Option Valuation of Flexible Investments:
The Case of a Scrubber for Coal-Fired Power Plant

by

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MIT-CEEPR 94-001WP

March 1994
OPTION VALUATION OF FLEXIBLE INVESTMENTS: THE CASE OF A SCRUBBER FOR COAL-FIRED POWER PLANT

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November 1993

This paper is derived from: Option Valuation of Flexible Investments: the Case of Environmental Investments in the Electric Power Industry, PhD thesis. MIT, Department of Nuclear Engineering, May 1992. I wish to thank my thesis supervisors, Professors Richard K. Lester, Robert S. Pindyck, and Stewart C. Myers for their constant guidance and support during the course of this work. Also, the financial support of the Nuclear Engineering Department and of the Center for Energy and Environmental Policy Research is gratefully acknowledged.

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ABSTRACT

Standard discounted cash flow methods are not well suited to the valuation of investments whose characteristics can be modified by the decision-maker after the initial investment decision has been made (multistage decision investments). For some problems of this type the theory of financial options offers a better alternative.

The theory is applied here to an existing coal-fired power plant that is required to comply with the new SO$_2$ emission limits introduced by the Clean Air Act Amendments of 1990. By assumption, the power plant operator can either purchase emission allowances from other utilities, or switch fuels to a lower-sulfur coal, or install an SO$_2$ emission reduction system (scrubber). The two main sources of uncertainties (future price of SO$_2$ allowances, and future difference between the price of high-sulfur and low-sulfur coals) are assumed to follow Wiener stochastic processes over time. A binomial model is developed to calculate the present value of the options to install the scrubber and/or switch coals. It is shown that the possibility of switching coal has little value to the utility in the case considered, but that the possibility of installing a scrubber reduces the net present cost of complying with the Clean Air Act SO$_2$ requirements. A parametric study is performed to estimate the influence of various model variables on the option present values. Also, the effect of future scrubber technology improvements is investigated. Finally, the model is used to obtain an investment criterion that specifies, ex-ante, the future conditions under which the scrubber should be installed or the fuel switched.

The investment case considered shows how contingent claim analysis can be applied in practice to evaluate realistic flexible investments. The results underline the need to take investment flexibility into account and the practical advantages of option valuation. They show that investment criteria can be substantially modified by the value of flexibility. Also, the binomial model for two underlying variables developed here is found to be quite intuitive and easy to apply numerically. It can also be used to determine investment criteria.
INTRODUCTION

Corporate investments are critical determinants of a company's future competitiveness. It is through investments that strategic decisions are implemented and that value is added to the firm. Investment decision-making, or capital budgeting, has therefore long been an important area of research in corporate finance. The objective is to provide managers with decision criteria that are simple enough to be easily implemented in day-to-day business situations. Most modern criteria require that the investment be "valued". Valuation techniques for investments in which only an initial decision is required (standard investments) are different from those for investments with multistage decision-making (flexible investments).

This paper examines the case of a scrubber investment to limit sulfur emissions of a coal-fired power plant, in the framework of the 1990 Clean Air Act Amendments. Section 1 shows how the most popular valuation techniques for standard investments may lead to incorrect decision-making for some multistage investments. An alternative method, the option valuation technique, is presented, and a simple example is used to illustrate its most interesting features. It is then shown that a substantial body of literature exists on the theoretical advantages of option analysis for real investment valuation, but that little work has been done on its practical applications. Section 2 briefly presents the 1990 Clean Air Act Amendments for sulfur emission reductions, and shows that the new legislation will have substantial implications for strategic decision-making by electric power companies. Section 3 then defines more precisely the investment situation of a coal-fired power plant that has to reduce its sulfur emissions and that can do so either by switching to a lower-sulfur coal or by installing an emission control system (scrubber). The modelling of the underlying stochastic variables of interest in this case is also discussed. The general continuous-time method usually used in the literature on option calculations is then presented, and
its applicability to the investment situation studied is discussed. Section 4 shows that a discrete-time method is more appropriate here, and the corresponding model is fully developed. Section 5 gives the main results of the analysis. The flexibility of the investment is evaluated in different situations, and a sensitivity analysis is performed. Finally, Section 6 offers general conclusions, and suggests directions for possible future work.

1. OPTION VALUATION OF FLEXIBLE INVESTMENT

1.1 Investment Flexibility

Discounted cash flow valuation methods can lead to erroneous decisions if the flexibility available to the investor is not recognised. Let an electric utility have the choice between building a coal-fired power plant and a plant fuelled by natural gas. Assume for simplicity that the utility is risk-neutral, and that the problem is a two-period one:

- at $t = 0$ the utility has to pay the capital costs of building the plant; and
- at $t = 1$ it has to pay a lump sum corresponding to its future fuel costs.

The utility's objective is to choose the alternative with minimal total costs. The coal-fired power plant is assumed to cost $100 million to build, versus $80 million for the natural gas plant. It is known at $t = 0$ that total coal costs at $t = 1$ will amount to $150 million, but there is uncertainty regarding the price of natural gas in the second period. The utility estimates that there is a 50% chance that its natural gas costs at $t = 1$ will be $400 million, and a 50% chance that they will only be $100 million. Since the utility is assumed to be risk-neutral, the expected values of the future cash-flows have to be discounted at the risk-free rate, whatever the inherent
risk of these cash flows. Let the risk-free interest rate be 10% between time $t = 0$ and time $t = 1$. Then:

$$NPV(\text{coal}) = -100 - 150/(1+0.1) = -236 \text{ million}$$
$$NPV(\text{gas}) = -80 - (0.5\times400 + 0.5\times100)/(1+0.1) = -307 \text{ million}$$

The utility should therefore invest in the coal-fired power plant. However, this conclusion does not hold if the utility has some "flexibility". Assume, for example, that at $t = 1$ it could add a coal gasifier costing $70$ million to the natural gas plant. This gasifier would allow the utility to burn coal fuel instead of natural gas.

Assume that the utility has chosen to build a natural gas-fired power plant at $t = 0$. If natural gas costs turn out to be $400$ million at $t = 1$, the utility obviously will not install the gasifier. But if they turn out to be $100$ million, the utility will be better off paying $70$ million for the gasifier plus $150$ million in coal costs, rather than paying $400$ million in gas costs. The NPV of the natural gas power plant investment in this case is:

$$NPV'(\text{gas}) = -80 - (0.5\times100 + 0.5\times(70+150))/(1+0.1) = -225 \text{ million}$$

Total costs for the natural gas case are therefore lower than for the coal case, and the utility should invest in the natural gas-fired plant at $t = 0$. Thus, the utility's optimal investment choice changes, because it now has some flexibility in the natural gas case that it does not have in the coal case. This flexibility is valuable, and its value in the case considered here is equal to: $307 - 225 = 82$ million. This simple example illustrates the importance of valuing investment flexibilities. This is what Kensinger [58] calls "valuing active management".

---

1 A risk-neutral decision-maker is indifferent between receiving an uncertain cash flow at time $t$ and receiving its expected value at $t$. Since expected values are certain, they have to be discounted at the risk-free rate.
1.2 Dynamic Discounted Cash Flow Method

The net present value of a flexible investment can be calculated with discounted cash flow methods by building a decision tree in which each node corresponds either to a decision choice for the investor, or to the realisation of an uncertain event (Magee [71]). The tree is solved step by step, starting with the last nodes and moving backward in time until the node corresponding to time \( t = 0 \) is reached (this was the method implicitly used in the example of the previous section).\(^2\) This method is sometimes called dynamic discounted cash flow. It is conceptually simple, but its actual implementation may lead to difficulties (Teisberg [116]). For one thing, very large trees may be cumbersome to solve. More fundamentally, the dynamic DCF method requires estimates of all relevant event probabilities. Such probabilities are usually highly subjective, and the decisions made can therefore be controversial. Also, the issue of risk has to be properly addressed. If the investor's utility function can be determined, decision analysis provides the best approach.\(^3\) However, this is unlikely to be the case for corporate investments by public companies. Alternatively, risk-return relationships of the CAPM-kind can sometimes be used in decision trees. However, for complicated trees this approach requires the determination of a very large number of \( \beta \)'s, and is often unlikely to be practical. In some cases, option theory can offer an interesting alternative to dynamic DCF methods (Trigeorgis and Mason [125]).

\(^2\) See Park and Sharp-Bette [89] for a more realistic example.

\(^3\) Abel [1] applies a related method to the valuation of the flexibility to choose different energy/capital mixes for the manufacturing of a given product. However he has to assume that the decision-maker is risk-neutral.
1.3 Option Approach

Call options (puts) are financial securities that give their owners the right to purchase (sell) a fixed amount of a specified underlying asset at a fixed price at any time on or before a given date (Cox and Rubinstein [24]).\(^4\) The electric utility case described in the previous section can be used to illustrate the applicability of option theory to the valuation of real investments.

**Option Analogy**

By investing in the natural gas-fired power plant the utility acquires not only the plant itself, but also an option to install the coal gasifier at time \( t = 1 \). As shown above, this option is valuable and should be taken into account. The case discussed here concerns the valuation at \( t = 0 \) of an asset \( Z \) (the option to build the gasifier) that is worth \( 400 - 150 - 70 = $180 \) million if the price of gas goes up at \( t = 1 \), and 0 if it goes down. Indeed, if the price of natural gas goes up, it was shown that the utility should install a gasifier: $400 million will be saved in natural gas costs, and the utility will only spend $150 million in coal costs, and $70 million for the gasifier itself. It was also shown that the utility should not install the gasifier if the price of gas goes down. The value of asset \( Z \) at \( t = 1 \) is thus contingent on the value of gas at the same time (option analysis is sometimes called *contingent claims analysis*).

**Arbitrage Approach**

Assume that the price of natural gas at \( t = 0 \) (which can be directly obtained from the market) corresponds to natural gas costs for the utility of $180 million at \( t = 0 \).\(^5\) Let the risk-free interest rate be 10%. Figure 1-1 shows different assets and their values at \( t = 0 \) and \( t = 1 \). Asset \( G \) represents the quantity of natural gas needed by the

\(^4\) Options that can only be exercised on a given date are said to be European, and options that can be exercised at any time before a given date are said to be American.

\(^5\) The $180 million is only a hypothetical amount, since the problem is formulated such that the utility does not pay fuel costs before \( t = 1 \).
utility for its future operation. Its value at $t = 0$ is $180$ million by assumption, and its value at $t = 1$ is either $400$ million if the price of gas goes up, or $100$ million if the price of gas goes down. Asset $Z$, the option to build the gasifier, has a present value $V_Z(0)$ which is the unknown of the problem. Its future value at $t = 1$ is contingent on the price of gas at that time, and is equal to $180$ million or $0$. Finally, asset $F$ is a risk-free asset of present value $10$ million. Its future value at $t = 1$ is $11$ million since the risk-free interest rate is $10\%$. 

<table>
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<tr>
<th>Asset</th>
<th>Value at $t=0$</th>
<th>Value at $t=1$</th>
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<tbody>
<tr>
<td>$Z$ (Gasifier Option)</td>
<td>$V_Z(0) = ?$</td>
<td>$400-150-70=180$ or $0$</td>
</tr>
<tr>
<td>$G$ (Natural Gas Costs)</td>
<td>$180$</td>
<td>$400$ or $100$</td>
</tr>
<tr>
<td>$P$ (Portfolio $= xG + yF$)</td>
<td>$180x + 10y$</td>
<td>$400x + 11y$ or $100x + 11y$</td>
</tr>
<tr>
<td>$F$ (Risk-free asset)</td>
<td>$10$</td>
<td>$11$</td>
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Figure 1-1: Arbitrage Method for Option Valuation
The basic idea of option valuation is to determine the present value of asset Z by constructing a portfolio P that includes only asset G and/or asset F, and whose value at $t = 1$ replicates the value of asset Z in all possible states of the world (Smith [111]). Let $y$ be the quantity of asset F and $x$ be the quantity of asset G in portfolio P. By definition, $x$ and $y$ are such that:

\[400x = 11y = 180\]
\[100x = 11y = 0\]

This gives: $x = 0.6$, and $y = -5.45$. Thus, a portfolio that consists in purchasing a quantity 0.6 of asset F, and in selling a quantity 5.45 of asset G has always exactly the same value as asset Z at $t = 1$. To avoid arbitrage possibilities, the present value of this portfolio P must be equal to the present value of asset Z. The present value of portfolio P is equal to the weighted sum of the present values of its components: $0.6 \times 180 - 5.45 \times 10 = 53.5$. Therefore, asset Z has a present value $V_Z(0) = \$ 53.5$ million. Since asset Z represents the opportunity for the utility to install a gasifier at time $t = 1$, $V_Z(0)$ represents the flexibility value of the investment.

**Advantages of the Option Approach**

Two of the most interesting features of option valuation for real investments are already apparent from this example:

1. The probability $p$ that the price of gas goes up was not needed to calculate the present value of asset Z. Instead, the market present value of natural gas costs was used as an indicator of the future behaviour of natural gas price. Thus,

---

6 A negative position in a portfolio is called a *short position*. It consists in borrowing the security in question and selling it.
7 Note that the present value of natural gas costs was not used in the dynamic DCF analysis of the same problem. Also, note that the flexibility value of the gasifier calculated by dynamic DCF was not
option valuation does not rely on explicit estimates of the probability of future events. Instead it uses direct market information that incorporates the market's estimates of these probabilities.

2. Also, option valuation does not depend on the investor's attitude toward risk. Indeed, the arbitrage argument is valid whatever the risk preference of the investor.

Contingent claims analysis can thus provide investors with a simple and powerful investment valuation method. The example just discussed shows that this method is suitable for investment valuations in situations where:

- Multistage decisions can be (or have to be) made by the investor; and
- These decisions ultimately depend on financial assets that are publicly traded in efficient markets.

1.4 Option Valuation of Real Assets in the Literature

There have been numerous articles on option theory in the financial literature since the seminal works of Black and Scholes [8] and Merton [81]. Some of these have dealt more specifically with the valuation of real assets like strategic investments or operational flexibilities.

Strategic Investment Valuation

Myers [84] suggests the use of option theory for the valuation of corporate investment opportunities, and Logue [70] stresses that it can be a particularly appropriate

equal to $53.5 million. The reason is that if $180 million really is the market present value for natural gas costs, the situation cannot be risk-neutral as was assumed for the DCF calculation, since:

$180 = (0.5 \times 400 + 0.5 \times 100)/1.1.$
technique for strategic investments. In fact, contingent claims analysis is seen by Myers [85] as a way to bridge the gap between financial and strategic analysis for capital budgeting. The idea is further developed by Kester [59], who recommends that investment opportunities be thought of as "options on the company's future growth".

R&D investments are essentially strategic investments. It is therefore not surprising that several articles in the literature study the use of option theory for R&D investment valuation (see, for example, Hamilton and Mitchell [45] and [83]). Several large manufacturing companies are also considering the use of option theory for the valuation of R&D investments (see, for example, Faulkner [36]). Sanchez [106] also uses contingent claims analysis, but more specifically to assess product development efforts in a strategic environment. Option theory has been used to value patents by describing them as options on technological innovations (Pakes [88]). Also, Competition Technology Corp. [22] and Kambil et al. [55] have suggested that information technology investments can be valued with a contingent claims approach. Finally, several authors have argued that options may offer a convenient way for scientists to convince financially-oriented managers of the long-term value of specific R&D investments (Naj [86]).

**Operational Investments**

Contingent claims analysis has also been used in the literature to value production or manufacturing flexibilities (referred to here as *operational flexibilities*). An overview of various operational flexibilities is given by Mason and Merton [76]. Whenever management has the possibility of modifying in any way the timing of a given investment, the corresponding flexibility has to be considered and valued. McDonald and Siegel [78] study the option to wait before investing in an irreversible project, and

---

8 Strategic investments are defined here as investments that are not valuable in themselves, but for the future opportunities they bring to the company.
derive an optimal investment rule. In another related paper the same authors focus on the option to shut down a facility, and use CCA to evaluate this option [79]. Brennan and Schwartz [13] study the value of a mine that can also be temporarily or permanently shut down. The main results of the analysis are generalised in Brennan and Schwartz [11]. Majd and Pindyck [73] consider the case of a plant that is being built over an extended period of time. The actual building rate depends on the stochastic behaviour of the underlying variable (the production value), and can be determined through option analysis. In another paper Majd and Pindyck [72] study how the optimal production timing of a competitive firm is modified if its production costs follow a learning curve.

The possibility of modifying the production capacity of a given facility can also be analysed as an option for the manufacturing company. Pindyck [95] shows how different production capacity choices may affect a company's value. The same idea is illustrated in a different context for a comparison between building a large coal-fired power plant and building several natural gas turbines (Pindyck [94]). In a similar vein, Thomas [120] studied the modularity advantage of gas-cooled nuclear reactors.

In some cases a given facility can produce different products, or a given company can use different facilities to produce a given product. These options also have to be valued. For example, Kulatilaka [65] and [66] use option analysis to show the importance of flexible product designs that allow easy product modification. Similarly, He and Pindyck [47] and Triantis and Hodder [123] analyse the advantage of having the capability to produce different outputs within a given facility. Kogut and Kulatilaka [60] study a multinational company that can coordinate different international subsidiaries, and show how this flexibility can lead to value-creation. The ability to manufacture in different countries gives a company the option to change the locus of production depending on currency exchange rates (Kogut and Kulatilaka [61]).
Pindyck [97] stresses that it is the irreversibility of most investments that creates option value. He shows how properties of irreversible investments can explain some of the behaviour of aggregate investment in the economy. In a different area, Teisberg [118] studies the option valuation of companies that operate in regulated environments, so that their losses and profits are limited (for an analogy, see Teisberg and Teisberg [119] on the option valuation of commodity purchase contracts with limited price risk). Virtually all of the literature reviewed here tries to show how contingent claims analysis can theoretically be used for real investment valuation. Little work has been done on the practical use of options in actual investment decision situations. The work presented in this paper was motivated by the need to show that option thinking can really be used in practice by companies for investment valuation and decision-making. To do so, the work will focus on a given investment situation in the electric power industry.

1.5 A Specific Investment Case

Option Valuation in the Energy Area

It seems quite clear that virtually any decision can be described in terms of options. However, and as described above, financial option theory is only really useful for investment situations that meet specific conditions. For several reasons the energy industry is an area of particular interest in this regard. Investments in that field are usually of substantial size and last over many years, which increases the chances of finding opportunities for management flexibility (and is also likely to increase the importance of taking the value of such flexibility into account). Equally important, natural gas, oil and coal are commodities publicly traded in large and reasonably efficient markets, and their prices are readily available. They can therefore easily be used as underlying variables for option analyses. Some articles on option theory in the energy sector have already been mentioned above. There are others: Graves et
al. [42] study the valuation of switching rights for natural gas pipelines. Paddock et al. [87] and Siegel et al. [110] value offshore petroleum leases by considering them as options to install a well and start exploitation if the price of oil is favourable. The energy area thus offers interesting opportunities to study the practical use of option valuation for real investments.

