Option Valuation of Flexible Investments:
The Case of a Coal Gasifier

by

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CONTENTS

ABSTRACT

This paper examines the use of contingent claim analysis to evaluate the option of retrofitting a coal gasifier on an existing gas-fired power plant in order to take advantage of changes in the relative prices of natural gas and coal. Commodity price changes over time were modelled by binomial stochastic processes, and the price of natural gas is first assumed to follow a Wiener process over time. The option to wait before retrofitting the gasifier was found to be very valuable to the utility. The volatility and convenience yield of natural gas prices were shown to have a strong influence on the exact option value. Uncertainties surrounding future gasifier capital costs proved to be less critical. The paper also examined the case where the price of natural gas follows a mean-reverting process over time, and found that the option value can be substantially affected.

1. Introduction

Traditional discounted cash flow techniques are not always very well adapted to the valuation of investments that involve multistage decision-making (*flexible investments*), and may even lead to erroneous investment decisions.[2] Option valuation (or *contingent claim analysis*) can provide an appropriate alternative in cases where the main sources of uncertainty affecting the future investment decisions are related to portfolios of assets (the *underlying variables*) that are traded on efficient financial markets. The application of option theory to the valuation of flexible investments has been seen as a way to bridge the gap between financial and strategic analysis for capital budgeting (Myers [29]), and a substantial body of literature shows how this can be done in principle.[3] However, little work has been done on the practical use of

[2] See Brealey and Myers [4] for an introduction to discounted cash flow methods.
[3] For typical examples, see McDonald and Siegel [27], Brennan and Schwarz [5], Majd and Pindyck [23], Kogut and Kulatilaka [20] or Teisberg [40].

option valuation in actual investment situations. The objective of this paper is to show how option thinking can be used in practice to evaluate a coal gasification investment by the operator of a natural-gas fired power plant. The choice of this example was motivated by: (i) the presence of well-defined underlying variables (the prices of natural gas and coal) related to widely traded commodities; (ii) the substantial size and duration of the investment, which are likely to lead to substantial option values; (iii) the flexibilities offered by the proposed investment; and (iv) its potentially important applications in industry.

2. Investment Case

2.1 Coal Gasification and its Benefits

Coal gasification has long been used in the chemical industry, but its application to power generation is still at the demonstration stage. The Texaco process (McCarthy and Engel [26]) and the Allis-Chalmers Kilngas process (Womeldorff [44]) seem to be the most promising techniques, and Beér [1] suggests that commercial gasifiers could become available to electric utilities in the next decade. Most gasification techniques presently under consideration entail reacting coal with steam and oxygen (or air) in a high-temperature environment (Rose [34]). In an integrated coal gasification combined-cycle (IGCC) plant, the synthesised gas is fed to a combustor that drives a gas turbine. Waste heat from the gas turbine is used to produce steam, which in turn powers a steam turbine. Electricity is generated by both the gas and steam turbines.[4] Spencer et al. [37] stress the high energy efficiency of the IGCC configuration. The possibility of building a central coal gasification plant serving several natural gas power plants is also being considered in the industry. Finally, repowering an existing natural gas power plant with IGCC is being actively considered. The gasifier

[4] See Weisman and Eckart [43] for a complete description of combined-cycle power plants.

substitutes for the plant's boiler, and the existing steam cycle and power generators need not be replaced (Siegel and Temchin [36]).

Coal gasification provides an environmentally attractive way to burn coal (Spencer et al. [37]). Sulphur and nitrogen emissions from coal gasifiers can easily be reduced, and water requirements are minimal (Makansi [24]). Also, coal gasifiers can be built in small modules of about 100 MWe, thus allowing more flexibility in capacity expansion (Siegel and Temchin [36]).

The environmental benefits and the capacity flexibility of coal gasification will not be evaluated here.[5] Instead, this paper will focus on the benefits brought by fuel flexibility: coal gasification can allow the operator of a natural gas-fired power plant to switch fuels from natural gas to coal, in order to take advantage of changes in gas or coal prices.[6] Makansi [24] writes that "a utility may elect to install a combined-cycle plant burning traditional fuels (...), then convert to coal-derived gas once it becomes economical to do so". This approach is being considered for a 2 x 385 MWe power plant in Martin, Florida, for a 4 x 125 MWe plant in Maryland, and for a 60 MWe in Richmond, Virginia (International Energy Agency [18]).

2.2 Option Description of the Investment Situation

This paper considers the case of an electric utility which is presently operating a natural gas combined-cycle power plant, and has the possibility of retrofitting a coal gasifier. The objective is to calculate the net present value of this gasifier investment.

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[5] An example of option valuation of power capacity flexibility is given by Thomas [41].

[6] The economic importance of fuel switching has long been recognized. For example Putnam, Hayes and Bartlett, Inc., and Energy Ventures Analysis [32] and Strategic Decision Group [38] describe the oil/gas switching capacity of the electric power industry as a key buffer to supply and price risks.

The economics of coal gasification have frequently been discussed in the literature (see, for example, Brooks and de Carbonnel [7], or Koppelman [21]). However, these studies focus on new IGCC power plants, and fail to recognise the flexibility value that coal gasification may bring to existing natural gas power plants: if natural gas prices remain low in the future (relative to coal prices), coal gasification may never be attractive to the utility. But if natural gas prices increase substantially, it might become advantageous to install a gasifier. Once installed, moreover, the coal gasifier need not necessarily be operated if natural gas prices decrease again. Thus, at $t = 0$, the utility owns an American compound option (to install the gasifier) on a continuous flow of European options (to operate the gasifier at a given time).[7] The American option to install the gasifier has an expiration date, T, which corresponds to the end of life of the existing power plant. Also, the European operating options obviously cease after time T. The value of the possibility to retrofit a gasifier is equal to the present value at $t = 0$ of the American compound option.

It will be assumed in this paper that the utility's natural gas and coal costs are proportional to the market prices of natural gas and coal, respectively. These two variables represent the main sources of uncertainty for the investment valuation and constitute the option's underlying variables.

2.3 Underlying Variables

Both natural gas and coal are traded on reasonably efficient markets, and their prices are therefore assumed here to follow random walks (i.e., no serial correlation between successive price changes). More specifically, the logarithms of price changes are assumed in Section 2 to 4 to follow Wiener processes, a common assumption in the

[7] By definition, a European option can only be exercised at the expiration date, while an American option can be exercised at any time before expiration (see Cox and Rubinstein [10] for details). A compound option is an option on an option.

option literature (Hull [16]).[8] This assumption is later relaxed in Section 5. Also, the price changes of natural gas and coal are found historically to be partially correlated,[9] which will have to be reflected in the modelling of their behavior over time. In the base-case model, the evolution of the two prices over time is therefore assumed to follow:

$$dC/C = \alpha_c \, dt + \sigma_c \, dz_c \tag{1}$$

$$dG/G = \alpha_G \, dt + \sigma_G \, dz_G \tag{2}$$

$$dz_c \, dz_G = \rho \, dt \tag{3}$$

where α_c (resp. α_G) represents the expected rate of increase of the price of coal (resp. natural gas) per unit time, σ_c (resp. σ_G) represents the standard deviation of the logarithmic change in coal (resp. natural gas) price per unit time, dz_c and dz_G are Wiener processes, and ρ is the correlation factor between the logarithm of the price changes.

