Modeling of and Experiments on Electromagnetic Levitation for Materials Processing

by

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Abstract

Electromagnetic levitation (EML) is an important experimental technique for research in materials processing. It has been applied for many years to a wide variety of research areas, including studies of nucleation and growth, phase selection, reaction kinetics, and thermophysical property measurements. However, the design of these systems has, for the most part, been empirical, and it will be shown that a more fundamental approach can provide benefits in a number of aspects, leading to a better design. The work presented here contributes to three aspects of levitation systems: modeling of electromagnetic effects, modeling of fluid flow characteristics, and experiments to measure surface tension and viscosity in microgravity.

In this work, the interaction between the electromagnetic field and the sample were modeled, and experiments to measure the surface tension and viscosity of liquid metal droplets were performed. The models use a 2-D axisymmetric formulation, and use the method of mutual inductances to calculate the currents induced in the sample. The magnetic flux density was calculated from the Biot-Savart law, and the force distribution obtained. Parametric studies of the total force and induced heating on the sample were carried out, as well as a study of the influence of different parameters on the internal flows in a liquid droplet.

The oscillating current frequency has an important effect on the feasible operating range of an EML system. Optimization of both heating and positioning are discussed, and the use of frequencies far from those in current use for levitation of small droplets provides improved results. The dependences of the force and induced power on current, frequency, sample conductivity, and sample size are given.

A model coupling the magnetic force calculations to a commercial finite-element fluid dynamics program is used to characterize the flows in a liquid sample, including transitions in the flow pattern. The dependence of fluid flow velocity on positioning force, sample viscosity, and oscillating current frequency is presented.

These models were applied to the design of thermophysical property measurements were performed in microgravity on the Space Shuttle. These experiments depend on careful control of the fluid flow in the sample, based on the MHD model presented. The measurements use the oscillating drop technique to provide very precise containerless measurement of surface tension, and the first containerless measurement of viscosity.

Results are presented for surface tension and viscosity of a Pd-18Si alloy for a large range of temperature, including both the superheated and undercooled regimes, as an example of the many data taken on many materials, including zirconium, steels, and modern metallic glass forming alloys.
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Chapter 1: Introduction

Section 1.1: Purpose

The work presented here consists of research on various aspects of electromagnetic levitation (EML) for measuring properties of liquid metals. The goal of this work is to improve both the design of the levitation systems themselves and the design of the experiments performed by EML. This goal is achieved by the detailed studies of the relevant parameters. Finally, results from Space Shuttle experiments reliant on these calculations are presented.

The important characteristics of the combination of an EML system and a particular sample material include: containment force, minimum and maximum temperature, and fluid flow in the liquid sample. The importance of containment force and independent temperature control is dictated by the need to process a variety of liquid metals and semiconductors over a wide temperature range. However, fluid flow is also a critical factor in many experiments; for example, turbulent eddies in the internal flow will provide an alternate mechanism for momentum transfer, making viscosity measurements impossible.

The main contributions detailed in this work are:

(1) A parametric study of the design parameters of an EML system. The dependence of positioning force, and induced heating are given with respect to current frequency, sample size, and sample conductivity, for both quadrupole (positioning) and dipole (heating) fields. A discussion of the results and conclusions of this study is presented in Chapter 3; some more detailed plots intended for reference in the design of the Auburn EML are also available in Appendix A.

(2) A parametric study of the flow in a levitated droplet. This study gives the dependence of the maximum fluid flow velocity in the droplet on oscillating current frequency, positioning force, and sample viscosity. The same tools are used to illustrate some of the transitions in fluid flow patterns, and may be used to extrapolate flow conditions, including a transition to turbulence, from one sample system to another.

(3) A successful containerless microgravity investigation of thermophysical properties of a Pd-Si alloy. This measurement is the first experimental validation of the oscillating droplet technique for measuring viscosity, and also provided very high-precision surface tension data on this alloy system. Also, when combined with the anomalous viscosity measurements obtained in the 1994 Shuttle flight [1], these measurements help define the feasible regime of internal flow for conducting these measurements.
Section 1.2: Background

Section 1.2.1: Electromagnetic Levitation

Electromagnetic levitation provides a means for positioning and containing a metal sample without contact by a crucible, first used by Muck [2]. The basis of EML is that eddy currents are induced in a conductive sample placed in an alternating electromagnetic field. These currents heat the sample, giving rise to the common name “levitation melting”. The interaction of the induced current with the magnetic field produces a localized Lorentz force. This force drives fluid flow in a liquid sample. When the applied magnetic field has a gradient, there is a net force exerted on the droplet; the volume integral of the local Lorentz force is non-zero.

