.

The motivation for our approach can be seen in the shortcomings of other methods. Consider first the most commonly used representation of the uncertainty underlying a project. This consists of a group of scenarios of the uncertain inputs to the analysis, often a "base case" supplemented by "high" and "low" cases to test extreme conditions. Scenario construction is a judgmental process, and careful definitions of the procedure are rare. No quantitative information need be given about the relative likelihood of the scenarios or about how those likelihoods might change over time with the arrival of new information. Usually, scenarios of underlying uncertain variables are run through the cash-flow model to compute cash-flow magnitudes, which are then are discounted to compute a "value" for each. It is up to the manager to interpret the risk-return tradeoffs implicit in the scenario format, and combine with judgment to determine a value for the project itself.

The cash-flow simulations themselves are well defined: they illuminate various regions of the uncertain range of cash-flow realizations. But the valuation numbers are problematic. The "value" attached to each scenario is best interpreted as an after-the-fact assessment of whether the project would be considered to have been worthwhile, conditional on the occurrence of that scenario and on the existence of alternative instruments with rates of return only at the chosen discount rate.

Often managers take such a set of "value" numbers and look at their central tendency as some measure of "return" and at their spread as a measure of project "risk". There are problems with this procedure quite
apart from the obscure meaning of the discounting within this after-the-fact comparison. First, if the discount rate already has a risk premium in it, the "return" number is already a "risk adjusted return." Therefore, the effects of uncertainty come into the "valuation" in two places: at the level of each "value" and in their spread. The individual impacts of these two effects and their interaction are not clearly stated. Second, even though the spread of scenario results may indicate something about the variance of cash-flows, risk should be related (roughly speaking) to the contribution of these cash-flows to uncertainty in the future value of the portfolio of assets held by project stakeholders. This contribution is usually measured by the co-variance of the value of the cash-flows with the value of the portfolio, not the variance of the cash flows themselves.

A related approach attempts to be more quantitative about the uncertainty. A probability is assigned to each of a small number of judgmental scenarios, or a formal model may be used to simulate a large sample of scenarios. Whatever the calculation, the representation of uncertainty is given by the joint probability distribution of the underlying uncertain variables.

Such a static probability distribution of underlying uncertain variables provides some quantification of the uncertainty, but it does not represent information flow and uncertainty resolution. When used to support simulation of project cash-flows, a static probability distribution provides just a quantification of the scenario-based procedure. As such it is well defined and may be of use in analyzing some current statistics of the cash-flow patterns.

When it comes to valuation, however, problems arise once again. There are two common approaches that use probability distributions. One method
(Hertz 1964) simply quantifies the distribution of the scenario "values," in the same manner as the cash-flow simulation quantifies the distribution of scenario cash-flows. Each "value" in the distribution is constructed from a single scenario. The "return" of the project is some central tendency of the distribution, and the "risk" is a spread. This approach suffers from all of the flaws of the scenario-based valuation method.

A second approach pays more attention to the fact that value is a property of the whole distribution. The expectation of the net cash-flows is discounted, almost always at a constant rate. Unfortunately, the discounting structure is usually only vaguely related to the uncertainty in the cash-flows and to the resolution of that uncertainty over time. At best the underlying rate is calculated by applying a one-period asset pricing model (such as the CAPM) to estimate a one-period risk premium for a portfolio of assets that has a risk structure "similar" to the project at hand. This risk premium is used to construct the discount rate (Brealey and Myers 1988, Ch.7).

The most important problems with this risk discounting framework arise because the representation of uncertainty is not strong enough to support the valuation method being used. The valuation is performed by replicating a one-period asset pricing model period by period. However, the representation of uncertainty does not allow for a description of the possible conditions in each of these periods. An implicit assumption is made that the structure of future conditions as seen from each time is the same (i.e., stationary). This structure of future conditions includes aspects of the economy, such as the term structure of interest rates and of excess expected returns for risk, and also specifics of the project, in particular the proportional amount of uncertainty in each expected net cash
flow (Fama, 1977).

The assumption that the uncertain future looks the same in each period is very restrictive; most real-life projects have a changing risk structure over time. Any project goes through various stages during which there are changes in the sources and magnitudes of underlying uncertainty, and in the filtering of this uncertainty through the evolving project structure, which may include changes in project operating leverage, operating flexibility, and the effects of complex tax systems. Section 4 illustrates these effects. Of course, a project could be force-fit into a framework in which there is no period-by-period variation in the structure of its uncertainty. But then it would be impossible to perform a detailed analysis of uncertainty and its relation to project value, which is precisely the type of analysis needed to support project decisions.

Now compare these methods with the representation of uncertainty used in most modern financial theory, and upon which our proposal is based. This consists of a model of the joint probability distribution of the uncertain inputs into the analysis, along with a quantitative description of the process by which this distribution might change over time with the arrival of new information. A model with these components, which we call an information model, can represent the resolution in a contingent fashion over time of the uncertainty in the inputs to the project cash-flow model. As shown below, the analysis and judgment required to estimate such a structure is similar to that required by conventional approaches to project evaluation.

