Uncertainty, Investment, and Industry Evolution

by

Ricardo J. Caballero and Robert S. Pindyck

MIT-CEEPR 92-009WP August 1992

UNCERTAINTY, INVESTMENT, AND INDUSTRY EVOLUTION*

Ricardo J. Caballero
and
Robert S. Pindyck

Massachusetts Institute of Technology and NBER

This draft: August 6, 1992

Abstract

We study the effects of aggregate and idiosyncratic uncertainty on the entry of firms, total investment, and prices in a competitive industry with irreversible investment. We first use standard dynamic programming methods to determine firms' entry decisions, and we describe the resulting industry equilibrium and its characteristics, emphasizing the effects of different sources of uncertainty. We then show how the conditional distribution of prices can be used as an alternative means of determining and understanding the behavior of firms and the resulting industry equilibrium. Finally, we use four-digit U.S. manufacturing data to examine some implications of the model.

JEL Classification Numbers: D92, E22

Keywords: Investment, sunk costs, industry evolution, competitive equilibrium.

*This research was supported by M.I.T.'s Center for Energy Policy Research, and by the National Science Foundation through Grant No. SES-8618502 to R. Pindyck and Grant No. SES-9208896 to R. Caballero. Caballero also acknowledges financial support from the Sloan Foundation and the National Bureau of Economic Research (through an Olin Fellowship). Our thanks to Brian Sliker for making available his data set on U.S. manufacturing industries, and to Yunyong Thaicharoen for his outstanding research assistance.

1 Introduction.

Most investment expenditures are at least in part irreversible, i.e., are sunk costs that cannot be recovered should market conditions change adversely. As a result, the cost of making an investment includes not only the expenditure itself, but also an opportunity cost associated with commiting resources rather than waiting for new information to arrive. A growing literature has shown how this opportunity cost can be evaluated, and demonstrated that it is highly sensitive to uncertainty over future project values, so that changing market conditions that affect the riskiness of future cash flows can have a large impact on investment spending. These results emphasize the role of uncertainty as a determinant of investment spending, and suggest that policies that reduce volatility (over, say, exchange rates, prices, or interest rates) may lower the required cost of capital.[1]

In most of the recent literature, the emphasis is on the investment decisions of an individual firm, rather than industry-wide investment and growth, and uncertainty is modelled by introducing an exogenous state variable (e.g., a demand or cost shift parameter, the price of the firm's output, or the interest rate) that follows some stochastic process. However, similar effects of uncertainty on investment can be found at the industry level. The reasons for these effects, however, may not be the same.

What always matters for investment are the distributions of future values of the marginal revenue product of capital — if these distributions are symmetric (and the firm is risk-neutral), increasing uncertainty will not affect investment. For a monopolist, irreversibility causes the distributions to be asymmetric because the firm cannot disinvest in the future if negative shocks arrive; hence the firm invests less today to reduce the frequency of bad outcomes in the future (i.e., the frequency of situations in which the firm has more capital than desired). On the other hand, in a competitive industry with constant returns to scale,

[1]McDonald and Siegel (1986) were among the first to demonstrate the implications of irreversibility for investment decisions. Other examples of this literature include Bertola and Caballero (1990), Dixit (1989b), Majd and Pindyck (1987), and Pindyck (1988). For an overview, see Dixit (1992) and Pindyck (1991). The earlier literature on investment under uncertainty, e.g., Hartman (1972) and Abel (1983), demonstrates how uncertainty will increase the expected value of a marginal unit of capital if the marginal revenue product of capital is a convex function of the stochastic variable (an implication of Jensen's inequality), and thereby increase investment.

the distribution of future marginal revenue products is independent of the firm's current investment. But this distribution is not independent of *industry-wide* investment if the elasticity of demand faced by the industry is less than infinite.

As a result, when studying irreversible investment in an industry context, it is important to distinguish between aggregate (i.e., industry-wide) and idiosyncratic (firm-level) shocks. To see this, consider idiosyncratic and aggregate shocks to productivity that are both symmetrically distributed. Although either type of shock might affect the expected future market price and hence the expected marginal revenue product of capital, the idiosyncratic shocks will lead to a symmetric probability distribution for the marginal revenue product.[2] Aggregate shocks, however, will not; although negative shocks can reduce the market price, positive shocks will be accompanied by the entry of new firms and/or expansion of existing firms, which will limit any increases in price. As a result, the distribution of outcomes for individual firms is truncated; negative shocks to productivity will reduce profits more than positive shocks will increase them, and irreversible investment will be reduced accordingly.[3] An important objective of this paper is therefore to clarify the different mechanisms through aggregate and idiosyncratic shocks interact with irreversibility in a competitive industry.

Uncertainty affects irreversible investment in two ways — first, through the effect of the firm's current investment on the expected path of its marginal revenue product of capital, and second, through the effects of competitors' investment on the path of the firm's marginal revenue product. Caballero (1991) has shown that with constant returns to scale, the importance of the first effect decreases as the demand curve facing the firm becomes more elastic, as long as the uncertainty is firm-specific. However, this does not mean that industry-level uncertainty will not affect industry investment and output in a competitive equilibrium. As shown by Pindyck (1992), irreversibility will have the same type of effect on industry investment as it would for a monopolist once one allows for entry of new firms or the expansion of existing ones. The reason for this is that irreversibility combined with the possibility of entry

[2]For simplicity, we are ignoring the effect of uncertainty through the convexity of the marginal revenue product of capital, as stressed by Hartman (1972).

[3]This is an example of the "bad news principle" discussed by Bernanke (1983).

affects the distribution of marginal revenue products seen by each individual firm. Hence another objective of this paper is to fully characterize the distribution of marginal revenue product and its evolution, and show how that distribution affects entry, investment, and the price level itself.

Our work extends and complements recent work by Dixit (1989a), Leahy (1991), and others. Dixit characterizes industry evolution in the presence of aggregate uncertainty by using dynamic programming methods to determine the entry and exit decisions of individual firms of discrete size. Leahy models an industry equilibrium in which price is endogenous, and shows that under reasonable assumptions, it is optimal for individual firms to make their investment decisions under the myopic assumption that price follows an exogenous lognormal random walk.[4] In the first part of this paper we follow an approach similar to that used by these authors, but emphasizing the effects on entry of different sources of uncertainty. We then go on to show how the conditional distribution of prices can be used as an alternative way to characterize the behavior of individual firms, and thereby determine the industry equilibrium. This provides insight into the nature of an equilibrium with irreversible investment.

We examine the different effects of idiosyncratic and aggregate uncertainty using a simple model of a competitive market in which firms have constant returns to scale and there is a sunk cost of entry. In the next section, we frame the model as a dynamic programming problem, and we obtain a solution and examine its properties. In Section 3 we re-frame the problem in terms of the conditional distribution of the marginal revenue product. We calculate the time path for this distribution, and show how it provides additional insight into the effects of uncertainty on investment and industry evolution. In Section 4 we use four-digit U.S. manufacturing data to examine some of the implications of the model, and to gauge the importance of uncertainty for industry investment. Section 5 concludes, and discusses some possible extensions of our work.

[4]In related studies, Lippman and Rumelt (1985) model a competitive industry equilibrium with free entry and exit, and Dixit (1991) characterizes the equilibrium for a competitive industry with irreversible investment and a price ceiling.

2 A Stylized Model.

We begin by constructing a highly stylized model in which the value of a marginal unit of capital is stochastic and exogenous. For simplicity, we formulate the model in a way that eliminates the conventional positive Jensen's inequality effect of uncertainty on the value of a marginal unit of capital that arises from the endogenous response of variable factors to exogenous shocks. This lets us focus on the way in which the effects of uncertainty are mediated through the equilibrium behavior of all firms.