This net force is used to levitate a sample against gravity, or to contain a sample against residual accelerations in a microgravity environment. The main difference between designs intended to operate in microgravity and those intended for ground-based applications is in the coil design. In a ground-based levitator, the greatest force is needed to oppose gravity, so these systems are designed with a large gradient in the vertical direction. In reduced-gravity experiments, however, the direction of the perturbations is variable, so the field gradient should ideally be the same in all directions. This condition is difficult to achieve with only one set of positioning coils; however, microgravity levitators need greater radial containment and less vertical containment than ground-based systems.

Electromagnetic levitation is widely applied in materials research. This technique is used in studies of nucleation and phase selection, reaction kinetics, and thermophysical property measurements, among other topics. These experiments take advantage of the fact that EML is a containerless process, eliminating the heterogeneous nucleation sources and chemical contamination that can be caused by a container. Some of the applications are detailed below, along with some of the relevant issues particular to each.

EML can be used to measure the surface tension and viscosity of a liquid metal by the oscillating droplet technique, which is described in §1.3.5. In a typical experimental cycle, shown in Fig 1.2.1, the sample is positioned, melted, and superheated. Then, the sample is allowed to cool, and squeezing pulses of the heating field are applied to excite surface oscillations. Eventually, the sample undercools and recalesces.

In this experiment, the most important issue is fluid flow. If the internal flow in the droplet should become turbulent, the turbulent eddies provide an alternate mechanism for damping of the surface
oscillations, and an anomalously high viscosity is measured. It is believed that turbulent internal flows were responsible for the anomalous results made on the IML-2 Space Shuttle mission in July 1994, when similar measurements were attempted[1]. Microgravity is critical to achieving laminar flow in these droplets; however, even in a microgravity levitator, modeling and careful design of experiments are important.

Other important issues include both the maximum and minimum temperature available for a given sample. Measurements of this type were performed in the TEMPUS EML facility on the MSL-1 Space Shuttle mission in 1997 on materials ranging in melting point from about 700°C to 1852°C, and on the Pd-Si alloy sample alone from about 600°C to over 1400°C. Such a wide temperature range requires independent control of heating and positioning of the sample.

Experiments on nucleation and phase selection employ a similar thermal profile to that used by the thermophysical property measurements (Fig. 1.2.1), except that the excitation pulses are not required. The sample is melted and undercooled, then either experiences spontaneous nucleation, or recalescence is triggered by a stimulus needle at the desired temperature. In practice, this class of experiments was often performed in conjunction with the oscillation measurements.

These experiments are also sensitive to fluid flow. Experiments on MSL-1 were performed to explore the effect of internal fluid flow on nucleation in zirconium by Bayuzick and Hofmeister of Vanderbilt University, and the effect of convection on phase selection and solidification velocity was a part of the investigation by Flemings and Matson of MIT. The author is collaborating with both groups to provide an estimate of the internal flow velocity conditions, using the models described in this thesis.
Fig 1.2.1: Typical thermal cycle for thermophysical property measurements. The sample is melted and superheated, then allowed to cool. As the droplet cools, the heating field is pulsed to excite surface oscillations. The sample recalesces and is fully solidified, then mechanically damped in preparation for the next cycle.

Section 1.2.2: Current Electromagnetic Levitation Systems

In the following chapters, reference is made to several different electromagnetic levitation systems. There are three different microgravity levitators mentioned: TEMPUS, MEL, and the Auburn EML, as well as a "typical" ground-based levitator.

It is difficult to characterize the class of ground-based levitators, but approximate ranges of the various design parameters follow. Most levitators are designed to handle samples of a few grams, and only ones using simple coils are considered here, as the area of flux concentrators is outside the scope of this work. The simple levitators typically use a few hundred amps at a frequency in the range of 10 kHz to 1.5 MHz, although Fromm and Jehn [3] report levitating a 20 mm diameter copper sphere with 4500A at 50 Hz! Also, ground-based levitators are usually of a single-frequency design, as lifting a sample against earth's gravity usually supplies more than enough heat to melt the sample.
TEMPUS is a German acronym for “Tiegelfreies ElektroMagnetisches Prozessieren Unter Schwerelosigkeit,” (which is “containerless electromagnetic processing under weightlessness” in English). This levitator has been used on three different Space Shuttle missions: the Second International Microgravity Laboratory (IML-2), the First Microgravity Science Laboratory (MSL-1), and the reflight of MSL-1, unofficially known as MSL-1R.

