THEORY OF THE DISPERSION OF MAGNETIC PERMEABILITY IN FERROMAGNETIC MATERIALS AT MICROWAVE FREQUENCIES

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In magnetic materials, it is generally accepted that the magnetic properties are determined by the size and shape of the crystallites, or the domains. The domains are small regions within a crystal that have a uniaxial magnetic anisotropy. The domains are separated by domain walls, which are regions of high magnetic stress. The macroscopic magnetic properties of a material are determined by the distribution and orientation of the domains.

The magnetic behavior of ferromagnetic materials can be understood in terms of the concepts of domain structure and domain wall motion. The magnetic properties of a material are determined by the size, shape, and orientation of the domains, as well as the interactions between the domains and the domain walls.

A computer evaluation of the experimental data on the permeability of ferromagnetic materials (including alloys and dusts) at frequencies between 100 cycles per second and 10,000 cycles per second has been published by Allanson. It is proposed here for the purpose of comparison with the theory developed below to consider briefly the results of measurements on iron, nickel, and cobalt at frequencies above 100 cycles per second, including some data not available to Allanson.

Experimental values for iron and steel are plotted in Fig. 1, as determined by Arkadijev, Hong and Jones, Potapenko and Sänger, Lindman, Hong and Gottlieb, Glatthor, and E. Maxwell. The curve labeled $\mu_p$ is drawn through the points for the experimental permeabilities deduced from resistive losses in a circuit element containing the ferromagnetic specimen, while the curve labeled $\mu_R$ is deduced from the reactance of the circuit element. The relationship between $\mu_p$ and $\mu_R$ is discussed in the next section.

The measurements of Lindman have not been taken into account in drawing the $\mu_p$ curve, since his values are far out of line with those of Hong and Jones, Hong and Gottlieb, Glatthor, and Potapenko and Sänger. The apparent discrepancy here may be due to real differences in the dimensions of surface domains or in the electrical conductivity of the surface layer of the specimen. Maxwell has studied the effect of surface finish on microwave attenuation in wave guides and finds considerable

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15. E. Maxwell, M.I.T. R.L. Report 854 (1946), unclassified. The values of the permeabilities credited to Maxwell were calculated from his values for the attenuation of 1.25 cm radiation in rectangular wave guides.
In the analysis, it was found that the data collected from the experimental setup was consistent with the theoretical predictions. The high precision of the equipment allowed for accurate measurements, which were corroborated by the results obtained in separate experiments. These findings are crucial for future research and development in the field.

**Figure 1:**

A close-up view of the experimental setup is shown above. The precision in the measurements is evident in the detailed view of the apparatus. Further analysis of these data is ongoing to refine the understanding of the underlying processes.
There is a central point of contention in the literature with regard to the connection between the permeability $\mu$ derived from resistive losses (as by measuring the $R$ in a coil or the attenuation of energy along a coax line) and the permeability $\mu$, obtained from reactance measurements (as by measuring the resonant frequency of a coil or the wave length of standing waves along a coax line). It does not appear to have been realized in the literature that the two sets of measurements inherently reveal different aspects of the very fundamental physical phenomena. In this section the fundamental philosophy will be developed which underlies the interpretation, in terms of permeability, of $R$, $L$, measurements in a dispersive region. The ideas were stimulated by a paper of Rayleigh 

The usual definition of initial permeability as the ratio of $B$ to $H$ for weak fields does not correspond at high frequencies to the quantities actually observed in experiments. In a dispersive region the value of the ratio $B/H$ may vary from point to point in the radiation field within the specimen both amplitude and phase. Detailed knowledge of this "permeability field", supposing it could be determined, would be less useful (for most purposes) than a convenient swamping up of the magnetic behavior of the material in terms of an effective permeability.

The natural and logical method of defining the effective permeability is as follows: The impedance

$$Z_{\text{calc}}(\nu,\omega) = (\text{calc}(\mu_1,\omega) + j\text{calc}(\mu_2,\omega))$$

of a circuit element containing the ferromagnetic material can in principle be calculated from Maxwell's equations in the usual form, given $\mu$ and $\omega$.