We consider a market with a large number of "productive units." Each productive unit might be a single firm, or individual firms might each own several productive units. These productive units are industry specific, so that their installation involves a sunk cost. Entry occurs when new productive units are added, either because new firms invest and enter the market, or existing firms invest in new capacity. What matters is that idiosyncratic shocks apply to these productive units individually, i.e., the units all have the same expected productivity, but will have randomly differing realized productivities.[5] To clarify the ways in which uncertainty affects investment, we will assume that the owners and managers of these units are risk-neutral.[6]

We will assume that these productive units are small enough and the number of them is large enough so that we can represent them as a continuum whose mass at time t is $N(t)$. Total industry output, $Q(t)$, is given by:

$$Q(t) = \int_0^{N(t)} A_i(t)\, di \tag{1}$$

where $A_i(t)$ is the output of productive unit i at time t. The A_i's are assumed to follow arbitrary and possibly correlated exogenous stochastic processes. We decompose these individual productivity variables into two parts, their average (the aggregate) and the remainders:

$$A_i(t) = A(t)a_i(t), \quad \text{such that } \int_0^{N(t)} a_i(t)\, di = N(t).$$

[5]One could think of these productive units as coffee trees, where each firm can plant one or more trees. While the expected productivity of each tree is the same, the realized productivity of each tree will differ.

[6]Then the investment rules we derive maximize firms' values in a competitive financial market, whether or not idiosyncratic or aggregate shocks are spanned by the set of traded assets in the economy.

Here $A(t)$ is the average productivity of the industry, so that $Q(t) = A(t)N(t)$, and $a_i(t)$ is the productivity of unit i relative to that of the industry as a whole.

We allow for one idiosyncratic and two aggregate sources of uncertainty. First, we let $a_i(t)$ and $A(t)$ follow separate stochastic processes, so that productivity has both an idiosyncratic and an aggregate component. Second, we introduce another source of aggregate uncertainty through the industry demand curve. Industry demand is taken to be isoelastic:

$$P(t) = M(t)Q(t)^{-1/\eta}, \tag{2}$$

where $M(t)$ is an exogenous stochastic process that captures aggregate shocks. We will assume that $M(t)$ follows a diffusion.

The measure of industry size, $N(t)$, increases with entry and decreases with "failures," i.e., the (involuntary) removal of productive units. We assume that the latter occurs at an exogenous proportional rate γ. At the level of an individual unit, a "failure" is a Poisson arrival, and the intensity of the Poisson process is γ.[7] Alternatively, we could have assumed a deterministic depreciation rate γ that applies to all units; our results (from eqn. (4) below onwards) would be the same.

To introduce irreversibility, we assume that entry of a productive unit requires a sunk cost F. Free entry determines that there are no profits to be made by adding another productive unit to the industry, so that:

$$F \geq \mathrm{E}_i \left\{ \mathrm{E}_0 \left[\int_0^\infty P(t)A_i(t)e^{-\delta t}\, dt \right] \right\}, \tag{3}$$

which holds with equality at all times in which there is entry. The parameter δ is the discount rate. Note that the expectation E_i is over all unit-specific uncertainty, which includes the stochastic productivity process $a_i(t)$ as well as the Poisson failure process for each unit. The expectation E_0 is over the distribution of the future marginal revenue product of capital, $P(t)A_i(t)$, and therefore accounts for the possible (irreversible) entry of new productive units. As will become evident, the ability to enter the industry reduces the probability of

[7]It would be more realistic, of course, to make the Poisson arrival rate depend on the age of the specific unit. However, that complicates the model but adds little additional insight.

good outcomes by truncating the upper part of the distribution for the aggregate component of $P(t)A_i(t)$, namely $P(t)A(t)$.

By Fubini's theorem and the construction of $A_i(t)$ we can pass the expectation operator E_i inside the integral in eqn. (3), so that it reduces to:

$$F \geq \mathrm{E}_0 \left[\int_0^\infty P(t)A(t)e^{-(\delta+\gamma)t}\, dt \right]. \tag{4}$$

Note that the only idiosyncratic effect that remains in (4) is the failure rate γ, and this is now indistinguishable from an industry-wide depreciation rate. Because the value of the output of each unit is linear in the output-specific stochastic state variable, we can eliminate all other idiosyncratic elements from the right-hand side of eqn. (3). This is an extreme result, and it holds because we have assumed that there is no *selective entry*; i.e. potential entrants cannot choose the distribution for their idiosyncratic shock.

Since $Q(t) = A(t)N(t)$, we can use the market demand equation to construct a measure of the value of output for an average productive unit. Letting $B(t)$ denote the average value of output:

$$B(t) \equiv P(t)A(t) = M(t)A(t)^{\frac{\eta-1}{\eta}}N(t)^{-\frac{1}{\eta}} \tag{5}$$

Because the industry size $N(t)$ is endogenous, $B(t)$ will follow a regulated stochastic process, where $N(t)$ regulates $B(t)$. Letting lower case letters represent the logarithm of the corresponding variable, we can write:

$$d\log B(t) \equiv db(t) = dm(t) + \left(\frac{\eta-1}{\eta}\right) da(t) - \frac{1}{\eta}dn(t). \tag{6}$$

In order to obtain analytical results that can be used to illustrate the implications of different sources of uncertainty, we will make the simplifying assumption that the aggregate stochastic state variables follow geometric Brownian motions. Thus, we write the dynamics of $m(t)$ and $a(t)$ as:

$$dm(t) = \left(\alpha_m - \tfrac{1}{2}\sigma_m^2\right) dt + \sigma_m dz_m(t) \tag{7}$$

$$da(t) = \left(\alpha_a - \tfrac{1}{2}\sigma_a^2\right) dt + \sigma_a dz_a(t) \tag{8}$$

We will also assume that the Wiener processes $dz_m(t)$ and $dz_a(t)$ are uncorrelated. (It is easy to relax this assumption.) Then $B(t)$ will follow a particularly simple regulated geometric

Brownian motion. Specifically, $B(t)$ will remain at or below a fixed upper boundary. This fixed boundary, which we denote by U, is yet to be determined as part of an industry equilibrium. Regulation is due to entry; when this is not occurring, $n(t) \equiv \log(N(t))$ will follow:

$$dn(t) = -\gamma dt,$$

and $b(t)$ is given by:

$$db(t) = \beta dt + \sigma_b dz(t), \tag{9}$$

where

$$\beta = \alpha_m - \tfrac{1}{2}\sigma_m^2 + \frac{\gamma}{\eta} + \frac{\eta-1}{\eta}\alpha_a - \frac{\eta-1}{2\eta}\sigma_a^2$$

and

$$\sigma_b = \sqrt{\sigma_m^2 + \left(\frac{\eta-1}{\eta}\right)^2 \sigma_a^2}\,.$$

This model is simple enough so that we can find a closed form solution for the optimal investment rule, i.e., for the upper boundary U. (Later we will see how the entire problem can be recast in terms of the conditional distribution of marginal revenue product.) Let $W(x)$ denote the value of entering the industry at $t=0$ when $b(0)=x$, so that $B(0)=e^x$:

$$W(x) = \int_0^\infty e^{-(\delta+\gamma)t} \mathrm{E}_0[B(t)|B(0)=e^x]\, dt. \tag{10}$$

By arbitrage, over an interval dt, the total expected return from being in the industry must be equal to $(\delta+\gamma)W dt$. This expected return has two components, an expected capital gain, $\mathrm{E}_0 dW$, and a flow of revenue $B(0)dt = e^x dt$. By Ito's Lemma, $\mathrm{E}_0 dW = \beta W'(x)dt + \frac{1}{2}\sigma_b^2 W''(x)dt$, so $W(x)$ must satisfy the following differential equation:

$$\tfrac{1}{2}\sigma_b^2 W''(x) + \beta W'(x) - (\delta+\gamma)W(x) + e^x = 0. \tag{11}$$

In addition, $W(x)$ must satisfy the following boundary conditions:

$$\lim_{x\to-\infty} W(x) = 0, \tag{12}$$

and

$$W'(u) = 0, \tag{13}$$

where $u = \log U$. Boundary condition (12) follows from the fact that 0 is an absorbing boundary for B. Condition (13) is the value matching/smooth pasting condition that holds at the trigger point u.