This levitator is a purely microgravity facility, which is unable to support a sample against Earth’s gravity. It uses two sets of coils to provide separation of heating and positioning. The TEMPUS MSL-1 coil configuration is shown in Figure 1.2.2 (a). MSL-1 TEMPUS can process metallic samples of 7 or 8 mm diameter, with melting points from about 700°C to over 1850°C. Under typical operating conditions, the positioning coils are operated continuously at about 200 amps at 160 kHz. Then, when the sample is stably positioned, the heating coil current, which is at 350 kHz, is increased from near zero to about 250 A to melt the sample. The sample melts and the liquid is superheated. Then the heating current is reduced, and the sample cools.

Another microgravity levitator is the Modular (or Marshall) Electromagnetic Levitator (MEL). MEL was a two-frequency levitator used for experiments on parabolic flights. This powerful levitator could position samples against gravity, despite its symmetric positioning coil design, shown in Fig. 1.2.2 (b). Typical values for positioning were 350A at 300kHz, and samples were melted with about 200A at 600 kHz.

The third microgravity levitator design is the subject of Chapter 3. This levitator is being built by Auburn University and has recently been named “Vulcan”. This device is being developed for use on the International Space Station. Our research group was awarded a contract to make recommendations about the design and operation of this device based on our modeling and experience with other systems. The current working coil system is shown in Fig. 1.2.2 (b), while recommendations for the operating parameters of this device are made in §3.3.
Section 1.2.3: Modeling of Electromagnetically Levitated Droplets

The history of modeling of the magnetic force and induced heating of levitated droplets begins with the work of Okress, et al. in 1952 [4]. This “dipole” model replaces the magnetic field of a spherical sample with that of a single current loop. The magnetization of a sphere in a uniform magnetic field is used to calculate the size and current in this loop. This model is useful to estimate the force on a sphere in a magnetic field with a small gradient, as when the space between the sample and the coils is large, despite the contradictory assumption of a uniform field in calculating the magnetization of the sample.

A model presented by Fromm and Jehn [3], based on an expression by Smythe [5, §10.08] for the power induced in a sphere in a uniform magnetic field gives a solution in similar form to that of Okress for the force. While the expression for induced power is exact for the case of a uniform field, that situation is only approximated in electromagnetic levitation systems.

Fromm and Jehn also provide a theoretical analysis of the ratio of positioning force to induced heating power, and conclude that a two-frequency levitation device is desirable for allowing some separation of the heating and positioning of samples. Their other contributions include summation rules for multiple source coils and experimental measurements of the force on non-spherical samples.

Rony provides detailed derivations for the model of Okress, et al. and Fromm and Jehn in his review work [6], as well as a survey of the properties of all metals with respect to levitation.
Holmes [7] uses this model to consider the stability criteria for levitation of a sphere, as well as providing expressions for the field of helical and conical source coils.

A major improvement to the “dipole” method of calculation of the magnetic force and induced heating of levitated samples came with the work of Brisley and Thornton in 1963 [8]. Their paper gives an exact solution for the total force on a sphere in a system of axially symmetric coils as a series of Bessel functions. Sathuvalli and Bayazitoglu [9] show that the “dipole” expression for force is identical to the first term of this series solution.

Lohöfer presented a representation of the induced power similar to Brisley and Thornton’s expression for force [10], extended this expression and Brisley and Thornton’s force calculations to arbitrary source current distributions [11], and found an expression for the impedance of the droplet [12], allowing measurement of the electrical conductivity of the sample by a levitation technique.

Li used Brisley and Thornton’s expressions for the vector potential in the drop to develop expressions for the current distribution in the sphere [13]. This allows the prediction of the temperature distribution and transient thermal response within the sphere. He also used a similar technique for the force distribution in the droplet, allowing estimation of the internal flow in a liquid sample to be calculated by the linearized form of the Navier-Stokes equations [14,15].

A third formulation for modeling the force on and heating of a levitated droplet, which is employed in this work, was developed in the research group of Prof. J. Szekely. This approach relies on discretizing the droplet into a series of computational elements and calculating their interaction, is called the method of mutual inductances and is derived in §2.2. This method was first applied to spherical droplets by El-Kaddah and Szekely in 1983 [16]. A coordinate transformation allowing application to non-spherical axisymmetric bodies was made by Zong, et al. [17-19]. Another improvement, a more exact treatment of the nearly rectangular cross-section of the computational elements adapted from [20,21] was added by Schwartz [1].