This calculation can actually be carried out in closed form for the important experimental geometries, such as a rectangular wave guide or coaxial line resonator. Suppose that the result of a series of measurements on the circuit element gives us experimental values.

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in general p is defined as the matrix which has the properties:

\[ p \cdot f = f \]  

and which is complex.

In this connection, it is important to note the fundamental role of the relationship of the matrix elements to the relative change of the reaction and the reverse, and to the reaction and the forward. The reason for this is that a serious distribution of the points in the matrix \( L \) is not the case with Mulliken-Simons. For this reason, a formalism is possible in the analysis theorem.

If a form \( p_{ij} \) of the system is to be taken to mean

\[ \phi_i \cdot V_{ij} \cdot \phi_j = \delta_{ij} \]

This relation determines the coordinate \( p_{ij} \) of the core, i.e., it determines only the core part of the total matrix, and the possibility of taking in only

\[ K_{\text{core}}(\omega_{ij}, \omega_{ij}) \]

that determines the core form \( p_{ij} \).

There is no difference in the core relation, between \( p_{ij} \) and the complex \( \omega_{ij} \). The complex \( \omega_{ij} \) has only the nature of the core part of the total complex, and \( p_{ij} \) has none of the information on the relative part of \( p_{ij} \).

The difference in the complex, which underlies the introduction of a complex correlation, is different from the correlation with the introduction of a complex correlation, in the Pearlman theorem. These are the reasons that Teller and others advocate the formalism of the complex correlation, which is now called the complex correlation function of the
where an estimation of increase in the intrinsic permeability of ferromagnetic material it is possible to have a microscopic permeability which varies from point to point in the material.

In the literature \( \mu_1 \) is sometimes called the "outer permeability" and may be denoted also by \( \mu_p \) or \( \mu_{\infty} \); \( \mu_2 \) is sometimes called the "inner permeability" and may be denoted by \( \mu_1 \) or \( \mu_0 \).
Theory of Domain Dynamics

In the low-frequency region the skin depth for field penetration is comparable to or smaller than the dimensions of a domain. For iron, with μ = 100 we have \( \delta^2 \approx 1.6 \times 10^{-2} \sqrt{f} \), where \( \delta \) is the skin depth in cm and \( f \) is the frequency in mc/sec. This gives the following values:

\[
\begin{array}{ccc}
\text{f (mc/sec)} & \text{10}^2 & \text{10}^4 \\
\text{5 (cm)} & 1.6 \times 10^{-4} & 1.6 \times 10^{-5} & 1.6 \times 10^{-5}
\end{array}
\]

whereas domain dimensions are estimated at \( 10^{-3} \) to \( 10^{-4} \) cm. It is therefore necessary to reconsider the conventional application of Maxwell's equations to the skin effect problem.

In the limiting case in which the surface energy of the domain boundaries is greater than the magnetization energy, the domain wall shifts as a whole in the direction of the applied field even if, due to the skin effect, the magnetic field only penetrates a short distance into the domain. If \( M \) is constant across the domain boundary (low frequency case), the average macroscopic magnetization \( \bar{M} \) for weak fields is given by

\[
\bar{M} = \gamma H
\]

In the usual definition of the initial susceptibility \( \gamma \). If now \( H \) varies across the boundary we suppose that the effective magnetization is given by the susceptibility times the average value of \( H \) across the boundary:

\[
\bar{M} = \gamma \left\langle 1/2d \int_{-d}^{d} H(y) dy \right\rangle
\]

This assumption is based on the physical concept that the boundary area where \( 2 \gamma H \) is integrated over the area of the boundary to give a force which shifts the boundary in the direction of the applied field up to where this force is balanced by the force exerted on the boundary by the internal stresses in the material. Here \( M_a \) is the effective magnetic moment for a domain. The distance the wall is moved is related to the macroscopic magnetization \( \bar{M} \) observed in an experiment at a particular frequency, the following parameter is introduced:

\[
\alpha = \frac{M_a}{\gamma H}
\]
of the domain thickness, so that the applied magnetic field only penetrates a little way from the domain boundary. The force on this boundary associated with the surface value of this field strength is correspondingly less than the force obtaining at low frequencies, so that the boundary is shifted by a reduced amount. To the field this reduced shift looks like a reduced permeability.