The reader can check that eqn. (11) has the following simple solution that satisfies the associated boundary conditions:

$$W(x) = \frac{e^x}{\delta + \gamma - \beta - \sigma_b^2/2} - \frac{e^u/\lambda}{\delta + \gamma - \beta - \sigma_b^2/2} e^{\lambda(x-u)}, \tag{14}$$

where

$$\lambda = \frac{-\beta + \sqrt{\beta^2 + 2(\delta + \gamma)\sigma_b^2}}{\sigma_b^2}. \tag{15}$$

A sufficient condition for the existence of this solution is that the discount rate be large enough so that the value of a unit remains bounded even if entry into the industry were prohibited throughout the future. Specifically, we require that $\delta + \gamma - \beta - \sigma_b^2/2 > 0$, i.e., $\delta > \alpha_m + \frac{\eta - 1}{\eta}(\alpha_a - \gamma) - \frac{\eta - 1}{2\eta^2}\sigma_a^2$. This ensures that $\lambda > 1$.

We can now determine U, the upper boundary of $B(t)$. If we had solved this as a central planning problem, we would determine U from the first-order ("super contact") condition that $W''(U) = 0$. Instead, we will follow Leahy (1991) and use the free entry condition, which in this case is $F = W(u)$. Hence:

$$\frac{U}{F} = \frac{\lambda}{\lambda - 1}(\delta + \gamma - \beta - \tfrac{1}{2}\sigma_b^2). \tag{16}$$

Because of free entry, $\mathrm{E}_0 \int_0^\infty B(t)e^{-(\delta+\gamma)t}dt = F$, where $t = 0$ is the time of entry. Since $U \geq \mathrm{E}_0[B(t)]$ for all t and $U > \mathrm{E}_0[B(t)]$ for all $t > 0$, we know that $\mathrm{E}_0 \int_0^\infty Ue^{-(\delta+\gamma)t}dt > F$. This is a result of irreversibility; there is an opportunity cost of investing now rather than waiting for new information. If firms could "uninvest" and recoup the cost F, we would instead have the standard Marshallian result that $\mathrm{E}_0 \int_0^\infty Ue^{-(\delta+\gamma)t}dt = F$.

For simplicity of exposition, in what follows we will assume that aggregate productivity is constant, so that $\alpha_a = \sigma_a = 0$. (Note that then $\sigma_b = \sigma_m$.) Recall that α_m and σ_b represent the mean and the standard deviation of the rate of growth of revenue per productive unit averaged over the industry when there is no entry. With tedious calculation, one can show that $\partial(U/F)/\partial\sigma_b > 0$ and $\partial(U/F)/\partial\alpha_m < 0$. A smaller value of α_m raises U/F because given any value of U/F, it implies a lower expected price and therefore less entry is needed to satisfy the zero profit condition. (This is discussed further below.) A higher value of σ_b, raises U/F by increasing the opportunity cost of investing, and thereby raising the threshold required for a firm to commit the sunk cost F. But note that it is only *aggregate* uncertainty that matters; U/F is unaffected by idiosyncratic shocks. Figure 1 shows this dependence of U/F on α_m and σ_b.

One can also show that $\partial(U/F)/\partial\eta > 0$, and $\partial(U/F)/\partial\delta > 0$. An increase in the elasticity of demand, η, implies that the potentially positive effect of the failing units on the price is reduced. This lowers expected revenue flow and hence raises the threshold required for investment. An increase in δ likewise raises the threshold by directly lowering the expected present value of returns and by increasing the opportunity cost of investing in the unit now, rather than waiting and discounting the expenditure F. As for $\partial(U/F)/\partial\gamma$, the discounting effect described above again holds (capital depreciates faster when γ is larger). However, there is an offsetting effect from the increased depreciation of the capital of other firms, which tends to raise the expected industry price as seen from the time of entry. The first effect dominates for most reasonable parameter values.

We can now describe the behavior of industry investment, output, and price in equilibrium. Suppose, for example, that aggregate demand increases. Then entry of new productive units will occur, so that price will rise only to the point that $P(t)A(t) = U$. Figures 2A, 2B, and 2C illustrate this by showing a particular sample path for industry evolution for two values of σ_m, .15 and .30. (In this simulation, the other parameters are $\eta = 2$, $\alpha_m = .02$, $\alpha_a = \sigma_a = 0$, $\gamma = .03$, $\delta = .06$, and $F = 100$.) The top graph shows the log of the stochastic driving force, $m(t)$. (The realization for $z(t)$ is the same for the two lines, but the values of σ_b are different.) Figure 2B shows the log of the number of productive units, $n(t)$. Note that when $m(t)$ is falling (e.g., between $t = 12$ and 18), there is little or no investment, so $n(t)$ falls due to failures (or depreciation). For $t > 18$, $m(t)$ is generally rising, and so entry occurs and $n(t)$ rises.

Figure 2C shows the realization for the log of price, $p(t)$. Note that $p(t)$ appears stationary; that is because we have set $\alpha_a = \sigma_a = 0$ for all t, so that $A(t) = 1$ always. (Hence price is equal to the average revenue per productive unit, which is the relevant state variable for the decision-making unit. In the more general case, $b(t)$ would follow the same pattern as in Figure 2C, and $p(t)$ would be the sum of $b(t)$ and a Brownian motion.) As the figure illustrates, during "recessions," i.e., when $m(t)$ is falling, price will also fall, and will fall farther when σ_b is larger. But during "good times," $p(t)$ is generally higher when σ_b is larger. The reason is that with a larger σ_b, there is a greater chance of deeper "recessions," so during "good times" firms wait longer before entering, n is smaller, and p is higher.

Underlying all of these results is a forecast of future revenues by firms that are considering entry. In fact, this forecast (which must take into account entry by other firms) completely determines the decision to enter. Hence, looking directly at the expected value of future revenues, and their dependence on the underlying parameters, helps to understand industry evolution. We turn to this in the next section.

3 The Price Distribution and Entry.

In the previous section we found the optimal investment rule in the standard way — by using dynamic programming to calculate the the firm's value function. In general, this approach is useful in that studying the local (in time) behavior of the value function allows one to fully characterize complex dynamic problems. Problems in which the optimal or competitive outcome consists of regulating a Brownian motion, as in the model developed in the previous section, are good examples of this. Value matching, smooth pasting and the Bellman equation are all intuitive properties arising from this local analysis.

However, dynamic programming sometimes conceals the economic intuition as to how changes in parameters affect optimal policies. For example, eqn. (10) defined the value function, $W(x)$, as the expected present value of the flow of marginal revenue product. Thus, any effects of changes in the variance or drift parameters on the value function, and therefore on the optimal investment rule, must come through their effects on either the path of the expected marginal revenue product or the discount rate. Those effect, however, remained somewhat hidden in our solution to the problem in the previous section. In this section we look at the expected path of marginal revenue directly and show how it is affected by the underlying parameters.[8]

To do this, we need to derive the conditional probability density for b, which we denote by $f(b,t)$. Since we know that a firm will enter only when $b(t) = u$, we can replace x by u in eqn. (10). Hence $f(b,t)$ is the probability density of b at a time t from the moment of entry, conditional on $b(0) = u$. As mentioned above, any effects of parameters such as β and σ_b on the entry point u and hence on price will occur through their effects on the path of the density $f(b,t)$, and in particular on the function:

$$\mathrm{E}[B(t)|B(0) = U] = \int_{-\infty}^{\log U} e^b f(b,t)\, db.$$

This expected value begins at the moment of entry at U, and then converges over time to the ergodic mean (see below):

$$\bar{B}_\infty \equiv \lim_{t\to\infty} E[B(t)|B(0) = U] = \frac{2\beta}{\sigma_b^2 + 2\beta} U. \tag{17}$$

If the discount and depreciation rates are small, we can accurately describe the impact of drifts and uncertainty on the equilibrium entry point, U, by determining their impact on the ergodic mean. It is straightforward to see that $\bar{B}_\infty$ rises with β and falls with σ_b; thus, by the free entry condition, U must fall with β and rise with σ_b.