We can illustrate an information model, and the way uncertainty is resolved as events occur, using a series of flips of a coin. In this case, the model describes the uncertain evolution in time of the probability for
each possible realization of the whole series of flips. This evaluation depends on the information given by each successive flip in the series. An example is the evolution of the probability that two flips of the coin will yield the outcome "Heads, Heads". As shown in Figure 1, this probability begins at $1/4$. If the uncertainty in the first flip is resolved to a realization of "Heads", then the probability of an ultimate "Heads, Heads" outcome goes to $1/2$. On the other hand, if the first flip is "Tails", the probability goes to 0. Finally, after the second flip, the uncertainty is resolved: the probability of "Heads, Heads" is 1 if both flips have been "Heads", and 0 otherwise.

In section 2 we show how such an information model can be formulated and estimated in practice. We illustrate the method in application to oil field development, and in the sample calculations oil prices are taken to be the only underlying uncertain variables. Section 3 then develops a valuation method based on our formulation of the information model. We show how the structure of the information model may be used to value the project as a portfolio of assets corresponding to the uncertain inputs into the analysis (e.g., oil-denominated bonds corresponding to oil prices). Then we show how these simpler underlying assets might be valued themselves. The result is that the necessary discount rate information comes from an analysis of the underlying assets. The discounting of the project itself is then an output rather than an input of the analysis. Section 4 shows what the results might look like for an actual oil development project, focusing on information about value and risk that may be calculated.

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2 Other variables, such as those representing cost or geological conditions, could also be treated as sources of uncertainty.
2. The Information Model

2.1 Terminology and Notation

Development of an information model requires careful definitions of concepts such as event, information, and time. Assuming that knowledge once gained is never lost, the information on hand at any time is at least as great and likely greater than that available before. The index \( s \) is used to represent the movement of the observer through time. Because it also represents the information that is available at a particular time, it is referred to as information time.

Some variables are defined in terms of particular events in time. An example is an oil price, where the event is the sale of a barrel of oil on a specified date. The price at each time of sale is a different variable. However, it is natural to think of the time series of oil prices as a group of variables with the same name, and to use the time of sale to distinguish among them. Generically these times that distinguish among events, which in turn define variables, can be thought of as variable definition times. They are denoted here by the subscript \( u \) (e.g., \( P_u \) for the price of oil at time \( u \)).

Cash-flow amounts in a cash-flow model demonstrate an important instance of such a variable-definition time index. Each amount has associated with it the particular time at which the cash actually flows. Borrowing from the terminology of the bond market, the variable definition time for a cash-flow amount is called its maturity time. Because the maturity time of a cash-flow is such an important variable definition time, it is given its own subscript, \( t \).

Some of the concepts needed for analysis use more than one notion of

\[ \text{A tilde over a variable indicates it is uncertain.} \]
time. An example is the expectation of a future oil price, $E_s(\bar{P}_u)$, which may be interpreted as "the expected magnitude of the oil spot price at (variable definition) time $u$, when the expectation is formed on the basis of all the information available at (information) time $s$." Another example is a cash-flow amount that is some function of $\bar{P}_u$, but which flows at a later (maturity) time. Tax-loss carry-forwards have this character. The amount of the cash-flow has two variable definition times: the (variable definition) time $u$ of the spot price used to define it, and the (maturity) time $t>u$ when it flows. Statistics of the variable, such as its expectation or median, may be examined for any (information) time $s$.

2.2 Specification

With these definitions, we can discuss the characteristics of an information model of oil prices, $\bar{P}_u$. At the point of project decision, at (information) time $s=0$, no one knows exactly what the oil price will be at (variable definition) times, $u>0$. Nonetheless it may be possible to summarize part of the available knowledge of future oil prices by some measure of central tendency in each, such as the current expected magnitude $E_0(\bar{P}_u)$, and to make qualitative statements about how the uncertainty in $\bar{P}_u$ might be resolved over (information) time.

For example, the process might be formulated as follows. At $s=0$, the expectation is $E_0(\bar{P}_u)$. When we arrive at (information) time $s=u$, the oil price will be known, and $E_{s-u}(\bar{P}_u)$ will simply equal the realization, $\bar{P}_u$. Over the period between $s=0$ and $s=u$, the expectation $E_s(\bar{P}_u)$ will follow some path, the precise trajectory depending on new information gained along the way. This new information about the oil market might be specified in various ways, but one simple model is that all the information needed to
determine the revision in expectations of future oil prices is provided by the most recent unanticipated change in the expectation of the current price.\footnote{More precisely, the new information arriving during the period from \( s \) to \( s+\Delta s \) is the revision over this period in the expectation of \( \tilde{P}_u \) where \( u=s+\Delta s \).}

To illustrate with a three period example, suppose at information time 0 the expected magnitudes of future oil prices are \( E_0(P_1) = $15 \), \( E_0(P_2) = $16 \), and \( E_0(P_3) = $18 \). At \( s=0 \), no one knows what information will become available between \( s=0 \) and \( s=1 \). Therefore the expectation as of (information) time \( s=0 \), of what the future will look like at \( s=1 \), is still $16 and $18. That is, \( E_0[E_1(P_2)] = $16 \), and \( E_0[E_1(P_3)] = $18 \). Now suppose there is an unanticipated price change in the period ending at (information) time 1, and \( P_1 \) turns out to be $13 instead of the previous $15 expectation. What will \( E_1(P_2) \) and \( E_1(P_3) \) be then? Will the revision in expectations from \( s=0 \) to \( s=1 \) be upward or downward? And by how much for period 2 and period 3?

