Flutter Boundary Prediction Using Experimental Data

by

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Abstract

Flight Flutter testing is an expensive and dangerous process, for the aircraft has to be flown close to the flutter boundary to get the reliable estimates of the actual flutter boundary. A new method that allows to obtain the flutter boundary estimates using experimental data taken at safe flight conditions is presented. The method employs both the analytical model of the aircraft’s flexible dynamics and the experimental measurements. The method proceeds as follows: 1) the time-frequency analysis method is used to obtain the transfer functions estimates. The structural modes observed in the experimental data are identified using the understanding of the aircraft’s physics; 2) the analytical model of the aircraft’s flexible dynamics is fitted with the experimental data, and the fitted model is used to obtain the flutter boundary prediction. The method is applied to the wing test article and to the F-18 System Research Aircraft. Flutter boundary predictions are computed for both cases.

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## Contents

1 Introduction ............................................. 15
   1.1 Motivation ........................................... 15
   1.2 Outline of the Thesis ................................. 17

2 A Method For the Transfer Function Estimation and Its Validation 21
   2.1 Motivation ............................................. 21
   2.2 Description of The Method ............................. 22
      2.2.1 Time-frequency Analysis of the Signal .......... 22
      2.2.2 Transfer Function Estimation Using Time-frequency Analysis 24
      2.2.3 Transfer Function Estimate Statistics .......... 26
   2.3 Validation of Time-frequency Analysis Method for Transfer Function
      Estimation Using Numerical Example .................. 29
      2.3.1 Bias and Variance Trade-off .................... 32
      2.3.2 Comparison with Other Existing Methods ........ 33
   2.4 Application to F18-SRA Structural Dynamics Identification .... 38

3 Fit of the Analytical Model Using the Experimental Data ............ 43
   3.1 Introduction ........................................... 43
   3.2 Proposed Method for the Fit of the Model and Flutter Boundary Pre-
      diction .................................................. 44
   3.3 Experimental Investigation ............................. 48
      3.3.1 Description of the Wing Test Article ............ 48
      3.3.2 Experimental Data Taken on the Wing ............ 48
3.3.3 The Theoretical Model for The Test Article ............... 49
3.3.4 Model Reduction ........................................ 49
3.3.5 The Compression Of The Available Data .................. 50
3.3.6 Fit of the Model and the Improved Flutter Boundary Prediction 51
3.4 Application of the Method to the F-18 SRA ................... 54
3.4.1 Analytical Model for the F-18 SRA ...................... 54
3.4.2 Data Analysis ............................................. 55
3.4.3 Analytical Model Fit ..................................... 63
3.4.4 The Flutter Boundary Prediction ........................ 65

4 Conclusions .................................................. 71
List of Figures

1-1 DEI exciter mounted on F18-SRA left wing. ......................... 17
1-2 The proposed flutter boundary prediction method. ............... 18

2-1 Bank of filters used in Time-frequency analysis. ............... 23
2-2 Time and frequency contents of F18-SRA right input (top) and left forward output (bottom). ................................. 24
2-3 Time-frequency analyses of right DEI exciter force and left forward wing tip accelerometer. Flight 0549, Mach=0.8, Altitude=40 000 feet. 25
2-4 Time-frequency analyses of left forward wing tip accelerometer and the denoised time-frequency decomposition. Flight 0549, Mach=0.8, Altitude=40 000 feet. ........................ 26
2-5 The windowing of input and output signals. ..................... 27
2-6 Transfer function for numerical example. ....................... 30
2-7 Top: Time-frequency analyses for nominal, noise-free input and output. Bottom: Time-frequency analysis and “denoised” time-frequency analysis of output for noise PSD = 0.5. ..................... 31
2-9 Windowing procedure for trade-off analysis. ..................... 33
2-10 RMS of transfer function estimates as a function of window size. Noise PSD is 0.5. Top: Estimate at 6.64 Hz. Bottom: Estimate at 7.42Hz. 34
2-11 The performance of different methods for the five realizations of noise with PSD of 0.5. From left to right, top to bottom: Time-frequency procedure; time-frequency analysis + frequency-domain subspace identification; simple Fourier analysis (just on realization is shown); Fourier analysis + frequency-domain subspace identification; Prediction Error Method; Welsh's periodogram with 1024 point window.


2-13 Transfer function estimates from 6 different data sets. Right exciter to forward left wingtip sensor. Fourier analysis.

2-14 Transfer function estimates. Right exciter to forward left wingtip sensor. For each of the identification methods 6 transfer function estimates from 6 different data sets are plotted on the same plot.

3-1 Original and reduced model: natural frequencies.

3-2 Original and reduced model: damping ratios.

3-3 Fit of the experimental transfer function by a state-space model.

3-4 Fit at three values of dynamic pressure: natural frequencies and damping ratios.

3-5 Fit at two values of dynamic pressure: natural frequencies and damping ratios.

3-6 Fit at one value of dynamic pressure: natural frequencies and damping ratios.

3-7 The first symmetric wing bending mode. Frequency range: 5.5 Hz - 7.5 Hz.

3-8 The first antisymmetric wing bending mode. Frequency range: 8 Hz - 10 Hz.
3-9 The first antisymmetric wing torsion mode. Frequency range: 15 Hz -17 Hz. ................................................. 57

3-10 The first symmetric wing outer torsion mode. Frequency range: 18 Hz - 20 Hz. ................................................. 58

3-11 Damping ratio estimation procedure for the flight data at Mach=0.8, 40 000 feet. (flight # 0549). Transfer functions shown are from left and right exciters to the left wing aft accelerometer. Dashed lines show the half power points. The dotted lines show the frequency of the peak. 59

3-12 Average values of the natural frequencies and damping ratios for the wing symmetric first bending mode. ................................. 59

3-13 Average values of the natural frequencies and damping ratios for the wing first antisymmetric bending mode. ................................. 60

3-14 Average values of the natural frequencies and damping ratios for the wing first antisymmetric torsion mode. ................................. 60

3-15 Average values of the natural frequencies and damping ratios for the wing symmetric outer torsion mode. ................................. 61

3-16 Natural frequencies and damping ratios of the four identified modes at Mach 0.8, 40 000 feet. Shown are the mean ± standard deviation intervals. Mode numbers: 1 - symmetric first wing bending; 2 - antisymmetric wing first bending; 3 - antisymmetric wing first torsion; 4 - symmetric wing outer torsion mode. ................................. 61

3-17 Natural frequencies and damping ratios of the four identified modes at Mach 0.8; 30 000 feet. Shown are the mean ± standard deviation intervals. Mode numbers: 1 - symmetric first wing bending; 2 - antisymmetric wing first bending; 3 - antisymmetric wing first torsion; 4 - symmetric wing outer torsion mode. ................................. 62
3-18 Natural frequencies and damping ratios of the four identified modes at Mach 0.8; 10 000 feet. Shown are the mean ± standard deviation intervals. Mode numbers: 1 - symmetric first wing bending; 2 - antisymmetric wing first bending; 3 - antisymmetric wing first torsion; 4 - symmetric wing outer torsion mode. 

3-19 Estimated natural frequencies and damping ratios of the first symmetric wing bending mode at different altitudes. Shown are the mean ± standard deviation intervals. 

3-20 Estimated natural frequencies and damping ratios of the first antisymmetric wing bending mode at different altitudes. Shown are the mean ± standard deviation intervals. 

3-21 Estimated natural frequencies and damping ratios of the first antisymmetric wing torsion mode at different altitudes. Shown are the mean ± standard deviation intervals. 

3-22 Estimated natural frequencies and damping ratios of the symmetric wing outer torsion mode at different altitudes. Shown are the mean ± standard deviation intervals. 

3-23 Changes in the natural frequencies and damping ratios of the structural modes for Mach 0.8 in the symmetric case. 

3-24 Changes in the natural frequencies and damping ratios of the structural modes for Mach 0.8 in the antisymmetric case. 

3-25 Results of the fit for the first antisymmetric wing bending mode. Shown are the experimental data (mean ± standard deviation interval) and the natural frequencies and damping ratios from the original and fitted models at different altitudes. 