If the discount and depreciation rates are large, the problem is more complicated because we cannot use the ergodic density to construct the right-hand side of (10). Instead we need to account for the whole path of $f(b,t)$. (Intuitively, we know that $f(b,t)$ must start as a spike at u when $t = 0$, and as t increases it must converge smoothly to the ergodic density.) Because $b(t)$ follows the diffusion equation (9), $f(b,t)$ must satisfy the Kolmogorov forward equation:

$$f_t(b,t) = \tfrac{1}{2}\sigma_b^2 f_{bb}(b,t) - \beta f_b(b,t) \tag{18}$$

(See Karlin and Taylor (1981).) Since $b(t)$ is regulated at u, the solution to this equation

[8]The effects of the discount rate are straightforward. We are not proposing that dynamic programming be replaced by the approach used in this section. Rather, we are trying to explain more fully why irreversible investment is depressed by industry-wide uncertainty.

must satisfy the following boundary conditions for $t > 0$:

$$f(u,t) = \frac{\sigma_b^2}{2\beta} f_b(u,t), \tag{19}$$

$$\lim_{b \to -\infty} f(b,t) = 0, \tag{20}$$

as well as the initial condition:

$$\int_{-\infty}^{x} f(b,0)\, db = \begin{cases} 0 & x < u \\ 1 & x = u \end{cases} \tag{21}$$

In Appendix A we derive the solution for $f(b,t)$ and show that it implies the following path for the expected marginal revenue product:

$$\mathrm{E}[B(t)|B(0) = U] = U\left[\frac{2\beta}{\sigma_b^2 + 2\beta} + e^{-\beta^2 t/2\sigma_b^2} \int_0^\infty \lambda(z) e^{-\sigma_b^2 z t/2}\, dz\right], \tag{22}$$

where

$$\lambda(z) = \frac{z^{1/2}}{(z + \beta^2/\sigma_b^4)(z + \beta^2/\sigma_b^4 + 2\beta/\sigma_b^2 + 1)} \tag{23}$$

We now substitute eqn. (22) back into eqn. (10) evaluated at $x = u$, which yields:

$$W(u) = \frac{U\phi(\beta,\, \sigma_b,\, \delta + \gamma)}{(\delta + \gamma)},$$

where

$$\phi(\beta,\, \sigma_b,\, \delta + \gamma) \equiv \frac{2\beta}{\sigma_b^2 + 2\beta} + \frac{2(\delta + \gamma)}{\pi} \int_0^\infty \frac{\lambda(z)}{\sigma_b^2 z + \beta^2/\sigma_b^2 + 2(\delta + \gamma)}\, dz. \tag{24}$$

The first term in the expression for $\phi(.,.,.)$ summarizes the impact of the various parameters on the ergodic mean, while the second term encompasses the the transition from the value of marginal revenue product at entry and its unconditional (ergodic) mean. Clearly, the latter will be relatively more important when firms give more weight to the short run, i.e. when $(\delta + \gamma)$ is large.

Given $\phi(.,.,.)$, the value of U can be found, as before, from the free entry condition:

$$U = F\frac{\delta + \gamma}{\phi(\beta,\, \sigma_b,\, \delta + \gamma)}. \tag{25}$$

One can check (numerically) that eqns. (16) and (25) are equivalent. We have again arrived at an expression for U (and thus the optimal investment rule), but this time by deriving the path for the expected marginal revenue product and utilizing the free entry condition.

Figure 3a shows $E[B(t)|B(0) = U]/U$ as a function of time for $\sigma_b = .10$, .15, and .20. (As before, the other parameter values are $\gamma = .03$, $\alpha_m = 0.02$, $\alpha_a = 0$, $\sigma_a = 0$, $\eta = 2$, $\delta = .06$, and $F = 100$.) Observe that with a higher value of σ_b, $E[B(t)|B(0) = U]/U$ falls farther. Because of the irreversibility of investment, the boundary U rises, but larger fluctuations in demand lead to periods of lower prices, so the ratio falls. Also, while the decline in this variable is greater when σ_b is large, it must fall farther, so that it takes longer to approach its steady-state value.

Figure 3b shows $E[B(t)|B(0) = U]$ for the same three values of σ_b. (Note that all of the curves intersect the vertical axis at U.) If the discount rate δ and depreciation rate γ were zero, these curves would all converge to the same value, and would not cross each other. The reason is that with no discounting, only the long-run steady-state matters, and not the transition to that steady-state. Different values of σ_b would result in different values of U such that the resulting ergodic means for the marginal revenue product would be the same. With discounting, however, the transition matters, so that the curves cross.

4 Investment in U.S. Manufacturing.

In this section we use data for two- and four-digit U.S. manufacturing industries to obtain measures of the key variables in our model. We then use the cross-sectional variation of these measures to examine the implications of the model, and to gauge the quantitative importance of uncertainty and irreversibility for investment.

Two implications of our model are particularly important. First, if an industry is reasonably competitive, it is aggregate uncertainty that should have the greatest effect on the trigger point U at which firms enter or expand their existing capacity. As shown earlier, this is because when investment is irreversible, aggregate uncertainty affects the distribution of prices faced by *all* firms, so that positive and negative shocks have an asymmetric effect on each firm's profits; shocks that are specific to a single firm, on the other hand, have symmetric effects on that firm's profits.[9] Second, the effect of uncertainty on investment is

[9]Such shocks can still affect investment, whether or not it is irreversible, but only insofar as they alter

mediated through its effect on the trigger point U. Increased uncertainty will increase U, so that firms require a higher marginal profitability of capital (and hence rate of return) before they are willing to invest. Hence tests of models of irreversible investment, and attempts to measure the effects of uncertainty, should be based on the behavior of U.

Unfortunately we cannot observe U directly. However, conditional on assumptions about the production technology and market structure, we can estimate our marginal profitability variable, $B(t)$, at least up to a scaling factor. As shown below, we can then use our estimates of $B(t)$ over certain intervals as a proxy for U.

Assuming the industry is competitive and the production function is Cobb-Douglas with constant returns to scale, we can express the output of a (price-taking) productive unit as:

$$Y(t) = A_t K_t^{\alpha} L_t^{\phi(1-\alpha)} M_t^{(1-\phi)(1-\alpha)} . \tag{26}$$

where α is the share of capital, and ϕ is the share of labor in a labor-materials composite which we will denote by H. Then, the marginal profitability of capital is given by:

$$\Pi_K = \alpha(1-\alpha)^{(1-\alpha)/\alpha} \left(\frac{P_t Y_t}{H_t^{1-\alpha} K_t^{\alpha}} \right)^{1/\alpha} P_{H,t}^{-(1-\alpha)/\alpha} , \tag{27}$$

where P_t is the price of output, and $P_{H,t}$ is the price of the labor-materials composite.

The marginal profitability of capital in this case is the value of output for an average productive unit, i.e., $B(t)$. We will work with $b(t) = \log B(t)$, which (again letting lower case letters represent logs of the corresponding variables) is given by:

$$b(t) = \log \left[\alpha(1-\alpha)^{(1-\alpha)/\alpha} \right] + \frac{1}{\alpha}[y_t - (1-\alpha)h_t - \alpha k_t] - \frac{1-\alpha}{\alpha} p_{H,t} + \frac{1}{\alpha} p_t . \tag{28}$$

We focus on investment and the marginal profitability of capital for 20 two-digit manufacturing industries. For each of these industries, we use data on the real value of output, real inputs of capital, materials, and labor, and the corresponding price deflators to obtain a time series for $b(t)$ over the 29 year period 1958–1986. We cannot measure $b(t)$ for the individual firms in these industries; instead we calculate a comparable series for $b(t)$ for each of 443 four-digit subsectors that make up the two-digit industries. We denote these series

---

the expected marginal revenue product of capital (as in the models of Hartman (1972) and Abel (1983)).

by $b_2(t)$ and $b_4(t)$ respectively. This approach is consistent with our model insofar as the price elasticities of demand for the outputs of the four-digit industries are large relative to those for the two-digit ones, and allows us to obtain measures of aggregate and idiosyncratic uncertainty and gauge their overall importance for investment with only moderate data requirements. The data and the calculation of the $b(t)$'s are discussed in Appendix B.[10]

