Project Evaluation:
A Practical Asset Pricing Method

by

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A PRACTICAL ASSET PRICING METHOD

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ABSTRACT

This paper presents a practical approach to project evaluation using techniques of modern financial economics, with a sample application to oil development under a complex tax system. The method overcomes shortcomings of conventional DCF methods which are either imprecise about the relation between economic value and uncertainty, or are rigid and unrealistic in the required assumptions about how a project's risks (and therefore its value) are influenced by market conditions, the project physical structure, and tax and contract provisions. It is based on the formulation and estimation of an "information model" which represents the resolution over time of uncertainties underlying a project (oil prices in the examples shown). The project can then be valued using derivative asset valuation, which replicates the consequences of a complex asset by a traded portfolio of simpler assets (in our case, riskless bonds and future claims on oil).

For ease of implementation, the method is designed to resemble current industry practice. The information model can be estimated using analysis and judgment similar to that applied in conventional evaluation. The formulation of decision alternatives, the selection of underlying uncertainties, and the design of a cash-flow model are the same as in standard DCF methods. Simulation and valuation results also can be presented in a familiar format. Restrictions must be placed on the "best" current asset pricing theory to achieve this convenient framework: the expected returns on the basic assets, which comprise the portfolios traded to replicate project cash flows, must be assumed to be known with certainty at the time of an evaluation.

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INTRODUCTION

Standard applications of discounted cash flow (DCF) to project evaluation have been criticized for inducing a bias against long-lived investments and for inadequacy in handling strategic aspects of project choice (Hayes and Garvin, 1982; Myers, 1984; MacCallum, 1987). More generally, it is well known that the risks of a project (and therefore its value) are influenced by market conditions, physical structure, and tax and contract provisions, but DCF does not assist much in understanding and calculating these effects. For some practitioners (e.g., Hayes and Garvin, 1982) the solution is to downplay financial analysis altogether. There are good reasons for the disquiet, but the proposed remedy overlooks opportunities to do better. Here we present a practical approach to project evaluation, using Derivative Asset Valuation, which can illuminate the effects of project structure on risk, and of risk on value. We illustrate the method in application to investments in oil production.

Concerns about DCF

To put the derivative asset approach in context, consider the characteristics of DCF methods in current use. Usually uncertainty is represented by a group of scenarios of the uncertain inputs to the analysis, often in the form of a "base case" supplemented by "high" and "low" alternatives. It is a judgmental process; careful definitions of the procedure are rare, and seldom is quantitative information given about the relative likelihood of the scenarios or how those likelihoods might change with the arrival of new information over time.

These scenarios of underlying uncertain variables are run through a cash-flow model to compute scenarios of cash-flow amounts, each of which is discounted using a common discount rate. The cash-flow simulations themselves are well defined: they illuminate various regions of the uncertain range of cash-flow realizations. But the valuation numbers are problematic. The "value" attached to each scenario may at best be interpreted as an after-the-fact assessment of whether the project would be considered worthwhile, conditional on the occurrence of that scenario and on the existence of alternative investments with rates of return only at the chosen discount rate.

Often managers take such a set of "value" numbers and interpret their central tendency as the expected "return" and their spread as a measure of project "risk", which adds another problem. If the discount rate already has a risk premium in it, then the return number is already adjusted for risk. The effects of uncertainty enter the valuation in two places, and their individual impacts and interaction are not clear. In its influence on value, furthermore, risk should be related to the contribution of cash-flows to uncertainty in the value of portfolios of assets held by project stakeholders. This contribution is measured by the *covariance* of project cash-flows with the portfolio values, not by the cash-flow variances themselves (Cox, Ingersoll, and Ross, 1985).

Some methods are more quantitative in their treatment of uncertainty. A probability may be assigned to each of a small number of judgmental scenarios, or a formal model may be used to simulate a large sample of scenarios distributed according to some probability measure. Such a model can be used to quantify the uncertainty underlying a project, for example by the estimation of current statistics of the cash-flows. When applied to valuation, however, problems again arise. One such method (Hertz, 1964) quantifies the distribution of the scenario "values" in the same manner as the cash-flow simulation quantifies the distribution of scenario cash-flows.[1] Each "value" in the distribution is constructed from a single scenario. The return attributed to the project is some central tendency of the distribution, and the risk is a spread. This approach is merely a quantification of the scenario-based valuation method, and it suffers from flaws already noted.

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1. Hertz measured the "value" of a project by its internal rate of return. More recent formulations use net present value, yielding an approach sometimes called the "expected net present value" method.

Another approach outlined in finance texts (e.g., Breeley and Myers, 1988) pays more attention to the fact that value is properly a function of the whole cash-flow distribution. The expectation of each net cash-flow is discounted, almost always at a constant rate. Frequently, however, the discounting is only vaguely related to the uncertainty in the cash-flows, to the resolution of that uncertainty over time, or to its relationship to macroeconomic uncertainties of interest to investors. At best, the discount rate is calculated by applying a one-period asset pricing model (such as the Capital Asset Pricing Model or CAPM) to estimate a one-period risk premium for a portfolio of assets that has a risk structure "similar" to the project at hand.

The problem is that a single-period representation of uncertainty often is inappropriate for the evaluation of projects that occur over multiple periods. The use of a single known discount rate involves an implicit approximation that key future conditions are known and stationary. These conditions include aspects of the economy such as the term structure of future interest rates and price of risks, and also specifics of the project such as the risk of each of the future net cash-flows (Fama, 1977). But the risk structure of most projects is not stationary: it evolves idiosyncratically as changes occur in the sources and magnitudes of the uncertainties underlying the project, and as this varying uncertainty is filtered through a changing project structure. In particular, it is necessary to consider the period-by-period variation in the structure of risk in order to give appropriate weight to long-term cash-flows (Laughton and Jacoby, 1991b) or to support analysis of operating flexibility (Laughton and Jacoby, 1991a; Laughton, 1991). Moreover, even in the absence of these considerations a constant discount rate framework can bias the comparison of project alternatives, as shown in Section 4.

An Approach Using Derivative Asset Valuation

A theory of valuation that will allow us to avoid these problems has been developed over the last two decades, based on a small set of propositions about the structure of financial markets and the information content of market prices. Often referred to as "options theory" because of its early application to options on common stock, its key proposition holds that valuation may be carried out, to a good approximation, as if financial markets were competitive and free of transaction barriers. In such a market, different assets with the same cash-flow consequences have the same price. Moreover, in such a market it is possible to replicate the cash-flow consequences, and thus the value, of a complex asset (such as an oil-field development lease) by executing a trading strategy in portfolios of simpler assets (such as riskless bonds and oil forward contracts).

Finally, all asset prices are determined by the risk preferences of investors, as reflected in markets. Thus the basic assets that provide information about risk discounting are those that have some direct interaction with future macroeconomic variables. For example, in our application the basic assets are forward oil contracts. These are related to corresponding future prices of oil, which are correlated with the state of the economy.

This set of ideas has had a profound effects on financial markets, such as those for options, futures, and collateralised securities. It gives traders the ability to calculate values of complex assets and to design hedging positions for firms seeking to extract value from their financial structures. As these ideas came into widespread use in the evaluation of financial assets, proposals were made (e.g., Breeden and Litzenberger, 1978; Myers, 1984; Brennan and Schwartz, 1985) to apply them in the evaluation of real assets, which are the main source of value in most situations. The natural resource sector has been a particular focus of this work because of the simple structure of many investments and the long lead times before returns are received, and because the main source of uncertainty, output price, is a good candidate for quantitative analysis.

Unfortunately, the rate of adoption of these new methods has been slow. Applications can require mathematical skills, such as the ability to formulate and solve differential equation problems (e.g., Brennan and Schwartz, 1985), that are not usually found in the planning and analysis divisions of resource companies. Also, the analysis may come in forms unfamiliar to managers used to DCF analyses (e.g., Breeden and Litzenberger, 1978), and often the calculations are formulated so as to require a drastic simplification in the description of project structure (e.g., Paddock, Siegel and Smith, 1988).