3-26 Results of the fit for the first antisymmetric wing torsion mode. Shown are the experimental data (mean ± standard deviation interval) and the natural frequencies and damping ratios from the original and fitted models at different altitudes.
3-27 Changes in the natural frequencies and damping ratios of the structural
modes for Mach 0.8 in the antisymmetric case after the fit. ........ 69
# List of Tables

2.1 Performances of different identification techniques on F18-SRA experimental data. ........................................ 42

3.1 Estimates of natural frequencies and damping ratios from the experimental data. ........................................ 51

3.2 Flutter boundary predictions from the fitted model. ........................................ 52

3.3 Symmetric and antisymmetric natural modes of the F-18 SRA and the corresponding modal frequencies. ........................................ 54

3.4 Results of the fit of the model. ........................................ 65

3.5 Comparison of the flutter boundaries from original and fitted analytical models. ........................................ 70
Chapter 1

Introduction

1.1 Motivation

Flutter is a dynamic instability, caused by the interaction of the structural dynamics of the aircraft and unsteady aerodynamic forces [1]. The theoretical mechanism of flutter is studied in detail in numerous references [2, 3, 4]. From a practical point of view flight flutter testing needs to be performed whenever a new aircraft is built or an existing aircraft is modified. The procedure of determining that the aircraft is free from flutter in the specified range of velocities and altitudes is called flight envelope clearance. Flight envelope clearance is an expensive and time consuming procedure, requiring a large number of flight tests. The current trend indicates that more and more flight time is required to get the flight envelope of an aircraft cleared [5]. For example, 489 tests were required to clear basic flight envelope for the F-14 aircraft, 278 were needed to clear the flight envelope of the F-15 [5]. Thus, more efficient and reliable methods of flutter testing could save a lot of flight time and correspondingly, money.

Flutter occurs when one of the elastic modes of the aircraft becomes unstable. The most reliable and commonly used indicator of the aircraft’s proximity to flutter is the damping ratio of the flexible modes of the aircraft. The standard technique for flutter testing has been to obtain the estimates of damping ratios of structural modes of the aircraft at several flight conditions and to extrapolate the obtained data [5].
The flutter boundary is then determined as a value of dynamic pressure for which the extrapolated curve of damping ratio crosses zero. However, the damping ratios are highly nonlinear in dynamic pressure, $\bar{q}$. Thus, to obtain reliable estimates of the flutter boundary, the aircraft should be flown close to the flutter boundary [5]. This is undesirable, because it can lead to the loss of pilot and the aircraft. New methods for the flutter boundary prediction are needed that will allow to predict the flutter boundary using the measurements taken at safe flight conditions, at low values of dynamic pressure.

An estimate of the flutter boundary can be obtained from the methods that solve the flutter equations of motion. The p-k iteration method is one of these methods [6]. The disadvantage of these methods is that they do not use experimental flight data, and thus can be used only as a mean to obtain a preliminary flutter boundary estimates.

The flutter testing poses problems in several areas. During the flight tests the aircraft’s flexible dynamics are excited by the input signal. The response of the aircraft is then measured and analyzed. The flutter boundary is determined using the measured input and output signals. Thus, there are three important areas in flutter testing:

First, the excitation to the system is important. The amplitude of the excitation signal should be high enough in order for the output signals to provide accurate information about the aircraft behavior. There are many excitation techniques used in flutter testing. These techniques include atmospheric turbulence, control surfaces pulses, stick raps, pyrotechnic bonkers [7, 5, 8, 9]. Recently, however, a large number of flight tests has been performed using the excitation system produced by Dynamic Engineering Incorporated. This excitation system uses wing tip mounted rotating slotted cylinders that create a force acting on the structure of the aircraft. This excitation system was tested at NASA Dryden [7] and was used in many envelope clearance programs [10, 11, 12]. Figure 1-1 shows the excitation system.

Second, the response of the structure of the aircraft to the excitation signal should be analyzed to extract the information about the natural frequencies and damping ra-
tios of the aircraft. This is often a challenging task, due to the proximity of structural modes in frequency domain. A variety of techniques exists to perform this analysis. A detailed review of a number of these techniques can be found in [13].

The third part of flutter testing is flutter boundary extrapolation from the analyzed data. Reference [13] describes some of the techniques that allow to do that.

1.2 Outline of the Thesis

This thesis presents the method that allows one to obtain the flutter boundary estimates, using the flight measurements taken at low values of dynamic pressure. The method consists of two major steps:

1) Processing and reduction of the experimental data.

2) Fit of the analytical model according to the experimental data and flutter boundary prediction from the fitted model.

Figure 1-2 shows the flutter boundary prediction method graphically.

The method uses an approach proposed in [14, 15, 16] to perform the first step. This approach allows one to obtain transfer function estimates from time-domain data. The method then uses the obtained transfer function estimates to produce the estimates of natural frequencies and damping ratios of the aircraft. The completion
of the second step - the fit of the analytical model, makes possible to extrapolate the natural frequencies and damping ratio estimates to predict the flutter boundary. This extrapolation allows one to obtain the flutter boundary estimates from the data collected at low dynamic pressure values.

The second chapter of the thesis describes the first step of the algorithm: the method for the transfer function estimation is presented. The validation of the method and comparison with other available methods using a numerical simulation and experimental F-18 SRA data is described.

In the third chapter the second step of the algorithm is described. Application to the wind tunnel entry is shown: refined flutter boundary estimates are obtained and are compared with the flutter boundaries determined experimentally. The application of the flutter boundary prediction algorithm to the F-18 System Research Aircraft is also presented. The analytical model for the F-18 SRA is described. The physics of the aircraft are used to establish the correspondence between the observed modes in the experimental transfer functions and the modes in the analytical model. The
analytical model of the F-18 SRA is fitted with the experimental data and the flutter boundary prediction is obtained from the fitted model.

Conclusions are drawn in Chapter 4.
Chapter 2

A Method For the Transfer Function Estimation and Its Validation

2.1 Motivation

As pointed out in Chapter 1, there are several steps in the process of flight flutter testing. The first step is to process and to reduce the data from the experimental flight measurements. This processing amounts to performing system identification on the experimental data. It can be done either in the form of transfer function estimation or in the form of derivation of the state space model that fits the data. The method described in this chapter is concerned with the estimation of the transfer functions of the system from the available data. The transfer function estimates can be used later to obtain the natural frequencies and damping ratios of the structural modes of the aircraft, as it will be shown in Chapter 3.

The method described in this chapter is applicable to flight tests where the frequency sweep excitation signal (chirp signal) is used. This kind of excitation signal often is used in flutter flight testing [10, 7, 11, 12]. The particular structure of the excitation signal can be exploited to perform “denoising” of the signal and to obtain
the transfer functions estimates from the “denoised” signals.

2.2 Description of The Method

The method for obtaining the transfer function estimates was developed and described in [14, 15, 16]. The idea of the method amounts to using the time-frequency representation of the signal to perform “denoising”. The frequency of the chirp signal is changing with time. Thus, a certain frequency is present in the chirp signal only at a certain period of time. Thus, a useful signal can be distinguished from noise, if this property of chirp signal is used.

2.2.1 Time-frequency Analysis of the Signal

Following the notation in [16], the excitation signal to the system is a frequency sweep:

\[ u(t) = A \sin(\omega(t)t + \psi) \]  \hspace{1cm} (2.1)

A discrete time, final length signal \( u_n \) is obtained by sampling:

\[ u_n = u(n\delta T) \]  \hspace{1cm} (2.2)

Function \( \omega(t) \) in (1) can be a linear or a logarithmic function of time. For example, in case of linear frequency sweep:

\[ \omega(t) = \dot{\omega}t + \omega_0 \]  \hspace{1cm} (2.3)

A bank of filters \( b_{\omega_0} \) is defined now as:

\[ b_{\omega_0}(\omega) = \hat{b}(\omega - \omega_0) \]  \hspace{1cm} (2.4)

Figure 2-1 shows the typical bank of filter used in time-frequency analysis.
Here, \( \hat{b} \) is a Discrete Time Fourier Transform [17] of passband filter template \( b \).
Now, define the time-frequency decomposition $\mathcal{T}\mathcal{F}$ of the signal $u$:

$$
\mathcal{T}\mathcal{F}(u, \omega_0, p) = \sum_{n=-\infty}^{\infty} u_n b_{\omega_0, p-n}
$$

$$
= (b_{\omega_0} * u)(p). \hspace{1cm} (2.5)
$$

This decomposition maps a 1-dimensional representation of the signal $u$ in the time domain to a 2-dimensional representation of the same signal in time and frequency domains.

The template filter was chosen to be a Morlet wavelet [18]:

$$
\hat{b}(\omega) = e^{-\lambda \omega^2} \hspace{1cm} (2.6)
$$

Parameter $\lambda$ defines the bandwidth of the template filter $b$. The bandwidth of the filter is kept constant for all filters in the filter bank.

Figure 2-2 shows the time and frequency contents of typical F18 SRA input and
output signals.

Figure 2-3 shows the 2-D and 3-D plots of time-frequency decompositions of the same signals.