Underlying these estimates will be some idea of the process by which expectations are revised:

- Is there an underlying upward drift in real oil price because of nonrenewable nature of the resource?
- Is uncertainty in price greater the farther away one looks into the future?
- Is there some cost floor which tends to put a lower boundary on price, and thus on expectations about price?
- From the demand side, is there a ceiling on the oligopoly price that can be sustained?

The specialization to one piece of information in each period of information time is for simplicity. A more realistic model involving two types of information, corresponding to long-term and short-term effects on oil prices, is used in Laughton (1988).
Are anticipated changes in the expectations of oil prices positively correlated with other measures of economic activity?

Answers to these types of questions constitute a qualitative description of the way uncertainty is resolved over time, and they set the outline of an information model of oil prices.

To formulate our quantitative model of such a process, we consider a circumstance in which the length of the (information) time periods $\Delta s$ is small and specify how the expectation of price for each future time is revised as a result of new information during each of these periods. We take these revisions of expectation to be normally distributed, each with a variance of the order of $\Delta s$. To apply this concept numerically, a normalized version of the information is constructed, denoted $\Delta \tilde{z}_{p,s}$, where $P$ identifies the information as affecting oil prices. The normalization commonly chosen is $E_s[\Delta \tilde{z}_s] = 0$ and $E_s[(\Delta \tilde{z}_s)^2] = \Delta s$.

Next, we specify the revision of expectations,

$$E_{s+\Delta s}(\bar{P}_u) - E_s(\bar{P}_u) = E_s(\bar{P}_u) \sigma_{p,u,P,s} \Delta \tilde{z}_{p,s} \quad s<u. \quad (1)$$

The revision for each price is proportional to the expectation of that price at the beginning of the period, $E_s(\bar{P}_u)$, and to the normalized information, $\Delta \tilde{z}_{p,s}$. The information proportionality constant $\sigma_{p,u,P,s}$ determines the scale of the revision in the expectation of $\bar{P}_u$ during the interval after time $s$ due to the arrival of information of type $P$. It is taken to be

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5 With only one bit of information $\Delta \tilde{z}_{p,s}$ per time period, the revisions of expectations for all prices are perfectly correlated with each other. With more than one information factor this would not be so.

6 This proportionality constant is hereafter simply called $\sigma$, with its subscripts taken to be understood.
known with certainty at all (information) times.

By generating a set of realizations of the sequence of random variables $\Delta \tilde{z}_{P,s}$, Equation 1 can be used to compute a set of dynamic paths in (information) time for $E_s(\tilde{P}_u)$, and a set of realizations of the prices, $\tilde{P}_u = E_{s-u}(\tilde{P}_u)$. With a large number of realizations it is possible to calculate statistics of the resulting sample that are close to the statistics of the true probability distribution of the oil prices.

2.3 Estimation

The parameters of the information model include the current term structure of expectations, $E_0(\tilde{P}_u)$, and the parameters in the proportional amount of uncertainty, $\sigma$. The parameters of $\sigma$ might be estimated from a statistical analysis of oil price history based on an assumption that the future will be similar to the past. Or estimates could be inferred from the judgments of oil market experts or others within the organization.

We illustrate the latter approach, whereby quantitative opinions are sought about specific statistics of the marginal probability distributions of oil prices. These estimates might be based on currently available information, or on sets of hypothetical information at future information times. For oil prices, a device we have found useful is "windows" of the distribution of each price (Jacoby and Paddock, 1985). The approach may be

If we use annual variables in the cash flow models, and if $\sigma$ is modelled not to vary in information time over the course of the year, then we can show (Laughton 1988) that we do not need to generate realizations for all small periods over a year but only for their sum, which we denote (for the year beginning at time $s$) by

$$z_{P,s} = \int_{s}^{s+1} \Delta \tilde{z}_{P,s}'$$

This annual information, $z_{P,s}'$, is a normal variable with unit variance.
illustrated using a simple three-period example. We have found it easier in practice to estimate the medians of the prices, \( M_0(\bar{P}_u) \), instead of the expectations in Equation 1. The first task then is to determine the term structure of medians, which is similar to asking for a "base case" scenario. A sample answer is shown in Figure 2.

Next we seek responses that can be used to determine the parameters of \( \sigma \) and check the validity of the model. One such set of data is the current upper and lower fractiles of the distribution for each price considered individually. In our oil price experiments, 0.1 and 0.9 fractiles have been used. These form a current window on each price, also shown in Figure 2.

If \( \sigma \) is constant, the \( F \)th fractile for the price at (variable definition) time \( u \) is \( M_0(\bar{P}_u) \times \exp[\sigma \sqrt{u} N(F)] \), where \( N(F) \) is the \( F \)th fractile of the standard unit normal probability distribution.

Each window contains two pieces of data (one each from the top and the bottom) about those combinations of the parameters of \( \sigma \) that determine the

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8 The two contain equivalent information when combined with \( \sigma \). If \( \sigma \) is constant, \( E_0(\bar{P}_u) = M_0(\bar{P}_u) \exp[1/2\sigma^2 u] \).