The fluid flow and free surface shape of levitated droplets have also been considered by a number of different authors, using a number of different methods. Mestel [22] reported a perturbational analysis of the free surface of a levitated drop, along with a semi-analytical calculation of the fluid flow in 1981. Sneyd and Moffatt [23] also calculated flow in a levitated sample, although in this case, the sample was a toroid supported between two coplanar concentric coils. Both use a simplified magnetic force of the form

\[ F = k \frac{B^2}{\mu_0 \delta} e^{\xi}. \]
An analytic model of the internal flow in a levitated droplet based on the force expression of Brisley and Thornton was presented by Li for a spherical sample in 1994 [14-15]. This model was extended by Zhang, Li, and Pang to slightly deformed samples by a perturbational method [24]. Another purely analytic model was presented by Bratz and Egry [25], which considers only surface oscillations of a droplet, and uses a magnetic pressure formulation for the levitation force.

For free surface shape of a levitated droplet, two models were used by Gagnoud, Etay, and Garnier [26] and by Schwartz [27,28]. The first is a “local” method, based on a magnetic pressure formulation, in which the force on each point on the surface of the droplet is balanced among internal pressure, magnetic pressure, and surface tension. Both also employed a variational formulation as an alternate approach, where the shape results from a minimization of the total potential energy of the droplet. The method of local normal force balance was applied to silicon in 1998 by Hahn et al. [29].

Numerical MHD models for the fluid flow in the levitated droplet have been presented by members of Prof. Szekely’s research group. El-Kaddah and Szekely calculated the flow in a levitated spherical droplet, with the forces coming from a mutual inductance formulation, and the fluid flow calculated by a finite difference method, with a turbulent kinetic energy - turbulent energy dissipation \((k - e)\) turbulence model for 1-g conditions [16] and also for zero-g conditions [30]. These calculations were extended to deformed droplets by Zong, et al. [17-19], who used a magnetic pressure formulation for free surface shape, and then calculated the fluid flow separately.

The next development was a true free-surface model by Schwartz and Szekely [1,31-33], who showed the effect of fluid flow on the shape of the free surface of a levitated droplet to be significant. They employed a uniform, enhanced viscosity to represent turbulent flows for comparison to shapes observed in earthbound levitation experiments, and predicted the extent of deformation to be expected in their microgravity experiments (IML-2, July 1994).

The purely laminar calculations presented in Chapter 4 use a similar formulation to Schwartz, but do not require any representation of turbulence, so the artificially enhanced viscosity is not needed.

Section 1.3: Surface Tension and Viscosity

Section 1.3.1: Theory of Surface Tension

Surface tension can be viewed in either of two equivalent ways: as the force which tries to collapse the interface between a liquid and its vapor, with units of N/m, or as the surface energy of this interface,
with units of $J/m^2$. These two views are different aspects of the same physical phenomenon. A survey of different theoretical formulations for surface tension may be found in Iida and Guthrie [34, pp.120-133], which is summarized below.

One important representation of this phenomenon is the formula by Fowler [35], based on the pair theory of liquids in statistical mechanics. This formula, given in equation 1.3.1, is based on a discontinuity in the density function $n_o(z)$, and neglecting effects of the gas phase.

\[
\gamma = \frac{m_o^2}{8} \int g(r) \frac{d\phi(r)}{dr} r^4 dr
\]  
(eq. 1.3.1)

According to Iida and Guthrie, this formulation often provides good agreement with experimental data, but there is considerable difficulty in obtaining reliable representations for the pair distribution function $g(r)$ and the pair potential $\phi(r)$. Data presented by Waseda and Suzuki [90] show that the temperature dependence of surface tension is only qualitatively described by this formula. The temperature dependence of the pair distribution function $g(r)$ is given by approximately by Fowler as $g(r) = e^{-\phi(r)/kT}$, while Waseda and Suzuki state that the pair potential $\phi(r)$ is not strongly affected by temperature.

Other purely theoretical formulations based on the hard-sphere and free electron models are also covered in this survey, but no information on their agreement with experiment is presented.

Semi-empirical formulae for surface tension are very useful for liquid metals. Iida and Guthrie give correlations for the melting-point surface tension of metals vs. melting point $T_m$ [34, p.129] and heat of evaporation at the melting point, $\Delta_f^0 H_m$ [34,p.131]. $V_m$ is the molar volume at the melting point; the subscript $m$ denotes quantities at the melting point.

\[
\gamma_m = 4.8 \times 10^{-8} \frac{RT_m}{V_m^\frac{3}{2}} \quad \text{(in SI units)}
\]

\[
\gamma_m = 1.8 \times 10^{-9} \frac{\Delta_f^0 H_m}{V_m^\frac{3}{2}}
\]  
(eq. 1.3.2)

These relations imply that $\Delta_f^0 H_m = 26.7RT_m$. However, this is not the correct form for Troughton's Law, which states that the heat of evaporation at the boiling point $\Delta_f^0 H_b$ is related to the boiling point $T_b$ by $\Delta_f^0 H_b = 91.2 \frac{J}{mol \cdot K} T_b$. 