To overcome these difficulties we have designed a user interface as similar as possible to familiar DCF methods. Several steps in the analysis are essentially the same as in current practice, including the formulation of decision alternatives, the determination of key uncertainties, and the layout of a cash-flow model. Moreover, the results come in a familiar form: simulations of project cash-flows and representations of their statistics, and calculation of the current value of the project cash-flow streams. To this familiar framework, we have added improvements made possible by the new theory. The basic change, which our proposal shares with all modern asset pricing methods, is the organization of scenarios into a tree that represents in a quantitative manner the way in which uncertainty about the project may be resolved as information arrives over time. This approach supports, as current DCF methods do not, both the calculation of economically well-defined notions of value in a multi-period setting and consistent analyses of future contingent management of a project.

We apply a restricted form of what we consider to be the "best" current asset pricing theory, which may be represented by the work of Cox, Ingersoll, and Ross (1985). The key restriction is that future expected returns for the basic assets (i.e., those in the portfolios traded to replicate project cash-flows) are assumed to be known with certainty at the time of an evaluation.[2] In this way we retain as much as possible of the power of the original theory while making the method accessible to analysts familiar with standard DCF methods. Accessibility is increased in two ways. First,

2. The actual returns may be uncertain; only the *expected* returns are modelled not to be so. Note also that this restriction applies only to the returns of the basic assets, not of the project itself.

calculations in this framework can be performed in a wide class of applications with complex cash-flow models, whereas general calculation tools have not yet been developed for the unrestricted theory. Second, the discount rate information required of managers (for discounting the basic assets) is like that required in DCF analyses in that the rates are taken to be known with certainty.

There are costs to this type of restriction. Future risk-free interest rates cannot be modelled to be uncertain, nor can the risk premia in the expected returns for holding the basic assets of the analysis.[3] These tradeoffs are appropriate, however, considering the desirability of a practical and accessible evaluation approach based on modern asset pricing. Decision support systems are best changed step by step (Keen and Scott Morton, 1978), with the valid intuition of one method carried over into the application of the next. The approach proposed here is a step that an organization might take away from scenario-based DCF, with its well-known limitations, toward better evaluation methods. As more calculation tools become available, and managers become familiar with modern asset pricing methods, other steps may follow.

To implement derivative asset valuation in this setting, two tasks of data gathering, analysis, and judgment are required. First, the uncertainty in oil prices, and the way it is resolved over time, must be represented in a quantitative model. The specification of such a model and procedures for the estimation of its parameters are discussed in Section 2. Second, the current value of the underlying assets (oil bonds in our example) must be determined. Several approaches to this task are discussed in Section 3, which also summarizes the mechanics of derivative asset valuation.

In Section 4 we apply DAV to a set of off-shore oil-field projects based on actual developments in the U.K. North Sea. The focus in the example is on the economic analysis of

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3. Other applications incorporate similar restrictions (Breeden and Litzenberger, 1978; Brennan and Schwartz, 1985). Our restrictions lead to a requirement that the basic macroeconomic variables be modelled to be lognormally distributed. Breeden and Litzenberger require the same for aggregate consumption, as Brennan and Schwartz do for the risk-adjusted distribution of macroeconomic variables.

project value, the risks that influence that value, and how both are influenced by the project structure. The results show that the U.K. tax system absorbs risk, especially because of the operation of its field-level profits tax. This feature makes the share of the cashflows received by the developer less risky than it otherwise would be. We also consider a series of fields of different sizes. There are economies of scale in development and production for the fields considered, and the revenue for each field is riskier than the costs. Therefore a smaller field has proportionally more volatile net cash-flows than a larger field, and higher rates should be used to discount its net cash-flows.

Typically an organization will discount the cash-flows of different projects with a common discount rate, regardless of project characteristics. If this is done for the fields in our example, the smaller fields will be over-valued relative to the larger fields. The derivative valuation approach determines, in a consistent manner, the appropriate discounting structures for projects with different characteristics, and we are therefore able to estimate the bias that common DCF methods would introduce into the evaluation.

Generalizations of the set of applications for which the method might be used are suggested among the concluding remarks in Section 5.

THE OIL PRICE MODEL

Our representation of the uncertain future is the same as that used in most modern financial theory: it consists of a probability measure for scenarios of uncertain inputs into the analysis, with a quantitative description of the process by which this measure will change over time with the arrival of new information. The process may be thought of as a tree-like grouping of the possible scenarios, where branching in the tree occurs as information arrives to distinguish among ever more refined sets of potential scenarios.[4] A model with these features is called an "information model", and in our example we apply this concept to oil prices. As shown below, the analysis and judgment required to estimate such a model is close to that required for standard DCF analysis.

Specification

Work with information models requires two time concepts, which we call "information time" and "variable definition time". Information time, noted by the index s, marks the movement of the observer through time. Variable definition times, on the other hand, distinguish among events which serve to define variables. An example of the latter is an oil price, where the event is the sale of a barrel of oil on a specified date. The oil price at each time of sale is a different variable. However, it is natural to think of the time series of oil prices as a group of variables with the same name, and to use the time of sale to distinguish among them. Variable definition times for oil prices are denoted by the subscript u (e.g., $\tilde{P}_u$ for the price of oil at time u).[5]

Cash-flow amounts in a cash-flow model demonstrate an important instance of a variable-definition time index. Each amount has associated with it the particular time at which the cash actually flows. Borrowing from the terminology of the bond market, the variable definition time for a cash-flow amount is called its "maturity time". Because the maturity time of a cash-flow is such an important variable definition time, it is given its own subscript t.

Some concepts needed for analysis use more than one notion of time. An example is the expectation of a future oil price, $E_s(\tilde{P}_u)$, which may be interpreted as the expectation at (information)

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4. For a description of the oil price model using tree structures and state pricing methods, see Jacoby and Laughton (1991).

5. A tilde over a variable indicates it is uncertain.

time s of the oil price at (variable definition) time u.

With these definitions, we can discuss the characteristics of an information model of oil prices. At a point of project decision, taken here to be at $s=0$, oil prices at times $u>0$ are not known. Nonetheless it may be possible to summarize available knowledge of future oil prices by some measure of central tendency in each, such as the expected magnitude, and to make statements about how the uncertainty in $\tilde{P}_u$ might be resolved over (information) time. At $s=0$ the expectation of the oil price at (variable definition) time u, $E_0(\tilde{P}_u)$, is some given number. When we arrive at (information) time $s=u$, the oil price will be known, and $E_{s=u}(\tilde{P}_u)$ will be the realization, $\tilde{P}_u$. Between $s=0$ and $s=u$, the expectation $E_s(\tilde{P}_u)$ will follow some path, the precise trajectory depending on the nature of new information gained along the way.

For our analyses we use a model based on the approximation that all the information needed to determine the revision in expectations of future oil prices is provided by the most recent unanticipated change in the expectation of the current price.[6] To illustrate the questions raised by such a model, consider how the term structure of oil price expectations might change from (information) time $s=0$ to $s=1$. At $s=0$ we do not know what information will become available between $s=0$ and $s=1$. However, we might ask how the revision in the expectation for each of the prices $E_1(\tilde{P}_u)$ - $E_0(\tilde{P}_u)$) for $u>1$, will depend on the (variable definition) time u of that price, given a particular realization of the price at time 1. To explore this issue, some particular questions include:

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6. More precisely, the new information arriving during the period from s to $s+\Delta s$ is the revision over this period in the expectation of $\tilde{P}_u$ where $u=s+\Delta s$. The specialization to one piece of information in each period is for simplicity. For a model involving two types of information, shorter and longer term oil market uncertainty, see Laughton (1988).