9 This exponential of a normal fractile comes from the log-normal structure of the price probability distributions. Equation (1) looks almost like the differential equation for an exponential function in \( z \). The expectation at an (information) time \( s \) in the future (that is not a small period of time away from the present) is distributed, as of the current (information) time, like the exponential of a normal variable, i.e., like a log-normal variable. The expectation of this log-normal variable is the current expectation of the price, and the variance of its corresponding normal variable, with \( \sigma \) constant, is \( \sigma^2 s \).

The distribution of the price \( \bar{P}_u \) is also the distribution of its expectation \( E_{s-u}(\bar{P}_u) \). The variance of the relevant normal variable, if \( \sigma \) be constant, would be \( \sigma^2 u \). Therefore \( \bar{P}_u \) would be proportional to \( \exp[\sigma \sqrt{u} \tilde{z}] \), where \( \tilde{z} \) is a unit normal variable. Because the realization of \( \bar{P}_u \) would in this case be an increasing function of the realization of \( \tilde{z} \), the fractiles of \( \bar{P}_u \) would be related to those of \( \tilde{z} \) by this same function. The result is the relation in the text.
overall uncertainty in each price. These estimates can thus be used both to estimate the overall uncertainty and to check whether its assumed specification is consistent with the responses. If the calculated windows do not diverge in unacceptable ways from those produced by the respondent, then the model structure would seem, thus far, to be plausible.

However, the oil price windows viewed as of (information) time s=0 may not test all aspects of the current probability distribution for each price. For example, they do not directly convey information about correlations that may exist between two or more future prices. Moreover, they do not test whether we have adequately captured the way in which uncertainty may be resolved over time. These issues can be studied by constructing windows from some imagined vantage point in the future, conditioned on scenarios of the prices between now and that point (Laughton 1988). Appropriate sets of such future window estimates are guaranteed to provide estimates for all the parameters of $\sigma$ and to provide tests of the specification of uncertainty.

2.4 **Strengths and Weaknesses**

Like the methods discussed in the Introduction, a formal information model has both strengths and weaknesses. It is quantitative and precise; it describes information flow explicitly and specifies the relation between information flow and uncertainty resolution. This is why the approach now dominates modern financial economics.

With no further elaboration the model can be applied to cash-flow simulation. Assume that we are only interested in prices at annual intervals, and that the proportional amounts of uncertainty, $\sigma$, do not vary

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10 In an application where $\sigma$ is constant over time, for example, one measure of the overall uncertainty in $P_u$ is $\sigma/\bar{u}$. 
over the course of each year in (information) time. A single oil price scenario can be calculated after drawing a realization of $\tilde{z}_{P,s}$ for each time s. This price series can be input to the cash-flow model to produce a scenario of cash-flow amounts. If this is done repeatedly with realizations of $\tilde{z}_{P,s}$ from the standard unit normal distribution, the result is a properly distributed sample of cash-flow scenarios. The sample mean of each cash-flow amount will be (for a large enough sample) a good estimate of its expected magnitude. Other sample statistics of the cash-flows can be calculated as well, such as current windows akin to those shown in Figure 1 for the underlying variables, or future windows conditional on a particular scenario up to the time of view. This provides a dynamic simulation environment that is much richer than that available from the group of cash-flow proformas that may be calculated from a scenario-based representation of uncertainty.

Also, as we show in the next section, the information model can support a well-defined process of economic valuation. Analogies can be used to set up a discounting framework, but these are drawn at the relatively simple level of underlying uncertain variables rather than at the more complex level of the project itself in all its detail.\textsuperscript{11} With this underlying valuation in place we can apply the theory behind options and futures valuation to determine how a particular cash flow model transforms the discounting of the relatively simple underlying risks into the discounting of the more complex risks of a project. Because there are no restrictions on the time pattern of the underlying risks, or on their transformation by the cash-flow model into project risks, we do not need to force the project

\textsuperscript{11} Moreover, there is no need for restrictive assumptions of stationarity in the uncertainty resolution, even at the level of these underlying variables.
risks into the restrictive pattern required by methods based on one-period asset pricing models. Moreover, the discounting structure at the project level is an output of the evaluation, not an input.

The use of a process representation has its costs, of course. Specification and estimation are more difficult than for static representations, and more judgment and information are required on the part of management. The strengths of the scenario description (primarily ease of use) and the corresponding weakness of the process description are important reasons why firms have tended in the past to use scenarios. The key task then of this and future work is to make the modelling of probabilistic processes as easy for managers in the future as is the modelling of groups of scenarios now.

3. Valuation

3.1 The Concept of Relative Valuation

The main advantage of the information model comes in application to valuation. Like many modern methods of asset pricing, our method applies the process of relative (or derivative) valuation to calculate the value of complex cash-flows. If the magnitude of each of the cash-flows associated with an asset (the derivative asset) is contingent only on the concurrent value of other assets (the underlying assets) then the value of the derivative asset can be calculated in certain circumstances relative to the values of underlying assets. The origin of this method is the classic work

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12 Also, for self-consistent valuation that is manageable, restrictions must be placed on the types of uncertainty models than can be considered, as discussed in Section 3.4.
of Black-Scholes and Merton on the valuation of options on common stock.\textsuperscript{13}

The derivative asset valuation is performed by replicating the value of the derivative asset at each (information) time by the value of a portfolio of the underlying assets, one portfolio for each time. Each successive replicating portfolio is constructed, without new financing, from the previous one by trading among the underlying assets in response to new information. The design of such a value-replicating trading strategy requires knowledge of the structure of the unanticipated changes (in response to the new information) of the value of each underlying asset. The valuation of the initial portfolio, and thus of the cash-flow claim, requires knowledge of the initial value of each underlying asset.