21
Surface tension is found to vary linearly with temperature. The surface tension is identically zero at the critical temperature $T_c$. The relationship between surface tension and temperature is Eötvös' law:

$$\gamma = \frac{k_r}{V^3}(T_c - T)$$

(eq. 1.3.3)

where $k_r$ is approximately $6.4 \times 10^{-8}$ (JK$^{-1}$ mol$^{-2}$) for all liquid metals[34, p.131], and again $V$ is the molar volume, but this time at temperature $T$.

Section 1.3.2: Theory of Viscosity

Viscosity is the resistance of a fluid to shear. For most liquids and gasses made up of small molecules, the relationship between shear rate and shear stress is linear (eq. 1.3.4). Fluids exhibiting this linear relationship between shear stress $\tau_{\alpha\beta}$ and the velocity gradient $\frac{dv_x}{dz}$ are called Newtonian. All liquid metals are believed to be Newtonian; however, large polymers and fluid mixtures such as semi-solid slurries have a much more complicated relationship between shear stress and shear rate.

$$\tau_{\alpha\beta} = -\mu \frac{dv_x}{dz}$$

(eq. 1.3.4)

The sign convention for $\tau_{\alpha\beta}$ is from ref. [89], and is chosen such that $\tau_{\alpha\beta}$ represents the viscous flux of $x$-momentum in the $z$-direction.

It should be noted that this relation is only applicable to laminar flows. For turbulent flows, the relation between shear stress and velocity gradient is of higher order, and is also strongly dependent on other factors such as the magnitude of the local velocity and the geometry of the flow system.

Iida and Guthrie [34, pp.167-189] provide a survey of the different theories used to describe the viscosity of liquid metals. The dominant theory for viscosity, based on the pair theory of liquids, was derived by Born and Green[36] (eq. 1.3.5):

$$\mu = \frac{2\pi}{15} \left(\frac{m}{kT}\right)^{1/2} n_o^2 \int_0^\infty g(r) \frac{\partial \phi(r)}{\partial r} r^4 \, dr$$

(eq. 1.3.5)

where $m$ is the mass of the atoms, $k$ is Boltzmann's constant, and $g(r)$ and $\phi(r)$ are, again, the pair distribution function and the pair potential, respectively. Values calculated from this formula generally coincide with experimental data, except for a few metals [34].
Other statistical mechanical formulations are less successful. Iida and Guthrie present comparisons for expressions from the “moment method” and from a sum of kinetic, hard-sphere, and soft attractions, neither of which can match experimental viscosity to a factor of 2. A pure hard-sphere model underestimates the viscosity by about 30% [34, pp.169-172].

A semi-empirical formula known as the Andrade equation gives the melting point viscosity \( \mu_m \) in terms of the molar mass \( M \) and the molar volume at the melting point \( V_m \):

\[
\mu_m = \frac{4 \nu m}{a} = 1.6 \times 10^{-4} \left( \frac{MT_m}{V_m^2} \right)^{\frac{1}{3}}
\]  

(eq. 1.3.6)

The factor \( \frac{4}{3} \) is an estimate made by Andrade [34], \( \nu \) is the characteristic frequency of oscillation of the atoms, and \( a \) is their spacing. This relationship is found to correlate very well with the reported data, although Battezzati and Greer [37] argue that the coefficient should be \( 1.88 \times 10^{-4} \). They report excellent agreement with the Andrade formula for most alloys as well, including metal-metalloid alloys such as Fe-Si and Cr-Ge, but not for glass-forming alloys such as Fe\(_2\)B and Fe\(_4\)P.

Iida, Guthrie, and Morita [38] use a more rigorous derivation based on the pair theory of liquids, which yields a similar result (eq. 1.3.7):

\[
\mu_m = 1.2 \frac{\nu m}{a}
\]  

(eq. 1.3.7)

The temperature dependence of viscosity is still somewhat disputed. The leading relations are an Arrhenius form, the similar form proposed by Andrade, and the Vogel-Fulcher-Tammann equation (eq. 1.3.8 a, b, and c, respectively).

\[
\mu(T) = A \exp \left( \frac{E}{RT} \right)
\]  

(a)

\[
\mu(T) = \frac{A}{\nu} \exp \left( \frac{C}{VT} \right)
\]  

(b)

\[
\mu(T) = A \exp \left( \frac{C}{T - T_0} \right)
\]  

(c)

Battezzati and Greer have found that the Arrhenius form reflects the behavior of most pure metals and alloys, but that the Vogel-Fulcher-Tammann form better reflects the behavior of glass forming alloys.
Another interesting result was noticed by Egry [39], that Fowler’s formula for surface tension (eq. 1.3.1) is similar in form to Born and Green’s formula for viscosity (eq. 1.3.5). The ratio of these expressions gives the ratio of surface tension to viscosity as:

\[ \frac{\gamma}{\mu} = \frac{15}{16} \frac{kT}{m} \]  

(eq. 1.3.9)

Egry later compared this expression to published values for pure metals [40], and found agreement near the melting temperatures to be within the range of reported values.