- Will the revisions of the expectations for oil prices at different times be proportionally the same in relation to the realized revision for the price at (variable definition) time $u=1$?
- If they are different, might the revisions differ in sign?
- If different, might the revisions be greater for prices in the near term than in the far term?

Underlying these the responses to these questions will be some notions about the behavior of oil markets and their influence on the way oil price expectations are revised over time:

- Is there an underlying upward drift in real oil price because of the nonrenewable nature of the resource?
- Is there some cost floor which tends to put a lower boundary on price, and thus on expectations about price?
- From the demand side, is there a ceiling on the oligopoly price that can be sustained?
- What is the time scale for the adjustment of supply or demand to shocks in the price for oil? Is it related to the time needed to bring new production capacity on stream or to change the stock of energy-using capital?
- How are unanticipated changes in the expectations of oil prices correlated with uncertainties about other measures of economic activity?

Answers to these types of questions constitute a qualitative description of the way uncertainty is resolved over time, and they set the outline for an information model of oil prices.

To formulate a quantitative version of such a model, consider a circumstance in which the length of the (information) time periods, Δs, is vanishingly small. The initial term structure of oil price expectations is an input to the process. During each period, the expectation of price for each future time is revised in response to the new information arriving during that period. For each period, we take these revisions of expectation to be determined by a single normally distributed variable with variance of order Δs. The indicator of motion through (information) time is $\Delta \tilde{z}_{OM,s}$.[7]

The subscript *OM* identifies the information as that emerging from oil markets, and the time s is the beginning of the period being considered. The normalization commonly chosen for these variables is $E_s[\Delta \tilde{z}_{OM,s}] = 0$ and $E_s[(\Delta \tilde{z}_{OM,s})^2] = \Delta s$. Because $\Delta \tilde{z}_{OM,s}$ represents new information in its period, it is independent of its counterparts in other periods.

The revision of expectations within each period may now be specified as

$$E_{s+\Delta s}(\tilde{P}_u) - E_s(\tilde{P}_u) = E_s(\tilde{P}_u)\sigma_{P_u,OM,s}\Delta \tilde{z}_{OM,s}, \tag{1}$$

which states that, in each period, the revision for each price is proportional to the expectation of that price at the beginning of the period, $E_s(\tilde{P}_u)$, and to the normalized information, $\Delta \tilde{z}_{OM,s}$. The volatility, $\sigma_{P_u,OM,s}$, determines the scale of the revision in the expectation of $\tilde{P}_u$ due to the arrival of new information of type *OM* during the interval after (information) time s.

A key restriction on the class of oil price models to be considered is that the volatility of the oil price expectation for any future period of (information) time is taken to be known with certainty at the time of the analysis. While σ may vary with (information) time and the (variable definition) time

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7. The important aspects of this specification are that the scale of the uncertain change in any variable, $\sqrt{\Delta s}$, is larger than the scale of any expected change, Δs, and that the probability of any variable change larger than order $\sqrt{\Delta s}$ is very small. Therefore the motion may be jittery but does not frequently have significant jumps.

of the price, it may not vary for a given information time across states of the oil market at that time.[8] (As shown below, this requirement follows from our assumption that expected rates of return of underlying assets are known with certainty.)

By generating a set of realizations of the sequence of random variables $\Delta \tilde{z}_{OM,s}$, Equation 1 can be used to compute a set of dynamic paths in (information) time for $E_s(\tilde{P}_u)$, and a set of realizations of the prices, $\tilde{P}_u = E_{s=u}(\tilde{P}_u)$. Although the theory is developed in terms of infinitesimal changes $\Delta \tilde{z}_{OM,s}$, practical calculations are done on the basis of an annual cash flow model. It can be shown (Laughton, 1988) that, if annual variables are used in cash-flow models, and σ is modelled not to vary in information time over the course of each year, then we do not need to generate realizations for all small periods over a year but only for their sum, which we denote (for the year beginning at time s) by

$$\tilde{z}_{OM,s} = \int_s^{s+1} \Delta \tilde{z}_{OM,s}.$$

This annual information, $\tilde{z}_{OM,s}$, is a normal variable with unit variance. Then in practice Equation 1 becomes

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8. For simplicity of exposition, we often take σ to be constant both over information time s and over oil price time u (at least for $s<u$: $s=0$ for $s>u$ when there is no more uncertainty about the price at u). In practice, σ might better be modelled as decreasing in u-s because, in any given period, we typically get more resolution about uncertainty of near-term prices than far-term prices. Information becomes staledated. In particular, short-run economic dislocations from longer-run equilibria occur more frequently than changes in long-run equilibria themselves, so prices exhibit reversion. If σ declines with the time to maturity of the cash-flow, then the per period discounting of a long-term cash-flow should be less than for a short-term cash-flow. Under these circumstances, constant per-period discounting introduces a bias against the long term. In Laughton and Jacoby (1991b) we show how to correct this flaw by using information models in which σ declines exponentially in time to maturity.

$$E_{s+1}(\tilde{P}_u) = E_s(\tilde{P}_u)\exp(-1/2\sigma^2_{P_u,OM,s} + \sigma_{P_u,OM,s}\tilde{z}_{OM,s}). \tag{1a}$$

With a large number of realizations it is possible to calculate statistics of the resulting sample that are close to the statistics of the true probability measure of the scenarios for oil prices.[9]

Estimation Methods

The parameters of the information model include the current term structure of expectations, $E_0(\tilde{P}_u)$, and the volatility parameters, σ.[10] The σ parameters might be estimated from a statistical analysis of oil price history based on an assumption that the future will be similar to the past; or estimates could be inferred from expert judgment. We illustrate the latter approach, in which quantitative opinions are sought about specific statistics of the marginal probability distributions of oil prices. These estimates might be based on currently available information, or on sets of hypothetical information at future information times. For oil prices, a device we have found useful is to examine "windows" of the distribution of each price (Jacoby and Paddock, 1985).

The first task is to determine the term structure of current expectations which, as noted earlier, is similar to asking for a "base case" scenario. In practice, we have found it easier to obtain

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9. Other applications of modern asset pricing (e.g., Brennan and Schwartz, 1985) formulate the information model in terms of movements in price itself rather than in the expectation of prices. Where σ is independent of u, our process stated in price terms is $\tilde{P}_{s+\Delta s} - \tilde{P}_s = \alpha_s P_s \Delta s + \sigma_s P_s \Delta\tilde{z}_{OM,s}$, where α_s is the proportional change over maturity time of the current expectation of the price when the maturity time is set to the information time being considered. That is, $\alpha_s = \partial u E_0(\tilde{P}_t)/E_0(\tilde{P}_t)\,|_{t=s}$ (Laughton and Jacoby, 1991a). For the case where σ is exponentially declining in u-s, see Laughton and Jacoby (1991b).

10. Where no confusion will result, the volatility is simply denoted σ, with its subscripts taken to be understood.

estimates of the current medians of the prices, $M_0(\tilde{P}_u)$, rather than the expectations, but the effect is the same.[11] Second, we seek responses that can be used to determine the parameters of σ and check the validity of the model. One such set of data is the current upper and lower fractiles (i.e., price windows) of the distribution for each price considered individually. In our oil price experiments, 0.1 and 0.9 fractiles have been used. If σ is constant, the *F*th fractile for the price at (variable definition) time u is $M_0(\tilde{P}_u)\exp[\sigma\sqrt{u}\,N(F)]$, where $N(F)$ is the *F*th fractile of the standard unit normal probability distribution.[12]

Each window contains two pieces of data, one each from the top and the bottom, about those combinations of the σ parameters that determine the overall uncertainty in each price. (In an application where σ is constant over time, for example, one measure of the overall uncertainty in $\tilde{P}_u$ is $\sigma\sqrt{u}$, which is the standard deviation of its logarithm.) These estimates can thus be used both to estimate the overall uncertainty and to check whether its assumed specification is consistent with the

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11. The two contain equivalent information when combined with σ. If σ is constant, then $E_0(\tilde{P}_u) = M_0(\tilde{P}_u)\exp[1/2\,\sigma^2 u]$.