2.2.2 Transfer Function Estimation Using Time-frequency Analysis

Figure 2-3 shows that the useful part of the signal can be separated from the noisy part of the signal in the time-frequency domain. More specifically, a straight dark line in the time-frequency decompositions of input and output signals corresponds to a linear frequency sweep, used as an input signal to the system. The time-frequency content of the signal at other frequencies at other times is then due to noise or the “secondary” harmonics. This interpretation suggests the following denoising procedure in time-frequency domain:
1) Perform a time-frequency decomposition of input and output signals to the system

2) Define a mask in time-frequency domain; keep the coefficients of the time-frequency decomposition as they are inside the mask, and set them to zero outside of the defined region

3) For each frequency point of interest, compute the transfer function estimates from the “denoised signals”

Figure 2-4 shows the time-frequency decomposition of the left forward tip accelerometer output signal for the F-18 SRA and the denoised time-frequency decomposition.

The described method for transfer function estimation makes possible to eliminate the noise in the signals that enters into the signals at the frequency and at the time different from the frequency and the time of a “useful” signal.
Figure 2-4: Time-frequency analyses of left forward wing tip accelerometer and the denoised time-frequency decomposition. Flight 0549, Mach=0.8, Altitude=40 000 feet.

2.2.3 Transfer Function Estimate Statistics

The procedure in the previous section can be understood as filtering the signal at the desired frequency and then windowing it. This procedure involves a trade-off between the amount of noise removed from the estimates and the amount of useful signal lost due to windowing. If the window that is used is too narrow then this will introduce a bias in the identification procedure. If the window is too large, then noise will be present, increasing the variance of the transfer function estimate.

Thus, the analysis of the trade-off should be done. For simplicity purposes, the system is assumed to be single-input, single-out (SISO). Both process and sensor noise are assumed to be independent white noise, such that the general input-output relationship in this case may be written

\[ y = h \ast u + h_w \ast w + v \]

(2.7)

\[ = y_{\text{nom}} + h_w \ast w + v, \]

where \( w \) and \( v \) are the process and sensor noise respectively. \( h_w \) is an additional transfer function from process noise to output and is a priori unknown.

- **Bias:**

Assuming that the noises \( w \) and \( v \) have zero mean, and in the absence of any prior knowledge of \( h \) and \( h_w \), the best estimate \( \hat{H} \) of transfer function \( H \) at a
given frequency $\omega_i$

$$\tilde{H} = \frac{Y_{\text{win}}}{U_{\text{win}}}$$  

(2.8)

In this equation, $Y_{\text{win}}$, $U_{\text{win}}$ are Discrete Time Fourier Transforms of the win-
dowed output and input signals correspondingly:

$$u_{\text{win}}[n] = \begin{cases} u[n], & n \in N_1 : N_2 \\ 0, & \text{otherwise.} \end{cases}$$  

(2.9)

Figure 2-5 shows the procedure of windowing.

$$y_{\text{win}}[n] = \begin{cases} y[n], & n \in N_1 : N_2 \\ 0, & \text{otherwise.} \end{cases}$$  

(2.10)

Since $w$ and $v$ are white, the expected value $E(\tilde{H})$ of $\tilde{H}$ is

$$E(\tilde{H}) = \frac{Y_{\text{nom,win}}}{U_{\text{win}}}$$  

(2.11)
The bias is then

\[
E(\tilde{H} - H) = \frac{Y_{nom,win}/U_{win} - Y_{nom}/U}{U_{win}}
\]

\[
= Y_{nom,win}/U_{win} - Y_{nom}/U + \frac{Y_{nom}(U_{win}-U)}{U_{win}}.
\]

(2.12)

Thus, the bias is the sum of two terms proportional to \(Y_{nom,win} - Y_{nom}\) and \(U - U_{win}\) respectively. Both terms get smaller as \(N_1\) gets smaller and \(N_2\) gets larger.

- **Variance:**

The variance \(\sigma\) of the estimate \(\tilde{H}\) satisfies by definition

\[
\sigma^2 = E\left( (\tilde{H} - E(\tilde{H}))(\tilde{H} - E(\tilde{H}))^* \right).
\]

(2.13)

In this case,

\[
\sigma^2 = E\left( (\tilde{N}_2 + H_w\tilde{N}_1)/U_{win} \right) ( (\tilde{N}_2 + H_w\tilde{N}_1)/U_{win})^*).
\]

(2.14)

Since the two noises \(w\) and \(v\) are uncorrelated and have zero mean, we obtain

\[
\sigma^2 = E\left( (\tilde{N}_2\tilde{N}_2^* + ||U_{win}||^2) + E((H_w\tilde{N}_1)(H_w\tilde{N}_1)^*/||U_{win}||^2),
\]

(2.15)

with

\[
E(\tilde{N}_2\tilde{N}_2^*) = E\left( \sum_{k=N_1}^{N_2} \sum_{l=N_1}^{N_2} n_2(k)n_2(l)e^{-i\omega_0(k-l)} \right)
\]

\[
= \sum_{k=N_1}^{N_2} \sum_{l=N_1}^{N_2} E(n_2(k)n_2(l))e^{-i\omega_0(k-l)}
\]

\[
= \sum_{k=N_1}^{N_2} \sum_{l=N_1}^{N_2} \delta(k - l)e^{-i\omega_0(k-l)}
\]

\[
= \sum_{k=N_1}^{N_2} N_{20}
\]

\[
= (N_2 - N_1)N_{20},
\]

(2.16)

where \(N_{20}\) is the power spectral density of \(n_2\). Similarly, we obtain

\[
E((H_w\tilde{N}_1)(H_w\tilde{N}_1)^*) = (N_2 - N_1)|H_w|^2 N_{10},
\]

(2.17)
where $N_{10}$ is the power spectral density of $w$.

Therefore, we obtain

$$\sigma^2 = \frac{(N_2 - N_1)(N_{20} + GN_{10})}{\|U_{win}\|^2}. \quad (2.18)$$

This term is proportional to $N_2 - N_1$ and inversely proportional to $U_{win}$. As a result, $\sigma^2$ tends to grow large as $N_1$ tends to 0 and $N_2$ grows. Conversely, if $N_2 - N_1$ gets too small, then $U_{win}$ may also tend to 0, and the behavior of $\sigma$ tends to a finite value.

### 2.3 Validation of Time-frequency Analysis Method for Transfer Function Estimation Using Numerical Example

To validate and compare the proposed method with the existing methods for transfer function estimation, a numerical example was tried: A simple, fourth order system, with the transfer function matching the transfer function from the right input to the left forward accelerometer for the F-18 SRA was used.

The transfer function of the system is:

$$H(s) = \frac{-200((s)^2 + 2s * 0.05 * 50.26 + (50.26)^2)}{(s^2 + 2s * 0.02 * 40.85 + 40.85^2)(s^2 + 2s * 0.02 * 56.56 + 56.56^2)}. \quad (2.19)$$

This system has two lightly damped poles at 6.5 and 9 Hz and an imaginary zero at 8 Hz. The transfer function of the system is shown in Figure 2-6.

The system was simulated for 30 seconds, using 200 Hz sampling frequency. The input that was used was a linear frequency sweep signal:

$$u = 1.5 \sin(2.51t^2) \quad (2.20)$$
Figure 2-6: Transfer function for numerical example.

The output of the system is then:

\[ y = h \ast u + w, \]  

(2.21)

where \( w \) is a discrete white Gaussian sensor noise.

The system was first simulated with noise levels equal to 0 and 0.5. The resulting time-frequency analyses of both input and output signals are shown in Figure 2-7.

As may be seen in Figure 2-7, the presence of noise significantly perturbs the time-frequency representation of the output signal. However, the output is still clearly visible. Based on this data only, it is therefore possible to "denoise" the output. In practice, this is done using a graphical display of the time-frequency analysis and by drawing a closed polygon around the area of greatest interest. The resulting "denoised" time-frequency representation is also given in Figure 2-7. Figure 2-8 shows the obtained estimated transfer function using the noise removal procedure, along with the transfer function estimates based on straight Fourier analysis. Considerable improvement may be seen.
Figure 2-7: Top: Time-frequency analyses for nominal, noise-free input and output. Bottom: Time-frequency analysis and “denoised” time-frequency analysis of output for noise PSD = 0.5.