It is of interest to consider as a simplified model a film one domain in thickness (Fig. 2), since in this case Maxwell's equations can be solved to give what is essentially the equation of motion of a domain boundary. The longitudinal extent of a domain along the surface of the film is supposed to be small in comparison with the thickness of the film. The domains are considered for simplicity to have only "180°" walls - that is, the domains are either magnetized in the direction of the applied field or in the opposite direction. The applied field is parallel to the surface of the film and is symmetric about the central plane of the film.

From Maxwell's equations
\[ \text{curl } \mathbf{j} = 4\pi \mathbf{M} / c \]  
\[ \text{curl } \mathbf{H} = -\mathbf{j} - 4\pi \mathbf{M} / c \]  
we get
\[ \text{curl curl } \mathbf{H} = \mathbf{j}(4\pi \mathbf{M} / c^2)(\mathbf{H} + 4\pi \mathbf{M}) \]  
for time dependence of the form \( \text{exp}(i\omega t) \). By the symmetry of the problem we have
\[ \text{curl curl } \mathbf{H} = -\mathbf{j}^2 \left( \frac{\partial^2 \mathbf{H}}{\partial y^2} \right) \]  
and hence
\[ \frac{\partial^2 \mathbf{H}}{\partial y^2} = \mathbf{j}(4\pi \mathbf{M} / c^2)(\mathbf{H} + 4\pi \mathbf{M}) \]  
Since \( 4\pi \mathbf{M} \) is constant with respect to \( y \), we can add this quantity \( \mathbf{M} \) to the left side without altering the value of the derivative there. In this definition \( \partial \mathbf{H} / \partial y = \mathbf{M} \) giving
\[ \frac{\partial^2 \mathbf{H}}{\partial y^2} = 16\pi \mathbf{M} / c^2 \mathbf{H} \]
which can be compared with the usual skin-effect equation

$$
\frac{d^2 \psi}{dy^2} = \frac{\mu \omega \sigma \psi}{\alpha^2} \frac{\partial^2 \psi}{\partial y^2}
$$

(14)

For convenience in working with Eq. (15) we shall hereafter omit the subscript \( y \) on \( E_x, E_y, \) and \( E_z \). We write

$$
\psi^2 = \frac{\mu \omega \sigma \psi}{\alpha^2} = 2y^2
$$

(15)

where \( \alpha \) is the skin depth for permeability unity. So (15) becomes

$$
\frac{d^2 \psi}{dy^2} = \psi^2
$$

(15a)

A symmetric solution of this equation is

$$
\psi = C \cosh \alpha y
$$

(16)

where \( C \) is a constant to be determined in terms of \( H(y) \), the magnetic field at the surface of the film. The definition of \( \alpha \) we have \( \psi = 8 - \frac{4\mu H}{\alpha} \), or using Eq. (7),

$$
H(y) = B(y) - \left( 4\mu/2d \right) \int_{-d}^{d} H(y) \, dy ;
$$

(17)

where \( H(y) \) is given by Eq. (16). The solution of this equation is found to be

$$
B(y) = C \cosh \alpha y - \left( 4\mu/2d \right) \int_{-d}^{d} \psi \cosh \alpha y \, dy
$$

(18)

where \( \psi \) is defined as \( \psi = 8 - \frac{4\mu H}{\alpha} \).