Equations (1) to (3) describe the evolution of the prices of gas and coal in a continuous time model. By similarity with the seminal work of Black and Scholes [2] and Merton [28], it is possible to show that the option value of interest in this paper verifies a set of partial differential equations. However, the investment case considered is too complicated to allow for a closed-form solution to these equations. They must therefore be solved numerically, which usually requires time discretisation.

[8] Wiener processes are random walks such that the return of an asset over period Δt is normally distributed, with means 0 and variance $\sigma\Delta t$. Let S be the price of such a financial asset, with value S(0) at $t = 0$. The behaviour of S over time is then given by: $dS/S = \alpha_S \, dt + \sigma_S \, dz$, where dz is a Wiener process, and where α_S and σ_S are constant. Stochastic calculus shows that the logarithm of S also follows a generalised Wiener process, and that changes of Log(S) over period Δt are normally distributed, with mean $(\alpha_S - \sigma_S^2/2)\,\Delta t$, and variance $\sigma_S^2\Delta t$. Therefore, the value of S at Δt is lognormally distributed, with mean $S(0)\exp(\alpha_S \Delta t)$. The expected value of S therefore increases exponentially with time, at a rate α_S per unit time.

[9] Both fuels are substitutes of each other in the energy sector.

Alternatively, the evolution of natural gas and coal prices over time can be described *from the outset* in a discrete-time model, and the option value is then calculated with simple recurrence relationships. This is the method adopted in this paper.

3. Binomial Method

3.1 Binomial Description of Wiener Process

The binomial method uses a binomial tree to describe the behaviour of the underlying variable(s) over time. It was first used for option valuation by Cox et al. [11] and by Rendelman and Bartter [33]. In this method time is modelled in discrete steps. Let t_o, ..., t_i, ..., t_N be the times at which the values of the underlying variables change.[10] A *period* will be defined as the interval between two successive times of the model. Periods in the binomial models are supposed to have the same size, $\Delta = t_{i+1} - t_i$. Also, the risk-free interest rate per unit time is assumed to remain constant over interval [0,T].

The binomial model implies that if an underlying variable has a value V at t_i, it can only take one of two values at t_{i+1} (noted V^+ and V^-). V^+ and V^- have to be properly chosen, so that, if the total number of periods N that correspond to interval [0,T] tends toward infinity, the probability distribution(s) of the underlying variable(s) over time converge toward generalised Wiener process(es). Cox and Rubinstein [10] derive the correct values of V^+ and V^- if there is only one stochastic underlying variable. They also suggest a way to adapt their model to the case in which there are two partially correlated underlying variables.[11] This is the approach that will be adopted here.

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[10] The notation will be: $t_o = 0$ and $t_N = T$.

[11] Binomial models for two stochastic variables were first investigated by Evnine [13].

The objective is to build a binomial tree which models the possible values of the fuel prices C and G over interval of time [0,T]. If there is only one stochastic underlying variable, the expressions giving V^+ and V^- as functions of V are independent of the period considered. With two stochastic variables, however, Cox and Rubinstein [10] suggest grouping the N periods that span interval [0,T] into n sets of 3 consecutive periods ($3 \times n = N$).[12] Let $C = C(3k)$ and $G = G(3k)$ be the values of the underlying variables at time t_{3k}, beginning of period $3k + 1$ ($0 \leq k \leq N/3-1$). Then the stochastic behaviour of C and G over the next three periods is described by the following process:

1. During period $3k + 1$, C is multiplied by u with probability p, or by d with probability $1 - p$, whereas G is multiplied by R, return of a riskless asset over an interval of time Δ. The resulting values at t_{3k+1} are $C(3k + 1)$ and $G(3k + 1)$.

2. During period $3k + 2$, $G(3k + 1)$ is multiplied by u' with probability p', or by d' with probability $1 - p'$, whereas $C(3k + 1)$ is multiplied by R. The resulting values at t_{3k+2} are $G(3k + 2)$ and $C(3k + 2)$.

3. During period $3k + 3$, $C(3k + 2)$ is multiplied by u''_C and $G(3k + 2)$ by u''_G with probability p'', or $C(3k + 2)$ is multiplied by d''_C and $G(3k + 2)$ by d''_G with probability $1 - p''$. The resulting values at t_{3k+3} are $C(3k + 3)$ and $G(3k + 3)$.

This three-period process is then repeated for $C(3k + 3)$ and $G(3k + 3)$. It is clear that periods 1, 4, 7, etc. model the volatility of coal price C, and periods 2, 5, 8, etc.

[12] Boyle [3] develops a slightly different method in which each price can be followed by five possible prices the following period. However, this method leads to very large price trees.

model the volatility of gas price G. Periods 3, 6, 9, etc. model the correlation between the two assets.

The objective is then to determine the various *u*'s, *d*'s and *p*'s such that, if the total number, *N*, of periods over interval [0,T] increases, the asset price changes over time described by the binomial tree approximate Equations (1) to (3). To do so, it is necessary to calculate the standard deviations, expected rates of increase and correlation factor of the underlying assets if they follow the binomial tree just described. The results can then be equated to the empirical values σ_G, σ_C, α_G, α_C, and ρ given by Equations (1) to (3).

It is possible to show that, provided that $\rho < \sigma_C/\sigma_G < 1/\rho$, a set of acceptable values for the u's and d's is:[13]

$$\text{Ln } u = (3T/N\ \sigma_C(\sigma_C-\sigma_G\rho))^{1/2} - T/N\ \delta_C \quad (4)$$

$$\text{Ln } d = -(3T/N\ \sigma_C(\sigma_C-\sigma_G\rho))^{1/2} - T/N\ \delta_C \quad (5)$$

$$\text{Ln } u' = (3T/N\ \sigma_G(\sigma_G-\sigma_C\rho))^{1/2} - T/N\ \delta_G \quad (6)$$

$$\text{Ln } d' = -(3T/N\ \sigma_G(\sigma_G-\sigma_C\rho))^{1/2} - T/N\ \delta_G \quad (7)$$

$$\text{Ln } u''_G = (3T/N\ \sigma_G\sigma_C\rho)^{1/2} - T/N\ \delta_G \quad (8)$$

$$\text{Ln } d''_G = -(3T/N\ \sigma_G\sigma_C\rho)^{1/2} - T/N\ \delta_G \quad (9)$$

$$\text{Ln } u''_C = (3T/N\ \sigma_G\sigma_C\rho)^{1/2} - T/N\ \delta_C \quad (10)$$

$$\text{Ln } d''_C = -(3T/N\ \sigma_G\sigma_C\rho)^{1/2} - T/N\ \delta_C \quad (11)$$

where δ_C (resp. δ_G) is the convenience yield of coal (resp. natural gas), defined as the difference between the commodity's expected rate of return, μ, required by investors who are willing to hold the commodity, and the expected rate of increase, α, of the commodity price. Convenience yields correspond to storage costs and benefits that include "the ability to profit from temporary local shortages, or the ability to keep a

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[13] See Herbelot [15] for the derivation.

production process running" (Hull [16]). They are similar to dividend yields for stocks, and are important for option valuation because the benefits they represent accrue to the owner of a commodity but not to the owner of an option on this commodity.