A project is a bundle of claims to individual cash-flows, and it can be treated as a derivative asset provided that the claim to each of its cash-flows is a derivative asset. The value of the project is just the sum of the value of its component cash-flows. One can then focus on single cash-flows and value them individually.

In the oil field development example used here, each uncertain cash-flow amount is contingent on the magnitude of various oil prices. Moreover, within an information model with oil prices as the only uncertain variables, the cash-flow amounts are contingent only on these oil prices. Each oil price, $P_u$, in turn, can be formulated as the terminal value of the asset that we call an \textit{oil bond}. Each oil bond is a claim to a cash-flow at some (maturity) time $t$, where the magnitude of the payment is the spot price

\textsuperscript{13} See Cox and Rubinstein (1985). Some technical assumptions are required about the ease of trading in financial markets, the homogeneity of traders' information and beliefs, and the smoothness or continuity of the effects of new information (Merton, 1977). Our information models satisfy these restrictions (Laughton, 1988). The interpretation of results depends to some extent on the degree to which real markets satisfy these assumptions.
\( P_u \) at some time \( u \) (less than or equal to \( t \)). These oil bonds, along with cash discount bonds, are the underlying assets. If the mechanism for implementing the relative valuation is known, then the residual problem is to find a way to establish the current values of the underlying assets and the structure of their unanticipated returns over time.

3.2 Valuation of Underlying Assets

Compared to conventional methods of oil field evaluation, our method specifies the effects of uncertainty on value at an earlier stage of the analysis: the economic analysis of risk is applied to the underlying uncertain variables rather than to the cash-flows. In conventional discounted cash-flow analysis the question is, "What is the right discount rate for the project?" The equivalent question in the derivative asset approach is, "What is the value today of claims in the future to cash or to future oil prices, and how is the uncertainty in the value of those claims resolved over time?"

We first make the assumption that bonds (oil or cash discount) can be valued as if future short-term nominal and real riskless rates were known. Real or nominal cash discount bonds may then be valued using a time series of short-term market rates to discount the cash-flow to be received. Information about nominal rates can be extracted from yields on relevant government obligations. Real rates can be calculated either by using some long-term average of past real rates or by setting up a model of future

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14 This is where many asset pricing models in modern financial economics diverge from each other. The more satisfying theories (e.g., Cox, Ingersoll and Ross, 1985) require computations that at present are too difficult to apply in practice. We attempt a compromise between the insights of the best theory and the practical needs of organizations currently using scenario methods (Laughton, 1988).
inflation rates and subtracting these from the nominal rates.

Turning to the valuation of oil bonds, it may be possible in some circumstances to use market data, for instance by using forward or futures prices. Alternatively, the value of hypothetical oil bonds might be based on the judgment of oil company managers or other experts. Finally, a discounting structure based on judgment could be used to convert expected oil prices into oil bond values. This last approach is illustrated here.

We have already determined the risk-free discounting structure that we wish to use to discount the passage of time. We focus then on risk discounting.

First, we assume that risk discounts can be based on a valuation framework in which there is only one type of risk, and that this risk arises from the correlation between the unanticipated change in value of the asset and the unanticipated change in a single "risk factor". An example of such a one-factor model in one time period is the simple capital asset pricing model (or CAPM) in which the risk factor is the value of a broad-based portfolio of assets. Second, we assume that future expected rates of return on the underlying risky assets (the oil bonds) are known with certainty.¹⁵

These assumptions allow us to calculate what we need: the expected rate of return at each relevant (information) time s on a claim to a cash-flow at each (maturity) time t with amount \( \bar{P}_u \) for each time \( u \leq t \). This rate is denoted \( R_s(\bar{P}_u,t) \). It is the sum of two terms: the short term risk-less rate of return, \( r_s \) (the return for time), and a risk premium which is the product of the price of risk at (information) time s for oil price

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¹⁵ The actual future returns are allowed to be uncertain, but the expected returns are not. This is similar to the assumption that future risk-free rates of return are known with certainty.
uncertainty, and the proportional amount of oil price uncertainty, \( \sigma \). In symbolic terms, we have

\[
R_s(\bar{P}_u,t) = R_s + b_{p,s} \sigma_{p,u,p,s} s<u<t, \tag{2}
\]

where \( b_{p,s} \) is the price of risk at (information) time \( s \) for the oil price uncertainty factor, \( P \).\(^{16}\) To insure that the expected rate of return is certain, we assume that its determinants (risk-free interest rates, \( r \), prices of risk, \( b \), and proportional amounts of uncertainty, \( \sigma \)) are also known with certainty.\(^{17}\)