Thus the constant \( C \) is given in terms of \( H(d) \) by

$$
C = B(d) - \left( 4\mu/2d \right) \int_{-d}^{d} \psi \cosh \alpha y \, dy
$$

(19)

The solution of the ordinary eddy current equation \( \psi = 8 - \frac{4\mu H}{\alpha} \) above for permeability \( \mu \) is

$$
B(y) = C \cosh \alpha y - \mu H(y)
$$

and

$$
\psi^2 = \frac{\mu \omega \sigma \psi}{\alpha^2}
$$

(20)
Now that we have the formal solutions of both the ordinary eddy current equation and the domain eddy current equation, we can go on to calculate \( \mu_0, \mu \), and \( \mu_0 \) by following the procedure outlined in Section III in calculating the impedances. It is not necessary to specialize the calculation for a particular cavity or line; we can work with the intrinsic surface impedance of the film, which is defined as:

\[
Z = \frac{E_{\text{line}}}{H_{\text{line}}} = \frac{E_x}{H_x} \tag{22}
\]

From Eq. (8):

\[
E_x = (c/4x_0) \frac{\partial H_y}{\partial y} \tag{23}
\]

so that:

\[
x = -\left(\frac{c}{4x_0}\right) \left(\frac{\partial y}{\partial H_x}\right)_{y=d} \tag{22a}
\]

For the ordinary eddy current equation we have, using Eq. (20):

\[
E_{\text{ord}} = -\left(\frac{c}{4x_0}\right) \frac{p \tan \theta d}{1 - (1 - 1/\mu_0)(\tanh \theta d/pd)} \tag{24}
\]

The two expressions for \( E \) are equal if \( g \) is chosen so that:

\[
g d \tan \theta d = \frac{pd}{1 - (1 - 1/\mu_0)(\tanh \theta d/pd)} \tag{25}
\]

That is, if the effective permeability \( \mu \) is chosen so that:

\[
\sqrt{\mu} \tan \theta d = \frac{pd}{1 - (1 - 1/\mu_0)(\tanh \theta d/pd)} \tag{26}
\]

Since \( p \) involves \( \sqrt{\mu} \), the value of \( \mu \) satisfying this equation will be complex.

Let us consider limiting cases of... \( \ldots \tag{27} \)
a) **Low Frequency**

Here $|pd| \ll 1$, so that we can replace $\tanh \phi / \phi$ by unity. This gives $\mu = \mu_0$, the correct low frequency value.

b) **Very High Frequency**

Here $|pd| \gg 1$, so that $\tanh \phi / \phi$ approaches zero. This gives $\mu = 1$ in agreement with the measurements of Hansen and Hubens.

Values of $\mu$ satisfying Eq. (27) for various values of $pd$ are given in Table 1. These values were calculated by cut-and-try methods with assistance from Kennelly's tables and the tables prepared by Lowan, Morse, Feshbach, and Haurwitz.

Theoretical values of $\mu_R$ and $\mu_L$ are also given in Table 1 according to the definitions, Eqs. (4) and (5), where we identify

$$
F_{\text{exp}} \leftrightarrow F_{\text{ord}}; \quad x_{\text{exp}} \leftrightarrow x_{\text{ord}}; \quad x_{\text{calc}} \leftrightarrow x_{\text{ord}};
$$

so that $\mu_R$ is the real number which satisfies the real part of Eq. (27)

$$
\text{Re} \left[ \sqrt{\mu_R} \tanh pd \sqrt{\mu_R} \right] = \text{Re} \left( \frac{\tanh pd}{1 - (1 - 1/\mu_0)(\tanh pd/\phi)} \right) \quad (27a)
$$

and $\mu_L$ is the real number which satisfies the imaginary part:

$$
\text{Im} \left[ \sqrt{\mu_L} \tanh pd \sqrt{\mu_L} \right] = \text{Im} \left( \frac{\tanh pd}{1 - (1 - 1/\mu_0)(\tanh pd/\phi)} \right) \quad (27b).
$$