12. This exponential of a normal fractile comes from the lognormal structure of the price probability distributions. Equation 1 looks almost like the differential equation for an exponential function in z. The expectation at (information) time s in the future (that is not a small period of time away from the present) is distributed, as of the current (information) time, like the exponential of a normal variable, i.e., like a lognormal variable. The expectation of this lognormal variable is the current expectation of the price, and the variance of its corresponding normal variable, with σ constant, is $\sigma^2 s$.

The distribution of the price $\tilde{P}_u$ is also the distribution of its expectation $E_{s=u}(\tilde{P}_u)$. The variance of the relevant normal variable, if σ is constant, is $\sigma^2 u$. Therefore $\tilde{P}_u$ is proportional to $\exp[\sigma\sqrt{u}\,\tilde{z}]$, where $\tilde{z}$ is a unit normal variable. Because the realization of $\tilde{P}_u$ would in this case be an increasing function of the realization of $\tilde{z}$, the fractiles of $\tilde{P}_u$ would be related to those of $\tilde{z}$ by this same function. The result is the relation noted.

responses.[13] Figure 1 below shows such windows for the oil price model used in our sample valuations.

Advantages and Difficulties

The use of a formal information model to represent the future has both strong and weak points, as does the use of a set of qualitative scenarios. The strength of the formal model is that it is quantitative and precise. It describes information flow explicitly and specifies the relation between information flow and uncertainty resolution. This is why the approach now dominates modern financial economics.

With no further elaboration the information model can be applied to cash-flow simulation. Equation 1 can be used to generate a sample of oil-price paths, all beginning at P_0, which can be input into the cash-flow model to produce a sample of scenarios for cash-flows. The sample mean of each cashflow amount will be (for a large enough sample) a good estimate of its expected magnitude. Other sample statistics of the cash-flows can be calculated as well, including current windows for project cash-flow variables akin to those shown for oil price in Figure 1. Or distant windows can be conditioned by an assumed time path of price realizations over the near term, thus providing a dynamic simulation environment which is richer than the cash-flow pro-formas calculated using a scenario-based representation of uncertainty.

Also, as shown in the next section, the information model can support a well-defined process of economic valuation. Analogies can be used to set up a discounting framework, drawn at the

13. The oil price windows viewed as of (information) time $s=0$ may not test all aspects of the set of probability measures for oil price scenarios. For example, they may not reveal possible correlations between two or more future prices, and they do not test whether the way in which uncertainty is resolved over time is adequately represented. These issues can be studied by constructing windows from postulated vantage points in the future, conditioned on scenarios of the prices (Laughton 1988).

relatively simple level of underlying uncertain variables rather than at the level of the project itself, in all its complexity. With this underlying valuation in place, the theory of replicating portfolios can be applied to determine how a particular cash-flow model transforms the discounting of the relatively simple risks of the basic underlying assets into the discounting of the more complex risks of a project. Because there are no restrictions on the time pattern of the underlying risks, or on their transformation by the cash-flow model into project risks, the project risks need not be forced into the restrictive pattern required by methods based exclusively on one-period asset pricing models (like the CAPM). Moreover, the discounting structure at the project level is not an input to the valuation but an output, as is required for analysis of the effects of risk on value.

The principal difficulty in using information models arise from the fact that their specification and estimation are somewhat more difficult than the creation of a relatively unstructured group of scenarios, and more explicit judgment and information are required on the part of management. The strengths of the scenario description, primarily familiarity and ease of use, and the corresponding shortcoming of the process description are important reasons why firms have thus far tended to use scenarios. A key current task is to make the modelling of quantitative trees of scenarios as familiar and easy for managers in the future as the modelling of ordinary groups of scenarios is now.

VALUATION

As noted in Section 1.2, if the magnitude of each of the cash-flows associated with an asset (called the derivative asset) can be determined by the contemporaneous value of other assets (called the underlying assets) then the value of the derivative asset can be calculated in certain circumstances from the values of the underlying assets.[14] This is done by creating a trading strategy in portfolios

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14. See Merton (1977) or Cox, Ingersoll, and Ross (1985). Some technical conditions are required about the ease of trading and information dissemination in financial markets, as mentioned in Section 1, and the smoothness or continuity of the effects of new information (Merton, 1977). Our information

of the underlying assets which is designed to replicate the cash-flows of the derivative asset, and thus its value. Application to oil investment can take advantage of the fact that, within the context of a fixed operating policy, a project is a portfolio of claims to individual cash-flows. Cash-flows by type and year can be valued individually, then summed to yield the value of the project.

In the oil-field development example used here, each uncertain cash-flow is contingent on the magnitude of various oil prices. Moreover, in a situation where oil prices are the only uncertain variables, the cash-flow amounts are contingent *only* on these oil prices. Each oil price can be formulated as the terminal value of an asset that we call an "oil bond." Each oil bond is a claim to a single cash-flow at some (maturity) time t, where the cash-flow amount is the spot price $\tilde{P}_u$ at some (variable definition) time $u \leq t$.[15] These oil bonds, along with ordinary cash discount bonds,[16] are the underlying assets of the valuation, and a key step in derivative asset valuation is to determine their future values.

Valuation of the Underlying Assets

Compared to methods of project evaluation now in common use, modern asset pricing

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models satisfy the continuity conditions because of the very small probability of large changes in information during any short period of time. The interpretation of the results of the derivative asset valuation depends on the degree to which real markets satisfy these conditions.

15. An oil bond is similar to a forward contract on oil. However, a forward contract is a mutual obligation to exchange a fixed amount of oil for a fixed amount of cash at a specified time in the future, while an oil bond results in an exchange of cash *now* for cash at some time t in the future, the future cash amount determined by the value at some time $u \leq t$ of a fixed amount of oil.

16. Cash discount bonds are like regular coupon bonds with the coupons stripped off, so the bond is a claim to only one payment at maturity. They are sold at a discount in relation to bonds with coupon payments, hence the name "discount bond."

methods specify the effects of uncertainty on value at an earlier stage of the analysis. The economic analysis of risk is applied to the uncertain inputs to the project cash flow model rather than to model net cash flow.[17] In conventional DCF analysis the question is, "What is the right discount rate for the project?" Where oil prices determine the uncertainties, the equivalent question in a derivative asset valuation is, "What is the value today of two claims, one to a unit of cash and the other to a unit of oil, that will mature at each time in the future; and how will the uncertainty in the value of those claims be resolved over time?"

In addressing these questions, we first assume that claims to oil or cash can be valued as if future short-term nominal and real risk-free rates of return were known. Real or nominal cash discount bonds may then be valued by using short-term real or nominal risk-free rates, denoted r_s, to discount the risk-free cash-flows. If these rates are constant, the value at (information) time s of the cash discount bond maturing at time t is

$$V_s(t) = \exp(-r(t-s)). \tag{2}$$

Information about nominal rates can be extracted from yields on relevant government obligations. Real rates can be calculated either by using some long-term average of past real rates or by setting up a model of future inflation rates and subtracting these from the nominal rates.

Several methods are available for establishing the current value of the oil bonds. It may be possible in some circumstances to use market data, for instance by using forward or futures prices, prices in the spot market for producing oil-fields, or prices for securities based on such fields.

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17. Lessard (1979) uses value additivity to shift discounting from the level of the project level to its components. His components are streams of cash-flows such as revenue, rather than smaller cash-flow units, and his discounting is based mainly on the one-period asset-pricing model mentioned in Section 1.1.

Alternatively, the judgment of managers or other experts may be applied to estimate hypothetical forward prices, which can be converted to bond values by discounting at the riskless rate. Or bond values may be estimated directly, by answering the question, "What would the company pay today for a barrel of oil to be delivered in a particular year in the future?" Finally, a discounting structure based on judgment can be used to convert expected oil prices into oil bond values. This last approach is illustrated here. Estimates made using this last approach can be used directly in the evaluation, or as a check whether other bond value estimates are consistent with the information model and notions of oil price risk.