If the expectation of the oil price, \( E_0(\bar{P}_u) \), is discounted using the expected rates of return established in Equation 2, the result is the value at (information) time \( s=0 \) of a claim to \( \bar{P}_u \) to be received at time \( t \). This value is denoted \( V_0(\bar{P}_u,t) \). For simplicity, assume that the short term risk-less rate is a constant, \( r \), the price of risk in the oil price uncertainty factor is a constant, \( b \), and the proportional amount of uncertainty in the oil prices is also a constant, \( \sigma \). Then the value would be\(^{18}\)

\[
V_0(\bar{P}_u,t) = E_0(\bar{P}_u)\exp[-(r + b\sigma)u]\exp[-r(t - u)]. \tag{3}
\]

\(^{16}\) In the CAPM, \( b_{p,s} \) is the product of an overall price of risk (the ratio of the excess expected return on a broadly based portfolio divided by the standard deviation of its to-be-realized return) multiplied by the correlation between the unanticipated change in the oil price uncertainty factor, \( \Delta Z_p \), and the (normalized) unanticipated change in the value of the broadly-based portfolio (Laughton and Jacoby 1988).

\(^{17}\) The certainty of \( \sigma \) puts constraints on the modelling of the underlying variables (Laughton 1988): at any time they must be distributed according to a joint lognormal distribution.

\(^{18}\) Equation 3 generalizes to situations in which the risk-less rates and the prices of risk are not constant in (information) time, where there is more than one uncertainty factor, and where \( \sigma \) is not constant as well.
The first discount factor is due to the expected returns for (information) times up to the (variable definition) time $u$ of the oil price, and the second factor is due to the expected returns for (information) times from the (variable definition) time $u$ of the oil price to the (maturity) time $t$ of the cash-flow. This expression may be reformulated as

$$V_0(P_u, t) = E_0(P_u) \exp(-bou) \exp(-rt),$$

where the first discount factor is the discount for risk and the second is for time.

We have described a procedure by which we may calculate the current value of oil and cash discount bonds. The only other input needed for derivative asset valuation is the structure of the unanticipated returns of the oil bonds. Because we have modelled the expected returns to be known with certainty, the only uncertainty in oil bond value comes from the uncertainty in the expectation of the corresponding oil price. As a result, the proportional uncertainty in the value of an oil bond is equal to that in the expectation of the associated oil price, $\sigma_{P_u, P, s} \Delta \tilde{Z}_{P, s}$.

### 3.3 Mechanics of Derivative Asset Valuation

Remember that the value of the portfolio of cash-flows from an investment decision may be determined by summing the value of the individual cash-flows that make it up. We therefore need only to consider the valuation of the project cash-flows, one at a time, and given our assumptions the mechanics are quite simple. An intermediate construct is created for each uncertain variable, which we shall call the "risk-adjusted" version of the variable. An example is the risk-adjusted oil price, $RA(P_u)$. 
There are five steps in the valuation procedure.

1) Each risk-discounted (or certainty equivalent) oil price\(^{19}\) may be considered to be the expectation at \(s=0\) of the risk-adjusted oil price, \(E_0(RA[P_u])\).

2) We determine the distribution of risk-adjusted oil prices by using the probabilistic process for the evolution of the expectation of \(E_s(RA[P_u])\), as described below.

3) We input these risk-adjusted underlying variables into the cash-flow model, scenario by scenario, to produce a distribution of risk-adjusted cash-flow amounts.

4) The expectation of each risk-adjusted cash-flow is the risk-discounted cash flow.

5) This expectation can be discounted for time to give its present value.

The key step is to find the appropriate distribution for the risk-adjusted oil prices. There are two parts to this. First, the theory behind dynamic portfolio replication shows that the appropriate definition of the unanticipated proportional change in the expectation of the risk-adjusted oil price is the same as the unanticipated proportional change in the risk-discounted oil price. Second, according to our model of risk discounting, the unanticipated proportional change in the risk-discounted oil price is the same in any situation at any (information) time as that in the expectation of the true oil price. Therefore the expectation of the risk-adjusted oil prices and of the true oil prices evolve in response to new information according to probabilistic processes that have the same

\(^{19}\) The certainty equivalent at (information) time \(s\) of a cash flow is the certain payment at the (maturity) time \(t\) of the cash flow, a claim to which has the same value at time \(s\) as the cash flow itself. In our example, for the cash flow maturing at time \(t\) and giving a payment \(P_u\), the certainty equivalent is \(CE_0(P_u,t) = E_0(P_u)\exp(-btu)\). Because we assume that future expected rates of return are known, this expression depends only on \(u\), the (variable definition) time of the oil price, and not on \(t\), the (maturity) time of the cash-flow. Therefore we abbreviate the notation to \(CE_0(P_u)\).
unanticipated proportional changes. The path of \( E_s(\bar{P}_u) \) is generated by

\[
E_{s+\Delta s}(\bar{P}_u) - E_s(\bar{P}_u) = E_s(\bar{P}_u) \sigma \Delta z_{P,s}.
\]

(5)

where the process begins from a current expectation that is already discounted for risk, or (in the constant \( b \) and \( \sigma \) case)

\[
E(G(\bar{P}_u)) = E(\bar{P}_u) \exp(-b\sigma u).
\]

The result is a probability distribution for the risk-adjusted prices that, for each price, is shifted downward relative to the true distribution by amount proportional to the risk discount factor for that price.\(^{20}\) This is the source of the risk discount in the risk discounted cash-flows that are derived.\(^{21}\)

\(^{20}\) The expected rate of change in the risk-discounted oil price is not zero but the risk discount rate for that oil price, \( b\sigma \). The risk-discounted oil price gradually approaches the expectation of the true oil price as the uncertainty in the oil price is resolved over time. The expected rate of change in the expectation of the risk-adjusted oil price is zero (by definition of the concept of an expectation). Therefore, the risk-adjusted oil price distribution retains the risk discount.