Problem Definition

The objective of this work is to show how contingent claims analysis can be used in practice to value a well-defined strategic investment situation. The case discussed concerns the strategic choices that coal-fired power plant operators will have to make in order to reduce the $SO_2$ emissions that result from burning coal containing sulfur. The way these choices are made is going to change substantially as a result of the enactment by the US Congress of the 1990 Clean Air Act Amendments (CAAA). These amendments call for a nearly 50% reduction in $SO_2$ emissions by coal-fired power plants by the year 2000, and for a fixed emission cap thereafter. However, instead of imposing strict emission limits on individual plants the CAAA introduce a system of tradable $SO_2$ emission rights (or allowances) (see Section 2 for details). A number of allowances corresponding to the maximum aggregate emission level of the utility industry will be distributed each year by the US government to power producers, but the power producers will be permitted to buy and sell the allowances among themselves. As discussed in Section 2, this tradable system should give power producers greater flexibility in the choice of their compliance strategies (Lamarre [68]).

This paper will show that such flexibility is valuable, and that contingent claims analysis may fruitfully be used to select a compliance strategy.\(^9\) More specifically, an hypothetical existing power plant that burns high-sulfur coal and that initially emits

\(^9\) Tilly [121] evaluated scrubbers as standard investments, but failed to recognise the value of flexibility.
more $SO_2$ than permitted (based on the number of allowances assigned to it) will be studied. To comply with the Clean Air Act Amendments, the utility is assumed to have three alternatives:

1. it can purchase additional allowances on the emission market; or
2. it can switch to a lower-sulfur coal; or
3. it can invest in a system that captures sulfur before emission (scrubber).

The optimal strategy may of course include any combination of these three alternatives. It will be shown in Section 3 that the preferred choice at any given time ultimately depends on the values at that time of two underlying stochastic variables: the $SO_2$ allowance price, and the price difference between low-sulfur and high-sulfur coals (*coal price premium*).

The installation of an emission control system is essentially an irreversible investment, and switching fuel is also unlikely to be a freely reversible process for the utility. Option values are therefore present in this problem, and contingent claims analysis will be used to value them. Section 3 will show how this can be done.

**2. SULFUR EMISSION REGULATION AND CONTROL**

**2.1 $SO_2$ Emissions by Coal-Fired Power Plants**

Coal accounts for 27% of the energy consumed in the United States, and 55% of the electricity generated (Yeager [131]). It also constitutes 95% of all US fossil fuel reserves, and most experts believe that it will remain a fuel of choice to meet the future energy needs of the country (White House [129]). However, the use of coal for power generation has serious environmental consequences. In addition to $NO_x$, 


CO₂ and particulate emissions. Coal combustion leads to the release of sulfur dioxide, which is thought to be responsible for acid rain. Public concerns about the environmental hazards of acid rain have grown in the past 20 years, leading to increased SO₂ emission regulation.

2.2 SO₂ Emission Control in the 1990 Clean Air Act Amendments

In October 1990 the US Congress overwhelmingly adopted a set of Amendments to the 1970 Clean Air Act that deal with the attainment and maintenance of air quality standards (smog), motor vehicles and alternative fuels, toxic air pollutants, and acid deposition. Most provisions of the Act use direct emission limits to impose new federal standards on urban smog, automobile exhaust, and toxic air pollution. The Amendments also address the issue of NOₓ emissions by electric utilities, but again rely on direct emission limitations. Power producers will have to modify their burners to reduce NOₓ emissions, but will not have much flexibility to do so. This work on option valuation will therefore not consider NOₓ emissions. For SO₂ emissions however, the legislator introduced an innovative system of tradable emission rights. Two phases can be distinguished:

- Phase I concerns only the 111 most polluting US coal-fired power plants. The Amendments stipulate that, starting in year 1995, these plants will get annual SO₂ allowances corresponding to an emission level of 2.5 lbs of SO₂ per MMBTU of fuel burnt (an SO₂ allowance is defined as a right to emit one ton of SO₂ over one year). The objective of Phase I is to reduce total SO₂ emissions by 3.5 million tons.

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10 Coals used by power plants in the United States usually contain between 0.5% and 4% of sulphur (either as inorganic or organic sulphur). During combustion the sulphur is oxidised into SO₂, and released in the flue gas. Electricity generation is responsible for about 70% of all sulphur emissions in the US, or approximately 23 million tons in 1990.
11 The short summary presented here is based on Pytte [100].
12 NOₓ emission limits are set between 0.45 and 0.5 lbs per MMBTU, depending on the type of boiler. This should lead to a 2 million ton NOₓ emission reduction from the 1980 10 million ton level.
13 It is worth noting that the 1990 Amendments require a preliminary study to be carried out on interpollutant trading of SO₂ and NOₓ emission rights.
tons. Also, a special provision grants a two year deadline extension to plants that choose scrubber technologies to reduce their emissions.\footnote{This provision was intended to favor control technologies that allow the use of high-sulphur coal.}

- Phase II applies to all coal- or oil-fired combustion devices, and starts in year 2000. The objective is to limit total $SO_2$ emissions to an annual level of 8.9 million tons. The general emission limit for units of more than 75 MWe and emitting more than 1.2 lbs per MMBTU is set at 1.2 lbs per MMBTU. For other units, the emission limit depends on the plant's age, type, and present emission levels.

According to the Amendments, every year each unit will receive a number of allowances that corresponds to its maximum allowed emissions. Plant owners will then be free to reduce their emissions below this limit and sell the surplus allowances, or alternatively they may purchase additional allowances and emit a correspondingly larger quantity of $SO_2$. Owners will also be free to bank allowances for future use (even between Phase I and Phase II). At the end of each year utilities that have emitted more sulfur than they have allowances will pay penalties of $2000 per excess ton of $SO_2$, and will need to reduce their emissions by the same amount the following year.\footnote{Emission monitoring will obviously be important, and Pavetto and Bae [92] provide an overview of the topic.}

Special provisions were included to accommodate Mid-Western plants, the state of Florida, and certain independent power plants. In order to guarantee independent power producers (IPP's) access to the new allowance market, the regulator will be allowed to conduct auctions of 150,000 allowances per year during Phase I, and 250,000 per year during Phase II.\footnote{IPP's usually do not have existing facilities and therefore would not receive an initial allowance distribution. However, the Acid Raid Advisory Committee of the EPA has agreed to give written allowance guarantees to IPP's.} During Phase II the regulator will also be able to sell up to 50,000 allowances directly (at a price of $1,500 per allowance).
2.3 Potential Implementation Problems

There are still a number of uncertainties regarding the implementation of the emission trading system (Kranish [62]), but the Amendments on sulfur emission reductions will likely represent a substantial departure from standard regulatory methods. The Administration estimates that emission right trading will save the industry about $30 billion over the next 20 years, and Cushman [26] reports a figure of $1 billion per year.\footnote{The total cost of the Act's acid rain provisions is believed to be between $5 and $7 billion per year (Edison Electric Institute, as cited by Yates [130]). However, Burkhardt [16] reports costs of only $700 million per year during Phase I, and $3.8 billion per year during Phase II.}

However, the success of this new market approach is not guaranteed in a highly regulated sector like the electric power industry. In the past, the EPA has had some unsuccessful experiences with emission trading. The so-called bubble rule allowed emission trading between different sources of a given firm. Also, offset policies and emission right banking were allowed under certain conditions. These programs were not particularly successful (especially for trade between firms), because regulatory uncertainties and strict state-imposed trading conditions hampered trading considerably (Hahn and Hester [44]). It is worth noting that \(SO_2\) allowances are not described as property rights by the CAAA, suggesting that the EPA could unilaterally cancel or modify the rules governing their use (Banfield [5] or Dudek [29]).

The behaviour of state regulatory commissions will be important in determining the success or failure of the 1990 Amendments with respect to sulfur emission control. State regulators are said to favour tradable emission rights by a wide margin, but would rather maintain some control on allowance trading (Badger [4]). As Stalon [112] notes, this could lead to conflicts between state and federal regulators, for instance if some states decide to forbid allowance sales to out-of-state utilities. Such restrictions would obviously undermine the efficiency of the trading system, but
Devitt and Weinstein [28] believe that in the long run state regulators will understand that the interests of their constituencies demand unrestricted allowance trading. In any case, Brusger and Platt [15] stress the need for utilities to frequently communicate with their regulators.

The future behaviour of public utilities with respect to SO$_2$ trading is also not entirely clear. The Wall Street Journal [127] noted that some of them are still sceptical about the new trading system. Some may be tempted to hoard allowances, especially if regulatory or market uncertainties are high. However, allowance auctions and direct sales are designed to kickstart trading (Hausker [46]).

Allowance trading is expected to eventually reach several billion dollars per year, and large financial institutions are considering trading SO$_2$ allowances. The Wall Street Journal [127] reported that a manufacturer of environmental equipment is thinking about accepting allowances for the purchase of scrubber technology. Also, the Chicago Board of Trade was supposed to start trading allowance forward contracts in 1993, and was considering the trade of allowance futures contracts (Passell [91]). SO$_2$ allowances are thus likely to behave like financial securities in the future. This should allow the use of option theory for the valuation of allowance-related investments.

Krupnick et al. [63] note that even if substantial trading fails to materialise a satisfactory level of environmental protection will be achieved. However, the social cost of regulation might be higher in this case than it would have been with an effluent fee approach. Success or failure of the trading system will also determine the future of other proposed regulations. Senators Gore and Wirth for instance have suggested the use of allowance trading for CO$_2$ emissions [126], and a similar bill (H.R. 776) is under consideration in the House. Also, Southern California is considering the same method to reduce the costs of limiting the emission of smog producing gases (Passell [90]).
2.4 Sulfur Emission Control Technologies

Coal-fired power plants that need to reduce their sulfur emissions in order to comply with the new Amendments can choose among several strategies that range from fuel switching to sulfur removal before, during, or after combustion. The capital costs and sulfur removal costs\(^\text{18}\) of the most significant of technologies are summarized in Table 2.1.\(^\text{19}\) More advanced clean coal technologies are described by Burr [17] but will not be discussed here. The applicability and cost of the various retrofit technologies presented in Table 2-1 are highly plant specific. The optimum strategy choice will therefore vary from one unit to another. However, some strategies are likely to be more attractive than others for typical power plants. The base-case model considered in this paper will focus on coal switching and wet scrubbing. Physical cleaning, dry scrubbing and furnace sorbent injection do not seem to be effective enough for a high-sulfur coal plant. Also, dry sorbent injection is not applicable to such a plant. Furthermore, gasification and fluidized bed technologies, although probably quite attractive for new plants, are too expensive for retrofit in most cases. By contrast, wet scrubbers are effective, and not too expensive. The industry has substantial experience with them, and some of the 1990 Amendment provisions tend to favour their use. A precise model of the investment situation will therefore now be developed which recognises the possibility of switching fuel to a low-sulfur coal and the possibility of installing a wet scrubber. These are the two most likely compliance strategies, as noted by Steen and Starheim [113] or Zimmermann [132].

\(^{18}\) The calculation of removal costs is based on the technology's capital, O&M and fuel costs, and on its economic impacts on plant operation. The sum of these costs is levelized over the life of the equipment, and the resulting value is divided by the amount of \(SO_2\) removed per unit time. It will be shown in this paper that removal costs are imperfect measures of the attractiveness of a technology, since they do not value its flexibility.

\(^{19}\) For costs in Europe see Sanyal [107].
<table>
<thead>
<tr>
<th>Technology</th>
<th>Capital costs ($/kWe)</th>
<th>Removal Costs ($/ton)</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fuel Switching</strong></td>
<td>5-30</td>
<td>300-1000</td>
<td>[99] [108] [31]</td>
</tr>
<tr>
<td><strong>Physical Cleaning</strong></td>
<td>10</td>
<td>250-500</td>
<td>[102]</td>
</tr>
<tr>
<td><strong>Wet FGD</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conventional</td>
<td>145-290</td>
<td>290-1300</td>
<td>[3] [69] [99] [102]</td>
</tr>
<tr>
<td>forced ox. lime</td>
<td>150-550</td>
<td>800-3000</td>
<td>[33]</td>
</tr>
<tr>
<td>Dual Alkali</td>
<td>140-170</td>
<td>330</td>
<td>[21]</td>
</tr>
<tr>
<td></td>
<td>160-240</td>
<td>400-600</td>
<td>[56]</td>
</tr>
<tr>
<td><strong>Dry FGD</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lime Spray Dryers</td>
<td>100-210</td>
<td>270-650</td>
<td>[32] [56] [69] [102] [122]</td>
</tr>
<tr>
<td><strong>Fluidized Bed</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Atmospheric</td>
<td>305-590</td>
<td></td>
<td>[108]</td>
</tr>
<tr>
<td><strong>Furn. Sorb. Inj.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LIMB</td>
<td>70-110</td>
<td>500-750</td>
<td>[56] [99] [2]</td>
</tr>
<tr>
<td>Advacate</td>
<td>100</td>
<td>475-730</td>
<td>[19]</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>220</td>
<td></td>
</tr>
<tr>
<td><strong>Dry Sorb. Inj.</strong></td>
<td>115</td>
<td></td>
<td>[43]</td>
</tr>
</tbody>
</table>

Table 2-1: Costs of Retrofit Sulfur Emission Control Technologies ($ 1990)
3. **SCRUBBER INVESTMENT PROBLEM**

This section presents the scrubber investment problem studied in Sections 4 and 5. It describes the corresponding model, and more particularly the stochastic underlying variables. It then presents an option description of the investment situation, and shows that neither the financial option literature nor a continuous-time approach can lead to an analytical solution.

### 3.1 Investment Problem Description

As explained in Section 2, the 1990 Clean Air Act Amendments have important consequences for coal-fired power plants. In order to comply with the new $SO_2$ emission regulation the operators of such plants have the choice between three basic alternatives:

- They can decide to purchase $SO_2$ allowances from other utilities. These allowances will be added to those obtained directly from the regulator, and will be used to cover the plant's sulfur emissions; or
- They can decide to install an $SO_2$ emission reduction system, like a scrubber. This scrubber would represent a substantial investment for the utility, but could reduce sulfur emissions below the level corresponding to the allowance the utility receives freely from the regulator. In such a case the utility would not need to purchase allowances on the market, and may even be able to sell the allowances it does not need; or
- Because the scrubber is a capital-intensive investment the utility may prefer to switch fuels to a lower-sulfur coal. Low-sulfur coals are more expensive than high-sulfur ones, but the utility would save in allowance costs, and could also be able to sell the allowances it does not need.
The 1990 Clean Air Act Amendments do not require that the utility select only one of these alternatives. Instead, the power plant operators may initially choose one strategy (for example, coal switching), and later change to another (for example, the scrubber investment) if allowance and coal prices are favourable. This flexibility gives rise to option value.

It is important to note that the actual number of allowances to which the utility is entitled each year is irrelevant for this problem. Even if the utility had enough allowances to cover its sulfur emissions, the allowances not used for its own operation could be sold. Their use therefore represents a loss of revenue for the utility. Such a utility should consider installing a scrubber or switching fuel, just as a utility without any allowances should. For clarity's sake, however, it will be assumed that the utility does not receive any allowances (i.e., it has to purchase the allowances it needs).

### 3.2 Model Assumptions

#### General Assumptions

The models discussed in Sections 4 and 5 incorporate the following simplifying assumptions:

1. The power plant burns coal, and has well-defined physical life, size (power), and fuel requirements.\(^\text{20}\) The utility that owns and operates it has a duty to supply electricity to its customers. It cannot temporarily or permanently stop production

---

\(^{20}\) Baylor [7] shows that utilities will probably not expand the life of coal power plants in the future.
before $t = T$, end of the power plant life (base load plant).\textsuperscript{21} It is also not permitted to build a replacement plant (for instance a natural gas plant).\textsuperscript{22}

2. At $t = 0$ the power plant burns a high-sulfur coal, whose characteristics are defined by its cost per MMBTU and sulfur content per MMBTU. There is initially no sulfur control device at the plant.

3. The utility is subject to the 1990 Clean Air Act Amendments (Phase II sulfur emission requirements) beginning at $t = 0$, and no other environmental regulation change is expected in the future. This means that allowances must be acquired for sulfur emissions in excess of 1.2 lbs per MMBTU.

4. $SO_2$ allowances can be freely traded on a national market. There is a single allowance spot price at any time, and purchase price is equal to sale price (allowance transaction costs are neglected).

5. The utility trades fuel and allowances at spot prices. Though utilities typically enter into long-term supply contracts for at least part of their requirements, this is the best assumption to determine the true financial costs and benefits at any given time of the various strategies available to the utility (because the value of long-term contracts at a given time depends in fact on the spot price).

6. Coal prices per MMBTU at a given location are assumed to depend solely on the coal's sulfur content per MMBTU. This is not an unrealistic assumption: the market for coal is quite competitive (Joskow [54]), and the other coal

\textsuperscript{21} The capacity factor is supposed to take into account the production interruptions required for maintenance or repair.

\textsuperscript{22} Electric power plants are very capital-intensive investments, and construction times are typically quite long. It is therefore unlikely that many utilities will prematurely scrap their existing coal-fired power plants.
characteristics, although important for technical reasons, are not usually translated into coal prices, because only the heat rate and the sulfur content make a significant difference to the utility's costs and revenues.  

7. All investment possibilities are evaluated against a reference case in which the utility burns high-sulfur coal from $t = 0$ to $t = T$, and purchases allowances to cover its needs. The cash flow of this reference case are used as benchmark, and all net present values, investment values or option values discussed in this paper correspond to incremental cash flows relative to this benchmark.

8. Following standard capital budgeting practices the financing of the various investment alternatives is not considered here. If need be, this could be evaluated separately to obtain an adjusted net present value (see Brealey and Myers [10]).

9. Taxes are neglected.

Base-Case Assumptions

In the base case model, the utility is assumed to be able to choose between three alternatives to comply with the Clean Air Act:

- It can install a scrubber, of well-defined capital costs, O&M costs, and sulfur removal rate. This can be done at any time between $t = 0$ and $t = T$, and the scrubber starts operation immediately after the utility decides to install it (i.e., construction delays are assumed away). This assumption is later relaxed. Once installed, the scrubber has to be constantly operated, and it can only operate with

---

23 Because of the competitiveness of the coal market, it is not necessary to account for transportation cost differences between various coals (as analysed by Sharp [109]): they are already included in the coal delivered price.
high-sulfur coal. The scrubber is assumed to have zero net salvage value at $t = T$.

- It can switch to a well-determined low-sulfur coal, defined by its sulfur content per MMBTU, by the marginal variable costs of operation it occasions, and by its price per MMBTU. The utility can switch between high and low-sulfur coals as often as it wants, but there is a switching cost (which could correspond to a contract termination penalty, for example) which has to be paid whenever the utility switches fuel. Switching decisions are also assumed to be implemented immediately.

- It can continue to purchase allowances to cover its excess sulfur emissions.