To calculate the bond values using a discounting approach we need, for each (information) time s, the expected rate of return on a claim to a cash-flow of amount $\tilde{P}_u$ for each (maturity) time $u > s$. This rate, denoted $R_s(\tilde{P}_u)$, is taken to be the sum of two terms: the short-term risk-free rate of return (the return for time) and a risk premium which takes account of the quantity of oil price risk and the market valuation of that risk:

$$R_s(\tilde{P}_u) = r_s + PRisk_{OM,s}\sigma_{P_u,OM,s} \qquad s < u. \tag{3}$$

Here r_s is the risk-free rate at time s. The term $PRisk_{OM,s}$ is the price at time s for the risk due to oil-market uncertainty OM.[18] We mentioned earlier that practical DAV calculations require that we

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18. This form of discounting can be based on a valuation framework which has only one class of risk arising from the correlation between the unanticipated change in the value of an asset and the unanticipated change in a single "risk factor". An example in one time period is the CAPM, in which the risk factor is the value of a broad-based benchmark portfolio of assets. In the CAPM, the price of oil-market risk, $PRisk_{OM,s}$, is the product of an overall price of risk, which is the ratio of the risk premium in the expected return on the benchmark portfolio divided by the standard deviation of that return, multiplied by the correlation between that return and the variable that represents new oil-market information, $\Delta\tilde{z}_{OM,s}$ (Laughton 1988).

model the future expected rates of return on these underlying risky assets as if known with certainty, and therefore we must treat their determinants (r, *PRisk*, and σ) as also known with certainty. Note that the certainty of σ is required only if the volatility results from uncertainty that has a non-zero price of risk, i.e., macroeconomic uncertainties rather than purely technical or physical ones. [19]

If the current expectation of the oil price, $E_0(\tilde{P}_u)$, is discounted using the expected rates of return established in Equation 3, the result is the value at (information) time $s=0$ of a claim to $\tilde{P}_u$ to be received at time u. For ease of exposition, we consider a situation in which r, *PRisk*, and σ are constant over time s.[20] Then the value can be expressed as follows:[21]

$$V_0(\tilde{P}_u, u) = E_0(\tilde{P}_u)\exp(-(r + PRisk\sigma)u).$$

Somewhat more convenient for our purposes, it may be restated as

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19. The certainty of σ puts constraints on the modelling of basic macroeconomic variables (Laughton, 1988). At any time they must be lognormally distributed.

20. The model may be generalized to situations in which (1) the risk-free rates and the prices of risk depend on the (information) time, (2) there is more than one uncertainty factor, and (3) σ depends on the (variable definition) time of the price being considered, the type of uncertainty, and the (information) time.

21. A cash flow at time t may depend on the oil price at time $u<t$ (e.g., a tax-loss carryforward). In the replication it is desirable to consider a bond maturing at time t and paying the price at time u. Equation 4 becomes $V_0(\tilde{P}_u, t) = E_0(\tilde{P}_u)\exp(-(r+PRisk\sigma)u)\exp(-r(t-u))$. The first factor discounts for both time and risk over the period until oil price $\tilde{P}_u$ is known, and the second discounts only for time, from time u of the oil price up to time t of the cash flow (a period over which there is no risk due to uncertainty in this oil price).

$$V_0(\tilde{P}_u,u) = E_0(\tilde{P}_u)\exp(-PRisk\sigma u)\exp(ru), \tag{4}$$

where the first discount factor is for risk and the second is for time. The product of the first two terms in (4) can be thought of as the risk-adjusted expectation of oil price. If the superscript *RA* is used to denote a risk-adjusted quantity, it can be expressed as

$$E_0^{RA}(\tilde{P}_u) = E_0(\tilde{P}_u)\exp(-PRisk\sigma u). \tag{5}$$

This risk-adjusted expectation is frequently called the "certainty equivalent" of the oil price. It also is the forward price that would appear in oil markets. This quantity plays a significant role in the detailed calculations of cash-flow value, to which we now turn.

The Mechanics of Valuation

The theory behind dynamic portfolio replication shows that the unanticipated proportional change in the risk-adjusted expectation of an oil price, $E_0^{RA}(\tilde{P}_u)$, evolves in response to new information according to the same probabilistic process as the unanticipated proportional change in the value of a claim to that price, $V_0(\tilde{P}_u)$ (Cox, Ingersoll, and Ross, 1985). And, in the absence of uncertainty in expected returns, the unanticipated proportional change in the true expectation of that oil price, $E_0(\tilde{P}_u)$, responds according to the same process.[22] Thus, for constant σ, the true probability distribution of oil price, $\tilde{P}_u$, is of the form

22. Again, for an exposition of these relationships using some simple tree diagrams and state pricing methods, see Jacoby and Laughton (1991).

$$\tilde{P}_u = E_0(\tilde{P}_u)\exp(-1/2\sigma^2 u)\exp(\sigma \int_0^u \Delta\tilde{z}) . \tag{6a}$$

The risk-adjusted distribution of the price is of the form

$$\tilde{P}_u^{RA} = E_0(\tilde{P}_u^{RA})\exp(-1/2\sigma^2 u)\exp(\sigma \int_0^u \Delta\tilde{z}) . \tag{6b}$$

This risk-adjusted price is then the basis of the value calculation. The steps are the following.

(1) Set up a sample of scenarios of the uncertain inputs into the cash-flow model (the risk-adjusted oil prices) using Equation 6b, beginning at $s=0$ with the initial risk-adjusted expectation given in Equation 5.

(2) Put these uncertain inputs into the cash-flow model, scenario by scenario, to produce a risk-adjusted sample of cash-flow amounts.

(3) Find the mean of this risk-adjusted sample for each cash flow. These are estimates of the risk-adjusted expectations or certainty equivalents of the cash flows.

(4) Discount using the risk-free rate to yield the current values of the cash flows.

Several approximations underlie the valuation that results from this procedure. Recall that the expected returns on the basic assets must be assumed known with certainty, which implies that r, *PRisk*, and σ must be approximated as certain. We should point out, however, that stronger versions of these approximations are required to justify any valuation based completely on DCF methods. Our method requires only that the macroeconomic variables that are uncertain inputs into the project cash-flow model be jointly lognormal. DCF requires at least that the additive components of the net cash-flows be jointly lognormal and, if discounting is done at the level of net cash-flow, that the net cash-flows be jointly lognormal as well.

Also, DAV requires that assets be continuously available for trade at unique prices. However, it should be noted that, while liquid markets for long-term oil bonds do not exist at present, oil price risk may be traded in impure forms in forward and futures markets for oil and in the markets for oil company securities and portions of developed fields or undeveloped leases. Moreover, much can be learned about a project by valuing it *as if* the company could sell claims to the underlying components of the project directly into oil bond markets instead of indirectly into the securities or asset markets.

APPLICATION TO OIL INVESTMENT

Implementation of the Method

The method is illustrated with a 300 million barrel (mmbbl) oil-field development based on an actual project in the U.K. sector of the North Sea, and then with a series of similar projects of different sizes. The proportional production profile and the pattern of real capital costs for each project are given in Table 1. The real fixed operating costs for each are $85 million per year during production. Real variable operating costs are $2 per barrel.

The tax system for the U.K. Outer Continental Shelf includes a Petroleum Revenue Tax (PRT) and a Corporate Income Tax (CIT). The PRT tax base is the project operating income less a deduction for capital recovery and a complex "oil allowance." There are restrictions on the PRT deduction, but carryforward is allowed. The PRT is a project-level tax and typically there are substantial PRT losses to carry forward from the initial investment. The tax base for the CIT is corporate-level income, and the contribution from the project is operating income less allowances for capital cost and PRT paid. In these sample calculations corporate income is assumed to be very large, so CIT shields are used immediately.[23]

Oil prices are the underlying uncertain variables for the project cash-flow model, and

23. For more details on this tax regime, see Jacoby and Laughton (1991).

uncertainty is modelled according to Equation 1a. Annual volatility σ is a constant 0.1 across prices and over (information) time. The real oil price medians are assumed to rise at 3% per year from a base of $18 per barrel. The resulting windows on the real oil price distributions are shown in Figure 1.