3.2 Recurrence Formulas

Option valuation with binomial models rests on the assumption that, at equilibrium, there cannot be arbitrage opportunities between the option itself, its underlying asset, and a riskless asset (this is also the basis for the partial differential equation method).[14] Let S is the present value of an underlying asset. It is assumed that next period the underlying asset value can move up to $S^+ = f\ S$ with probability π, or down to $S^- = g\ S$ with probability $1 - \pi$. Let Y be the present value of a derivative asset that takes value Y^+ if the underlying asset value moves up, and Y^- if it moves down.[15] R is the return of a risk-free asset.[16] It is assumed that S, f, g, R, Y^+ and Y^- are known, and the objective is to calculate Y.

Let a portfolio P consist of $(Y^+ - Y^-)/(f-g)/S$ shares of the underlying asset, and $(fY^- - gY^+)/(f - g)/R$ of a riskless bond (of present value 1 and return R). It is easy to check that the portfolio value next period will be Y^+ if the underlying asset price moves up, and Y^- if it moves down. The portfolio considered and the derivative asset will therefore always have same value next period. They must then have the same present value, in order to avoid arbitrage opportunities. The present value of the portfolio is equal to the weighted sum of the present values of its various components. Hence:

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[14] An arbitrage opportunity is defined as an opportunity to make a risk-free instantaneous profit. Arbitrage situations cannot persist at equilibrium, because investors take advantage of them as soon as they appear.

[15] If Y were the value of a European call option with exercise price K in a one-period model, Y^+ and Y^- would be given by: $Y^+ = max(0, fS - K)$, and $Y^- = max(0, gS - K)$.

[16] It is necessary to have $g < R < f$ for the problem to be interesting. Otherwise, it is easy to show there would be arbitrage opportunities.

$$Y = (Y^+ - Y^-)/(f-g)/S \times S + (fY^+ - gY^+)/(f-g)/R \times 1 \qquad (12)$$

If $h = (R-g)/(f-g)$, then:

$$Y = 1/R\,(\,h\,Y^+ + (1-h)\,Y^-) \qquad (13)$$

Equation (13) gives the value Y at time t_i of a derivative asset as a function of its two possible values at time t_{i+1}. It is the basic relationship used for option valuation in binomial models. Equation (13) does not depend on π, which means that the present value of the option is not explicitly dependent on the probability that the underlying asset moves up or down (which is why p, p' and p" did not have to be specified in Section 3.1). h is often called *pseudo-probability* because it has many characteristics of a probability, but usually does not correspond to any true probability.[17]

For binomial trees of more than one period, equation (13) can be used recurrently. If the option value is known at the end of the interval considered (expiration time), it can be calculated at any prior time by recurrence.

In the case of the gasifier, let W be the investment value at t if the gasifier is already in place, and let W_p be its value at t_i if the gasifier is not yet installed. Then, if the two possible states of the world at t_{i+1} are noted respectively "+" and "-", it is easy to show that:

$$W = \Lambda_{IGCC}\Delta + 1/R\,(hW^+ + (1-h)\,W^-) \qquad (14)$$

$$W_{P1} = 1/R\,(hW_P^+ + (1-h)\,W_P^-) \qquad (15)$$

$$W_{P2} = -\,q_{IGCC}\,Q + 1/R\,(hW^+ + (1-h)\,W^-) \qquad (16)$$

$$W_P = \max(W_{P1}, W_{P2}) \qquad (17)$$

[17] In a risk-neutral world however, it is obvious that $h = \pi$.

where $\Lambda_{IGCC}\Delta$ represents the net cash-flow to the utility of operating the gasifier over interval $[t_i, t_{i+1}]$,[18] and where $q_{IGCC}Q$ represents the gasifier capital cost at time t_i. Also, R is the risk-free return, and h is the pseudo-probability over the interval considered.[19]

Equation (14) gives the gasifier investment value at t_i as a function of the investment value at t_{i+1} in the case where the gasifier is already in place at t_i. Equations (15) to (17) give the investment value at t_i in the case where the gasifier is not yet in place at t_i. Equation (17) represents the choice for the utility between installing the gasifier (in which case $W_P = W_{P2}$), or not ($W_P = W_{P1}$). Since the investment value at $t=T$ is easy to calculate for all values of G and C, it is possible to use equations (14) to (17) to obtain the investment values at the previous period, then at the period before, etc., until one obtains the investment value at $t=0$.

Let $Y_{N/T}$ be the option value at $t=0$ calculated numerically with the binomial model in the case where there are N/T periods per unit time. It was found empirically that $Y_{N/T}$ varies approximately linearly with T/N (a direct result of the discretization of time). This linear effect simplified the calculation: the option was estimated for N/T=3 and N/T=4, and it was then possible to estimate the option value in a continuous time (N/T = ∞) by extrapolation.

4. Model Results for Wiener Process

4.1 Numerical Assumptions

It is assumed that the natural gas plant is already operating, and has well-defined life (20 years), power (536 MWe), and fuel requirements (natural gas costs of

[18] $\Lambda_{IGCC}\Delta$ is therefore equal to the natural gas costs saved over interval $[t_i, t_{i+1}]$ by operating the gasifier, minus the coal requirements and gasifier O&M costs over the same period.

[19] As shown by Teisberg [39], depending upon the interval consider, h is given by: (R-d)/(u-d), or (R-d')/(u'-d'), or (R-d")/(u"-d"), calculated with $\delta_C = \delta_G = 0$.

\$ 76.3 million/year at $t = 0$, corresponding to a delivered natural gas price of \$ 2.20/MMBTU).[20] The power plant has to be operated without interruption as a baseload unit until the end of its life. The utility makes fuel transactions at the market spot price, and does not enter into long-term contracts for its natural gas or coal fuel purchases.[21]

The utility which operates the natural gas-fired power plant is assumed to have the possibility of installing a gasifier at any time between $t = 0$ and $T = 20$ years, the end of the power plant life. If the gasifier is installed and operating, coal fuel requirements are assumed to amount to \$ 49.3 million/year at $t=0$, corresponding to a delivered coal price of \$1.5/MMBTU. In the base-case model, the utility is assumed to have the possibility of switching back to natural gas, after the gasifier is installed (i.e., the gasifier does not have to be continuously operated). This operating flexibility will be referred to as the option to interrupt the gasifier. Also, in the base-case model the gasifier capital costs are assumed to be equal to \$ 504.3 million at $t=0$, and to increase exponentially with time at a constant rate of 2%/year.[22] The gasifier is assumed to have zero salvage value at time T. Also, its O&M costs are assumed to be equal to \$ 14 million/year at $t=0$, and to increase exponentially over time at a constant rate of 2%/year. These costs are assumed to include any change in the O&M costs of the rest of the plant caused by the installation of the gasifier or by the use of coal instead of natural gas. For example, these O&M costs are assumed to take into account the decline in thermal efficiency that follows the installation of a gasifier (International Energy Agency [17]).