Recall that $b_2(t)$ is a regulated process, but when it is below the boundary $u = \log U$ it follows the Brownian motion of eqn. (9), which has a stochastic term $\sigma_b dz(t)$ that incorporates both demand and productivity shocks. We estimate σ_b directly from our time series for $b_2(t)$. Specifically, we calculate sample standard deviations of $b_2(t)$ for each of the 20 two-digit industries, which we denote by SDB2; this is our measure of aggregate uncertainty. Next, we calculate sample standard deviations of $b_4(t)$ for each of the 443 four-digit industries, and then average them across those industries that make up the corresponding two-digit industry. These numbers, denoted by SDB4, are the measures of *total* (aggregate and idiosyncratic) uncertainty at the relatively disaggregated level of four-digit industries.[11]

These numbers are shown in Table 1. Observe that the average four-digit standard deviation is typically two or three times as large as the corresponding two-digit standard deviation; as we would expect, there is more idiosyncratic than aggregate uncertainty over the marginal profitability of capital.[12] The two-digit standard deviations are on the order of 10 percent per year (consistent, for example, with the annual standard deviation of real returns on the New York Stock Exchange Index of 20 percent per year and an average debt/equity ratio of one). Also shown are the cross-sectional sample standard deviations of the four-digit SDB4's corresponding to each two-digit industry. Note that the standard deviations vary considerably across both four-digit and two-digit industries. We can exploit this variation to determine the impacts of SDB2 and SDB4 on investment.

[10]We used a data base assembled by Brian K. Sliker, who graciously made it available for our use. We included only 443 of the 450 four-digit SIC industries because of missing data in seven of the industries.

[11]Our estimator SDB2 is biased downwards from the true standard deviation because $b_2(t)$ is a regulated process. Eqn. (9) applies when it is not regulated, but our sample standard deviations include periods of regulation.

[12]Let SDB_I denote the standard deviation of idiosyncratic shocks. Then $\mathrm{SDB}_I = \sqrt{\mathrm{SDB4}^2 - \mathrm{SDB2}^2}$. If $\mathrm{SDB4} = .2$ and $\mathrm{SDB2} = .1$, $\mathrm{SDB}_I = .17$.

Changes in the volatility of $b_2(t)$ should not affect investment directly; instead they should affect the trigger point u. Although u is not observable, we can use extreme values of $b_2(t)$ as a proxy; since u is the upper barrier for $b_2(t)$, $b_2(t)$ should be close to u when it is large relative to its average value. We use three variables, all computed relative to the industry mean of $b(t)$, as measures of u at the two-digit level: (i) the maximum of $b_2(t)$ over the 29 years of data, denoted by DBMAX; (ii) the average of the top decile (three observations) of the 29 annual values of $b_2(t)$, denoted by DBDEC; and (iii) the average of the top quintile (six observations), denoted by DBQUINT. We average over several extreme values and use DBDEC and DBQUINT rather than just DBMAX because $b_2(t)$ may in practice rise above u temporarily if there are lags in investment, if there are predictable temporary increases in $b_2(t)$, or if firms do not always optimize. We compute these variables relative to the mean because $b(t)$ is identified only up to a constant, which may differ across sectors.[13]

Table 2 shows cross-section regressions of DBMAX, DBDEC, and DBQUINT against SDB2, SDB4, and a constant. These regressions are consistent with a basic prediction of the model — that aggregate uncertainty should have the greatest effect on the trigger point u (and hence on investment). The coefficient of SDB2 is much larger than that of SDB4, and is significant at the 5 percent level when DBDEC or DBQUINT is the dependent variable. Also note that both the R^2 and the t-statistic on SDB2 increase as we average over more extreme values of $b(t)$ when constructing our proxy for u.

One problem with these regressions is that a higher standard deviation in the distribution of b's will by itself imply an increase in the extreme values of b even if the model were not valid. To deal with this possibility, we generate alternative measures of u based on the behavior of investment itself. We calculate and order a series for $\Delta K(t)$, the change in the real capital stock, and (for each industry) find the t_1 for which $\Delta K(t)$ is a maximum. We then take $b(t_1)$, subtract the mean of $b(t)$, and use the resulting number as a proxy for u. We denote this by DBKMAX. We then find the times t_1, t_2, and t_3 corresponding to the

[13]Note from eqn. (17) that u minus the mean of b is affected by uncertainty in the same qualitative way as is u itself, because the mean is much less sensitive to uncertainty than is u. When the discount rate is zero, the mean of b is unaffected by uncertainty.

three largest values of $\Delta K(t)$, and again find and average the corresponding values of $b(t)$; the resulting variable is denoted DBKDEC. Finally, we generate DBKQUINT using those b's corresponding to the top quintile of the ΔK's.

Table 2 shows regressions these investment-based measures of u on SDB2 and SDB4. Neither volatility variable is significant when DBKMAX or DBKDEC is the dependent variable, but SDB2 is significant (and the R^2 increases dramatically) when DBKQUINT is used. This again supports the notion that aggregate uncertainty has the greatest effect on the investment threshold.

We can use these results to gauge the effect of uncertainty on investment. We will focus on the regressions in Table 2 in which DBQUINT and DBKQUINT are the dependent variables, because these have the highest R^2's, most significant coefficients on SDB2, and are more robust (although downward biased) estimates of U. The coefficients on SDB2 provides measures of the semi-elasticity $\Delta \log(U/F)/\Delta\sigma_b$. These two regressions put this semi-elasticity in the range of 1.2 to 1.8. Thus an increase in the annual standard deviation of the marginal profitability of capital at the two-digit level from, say, .1 to .2 should increase the required return on investment by 12 to 18 percent (so that if the required return was 30 percent, it should rise to about 34 or 36 percent). This is a sizable but not overwhelming effect of uncertainty. It is consistent with the simulated elasticities of the model illustrated in Figure 1, but is less than predictions based on analyses of individual projects, e.g., by McDonald and Siegel (1986), Majd and Pindyck (1987), and others.

5 Conclusions.

In a competitive equilibrium, uncertainty over market demand or average productivity affects irreversible investment through the feedback of industry-wide capacity expansion and new entry on the distribution of prices. If demand increases, existing firms will expand or new firms will enter until the market clears. From the point of view of an individual firm, this limits the amount that price can rise under good industry outcomes. But if investment is irreversible, there is no similar mechanism to prevent price from falling under bad outcomes.

Each firm takes price as given, but knows that the distribution of future prices is affected by the irreversibility of investment industry-wide, which leads it to raise the trigger point at which it is willing to invest. Idiosyncratic shocks, which affect only an individual firm, do not induce entry and thus should have less impact on the firm's willingness to invest.

We have tried to clarify these channels through which aggregate and idiosyncratic uncertainty affect investment and industry evolution. Our model is simple enough so that it can be solved using standard dynamic programming methods, but we have emphasized the effects of uncertainty on the conditional distribution of prices, and shown how this distribution can be derived and used as an alternative means of determining and understanding the behavior of firms and the resulting industry equilibrium. We have also seen that the basic implications of the model are roughly consistent with the data for U.S. manufacturing industries, and we have obtained estimates of the semi-elasticity $\Delta \log(U/F)/\Delta\sigma_b$ that are moderate in size but plausible.

It is useful to compare our model with standard NPV models of investment based on the CAPM. In those models, it is systematic (economy-wide), and not non-systematic uncertainty that affects the discount rate. In our model, aggregate uncertainty, which is related to but not the same as systematic uncertainty, increases the trigger point, which corresponds to a higher required rate of return. Thus the mechanisms are very different, but the effects of different sources of uncertainty are similar as in NPV-CAPM models. We have ignored CAPM effects; they will magnify the effects of aggregate uncertainty that we have derived.

Our model is highly stylized, and makes a number of simplying assumptions. Some of them are important and should be kept in mind when interpreting our results. First, if there is a flexible factor, or if the firm can costlessly and temporarily shut down when price falls below variable cost, the marginal profit function will be convex in price and in exogenous productivity. Then for an industry of fixed size, an increase in idiosyncratic uncertainty will raise the present value of an additional unit of capital, and so to preserve the zero-profit condition, the trigger point at which entry occurs must decline. In this case, idiosyncratic uncertainty can lead to an increase in investment.