The relationship between surface tension and viscosity should be expected from their representations under the pair theory of liquids. Both properties are related to doing work against the pair potential: to create new surface, in the case of surface tension, and to displace the neighboring atoms in the case of viscosity.

For deep undercoolings, however, this expression does not match the reported data. This discrepancy should be expected, since the expressions of both Fowler and Born and Green consider only interactions between pairs of atoms. The higher-order interactions become more important for undercooled metals, and are especially important near the glass transition.

Section 1.3.3: Experimental Measurement of Surface Tension

A collection of published data for surface tension of pure metals is found in lida and Guthrie [34]. They also provide data on some alloy systems and molten salts.

The surface tension of liquid metals can be measured in a large number of ways. Each measurement technique has its own benefits and problems, however. For example, contamination, especially by oxygen, sulfur, nitrogen, selenium, or tellurium, can have a dramatic effect on surface tension, even in very small amounts ([34], pp. 136-139 for quantitative changes).

Some of the methods are summarized below, along with important benefits and limitations. These summaries are based on ref. [34] unless otherwise noted.

The most commonly used techniques for measuring surface tension in liquid metals is the sessile drop method. In this method, a drop of the liquid metal is placed on a substrate, and its cross section is measured optically. Great care must be taken in the choice of the substrate and handling of the droplet to prevent contamination. Also, obtaining good cylindrical symmetry of the droplet is important to the accuracy of the measurement.
Another method common in measurements on metals is the maximum bubble pressure method. In this method, a capillary is immersed in the fluid, and the pressure required to separate bubbles from the tip of the capillary is measured. This method provides clean, fresh surfaces for each measurement, but the data must be corrected to compensate for the differing pressure head on the different parts of the bubble.

The maximum drop pressure method is similar, but a drop of the liquid to be measured is forced into a gas, instead of forming a gas bubble in the liquid. The advantages of this method are also similar to the maximum bubble pressure method. This method has been used up to about 1000K, but difficulty was reported in 1921 in extending this method to higher temperatures.

The pendant drop and drop weight methods have been used to obtain melting-point surface tensions for highly reactive metals at high temperature. The pendant drop analysis is similar to the sessile drop, and the analysis of the drop weight method is straightforward. Contamination from a capillary can be eliminated by using a rod or tube of the same material as the drop, but this approach is limited to measurement at the melting point and also to alloys with a narrow melting range.

The capillary method of measuring surface tension is not widely used for liquid metals because of susceptibility to contamination and difficulty in measuring the contact angle.

Recently, techniques involving capillary wave techniques have been gaining attention. Experiments using macroscale waves are reported [41], but also measurements based on tiny thermally excited surface waves called “ripplons”, which are typically about 1nm amplitude and 100 µm in wavelength [42,43]. These ripplon techniques have great potential for measurement of liquid metals, but to date have only been used on model systems such as water and water/ethanol.

Finally, there is the oscillating droplet technique, which is used in this study. This technique is explained in detail in §1.3.5.

Section 1.3.4: Experimental Measurement of Viscosity

An extensive review of published values of viscosity of pure metals and binary alloys was compiled by Battezzati and Greer [37]. More viscosity data for pure metals are found in [34] and [44], and for semiconductors in [45].
Measurement of the viscosity of liquid metals is also performed by many different methods. The following summaries are adapted from ref. [34], except as noted.

Perhaps the most straightforward is the rotational viscometer, where the fluid occupies a space between two coaxial, rotationally symmetric bodies, and the torque resulting from an applied shear rate is measured. Variations include rotating spheres, cylinders, and disks, and some devices rotate the inner body and others the outer body, but all are simple in theory. In practice, there is difficulty in applying this technique to liquid metals, both due to the difficulty in choosing a material to contact the metal, and also in the precise alignment required for measuring small viscosities.

The oscillating vessel (cylinder, cup, sphere) is the most commonly used. In this method, a vessel is filled with liquid and set in rotational oscillation. The logarithmic decrement of the oscillation is measured, and leads to the viscosity. Some theoretical difficulties exist in the analysis of results from this system, but a number of approximate relations give acceptable results.