Financial differences between these three alternatives are related to the number of allowances that need to be purchased, and the capital, O&M, switching and fuel costs involved. In the base-case model, the stochastic variables are the allowance price and the coal prices. Since all investments are evaluated with respect to a reference situation in which the utility continues to burn high sulfur coal, the relevant underlying coal price variable is the difference between low-sulfur and high-sulfur coal prices (i.e., the coal price premium).

3.3 Stochastic Behaviour of Financial Assets

It was shown in Section 2 that $SO_2$ allowances are likely to be traded like financial assets. It will be shown below that the coal price premium and allowance price are likely to be strongly correlated, so that it is assumed in this section that coal price premiums will also behave like financial assets.
Random Walk Assumption

The behaviour of financial asset prices over time has been extensively studied in the financial literature. It has been observed that the probability distribution of asset returns at a future date $t$ depends only on the asset present price, and not on past asset prices. There is thus no serial correlation between the successive price changes of a given asset over time. The asset price follows a random walk, and the asset market is said to be weakly efficient.\(^{24}\) Fama and Miller [35] were one of the first to test empirically the behaviour of stock prices over time, and to show that they follow random walks. Most of the literature follows Fama and Miller, although some studies have found that series of asset prices over extensive periods of time may not always be random (see Taylor [115] for instance).

The Wiener Process

One of the simplest mathematical description of a random walk is the Wiener process. It assumes that the return of an asset over period $\Delta t$ is normally distributed, with means 0 and variance $\sigma \Delta t$. Most of the option literature assumes that the returns of financial assets follow generalised Wiener process (Hull [51]). Let $S$ be the price of such a financial asset, with value $S(0)$ at $t = 0$. The behaviour of $S$ over time is then given by the following stochastic equation:

$$\frac{dS}{S} = \alpha_S \, dt + \sigma_S \, dz \quad (3.1)$$

where $dz$ is a Wiener process, and where $\alpha_S$ and $\sigma_S$ are constant. If $S$ follows the generalised Wiener process given by equation 3.1, stochastic calculus shows that the logarithm of $S$ also follows a generalised Wiener process, and that changes of $\text{Log}(S)$ over period $\Delta t$ are normally distributed, with mean $(\alpha_S - \sigma_S^2/2) \Delta t$, and variance $\sigma_S^2 \Delta t$. Therefore, the value of $S$ at $\Delta t$ is lognormally distributed, with mean $S(0) \exp(\alpha_S \Delta t)$.

---

\(^{24}\) Strong efficiency requires that the market price fully reflects all publicly available information about the asset.
The expected value of $S$ therefore increases exponentially with time, at a rate $a_S$ per unit time.

In the base-case model, allowance price and coal price premium will be assumed to follow generalised Wiener processes of the type described by equation 3.1.25

### 3.4 Stochastic Processes for the Allowance Price and Coal Price Premium

#### Variances

It will be shown below that options on assets that follow generalised Wiener processes do not depend on the expected rate of increase $a$ of the asset price.26 By contrast, the standard deviation $\sigma$ of the underlying asset price return per unit time is very important. In most option calculations, $\sigma$ can be estimated from the past behaviour of the asset price (this assumes that $\sigma$ will remain the same in the future). In the case of allowances, there is obviously no past price history. There is not even a price yet. Estimates from the literature will have to be used.

The Edison Electric Institute (as cited by Phelps [93]) gives estimates of the initial allowance price of $500-600 (in $ 1990), and several other studies agree with this estimate (see ICF [52]). The present allowance price will therefore be assumed to be $500. To obtain an estimate of $\sigma_A$, the standard deviation of the instantaneous allowance price return per unit time, it is assumed that there is a 90% chance that the allowance price will remain between $500/3$ and $500 \times 3$ in the next 30 years.27 The 90% confidence interval of a normal distribution of standard deviation $\sigma$ is $[-1.65\sigma, 1.65\sigma]$. Since $\ln(1500/500) = 1.1$, the variance over 30 years of the $SO_2$ allowance return is $(1.1/1.65)^2 = 0.443$. The variance over one year is then $0.443/30$, which

---

25 Pindyck [94] actually shows that oil prices over 20 years can be described as generalised Wiener processes.

26 This result is related to the observations made in section 1 that option values in two-period binomial models do not depend on the probability that the underlying asset value goes up or down.

27 This upper limit of $1500 is specified by the Clean Air Act Amendments.
corresponds to a standard deviation, $\sigma_x$, of about 12%. This is obviously a very unreliable figure, and it is only chosen for illustrative purposes in the base-case model. Substantially different standard deviation values will also be tested.

Past coal price premiums have been reported in the literature (Resource Dynamics Corp. [104]). However, the introduction of allowance trading will certainly substantially modify the behaviour of coal prices, so that it would be unwise to use past price behaviour as an indicator of future price behaviour. Instead, it will be shown below that allowance price and coal price premium returns are likely to have similar variances. An arbitrary value of 14% per year will then be chosen for $\sigma_D$, the standard deviation of the instantaneous return of the coal price premium.

Convenience Yields

Standard valuation methods for stock options have to be adapted when the stock pays dividend. Similarly, it is necessary to consider convenience yields when calculating options on commodities. Convenience yields can be defined by a very simple relationship. Let $\mu$ be the rate of return of the commodity value, as required by investors who are willing to hold the commodity. $\mu$ is a function of the commodity's risk level. Let $\alpha$ be the expected rate of increase of the commodity price. The convenience yield is then defined by:

$$\delta = \mu - \alpha \tag{3.2}$$

It corresponds to storage costs and to benefits that include "the ability to profit from temporary local shortages, or the ability to keep a production process running" (Hull [51]). In this paper convenience yields are assumed to be constant, an assumption often made in the option literature.

---

28 For instance, $\mu$ can be deducted from the commodity's $\beta$ with the CAPM.
Convenience yields are important for option valuation, because the benefits that correspond to the convenience yield accrue to the owner of a commodity, but not to the owner of an option on this commodity. In order to calculate the convenience yield of the underlying assets, several approaches are possible. One consists in simply applying equation 3.2. However, this requires an estimate of the expected rate of increase of the commodity value, and such estimates are usually quite unreliable.\(^{29}\)

Another method is based on the present value of future contracts on the commodity.\(^{30}\)

Let \( F \) be the value of a futures contract on commodity \( S \). The owner of a futures contract agrees at \( t = 0 \) to purchase at a future date \( t \) a fixed quantity of commodity \( S \), for a price \( F \) (this is different from an option contract, because the holder of a futures contract has to purchase the commodity at \( t \)) (Duffie [30]). Since both parties to the futures contract agree on price \( F \), the present value of future payment \( F \) must be equal to the present value of the future delivery of good \( S \). Assume that the commodity price follows a generalised Wiener process, of expected rate of increase \( \alpha_s \). The future payment \( F \) is certain, and should therefore be discounted at the risk-free interest rate. The future value of the commodity is uncertain. Its expected value at \( t \) is \( S(0) \exp(\alpha_s t) \), and should be discounted at a risk-adjusted discount rate. The risk-adjusted discount rate equals the required rate of return \( \mu_S \) on assets of similar risks. Hence:

\[
F \exp(-rt) = S(0) \exp(\alpha_s t) \exp(-\mu_S t)
\]  

(3.3)

Since \( \delta_S = \mu_S - \alpha_s \), we obtain:\(^{31}\)

\[
F = S(0) \exp(r - \delta_S) t
\]  

(3.4)

\(^{29}\) One possible method would be to use Hotelling's theoretical result that the unit price of an exhaustible natural resource, less the marginal cost of extracting it, tends to rise over time at a rate equal to the return of comparable capital assets (Hotelling [50]). However, as discussed by Miller and Upton [82] empirical tests of Hotelling's principle have not always been successful.

\(^{30}\) See Brown and Errera [14] for an introduction to energy futures.

\(^{31}\) Brennan and Schwarz [13] derive a more general partial differential equation between futures and spot prices, for the case in which the convenience yield depends on the spot price.
If future contracts are publicly traded, it is easy to derive the convenience yield $\phi_s$ from equation 3.4. This was done for natural gas and oil (natural gas futures were first traded on the New York Mercantile Exchange in April 1990 (Rosekrantz [105]). In 1990, futures contracts gave convenience yields of about 7% per year for both fuels. However, because of the invasion of Kuwait by Iraq, 1990 was hardly a typical year for fossil fuel trading. In fact, Paddock et al. [87] earlier found a convenience yield of 4% for oil in 1980. Convenience yields thus vary over time. For example Heinkel et al. [48] and Cho and McDougall [20] find that high levels of inventory lead to low convenience yield values. Gibson and Schwarz [41] find that oil convenience yields vary randomly around an average value of 18% per year. In any case, there are no publicly traded futures contracts on coal.\textsuperscript{32} Hence, we will choose a value of 5% per year for coal, but test other values as well. If it is assumed that high-sulfur and low-sulfur coals have the same convenience yield, it is obvious that the coal price premium will also have a convenience yield of 5% per year. Also, if allowance price and coal price premium are strongly correlated, they are likely to have similar convenience yields. Therefore, a convenience yield of 5% will be chosen for the allowance price.\textsuperscript{33} Different values will also be tested, of course.

3.5 Relationship between Allowance Price and Coal Price Premium

In this section a simple relationship between allowance price $A$ and coal premium $D$ is derived and discussed. The focus is on the correlation factor between the logarithmic changes of allowance price and coal price premium.

\textsuperscript{32} The trading of futures contracts in a commodity exchange is only possible for well standardised commodities. This is not the case for coal, a fuel with many different varieties.

\textsuperscript{33} Since SO$_2$ allowances are not commodities and do not pay dividend it might seem reasonable to assume that they will not have a convenience yield. However, because of regulatory uncertainties, electric utilities are likely to hold more allowances than would be financially optimum. This is equivalent to saying that there is a convenience yield associated with the SO$_2$ allowances.
No-Switching Cost Case

Let a given utility have the opportunity to switch coal in order to reduce \( SO_2 \) emissions (the utility may also have other compliance alternatives, but this is irrelevant here). Let each coal be defined by \( x \), the sulfur emissions that this coal would release per year if it were burnt by the utility's power plant. If the utility selects the coal that corresponds to emission level \( x \), it will have to pay \( P(x) \) in coal purchases, and \( x A \) in allowance costs. It is first assumed that there are no switching costs between different coals. At time \( t \), the utility therefore chooses the sulfur content, \( x \), that minimises \( P(x) + x A \) (Weinstein [128]). Hence:

\[
\frac{\partial (P(x) + x A)}{\partial x} = 0
\]  

(3.5)

Therefore:

\[
\frac{dP}{dx} = -A
\]  

(3.6)

Let it be assumed that only two different coals can be used by the power plant considered, and that these yields sulfur emissions \( x_1 \) and \( x_2 \) respectively (with \( x_1 > x_2 \)). Equation 3.6 shows that, at equilibrium, the difference in price, \( D \), between these two coals satisfies the relation:

\[
D = P(x_2) - P(x_1) = A(x_1 - x_2)
\]  

(3.7)

In this simple model coal price premium and allowance price are proportional to each other. Over time, they are therefore perfectly correlated, and their relative changes over time are also perfectly correlated.\(^{34}\) Note that equation 3.7 still holds if the utility has other compliance strategies to choose from, in addition to switching coal or purchasing allowances.

\(^{34}\) Because, \( dD/D = dA/A \).
The previous derivation does not hold if the utility faces coal switching costs. However, equation 3.7 may still be true, provided that there are other utilities with zero switching costs that are able to affect coal and allowance prices.

**A-D Relationship with Switching Costs**

If all utilities face non-zero switching costs, the coal price premium, allowance price, and optimum strategy have to be determined simultaneously. This requires a modelling of the whole coal-burning power industry that is beyond the scope of this paper. To develop some qualitative insights into the effect of switching costs, however, a highly simplified dynamic model was developed consisting of three linear equations:

- Equation 3.8 describes the effect of allowance price $A$ and coal price premium $D$ on the strategic choices of the coal industry;
- Equation 3.9 describes the effect of coal demand on the coal price premium; and
- Equation 3.10 describes the allowance market.

For simplicity, it is assumed that the whole coal-fired power industry can only choose between two types of coal (high-sulfur and low-sulfur, as described in the previous section). $y(i)$ represents the proportion of the industry that burns high-sulfur coal at time $i$.

If $D(i) > A(i) \times (x_1 - x_2)$, low-sulfur coal becomes relatively more expensive than allowances, and some utilities will switch to high-sulfur coal in the next period. Hence:

$$y(i + 1) = y(i) + x(D(i) - A(i))$$

(3.8)
Where $\kappa$ is constant, and measures how responsive the industry is to changes in allowance price or coal price premium. $\kappa$ is obviously related to switching costs. If more plants switch to high-sulfur coal, the coal price premium goes down. This can be modelled by:

$$D(i) = \varepsilon - \varsigma y(i) \quad (3.9)$$

where $\varepsilon$ and $\varsigma$ are constant characteristics of the coal markets.

If more plants switch to high-sulfur coal, the demand for allowances increase, and the supply decreases.\(^{35}\) Hence, the allowance price goes up, which can be simply modelled by:

$$A(i) = \eta y(i) + \xi \quad (3.10)$$

where $\eta$ and $\xi$ are constant characteristics of the allowance market.

Equations 3.8, 3.9 and 3.10 can easily be solved simultaneously over time. Variable $y$ will reach a stable equilibrium provided that:

$$0 < \kappa < 1/(\varsigma + \xi) \quad (3.11)$$

The time it takes for the system to return to equilibrium after a perturbation depends on the value of $\kappa$. It is instantaneous if $\kappa = 1/(\varsigma + \xi)$, and infinitely long if $\kappa = 0$. This provides an easy way to relate $\kappa$ to overall switching costs. $\kappa = 0$ corresponds to infinite switching costs, and $\kappa = 1/(\varsigma + \xi)$ corresponds to zero switching costs.

The simple model formulated here shows that the correlation factor $\rho$ between $\log(A(i+1)/A(i))$ and $\log(D(i+1)/D(i))$ is always equal to 1. In order to study the

\(^{35}\) The supply of allowances is defined here as the number of allowances that utilities are willing to sell to other utilities rather than keep for their own use. It is not the (constant) number of allowances supplied each year by the regulator.
influence of the switching costs on \( p \), a stochastic \( \psi \) term is introduced into equation 3.10. \( \psi \) can be thought of as representing the influence of other compliance strategies on the allowance market. For simplicity it is assumed that \( \psi \) is uncorrelated with any other variable of the model. Equation 3.10 is now:

\[
A(i) = \eta y(i) + \xi + \psi(i)
\] (3.12)

where \( \psi \) is a random variable over time, with a uniform probability distribution over \([0, \psi_{\text{max}}]\). The higher \( \psi_{\text{max}} \) is relative to \( \xi \), the more important other compliance strategies are in determining the equilibrium allowance price.

The model was tested for various values of the parameters. It was found that \( \rho \) decreases as the switching costs increase. The higher the switching costs, the more "decoupled" are the allowance and coal markets.\(^{36}\) Also, for a given level of switching costs, \( \rho \) decreases as \( \psi_{\text{max}} \) increases.\(^{37}\) Hence, the more important other compliance strategies are, the less correlated are allowance and coal premium prices.

Finally, the influence of switching costs on \( \rho \) (as measured by the slope of curve \( \rho \) vs. \( \kappa \)) decreases as \( \psi_{\text{max}} \) increases. If other compliance strategies are important, the effects of switching costs on \( \rho \) become less important.

**Conclusion on A-D Correlation**

The dynamic model described here is not meant to be realistic. However, its results are reasonable, and provide an illustration of how switching costs, coal price premium and allowance price may interact, as well as an understanding of how other compliance strategies may influence the result. In the base-case model considered in this paper, switching costs are not zero, but are substantially lower than scrubber capital costs or fuel cost premiums (see Section 5 for numerical values). It will

---

\(^{36}\) Because the model is too simple, one does not exactly get \( \rho = 1 \) for zero switching costs.

\(^{37}\) However, if \( \psi_{\text{max}} \) is too large the system may diverge, and never reach equilibrium. This is also a consequence of the extreme simplicity of the model chosen.
therefore be assumed that the correlation factor between the two instantaneous underlying asset returns is lower than 1, but close to 1. A value of 0.8 will be selected for base-case calculations.

3.6 Strategy Choice in a Non-Flexible Model

In the case in which the utility has to decide at $t = 0$ what compliance alternative to use, and cannot change strategy thereafter, there is no flexibility, and the investment alternatives can be evaluated in a straightforward manner. It is convenient to estimate the levelized control cost of a given alternative over the power plant life, and to compare it with the amount of allowances saved per unit time. Figure 3-2 shows various conceptual compliance alternatives plotted on a graph giving their levelized control cost (in $ per year) vs. the corresponding $SO_2$ emission level. The efficient frontier represents the set of alternatives that have the lowest levelized cost for a given emission level.

At time $t = 0$, the utility should choose the alternative that minimises the sum of the levelized control costs plus the levelized allowance costs (equal to the emission level multiplied by the levelized allowance price). If the levelized allowance price is $A_1$ (in $/ton of SO_2$ per year), figure 3-2 shows that compliance alternative Y should be selected. It is the strategy that has the lowest total cost to the utility. If the allowance price is $A_2$ (with $A_2 > A_1$), the optimal strategy is X, which is more costly but also more effective than strategy Y. Figure 3-2 shows that high allowance prices make low emission strategies more attractive, a result that is quite intuitive.

This simplistic model thus provides an illustration of the importance of the allowance price for the choice of a compliance strategy. However, it is not a satisfactory strategy choice method for realistic investment situations, because it does not take the
value of flexibility into account. In such cases, option modelling is a superior approach.\textsuperscript{38}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3-2.png}
\caption{Influence of Allowance Price on Optimal Strategy Choice}
\begin{flushright}
No-Option Case
\end{flushright}
\end{figure}

3.7 Option Description of the Scrubber Investment Problem

No Option to Switch

It is first assumed that in order to reduce its $SO_2$ emissions, the utility has the option to install a scrubber, but does not have the option to switch fuel. The possibility of installing a scrubber before the end of the power plant's lifetime can be described as an American option, with an exercise price equal to the capital cost of the scrubber. Once installed, the scrubber need not be operated continuously. Its value can

\textsuperscript{38} See Section 1 for a discussion of why decision tree analysis does not apply well to flexible corporate investments.
therefore be computed as the sum of a continuous series of European options that correspond to the options to operate at a given time before the end of the power plant's life. The investment flexibility can therefore be valued as an American compound option on a continuous series of European options.\textsuperscript{39}

McDonald and Siegel \textsuperscript{[77]} calculate the present value of a European call option on a stock that pays dividends. The value of the installed scrubber at time \( t \) could be obtained from their result by integration. However, the corresponding variable would not follow a generalised Wiener process. To see this, one need only note that its value must be equal to zero at the end of the power plant's life. The work of Geske \textsuperscript{[40]} on compound options would therefore not be applicable here, and the investment value cannot be calculated directly from the financial option literature, even if the utility does not have the option to switch fuel.