[20] The gasifier and power plant characteristics used in this chapter (capital costs, O&M costs, fuel requirements at $t = 0$, etc.) were obtained from industry sources.
[21] Even if the utility enters into long-term contracts for its fuel purchases, the economic attractiveness of the gasifier investment should be calculated using the fuel's expected spot prices.
[22] In a modified version of the base-case model the gasifier costs are later assumed to be stochastic and partially correlated with natural gas prices.

In the base-case model, it is assumed that there are no construction delays between the time the utility decides to install the gasifier and the time it starts operation (this assumption will also be relaxed in a modified version of the model).

The standard deviations of price returns are obtained from the past behaviours of both prices (as given by the Energy Information Administration's *Natural Gas Monthly* [12], and by Manthy [25]). The standard deviation of the natural gas (resp. coal) price return was found to be 12.6%/year (resp. 7.0%/year). The correlation factor between the returns of the two underlying variables was also estimated from their past behaviour. Its value was found to be 0.4. Market estimates of the convenience yields associated with natural gas and coal are difficult to obtain.[23] Instead, an arbitrary value of 5%/year was used for both fuels in the base-case model, but the effects of other values were also assessed (see discussion in Section 4.3 below).

The risk-free interest rate per unit time was assumed to be constant over the life of the investment, and equal to 9.27%/year (the yield of a 20-year Treasury bond, as of May 1, 1990).

4.2 Option to Wait and Option to Interrupt

In the base-case model the utility has both the option to wait before installing the gasifier, and the option to interrupt its operation once it is installed (in order to burn natural gas again instead of coal). The corresponding investment value at $t = 0$ is found to be equal to $ 24.2 million. By comparison, if the utility installed the gasifier at $t = 0$, and operated it continuously until $T = 20$ years, the investment present value would be -$ 310.5 million. This value of - $ 310.5 million was obtained by modifying the binomial model slightly, but could also have been calculated directly as

[23] The most reliable method consists in using the relationship between futures value F, and spot price S of a commodity: $F = S \exp(r-\delta)t$, where r is the risk-free discount rate per unit time (see Brennan and Schwarz [6] for a derivation).

a standard NPV, since the utility has no option in this case. This net present value at $t = 0$ is equal to the sum of the natural gas costs saved over the investment life, minus the capital costs at $t = 0$, minus the coal and O&M costs over the investment life:

$$\begin{aligned} NPV_{IGCC} &= \int_0^{20} 76.3\, e^{-0.05t} dt - 504.3 - \int_0^{20} 49.3\, e^{-0.05t} dt \\ &\quad - \int_0^{20} 14.0\, e^{0.02t} e^{-0.0927t} dt \\ &= -\$\, 310.5\ million \end{aligned} \tag{18}$$

Obviously, if the utility can only choose between installing the gasifier at $t = 0$ and never installing it, it will prefer never to install it (-310.5 < 0). It is the possibility of installing the gasifier in the future which is valuable at t = 0 for the utility.

In order to assess the relative importance of the option to wait before installing the gasifier and the option to interrupt its operation, the investment value can be calculated for the case in which the utility can install the gasifier at any time before $T = 20$ years, but has to operate it continuously once installed. It is found that in this case the investment value at $t = 0$ is equal to \$ 24.1 million. This represents only a 0.4% decline from the case in which the utility has both options. The conclusion is that in almost all states of the world, the gasifier will be operated once installed.[24] This reflects the fact that, if a decision to install the gasifier has been made, natural gas costs are unlikely to fall below the sum of coal costs plus gasifier O&M costs.

It is also interesting to calculate the investment present value if the utility must install the gasifier at $t = 0$, but can choose whether or not to operate it. The calculation shows that the investment value in this case is equal to - \$ 271.7 million, which is 12% higher than the investment value in the case where the utility has neither the

[24] This result was obtained by assuming that there were no "switching costs" associated with interrupting and/or restarting gasifier operation. It would, of course, hold even more strongly if there were such switching costs.

option to wait nor the option to interrupt. Here again, the utility will prefer never to install the gasifier, rather than to install it at $t = 0$ (-271.7 < 0).

The investment values in the four cases just described are compared in Table 1. The table shows that the value of the option to operate the gasifier or not once it is installed is only significant if the utility does not have the option to wait before installing it. This result illustrates the importance of option interactions: the combined value of two options is generally not equal to the sum of the two separate option values (a result in agreement with Trigeorgis [42]). In the rest of this paper the utility is assumed to have the option to wait before installing the gasifier. Therefore, the possibility of interrupting the gasifier's operation will have little value to the utility. This means that the installed gasifier can be assumed to operate continuously until the end of power plant life.

Table 1: Gasifier Investment Value in Four Different Cases

Option(s) Available to the Utility	Investment Value
Option to Wait and Option to Interrupt	+ $ 24.2 million
Option to Wait Only	+ $ 24.1 million
Gasifier Installed at $t = 0$; Option to Interrupt Only	- $ 271.7 million
Gasifier Installed at $t = 0$; No Option	- $ 310.5 million

4.3 Influence of Model Parameters

The sensitivity of the investment value to the various parameters of the base-case model can be described by the elasticity of the investment value relative to the parameter considered. Table 2 gives these elasticities around the base-case values. It shows that the convenience yield of natural gas has a strong influence on the final investment value: for example, if δ_{gas} is equal to 4%/year instead of 5%/year, the investment value in the base-case model increases from $ 24.2 million to $ 45.5 million. The plant life is also found to have a significant effect on the investment value, a result connected to the relatively low convenience yield values.

It is worth noting that the importance of the various model parameters reported in Table 2 does not always correspond to the results of the scrubber investment case analysed in Herbelot [15]. For example, O&M costs appear to have less effect on the investment value here than in the scrubber case, probably because their value (relative to the capital costs) is higher in the scrubber case. Similarly, the power plant life is a more significant parameter for the gasifier investment, probably because the yearly operating benefits (relative to the capital costs) are higher in the gasifier case.