Second, we have ignored abandonment. Suppose a productive unit can be scrapped at any time for some positive scrap value. This sets a floor on the value of the unit; once the combination of price and the productivity of the unit (which includes an aggregate and idiosyncratic component) reaches the point at which the unit's value equals this scrap value, it will be scrapped. This possibility raises the value of the unit for any combination of price and expected productivity, which lowers the entry point u, and hence reduces the effect of aggregate uncertainty described by our model. Also, an increase in idiosyncratic uncertainty will raise the value of the unit. The reason is that potential entrants cannot know what their relative productivity will be until they enter. However, exit is done selectively, i.e., when idiosyncratic productivity is low *ex post*. Selective exit raises the value of a unit, lowering the critical cutoff point for entry. Hence a scrap value alters the effects of both aggregate and idiosyncratic uncertainty. It reduces the negative effect of aggregate uncertainty and creates a positive effect of idiosyncratic uncertainty. Also, the combination of faster entry and the incentive to exit when conditions are bad tends to reduce the variability of price.[14]

A price floor will have an effect similar to that of a scrap value, but only for aggregate uncertainty. It also reduces the negative effect of aggregate uncertainty on entry by limiting one of the two possible reasons for bad aggregate outcomes. (Bad aggregate outcomes that are due to a decline in average industry productivity are still possible.) However, a price floor will not alter the effect (or lack thereof) of idiosyncratic uncertainty.

[14]This is strictly correct only when $A(t)$ is stationary (possibly around a deterministic trend), since otherwise the variance of price becomes infinite. But even if $A(t)$ had a stochastic trend component, the statement would hold for finite intervals.

Table 1 — **Basic Statistics for Two- and Four-Digit Industries**

SIC	NOB	SDB2	MEAN of SDB4	SD of SDB4
20	47	0.058	0.246	0.162
21	4	0.104	0.451	0.567
22	30	0.118	0.366	0.192
23	33	0.076	0.304	0.124
24	17	0.168	0.327	0.091
25	13	0.125	0.258	0.144
26	17	0.113	0.217	0.075
27	17	0.061	0.192	0.135
28	28	0.088	0.224	0.110
29	5	0.201	0.256	0.063
30	6	0.127	0.242	0.134
31	11	0.093	0.231	0.084
32	27	0.099	0.236	0.124
33	26	0.250	0.506	0.385
34	32	0.123	0.277	0.121
35	44	0.160	0.301	0.097
36	39	0.147	0.264	0.095
37	15	0.184	0.403	0.135
38	12	0.105	0.220	0.076
39	20	0.109	0.255	0.079

Note: NOB is the number of 4-digit industries in each 2-digit industry; "MEAN of SDB4" is the cross-sectional sample mean of the 4-digit sample standard deviations of $\Delta b(t)$ corresponding to the 2-digit industry; and "SD of SDB4" is the cross-sectional sample standard deviation of the 4-digit sample standard deviations of $\Delta b(t)$ corresponding to the 2-digit industry.

Table 2 — Cross-Section Regression Results

Dependent Var.	Const.	SDB2	SDB4	R^2
DBMAX	-.0070	1.864	0.454	.264
	(.0913)	(1.078)	(0.611)	
DBDEC	-.0073	2.269*	0.091	.296
	(.0675)	(0.881)	(0.419)	
DBQUINT	.0041	1.873*	0.019	.359
	(.0488)	(0.615)	(0.294)	
DBKMAX	.0794	.4420	-.0019	.025
	(.1173)	(1.087)	(.6055)	
DBKDEC	.0569	.6310	-.0105	.083
	(.1097)	(.9229)	(.4979)	
DBKQUINT	-.0478	1.2147*	.0962	.469
	(.0765)	(.3934)	(.2324)	

Note: SDB2 is the sample standard deviation of $b(t) = \log B(t)$ for each 2-digit industry, and SDB4 is the average sample standard deviation of $b(t)$ for the 4-digit industries that comprise the 2-digit industry. Standard errors corrected for heteroscedasticity are shown in parentheses. A * denotes significance at the 5 percent level.

Appendix

A. The Density Function and Conditional Expectation of $B(t)$.

Let $y \equiv b - u$, and $g(y,t)$ be the density of y at time t, so that $f(b,t) = g(b-u,t)$. In this setup, finding the path of the conditional density of $b(t)$ amounts to solving the problem defined below by $(A.1)$ to $(A.6)$:

$$g_t(y,t) = \frac{1}{2}\sigma_b^2 g_{yy}(y,t) - \beta g_y(y,t), \tag{A.1}$$

$$g(0,t) = \frac{\sigma_b^2}{2\beta} g_y(0,t), \tag{A.2}$$

$$\lim_{y \to -\infty} g(y,t) = 0, \tag{A.3}$$

$$g(y,t) \geq 0 \qquad \forall y \leq 0 \text{ and } t \geq 0, \tag{A.4}$$

$$\int_{-\infty}^{0} g(y,t)\,dy = 1 \qquad \forall t \geq 0, \tag{A.5}$$

$$\int_{-\infty}^{x} g(y,0)\,dy = \begin{cases} 0 & x < 0 \\ 1 & x = 0 \end{cases}. \tag{A.6}$$

A solution to a similar problem, although with a different initial condition, can be found in Bertola and Caballero (1990). Here we only outline the basic steps of the solution, which is obtained by the method of Separation of Variables.

Writing the solutions of the homogeneous problem as: $g(y,t) = T(t)Y(y)$, decomposes the problem into two ordinary differential equations:

$$T'(t) + \lambda T(t) = 0, \tag{A.7}$$

$$Y''(y) - \frac{2\beta}{\sigma_b^2} Y'(y) + \frac{2\lambda}{\sigma_b^2} Y(y) = 0, \tag{A.8}$$

subject to the boundary conditions above, with λ a constant. The solution method has the following steps: First, find the values of λ for which the homogenous problem has a solution. Second, characterize each of these solutions. And third, combine these solution to satisfy the in-homogenous initial condition.

The characteristic equation of $(A.8)$ has real solutions for $\lambda \leq \theta\beta/4$, where $\theta \equiv 2\beta/\sigma_b^2$. It is easy to verify that the only real solution that satisfies the homogenous boundary conditions occurs when $\lambda = 0$, which yields the particular solution:

$$Y(y; \lambda = 0) = \theta e^{\theta y}. \tag{A.9}$$

On the other hand, there is a continuum of solutions for values of $\lambda > \theta\beta/4$, which take the form:

$$Y(y; \psi) = B(\psi) e^{\frac{\theta}{2} y} \left(\cos \psi y + \frac{\theta}{2\psi} \sin \psi y \right), \tag{A.10}$$

where

$$\psi \equiv \sqrt{\frac{\theta\lambda}{\beta} - \frac{\theta^2}{4}}.$$

The coefficients $B(\psi)$ are identified by the initial condition, yielding:

$$B(\psi) = \frac{2}{\pi} \frac{\psi^2 \sigma_b^4}{\psi^2 \sigma_b^4 + \beta^2}. \tag{A.11}$$

Combining $(A.9)$, $(A.10)$ and $(A.11)$ we obtain the solution for $g(y,t)$:

$$g(y,t) = \theta e^{\theta y} + \frac{2}{\pi} e^{-\frac{\beta\theta}{4} t + \frac{\theta}{2} y} \int_0^\infty \frac{\psi e^{-\frac{\beta\psi^2}{\theta} t}}{(\psi^2 + \theta^2/4)} \left(\psi \cos \psi y + \frac{\theta}{2} \sin \psi y \right) d\psi. \tag{A.12}$$

The expression for the conditional expectation shown in the text is is now easily obtained by solving the integral in the expression:

$$\mathrm{E}[B(t)|B(0) = U] = U \int_{-\infty}^{0} e^y g(y,t)\, dy.$$

B. The Data and Calculation of $b(t)$.

Our raw database was originally developed by Brian K. Sliker at M.I.T., and is used with his permission. We calculate $b(t)$ based on eqn. (28) using two- and four-digit SIC data for the real value of output ($ROUTPUT$), real inputs of capital (RK), materials ($RMAT$), and labor hours ($TOTHRS$), and the corresponding price deflators. $TOTHRS$ is the sum of hours for production workers ($PWHRS$) and and non-production workers ($NPWHRS$), where the latter is estimated as the product of non-production worker employment ($NPWEMP$) and average hours per employee for production workers (the mean of $PWHRS/PWEMP$).