The valuation of claims to oil in the future is based on Equation 3. In this example, however, we do not start with the component parts and build up an estimate of $R_s(\tilde{P}_u, u)$. We go about the task the other way around, moving directly to an estimate of an "average" expected rate of return using information and judgment about the oil industry, and then checking for consistency with the model of expected returns underlying Equation 3 (with constant *PRisk*).

Typically in the oil industry, development projects are evaluated at annual discount rates in a range around 10% real. We take this rate to be a good representation of the "average" return for time and risk in these projects. Then we observe that these risks consist, in the main, of underlying oil-price risks imbedded in the revenues which are levered by less risky capital and operating costs. We make a rough correction for this leverage effect, and estimate that the appropriate "average" discount rate for an "average" pure oil claim is around 7% per year real. With this estimate we calculate the oil bond values, which are shown in Figure 2.

Are these estimates of the discounting of oil bonds plausible? With the real risk-free rate at 3% per year, the risk premium on the oil bonds is about 4% per year. With $s=0.1$, Equation 3 gives *PRisk*=0.4 in annual terms.[24] One rough check on this result is to estimate what parameters would be consistent with this level of *PRisk* under a one-period CAPM valuation framework. A commonly accepted figure for the price of risk of the market portfolio is 0.5 in annual terms (Ibbotson

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24. These quick calculations are possible because the information structure has only one source of uncertainty, and r, *Prisk* and σ are all constant in this example.

Associates, Inc., 1989). Such an estimate would imply a correlation of 0.8 between changes in the oil price uncertainty factor and changes in the value of the market portfolio. This number seems reasonable for periods when oil markets are strongly influenced by demand conditions, as we expect they will be for the coming decade. However, under conditions of supply-side shock, as existed twice in the 1970s, this correlation could be negative. Unfortunately, the valuation method requires that this parameter be known with certainty, and we cannot make it depend on uncertain future conditions.[25]

A more complete study of the issue would include further analysis of the parameters used above and of the resulting bond values, perhaps supported by analysis of financial market data. In the implementation of this procedure, the debate and discussion of these estimates of oil bond value are a critical aspect of the process of project assessment. Analysis and judgment thus are focused, as they should be, on the heart of the issue of oil-field development: the worth today of claims to future oil production.

One final set of parameters is the time series of inflation factors, which is taken to be generated by an inflation rate of 5% per year. Combined with the real risk-free rate of 3% per year, nominal return to risk-free bonds is 8% per year.

Sequences of the information variables, $z_{OM,s}$, are generated, and they are used to compute both a sample distributed according to the true measure of scenarios of oil prices and a sample for the risk-adjusted measure. The resulting samples are then applied in the two forms of analysis mentioned above: simulation and valuation. For an exploration of some of the simulation results this method makes possible, see Laughton (1988). We now turn to the detailed examples of valuation.

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25. There are two solutions if this appears to be a problem. An analyst may take some average and pretend it is not uncertain (as we do) or, if this approximation appears too damaging, formulate the model with supply and demand determinants as underlying variables.

Results for the 300 mmbbl Field

Figure 3 shows the current value of the claims to each of various cash-flows, year by year, that make up the 300 mmbbl project. Table 3 shows the totals. The two tax obligations are considered separately, so that their timing and magnitude can be clearly seen. Information of this type should improve understanding of project economics and help in fiscal planning. DCF methods do not support such a detailed analysis of the components of value because, at best, the discounting structure is valid for the net cash-flows, not for its components.

Derivative asset valuation also can be used, like an "X-ray machine", to study the internal risk structure of a project. Figure 4 and Table 2 use the current real equivalent constant discount rate (ECDR) as a measure of the riskiness of these cash-flows. The current real ECDR of a cash-flow or of a cash-flow stream is that constant discount rate which, when applied to the "deflated" or "real" (as opposed to "inflated" or "nominal") current expected magnitude of that cash-flow or cash-flow stream, produces its current value.[26]

We note first in Figure 4 and Table 2 that the current real ECDR is 7% per year for revenues of all maturities and for the revenue stream as a whole. These cash-flows are proportional to the oil price, so this is simply our oil bond discounting specification coming back in another form. Moreover, the current real ECDR of the risk-free costs (also shown in Figure 4) is the real risk-free rate of 3% per year for each cost and for the stream of costs. The risk of the pre-tax cash-flow during production is higher than that of gross revenue because of leverage effects of costs. For the first few

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26. This simple way of displaying risk breaks down for single cash-flows if $E_0(\tilde{X},t)$ is of opposite sign to $V_0(\tilde{X},t)$, or either is equal to zero. In our example, the revenue is zero in years years 0 to 3 and from year 15 on, and the pre-tax cash-flow is zero in the post production years. The after-tax and pre-tax cashflows have expectation and value with opposite signs in the last year of production (year 14). The ECDR for a cash-flow stream may also not exist if the polynomial equation that determines it has no positive real roots, a problem similar to that posed by internal rate of return calculations.

years of production, the leverage is relatively weak because the value of the claims to the costs are low in relation to the value of the claim to the revenues. But leverage increases as oil output declines, increasing fixed operating costs as a fraction of revenue.

These results illustrate one of the aspects of our critique of DCF analysis. A constant discount rate implies a flat ECDR structure. Figure 3 shows that the very nature of the project rules against a constant rate. The choice of any constant rate as a surrogate for the more complex process requires a complicated weighing of effects, which cannot be undertaken without analyzing the project first.

Figure 4 and Table 2 also show the risk in the after-tax net cash-flows. The effect of the tax system on the ECDRs is striking. The current real ECDRs are lower after-tax than pre-tax, indicating that the tax payments are more risky than the pre-tax cash-flows. Moreover, in some years a great deal of the oil market risk is transferred to the government, as indicated by the deep drop in the current real ECDR of the net-tax flows in years 6 and 7. The drop occurs because, with this project and outlook for oil price valuation, the PRT tax loss carryforwards are likely (on a risk-adjusted basis) to run out in these years, making the actual amount of PRT collection highly dependent on the uncertain path of oil prices and therefore highly risky.

A Comparison of Fields of Different Size

Figure 5 shows the ECDRs of the cash-flow streams of projects with the same capital, fixed and unit variable operating costs, and the same proportional production profile, but with field sizes ranging from 150 mmbbl to 300 mmbbl. The revenue and cost ECDRs are constant across field size at 7% per year and 3% per year, revealing once again the oil bond and cash bond discounting.

Note that the ECDR of the pre-tax, tax and net cash-flow streams all decrease with field size at a decreasing rate. This happens because the leverage noted above decreases as field size increases and the fixed costs become smaller relative to the revenues. The taxes are more levered than the pre-tax stream for all field sizes because of all the special leverage-increasing effects that are not in the pre-tax stream. As a result the net cash-flow stream is less levered than the pre-tax stream for all field sizes. Moreover, although the spread in the ECDR between the pre-tax and the tax cash-flows decreases with field size, the increase in the relative value of the taxes causes the ECDR of the net cash-flow stream to fall more than that of the pre-tax stream.

Finally, the PRT leverage (not shown) is the highest of all the streams, and decreases dramatically from 0.345 per annum at 150 mmbbl to 0.120 at 300 mmbbl as the effect of various leverage-increasing features that are special to the PRT, such as the oil allowance, uplift and tax loss carryforwards, become less important with large fields. The CIT also is more levered than the pre-tax cash-flows for small fields because of the capital allowances. However, in this case it becomes less levered than the pre-tax stream for large fields because the PRT deduction also becomes larger and soaks up risk.