Table 2: Elasticity of the Gasifier Investment Value Relative to Various Model Parameters

Model Parameter	Elasticity
Natural Gas Convenience Yield	-4.4
Plant Life	+3.4
Natural Gas Volatility	+1.7
Coal Convenience Yield	+1.3
Risk-free Interest Rate	+1.2
Gasifier O&M Costs	-0.5
Gasifier Cost Escalation Rate	-0.2
Coal Volatility	+0.05

4.4 Results of the Modified Models

The base-case model described in the previous section was modified to make it more realistic. In particular, the effects on the investment value of construction delays, deterministic changes in the gas price volatility, uncertainty of the gasifier capital cost, and mean-reversion of natural gas prices were investigated.

Gasifier Construction Lead-Time

The first modified model considers the case in which there is a time delay between the moment the utility decides to install the gasifier and the time the gasifier starts operation. Figure 1 gives the investment value at $t = 0$ for construction lead-times ranging from 0 to 5 years.

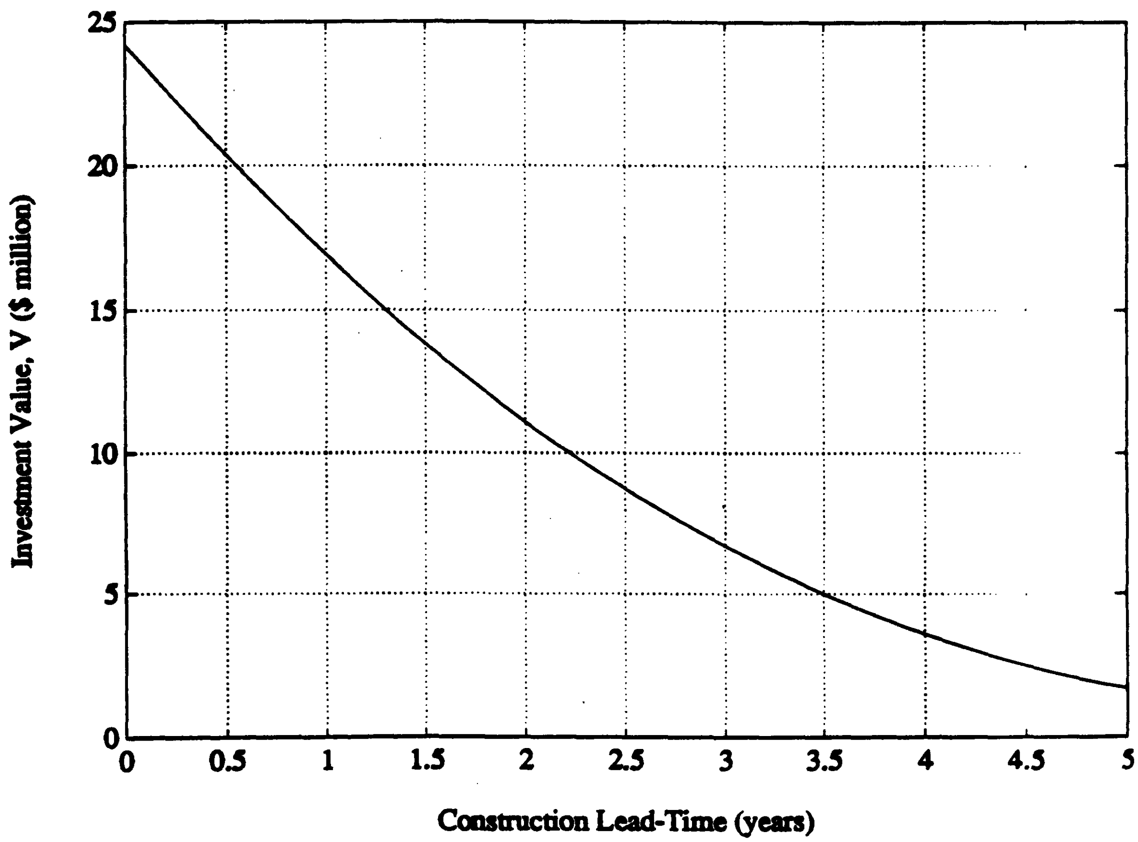

Figure 1: Effect of Construction Lead-Time on Gasifier Investment Value

Note that for zero construction lead-time the modified model gives an investment value of \$ 24.2 million, as expected. For realistic construction lead-times of 3 years or so, the investment flexibility value is virtually zero. It is thus very important to determine carefully how long the gasifier installation might take. This would be even more true if power plant operation had to be interrupted during gasifier construction.

Future Change in Natural Gas Price Volatility

Table 2 showed that the volatility of natural gas prices is a significant determinant of the gasifier investment value at $t = 0$. The influence of an expected future change in this volatility was estimated in a modified version of the base-case model. It was assumed that at a given future time, l, the volatility of natural gas prices suddenly increases from 12.6%/year to 25%/year, and remains at that level for the rest of the power plant life. Figure 2 gives the investment value at $t = 0$, as a function of time l. As expected, for $l = 20$ years, the investment value is equal to the base-case value of \$24.2 million. Similarly, for $l = 0$, the investment value is equal to \$ 123 million, which corresponds to a constant natural gas price volatility of 25%/year over the life of the power plant. Figure 2 shows that if the volatility change occurs after year 12, the investment present value changes by less than 15%. Since the volatility jump chosen was very substantial (doubling of the standard deviations), it is reasonable to assume that the present value of the investment in most cases will not be strongly influenced by changes in the standard deviation occurring a decade or more hence.[25]

Stochasticity of Gasifier Capital Cost

Coal gasification is a relatively new technology for electric power plants, and the future capital cost of a gasifier is not known with certainty. In a modified version of the base-case model it is assumed that the gasifier capital cost follows a modified

[25] Jensen Associates [19] report that forecasts of natural gas prices in 2010 range between \$4.6 and 414.8/MMBTU in the literature (for a present price of \$2.30/MMBTU in 1990). If this corresponds to a 99% confidence interval and the distribution of the price return is normal, the standard deviation for natural gas is equal to 19%/year. The value of 25%/year chosen here is therefore larger than is reasonable to expect.

Wiener process over time, and is partially correlated with natural gas prices.[26] The standard deviation of the gasifier capital costs is assumed to be 5%/year. For a capital cost of \$ 504.3 million at $t = 0$, this volatility corresponds to a 90% confidence interval of \$ 287-722 million for gasifier capital cost 20 years from now.[27]