Figure 2-8: Transfer function estimates. Left: Solid: Proposed approach. Dashed: actual transfer function. Right: Estimate based on Fourier analysis. Noise PSD is 0.5.
2.3.1 Bias and Variance Trade-off

For moderate noise levels, the previous paragraph shows that time-frequency analysis combined with appropriate graphical interfaces allows the user to isolate the part of the input and output signals that bear the most information. However, this procedure is subjective and based on visual signal evaluation only. In particular, it is important to demonstrate that the subjective choices actually are supported by objective facts. As shown in section 2.2.3, choosing a "narrow" polytope to clean both inputs and output signals introduces a bias in the transfer function estimate, because of excessive input and output signal truncation, even in the absence of noise. Conversely, choosing a "wide" polytope increases the variance of the transfer function estimate $\tilde{H}$, by allowing too much noise to enter the system. In general, the user wishes to find an optimal trade-off between these two difficulties by minimizing, for example, the RMS transfer function deviation, given by

$$RMS^2 = \text{bias}^2 + \text{variance}^2.$$  

This issue is explored at two specific frequencies, 6.64 Hz and 7.42 Hz. These frequencies were chosen because they correspond to resonant and non-resonant frequencies respectively, as shown in Figure 2-6. The trade-off study was performed as follows: For any frequency $\omega$, consider a time window centered around the maximum amplitude of the filtered output $\mathcal{T}\mathcal{F}(y, \omega)$, and the corresponding truncated signals $\mathcal{T}\mathcal{FW}(y, \omega)$ and $\mathcal{T}\mathcal{FW}(u, \omega)$, as shown in Figure 2-9.

The trade-off study looks at the transfer function variance and bias versus the window size. The RMS of the estimated transfer function is easy to compute either experimentally or via analytical calculations and is plotted as a function of the size of the window in Figure 2-10. The window sizes that yield the smallest RMS are 5.87 sec for the transfer function estimate at 6.64Hz and 2.33 sec at 7.42Hz. A look at Figure 2-7 confirms that the graphical "cleaning" procedure approximately follows these indications, with larger windows being used for resonant frequencies and narrower windows for nonresonant frequencies.
2.3.2 Comparison with Other Existing Methods

Many methods exist to identify dynamical systems. The goal of this paragraph is to compare the relative performance of the estimation procedure based on time-frequency analysis with these methods. It is worthy to note that such comparisons are only indicative of the value of the proposed method. The benchmark identification methods have been chosen from three distinct groups:

- **Direct estimation methods:**
  These comprise direct transfer function evaluation by using Welch’s average periodogram method in [17, 19] and implemented in the MATLAB signal processing toolbox (spectrum command). The proposed time-frequency analysis described in this paper naturally falls in this category since it may be indeed seen as a smart windowing technique. For the purpose of benchmarking, Welch’s averaged periodogram method was used with overlapping rectangular windows of full size (which is the classical Fourier analysis) and 1024 points, respectively.

- **Parametric estimation methods**
  These methods are described in detail in the book by Ljung [20] and comprise very well-known methods such as ARMA [21] or Prediction-Error Methods. In
Figure 2-10: RMS of transfer function estimates as a function of window size. Noise PSD is 0.5. Top: Estimate at 6.64 Hz. Bottom: Estimate at 7.42 Hz.
this thesis, the MATLAB implementation of the Prediction-Error Method was used [22]. The order of the system was chosen to be 4.

- **Subspace identification methods**

  These methods have arisen recently and are applicable both in the time-domain [23] and in the frequency domain [24]. In this thesis, the N4SID algorithm presented in [23] is used. The frequency-domain algorithm presented in [24] is also used. It takes as an input either the estimated transfer functions obtained with simple Fourier analysis or with the proposed time-frequency based approach. The latter combination is meant to illustrate how the proposed method may be combined with existing identification procedures.

  The numerical system shown in Figure 2-6 was simulated using the previously defined input and a discrete white sensor noise of amplitude 0.5. The corresponding transfer function estimates are shown in Figure 2-11. It may be seen that the time-frequency analysis gives accurate information about transfer function magnitudes and resonant frequencies. Combining the time-frequency approach with the frequency-domain subspace identification method also yields good results, although not as good. Note however this estimate provides a finite-dimensional representation of the system, which may be useful for several applications, including control. Note also, that the Prediction Error Method gives accurate transfer function estimates. Note that N4SID is absent from the comparative plots because the results it provided were very inaccurate.

  A more quantitative performance evaluation was performed as follows: Five output noise power spectral densities (0.1, 0.5, 1, 1.5 and 2) were chosen. For each power spectral density, five numerical simulations were performed with different noise realizations. Each of these data was used to identify transfer functions using the all of the above-mentioned identification methods. The identification accuracy was then evaluated with the following normalized performance measure, covering the frequency range from 5 to 12 Hz:
Figure 2-11: The performance of different methods for the five realizations of noise with PSD of 0.5. From left to right, top to bottom: Time-frequency procedure; time-frequency analysis + frequency-domain subspace identification; simple Fourier analysis (just on realization is shown); Fourier analysis + frequency-domain subspace identification; Prediction Error Method; Welsh’s periodogram with 1024 point window.

\[
J = \frac{\sqrt{\int_{\omega=10\pi}^{24\pi} (\hat{H}(j\omega) - \hat{H}(j\omega))(\hat{H}(j\omega) - \hat{H}(j\omega))^* d\omega}}{\sqrt{\int_{\omega=10\pi}^{24\pi} \hat{H}(j\omega)\hat{H}(j\omega)^* d\omega}},
\]

(2.22)

where \( \hat{H} \) the original (known) transfer function and \( \hat{H} \) is transfer function estimate. The cost \( J \) was calculated for all 5 simulations and averaged. The resulting relative performances are shown in Figure 2-12.

The reader can see that time-frequency analysis and the prediction error method obtain similar performance over noise ranges of practical interest.
2.4 Application to F18-SRA Structural Dynamics Identification

The F-18 SRA is the aircraft that NASA Dryden Flight Research Center uses for several flight testing programs. Optical sensing, new actuation concepts, smart structures, and advanced airdata and flight control systems were tested on this aircraft [10].

The proposed time-frequency analysis is now illustrated using experimental data from the F18-SRA, as it was done in [16]. For all practical purposes, the F18-SRA may be considered as a 2-input, 4-output system, where the only informative outputs are the forward and aft wing tip accelerometers. In this section, we are interested in determining the transfer functions from left and right exciters to the forward left wingtip accelerometer, over the frequency range spanning from 5 to 12 Hz.

Six independent experimental measurements are available for the F18-SRA flying at 40,000 feet and Mach 0.8. These data consisted of ascending and descending frequency sweeps. Each set of experimental data consisted of a symmetric and an antisymmetric run (whereby wingtip actuators were excited either in phase or out-of-phase). These experimental data were used separately and the obtained results were cross-checked. The goal of this paragraph is to compare the relative performance of the estimation procedure based on time-frequency analysis with other methods. The data were detrended prior to any further processing. The methods were applied to both unfiltered and filtered (in the 5-12Hz range) data and only the best of the two respective results were recorded.

It is, however, worthy to note that such comparisons are only indicative of the value of the proposed method. The same benchmark identification methods as for numerical simulation section have been chosen:

- Direct estimation methods:

For the purpose of benchmarking, only full size windows (corresponding to classical Fourier analysis) were used. Smaller sized windows[19] yielded no coherent results and were therefore discarded.
- **Parametric estimation methods:**

  These methods did not easily handle multiple data sets (corresponding to separate symmetric and antisymmetric runs). They were therefore discarded for this specific application.

- **Subspace identification methods**

  For the frequency-domain subspace identification procedure, and following the notation in McKelvey *et al.* [24], the parameters were set as follows: system order equal to 4, number of block rows $i$ equal to 10. This subspace identification algorithm uses transfer function samples as input and produces a state-space model. The chosen transfer functions were from the time-frequency based estimation procedure and the classical Fourier analysis, respectively.

  For the multivariable, time-domain subspace identification procedure N4SID (available on MATLAB [25]), the F18-SRA experimental data spanned a larger frequency range than necessary. As a consequence, the presence of resonant modes outside this range resulted in an increased number of states. The “best” system order was determined to be 14. In addition, and following standard notation [23], the number of block rows $i$ was chosen to be 20. N4SID had to be modified to account for the presence of multiple data sets (symmetric and antisymmetric runs) [26].

  The time-frequency estimation procedure relied on experimental data provided by NASA-Dryden exclusively. No *a priori* model of the aircraft’s flexible dynamics was used. Each of the six data sets was de-noised following the procedures explained earlier. Denote by $\hat{h}_{l,i}$ and $\hat{h}_{r,i}$, $i = 1, \ldots, 6$ the estimated transfer functions from left and right inputs to left forward sensor, respectively. For each of the transfer function estimation methods, the six transfer function estimates $\hat{h}_{r,i}$, $i = 1, \ldots, 6$ were obtained and are plotted in Figs. 2-13, 2-14.

  Similar plots were obtained for $\hat{h}_{l,i}$, $i = 1, \ldots, 6$. The quality of transfer function estimates may be evaluated visually and reveals no important difference between
the results obtained using the time-frequency analysis, time-frequency analysis completed with subspace identification as presented in McKelvey et al. [24] and N4SID. One may notice, however, that transfer function phases are more repeatable for the time-frequency based approaches. These three estimates clearly outperform straight Fourier analysis and Fourier analysis combined with frequency-domain subspace identification.