2. A. N. Lowan, P. M. Morse, A. Feshbach, and E. Haurwitz, *Tables for Solutions of the Wave Equation for Rectangular Boundaries Having Finite Impedance*, Applied Mathematics Panel Note No. 18; Section No. 8.1: sr1046 - 2049 (June, 1948); unclassified.
<table>
<thead>
<tr>
<th>Frequency (mc/sec)</th>
<th>Parameter</th>
<th>Permeability $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(\mu/\mu_N)^2$</td>
<td>Amplitude</td>
</tr>
<tr>
<td>82</td>
<td>0.1</td>
<td>104</td>
</tr>
<tr>
<td>334</td>
<td>0.2</td>
<td>127</td>
</tr>
<tr>
<td>750</td>
<td>0.3</td>
<td>39</td>
</tr>
<tr>
<td>2080</td>
<td>0.5</td>
<td>35</td>
</tr>
<tr>
<td>4570</td>
<td>0.75</td>
<td>16</td>
</tr>
<tr>
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<td>9</td>
</tr>
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<td>75000</td>
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<td>2.6</td>
</tr>
<tr>
<td>750000</td>
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<td>1.6</td>
</tr>
<tr>
<td>2100000</td>
<td>5.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

In Fig. 4 $\mu_R$ and $\mu_L$ are plotted together with the smoothed experimental curves for iron taken from Fig. 1. The arbitrary constant $k_d$, which is the thickness of the domain film model, has been taken to be $2.48 \times 10^{-4}$ cm; this value was chosen to make the half-value points on the experimental and theoretical curves coincide.

It is seen that the theory predicts the order of magnitude of the spacing between the $\mu_R$ and $\mu_L$ curves correctly. The general nature of the theoretical permeability change is in accordance with the experimental data, but the slopes of the theoretical curves are steeper than the experimental. The thickness of the film is within the limits of reasonable estimates of domain dimensions, although the thickness is somewhat on the small side.

The discrepancy in the slopes is most likely to be accounted for by local variations in domain dimensions and thus permeability, since these variations will act to smear out the dispersive region. The absence of the hump predicted for the $\mu_R$ curve on the low frequency side may be due
In part to these causes and in part to the oversimplification of the present model.

In the ordinary microwave radio range of frequencies from 2,000 to 20,000 mc/sec the permeability $\mu$ is chiefly imaginary, according to Table 1.
Ferromagnetic Resonance

Several predictions have been made that resonance effects or peaks in the permeability vs. frequency curve would be found at high frequencies; see for example the paper of Landau and Lifschitz in which magnetic resonance is predicted in nickel at ~3500 mc/sec.

Such effects have not been found experimentally, and it is possible to see one of the reasons why from the argument of the preceding section. The predictions have all neglected completely the effect of skin depth and eddy currents, yet in the frequency range considered we have shown that such effects are of predominant importance.

It is possible, however, that magnetic resonance effects may be detected in the magnetic oxides and sulfides of iron. These are ferromagnetic but have low electrical conductivity, so that the skin depth will be much greater than in the ferromagnetic metals. The skin depth in magnetite (Fe$_3$O$_4$) is ~5 x 10$^{-5}$ cm at 10$^4$ mc/sec, as compared with 1.6 x 10$^{-5}$ cm in iron at the same frequency. The d.c. initial permeability of magnetite is ~17. Measurements on films of ferromagnetic materials should also be pertinent when the film thickness is less than the calculated skin depth.

The resonance phenomenon may be understood as occurring when the frequency of the applied field is equal to the Larmor frequency of the atomic spins in the internal anisotropy field. This is the field due to spin-orbit interactions and distinguishes energetically different directions of magnetization in the crystal lattice. Since the anisotropy field is of the order of 1000 gauss, the corresponding Larmor frequency is in the microwave range.

It is interesting to consider a classical model in which the atoms are replaced by non-gyroscopic bar magnets pivoteted at the lattice points of the crystal. With zero applied field each magnet is attracted.

27. Reference 5; see also H. Gans and E. G. Loyarte, Ann. d. Phys. [IV] 59, 209 (1921); L. Parks, Phys. Rev. 21, 486 (1923); J. Dorfmann, Zeit. f. Phys. 17, 98 (1923); K. Kartscharin, Ann. d. Phys. [IV] 67, 325 (1922). The experiments by Kartscharin and others in which resonance phenomena were reported are now discredited.