\textbf{Option to Switch Fuel or to Scrub}

The problem is further complicated by the possibility of switching coal. In this case the operating options after the scrubber is installed are like options on the maximum of two assets. The first asset corresponds to the benefits to the utility of operating the scrubber with high-sulfur coal, and the second asset corresponds to the benefits of burning low-sulfur coal without operating the scrubber. The valuation of options on the maximum or minimum of two assets was studied by Stulz \textsuperscript{[114]}. Also related to the case considered here are the papers of Margabe \textsuperscript{[75]} and Fischer \textsuperscript{[37]} on the valuation of European calls with stochastic exercise prices.\textsuperscript{40} However, none of these papers is general enough to be used here. In particular, they do not consider compound options. This is also true of McDonald and Siegel \textsuperscript{[78]}, who value a production facility with infinite life. Triantis and Hodder \textsuperscript{[123]} evaluate a facility that can produce different outputs. However, they neglect switching costs between

\textsuperscript{39} A compound option is an option on an option.

\textsuperscript{40} Margabe's exchange option is really a special case of Fischer's problem, when the asset that determines the exercise price does not have payouts.
production modes in order to obtain an analytical solution of the value of flexibility. Since there is therefore no irreversibility in their model, their results cannot be used here.

The theoretical work most closely related to the problem considered here is the article by Carr [18] which considers the valuation of a European compound exchange option. The exercise of such an instrument involves delivering one asset in return for an exchange option. To keep the analysis tractable Carr assumes that the asset delivered is the same for both the first and second exchanges. Also, the assets considered by Carr do not make payouts over the life of the options. Unfortunately, neither assumption holds in the investment situation of interest in this paper. The investment problem defined in Section 3 is thus too complex to be solved by a direct application of the theoretical literature on financial options. A valuation model will have to be developed instead.

3.8 Option Valuation in a Continuous-Time Model

Most of the literature on option valuation employs a continuous description of time (see, for example, Fischer [38]). The same method will be used here to try to evaluate the investment flexibility. For simplicity, it will be assumed that the utility cannot switch fuel. Even then, it will be shown that no analytical solution can be found.

Derivation of the Partial Differential Equation (PDE)

The utility is assumed to have the option to install the scrubber at any time between \( t = 0 \) and \( t = T \) (end of power plant life), and the option to operate it or not once installed. The only underlying variable is the allowance price \( A \), whose value is assumed to follow equation 3.1. Let \( W(A,t) \) be the investment value at time \( t \) if the scrubber is already in place and if the allowance value is \( A \). If the scrubber operates
at time \( t \), the cash flow per unit time \( \Lambda_{scr}(A,t) \) is equal to the value of the allowances saved by scrubbing, minus the operating costs of the scrubber. It is assumed that the utility can freely decide at each time \( t \) whether to operate the scrubber or not. The actual investment cash flow per unit time after the scrubber is installed is therefore \( \max(\Lambda_{scr}(A,t), 0) \).

Let us consider a portfolio that consists in purchasing \( W \) of the investment considered (long position), and selling \( \partial W / \partial A \) allowances (short position). The value, \( P \), of this portfolio is obviously \( W - \partial W / \partial A A \). Since the investment brings \( \max(\Lambda_{scr}(A,t), 0) \) per unit time, the instantaneous change in portfolio value appears to be: \( dW - \partial W / \partial A dA + \max(\Lambda_{scr}(A,t), 0) dt \). However, short selling a stock requires the payment of dividends to the person from whom the stock was borrowed. Similarly here, the portfolio owner has to pay convenience yield to be able to short-sell allowances. Therefore, the true instantaneous portfolio value change is:

\[
dP = dW + \max(\Lambda_{scr}(A,t), 0) dt - \partial W / \partial A dA - \delta_A A \partial W / \partial A dt \tag{3.13}
\]

It is shown below that this value change is risk-free. Hence:

\[
dP = r_F dt P = r_F dt (W - A \partial W / \partial A) \tag{3.14}
\]

Since \( A \) is a stochastic process, \( W(A,t) \) is a function of a stochastic process. A lemma of stochastic calculus (called Ito's Lemma) gives the expression of the differential element \( dW \):\(^{41}\)

\[
dW = \partial W / \partial t dt + \partial W / \partial A dA + 1/2 \sigma_A^2 A^2 \partial^2 W / \partial A^2 dt \tag{3.15}
\]

Combining equations 3.13, 3.14 and 3.15 gives:

\[
(3.16)
\]

\(^{41}\) See Malliaris and Brock [74] for a description of Ito's lemma.
which is the basic partial differential equation obtained with the continuous-time method. Note that the terms in \( dA \) cancelled out, showing that the portfolio of value \( P \) was indeed risk-free. Provided that the boundary conditions are properly specified, equation 3.16 can be solved to obtain the expression of \( W \) as a function of \( A \) and \( t \).

Another partial differential equation can also be derived to give the value \( V(A, t) \) of the investment if the scrubber is not in place yet. This new equation is similar to equation 3.16, but the term \( \max(W_{\text{scr}}(A,t), 0) \) disappears because there is no scrubber yet:

\[
\frac{1}{2} \sigma^2 \frac{\partial^2 V}{\partial A^2} + (r_F - \delta_A) A \frac{\partial V}{\partial A} - r_F V + \frac{\partial V}{\partial t} = 0
\]

At \( t = 0 \) the investment value of interest in this paper would then be given by \( V(A(0), 0) \). However, the boundary conditions of equation 3.17 are functions of \( W \). Hence, solving equation 3.17 requires first solving equation 3.16.

**Solving the Partial Differential Equations**

As far as we know equations 3.16 and 3.17 do not have analytical solutions.\(^{42}\) A difficulty is the explicit presence of time in equations 3.16 and 3.17. If the power plant did not have a well defined end of life, the equations given above could be simplified, and solved analytically. Brennan and Schwarz [13] study the somewhat related problem of a mine that can be temporarily or permanently shut down. The mine value depends on the future differences between the value of its production (which is assumed to follow a generalised Wiener process), and its (deterministic) costs. The authors show that if the inventory of the commodity is infinite, and if all costs increase at a constant inflation rate, the mine value does not depend explicitly on time. This allows them to find an analytical solution to the problem. Pindyck [96] is

\(^{42}\) Kulatilaka [64] also found that option interdependencies require numerical methods.
also able to obtain an analytical solution to a similar investment problem because, once again, time is not an explicit variable. In a different context, Majd and Pindyck [73] study the optimal investment timing for a factory that takes time to build. In this case, the investment value depends explicitly on both the value of the completed factory, and on the amount of investment remaining for completion. As a consequence, the investment value is given by a partial differential equation (equation 2.a of [73]) similar to equations 3.16 and 3.17, and has to be solved numerically. Thus, even if the utility does not have the option to switch fuel, the investment value cannot be calculated analytically.

The advantage of the continuous-time method described above is that it can lead to analytical expressions for the option value. However, this is only the case if the problem is simple enough, so that the partial differential equation(s) can be solved analytically. In more realistic problems like the one considered in this paper, the partial differential equation has to be solved numerically.43 If this is the case, a discrete-time valuation method known as the binomial approach might be preferred to the continuous-time PDE approach.

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43 Brennan and Schwarz [12], Kemna [57] and Meehan [80] discuss the use of finite-difference methods to solve the PDE. Also, Barone-Adesi and Whaley [6] develop an approximate method for options with short expiration times.
4. BINOMIAL MODEL FOR SCRUBBER INVESTMENT VALUATION

In this section, a discrete-time binomial model is developed to calculate the present value of the investment case described in Section 3.

4.1 Binomial Model for the Base-Case Problem

Principle of Binomial Model: The binomial method uses a binomial tree to describe the behaviour of the underlying variable(s) over time. It was first used for option valuation by Cox et al. [25] and by Rendelman and Barter [103]. In the binomial method time is modelled in discrete steps. Let \( t_0, \ldots, t_i, \ldots, t_N \) be the discrete model times at which the values of the underlying variable change.\(^{44}\) A period will be defined as the interval between two successive times of the model. Periods in the binomial models are supposed to have the same size, \( \Delta = t_{i+1} - t_i \). Also, the risk-free interest rate per unit time is assumed to remain constant over the interval \([0,T]\).

The binomial model says that if an underlying variable has a value \( V \) at \( t_i \), it can only take one of two values \( V^+ \) and \( V^- \) at \( t_{i+1} \). \( V^+ \) and \( V^- \) have to be properly chosen, so that, if the total number of periods \( N \) that correspond to interval \([0,T]\) tends toward infinity, the probability distribution(s) of the underlying variable(s) over time converge toward generalised Wiener process(es). Cox and Rubinstein [24] derive the correct values of \( V^+ \) and \( V^- \) if there is only one stochastic underlying variable. They also suggest a way to adapt their model to the case in which there are two partially correlated underlying variables.\(^{45}\) This is the approach that will be adopted here.

\(^{44}\) The notation will be: \( t_0 = 0 \) and \( t_N = T \).

\(^{45}\) Binomial models for two stochastic variables were first investigated by Evnine [34].
Binomial Description for Two Stochastic Variables (No Convenience Yields)

It is first assumed for simplicity that the two underlying assets do not have convenience yields. Let their price behaviour over time be described by the following stochastic processes:

\[
\begin{align*}
\frac{dA}{A} &= \mu_A \, dt + \sigma_A \, dz_A \quad (4.1) \\
\frac{dD}{D} &= \mu_D \, dt + \sigma_D \, dz_D \quad (4.2) \\
dz_A \, dz_D &= \rho \, dt \quad (4.3)
\end{align*}
\]

These equations correspond to the assumption made in Section 3 that the logarithmic changes of variables \(A\) and \(D\) follow generalised Wiener processes, and are partially correlated (with a correlation factor \(\rho\)).

The objective here is to build a binomial tree which models the possible values of variables \(A\) and \(D\) over interval of time \([0,T]\). If there is only one stochastic underlying variable, the expressions giving \(V^+\) and \(V\) as functions of \(V\) are independent of the period considered. With two stochastic variables, however, Cox and Rubinstein [24] suggest grouping the \(N\) periods that span interval \([0,T]\) into \(n\) sets of 3 consecutive periods \((3 \times n = N)\). Let \(A = A(3k)\) and \(D(3k)\) be the values of the underlying variables at time \(t_{3k}\), beginning of period \(3k + 1\) \((0 \leq k \leq N/3-1)\). Then the stochastic behaviour of \(A\) and \(D\) over the next three periods is described by the following process (illustrated by Figure 4-1):

1. During period \(3k + 1\), \(A\) is multiplied by \(u\) with probability \(p\), or by \(d\) with probability \(1 - p\), whereas \(D\) is multiplied by \(R\), return of a riskless asset over an interval of time \(\Delta\). The resulting values at \(t_{3k + 1}\) are \(A(3k + 1)\) and \(D(3k + 1)\).

---

46 The model will be modified in a next section to account for convenience yields.
47 Boyle [9] develops a slightly different method in which each price can be followed by five possible prices the following period. However, this method leads to very large price trees.
2. During period $3k + 2$, $D(3k + 1)$ is multiplied by $u'$ with probability $p'$, or by $d'$ with probability $1 - p'$, whereas $A(3k + 1)$ is multiplied by $R$. The resulting values at $t_{3k+2}$ are $A(3k + 2)$ and $D(3k + 2)$.

3. During period $3k + 3$, $A(3k + 2)$ and $D(3k + 2)$ are both multiplied by $u''$ with probability $p''$, or by $d''$ with probability $1 - p''$. The resulting values at $t_{3k+3}$ are $A(3k + 3)$ and $D(3k + 3)$.

Figure 4-1: Three-Period Binomial Tree for Two Stochastic Variables
This three-period process is then repeated for $A(3k + 3)$ and $D(3k + 3)$. It is clear that periods 1, 4, 7, etc. model the volatility of the allowance price, and periods 2, 5, 8, etc. model the volatility of the coal price premium. Periods 3, 6, 9, etc. model the correlation between the two assets.

The objective is then to determine the various $u$'s, $d$'s and $p$'s such that, if the total number, $N$, of periods over interval $[0,T]$ increases, the asset price changes over time described by the binomial tree approximate equations 4.1 to 4.3. To do so, it is necessary to calculate the standard deviations, expected rates of increase and correlation factor of the underlying assets if they follow the binomial tree just described. The results will be equated to the empirical values $\sigma_A$, $\sigma_D$, $\mu_A$, $\mu_D$, and $\rho$ given by equation 4.1 to 4.3. Note that there is no reason to expect that there will be only one possible set of values for the $u$'s, $d$'s and $p$'s.

**Variances, Means and Covariances for a Three-Period Model**

Since equations 4.1 to 4.3 involve $dA/A$ and $dD/D$, the focus will be on the logarithmic changes of the asset values. Let $A_o = \ln(A(3k+3)/A(3k))$ and $D_o = \ln(D(3k+3)/D(3k))$, where $A(i)$ and $D(i)$ represent the value at time $t_i$ of the underlying assets (as shown on Figure 4-1). $A_o$ and $D_o$ represent the logarithm of the relative changes of $A$ and $D$ over one of the $n$ sets of three consecutive periods. The objective is to find the variances, means and covariances of $A_o$ and $D_o$. Both processes can be divided into subprocesses. Let $A_1 = \ln(A(3k+2)/A(3k))$, $A_2 = \ln(A(3k+3)/A(3k+2))$, $D_1 = \ln(D(3k+2)/D(3k))$, and $D_2 = \ln(D(3k+3)/D(3k+2))$. It is clear that:

\[
A_o = A_1 + A_2 \quad (4.4)
\]
\[
D_o = D_1 + D_2 \quad (4.5)
\]
\[
A_2 = D_2 \quad (4.6)
\]
Let us further define $x = \ln(u)$, $y = \ln(d)$, $x' = \ln(u')$, $y' = \ln(d')$, $x'' = \ln(u'')$, and $y'' = \ln(d'')$. Let $\lambda = \ln(R)$. $A_1$, $A_2$, $D_1$ and $D_2$ can then be described by the following processes:

$$A_1 = x + \lambda \text{ with probability } p$$
and
$$y + \lambda \text{ with probability } 1-p$$

$$D_1 = x' + \lambda \text{ with probability } p'$$
and
$$y' + \lambda \text{ with probability } 1-p'$$

$$A_2 = D_2 = x'' \text{ with probability } p''$$
and
$$y'' \text{ with probability } 1-p''$$

$(A_1, A_2)$, $(A_1, D_1)$, $(A_1, D_2)$, $(D_1, A_2)$, and $(D_1, D_2)$ are all pairs of independent processes. Therefore, given equations 4.4 to 4.6:

$$\sigma^2_{A_0} = \sigma^2_{A_1} + \sigma^2_{A_2} \quad (4.7)$$
$$\sigma^2_{D_0} = \sigma^2_{D_1} + \sigma^2_{D_2} \quad (4.8)$$
$$\text{cov}(A_0, D_0) = \text{cov}(A_2, D_2) = \sigma^2_{A_2} \quad (4.9)$$
$$\mu_{A_0} = \mu_{A_1} + \mu_{A_2} \quad (4.10)$$
$$\mu_{D_0} = \mu_{D_1} + \mu_{D_2} \quad (4.11)$$

The variances and means of simple processes like $A_1$, $A_2$, $D_1$ and $D_2$ are easy to calculate. The variances, means and covariance of $A_0$ and $D_0$ are then:

$$\sigma^2_{A_0} = p(1-p)(x-y)^2 + p''(1-p'')(x''-y'')^2 \quad (4.12)$$
$$\sigma^2_{D_0} = p'(1-p')(x'-y')^2 + p''(1-p'')(x''-y'')^2 \quad (4.13)$$
$$\text{cov}(A_0, D_0) = p''(1-p'')(x''-y'')^2 \quad (4.14)$$
$$\mu_{A_0} = p x + (1-p) y + p''x'' + (1-p'') y'' + \lambda \quad (4.15)$$
$$\mu_{D_0} = p'x' + (1-p') y' + p''x'' + (1-p'') y'' + \lambda \quad (4.16)$$
Complete Specification of the Binomial Tree

In the binomial model both underlying variables follow \( n \) successive and independent stochastic processes over interval \([0,T]\). Each of these \( n \) processes is identical to process \( A_0 \) (for the allowance price) and \( D_0 \) (for coal price premium). In fact, the logarithms of the total returns \( \ln(A(N)/A(0)) \) and \( \ln(D(N)/D(0)) \) are equal to the sum of these \( n \) processes of means, variances and covariance given by equations 4.12 to 4.16. Hence, \( \ln(A(N)/A(0)) \) (resp. \( \ln(D(N)/D(0)) \)) has a variance \( n \times \sigma_{A_0}^2 \) (resp. \( n \times \sigma_{D_0}^2 \)) and a mean \( n \times \mu_{A_0} \) (resp. \( n \times \mu_{D_0} \)). The covariance of the two processes is \( n \times \text{cov}(A_0,D_0) \). As \( n \) increases (for a fixed \( T \)), the probability distributions of \( \ln(A(N)/A(0)) \) and \( \ln(D(N)/D(0)) \) must tend toward the probability distributions defined by equations 4.1 to 4.3. Hence:

\[
\lim_{n \to \infty} n \sigma_{A_0}^2 = \sigma_A^2 T \tag{4.17}
\]

\[
\lim_{n \to \infty} n \sigma_{D_0}^2 = \sigma_D^2 T \tag{4.18}
\]

\[
\lim_{n \to \infty} \text{cov}(A_0,D_0) = \rho \sigma_A \sigma_D T \tag{4.19}
\]

\[
\lim_{n \to \infty} n \mu_{A_0} = \mu_A T \tag{4.20}
\]

\[
\lim_{n \to \infty} n \mu_{D_0} = \mu_D T \tag{4.21}
\]

It is now necessary to find at least one set of \( x \)'s, \( y \)'s and \( p \)'s that satisfy equations 4.17 to 4.21. By analogy with Cox and Rubinstein [24], it is assumed that: \( x = X(3T/N)^{1/2}, \ y = Y(3T/N)^{1/2}, \ x' = X' (3T/N)^{1/2}, \ x'' = X'' (3T/N)^{1/2}, \ y'' = Y'' (3T/N)^{1/2} \). It is also assumed that: \( p = 1/2 + p_o (3T/N)^{1/2}, \ p' = 1/2 + p'_o (3T/N)^{1/2}, \) and \( p''=1/2 \). Also, if \( r_F \) is the continuous interest rate per unit time, it is obvious that:

\[
\lambda = r_F T/N \tag{4.22}
\]
Plugging equations 4.12 to 4.16 into equations 4.17 to 4.21, and replacing the x's and y's by their expressions with X's and Y's, it is possible to show that the X's, Y's and p's must verify the following relationships:

\[
\begin{align*}
&\frac{1}{4} (X - Y)^2 + \frac{1}{4} (X'' - Y'')^2 = \sigma_\text{A}^2 \\
&\frac{1}{4} (X' - Y')^2 + \frac{1}{4} (X'' - Y'')^2 = \sigma_\text{D}^2 \\
&\frac{1}{4} (X'' - Y'')^2 = p \\
&X + Y + X'' + Y'' = 0 \\
&X' + Y' + X'' + Y'' = 0 \\
&p_\circ (X - Y) + r_\text{F} = \mu_\text{A} \\
&p'_{\circ} (X' - Y') + r_\text{F} = \mu_\text{D}
\end{align*}
\]

(4.23)
(4.24)
(4.25)
(4.26)
(4.27)
(4.28)
(4.29)

It is assumed that: \( Y = -X \), \( Y' = -X' \), and \( Y'' = -X'' \). Provided that \( \rho < \sigma_\text{A}/\sigma_\text{D} < 1/\rho \), the solutions to equations 4.23 to 4.29 are:

\[
\begin{align*}
X &= (\sigma_\text{A}^2 - \sigma_\text{A}\sigma_\text{D}\rho)^{1/2} \\
X' &= (\sigma_\text{D}^2 - \sigma_\text{D}\sigma_\text{A}\rho)^{1/2} \\
X'' &= (\rho\sigma_\text{A}\sigma_\text{D})^{1/2} \\
p_\circ &= \frac{1}{2}\frac{\mu_\text{A} - \frac{r_\text{F}}{\rho}}{\left(\sigma_\text{A}^2 - \sigma_\text{A}\sigma_\text{D}\rho\right)^{1/2}} \\
p'_{\circ} &= \frac{1}{2}\frac{\mu_\text{D} - \frac{r_\text{F}}{\rho}}{\left(\sigma_\text{D}^2 - \sigma_\text{D}\sigma_\text{A}\rho\right)^{1/2}}
\end{align*}
\]

(4.30)
(4.31)
(4.32)
(4.33)
(4.34)

The \( u \)'s, \( d \)'s and \( R \) that define the binomial tree are therefore given by:

\[
\begin{align*}
u &= e^{\sqrt{\sigma_\text{A}(\sigma_\text{A} - \sigma_\text{D}\rho)}\sqrt{\frac{t}{r}}} \\
d &= \frac{1}{u} \\
u' &= e^{\sqrt{\sigma_\text{D}(\sigma_\text{D} - \sigma_\text{A}\rho)}\sqrt{\frac{t}{r}}} \\
d' &= \frac{1}{u'} \\
u'' &= e^{\sqrt{\rho\sigma_\text{A}\sigma_\text{D}}\sqrt{\frac{t}{r}}} \\
d'' &= \frac{1}{u''} \\
R &= e^{r\frac{t}{r}}
\end{align*}
\]

(4.36)
(4.37)
(4.38)
(4.39)
(4.40)
(4.41)
(4.42)
Equations 4.36 to 4.42 completely define the binomial tree for two stochastic variables without convenience yields. These equations are only valid if \( \rho < \frac{\sigma_A}{\sigma_D} < \frac{1}{\rho} \). However, it was shown in Section 3 that \( \sigma_A \sim \sigma_D \), so that this condition is likely to be verified for the investment situation considered here.