Other methods include the capillary method, where the rate of flow caused by a measured pressure drop is measured, and the oscillating plate method, where the drag force on a flat plate immersed in the fluid is measured. Also, Nishio and Nagasaka state that it is possible to measure the viscosity of a liquid from the same ripplons analyzed to measure surface tension [43].

Finally, the oscillating drop method is described in the following section.

Section 1.3.5: Oscillating Drop Method for Surface Tension and Viscosity

The measurement of surface tension and viscosity by the oscillating drop technique relies on the fact that the resonant frequency of surface oscillations on a liquid sphere is determined by the surface tension of the liquid, and that, given no other mechanism of damping, the damping of these oscillations is related to the viscosity. Evaluation of these properties from the oscillation spectra is based on solutions from classical fluid mechanics (eq. 1.3.10), by Rayleigh [46] for surface tension and Lamb [47] for viscosity.

Rayleigh
\[ \omega_0^2 = \frac{l(l-1)(l+2)\gamma}{\rho R_s^3} \]  

Lamb
\[ \tau_l = \frac{\rho R_s^2}{(l-1)(2l+1)\mu} \]
where $\omega_1$ is the angular frequency of oscillation mode $1$, for a droplet of surface tension $\gamma$, viscosity $\mu$, density $\rho$, and radius $R_o$. A more detailed treatment of the theory and practice of these measurements is deferred to Chapter 5.

The measurement of the surface tension of liquid metals using this technique was first reported by Fraser, et al. [48] in 1971. Since then many authors, some of whom are listed in [49-54], have applied this measurement technique to liquid metals with electromagnetic levitation, and one with acoustic levitation [55], all with varying degrees of success.

Most results overestimate the surface tension by a significant fraction. This difference was at first attributed to cleaner surfaces of the levitated droplets, but a dependence of the apparent surface tension on the mass of the sample indicated that other factors were at work, as well.

Improvements were made by Soda, et al. [49], who reported the effect of large oscillation amplitudes on the measurements, Sauerland, et al. [52,53], who give a method of identifying the oscillation modes associated with split peaks in the oscillation spectrum.

The work by Cummings and Blackburn [56] allows for correction of the oscillation frequency to account for the peak splitting due to the eccentricity of drops levitated on the ground, and also for the effect of the magnetic force field on the oscillations. This correction was found to eliminate the dependence on sample mass previously seen in these measurements [52], and also brings the surface tension data into agreement with measurements by other techniques [52, 54].

The Cummings and Blackburn correction was further validated by comparison to microgravity experiments conducted in July 1994 as part of the IML-2 Space Shuttle mission by the teams of Szekely and Egry [1,57-59]. It was found that corrected 1-g data from oscillation experiments matched microgravity experimental results within the scatter of the microgravity results.

The greatest limitation of the levitation technique for measurement of surface tension is the need to correct the measured oscillation frequency for the effect of the magnetic field. This technique does, however, allow measurements of undercooled and superheated liquid metals and semiconductors without contamination or nucleation from a container. Also, the measurements may be performed quickly, and at a temperature in the accessible range above about $500^\circ$C.

The history of viscosity measurement by the oscillating drop technique is much simpler. Previous attempts to measure viscosity by this method have given anomalously high results. This discrepancy is
believed [1] to be caused by turbulent internal flows in the sample, driven by the positioning forces. Other reports [60] of attempts to measure viscosity with this method using electrostatic levitation report a similar discrepancy, perhaps due to Marangoni convection.

The measurements reported here, which were conducted on the MSL-1 Space Shuttle missions in April and July 1997, represent the first successful application of this technique to the measurement of the viscosity of liquid metals. These experiments were carried out by Prof. M. Flemings, Dr. Gerardo Trapaga, and the author, in collaboration with Prof. Ivan Egry and Dr. Georg Löhöfer of the DLR in Cologne, Germany.

The greatest limitation of this technique is the absolute requirement for laminar flow. Many different driving forces for internal flow, including electromagnetic force, natural convection, and Marangoni convection, must all be reduced or eliminated in order to perform this measurement. Therefore, measurements using this technique require great care be taken, even in microgravity.

These viscosity measurements do, however, provide the advantages of freedom from contamination and the ability to measure undercooled samples because of the lack of a container.

Section 1.4: Organization

The following chapters discuss the derivation, results, and benefits of magnetic and magnetohydrodynamic models for a levitated droplet, and the results of an experiment which depends on those models.