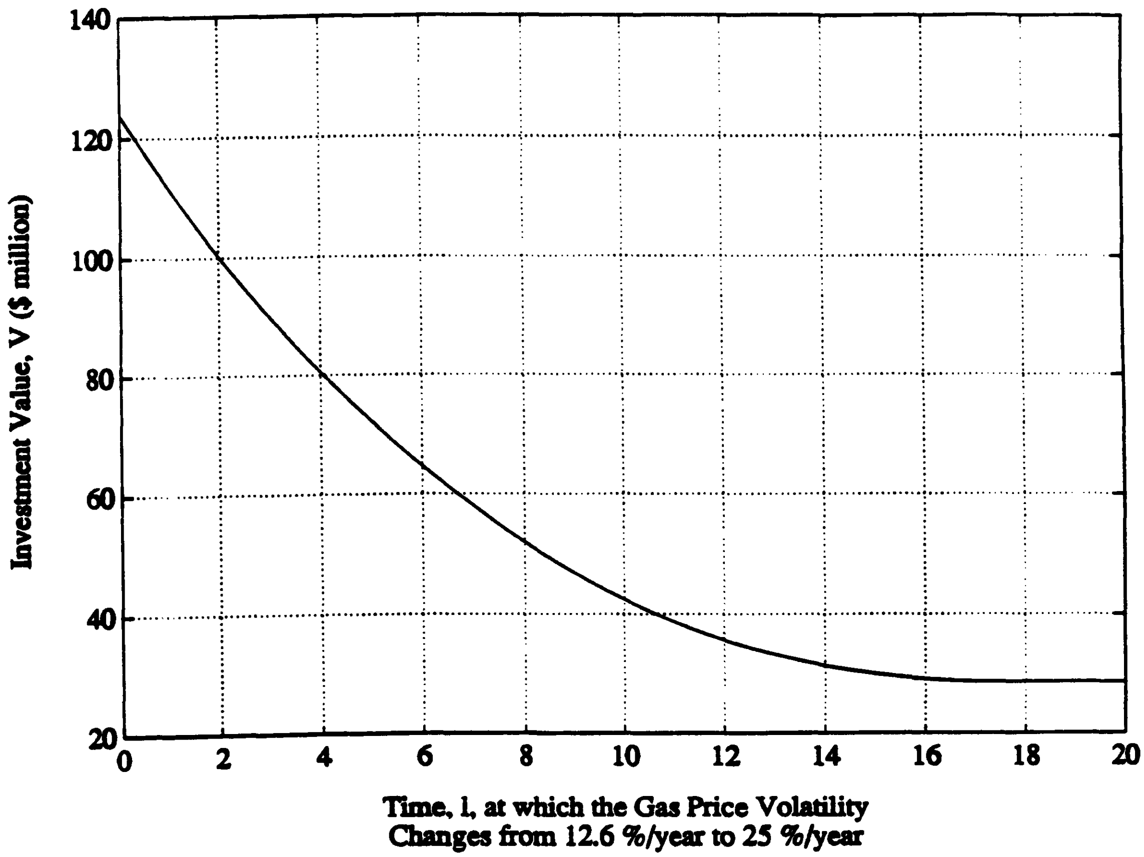

Figure 2: Effect on the Gasifier Investment Value of a Future Change in the Volatility of Natural Gas Price

Because the binomial model developed here can only include two stochastic variables, it is necessary to assume in this modified model that coal prices increase deterministically. Let α_{coal} be the corresponding rate of exponential increase per year. Since the rate of return over one year of a risk-free asset is given in equilibrium by the risk-free interest rate r_F, it is necessary to have:

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[26] It is likely that the uncertainty surrounding the gasifier capital cost decreases with time. The Wiener process assumption is only made for simplicity.

[27] See, for example, Ross [35] for a table of normal distribution.

$$\alpha_{coal} = r_F - \delta_{coal} \tag{19}$$

where δ_{coal} is the convenience yield of coal.

It is important to note that if coal prices are stochastic but uncorrelated with the other two stochastic variables of the problem, the general model can still be used because only the expected value of uncorrelated stochastic variables matters for option valuation. In such a case only a slight modification of the model is required: if the decision-maker is not risk-neutral and coal prices are stochastic, equation (19) has to be replaced by:

$$\alpha_{coal} = \mu_{coal} - \delta_{coal} \tag{20}$$

where μ_{coal} is the required rate of return of a financial security of risks similar to those associated with coal prices. The determination of α_{coal} therefore requires the use of a risk-return relationship of the CAPM type. For simplicity, it will be assumed in this section that coal prices are deterministic (or, equivalently, that the utility is risk-neutral), so that Equation (19) holds. For consistency with the base-case model, it is also assumed that the convenience yield associated with the gasifier capital cost is equal to the difference between the risk-free interest rate and the rate of increase of the gasifier capital cost in the base-case model (hence, δ_{IGCC} is assumed to be equal to 9.27 - 2 = 7.27%/year).

Figure 3 gives the investment value if the correlation factor between the gasifier capital cost and the price of natural gas ranges from 0 to 0.3. For a zero correlation factor, the gasifier capital costs are uncorrelated with natural gas prices. The investment value is then equal to the base-case model when there is no correlation between natural gas and coal prices ($ 30.1 million). Figure 3 shows that the investment value is not very sensitive to the correlation factor between natural gas prices and gasifier capital costs. It is therefore reasonable to assume that they are uncorrelated. Moreover, since only the expected value of uncorrelated stochastic variables matters for option valuation, it is reasonable to treat the gasifier capital cost as a deterministic variable.