Note that we have only proved that the means, variances and covariance of \( \ln(A(N)/A(0)) \) and \( \ln(D(N)/D(0)) \) tend toward the values required by equations 4.1 to 4.3. It is also possible to show that, as \( n \) tends toward infinity, their probability distributions tend toward normal distributions, as required. To show this, a special case of the central limit theorem can be used. The demonstration is similar to the one given by Cox and Rubinstein [24] and will not be repeated here.

To summarise, a binomial description of the stochastic behaviours of assets \( A \) and \( D \) over time has been found, that tends toward the model described by equations 4.1 to 4.3 when the total number of periods per unit time tends toward infinity. It is now necessary to calculate the option values in this framework.

**Recurrence Relationship for One-Period Model**

Option valuation with binomial models rests on the assumption that, at equilibrium, there cannot be arbitrage opportunities between the option itself, its underlying asset, and a riskless asset (this is also the basis for the PDE method).\(^{48}\) Let us consider the one-period model described by Figure 4-2. \( S \) is the present value of the underlying asset. It is assumed that next period the underlying asset value can move up to \( S^+ = f S \) with probability \( \pi \), or down to \( S^- = g S \) with probability \( 1 - \pi \). Let \( Y \) be the present value of a derivative asset that takes value \( Y^+ \) if the underlying asset value

\(^{48}\) An arbitrage opportunity is defined as an opportunity to make a risk-free instantaneous profit. Arbitrage situations cannot persist at equilibrium because investors take advantage of them as soon as they appear.
moves up, and \( Y^+ \) if it moves down. It is assumed that \( S, f, g, R, Y^- \) and \( Y^+ \) are known, and the objective is to calculate \( Y \).

The approach used here is the same as was used in Section 1 to calculate the flexibility value of a gasifier investment. Let a portfolio \( P \) consist of \( \frac{Y^+ - Y^-}{f-g} \) shares of the underlying asset, and \( \frac{(fY^- - gY^+)/(f-g)R}{S} \) of a riskless bond (of present value \( I \) and return \( R \)). It is easy to check that the portfolio value next period will be \( Y^+ \) if the underlying asset price moves up, and \( Y^- \) if it moves down. The portfolio considered and the derivative asset will therefore always have same value next period. They must then have the same present value, in order to avoid arbitrage opportunities. The present value of the portfolio is equal to the weighted sum of the present values of its various components. Hence:

\[
Y = \frac{Y^+ - Y^-}{f-g} S + \frac{fY^- - gY^+}{f-g} R \times I
\]  

(4.43)

If \( h = \frac{(R-g)}{(f-g)} \), then:

\[
Y = \frac{1}{R} \left( h Y^+ + (1-h) Y^- \right)
\]

(4.44)

Equation 4.44 gives the value \( Y \) at time \( t_i \) of a derivative asset, as a function of its two possible values at time \( t_{i+1} \). It is the basic relationship used for option valuation with the binomial model. As noted in Section 1, equation 4.44 does not depend on \( \pi \). The present value of the option is not explicitly dependent on the probability that the underlying asset moves up or down. \( h \) is often called pseudo-probability because it has many characteristics of a probability, but usually does not correspond to any true probability.\(^5\)

\(^49\) If \( Y \) were the value of a European call option with exercise price \( K \) and exercise time \( t = 1 \), \( Y^- \) and \( Y^+ \) would be given by: \( Y^- = \max(0, fS - K) \), and \( Y^+ = \max(0, gS - K) \).

\(^50\) It is necessary to have \( g < R < f \) for the problem to be interesting. Otherwise, it is easy to show there would be arbitrage opportunities.

\(^51\) In a risk-neutral world however, it is obvious that \( h = \pi \).
For binomial trees of more than one period, equation 4.44 can be used recurrently. If
the option value is known at the end of the interval considered (expiration time), it
can be calculated at any prior time by recurrence.

\[ S \begin{cases} fS \\ gS \end{cases} \]

\[ Y = ? \begin{cases} Y^+ \\ Y^- \end{cases} \]

\[ 1 \longrightarrow R \]

**Figure 4-2: One Period Valuation Model**

**Adding Convenience Yields to the Model**

Let us now assume that part of the return to the holder of the underlying asset \( S \) is
given as a convenience yield, rather than as an increase in asset price. The binomial
model previously derived has to be modified.\(^{52}\) Let:

\(^{52}\) See McDonald and Siegel [77] for a discussion of convenience yields in a continuous-time model.
be the new stochastic description of the underlying asset price (allowance price or coal price premium), with $S$ the asset price at $t_i$. Since, by assumption, the underlying asset total return (price increase plus convenience yield) has not changed, the asset value at $t_{i+1}$ is still either $fS$ or $gS$, with $f$ and $g$ given by the equations derived in the no-convenience case. However, comparing equation 4.45 with equation 4.1 or 4.3 shows that the asset price at $t_{i+1}$ is now either equal to $f \exp(-\delta_S T/N)S$ or to $g \exp(-\delta_S T/N)S$ (Teisberg [117]). Since the underlying asset total return remains the same, the no-arbitrage portfolio approach leads to the same one-period recurrence relationship for the derivative asset (equation 4.44).

Thus, the inclusion of convenience yields in the model only requires that the relative jumps of the underlying asset value be multiplied by $\exp(-\delta_S T/N)$. In the investment case considered here, the new tree parameters are therefore:

$$
\begin{align*}
    u_\delta &= e^{\sqrt{\sigma^2 - \sigma_D^2}} \sqrt{\delta_S} e^{-\delta_S T/N} \\
    d_\delta &= e^{-\sqrt{\sigma^2 - \sigma_D^2}} \sqrt{\delta_S} e^{-\delta_S T/N} \\
    u'_{\delta A} &= e^{\sqrt{\sigma^2 - \sigma_{A D}^2}} \sqrt{\delta_S} e^{-\delta_A T/N} \\
    d'_{\delta A} &= e^{-\sqrt{\sigma^2 - \sigma_{A D}^2}} \sqrt{\delta_S} e^{-\delta_A T/N} \\
    u''_{\delta A} &= e^{\sqrt{\sigma^2 - \sigma_{A D}^2}} \sqrt{\delta_S} e^{-\delta_A T/N} \\
    d''_{\delta A} &= e^{-\sqrt{\sigma^2 - \sigma_{A D}^2}} \sqrt{\delta_S} e^{-\delta_A T/N} \\
    u''_{\delta D} &= e^{\sqrt{\sigma^2 - \sigma_{D A}^2}} \sqrt{\delta_S} e^{-\delta_D T/N} \\
    d''_{\delta D} &= e^{-\sqrt{\sigma^2 - \sigma_{D A}^2}} \sqrt{\delta_S} e^{-\delta_D T/N}
\end{align*}
$$

Note that it is now necessary to distinguish between $u''_{\delta A}$ and $u''_{\delta D}$, because $\delta_A$ and $\delta_D$ are not necessarily equal. Note also that it is no longer true that: $d_\delta = 1/u_\delta$. 

\[dS/S = (u_\delta - \delta_\delta)dt + \sigma_\delta dz\] (4.45)
Recurrence Formulas for Base-Case Investment

The utility considered here has the choice between installing a scrubber and switching to a low-sulfur coal. The scrubber total capital costs are \( q_{\text{scr}} Q \), and the capital costs of switching are \( q_{\text{sw}} Q \), (\( Q \) is the plant size in kWe). The benefits per unit time of running the scrubber are \( \Lambda_{\text{scr}} \), defined in a previous section. The benefits per unit time of burning low-sulfur coal are denoted as \( \Lambda_{\text{sw}} \), and are equal to the value of the allowances saved by using low-sulfur coal instead of high-sulfur coal, minus the difference in coal costs and O&M cost for low-sulfur and high-sulfur operation. For simplicity, it was assumed in the base-case that the scrubbing or switching decisions are immediately implemented (no construction delays), and that the scrubber has to be operated with high-sulfur coal once installed. Both underlying variables (allowance price and coal price premium) follow the stochastic processes described by equations 4.1 to 4.3. The binomial tree and the recurrence formula developed above can then be used to calculate the investment present value.

Each node of the binomial tree corresponds to a given time \( t_i \), and to given values \( A \) and \( D \) of the underlying variables. It will be called state \( (t_i, A, D) \). The objective here is to find recurrence relationships similar to equation 4.44 that give the investment value in state \( (t_i, A, D) \) as a function of the investment values in the two states \( (t_{i+1}, A^+, D^+) \) and \( (t_{i+1}, A^-, D^-) \) that follow state \( (t_i, A, D) \) in the binomial tree (see Figure 4-3). Assume that at time \( t_i \) the utility decides what strategy to use over period \( [t_{i+1}, t_{i+2}] \), so that the strategy in use over period \( [t_i, t_{i+1}] \) has already been decided at time \( t_{i-1} \). Let \( V \) be the investment value in state \( (t_i, A, D) \) if the utility has previously decided to burn high-sulfur coal over period \( [t_i, t_{i+1}] \), and does not have a scrubber installed. Let \( Z \) be the investment value in state \( (t_i, A, D) \) if the utility has decided to burn low-sulfur coal over period \( [t_i, t_{i+1}] \), and does not have a scrubber installed. And let \( W \) be the investment value in state \( (t_i, A, D) \) if the utility has a scrubber operating over period \( [t_i, t_{i+1}] \). \( V^+, Z^+ \) and \( W^+ \) are the same values in state \( (t_{i+1}, A^+, D^+) \), and \( V^-, Z^- \) and \( W^- \) are the same values in state \( (t_{i+1}, A^-, D^-) \). Finally,
let $h$ be the pseudo-probability to be used for period $[t_i, t_{i+1}]$. The exact expression of $h$ depends on the period considered. If $i = 3k$ (k integer), the allowance price in the only variable to move stochastically over the period considered. Therefore $h$ should be calculated as:

$$h_1 = \frac{R - d}{u - d}$$

(4.54)

If $i = 3k + 1$ (k integer), only the coal price premium changes stochastically over the period considered. Therefore, $h$ should be calculated as:

$$h_2 = \frac{R - d'}{u' - d'}$$

(4.55)

Figure 4-3: Total Binomial Tree
Finally, if \( i = 3k + 2 \) (k integer), both \( A \) and \( D \) move stochastically over the period considered, and \( h \) should be calculated as:

\[
h_3 = \frac{(R - d''')}{(u''' - d''')}
\]  \hspace{1cm} (4.56)

where \( u, d, u', d', u'', d'' \), and \( R \) are given by equations 4.36 to 4.42.

**Calculation of Z:** Assume that the utility is burning low-sulfur coal over period \([t_i, t_{i+1}]\), and does not have a scrubber installed. The investment value \( Z \) in state \((t_i, A, D)\) depends on the optimum strategy to be chosen at time \( t_i \) for period \([t_{i-1}, t_{i+2}]\).

- If it is better to keep on burning low-sulfur coal during the period \([t_{i+1}, t_{i+2}]\), the investment value at \( t_{i+1} \) will be \( Z^+ \) or \( Z^- \), and \( Z \) is given by:

\[
Z_1 = \Lambda_{sw_i}(t_i, A, D) \Delta + 1/R (h Z^+ + (1-h) Z^-)
\]  \hspace{1cm} (4.58)

which represents the marginal benefits for the utility of burning low-sulfur coal over period \([t_i, t_{i+1}]\), plus the value at \( t_i \) of a derivative asset that can take values \( Z^+ \) or \( Z^- \) at time \( t_{i+1} \).

- If it is better to switch to high-sulfur coal and buy allowances for period \([t_{i+1}, t_{i+2}]\), the investment value at time \( t_{i+1} \) will be \( V^+ \) or \( V^- \), and \( Z \) is given by:

\[
Z_2 = \Lambda_{sw_i}(t_i, A, D) \Delta - q_{sw_i} Q + 1/R (h V^+ + (1-h) V^-)
\]  \hspace{1cm} (4.58)

which represents the benefits of burning low-sulfur coal for period \([t_i, t_{i+1}]\), minus the capital cost of switching fuel, plus the value at \( t_i \) of a derivative asset that can take values \( V^+ \) or \( V^- \) at time \( t_{i+1} \).
• Finally, if it is better to build a scrubber and start operating it at time $t_{i-j}$, the investment value at $t_{i+j}$ will be $W^+$ or $W^-$, and $Z$ is given by:

$$Z_3 = \Delta_{w_1}(t_i, A, D) - q_{scr} Q + \frac{1}{R} (h W^+ + (1-h) W^-) \quad (4.59)$$

which represents the benefits of burning low-sulfur coal for period $[t_i, t_{i+j}]$, minus the cost of installing the scrubber, plus the value at $t_i$ of a derivative asset of value $V^+$ or $V^-$ at time $t_{i+j}$.

At time $t_i$, the utility will choose the strategy with the highest value, so that:

$$Z = \max(Z_1, Z_2, Z_3) \quad (4.60)$$

Equations 4.57 to 4.60 give $Z$ in state $(t_i, A, D)$ as a function of $V^+$, $V^-$, $Z^+$, $Z^-$, $W^+$, and $W^-$ at time $t_{i+j}$.

**Calculation of $V$:** If the utility burns high-sulfur coal during period $[t_i, t_{i+j}]$, and does not have a scrubber installed, the investment value $V$ in state $(t_i, A, D)$ is given by:

$$V_1 = \frac{1}{R} (h V^+ + (1-h)V^-) \quad (4.61)$$

$$V_2 = -q_{w_1} Q + \frac{1}{R} (h Z^+ + (1-h) Z^-) \quad (4.62)$$

$$V_3 = -q_{scr} Q + \frac{1}{R} (h W^+ + (1-h)W^-) \quad (4.63)$$

$$V = \max(V_1, V_2, V_3) \quad (4.64)$$

by analogy with the previous case.
Calculation of \( W \): If the utility is already operating a scrubber during period \([t_i, t_{i-1}]\), the calculation of \( W \) is simple, because it was assumed that once the scrubber is installed the utility has to run it. Hence, \( W \) in state \((t_i, A, D)\) is given by:

\[
W = \Lambda_{scr} \Delta + 1/R (h W^+ + (1-h)W^-)
\]  

(4.65)

which represent the marginal benefits of operating a scrubber over period \([t_i, t_{i+1}]\), plus the value at \( t_i \) of a derivative asset that takes values \( W^+ \) or \( W^- \) at \( t_{i+1} \).

Equations 4.57 to 4.65 thus give the investment values \( V, W, \) and \( Z \) in state \((t_i, A, D)\) as functions of the investment values \( V^+, V, W^+, W^- \) and \( Z^- \) in states \((t_{i+1}, A^+, D^+ \) and \((t_i, A^-, D^- \). The investment values can easily be calculated at time \( T \), end of the power plant life:

\[
V(T, A, D) = 0
\]  

(4.66)

\[
W(T, A, D) = \Lambda_{scr}(T, A, D)\Delta
\]  

(4.67)

\[
Z(T, A, D) = \Lambda_{scr}(T, A, D)\Delta
\]  

(4.68)

By recurrence, the investment values can then be calculated in any state prior to \( t = T \). In particular, \( V \) can be calculated at \( t = 0 \), which gives the flexibility present value if the utility starts our burning high-sulfur coal, and does not have a scrubber installed at \( t = 0 \).