A quantitative measure of identification performance was performed as follows: Following recommended practices [20], the transfer function evaluated for each data set was tested against the other five data sets. Specifically, for each of the six data sets, the Fourier transform of the left and right DEI exciter inputs $u_{l,i}$ and $u_{r,i}$ and the forward left wingtip accelerometer output $y_i$, $i = 1, \ldots, 6$ were computed over the frequency range from 5 to 12 Hz.

The ability of a given transfer matrix $[\hat{h}_{l,i}, \hat{h}_{r,i}]$ to repeat a specific data set $u_{l,j}$, $u_{r,j}$, $y_j$ is quantified by the cost function

$$J_{ij} = \sqrt{\int_{\omega=10\pi}^{24\pi} \left[ \hat{y}_j(\omega) - \left[ \hat{h}_{l,i}(\omega) \hat{h}_{r,i}(\omega) \right] \left[ \hat{u}_{l,j}(\omega) \hat{u}_{r,j}(\omega) \right]^T \right]^2 d\omega. \quad (2.23)$$

Figure 2-13: Transfer function estimates from 6 different data sets. Right exciter to forward left wingtip sensor. Fourier analysis.
Figure 2-14: Transfer function estimates. Right exciter to forward left wingtip sensor. For each of the identification methods 6 transfer function estimates from 6 different data sets are plotted on the same plot.
Identification procedure | Cost
--- | ---
Proposed approach | 1.9039
Fourier analysis | 2.9799
Proposed approach and freq. subspace ID | 1.9815
Fourier and freq. subspace ID | 2.9355
Time-domain subspace ID | 1.9927

Table 2.1: Performances of different identification techniques on F18-SRA experimental data.

For each optimization method, the following average performance index

\[
J = \frac{1}{30} \sum_{i=1}^{6} \sum_{j=1, j \neq i}^{6} J_{ij}
\]  

was computed and reported in the table 2.1.

It may be seen that the proposed approach performs best by this measure of performance. Note however again that such comparisons are necessarily subjective. A more constructive message is that time-frequency analysis can be included with and benefit to any of the above-mentioned methods, as has been shown for the case of frequency-domain, subspace identification.
Chapter 3

Fit of the Analytical Model Using the Experimental Data

3.1 Introduction

The problem of system identification amounts to finding a model that reproduces a given set of the measured outputs of the system in response to a given set of input signals in the best way. There are two classes of system identification: input-output identification and physical identification [27]. Input-output identification is concerned with reproducing the behavior of the system without specific limitation on the model that is used to represent the system. On the other hand, the physical identification problem is about finding more accurate values of the physical parameters in an already available model of the system.

An example of physical identification is a problem of obtaining more accurate values of structural matrices in finite element models. Another example is ground testing for a space structure. In the latter case, the physical identification is used to obtain the correct model of a system in a certain environment, using the measurements taken in that environment, with a goal to use the obtained model in a different environment.

The flutter boundary prediction problem can be viewed as an identification problem, if flight measurements are used to get the flutter boundary estimates. In the
context of physical identification, the flutter boundary prediction problem amounts to finding the best values for certain physical parameters in the model that describes the flexible dynamics of an aircraft, for which the flutter boundary is to be determined.

The method proposed in this thesis amounts to performing the physical identification of the flexible dynamics of the aircraft in the environments for which the measurements are available. The identified model is then used to predict the flutter boundary.

### 3.2 Proposed Method for the Fit of the Model and Flutter Boundary Prediction

The proposed method is an extension of physical identification approach to the flutter boundary prediction problem. For the method to be applied, a preliminary model of the system and experimental measurements must be available. The appropriate analytical model can be obtained from the combination of finite element analysis and approximation of unsteady aerodynamic forces, as it was done in [28], [29]:

\[ M\ddot{\eta} + C\dot{\eta} + K\eta + \bar{q}Q(\eta)\eta = 0. \quad (3.1) \]

The unsteady aerodynamic forces can be approximated by Padé polynomials:

\[ Q(\eta) = A_0 + ikA_1 + (ik)^2A_2 + \frac{ik}{ik + \beta_1}A_3 + \frac{ik}{ik + \beta_2}A_4 + \cdots. \quad (3.2) \]

Equation (1) can then be written as

\[
\begin{align*}
(\hat{K} + \bar{q}A_0)\eta + (\hat{C} + \bar{q}\frac{\phi}{2V}A_1)\dot{\eta} + \\
(\hat{M} + \bar{q}(\frac{\phi}{2V})^2A_2)\ddot{\eta} + \bar{q}A_3\dot{x}_1 + \bar{q}A_4\dot{x}_2 + \cdots = 0,
\end{align*}
\]

or

\[
\hat{K}\eta + \hat{C}\dot{\eta} + \hat{M}\ddot{\eta} + \bar{q}A_3\dot{x}_1 + \bar{q}A_4\dot{x}_2 + \cdots = 0. \quad (3.4)
\]
This can be written equivalently in the state space formulation:

\[
\begin{pmatrix}
I \\
\tilde{M} \\
I \\
I
\end{pmatrix}
\begin{pmatrix}
\dot{\eta} \\
\ddot{\eta} \\
\dot{x}_1 \\
\dot{x}_2
\end{pmatrix}
= \begin{pmatrix}
0 & I & 0 & 0 \\
-\tilde{K} & -\tilde{C} & -\tilde{q}A_3 & -\tilde{q}A_4 \\
0 & I & -\frac{V}{b}\beta_1 I & 0 \\
0 & I & 0 & -\frac{V}{b}\beta_2 I
\end{pmatrix}
\begin{pmatrix}
\eta \\
\dot{\eta} \\
\dot{x}_1 \\
\dot{x}_2
\end{pmatrix}, \quad (3.5)
\]

or

\[
\begin{pmatrix}
\dot{\eta} \\
\ddot{\eta} \\
\dot{x}_1 \\
\dot{x}_2
\end{pmatrix}
= A(\tilde{q})
\begin{pmatrix}
\eta \\
\dot{\eta} \\
\dot{x}_1 \\
\dot{x}_2
\end{pmatrix}. \quad (3.6)
\]

For a fixed Mach number the resulting model is linear in \(\tilde{q}\), dynamic pressure. Thus, the model developed to represent the flexible dynamics of the aircraft for a certain Mach number and certain values of dynamic pressure, can be used to represent the system for flight conditions with the same Mach number but with a different value of dynamic pressure.

Once the model of the system and the flight measurements are available, the physical system identification can be performed. The system identification amounts to minimizing the discrepancy between the model and the measured inputs and outputs of the system.

To define the discrepancy quantitatively a cost function has to be introduced. Several choices of the cost functions can be made for the flutter boundary prediction problem. In the work by Duchesne [15] the cost function was defined as an \(H_2\) norm of the difference between the transfer functions obtained from the model and from the measured inputs and outputs. The \(H_2\) norm of a transfer function \(G(j\omega)\) is defined in the following way:

\[
||G||_2^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{trace} [G(j\omega)G^*(-j\omega)] \, d\omega. \quad (3.7)
\]
However, this choice of the cost function did not provide a satisfactory match of the natural frequencies and damping ratios of the structural modes in the experimental transfer functions. Thus, a different cost function is chosen for the proposed method. In fact, the flutter mechanism is a coupling of structural modes, and the flutter happens when one of the structural modes becomes unstable. Thus, natural frequencies and damping ratios of the structural modes of the aircraft are reliable indicators of the proximity of the aircraft to the flutter.

The eigenvalues of the matrix $A(\alpha_{ij}, \bar{q})$ are found from the following equation:

$$|A(\alpha_{ij}, \bar{q}) - \lambda_k I| = 0. \quad (3.8)$$

In this equation $k = 1 \ldots K$, where $K$ is the dimension of $A(\alpha_{ij}, \bar{q})$ matrix.

In the equation 3.8, $\alpha_{ij}$ are the coefficients of the matrix $A(\bar{q})$ that are fitted in the optimization problem.

The estimates $\tilde{\lambda}_k(\bar{q}_l)$ of the eigenvalues of the matrix $A(\alpha_{ij}, \bar{q})$ are obtained from the experimental flight data in the following way:

$$\tilde{\lambda}_k(\bar{q}_l) = \omega_k(\bar{q}_l)g_k(\bar{q}_l) + j\omega_k(\bar{q}_l)\sqrt{1 - g_k^2(\bar{q}_l)}, \quad (3.9)$$

where the values $\omega_k(\bar{q}_l)$ and $g_k(\bar{q}_l)$ are the experimental estimates of the natural frequencies and damping ratios of the flexible modes, obtained from the experimental data for $L$ different flight conditions.