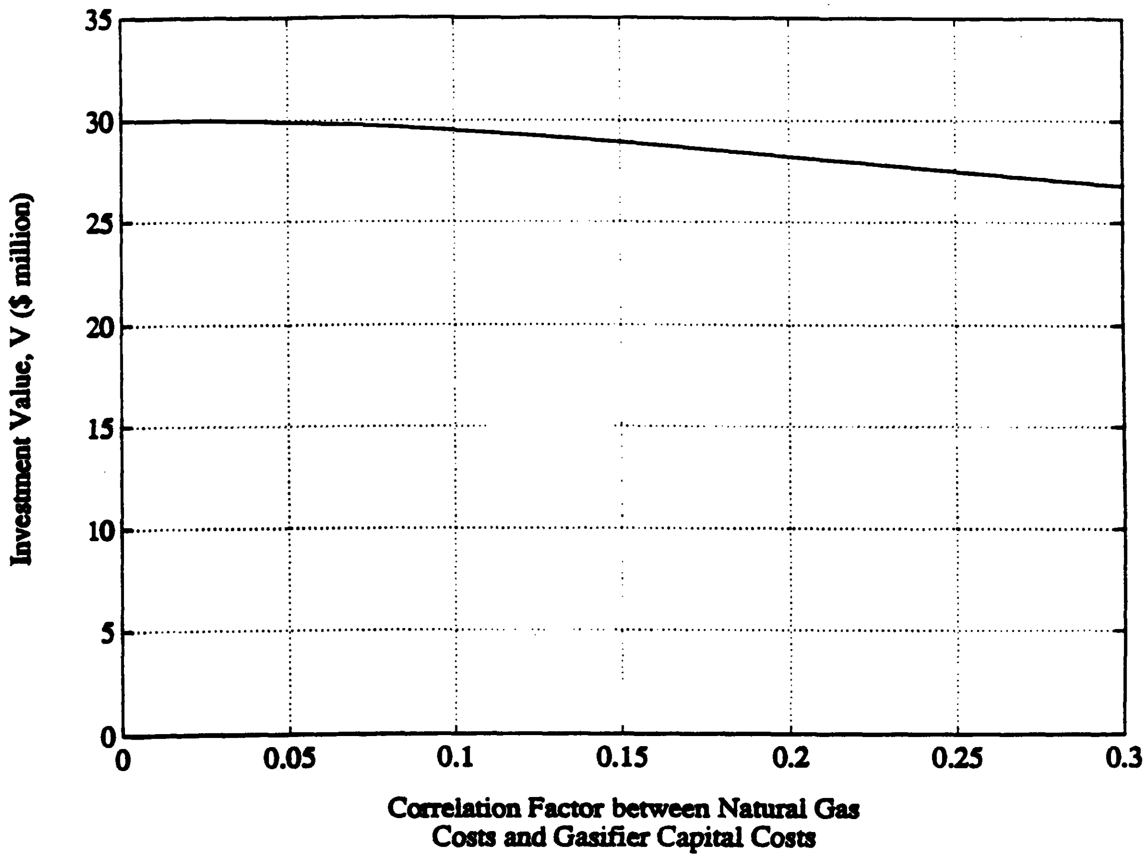

Figure 3: Effect of the Correlation Factor between Natural Gas Price and Gasifier Capital Cost Return on the Gasifier Investment Value

5. Mean-Reverting Stochastic Process

5.1 Mean-Reverting Natural Gas Prices

In all preceding option calculations the stochastic variables were assumed to follow modified Wiener processes over time (a common practice in the financial option literature). Such an assumption may be legitimate for stock prices, but there are some indications that commodity prices have a long-term tendency to revert to a "mean" value. For example, Pindyck and Rubinfeld [31] statistically reject the hypothesis that the price of crude oil followed a modified Wiener process over the last 80 years or so (however, Pindyck [30] also shows that it is not possible to reject this hypothesis for crude oil prices over the last 20 years). It was therefore decided to study whether the price of natural gas, P, follows the mean-reverting process given by:

$$dP/P = k\,Ln\,(P_o/P)dt + \sigma_P\,dz \tag{21}$$

where dz is a Wiener process, σ_P is the instantaneous standard deviation of the price return, P_o is the "mean price", and k is the *mean-reverting coefficient*. For $k = 0$, Equation (21) describes a Wiener process. For $k > 0$, the price of natural gas tends towards P_o. The larger the magnitude of k, the stronger this tendency. To see if natural gas prices follow Equation (21), their behaviour over the last 70 years was obtained from Manthy [25] and the Energy Information Administration [12]. It is possible to show from Equation (21) that if P_j is the price of natural gas at year j, and P_{j-1} is the price at year j - 1:

$$Ln\,(P_j) = (1 - exp(-k))\,Ln\,(P_0) + exp(-k)\,Ln\,(P_{j-1}) + \epsilon_j \tag{22}$$

where ϵ_j is normally distributed, of mean zero, and independent of ϵ_i for $i \neq j$. $Ln(P_j)$ was therefore regressed against $Ln(P_{j-1})$. The slope of the best fit line was found to be $u = 0.968$, which corresponds to a value of k of 0.032. The standard deviation of the slope estimate was found to be $s = 0.012$. It is then possible to test whether the value of the slope is significantly different from 1 (a slope of 1 corresponds to the Wiener process, because it occurs when $k = 0$). If the slope estimate follows a t-distribution of expected value 1, the t-value is given by (u-1)/s = 2.6, which is significant at the 5% level, but not at the 1% level (for 68 degrees of freedom). The price of natural gas over the last 70 years is therefore unlikely to follow a Wiener process (although it is impossible to reject this possibility completely). It was therefore decided to calculate the gasifier investment value in the case where natural gas prices follow the mean-reverting stochastic process described by Equation (21). For simplicity the price of coal was assumed to increase deterministically at a rate of 4.27%/year.

5.2 Binomial Model Modification

Cox and Ross [9] studied the valuation of options on stocks whose prices do not follow modified Wiener processes. However, they focused on jump processes and did not consider mean-reverting processes. The same is true of Hull [16]. However, Teisberg [39] has derived a binomial model for option valuation when the stochastic variable follows a mean-reverting process. Her model was used for this work. If the price of underlying asset is P_j at time j, the two possible prices at time $j + 1$ in the binomial model are:

$$P^{+}_{j+1} = P_j \exp(\sigma_P (n/T)^{1/2}) \exp(-kn/T \operatorname{Ln}(P_0/P_j)) \tag{23}$$

$$P^{-}_{j+1} = P_j \exp(-\sigma_P (n/T)^{1/2}) \exp(-kn/T \operatorname{Ln}(P_0/P_j)) \tag{24}$$

where n/T is the number of periods per unit time of the binomial model. Note that when the underlying asset price follows a Wiener process (i.e. when $k = 0$), the number of possible asset prices in the binomial model after j periods is just j; the value after two periods when the price goes up after the first period and then down after the second period is equal to the value after the price goes down and then up. This is no longer the case with mean-reverting processes, and the number of possible prices at period j is equal to 2^j. This may cause numerical difficulties in some cases, and makes the binomial approach less attractive for mean-reverting processes. Other methods, like the one developed by Kulatilaka and Marcus [22] may be more appropriate.[28]

5.3 Mean-Reverting Model Results

Figure 4 shows the investment value at time $t = 0$ for various values of the mean-reverting coefficient k.[29] For $k = 0.03$, the investment value is found to be \$ 15.0 million, vs. \$ 30 million for the Wiener process case (with no correlation between natural gas and coal prices since coal is supposed to be deterministic in this section). In an investment case concerning the choice between one large coal power plant and several small oil turbines, Pindyck [30] also found that the value of the option to wait declined as the mean-reverting coefficient of oil prices increased. The magnitude of the decline is hard to compare with our results because Pindyck focuses on the minimum oil price to invest in the coal power plant, rather than on the investment present value.

To gain insight into the strength of the mean-reverting effect for $k = 0.03$, it is useful to relate the mean-reverting coefficient k, to the "half-time", $t_{1/2}$, the time taken for the price of natural gas to reach a value 1.5 P_0 after a sudden jump in price from P_0 to $2P_0$ (for this calculation it is assumed that $\sigma_P = 0$). The calculation shows that:

$$t_{1/2} = 1/k \, Ln \, (Ln(2) \, / \, Ln(1.5)) = 0.54 \, / \, k \tag{25}$$

As expected, the higher the value of k, the faster the natural gas price reverts to its mean value. Equation (25) predicts a "half-time" of 17 years for a mean-reverting

[28] Kulatilaka's method is based on Constantinides [8]. It entails replacing the required return of a risky asset, μ, by $\mu - \lambda\sigma\rho$, where λ is the market price of risk, σ the variance of the asset return, and ρ its correlation factor with the market. Constantinides shows that the risk-free discount rate can then be used to discount all claims contingent on this asset.

[29] It was possible to use Teisberg's method for these calculations because of the relatively simple model chosen.

coefficient of 0.03. Over a period of 20 years the mean-reverting effect is therefore probably barely detectable.