\(^{21}\) An example of the risk-discounting steps of the derivative asset valuation process is the valuation of the revenue for (maturity) time \( t \), \( \text{REV}_t \). Once a sample of the risk-adjusted oil price, \( \text{RA}[\bar{P}_{t_k}], k=1,\ldots,K \), is created, the associated revenue amounts can be computed by applying the cash-flow model to each member of the oil price sample. This risk-adjusted revenue sample is shifted downward relative to the "true" revenue because of the downward shift in the prices from which it is derived. The mean of a large sample of the risk-adjusted revenue is a good estimator of the expectation, which according to theory is the risk-discounted revenue. Because the risk-adjusted revenue sample is shifted down, so is its mean. It is this downward shift in the mean of the risk-adjusted distribution that produces the risk-discounting.
3.4 Review of Approximations

Several approximations underlie this method. First, the method used to value the underlying assets requires that certain key variables be known with certainty, including future risk-less interest rates, future prices of risk, and the proportional co-variances among future changes in the expectation of underlying variables. However, it should be noted that stronger versions of our underlying valuation approximations are be required to justify any properly organized DCF valuation method. Therefore, our method is more generally applicable than the typical DCF method.

Second, derivative asset valuation requires that assets be continuously available for trade at unique prices. However, it should be noted that, while liquid markets for oil bonds do not exist at present, oil price risk may be traded in impure forms in forward and futures markets for oil and in the markets for oil company securities. Moreover, much can be learned about a project by valuing it as if the company could sell claims to the underlying components of the project directly into oil bond markets instead of indirectly into the securities markets.

4. AN APPLICATION TO OIL DEVELOPMENT

4.1 Implementation of the Method

The method is illustrated by applying it to a 300 million barrel (mmbbl) oil development project in the U.K. sector of the North Sea. The tax system for the U.K. Outer Continental Shelf includes a Petroleum Revenue Tax (PRT) and a Corporate Income Tax (CIT). Oil prices are the underlying

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22 The PRT tax base is the project operating income less a deduction for capital recovery, and less a complicated "oil allowance." The PRT tax rate is 75%. The tax base for the CIT is corporate-level income, and the contribution of the project is its operating income less an allowance for capital cost and a deduction of PRT paid. The current CIT rate is 35%. In
uncertain variables for the project cash-flow model. The uncertainty is modelled according to the probability process defined in Equation 1. The concept of the expected magnitudes of future oil prices $E_0(\overline{P}_u)$ is illustrated in Figure 2. The standard deviation of oil prices, $\sigma$, is assumed to be constant across prices and over (information) time and equal to 0.1 in annual terms.

The valuation of claims to oil in the future is based on Equation 2, which states the expected return on an oil bond. In this example, however, we do not start with the component parts and build up an estimate of $R_s(\overline{P}_u,t)$. We go about the task the other way around, moving directly to an estimate of an "average" expected rate of return using information and judgment about the oil industry, and then checking for consistency with the model of expected returns underlying Equation 2 (with constant $b$).

Typically in the oil industry, development projects are evaluated at annual discount rates in a range around 10% real; we assume this rate is a good representation of the "average" costs of time and the risks of these projects. Then we observe that these risks consist, in the main, of underlying oil price risks leveraged (i.e., made still larger) by capital and operating costs. We make a rough correction for this leverage effect, and estimate that the appropriate "average" discount rate for an "average" pure oil claim is around 7% per year real.\(^{23}\) With this estimate we calculate the oil bond values, shown in Figure 2.

Are these estimates plausible? Under our assumptions the real riskless rate is about 3% per year, so the risk premium on the oil bonds is

\[^{23}\] A glance at Figure 4 will reveal that the process of "un-levering" is approximate, because the leverage is not constant during the project life.
about 4% per year. With $\sigma=0.1$ in annual terms, Equation 2 gives a risk price $b=0.4$ in annual terms for the risk in the oil price factor.\textsuperscript{24} One rough check on this result is to estimate what parameters would be consistent with this level of $b$ under a one-period CAPM valuation framework. A commonly accepted figure for the price of risk of the market portfolio is 0.5 in annual terms (Brealey and Myers 1988, Ch.7). Such an estimate would imply a correlation of 0.8 between changes in the oil price uncertainty factor and changes in the value of the market portfolio. This number seems reasonable for periods when oil markets are strongly influenced by demand conditions, as we expect they will be for the coming decade. However, under conditions of supply-side shock, as existed twice in the 1970s, this correlation could be negative. Unfortunately, the valuation method requires that this parameter be known with certainty.\textsuperscript{25}

A more complete study of the issue would include further analysis of the parameters used above and of the resulting bond values, perhaps supported by analysis of financial market data. In the implementation of this procedure, the debate and discussion of these estimates of oil bond value are a critical aspect of the process of project assessment. Analysis and judgment thus are focused, as they should be, on the heart of the issue of oil investment, which is what claims to future production are actually worth in cash terms today.