4.2 Modifications to the Base Case Model

One of the greatest advantages of the binomial model is that is can easily be adapted to make it more realistic. To illustrate this, it is now assumed that the scrubber need not be operated continuously once installed. The underlying variable model remains unchanged, and only the recurrence relationships have to be modified. Assume that
two additional modes of operation of the scrubber are introduced. One corresponds to
the case in which the scrubber is in place but is not operated, and the utility burns
high-sulfur coal (investment value \( K \)). The other mode corresponds to the case in
which the scrubber is in place but is not operated, and the utility burns low-sulfur coal
(investment value \( L \)). Restarting the scrubber is assumed to cost \( q_{\text{res}} Q \). The
derivation of the recurrence relationships needed for computer coding is then
straightforward. Equation 4.65 has to be replaced by:

\[
W_1 = \Lambda_{\text{scr}} \Delta + 1/R (h W^+ + (1-h)W^-) \tag{4.69}
\]
\[
W_2 = \Lambda_{\text{scr}} \Delta + 1/R (h K^+ + (1-h) K^-) \tag{4.70}
\]
\[
W_3 = \Lambda_{\text{scr}} \Delta + 1/R (h L^+ + (1-h) L^-) \tag{4.71}
\]
\[
W = \max(W_1, W_2, W_3) \tag{4.72}
\]

Furthermore:

\[
K_1 = 1/R (h K^+ + (1-h) K^-) \tag{4.73}
\]
\[
K_2 = -q_{\text{rest}} Q + 1/R (h W^+ + (1-h) W^-) \tag{4.74}
\]
\[
K_3 = -q_{\text{swi}} Q + 1/R (h L^+ + (1-h) L^-) \tag{4.75}
\]
\[
K = \max(K_1, K_2, K_3) \tag{4.76}
\]

and:

\[
L_1 = \Lambda_{\text{swi}} \Delta + 1/R (h L^+ + (1-h) L^-) \tag{4.77}
\]
\[
L_2 = \Lambda_{\text{swi}} \Delta - q_{\text{rest}} Q + 1/R (h W^+ + (1-h) W^-) \tag{4.78}
\]
\[
L_3 = \Lambda_{\text{swi}} \Delta - q_{\text{swi}} Q + 1/R (h K^+ + (1-h) K^-) \tag{4.79}
\]
\[
L = \max(L_1, L_2, L_3) \tag{4.80}
\]

Adapting the binomial model to different operational flexibilities is therefore
straightforward. By comparison the interruptable scrubber case would require the
resolution of 5 partial differential equations (one per operation mode) if the
continuous-time method were used. This would probably be quite difficult.
4.3 Binomial Model Computational Speed

The main computational disadvantage of the binomial method is that it requires a large number of periods per unit time to converge toward the continuous case. Also, adding new operating modes slows down somewhat the computation of the investment value. Fortunately the binomial model developed above exhibits an interesting property which notably improves computation speeds.

Let $Y(N/T)$ be an option value at $t = 0$ calculated numerically with the binomial model, in the case where there are $N/T$ periods per unit time. It turns out empirically that $Y(N/T)$ varies approximately linearly with $N/T$. As an example, Figure 4-4 gives the investment present value $V$ in the base case, as a function of $T/N$. The linearity is clear. The same effect was observed in many different cases: for different numerical values, different options, more complicated models, and also in cases where an analytical expression was available for the option value in the continuous case (European call option of Black and Scholes [8]). In all cases there was strong linearity (deviation of less than a percent from the best fit line). Therefore the option value $Y_{\infty}$ in the continuous case can be obtained from the binomial model by:

$$Y_{\infty} = 4 \times (Y_4) - 3 \times (Y_3)$$  \hspace{1cm} (4.81)

where $Y_4$ (resp. $Y_3$) is the option value calculated by the binomial model with 4 (resp. 3) periods per year. In cases where $Y_{\infty}$ could be obtained independently of the binomial model equation 4.81 was verified to within less than a percent or so.

The linearity effect described here is directly related to the discretization of time. In fact, it is possible to show that the present value of a uniform continuous cash-flow between times $t_0$ and $t$, in the future verifies the same property (it is actually a first-order approximation). To make sure that the same first-order approximation holds for
options, equation 4.81 had to be tested every time substantial modifications were made to the base-case model, or to the numerical values used. All option values given in Section 5 were obtained by first calculating $Y_4$ and $Y_3$, and then using equation 4.81.

![Figure 4-4: Linearity of the Option Value with the Period Length](image)

\[ V = 14.952 - 13.43 \frac{T}{N} \]

\[ (-113) \quad (514) \]

\[ R = 0.99985 \]

Figure 4-4: Linearity of the Option Value with the Period Length
Conclusions on the Binomial Method

A binomial model was derived to describe the time behaviour of two partially correlated underlying asset variables that follow generalised Wiener processes.\textsuperscript{53} It was shown that the recurrence relationships used to calculate the investment's option values can easily be modified to model different operational flexibilities. Also, the binomial model does not require the use of sophisticated stochastic calculus, and is more intuitive than the continuous-time model. It is therefore likely to be more acceptable to corporate managers. The binomial method is also relatively fast (especially if equation 4.81 can be used).\textsuperscript{54} It was therefore preferred to the PDE method for the option calculations of Section 5.

\textsuperscript{53} Rajan [101] derived the same model independently.
\textsuperscript{54} Geske and Shastri [39] find that discrete-time methods for option calculations are not necessarily more time or memory consuming than continuous-time methods.
5. SCRUBBER MODEL RESULTS

5.1 Numerical Value Assumptions

The notations and numerical values of the variables used in the base-case model are presented in Table 5.1.

Power Plant Assumptions

The coal-fired power plant considered in this paper is based on an existing facility, described by Hill [49]. It is a 536 MWe power plant with a capacity factor of 80% and a heat rate of $8.98 \times 10^{-3}$ MMBTU/kWh. It started operation in 1968 and can operate until 2020.\footnote{It is not unusual for coal power plants to operate for more than 50 years. In fact, ICF [53] assumes in one of its models that the standard life of a coal-fired power plant is 55 years.} The power plant considered is a phase II power plant, which means that, starting in year 2000, it will receive yearly $SO_2$ allowances that correspond to an emission level of 1.2 lbs/MMBTU. After year 2000 it will need to purchase additional allowances for $SO_2$ emissions above that level. The period of interest here is therefore from year 2000 to year 2020, the end of the power plant life. However, for ease of presentation it is assumed in this section that phase II of the Clean Air Act Amendments starts in year 1990 (instead of 2000), and that the power plant life ends in year 2010 (instead of 2020). The "present" time ($t = 0$) can then be defined as year 1990. At $t = 0$ the power plant is assumed to use a coal that releases 3.3 lbs of $SO_2$ per MMBTU of fuel burnt, and to have to reduce its emissions to 1.2 lbs/MMBTU.\footnote{Since extra allowances can always be sold on the market, the model developed here can easily be applied to power plants that already emit less than 1.2 lbs/MMBTU at $t = 0$.}
Scrubber Assumptions

The scrubber assumptions are derived from ICF [53], which gives scrubber capital costs of $138/kWe in 1986 dollars. This value is converted into 1990 dollars, and a 30% cost adder is added to account for the cost of retrofitting. 1990 capital costs are then $200/kWe. ICF [53] also reports fixed O&M costs of 5.88 $/kW-year and variable O&M costs of 1.05 mills/kWh. Again, these figures are converted into 1990 dollars, and a 30% retrofit factor is added to the fixed O&M costs. Also, the scrubber is assumed to consume 2.5% of the electricity produced by the power plant. This is also added to the scrubber O&M costs. The total O&M cost in 1990 then amounts to 7.3 mills/kWh.

Coal Switching Assumptions

The low-sulfur coal available to the utility is assumed to release 1.0 lbs of $SO_2$ per MMBTU. In 1990, the coal price premium is assumed to be $0.45/MMBTU, which is the average coal price premium for Ohio bituminous coals of 3.3 lbs/MMBTU and 1.0 lbs/MMBTU reported by Resource Dynamics Corp. [104]. Since the power plant is initially designed for high-sulfur operation, switching to a low-sulfur coal increases O&M and capital costs. Additional O&M costs for coal receiving and handling, boiler operation, and transportation are assumed to amount to 0.5 $/MMBTU with low sulfur coal (Kumar et al. [67]). Also, boiler or particulate matter control upgrades are assumed to cost $25/kW (Kumar et al. [67]).

Financial Assumptions

As recommended by many practitioners, current (nominal) dollars are used here (Copeland et al. [23]). In the base-case model, the risk-free interest rate is assumed to be constant over the 20 years of plant operation, and equal to 9.27% per year, the yield of a 20-year Treasury bond (Wall Street Journal, May 1, 1990). Also, the

---

57 Throughout this article the producer price index (as given by RCG/Hagler, Bailly, Inc. [102]) is used to convert past prices into 1990 dollars.

58 Electricity is assumed to cost 10 c/kWh (Boston Edison's average electricity sale price in 1990).
various capital costs, switching costs, and O&M costs are assumed to escalate over the 20-year life of the plant at a rate of 2% per year.\textsuperscript{59}

### Table 5-1: Scrubber Investment Base-Case Assumptions and Notation

<table>
<thead>
<tr>
<th>Correlation Factor</th>
<th>( \rho = 0.8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SO(_2) Allowance</strong></td>
<td></td>
</tr>
<tr>
<td>Present Value</td>
<td>( A(0) = $ 500/\text{ton} )</td>
</tr>
<tr>
<td>Volatility</td>
<td>( \sigma_A = 0.12/\text{year} )</td>
</tr>
<tr>
<td>Convenience Yield</td>
<td>( \delta_A = 0.05/\text{year} )</td>
</tr>
<tr>
<td><strong>Coal Price Premium</strong></td>
<td></td>
</tr>
<tr>
<td>Present Value</td>
<td>( D(0) = $ 0.45/\text{MMBTU} )</td>
</tr>
<tr>
<td>Volatility</td>
<td>( \sigma_D = 0.14/\text{year} )</td>
</tr>
<tr>
<td>Convenience Yield</td>
<td>( \delta_D = 0.05/\text{year} )</td>
</tr>
</tbody>
</table>

#### Financial Assumptions

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference date</td>
<td>January 1, 1990</td>
</tr>
<tr>
<td>Risk-free interest rate</td>
<td>( r_F = 9.27%/\text{year} )</td>
</tr>
<tr>
<td>Escalation rate</td>
<td>( a = 2.0%/\text{year} )</td>
</tr>
</tbody>
</table>

#### Power Plant Assumptions

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Plant Size</td>
<td>( Q = 536,000 \text{kWe} )</td>
</tr>
<tr>
<td>Remaining Plant Life</td>
<td>( T = 20 \text{ years} )</td>
</tr>
<tr>
<td>Capacity Factor</td>
<td>( \eta = 80% )</td>
</tr>
<tr>
<td>Heat Rate</td>
<td>( \gamma = 8.98 \times 10^{-3} \text{ MMBTU/kWh} )</td>
</tr>
<tr>
<td>High-Sulfur Coal</td>
<td>( x_H = 3.3 \text{ lbs } SO_2/\text{MMBTU} )</td>
</tr>
</tbody>
</table>

#### Scrubber Assumptions

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Costs</td>
<td>( q_{scr} = $ 200/\text{kWe} )</td>
</tr>
<tr>
<td>Total O&amp;M Costs</td>
<td>( M_{scr} = 7.3 \times 10^{-3} \text{ /kWh} )</td>
</tr>
<tr>
<td>( SO_2) Removal Rate</td>
<td>( b = 90% )</td>
</tr>
</tbody>
</table>

#### Switching Costs Assumptions

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable Switching Costs</td>
<td>( M_{swi} = $ 0.5/\text{MMBTU} )</td>
</tr>
<tr>
<td>Switching Capital Costs</td>
<td>( q_{swi} = $ 25/\text{kWe/switch} )</td>
</tr>
<tr>
<td>Low-Sulfur Coal</td>
<td>( x_L = 1.0 \text{ lbs } SO_2/\text{MMBTU} )</td>
</tr>
</tbody>
</table>

\textsuperscript{59} If it is assumed that the annual riskless interest rate in constant dollars is 2.5\% (Copeland et al. [23]), the Treasury-bond yield given here leads to a market-expected inflation rate of 6.6\% per year. The escalation rate chosen here is substantially lower, because it is assumed that as a result of technological progress environmental equipment costs will escalate at a lower rate than inflation.
5.2 Preliminary Calculations

Using the notation of Table 5.1, it is possible to express the benefits per unit time, noted $\Lambda_{scr}$, associated with operating the scrubber in the base-case model. These benefits are equal to the value of the allowances saved by scrubbing, minus the scrubber operating costs. Let $B$ be the total number of MMBTU required by the plant for one year of operation. $B$ is given by:

$$B = Q \times \eta \times 365 \times 24 \times \gamma = 33.7 \times 10^6 \text{ MMBTU/year} \quad (5.1)$$

In the base-case model the utility is assumed to operate the scrubber only with high-sulfur coal. The total quantity of $SO_2$ emissions saved by scrubbing therefore equals $B \times x_H \times b$ pounds per year. The value of these avoided emissions at time $t$ is: $B \times x_H \times b/2000 \times A(t) \ ($/year). The scrubber O&M costs are given by: $M_{scr} \ exp(at) \times B/\gamma$ ($$/year). Hence, the benefits at time $t$ of using the scrubber with high-sulfur coal are (in dollar per year):

$$\Lambda_{scr} = -M_{scr} \ exp(at) \ B/\gamma + B \ b \ x_H \ A(t) / 2,000 \quad (5.2)$$

Similarly, the benefits to the utility of burning low-sulfur coal instead of high-sulfur coal can be evaluated. The number of allowances saved in this case is $B \times (x_H - x_L)/2000$. The marginal costs of burning low-sulfur coal include the coal price premium, plus the additional O&M costs. The total benefits at time $t$ of burning low-sulfur coal are then (in dollar per year):

$$\Lambda_{rel} = -B \ (D + M_{rel} \ exp(at)) + B \ (x_H - x_L)A(t) / 2,000 \quad (5.3)$$

Maximum Compliance Costs
The new Clean Air Act $SO_2$ requirements may represent a substantial cost for the utility considered here. This cost is maximum if the utility does not have the option to switch fuel, or the option to install an emission control device. This maximum net present cost to the utility, $C_0$, of the 1990 sulfur amendments is equal to the net present worth of all allowance purchases for the 20 remaining years of power plant operation.

Since the utility burns high-sulfur coal, but is only allowed emissions of $B \times 1.2$ pounds of $SO_2$ per year, the total cost per year, $C_{tot}$, is:

$$C_{tot}(t) = \frac{B(x_H - 1.2)A(t)}{2000}$$  \hspace{1cm} (5.4)$$

$C_0$ is then obtained by integrating $C_{tot}$ over the next twenty years of operation. The expected allowance price at time $t$ is given by: $A(0) e^{\alpha_A t}$, where $\alpha_A$ is the expected rate of increase of the allowance price ($\alpha_A$ is not known). This expected value should be discounted by the risk-adjusted discount rate for the allowance, $\mu_A$, which is equal to $\alpha_A + \delta_A$ (see equation 3.2). Hence, $C_0$ is given by:

$$C_0 = \int_{t=0}^{20} \frac{B \times (x_H - 1.2)}{2000} \times A(0) e^{-\mu_A t} dt$$  \hspace{1cm} (5.5)$$

This value can easily be calculated using the data of table 5.1:

$$C_0 = -$223.7 million$$  \hspace{1cm} (5.6)$$

If the utility has no opportunity to switch fuel or to install a control device, the Clean Air Act $SO_2$ provisions will cost $223.7 million. $C_0$ will be referred to as the maximum compliance cost in this paper. The maximum compliance cost is found to depend strongly on the value of the allowance price convenience yield. For instance,
if $\delta_\lambda = 0$ it is equal to -$353.8$ million, and if $\delta_\lambda = 0.10$ it is equal to -$153.0$ million only.

**Investment Value Definition**

The maximum compliance cost corresponds to the benchmark investment situation defined in Section 3, against which all the investment values in the present section are calculated. Investment values are thus defined in this section as the difference between the actual net present compliance costs of the Amendments in the case considered and the maximum compliance cost $C_\alpha$ (see Figure 5-1). For example, if in a given case the investment value is found to be worth $20$ million, the actual cost of complying with the Title IV $SO_2$ provisions of the Clean Air Act Amendments is $223.7 - 20 = 203.7$ million. This definition of the investment value best corresponds to the flexibility value of interest in this work.

Investment values in the following sections are always calculated at $t = 0$. It is assumed that the utility initially burns high-sulfur coal, and does not have a scrubber installed. Investment values in this section are therefore denoted $V$, in conformance with the notation of Section 4. The specific operating model in which the investment value is calculated is represented by a subscript.

![Diagram](image)

*Figure 5-1: Definition of Investment Value in the Scrubber Investment Case*
Thus,

- $V_{\text{out}}$ is defined as the investment value if the utility can install a scrubber, and switch coal (case 1);
- $V_{\text{swi}}$ is defined as the investment value if the utility can switch fuel back and forth between high- and low-sulfur coals, but cannot install any emission control device (case 2); and
- $V_{\text{scr}}$ is defined as the investment value if the utility can install a scrubber, but always has to burn high-sulfur coal (case 3).

Note that in the base-case model (case 1) a scrubber, once installed, has to be operated, and cannot use low-sulfur coal.

**Scrubber Investment Value in No-Flexibility Case**

It is interesting to calculate the NPV of the scrubber investment if the utility can only install the scrubber at $t = 0$, and has to burn high-sulfur coal. It is noted $\text{NPV}_{\text{scr}}$, and is calculated with the standard discounted cash flow method since there is no option available. The total capital costs at $t = 0$ are equal to $q_{\text{scr}} \times Q$. Scrubber O&M costs are assumed to escalate deterministically (at a rate $a$ of 2% per year), and should therefore be discounted at the risk-free rate. By contrast, the value of the allowances saved by scrubbing have to be discounted at their risk-adjusted discount rate, $\mu_A$. Therefore, the standard net present value, $\text{NPV}_{\text{scr}}$, of the scrubber investment at $t = 0$ is equal to:

$$
\text{NPV}_{\text{scr}} = -q_{\text{scr}} \times Q - \int_{t=0}^{20} \frac{B}{\gamma} M_{\text{scr}} e^{at} e^{-r_s t} dt + \int_{t=0}^{20} \frac{B b x_H}{2,000} A(0) e^{at} e^{-\mu_A t} dt \quad (5.7)
$$

Note that $\alpha_A - \mu_A = -\delta_A$, so that $\text{NPV}_{\text{scr}}$ is easy to calculate:

$$
\text{NPV}_{\text{scr}} = -$79.9 million \quad (5.8)
$$
Since $NPV_{scr} < 0$, the utility should not install the scrubber at $t = 0$. If the utility were to install the scrubber at $t = 0$ anyway, the actual net present cost of compliance would be: $C_0 + NPV_{scr} = -303.6$ million.

### 5.3 Initial Results

**Investment Values**

Investment values $V_{tot}$, $V_{swi}$ and $V_{scr}$ were calculated with the binomial model of Section 4 and the numerical values of Table 5.1:

\[
V_{tot} = 14.87 \text{ million} \tag{5.9}
\]
\[
V_{scr} = 14.83 \text{ million} \tag{5.10}
\]
\[
V_{swi} = 0.98 \text{ million} \tag{5.10}
\]

The option to scrub and/or to switch coal therefore reduces the cost of complying with the Title IV provisions by about 7% (from $223.7$ to $208.8$ million), a relatively small but not negligible change. $V_{tot}$, $V_{scr}$ and $V_{swi}$ take on positive values because they represent options: the utility could always prefer to burn high-sulfur coal, in which case the investment value as defined in the previous section would be zero. $V_{tot}$ is larger than both $V_{scr}$ and $V_{swi}$, because each of the flexibilities available in cases 2 and 3 are also available in case 1. $V_{swi}$ is found to be quite low, which means that the option to switch coal is worth little. This is because the market prices of high- and low-sulfur coals are by assumption nearly equal to the value of the allowances saved by switching coals. As a consequence, $V_{tot} \sim V_{scr}$. Finally, it is interesting to note that $V_{tot} < V_{swi} + V_{scr}$\(^{60}\) in some states of the world it is impossible to take advantage of both the option to switch and the option to scrub. For instance, for high allowance prices it might be interesting to scrub, and interesting to switch fuel, but in

---

\(^{60}\) Trigeorgis [124] also found that the combined value of several options is generally lower than the sum of the separate option values.
the base-case model the utility is assumed to lose the option to switch fuel once the scrubber has been installed.