This general decline in net ECDRs with field size means that the larger fields will be undervalued relative to the smaller fields if all are discounted with the same discounting structure, as they would be using standard DCF methods.[27] To demonstrate this effect, Figure 6 compares the field value calculated using our proposed approach with the value using a DCF method that discounts the true expectation of the net cash-flows at 10% per year. The conclusion is that a dependence on standard DCF methods can lead to a bias against the "now or never" development of larger as opposed to smaller fields if there are economies of scale in development and if revenues are more

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27. The purely value-additive method of Lessard (1979) can deal with the pre-tax part of this particular problem, but it does not provide a means of determining the effects on appropriate discounting of the complicated nonlinear and intertemporal effects in the taxes.

risky than costs.[28] This problem may be corrected by the use of modern asset pricing insights such as those embodied in our proposed approach to project evaluation.

CONCLUSIONS

By means of a carefully-chosen set of restrictions, we have demonstrated how to construct a method of project evaluation that preserves the essence of modern concepts of asset pricing, but that also yields a set of calculatons and supporting judgments that are easily accessible to operating managers brought up on standard DCF. The method removes the "black box" characteristics that hamper the usefulness of many applications of options methods. The resulting transparency and ease of implementation should facilitate the transfer to real assets of ideas that have proved so fruitful in application to financial assets.

Because the purpose of this paper is the illustration of the method itself, the market and project structure and the tax system behavior have been kept simple. Other characteristics can be analyzed using this approach, including the effects of different oil price regimes, production profiles or cost structures, the effects of an uncertain tax-loss carry-forward status for the developer, or the value of a tax change such as an increase in the oil allowance or the uplift. All of these would have implications both for expected cash flows and for implicit discounting. DCF methods would not help much to determine the effects on discounting, while derivative asset valuation methods would capture the effect on both.

The oil price model used to illustrate the method also is a simple one; more realistic versions would likely contribute additional insight. An important example (one that is easy to implement) is the adjustment of the model to reflect reversion, which can correct for a bias against long-term

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28. Use of some other fixed discount rate only shifts the DCF curve roughly in parallel, leaving the bias the same.

investments that can occur in methods using constant per-period discounting. Also, we have considered only one underlying source of uncertainty: oil price uncertainty in the revenue. Uncertainties in costs or in recoverable reserves could be added.

Finally, this implementation of derivative asset valuation opens the door to analysis of projects with operating flexibility, such as the ability to stop and restart production, delays in construction, phased development, abandonment, or technology switching. The best management policy might be computed completely within the framework developed above, although these applications likely will prove quite limited. Fortunately, there are a wide range of applications for which trial policies can be formulated on the basis of judgment or the results of other analyses. The methods in this paper can then be used to select the best from among those tested. Thus the approach here is a practical step toward new approaches to the design, planning, and management of projects, as well as to the correction of damaging shortcomings of methods now in common use.

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Table 1. Capital Costs and Proportional Production Profile

Time since start	Capital Cost ($mm)	Proportional Production Profile (1/year)
0	112	0.00
1	389	0.00
2	320	0.00
3	278	0.00
4	84	0.11
5	24	0.17
6	27	0.17
7	0	0.17
8	0	0.12
9	0	0.08
10	0	0.06
11	0	0.04
12	0	0.03
13	0	0.03
14	0	0.02

Table 2. Project and Cash-Flow Stream Value and Risk (300 mmbbl field)

Stream	Value ($mm)	ECDR (1/year)
After-tax CF	668	0.075
Pre-tax CF	1842	0.092
Revenue	4205	0.070
Cost	2363	0.030
PRT	697	0.105
CIT	477	0.083

Figure 1
Current 0.5 (Median), 0.9, and 0.1 Fractiles
of Future Real Oil Prices

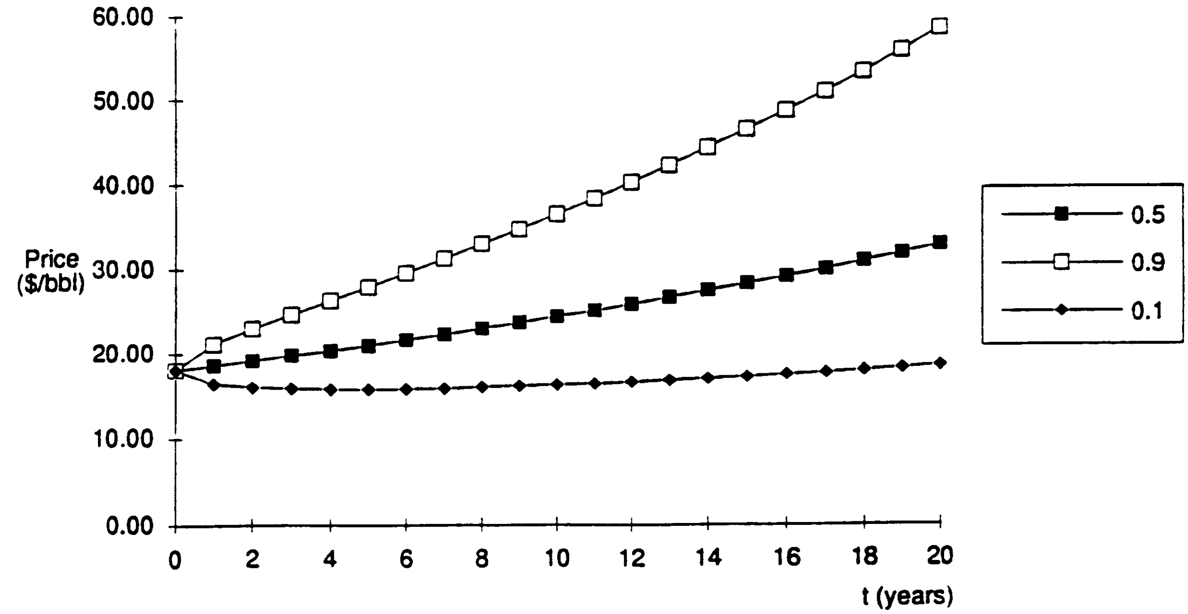

Figure 2
Current Real Oil Price Expectations,
Forward Prices, and Oil Bond Values