We calculate the labor and materials shares by setting α_L and α_M equal to the mean values of $TLC/NOUTPUT$ and $NMAT/NOUTPUT$ respectively, where TLC is total labor costs, $NMAT$ is the nominal value of materials inputs, and $NOUTPUT$ is the nominal value of output. Letting $\phi = \alpha_L/(\alpha_L + \alpha_M)$, we then compute the Solow residual:

$$a_t = y_t - (1 - \alpha_K)h_t - \alpha_K k_t \,,$$

where $y_t = \log(ROUTPUT)$, $h_t = \phi l_t + (1 - \phi)m_t$, $l_t = \log(TOTHRS)$, $m_t = \log(RMAT)$, and $k_t = \log(RK)$. Finally, $b(t)$ is given by:

$$\begin{aligned} b(t) \;&=\; \log\left[(1-\alpha_K)^{(1-\alpha_K)/\alpha_K}\alpha_K\right] \;+\; (1/\alpha_K)a_t \\ &\quad - \frac{1-\alpha_K}{\alpha_K}\left[\phi(\log TLC - l_t) + (1-\phi)\log PMAT - \log POUTPUT\right] . \end{aligned}$$

References

Abel, Andrew B., "Optimal Investment Under Uncertainty," *American Economic Review*, March 1983, **73**, 228–33.

Bernanke, Ben S., "Irreversibility, Uncertainty, and Cyclical Investment," *Quarterly Journal of Economics*, Feb. 1983, **98**, 85-106.

Bertola, Giuseppe, and Ricardo Caballero "Irreversibility and Aggregate Investment," unpublished, Princeton University, 1990.

Caballero, Ricardo J., "Competition and the Non-Robustness of the Investment-Uncertainty Relationship," *American Economic Review*, March 1991.

Dixit, Avinash, "Hysteresis, Import Penetration, and Exchange-Rate Pass-Through," *Quarterly Journal of Economics*, May 1989a, **104**, 205-228.

Dixit, Avinash, "Entry and Exit Decisions under Uncertainty," *Journal of Political Economy*, June 1989b, **97**, 620-638.

Dixit, Avinash, "Irreversible Investment with Price Ceilings," *Journal of Political Economy*, June 1991, **99**.

Dixit, Avinash, "Investment and Hysteresis," *Journal of Economic Perspectives*, Winter 1992, **6**, 107-132.

Hartman, Richard, "The Effects of Price and Cost Uncertainty on Investment," *Journal of Economic Theory*, Oct. 1972, **5**, 258-266.

Karlin, Samuel, and Howard M. Taylor, *A Second Course in Stochastic Processes*, New York: Academic Press, 1981.

Leahy, John, "The Optimality of Myopic Behavior in a Competitive Model of Entry and Exit," Discussion Paper No. 1566, Harvard University, August 1991.

Lippman, Steven A., and R. P. Rumelt, "Industry-Specific Capital and Uncertainty," unpublished, UCLA, Sept. 1985.

Majd, Saman, and Robert S. Pindyck, "Time to Build, Option Value, and Investment Decisions," *Journal of Financial Economics*, March 1987, **18**, 7–27.

McDonald, Robert, and Daniel R. Siegel, "The Value of Waiting to Invest," *Quarterly Journal of Economics*, Nov. 1986, **101**, 707-728.

Pindyck, Robert S., "Irreversible Investment, Capacity Choice, and the Value of the Firm," *American Economic Review*, Dec. 1988.

Pindyck, Robert S., "Irreversibility, Uncertainty, and Investment," *Journal of Economic Literature*, September 1991.

Pindyck, Robert S., "A Note on Competitive Investment Under Uncertainty," *American Economic Review*, forthcoming, 1992.