Since several of the numerical values chosen for the investment calculations are not known with a high degree of certainty, it is important to estimate the sensitivity of the investment values to the various parameters. Whenever possible, this sensitivity study should take into account the fact that model parameters may not always change independently from each other (Brealey and Myers [10]).

**Influence of Convenience Yields**

*Influence of \( \delta_A \):* Figure 5-2 gives the influence of the allowance price convenience yield, \( \delta_A \), on the investment value in cases 1 and 3 (the investment value \( V_{tot} \) in case 2 is virtually equal to \( V_{tot} \)). Figure 5-2 shows that the investment values decline when \( \delta_A \) increases. This result was expected: everything else being equal, allowance prices are lower with higher \( \delta_A \), and options to save allowances are therefore less valuable. Figure 5-2 shows that if the value of \( \delta_A \) changes from 5% to 4%, the investment value \( V_{tot} \) virtually doubles. The influence of \( \delta_A \) is strong because it plays a role similar to that of a discount rate for the future allowances saved, as equation 5.7 shows in the no-flexibility case. This strong influence of \( \delta_A \) is quite unfortunate, since its actual value is not well known at this stage. However, once the trading of allowances and allowance futures starts, it should be possible to obtain a better estimate of \( \delta_A \). The elasticity of \( V_{tot} \) with respect to \( \delta_A \) and to various other parameters discussed below is given in Table 5.2.61

*Influence of \( \delta_D \):* If the convenience yield of the coal price premium is zero instead of 5%, the investment value in the base-case is found to be: \( V_{tot} = \$14.84 \) million.

---

61 This elasticity is defined by \( \frac{X dV_{tot}}{V_{tot} dX} \), where \( X \) is the value of the parameter considered. The elasticity was estimated for small values of \( dX \). It is only valid in the region of the base-case value of \( V_{tot} \).
instead of $14.87 million. Thus, contrary to the effect of the allowance convenience yield, the investment value decreases if $D$ decreases. This is reasonable: the higher $D$ is, the lower the coal price premiums are, and hence the cheaper it is to switch fuel.

**Influence of Correlation Factor**

Figure 5-3 gives the influence on the investment value of the correlation factor, $\rho$, between the two underlying asset prices. It is found that the lower the correlation value is, the higher the investment value. This was expected, because for low correlation factors the two underlying variables can have more extreme values in a given state of the world. And extreme values of the underlying asset (both very low and very high) increase the option value, as shown by Cox and Rubinstein [24].

![Figure 5-2: Effect of the Allowance Convenience Yield on the Scrubber Investment Value.](image)
Figure 5-3 shows that $\rho$ has a substantial effect on $V_{swi}$ but not on $V_{tot}$ or $V_{scr}$. This is not surprising since switching fuel is not very important in the base-case model. It is fortunate that $\rho$ does not have a strong impact on $V_{tot}$, since the correlation factor cannot be obtained empirically yet.

Figure 5-3: Effect of the Correlation Factor on the Scrubber Investment Value.
Influence of Standard Deviations of Underlying Variable Returns

The effects of $\sigma_A$ and $\sigma_D$ on the investment value were first studied separately. $\sigma_D$ was found to have little influence on $V_{tot}$ (which could have been expected since the option to switch is not very valuable in the base-case) and only marginally more on $V_{swi}$. By contrast, $\sigma_A$ was found to have a strong effect on both values. For instance, if $\sigma_A$ increases from 12\%/year to 14\%/year, $V_{tot}$ increases from $14.9$ to $19.6$ million, and $V_{swi}$ increases from $1.0$ to $2.3$ million. For both $\sigma_A$ and $\sigma_D$, the higher the standard deviation is, the higher the investment value. This result is similar to that reported by Cox and Rubinstein [24] for stock options: call and put values increase with the standard deviation of their underlying asset. This is related to the previous remark that option values increase when the underlying asset prices can take more extreme values.

Studying the effect of $\sigma_A$ separately from that of $\sigma_D$ is somewhat unjustified, since it was shown in Section 3 that there is likely to be a strong relationship between the two values in the future. Figure 5-4 therefore gives the investment values $V_{tot}$ and $V_{swi}$ as functions of $\sigma_A = \sigma_D$. The figure shows that, as expected, the investment value increases when the standard deviations increase, and that the influence of the standard deviation is substantial.

Influence of Power Plant Lifetime

If the power plant and scrubber can operate for more than 20 years, the investment value should increase, just as the value of a European call option increases with the expiration date (Cox and Rubinstein [24]). Indeed, Figure 5-5 shows that the investment value in the base-case increases from $14.9$ million to $39.7$ million if the lifetime increases from 20 to 30 years.

Thus, had the power plant been assumed to have an infinite life in order to simplify the calculation in the continuous-time method of Section 4, a significant error would
have been introduced. This may seem surprising, since many real option studies assume an infinite-life investment (e.g. a plant or a mine).

Figure 5-4: Effect of the Standard Deviations on the Scrubber Investment Value.
This apparent paradox can be explained by using a note in Majd and Pindyck [73]. These authors assume that the value of $E$ of a plant's production follows a generalised Wiener process over time with a convenience yield $\sigma_E$ ($E$ in dollars per unit time). It is then straightforward to show that, in the absence of the option to interrupt production before the end of the plant's life, the value $U(t)$ at $t$ of an installed plant is given by:

$$U(t) = \frac{E(t)}{\delta_E} \left(1 - \exp(-\delta_E T)\right)$$

(5.12)
where $T$ is the plant's remaining lifetime. $U(t)$ therefore tends toward $E(t)/\delta_E$ as $T$ tends toward infinity. However, equation 5.12 is only valid if the convenience yield is not zero. If $\delta_E = 0$, $U(t)$ is given by:

$$U(t) = TE(t)$$

(5.13)

In this case the plant value increases linearly with $T$, and tends toward infinity. These results are illustrated by Figure 5-6.

![Graph of Plant Value as Function of the Plant's Remaining Life](image)

**Figure 5-6: Plant Value as Function of the Plant's Remaining Life**

As shown by Cox and Rubinstein [24], the value of a call option tends toward the value of its underlying asset when the expiration date $T$ tends toward infinity. Therefore, as $T$ tends toward infinity, the value of an option on the plant considered by Majd and Pindyck tends toward infinity if the convenience yield is zero, and
toward a finite limit if the convenience yield is not zero. (The lower the convenience yield, the higher this limit.) In the case considered in this work convenience yields are around 5%/year, which means that the investment value only reaches its limiting value for very large $T$'s. This explains why the investment values for $T = 20$ years and $T = 30$ years are so strikingly different.

Influence of Capital Costs and Switching Costs

**Scrubber Capital Costs:** The influence of the scrubber capital cost, $q_{scr}$, on the investment value was found to be significant in the base-case model. If the capital cost is $300$/kWe (instead of $200$/kWe) $V_{tot}$ decreases from $14.9$ million to $7.9$ million. If the capital cost reaches $500$/kWe, the investment value declines to $2.3$ million. As expected, changes in $q_{scr}$ do not affect $V_{swi}$. Also, as the scrubber capital cost declines to zero, the investment value in cases 1 and 2 converge. This is reasonable: it is usually cheaper to install a (free) scrubber than to switch fuel, so that the option to switch fuel is virtually worthless. By contrast, if capital costs are $500$/kWe, there is a significant difference between $V_{tot}$ and $V_{scr}$, because the option to switch is becoming relatively more valuable.

**Switching Costs:** As expected, switching costs are found to have no effect on $V_{scr}$, little effect on $V_{tot}$, but substantial effects on $V_{swi}$. For instance, switching costs of zero (instead of $25$/kWe/switch) increase $V_{tot}$ from $14.9$ to $15.0$ million, and $V_{swi}$ from $1.0$ to $1.8$ million. The negligible impact of the switching cost on $V_{tot}$ suggests that utilities are unlikely to make very different investment decisions whether they face high or low switching costs. The assumption made in Section 3 that switching costs are low is thus likely to lead to results that are not very different from reality.
Influence of Operating Costs

Scrubber operating costs, $M_{scr}$, were found to have a very substantial influence on the investment value $V_{tot}$. Obviously, the lower $M_{scr}$ is, the higher $V_{tot}$. For instance, if the operating costs are changed from 7.3 mills/kWhe to 5.0 mills/kWhe, the investment value increases from $14.9 million to $37.2 million, a very large change.

The reason for this strong effect is that the option to scrub is the most important flexibility for the utility in the base-case model, and the scrubber O&M costs are a substantial component of $\Lambda_{scr}$, the net benefit to the utility if it operates a scrubber.

Influence of Risk-Free Interest Rate and Escalation Rate

Risk-Free Interest Rate: The risk-free interest rate was also found to have a strong influence on the investment values considered here. For instance, if the rate is 7%/year instead of 9.3%/year, $V_{tot}$ declines to $4.6 million, and $V_{swi}$ to $0.3$ million. Conversely, if the rate is assumed to be 10% per year, the investment value $V_{tot}$ increases to $20.2$ million, and $V_{swi}$ to $1.5$ million.

It is interesting to note that, just as with European call options, the investment option value increases as the risk-free interest rate increases. The reason is that a higher risk-free interest rate decreases the present value of the exercise price (here, the scrubber capital costs or switching costs), whereas the present value of the benefits remains fixed, because it is related to the present value of the underlying assets (Cox and Rubinstein [24]). Another way to demonstrate the same result is to look at the single-period model of Section 4.1. It is easy to calculate from equation 4.44 that:

$$\frac{\partial Y}{\partial R} = \frac{(f Y^+ - g Y^-)}{(f-g) / R^2}$$

(5.14)

where $Y$ is the investment value at $t = 0$, $Y^+$ and $Y^-$ are its two possible values at $t = 1$, $R$ is the risk-free return, and $f$ and $g$ are the two possible returns of the

62 By contrast the present value of a standard investment usually decreases as the interest rate increases.
underlying asset over the period considered. As the number of periods per unit time in the binomial model increases, \( f \) and \( g \) tend toward \( l \), so that \( \epsilon Y / CR \) tends toward \( (Y^- - Y^+)/ (f-g)/R^2 \), which is positive.\(^{63}\)

**Deterministic Non-Constant Risk-Free Interest Rate:** The base-case model was slightly modified to test whether the assumption that the interest rate is constant over time was justified. The expected value of the risk-free interest rate over the next twenty years was estimated from the yields of Treasury bills and bonds.\(^{64}\) Let \( X_i \) and \( X_{i+1} \) be the present yields of bonds of respective maturity \( i \) and \( i + 1 \) years. The market estimate, \( r_F(i + 1) \), of the risk-free interest rate for year \( i+1 \) can be estimated with the equation:

\[
(1 + X_{i+1})^{i+1} = (1 + X_i)^i \times (1 + r_F(i + 1))
\]

(5.15)

The interest rate \( r_F \) in the base-case model was then replaced by the \( r_F(i) \) given by the market. The investment value \( V_{tot} \) was found to vary by less than 0.5%. Therefore, it is legitimate to assume that the risk-free interest rate is constant over \([0,T]\), and equal to the yield of a Treasury bond for \( T \) years.

**Escalation Rate:** The elasticity of the investment value \( V_{tot} \) relative to the rate, \( a \), at which capital costs, switching costs, and O&M costs escalate over the next 20 years was found to be low (see Table 5.2). However, \( a \) is not known with a high degree of precision, and if it were equal to 6\% instead of 2\%, the investment value in the base-case would decrease from \$14.9 million to \$1.0 million. It is therefore important to estimate the escalation rate as precisely as possible.

---

\(^{63}\) Note however that this result does not necessarily hold if \( Y^+ \) or \( Y^- \) depend on \( R \). In the case considered in this section, the discounted value of the scrubber O&M costs depends on \( R \), but is not very substantial.

\(^{64}\) Note that risk-free interest rates are still considered to be deterministic, although not constant.
Influence of Scrubber Sulfur Removal Rate

The scrubber's sulfur removal rate, $b$, was found to have a significant impact on the investment value $V_{tot}$. If $b$ is increased from 90% to 100%, the investment value increases from $14.9$ million to $24.4$ million. This result suggests that more efficient removal technologies might change the investment value substantially. Therefore, the possibility that new removal technologies might appear in the future is studied in the next section, and the impact on the investment value is evaluated.

Table 5.2: Elasticities of the Scrubber Investment Value Relative to Various Model Parameters

<table>
<thead>
<tr>
<th>Model Parameter</th>
<th>Elasticity of $V_{tot}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scrubber Removal Rate, $b$</td>
<td>+ 5.8</td>
</tr>
<tr>
<td>Scrubber O&amp;M Costs, $M_{scr}$</td>
<td>- 4.7</td>
</tr>
<tr>
<td>Risk-Free Interest Rate, $r_F$</td>
<td>+ 4.5</td>
</tr>
<tr>
<td>Allowance Convenience Yield, $\delta_A$</td>
<td>- 4.1</td>
</tr>
<tr>
<td>Standard Deviation, $\sigma_A = \sigma_D$</td>
<td>+ 2.0</td>
</tr>
<tr>
<td>Scrubber Capital Costs, $q_{scr}$</td>
<td>- 1.0</td>
</tr>
<tr>
<td>Power Plant Lifetime, $n$</td>
<td>+ 0.8</td>
</tr>
<tr>
<td>Escalation Factor, $a$</td>
<td>- 0.5</td>
</tr>
<tr>
<td>Correlation Factor, $\rho$</td>
<td>- 0.2</td>
</tr>
<tr>
<td>Switching Costs, $q_{swi}$</td>
<td>$\sim 0$</td>
</tr>
<tr>
<td>Coal Price Premium Convenience Yield, $\delta_D$</td>
<td>$\sim 0$</td>
</tr>
</tbody>
</table>
Overall, the sensitivity study performed in this section shows that the parameters describing the stochastic behaviour of the allowance price (price volatility and convenience yield) have a strong influence on the investment value. Also, the scrubber's removal rate and O&M costs are more important than its capital costs (higher elasticity of the investment value). As could have been expected, the parameters related to coal switching do not appear to be important.

5.4 Modification of the Scrubber Base-Case Model

This section presents several modifications to the base-case model in order to try to make it more realistic. In particular, the effect on the investment value of new scrubber technologies, of construction delays, and of different scrubber operational flexibilities will be assessed.

Influence of New Technologies

The first modification consists in considering the possibility that cheaper or more efficient sulfur removal retrofit technologies might appear in the future. The possibility of such a development might increase the value of the option to wait before installing a scrubber. Two different future retrofit technologies are considered here. Furnace Sorbent Injection (FSI) technology is reported to be generally cheaper but also less effective than wet scrubbers. By contrast, advanced scrubbers (as defined by Chomka et al. [21]) have the potential to be both more effective and cheaper than standard scrubbers. Table 5.3 summarises the cost and effectiveness assumptions for these two new technologies.

In the first modified version of the base-case model, the utility was assumed to have the possibility (in addition to the usual three compliance alternatives) of installing an FSI control system at any time after time $j$, when the new technology becomes available on the market. The rest of the base-case model was not modified. It was
found that the investment value does not change much, irrespective of the date of FSI availability, $j$. The explanation for this is that most of the investment value calculated in the base-case model comes from states of the world in which the allowance price increases a lot. But in such states of the world the lower removal rate of FSI technology makes it uncompetitive with scrubber technology.

Table 5.3: Cost and Effectiveness Assumptions for Future Sulfur Removal Technologies

<table>
<thead>
<tr>
<th></th>
<th>Standard Scrubber</th>
<th>Furn. Sorb. Inj.</th>
<th>Advanced Scrubber</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Costs ($/kWe)</td>
<td>200</td>
<td>100</td>
<td>140</td>
</tr>
<tr>
<td>O&amp;M Costs (mills/kWhe)</td>
<td>7.3</td>
<td>7.3</td>
<td>6.5</td>
</tr>
<tr>
<td>SO$_2$ Removal Rate</td>
<td>0.9</td>
<td>0.60</td>
<td>0.98</td>
</tr>
<tr>
<td>Reference</td>
<td>[86]</td>
<td>[199]</td>
<td>[5] and [35]</td>
</tr>
</tbody>
</table>

If the advanced scrubber technology is substituted for FSI in the modified model, the investment value $V_{tot}$ changes substantially, as illustrated in Figure 5-7. As expected, the sooner the advanced scrubber becomes available, the higher the investment value. If the date on which the advanced scrubber becomes available is $j = 20$ years, the utility will have no opportunity to use it. Therefore the value of $V_{tot}$ in Figure 5-7 for $j = 20$ years is just the base-case value, $\$14.9$ million. If $j = 0$, the advanced scrubber is available immediately. Since the advanced scrubber is both cheaper and more effective than the standard one, the opportunity to install the old scrubber is then worthless to the utility, and the investment value can be calculated with the old model.
(by substituting the characteristics of the advanced scrubber for those of the old one).
Both the modified model for $j = 0$ and the base-case model with the new scrubber values give the same result: $42.5$ million.

Figure 5-7: Effect of the Availability of an Advanced Scrubber Technology

Influence of Construction Delays
Another unrealistic feature of the base-case model is the assumption that the scrubber can be installed instantaneously. In practice, there is a delay between the time that decision to install the scrubber is made and the time that it starts operating. The base-case model needs to be substantially modified to take this effect into account.
For simplicity, the option to switch fuels is not considered in this modified model. Let \( w \) be the number of periods corresponding to the length of the construction delay.

In the no-delay model without the switching option, each state of the world \((t_k, A, D)\) is assigned investment values \( V(t_k, A, D) \) and \( Z(t_k, A, D) \) (see Section 4 for notation).

Here, it is necessary to add \( w + 1 \) investment values, noted \( V_0(t_k, A, D) \) with \( 0 \leq \varnothing \leq w \). \( V_0(t_k, A, D) \) corresponds to the investment value if the decision to install a scrubber has previously been made, and the scrubber is to start operating in \( w - \varnothing \) periods (i.e. at time \( t_{k+\varnothing} \)). The longer the time delay, the more additional investment values \( V_0 \) there are.

For a construction lead-time of 2 years, the investment value is found to decrease from $14.3 million to $9.3 million. For a construction period of 5 years, the investment value is only $3.0 million. Construction delays are therefore important characteristics of the model. Their effect on the optimal investment strategy is described below.