$L$ values of dynamic pressure, $\bar{q}_l$, $l = 1 \ldots L$, correspond to $L$ flight conditions for which the flight data were gathered.

The cost function is then defined as the sum of the distances between the locations of the poles of the structural modes of interest in the model and in the measured data:

$$J(\alpha_{ij}) = \sum_{l=1}^{L} \sum_{k=1}^{K} \left| \lambda_k(\alpha_{ij}, \bar{q}_l) - \tilde{\lambda}_k(\bar{q}_l) \right|. \quad (3.10)$$

The minimum of the cost function defined in 3.10 must be found to solve the optimization problem.

Now that the cost function to minimize is defined, the parameters that are to be
changed to minimize the cost function have to be determined. The model parameters that can be changed are the coefficients of structural matrices (stiffness, damping and mass matrices) and coefficients of matrices used to approximate the unsteady aerodynamics. The coefficients of the structural matrices are determined from the finite element analysis and from the ground vibration tests. The ground vibration tests can be seen as the physical identification of the structural parameters of the system with the measurements that are taken at zero dynamic pressure (ground condition).

The parameters that should be changed during the identification with in-flight data are then the coefficients of matrices that approximate the unsteady aerodynamics. This is an approach similar to that employed by Nissim and Gilyard [30]. The physical identification of those parameters reduces to finding a minimum to the cost function, specified above, with respect to the coefficients of the matrices approximating the unsteady aerodynamics. The quasi-Newton method can be used to solve the optimization.

Once the optimization is performed and the optimal values of the parameters are found, the identified model can be used for the flutter boundary prediction. The smallest value of dynamic pressure that makes the system unstable can be found, using binary search. This value of the dynamic pressure is the determined flutter boundary. Figure 1-2 in Chapter 1 illustrates the flutter boundary prediction method. In this figure, the round boxes denote the part of the information that is already available, before the flutter boundary prediction method is applied. This information consists of modal matrices from the finite element model, approximation of unsteady aerodynamics and time histories of the experimentally measured inputs and outputs to the system.

The information in the rectangular boxes is obtained using the steps of the flutter boundary prediction algorithm from the information in the round boxes.

A limitation of this method is the possibly large number of parameters involved in optimization procedure. This limitation will be illustrated in more details when the applications of the method are considered. The main advantage of the method is that it allows one to obtain the flutter boundary estimate using the flight data, taken at
flight conditions that are still far from the flutter boundary. Thus, the risk associated with flying the aircraft close to the flutter boundary, as should be done to determine the flutter boundary according to traditional flutter clearance methods [13], can be reduced.

### 3.3 Experimental Investigation

#### 3.3.1 Description of the Wing Test Article
As an application of the proposed method, the wing test article example is described. The wing that is presented here was used in [29]. The test article was designed to enable the bend-twisting coupling. The main structural element of the wing was a graphite/epoxy, aluminum honeycomb sandwich structure. 15 piezoelectric actuators were used to provide the excitation to the test article. The actuators could be used either individually, or in groups. Five different combinations of actuators provided 5 different groups for the excitation of the wing. The sensor mechanism included 10 strain gauges and 4 accelerometers. Frequency sweep excitation signals were used for the testing described in the next section.

#### 3.3.2 Experimental Data Taken on the Wing
The wing was tested in the wind tunnel to determine the flutter boundary experimentally. The wind tunnel was operated at several values of dynamic pressure, and for each test time histories of the actuators and sensors were recorded. The data were collected at 50, 60, 65, 70 and 75 psf. The flutter boundary was determined to be 82 psf by extrapolating the readings of the on-line frequency analyzer.

The raw time-history data from the wind tunnel test were compressed by obtaining the transfer function estimates from individual actuators and actuator groups to strain gauges and accelerometers. The transfer function estimates obtained in [29] were used.
3.3.3 The Theoretical Model for The Test Article

The analytical model in [29] was developed to accurately capture the dynamics of the first 3 structural modes. 20 modes were extracted from the finite element. Only 8 lowest frequency modes were kept for the aeroelastic model. 2 lags were used for the approximation of the aerodynamic force. The approximating aerodynamic matrices were computed for the first 4 structural modes.

3.3.4 Model Reduction

For the purpose of improved flutter boundary prediction, the analytical model has been fitted with the estimates of modal parameters for the first 3 structural modes. The stiffness and damping matrices in the analytical model are fitted so that the analytical model at 0 dynamic pressure is in accordance with the results of the experiment. This fit is identical to the fit of the model, according to the results of ground vibration tests.

To simplify the optimization procedure, the number of parameters to fit should be reduced, if possible. This is done for the analytical model, by computing the aerodynamic approximation matrices only for the first 3 structural modes. The number of structural modes is also reduced to simplify the computation of the cost function. The simplified model is tested to see if its behavior changes drastically form the behavior of the original analytical model. The values of natural frequencies and damping ratios for the first 3 structural modes are compared with the original and reduced analytical models in Figure 3-1 and Figure 3-2.

From Figures 3-1, 3-2 it is clear that the behavior of the model does not change significantly. With 2 aerodynamic lags to approximate the generalized aerodynamic forces, the number of coefficients in 5 aerodynamic approximation matrices is 45, with the size of each individual matrix being $3 \times 3$. This dimension of the matrices is stipulated by the fact that generalized aerodynamic forces are computed only for 3 structural modes.
3.3.5 The Compression Of The Available Data

The available experimental transfer functions were used to obtain the estimates of modal frequencies and damping ratios for the first three structural modes. To estimate the natural frequencies and damping ratios the transfer functions were fitted with the minimum phase state space models, using the routine from Matlab’s μ-Analysis and Synthesis Toolbox [31]. The damping ratios and natural frequencies frequencies were obtained from the poles of the identified state space representations of transfer functions.

Table 3.1 contains the averages of the estimates of natural frequencies and damping ratios of the first three structural modes. In this table $q$ is the dynamic pressure, and $w_i, c_i, i = 1 \ldots 3$, are the estimates of natural frequencies and damping ratios of the first three structural modes.

In Figure 3-3 is shown the typical approximation of the magnitude of the experimental transfer function by the magnitude of the transfer function of the minimum
Table 3.1: Estimates of natural frequencies and damping ratios from the experimental data.

<table>
<thead>
<tr>
<th>$\bar{q}$ (psf)</th>
<th>$w_1$</th>
<th>$g_1$</th>
<th>$w_2$</th>
<th>$g_2$</th>
<th>$w_3$</th>
<th>$g_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>3.8673</td>
<td>0.0829</td>
<td>11.8748</td>
<td>0.0577</td>
<td>17.6739</td>
<td>0.0424</td>
</tr>
<tr>
<td>60</td>
<td>4.2784</td>
<td>0.0802</td>
<td>10.8004</td>
<td>0.0584</td>
<td>17.1191</td>
<td>0.0428</td>
</tr>
<tr>
<td>65</td>
<td>4.5968</td>
<td>0.0592</td>
<td>10.1733</td>
<td>0.0656</td>
<td>17.1040</td>
<td>0.0465</td>
</tr>
</tbody>
</table>

3.3.6 Fit of the Model and the Improved Flutter Boundary Prediction

Implementation of the quasi-Newton method in Matlab’s optimization Toolbox [32] was employed to perform the optimization. Three different optimization procedures were performed: one in which the experimental measurements for all flight conditions were used ($\bar{q}=50,60,65$ psf); another one in which the flight measurements for only two flight conditions were used ($\bar{q}=50,60$ psf); and the last one in which only the measurements for one flight condition were used ($\bar{q}=50$). In terms of formula in the equation 3.10 these three cases correspond to the value of $L$ being 3, 2, or 1, implying that the poles for three, two or one value of dynamic pressure are fitted.

The results of optimization are presented in the Table 3.2.

Here, $L$ is the same as in the equation 3.10 for the cost function above. The predicted dynamic pressure of flutter is denoted $q_F$. The flutter boundary determined from the on-line frequency analyzer readings is denoted as $\bar{q}_F$. Figure 3-4,
<table>
<thead>
<tr>
<th>L</th>
<th>initial cost</th>
<th>final cost</th>
<th>number of iterations</th>
<th>Predicted flutter boundary: $q_f (psf)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>49.94</td>
<td>9.13</td>
<td>3349</td>
<td>97.2</td>
</tr>
<tr>
<td>2</td>
<td>27.73</td>
<td>8.25</td>
<td>1902</td>
<td>85.3</td>
</tr>
<tr>
<td>3</td>
<td>9.55</td>
<td>1.43</td>
<td>2819</td>
<td>85.1</td>
</tr>
</tbody>
</table>

Table 3.2: Flutter boundary predictions from the fitted model.