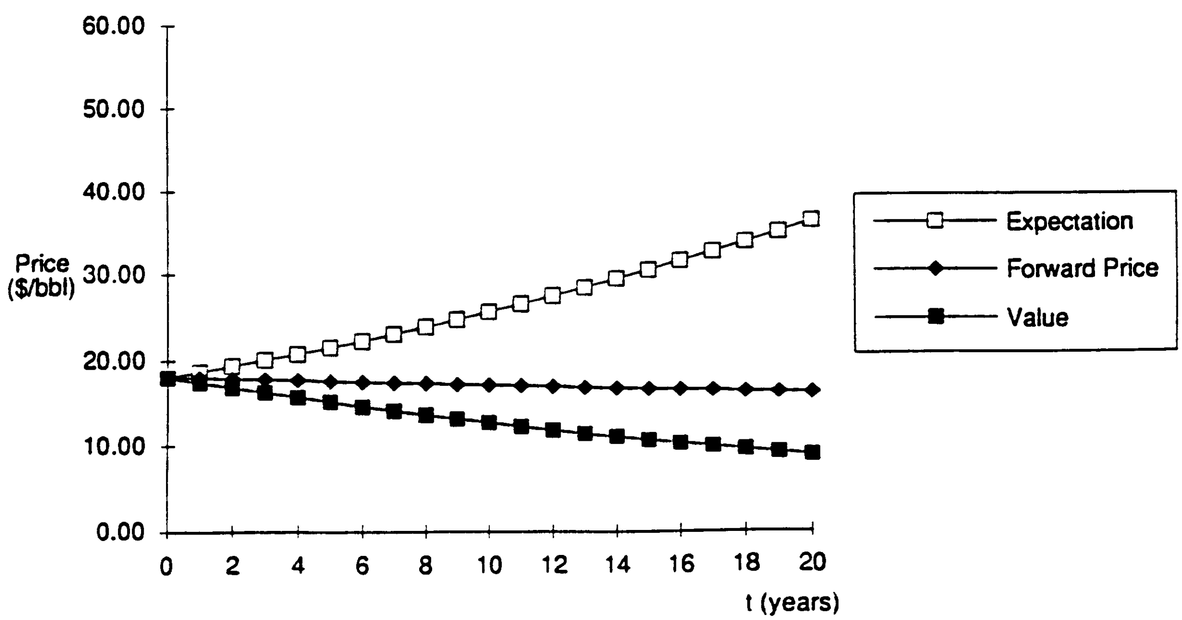

Figure 3
Cash-Flow Values for the 300 mmbbl Field

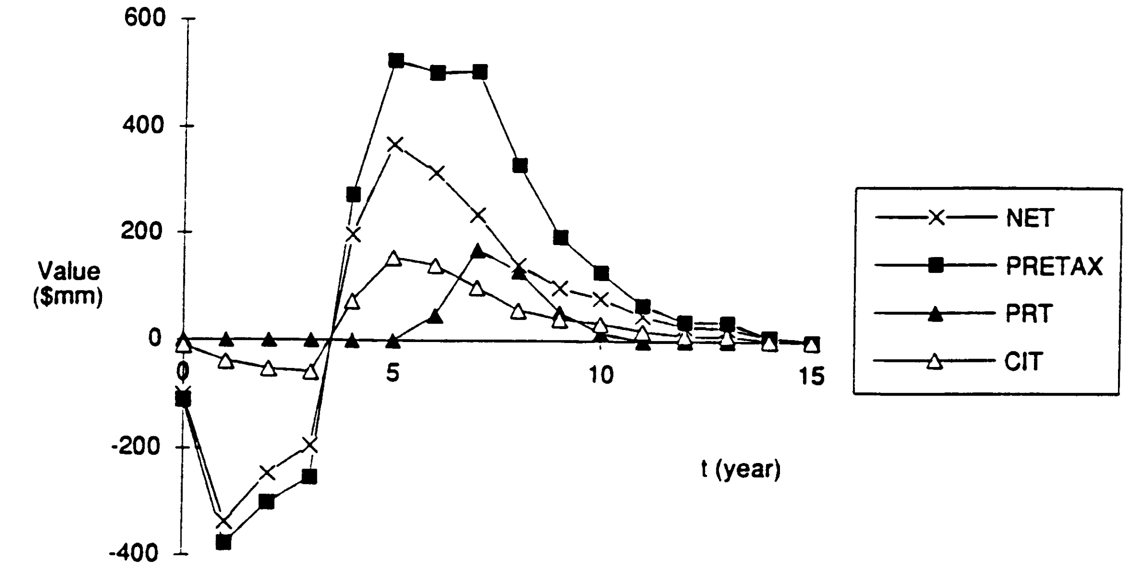

Figure 4
Cash-Flow ECDRs for the 300 mmbbl Field

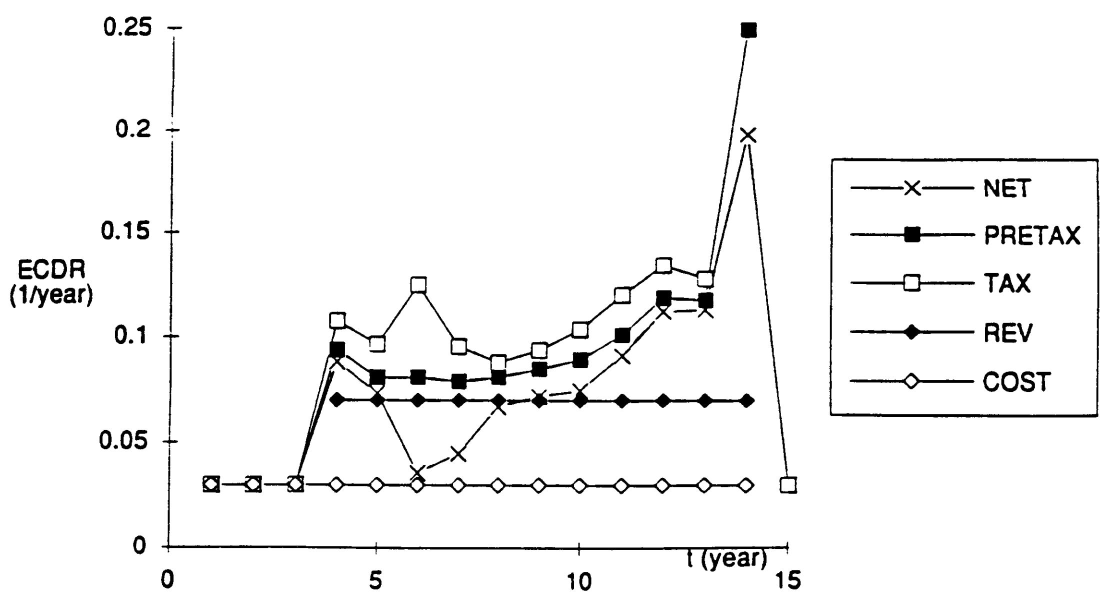

Figure 5
ECDR vs. Field Size
Operating and Tax Leverage

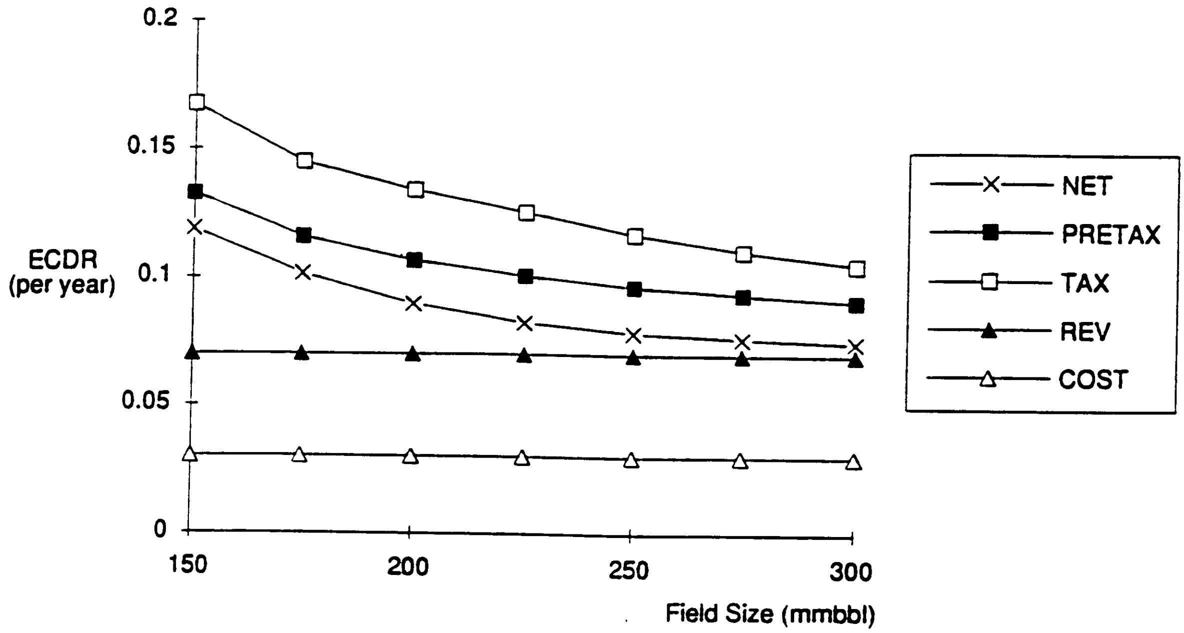

Figure 6
Current Value vs. Field Size
Proposed Method vs. 10% DCF