**Influence of Scrubber Salvage Value**

The base-case model can easily be modified to account for the salvage value of the scrubber. Equation 4.67, which gives the option value at the beginning of the last period, is modified as follows:

\[
W(T, A, D) = \Lambda_{scr}(T, A, D) + Salvage \text{ Value}
\]  

(5.16)

If the salvage value at time \( T \) is equal to 20% of the scrubber capital costs (or $21.4 million), the investment value increases from $14.9 million to $16 million, a 7.2% increase only. No discussion of the scrubber salvage values was found in the literature, and the effects are neglected in the rest of this section.
Influence of Different Flexibilities

*Scrubber Interruptability:* The model developed in Section 4 for the case in which scrubber operation can be interrupted and resumed was also evaluated. It was assumed that there were no capital costs associated with interrupting scrubber operation, but that scrubber restart would cost $5/kWe. The investment value $V_{tot}$ was found to be only 2% higher than in the base-case. Even with zero restart costs, the investment value was still less than 2.5% higher than in the base-case. The option to interrupt and resume operation of the scrubber was thus not found to be very valuable.

*Scrubber Operation with Low-Sulfur Coal:* Next, the base-case model was modified to allow the utility to burn low-sulfur coal even when the scrubber is operating. It was assumed that the utility with a scrubber installed could switch back and forth from low to high-sulfur fuel, just as it can with no scrubber. The resulting investment value was virtually unchanged, which is not surprising, since coal switching was not found to be a very valuable option in the previous models.

*Coal Switching Modifications:* In still another modification of the base-case model, it was assumed that switching coal was substantially more expensive the first time than on subsequent occasions (this would be the case if the utility had to make plant modifications the first time). The results show that subsequent switching costs are reduced to zero and the initial switching cost remains at $25/kWe, the option value only increases by about 0.5% (it was found in a previous section that if all coal switching costs are zero, $V_{tot}$ increases by about 1%).
5.5 Investment Criterion

Investment Decision at Time t

The binomial model used for the calculations presented so far can easily be used for investment decision-making. In fact, and as explained in Section 4, the investment value in a given state of the world is calculated by the binomial model as the maximum of the values of all strategic alternatives available to the utility. For instance, at \( t = 0 \) in the base-case model:

\[
V_1 = \$ 14.9 \text{ million} \tag{5.17}
\]

\[
V_2 = -\$ 12.0 \text{ million} \tag{5.18}
\]

\[
V_3 = -\$ 79.9 \text{ million} \tag{5.19}
\]

\[
V_{10} = \max(V_1, V_2, V_3) = \$ 14.0 \text{ million} \tag{5.20}
\]

\( V_1 \) is the investment value if the utility does not install the scrubber or switch fuel at \( t = 0 \), \( V_2 \) is the investment value if the utility decides to switch fuel at \( t = 0 \), and \( V_3 \) is the value if the utility decides to install a scrubber at \( t = 0 \). Obviously, at \( t = 0 \) the utility should choose alternative 1 (no coal switching, and no scrubber installed). Similarly, for any state of the world \((t_k, A, D)\), the binomial model calculates the optimal strategy choice for the utility. The model developed in Section 4 can therefore easily be used for investment decision-making.

It will also be useful to obtain an ex-ante investment criterion that tells the utility at \( t = 0 \) under what future conditions it should install the scrubber.

No Switching Case

Such a criterion can easily be found in the case where the utility is assumed to have only the option to install the scrubber. In this case, the only underlying variable is the allowance price, and clearly if it is worth installing a scrubber at time \( t \) for an
allowance price $A$. It is also worth installing the scrubber at time $t$ for all allowance prices higher than $A$. It is easy to deduce from the binomial option model the minimum value $A_{crit}(t)$ such that the utility should install the scrubber at $t$ (provided it is not already in place). Figure 5-8 gives $A_{crit}(t)$ for the base-case model, and $A'_{crit}(t)$ for the case in which the scrubber takes two years to install. As expected $A_{crit}(t)$ and $A'_{crit}(t)$ are increasing functions of time. $A_{crit}(t)$ approaches infinity as $t$ approaches 20 years, and $A'_{crit}(t)$ approaches infinity as $t$ approaches 18 years. The less time there is to operate the scrubber, and the higher the allowance price has to be to justify installing it.

The dotted curves of Figure 5-8 correspond to exponentially increasing allowance prices. Figure 5-8 thus shows that if the allowance price turns out to increase exponentially at a rate of 10% per year, the utility will install the scrubber a little before year 9 if it can start operation immediately, and at about year 7 if it takes two years to build. Figure 5-8 also shows that if the allowance price increases at a rate of less than 7.2% per year, the utility will never install the scrubber. It is important to recognise that these exponential curves do not describe information available ex-ante. If the utility knows ex-ante that the allowance price will increase exponentially at a given rate, there is no need for a binomial option model, because there is no uncertainty.

It is possible to calculate a minimum allowance price $A_{NPV}(t)$ for installing the scrubber at time $t$, if there is no option to wait (i.e. if the utility can only install the scrubber at $t$, or never). Similarly, a critical value $A'_{NPV}(t)$ can be defined if there is no option to wait, and the scrubber takes two years to install. $A_{NPV}(t)$ and $A'_{NPV}(t)$ are obtained from standard NPV calculations of the scrubber investment, and are given in Figure 5-9. As expected, $A_{NPV}(0) > \$ 500$, which shows again that the utility should not install the scrubber at $t = 0$. 
Figure 5-8: Simple Scrubber Investment Criterion (no possibility of switching coals).

At any given time, $A_{\text{crit}}(t)$ is higher that $A_{\text{NPV}}(t)$ (and $A_{\text{crit}}'(t)$ is higher than $A'_{\text{NPV}}(t)$) because a utility which uses an option model to make its scrubber investment decision will wait for a higher price before installing the scrubber. This reflects the irreversibility of the scrubber investment. If the utility does not install the scrubber, it retains a valuable option to do so at a future date. If it installs the scrubber, it forgoes this option. It also seems reasonable that, as time tends toward 20 years, the difference between $A_{\text{crit}}$ and $A_{\text{NPV}}$ diminishes: the option to wait is less and less valuable as the expiration date gets nearer (similarly for $A_{\text{crit}}'$ and $A'_{\text{NPV}}$).
Investment Criterion in the Base-Case Model

In the base-case model, the utility has both the option to install a scrubber and the option to switch fuel. There are two underlying variables, so that it is not possible to find a simple critical value of the allowance price as in the previous section. However, it is possible to represent on a two-dimensional diagram the combination of allowance price (A) and coal price premium (D) that corresponds to a given optimal compliance strategy at a given time, \( t_o \). Figure 5-10 gives such diagrams for \( t_o = 5 \) years and \( t_o = 10 \) years (the case \( t_o = 0 \) is obviously uninteresting, since the values of the underlying values are well known, and the optimal investment policy is straightforward). Figure 5-10 shows that the price points for the underlying asset value A and D can divided into three regimes, depending on which of the three
compliance methods is preferable. The diagrams are somewhat different if, at \( t = t_0 \), the power plant is burning high-sulfur coal (dotted lines) or low-sulfur coal (full lines).

Figure 5-10 shows that for high allowance prices, it is best to install a scrubber, unless coal price premiums are very low. This is especially true if the utility is burning high-sulfur coal (see case \( t_0 = 10 \) years). Similarly, if coal price premiums are very high, there is a cut-off allowance price value above which it is better to install a scrubber (and burn high-sulfur coal), and below which it is better to simply burn high-sulfur coal. It is possible to check that these cut-off values for \( t_0 = 5 \) years and \( t_0 = 10 \) years correspond to the critical value \( A_{cr} \) at 5 and 10 years given by Figure 5-8. This is reasonable since if coal price premiums are very high the option to switch coal is virtually worthless. Also note that if the utility already burns low-sulfur coal, the cut-off allowance value is lower, which means that the utility will install the scrubber sooner than if it burns high-sulfur coal. This is also a reasonable result, since there is a cost to changing compliance policy. Similarly, there is a large region of the diagram for \( t_0 = 10 \) years (around \( A = 1000 \) and \( D = 0.6/\) MMBTU) in which the utility should keep on burning the same fuel.

Note that Figure 5-10 was obtained from the binomial model, which means that for a given \( t_0 \) not all combinations of allowance price and coal price premium are available. This explains why the curve is incomplete for \( t_0 = 5 \) years. It also explains why the curves given are not always very precise. For instance, it was found that the cut-off allowance value for high coal price premiums was between \$1070 and \$1140 for \( t_0 = 10 \) years, rather than exactly \$1100 as predicted by Figure 5-8.

The diagrams of Figure 5-10 and the curves of Figures 5-8 and 5-9 show that the binomial model provides a convenient way to obtain simple investment criteria.
Figure 5-10: Scrubber Investment Criterion in the Base-Case Model
6. CONCLUSIONS

6.1 Scrubber Investment for Compliance with Clean Air Act

This work has analysed the strategies available to a coal power plant that needs to comply with the 1990 Clean Air Act Amendments. It was found that the option to switch fuel was not very valuable for the utility in the base-case model, probably because the relative market prices of high- and low-sulfur coals at equilibrium will likely equal the value of the corresponding allowances saved because of reduced emissions. By contrast, the option to wait before installing the scrubber was found to be important, thus illustrating what Trigeorgis and Mason [125] call the asymmetry of investment decisions. The results show that there is a large difference between the investment value of the scrubber when the flexibility is taken into account ($14.8 million) and the conventionally determined NPV of the scrubber investment at $t = 0$ (-$79.9 million).

The values of both the standard deviation and the convenience yield associated with the $SO_2$ allowance price were found to have a strong influence on the investment value. This result shows the need for a precise description of the main stochastic variable. By contrast, the parameters describing the stochastic behaviour of the coal price premium did not appear to be important at all, and neither did the correlation factor between the two underlying variables. This is obviously related to the low importance for the utility of the option to switch fuel in the base-case model.

As far as the scrubber itself is concerned, O&M costs, sulfur removal rate, and construction delays were found to be the most significant model parameters. The scrubber’s capital costs and salvage value appeared relatively less important here.
The possibility that technology improvements might change the scrubber's future costs and/or performance was illustrated in one of the modified models of Section 5. The future availability of cheaper and more efficient scrubbers was found to increase the value of the option to wait by a factor of two (provided that these advanced scrubbers became available in the following 10 years or so of the power plant life). Finally, most of the scrubber operating flexibilities considered in Section 5 were found to have little impact on the investment present value. This was the case for the possibility of interrupting scrubber operation, and for the possibility of combining scrubber operation and fuel switching.

6.2 Contingent Claims Analysis for the Valuation of Real Flexible Investments

The particular environmental investments discussed in this work suggest a general procedure to determine whether option valuation can be used with a given real investment. They also show how to apply it in practice.

1. It is first necessary to determine the main sources of uncertainty (underlying variables) affecting the investment's future cash flows. In many cases there will only be one or two such sources. In the energy sector the sources of uncertainty might be fuel or allowances prices, and in the manufacturing sector they might be the values of the process's inputs or outputs. If there are additional sources of uncertainty that are uncorrelated with the main ones, it is usually possible to replace them by deterministic variables of properly chosen characteristics (see Section 5 for details).

2. Once it is clear that the investment is uncertain, the main flexibilities available to the investment decision-maker should be determined. Only those flexibilities that are functions of the main sources of uncertainty defined in step 1 are relevant here. In almost every investment situation there will be the flexibility
to delay the investment, and/or prematurely abort it. In some cases the flexibilities to momentarily interrupt the investment, or to modify the investment's cash flows (for instance by changing the investment's inputs or outputs) will also need to be considered. 65

3. Contingent claims analysis will only be applicable for the valuation of the investment in question if the main sources of uncertainty found in step 1 are "spanned" by the existing financial market. This requires that portfolios of existing financial assets can be found that are perfectly correlated with the underlying variables. Otherwise, the no-arbitrage portfolio approach that is used for option valuation may not apply. Also, the existence of a duplicating financial portfolio will simplify the modelling of the underlying asset value over time.

Contingent claims analysis can only be used for the valuation of real flexible investments if steps 1, 2 and 3 above can be successfully completed.

4. After deciding that option valuation is the preferable method for calculating the investment value, it is necessary to choose the stochastic processes that best describe the behaviour (or past behaviours of the duplicating portfolios) and by checking that their future behaviours are unlikely to be substantially different from what they have been in the past (for example, there should not be any fundamental changes expected in the industry). In many cases, modified Wiener processes will be acceptable descriptions of the changes over time of the underlying asset values. This might especially be true if the underlying variable is a stock price. In some cases, however, other processes may be more appropriate, for example for commodities (see Taylor [115]). In all cases great care should be taken to obtain the most accurate stochastic description. For

65 The flexibility to invest in a follow-up project will be discussed below.
example, a reliable estimate of the convenience yields seems to be required for a precise calculation of claims that are contingent on commodity prices.

5. If the problem considered is simple enough (one underlying variable, one or two simple flexibilities available to the decision-maker), or can be simplified without serious loss of realism, it is probably better to adopt a continuous description of time, and to derive the partial differential equation(s) that can be solved for the investment present value (this continuous-time method was presented in Section 3). If the investment is indeed simple enough, it should be possible to obtain an analytical solution to the partial differential equation(s), and hence to get a closed-form expression for the investment value.

6. If the investment cannot be simplified, the continuous-time approach will require a numerical method to solve the partial differential equation(s) for example, the finite difference method described by Press et al. [98]. Since such methods involve approximations and computer calculations, it might be preferable to adopt a discrete-time description of the behaviour of the underlying asset value at the outset, even if a large number of periods per unit time is necessary to approximate a continuous-time behaviour.66

7. If a discrete-time method is chosen, a binomial description of the underlying asset price behaviour over time is recommended, especially if there are one or two underlying assets, and if they follow Wiener processes. The binomial tree of Section 4 can be used for two partially correlated stochastic variables. If there were three partially correlated stochastic variables in the problem it would probably be feasible to build a binomial tree that consists of sets of six consecutive periods. During the first three periods only one asset moves

66 In fact, the underlying asset value is never available in a strictly continuous manner over time anyway.
stochastically (first asset 1, then asset 2, and finally asset 3), and during the last three periods of each set two variables move stochastically together (first assets 1 and 2, then assets 1 and 3, and finally assets 2 and 3). However, it is very likely that one or two of the three underlying variables are either weakly correlated with the other(s), or do not have a strong influence on the investment value. Admittedly, it might be difficult to determine before the calculation whether a given underlying variable has a strong influence on the investment value or not. However, in some cases the economics of the problem may help. In the scrubber case of Section 5, for example, low coal switching costs for some utilities make it likely that coal price premiums will be virtually equal to the value of the allowances saved, thus making the option to switch coal virtually worthless. Overall, it should thus usually be possible to reduce the number of partially correlated underlying variables to one or two.

8. After obtaining the binomial tree that describes the behaviour over time of the underlying asset(s), the various flexibilities available to the decision-maker should be translated into recurrence relationships giving the investment value at a given node of the tree at period \( j \), as a function of the investment value at the two possible following nodes at period \( j + 1 \). In most cases the flexibility itself will appear in these relationships as a maximum or minimum expression of the various possibilities available to the decision-maker at the node considered (see Section 4 for an example).

9. The investment value should then be calculated at all nodes of the binomial tree that correspond to the last period of the investment life. Using the recurrence relationship derived in step 8, it will then be possible to calculate the investment value at all nodes at the penultimate period, then at all nodes the period before, etc., until the investment value at time \( t = 0 \) is obtained.
10. If the investment problem consists in deciding whether and when to make a
certain decision (for example, installing a scrubber), it is sometimes possible to
use the binomial approach to obtain at time $t = 0$ an investment criterion for
making this decision in the future. This should be especially easy if there is
only one underlying variable, and if at each time there is a maximum (or
minimum value of the underlying variable above which (below which) it is
preferable to make the decision in question.

6.3 Advantages of the Binomial Method

The binomial approach used in this work was found to be easy to code, and fairly
intuitive. This makes it particularly well-suited to business applications. Moreover,
the example chosen showed how versatile a method it is. Numerous modifications to
the basic model were made, in order to investigate various operating flexibilities. In
most cases these modifications required only changes in the recurrence relationships
for the investment value, or changes in the investment value at the end of the
investment period. In some cases the changes were more substantial, for example to
calculate the investment value with construction days. But even in this case, the
binomial model describing the behaviour of the two underlying stochastic variables
did not have to be modified. The binomial description of the two partially correlated
Wiener processes that was derived in Section 4 could thus be used for the valuation of
any investment contingent on two partially correlated variables that follow modified
Wiener processes.67

67 Provided that $\rho_{12} < \sigma_1/\sigma_2 < 1/\rho_{12}$, where $\rho_{12}$ is the correlation factor between the processes,
and where $\sigma_1$ and $\sigma_2$ are the standard deviations of the two modified Wiener processes.
6.4 Suggestions for Future Work

Stochastic Convenience Yields
This work has shown the importance of convenience yields for the valuation of options contingent on commodities like fossil fuels. Oil convenience yields have been shown to vary stochastically over time and to be partially correlated with oil spot prices (Gibson and Schwartz [41]). It would be interesting to see if the results can be generalised to other commodities. If this were the case, the binomial model used here would have to be adapted to take this effect into account and see whether it substantially changes the value of the flexibility.

Option Valuation of R&D Investments
Another possible application of contingent claims analysis concerns strategic investments like R&D investments in process or product innovation. Such investments are valuable not in themselves (on their own they are virtually worthless) but for the option they give the decision-maker to invest in future profitable follow-up projects. Even though many studies in the management of technology warn against the dangers of a linear model of innovation (where research leads to development, which in turn lead to production), it might be helpful to think of research investments as options on development investments, and of development investments as options on production investments.

In many cases, however, the underlying asset (for example a new plant producing the new product) may hardly be spanned by existing financial assets. In fact, how could a truly new product be spanned by existing financial assets? Should one instead consider the products it replaces? Or substitute products? Is it reasonable to consider the industry as a whole as a proxy for the duplicating portfolio? Because of such issues, option thinking might turn out to be more helpful to evaluate research
investments that are targeted at incremental innovations, in which case the new product is not too different from the one it replaces. \(^{68}\)

Another issue of potential importance with the valuation of R&D investments is the possibility that game-theory type situations might arise, for example if several competitors hold options on a single asset. This kind of difficulty is less likely to be encountered with development investments than with research investments, and less with process innovation investments than with product innovation investments, simply because potential profits are more easily made proprietary in the former cases than in the latter. Thus, even if option valuation for strategic investments like R&D investments is quite clear in principle, much research remains to be done to show that it can actually be used in practice.

The same can be said for environmental investments. Public demand for environmental protection is likely to increase in the future, and tradable emission systems are likely to become a standard approach for environmental regulation. They will give companies flexibility in the choice of their compliance strategies. Option valuation is therefore likely to be useful for the valuation of environmental investments in the future. Its usefulness may in fact go beyond the energy sector, and beyond environmental concerns. Dertouzos et al. [27] recommend that American industrial companies "emphasise product variety and manufacturing flexibility in the development of production systems". Contingent claim analysis may, in some cases, offer an interesting way to support quantitatively this recommendation.

\(^{68}\) However, the investment in this case may be quite small.
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