FIG 1: DEPENDENCE OF U/F ON σ_b AND α_m

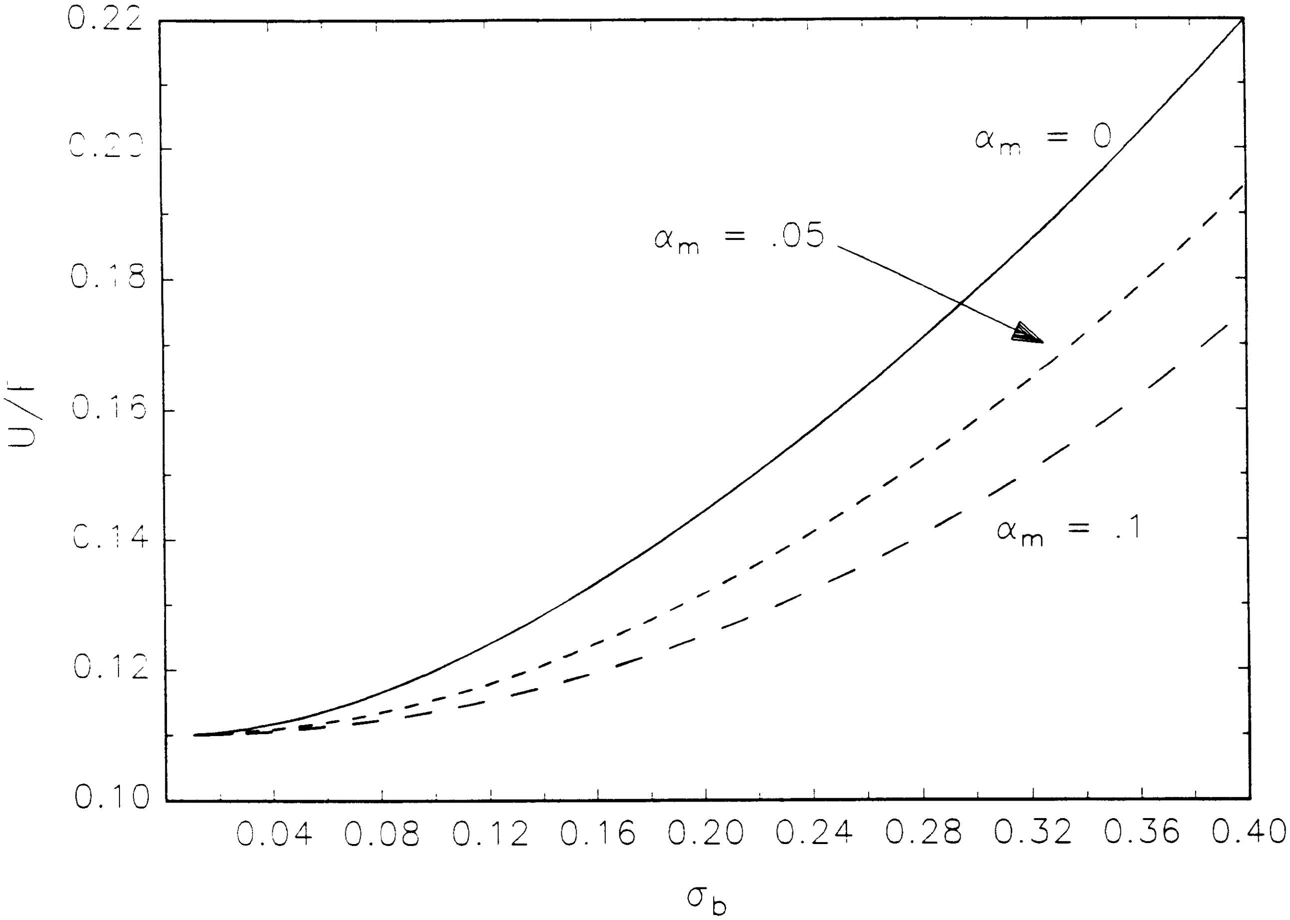

FIG. 2A: LOG DRIVING FORCE

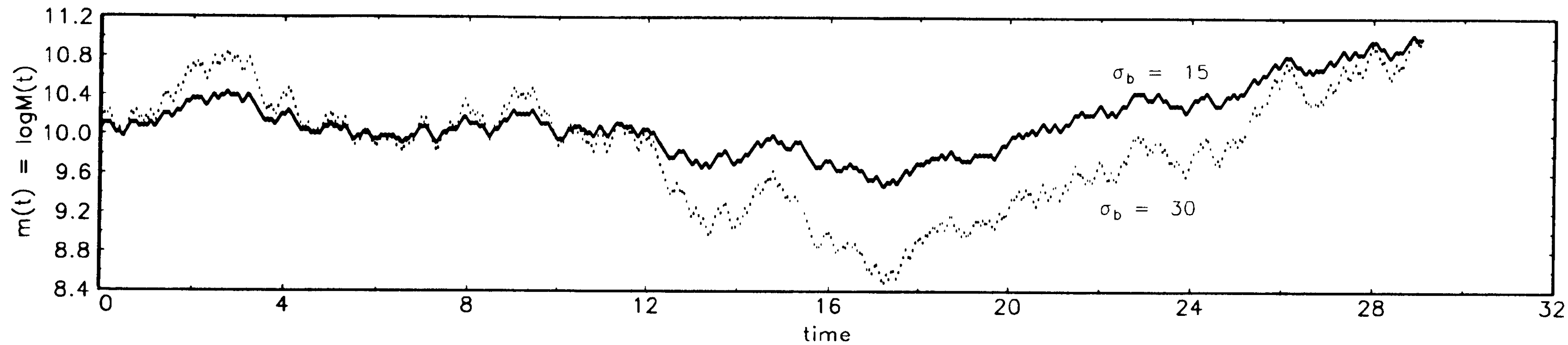

FIG. 2B LOG OF NUMBER OF PRODUCTIVE UNITS

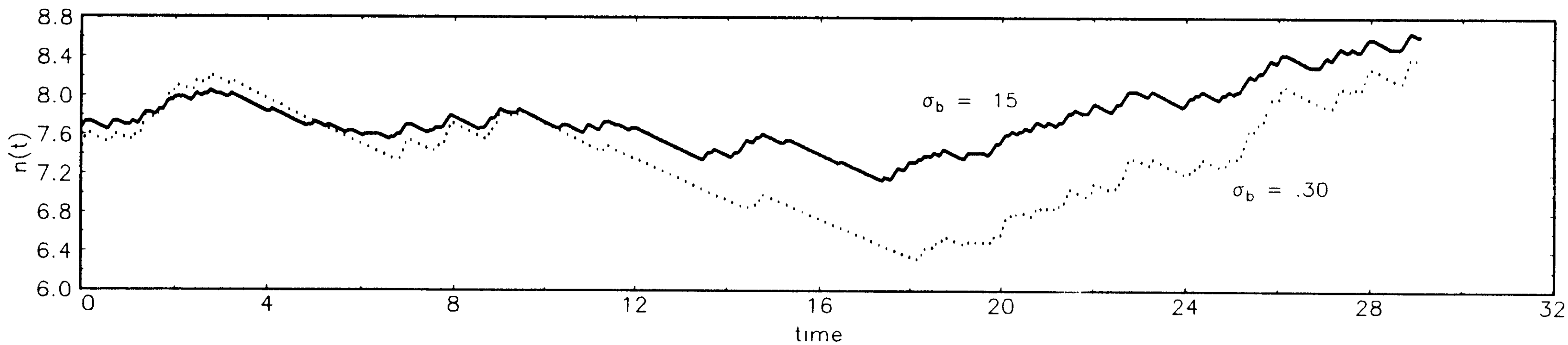

FIG. 2C: LOG OF PRICE

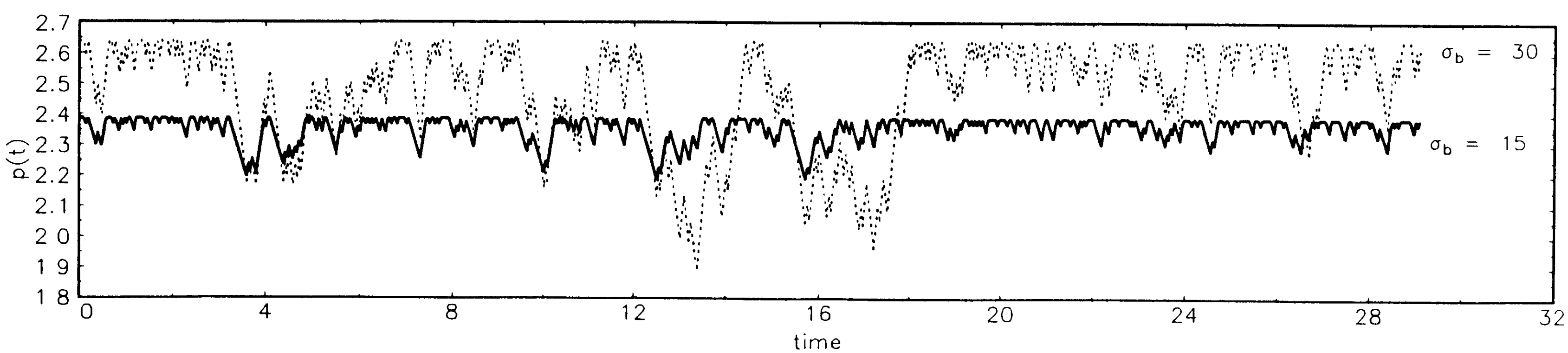

FIG. 3A: E{B(t)|B(0)=U}/U

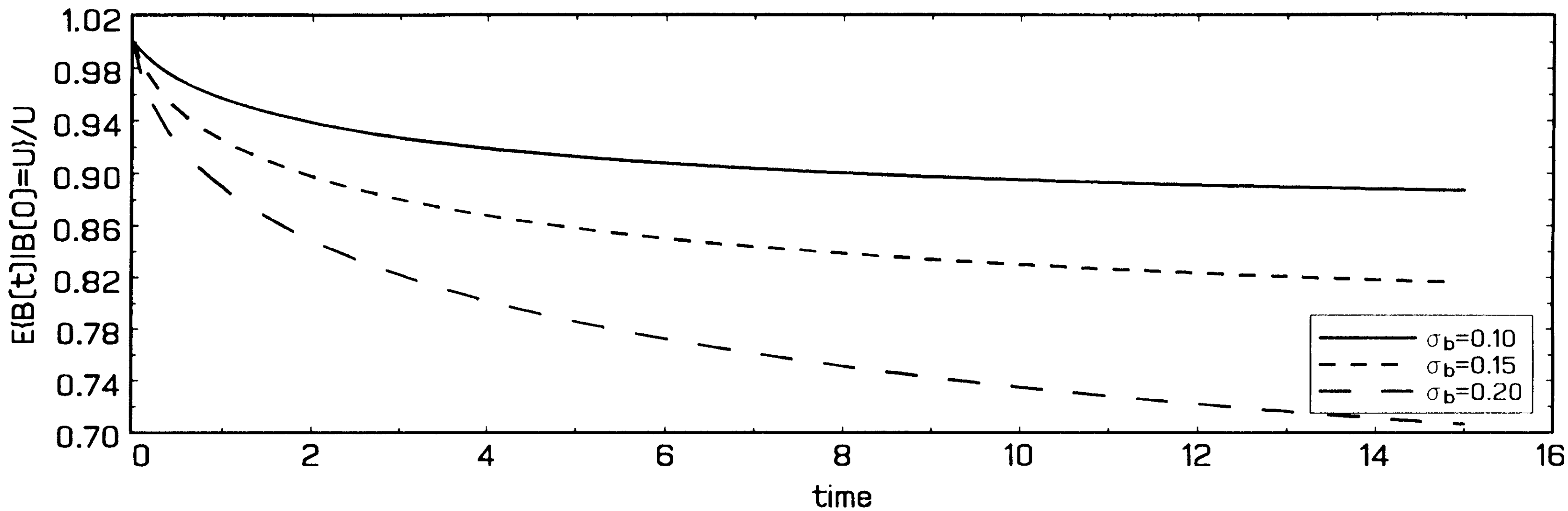

FIG. 3B: E{B(t)|B(0)=U}