Thus, it was found that, even though modified Wiener and mean-reverting processes may both be acceptable descriptions of the price behaviour of natural gas over 20 years, they lead to option values that can differ by a factor of 2. This result clearly shows the importance for option valuation of describing the stochastic behaviour of the underlying variable as accurately as possible.

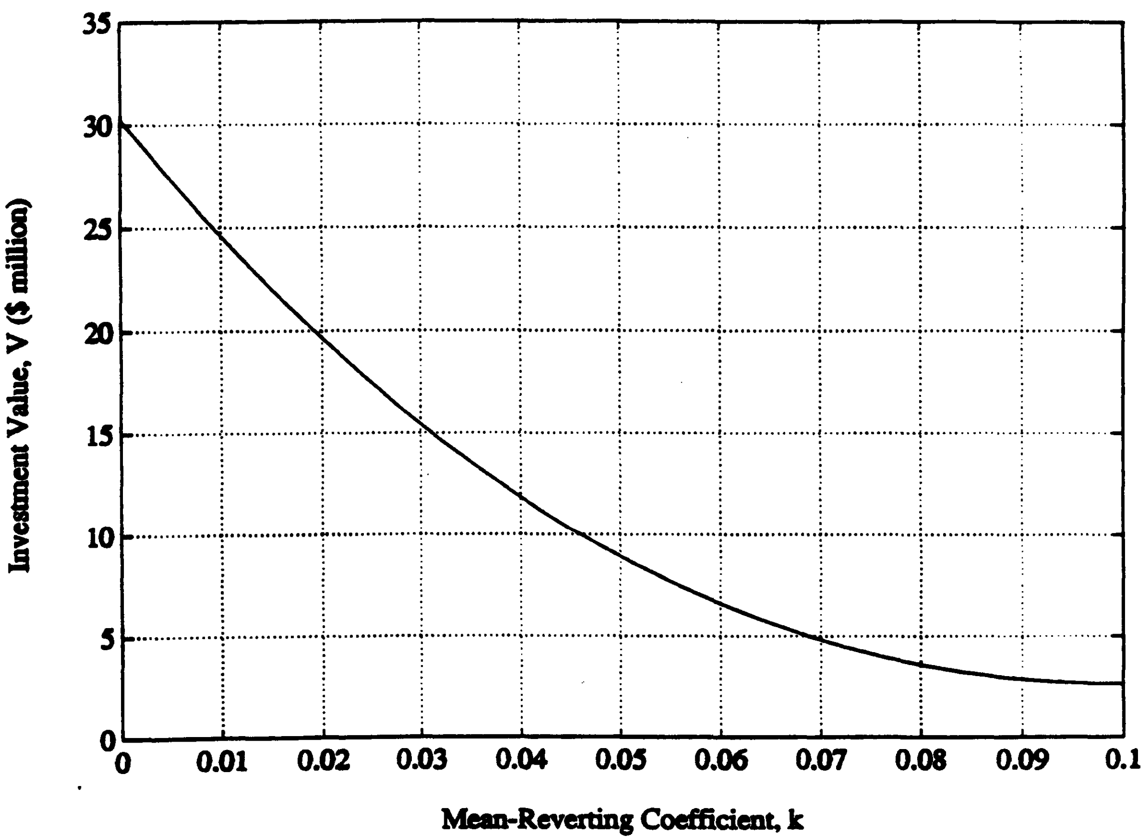

Figure 4: Effect of Mean Reverting Coefficient on Gasifier Investment Value

6. Conclusions

The present value of the gasifier investment considered in this paper (including the option to wait before installing the gasifier, and the option to interrupt its operation) was found to be substantial. By contrast, the calculation showed that installing the gasifier at $t = 0$ was not attractive to the utility, even if the gasifier's operation could be interrupted in order to switch fuel back to natural gas. It was the option to wait before installing the gasifier which proved very valuable.

As in the scrubber case analysed in Herbelot [15], the volatility and convenience yield of the main stochastic underlying variable (in this case the price of natural gas) were found to have a strong influence on the investment value. This was especially true for the volatility value in the first years of the investment. It was possible to show that a doubling of the volatility in the last ten years of the power plant life did not substantially increase the investment value at $t = 0$.

The form of the stochastic equation chosen to describe the price behaviour of natural gas over time was also found to be important. In the case of a particular mean-reverting process the flexibility value was found to be half what it was in the case of the Wiener process, even though mean-reversion was not particularly strong. This is an unfortunate result because it is statistically difficult (on the basis of the data available) to decide the type of stochastic process that fuel prices actually follow.

Another critical determinant of the flexibility value was the time it would take for the gasifier to begin operation after the installation decision had been made by the utility. This particular parameter would lose some of its importance if the power plant had a longer remaining life than assumed in the present model.

Finally, the uncertainty surrounding future gasifier capital costs was not found to be very important, even though the value at $t = 0$ of the gasifier cost was found to have a more significant influence on the investment present value than in the scrubber case of Herbelot [15].

Advantages of the Binomial Method

The binomial approach used in this paper was found to be easy to code, and fairly intuitive, which makes it particularly well-suited to business applications. Moreover, the example considered showed how versatile the method is. Numerous modifications to the basic model were made, in order to investigate various operating flexibilities. In most cases these modifications required only changes in the recurrence relationships for the investment value, or changes in the investment value at the end of the investment period. In fact, the same model was used for the scrubber model of Herbelot [15], even though the investment situation was very different. The binomial description used here could thus be applied to the valuation of any investment contingent on two partially correlated variables that follow modified Wiener processes.[30] The binomial method may not, however, be the most appropriate way to calculate option values if the underlying variables follow mean-reverting processes.

Suggestions for Future Work

This paper has shown the importance of convenience yields for the valuation of options contingent on commodities like fossil fuels. Oil convenience yields have been shown to vary stochastically over time and to be partially correlated with oil spot prices (Gibson and Schwartz [14]). It would be interesting to see if the results can be generalised to other commodities. If this were the case, the binomial model used here would have to be adapted to take this effect into account and see whether it substantially changes the value of the flexibility. Further research is also needed for

[30] Provided that $\rho_{12} < \sigma_1 / \sigma_2 < 1/\rho_{12}$, where ρ_{12} is the correlation factor between the processes, and where σ_1 and σ_2 are the standard deviations of the two modified Wiener processes.

mean-reverting stochastic processes: the binomial model used here was not found to be particularly convenient for such processes because of the large size of the binomial tree if n/T becomes too large.

Another possible application of contingent claims analysis concerns strategic investments like R&D investments in process or product innovation. Such investments are valuable not in themselves (on their own they are virtually worthless) but for the option they give the decision-maker to invest in future profitable follow-up projects. Even though many studies in the management of technology warn against the dangers of a linear model of innovation (where research leads to development, which in turn lead to production), it might be helpful to think of research investments as options on development investments, and of development investments as options on production investments. However, much research remains to be done to show that it can actually be used in practice for such cases.

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