Figure 3-4: Fit at three values of dynamic pressure: natural frequencies and damping ratios.

Figure 3-5 and Figure 3-6 show the locus of natural frequencies and damping ratios of the first three structural modes with change in dynamic pressure for three different optimization procedures.

According to the data, the flutter mechanism of the wing is the coupling of the first and second structural modes, with the first mode going unstable. However, the theoretical model alone predicts that it is the second mode that goes unstable. While the fit of the model improves the prediction of the flutter boundary, it is also

Figure 3-5: Fit at two values of dynamic pressure: natural frequencies and damping ratios.
interesting to note that fit of the model at three flight conditions changes the flutter mechanism of a model: the first mode, as the experimental data predicted, is going unstable.

The original model predicts the flutter at 127 psf, while the actual flutter boundary value, obtained from the on-line frequency analyzer readings approximation was determined to be 82 psf. Thus, analytical model overpredicts the flutter by 45 psf, while the model fitted with the largest amount of data gives more conservative flutter prediction at 85.1 psf, which is 3.1 psf overprediction. If only data at 50 and 60 psf are used, the flutter is overpredicted at 85.3 psf, which is still closer to the actual flutter boundary than the prediction from the original analytical model. This case is especially interesting, because for the values of dynamic pressure at 50 and 60 psf, the damping ratios are still going up, while 65 psf is the first value of dynamic pressure when damping ratios decrease in comparison to their values at previous value of dynamic pressure. While the stability of the system is still increasing, one can already obtain precise flutter boundary estimates.

For the case of optimization only at one value of dynamic pressure (50 psf), the flutter boundary is predicted at 97.2 psf, which is still a better prediction than the analytical model alone gives.
3.4 Application of the Method to the F-18 SRA

3.4.1 Analytical Model for the F-18 SRA

The analytical model for the F-18 SRA was developed in the manner, similar to that for the wind tunnel article in Chapter 4. The model contains 28 structural modes: 14 symmetric and 14 antisymmetric modes. The Table 3.3 describes the structural modes of the F-18 SRA and shows the frequencies of the modes.

<table>
<thead>
<tr>
<th>Symmetric mode</th>
<th>Hz</th>
<th>Antisymmetric mode</th>
<th>Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wing first bending</td>
<td>5.59</td>
<td>Fuselage first bending</td>
<td>8.15</td>
</tr>
<tr>
<td>Fuselage first bending</td>
<td>9.30</td>
<td>Wing first bending</td>
<td>8.84</td>
</tr>
<tr>
<td>Stabilator first bending</td>
<td>13.21</td>
<td>Stabilator first bending</td>
<td>12.98</td>
</tr>
<tr>
<td>Wing first torsion</td>
<td>13.98</td>
<td>Wing first torsion</td>
<td>14.85</td>
</tr>
<tr>
<td>Vertical tail first bending</td>
<td>16.83</td>
<td>Vertical tail first bending</td>
<td>15.61</td>
</tr>
<tr>
<td>Wing second bending</td>
<td>16.95</td>
<td>Wing second bending</td>
<td>16.79</td>
</tr>
<tr>
<td>Wing outboard torsion</td>
<td>17.22</td>
<td>Fuselage second bending</td>
<td>18.62</td>
</tr>
<tr>
<td>Fuselage second bending</td>
<td>19.81</td>
<td>Trailing edge flap rotation</td>
<td>23.47</td>
</tr>
<tr>
<td>Trailing edge flap rotation</td>
<td>23.70</td>
<td>Fuselage torsion</td>
<td>24.19</td>
</tr>
<tr>
<td>Stabilator fore and aft</td>
<td>28.31</td>
<td>Launcher rail lateral</td>
<td>24.35</td>
</tr>
<tr>
<td>Wing second torsion</td>
<td>29.88</td>
<td>Stabilator fore and aft</td>
<td>28.58</td>
</tr>
<tr>
<td>Fuselage third bend, aileron rotation</td>
<td>33.44</td>
<td>Wing second torsion</td>
<td>29.93</td>
</tr>
<tr>
<td>Aileron torsion</td>
<td>38.60</td>
<td>Aft fuselage torsion</td>
<td>37.80</td>
</tr>
<tr>
<td>Stabilator second bending</td>
<td>43.17</td>
<td>Wing pitch</td>
<td>39.18</td>
</tr>
<tr>
<td>wing third bending</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3: Symmetric and antisymmetric natural modes of the F-18 SRA and the corresponding modal frequencies.

The values of stiffness, mass and damping matrices were obtained by NASA from the finite element analysis [33]. The unsteady aerodynamic forces were computed using panel methods. A program STARS [34] developed at NASA Dryden was used to solve subsonic aerodynamic equations. Thus, the following system of equation was obtained to describe the flexible dynamics of the aircraft [28]:

\[
M \ddot{\eta} + C \dot{\eta} + K \eta + \ddot{q}Q(k)\eta = 0
\]  

(3.11)

The unsteady aerodynamic forces were approximated with a state space system:
\[
Q(s) = \begin{bmatrix} A_Q & B_Q \\ C_Q & D_Q \end{bmatrix} = C_q(sI - A_Q)^{-1}B_Q + D_Q. \tag{3.12}
\]

Thus, the state space equations for the flexible dynamics have the following form:

\[
\begin{pmatrix}
\dot{\eta} \\
\ddot{\eta} \\
\dot{x}
\end{pmatrix} =
\begin{pmatrix}
0 & I & 0 \\
-M^{-1}(K + qD_Q) & -M^{-1}C & -\bar{q}M^{-1}C_Q \\
0 & 0 & A_Q
\end{pmatrix}
\begin{pmatrix}
\eta \\
\dot{\eta} \\
x
\end{pmatrix} \tag{3.13}
\]

### 3.4.2 Data Analysis

**Identifying the Correspondence between the Peaks of the Transfer Functions and Structural Modes in the Analytical Model**

From the database of flight data available for the F-18 SRA, the flight data for Mach number of 0.8 were chosen. The signal to noise ratio was relatively high in these data in comparison to the data at other Mach numbers. Data for three altitudes were available: 10 000, 30 000 and 40 000 Feet. For each of the data sets the transfer functions from left and right exciters to all 10 accelerometers were estimated. The structural modes that were observed in the experimental transfer functions had to be identified now to establish the correspondence between the structural modes in the analytical model and the structural modes in the experimental transfer functions.

The analytical model predicted the frequencies of the structural modes. These frequencies were compared with the frequencies of the peaks, observed in the experimental transfer functions. The closest predicted frequencies to the frequency of the peak in the experimental transfer functions provided the possible candidates.

As the next step, the physics of the aircraft had to be used for the identification of the structural modes. The fact that bending and torsion modes have different physical nature and that symmetric and antisymmetric modes have different physical nature was used.

With the measurements of wing forward and wing aft sensors (left and right wings) it is possible to identify the following structural modes based on the following
reasoning:

1) Symmetric bending mode corresponds to a synchronous motion of all four sensors, meaning that all transfer functions from either left or right exciter to all four sensors are in phase. The circles obtained by plotting the transfer functions in the Nyquist plane are in phase.

2) Antisymmetric bending corresponds to the motion of the sensors when all left side sensors move synchronously, out of phase with all right side sensors.

3) Symmetric torsion corresponds to the motion, when all forward sensors move in phase and they all are out of phase with aft sensors.

4) Antisymmetric torsion corresponds to synchronous motion of left aft and right forward sensor. They are moving out of phase with right aft and left wing sensors.

Using the analysis described above, the four out of five tracked modes in the experimental transfer functions were identified as first symmetric wing bending, first antisymmetric wing bending, first symmetric wing outer torsion and first antisymmetric wing torsion. The figures 3-7, 3-8, 3-9, 3-10 illustrate the methods that were used to identify the structural modes.

The fifth mode was identified as an antisymmetric torsion. However, there is no antisymmetric torsion mode in the analytical model with the frequency close to the
Figure 3-8: The first antisymmetric wing bending mode. Frequency range: 8 Hz - 10 Hz.

Figure 3-9: The first antisymmetric wing torsion mode. Frequency range: 15 Hz - 17 Hz.
Figure 3-10: The first symmetric wing outer torsion mode. Frequency range: 18 Hz - 20 Hz.

frequency of the peak found in the experimental transfer functions. Thus, the fifth mode was not used in the fit of the finite element model.

Estimation of Natural Frequencies and Damping Ratios of the Structural Modes

For each of the transfer functions, the damping ratios of all the four identified modes were estimated. It should be noted that in some transfer functions some of the four identified modes were not observed. The damping ratios were estimated, using the half-power point method:

\[ g = \frac{(\omega_l - \omega_r)}{2\omega_p}. \]  

(3.14)

Here, \( \omega_l \) and \( \omega_r \) are left and right half power point frequencies and \( \omega_p \) is the frequency of the peak. Figure 3-11 shows the procedure of damping ratio estimation.