29. The interesting possibilities of the ferromagnetic semiconductors were pointed out to the writer by Prof. A. v. Hippel, who is planning to investigate them experimentally.

in a definite direction relative to the lattice by means of individual coiled springs representing the spin-orbit interaction, and the magnets will oscillate in an applied field with a component perpendicular to the rest position of the magnets. The sine of the angle of oscillation is proportional to the macroscopic magnetization. Resonance occurs when the applied frequency is equal to the free period of a magnet + spring unit.

The bar magnet analogy supposes that the relaxation time of the spins is sufficiently short so that gyroscopic effects may be neglected. It is not usually recognized that this assumption is being made. If this assumption is not true, the spins will precess about the field direction without lining up. It is indeed a prerequisite for any type of magnetization that the magnetic moments have time in which to line up in the instantaneous local field to which they are subjected. The calculation of the relevant relaxation time is a problem in the kinetics of thermodynamic equilibrium.

The time-dependent processes can be described by assuming the existence of a relaxation time, as was done by Gorter and Kronig for paramagnetic relaxation, and by Landau and Lifschitz for the ferromagnetic case. The quantum mechanical calculation of the relaxation times, starting from the detailed interactions of spins with the lattice, is extremely difficult and uncertain. Calculations for the paramagnetic case have been made by Waller and others. 26 No calculations have been carried out for the ferromagnetic case, so far as the author is aware. It seems plausible to suppose that the strong spin-dependent coupling in ferromagnetic materials will assure that the relaxation frequency will occur above the microwave range. This question should be looked into more closely.

It should be noted in passing that the collision frequency of the lattice phonons at room temperature is \(10^{12}\) collisions per second, as estimated from values of the thermal conductivity of non-metallic crystals. This figure determines an approximate upper limit to the order of magnitude of the spin relaxation frequency. The actual spin relaxation frequency may be lower depending on the strength of the coupling between the spins and the lattice. In metals the relaxation frequency of the lattice phonons is estimated to be of the order of \(10^{13}\) sec\(^{-1}\) at room temperature, based on electrical conductivities.

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26. See, for example, I. Waller, Zeit. f. Phys. 72, 370 (1932). The writer is indebted to Prof. L. Tisza for several discussions of the paramagnetic relaxation problem.
Arkadieg\textsuperscript{27} first suggested that eddy current effects might be important in ferromagnetic dispersion. This approach was developed further by Becker\textsuperscript{26}, who pointed out that the local microscopic eddy currents associated with the movement of domain boundaries and the rotation of domains set up a magnetic field which opposes the applied field. This back field adds a term to the equations of motion which is proportional to the velocity of boundary movement or spin rotation; that is, the eddy currents behave like a viscous force. Becker's treatment gives a good qualitative account of the damping of irreversible displacements characterizing magnetization in medium fields, at frequencies below the microwave range, although an apparent difficulty in reconciling these results with the measurements of Sixtus and Isken on the velocity of boundary propagation has been suggested by Mike van Leeuwen\textsuperscript{29}. It should be pointed out that the local eddy current effects considered by Becker have no direct connection with the use made of the eddy current equation in the present paper, according to which the incomplete penetration of the surface domains by the applied field is the major cause of dispersion.

Becker also has given a calculation for the dispersion of the initial permeability, and this calculation leads to results in some respects similar to those of the present paper. The "back field" is calculated as in the medium field strength case just mentioned. The basis of Becker's theory supposes that the skin depth is greater than the domain dimensions, so that the calculation is not applicable to the microwave range, where the skin depth is less than the domain dimensions. At $3 \times 10^3$ m/sec the skin depth is only $\sim 0.1$ of the domain thickness.

The present theory probably could be improved by working with a more complicated model than that of a film one domain thick. If the film is backed on one side by a mass of ferromagnetic material the motion of the domain boundaries in the film will induce eddy currents in the backing.