The information structure is implemented by generating sequences of

\textsuperscript{24} These quick calculations are possible because the information structure has only one source of uncertainty, $\sigma$ is constant over time, and $r_s$ is very nearly constant.

\textsuperscript{25} There are two solutions to this problem: take some average and pretend it is not uncertain (as we do) or, if this approximation appears too damaging, formulate the model with supply and demand determinants as underlying variables, as mentioned in Section 1.
unit normal random variables, \( \tilde{z}_{p,s} \), and using them in two versions of the model of the information structure given by Equation 1: one for true oil prices, the other for risk-adjusted oil prices. The resulting samples are then applied in the two forms of analysis discussed above: simulation and valuation. For an exploration of some of the simulation results this method makes possible, see Jacoby and Laughton (1988) and Laughton (1988).

4.2 Valuation Results

Figure 4 shows the value of the claims to each of the various cash-flows, year by year, that make up this project. The series of pre-tax values is negative for the investment in the early years, and positive after the oil production starts in 1993. The two tax obligations are accounted for separately, so that their timing and magnitude can be clearly seen. The value of the after-tax net cash-flow is also shown. Information of this type should improve understanding of project economics and help in fiscal planning. Conventional DCF methods do not support such a detailed analysis of the components of value because, at best, the discounting structure might be valid for the net cash-flows, but not for its components.

Derivative asset valuation can also be used to study the internal risk structure of a project; it is an "X-ray machine" for risk in the project evaluation tool kit. The lower half of the Figure 5 shows the expected magnitudes of cash-flows for this same project: gross revenue, pre-tax cash-flow, and cash-flow net of tax. The upper part uses the current total real yield as a measure of the riskiness of these cash-flows. The current total real yield is that discount rate which, when applied to the inflation-adjusted current expected magnitude of a cash-flow, produces its current
Note first that the yield for gross revenue is approximately 7% per year for revenues of all maturities. (For years 1989 to 1992, the amount of revenue is zero and its current total real yield is undefined.) These cash-flows are proportional to the oil price, so this is simply our bond discounting specification coming back in another form. The risk of the pre-tax cash-flow during production is higher than that of gross revenue because of leverage effects of capital and operating costs. In the first few years of production, the leverage is relatively weak because costs are low in relation to expected revenue, as can be seen in the lower part of the figure. The leverage increases as oil output declines because costs rise as a fraction of gross revenue.

These results illustrate one of the concerns of our critique above of standard discounted cash-flow analysis. The near-universal practice is to apply a constant discount rate, which implies a flat yield structure. Figure 5 shows that the very nature of the project rules against a constant rate. The choice of any constant rate as a surrogate for the more complex process requires a complicated weighing of effects, which cannot be undertaken without analyzing the project first. As noted earlier, the discounting structure of a project is more properly an output of the analysis than an input, reflecting both the discounting of underlying uncertainties and the way that project structure shapes the underlying risks.

26 The current total real yield of a cash flow at time t with real amount $\bar{X}$ is $\text{TRY}_0(\bar{X},t)$ where $V_0(\bar{X},t) = E_0(\bar{X}) \exp(-\text{TRY}_0(\bar{X},t)t)$. This simple way of displaying risk breaks down if, as is possible, $V_0(\bar{X},t)$ is of opposite sign to $E_0(\bar{X},t)$, or either is equal to zero.

27 The amount of each of the pre-tax (and after-tax) cash-flows that occurs before the start of production is modelled to be known with certainty. The current total real yield is therefore just the real risk-free yield of about 3% per year.
into investment risks.

The figure also shows the risk adjustment for after-tax cash-flows. The effect of the tax system on the pre-tax yields is striking. First, the yields are lower after-tax than pre-tax, indicating that the tax payments are more risky than the pre-tax cash-flows. Moreover, in some years a great deal of the price risk is transferred to the government, as indicated by the deep drop in the current yields of the net-tax flows 1995 and 1996. The drop occurs because of the peculiar structure of the tax shields in the U.K. system. With this size of project, the capital cost shields for the PRT are likely to run out in these years, making the amount of PRT collection highly dependent on the uncertain path of oil prices and therefore highly risky.

Other characteristics of project structure and tax system behavior can be analyzed using this approach, for example by comparing projects of different size or firms with differing composition of other income. The probabilistic process used for illustration is a simple one; more realistic versions would likely contribute additional insight. Also, we have considered only one underlying source of uncertainty: oil price uncertainty in the revenue. Uncertainties in costs or in recoverable reserves could be added.

Finally, this method can be used to analyze projects with operating flexibility, such as the ability to stop and restart production, delays in construction, abandonment, or technology switching. The operating rules might be computed within the framework developed above, although these applications likely will prove quite limited. Fortunately, there are a wide range of applications for which such rules can be formulated on the basis of

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28 For some initial work on the use of these methods to analyze uncertain tax systems (i.e., political risk), see Cavoulacos et al. (1989).
judgment or the results of other analyses, opening the way to new approaches to these important problems.

REFERENCES


Figure 1. Probabilistic Process for the Conditional Probability of "Heads Heads" in 2 Coin Flips
Figure 2. Current Medians and Windows of Oil Price

* = median
Figure 3. Oil Prices and Bond Values
Figure 4. Value of Cash Flows
Figure 5. Project Risk Structure