Figures 3-12, 3-13, 3-14, 3-15 show the average values of the estimated natural frequencies and damping ratios for the four structural modes for different flight conditions.

Figures 3-16, 3-17, 3-18 show the frequencies and damping ratios of the four estimated modes:
Figure 3-11: Damping ratio estimation procedure for the flight data at Mach=0.8, 40 000 feet. (flight # 0549). Transfer functions shown are from left and right exciters to the left wing aft accelerometer. Dashed lines show the half power points. The dotted lines show the frequency of the peak.

Figure 3-12: Average values of the wing symmetric first bending mode. natural frequencies and damping ratios for the wing symmetric first bending mode.
Figure 3-13: Average values of the natural frequencies and damping ratios for the wing first antisymmetric bending mode.

Figure 3-14: Average values of the natural frequencies and damping ratios for the wing first antisymmetric torsion mode.
Figure 3-15: Average values of the natural frequencies and damping ratios for the wing symmetric outer torsion mode.

Figure 3-16: Natural frequencies and damping ratios of the four identified modes at Mach 0.8, 40 000 feet. Shown are the mean ± standard deviation intervals. Mode numbers: 1 - symmetric first wing bending; 2 - antisymmetric wing first bending; 3 - antisymmetric wing first torsion; 4 - symmetric wing outer torsion mode.
Figure 3-17: Natural frequencies and damping ratios of the four identified modes at Mach 0.8; 30 000 feet. Shown are the mean ± standard deviation intervals. Mode numbers: 1 - symmetric first wing bending; 2 - antisymmetric wing first bending; 3 - antisymmetric wing first torsion; 4 - symmetric wing outer torsion mode.

Figure 3-18: Natural frequencies and damping ratios of the four identified modes at Mach 0.8; 10 000 feet. Shown are the mean ± standard deviation intervals. Mode numbers: 1 - symmetric first wing bending; 2 - antisymmetric wing first bending; 3 - antisymmetric wing first torsion; 4 - symmetric wing outer torsion mode.
3.4.3 Analytical Model Fit

After the estimates of the natural frequencies and damping ratios of the structural modes are obtained, the analytical model can be fit with those estimates. Figures 3-23, 3-24 show the changes in natural frequencies and damping ratios of the structural modes, as the dynamic pressure grows, according to the to the analytical model. The Mach number used in the model is 0.8.

Only antisymmetric dynamics case will be considered, even though the symmetric case is handled analogously.

The reduction of the model is performed to reduce the number of parameters in optimization: the aerodynamics matrices $A_q, B_q, C_q, D_q$ are computed only for the first five structural modes of the model. Only the first five structural modes of the model are kept in the reduced model. The number of coefficients in the matrices
Figure 3-20: Estimated natural frequencies and damping ratios of the first antisymmetric wing bending mode at different altitudes. Shown are the mean ± standard deviation intervals.

Figure 3-21: Estimated natural frequencies and damping ratios of the first antisymmetric wing torsion mode at different altitudes. Shown are the mean ± standard deviation intervals.
Figure 3-22: Estimated natural frequencies and damping ratios of the symmetric wing outer torsion mode at different altitudes. Shown are the mean ± standard deviation intervals.

\(A_q, B_q, C_q, D_q\) is 95. Not all these coefficients are fitted in the optimization process. Only 2 structural modes were observed in the experimental transfer functions. Thus, only the corresponding coefficients of \(Q(s) = C_q(sI - A_q)^{-1}B_q + D_q\) are fitted in the optimization process. The quasi-Newton method from the Matlab’s optimization toolbox [32] was used again to produce the results, described in the Table 3.4.

<table>
<thead>
<tr>
<th>number of iterations</th>
<th>initial cost</th>
<th>final cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1400</td>
<td>50.67</td>
<td>13.77</td>
</tr>
</tbody>
</table>

Table 3.4: Results of the fit of the model.

Figures 3-25, 3-26 show the results of the fit for the wing first antisymmetric bending anf first antisymmetric torsion modes.

### 3.4.4 The Flutter Boundary Prediction

The changes of the natural frequencies and damping ratios in the fitted model are shown in the Figure 3-27. In this figure the first five antisymmetric structural modes of the F-18 SRA are shown. Also shown are the experimental estimates of the nat-
Figure 3-23: Changes in the natural frequencies and damping ratios of the structural modes for Mach 0.8 in the symmetric case.
Figure 3-24: Changes in the natural frequencies and damping ratios of the structural modes for Mach 0.8 in the antisymmetric case.
Figure 3-25: Results of the fit for the first antisymmetric wing bending mode. Shown are the experimental data (mean ± standard deviation interval) and the natural frequencies and damping ratios from the original and fitted models at different altitudes.

Figure 3-26: Results of the fit for the first antisymmetric wing torsion mode. Shown are the experimental data (mean ± standard deviation interval) and the natural frequencies and damping ratios from the original and fitted models at different altitudes.
ural frequencies and damping ratios of the first wing bending and first wing torsion structural modes for altitudes of 10 000, 30 000 and 40 000 feet. From Figures 3-25, 3-26, 3-27 one can see that experimental frequencies are matched more precisely than the experimental damping ratios. As the result of the fit, the frequency of the first wing bending mode increases. This apparently leads to the coupling of the first and second modes at about 1500 $\text{lbf/ft}^2$ - the value of dynamic pressure $\bar{q}$, roughly corresponding to the flutter boundary. The two lower plots in the Figure 3-27 show the changes in the damping ratios of the structural modes of the fitted model. The plot on the right side is the increased version of the plot on the left side.

The fitted model is used to predict the flutter boundary by finding the smallest value of $\bar{q}$ - dynamic pressure, that drives the system unstable. This value is shown
in Table 3.5 together with the frequency of flutter.

<table>
<thead>
<tr>
<th></th>
<th>original model</th>
<th>fitted model</th>
</tr>
</thead>
<tbody>
<tr>
<td>dynamic pressure (lb/ft²)</td>
<td>4556</td>
<td>1535</td>
</tr>
<tr>
<td>altitude (ft)</td>
<td>-50623.3</td>
<td>-13964.5</td>
</tr>
<tr>
<td>frequency (Hz)</td>
<td>7.596</td>
<td>9.016</td>
</tr>
</tbody>
</table>

Table 3.5: Comparison of the flutter boundaries from original and fitted analytical models.

It is interesting to note that the flutter mechanism has changed: in Figure 3-24 showing the unfitted model, the flutter was caused by the antisymmetric fuselage first bending mode going unstable. At the same time, as we see in the fitted model (Figure 3-27), the mode that becomes unstable after the fit is the wing antisymmetric first bending mode. The flutter mechanism in the fitted model agrees with the results of the p-k analysis [35] for the antisymmetric case at Mach number of 0.8, even though the flutter boundary prediction from the fitted model is much more conservative.
Chapter 4

Conclusions

In this thesis, a methodology for flutter boundary prediction was described. This methodology uses experimental flight data and is intended to be able to provide the flutter boundary estimates, using the analytical model and the data taken at the safe flight conditions. The methodology consists of physical identification of certain parameters in the analytical model. This identification is performed using optimization routine that tries to minimize the difference in behavior of analytical model and the measured experimental data. The difference is measured as the sum of distances between the poles of the system in the analytical model and the experimental data.

The difficulty of the method lies in the large number of parameters in the optimization procedure. This number can be decreased by taking the reduced model of the aircraft. The reduced model can be obtained by computing the unsteady aerodynamic forces only for the structural modes that are important in the flutter mechanism.

The experimental data are used to compute the transfer function with time-frequency analysis method, which employs the fact that the excitation signals to the systems are frequency sweeps. The time-frequency analysis method was compared with the state of the art identification methods and was found to perform well. The implications of choosing different time windows in the method were analyzed in terms of bias and the variance of the estimate of the transfer function.
The method for the flutter boundary prediction was applied to the wing test article, and the results were shown to be close to the actual flutter boundary. These results were an improvement in comparison to the results obtained just with the analytical model.

The method was then applied to the F-18 SRA. To do that, the transfer functions for the F-18 SRA had to be analyzed, using the understanding of the physics of the aircraft, to identify the correspondence between the structural modes observed in the transfer functions and the structural modes in the analytical model.

The natural frequencies and damping ratios of the structural modes of the aircraft were estimated from the transfer functions.

The analytical model of the F-18 SRA was fitted with these estimates. The flutter boundary was found as the minimum dynamic pressure driving the fitted system unstable.
Bibliography