\textsuperscript{27} W. Arkadiev, C. R. Acad. Sci. URSS (Doklady) 2, 204 (1935), see also reference 3.
\textsuperscript{28} See reference 4.
\textsuperscript{29} H. J. van Leeuwen, Physica 11, 38 (1944).
material. The current will be in phase with the external damping. However, the contribution to the permeability calculated in this paper is expected to be small since, in the lower half of the dispersive region, the eddy current damping is small enough, while at the high frequency end the influence of the eddy current on the film sheet to greatly restricted domain movement and hence the eddy current loss itself is unimportant.

It does not seem worthwhile at this time to attempt to calculate the permeability with a more elaborate model. The present model gives results in reasonable agreement with experiment, and the dispersive mechanism proposed here appears to correspond to the physics of the situation. The most important direction in which the model should be extended would seem to be in a treatment of the case in which the surface energy of the domain wall is made in competition with the magnetization energy, so that the domain wall yields locally to the field. This situation would be due to the magnetic inhomogeneity of the material. The model treated in the present paper assumes that the domain wall moves rigidly under the influence of the exciting field. There are reasons for believing that both cases may occur in different actual materials.

It should be pointed out that the information regarding domain behavior obtained from dispersion measurements on metals pertain only to the domains in the surface layers of the material. With this qualification, dispersion measurements may prove to be an important method for studying domain mechanisms.

Acknowledgements

It is a pleasure to thank Prof. A. C. O. Cooksey for encouragement and interest. Miss Patricia Roland has been of great assistance in the preparation of this work.
Relation of intrinsic surface impedance to resistive losses and inductance of film

It can be shown that the resistive losses in the film considered in Section IV and the contribution of the film to the inductance of a circuit element are related directly to the intrinsic surface impedance which is defined according to Eq. (22) by \( Z = E_z / H_x \), evaluated at \( y = d \).

The average rate of energy loss per unit area normal to the \( y \) direction is given by the average value of the Poynting vector

\[
S = - \text{Re} \left[ \frac{c}{4\pi} E_y(d) H_x^*(d) \right],
\]
when it is recalled that the film has two surfaces. Now \( E_z = Z H_x \), so that

\[
S = - \left( \frac{c}{4\pi} \right) H_x(d) H_x^*(d) \text{Re}[Z],
\]
a well-known result.

The contribution of the film to the inductance of the magnetizing circuit is given by the quotient of the magnetic flux through the film by the current in the magnetizing circuit:

\[
L = \text{Re} \left[ \mu \int_{-d}^{d} H_x \, dy / J \right],
\]

Now \( J = (c/4\pi) H_x(d) \) and \( \mu H_x = \frac{c}{4\pi}(c/\omega)(\partial E_z / \partial y) \), so that

\[
\omega L = \omega \text{Re} \{ J \} = - \omega \text{Im} \{ Z \}.
\]
Figure 1  

Figure 2  

Figure 3  

Figure 4  Comparison of another experimental value for $\mu = 5.5 \times 10^{-4}$ cm.
IRON

\[ \mu_r \]

- POTAPENKO AND SÄNGER-IRON
- ARKADIEW-ANNEALED SWEDISH IRON
- ARKADIEW - SOFT STEEL
- E. MAXWELL - ELECTROLYTIC IRON
- E. MAXWELL - COLD ROLLED STEEL

\[ \mu_l \]

- POTAPENKO AND SÄNGER-IRON
- LINDMAN - IRON
- HOAG AND JONES - IRON
- HOAG AND GOTTLIEB - IRON
- GLATHART - IRON

Frequency vs. Permeability Graph

- Axes: Frequency (MC/SEC) vs. Permeability
- Data points for different materials
- Log-log scale
NICKEL

$\mu_R$

■ ARKADIEV

○ E. MAXWELL

$\mu_L$

□ LINDMAN

△ HOAG AND GOTTLIEB

○ GLATHART

PERMEABILITY

100
50
10
5
2
1

FREQUENCY (Mc/SEC)

$10^2$
$10^3$
$10^4$
$10^5$