Chapter 2 contains a discussion of the relevant issues to magnetic model, including the criteria for choosing a method of modeling the interaction of the droplet with the magnetic fields and the types of results required of the model. A derivation of the method chosen for this study is given, along with the asymptotic relations predicted by a more simple model.

Chapter 3 involves calculated results from the design of a particular levitation system. Results which are applicable to the particular requirements of this system are presented, along with a methodology and some results which are more generally applicable to the design of levitation systems. These results are summarized from a parametric study, which is the source of the plots included as the appendix.

Chapter 4 presents results of parametric magnetohydrodynamic calculations on levitated droplets. Changes in dominance and in the qualitative flow pattern are presented, as well as results of a parametric
study of the effect of changing acceleration, viscosity, and frequency on the internal flows in the levitated sample.

Chapter 5 presents the results of an experiment which is dependent on these calculations. The surface tension and viscosity of a liquid metal alloy are measured by a containerless microgravity technique, which avoids the turbulent internal flows common in levitated droplets on the ground.

Finally, the conclusions drawn from this research are summarized in Chapter 6, and recommendations for future work are made in Chapter 7.
Chapter 2: Magnetic Model

Section 2.1: Summary of Methods

Section 2.1.1: Introduction

A good electromagnetic model for the behavior of a levitated droplet is an essential part of the effort to design successful experiments and to suggest improvements to the design of the levitator itself. The model must predict the response of a droplet to a levitator with multiple coils operating at different frequencies and phases. It must be able to address the deformation of liquid droplets, in order to assess equilibrium shape and determine the required pulse height for oscillation experiments.

The electromagnetic model must provide information about the total force and force gradient experienced by the sample, so that its stability may be evaluated for various operating conditions. Also, the Joule heat applied to the droplet is required for planning the thermal cycles. Not only the total heat and force are required, but also their distribution to allow calculation of the droplet’s internal flows and heat transfer. The force distribution also allows the evaluation of the effect of the magnetic fields on the droplet’s surface oscillation frequency.

Some desirable characteristics of the magnetic model are that it be versatile and adaptable, able to handle similar problems in induction heat-treatment, induction melting, and induction stirring. Also, the model must be easily automated and fast enough to readily run hundreds or thousands of cases at a time on a computer workstation for evaluation of the effect of changes in frequency, droplet conductivity, sample size, etc. on the droplet response. Specific areas addressed in this chapter include effects on total force and force gradient, volumetric force distribution, induced heating, and positioning efficiency (force per unit heat). Finally, the model should provide easy integration into any fluid dynamics packages, such as the commercial codes FIDAP (used in Chapter 4), and PHOENICS.
Fig 2.1.1: TEMPUS coil configuration and typical operating parameters.

For the case of TEMPUS, shown in Fig. 2.1.1, the model must represent a quadrupole positioning field produced by six coils, and a dipole heating field produced by six additional coils. There is a large range in the operational parameters such as coil current and process environment to accommodate the large range of sample materials.

A number of different possible approaches were considered for modeling the thermal and mechanical response of the droplet to the source currents. The relative merits of these approaches are detailed in §2.1.3, subject to the parameters discussed in §2.1.2.

Section 2.1.2: Relevant Timescales and Dimensionless Parameters.

There are a number of representative times that are important in characterizing a system. Some of the relevant times are listed below in Table 2.1.1, with the symbols defined in Table 2.1.2. The “typical” values listed are for a 7 mm diameter nickel sample. Most of nickel’s properties are representative of the samples flown on TEMPUS, but its melting-point viscosity, while typical of transition metals, is very different from that of the deep eutectics that make up a large fraction of the TEMPUS samples.
Characteristic Time | Definition | Value (typical)
--- | --- | ---
electromagnetic wave transit time | $\tau_{em} = \frac{l_0}{c}$ | $1.17 \times 10^{-11}$ s
charge relaxation time | $\tau_e = \frac{\varepsilon}{\sigma_{el}}$ | $7.38 \times 10^{-18}$ s
magnetic diffusion time | $\tau_m = \frac{\mu \sigma_{el} l^2}{\rho}$ | $1.85 \times 10^{-3}$ s
viscous relaxation time | $\tau_v = \frac{\rho l^2}{\eta}$ | $2.45 \times 10^{-1}$ s
material transit time | $\tau_{conv} = \frac{l_0}{U_o}$ | $1.75 \times 10^{-4}$ s
reciprocal oscillation frequency | $\tau_o = \frac{1}{\omega}$ | $1.12 \times 10^{-4}$ s (TEMPUS positioner)

Table 2.1.1: Characteristic timescales relevant to electromagnetic levitation systems. After [66]. Symbols and typical values are given below.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_0$</td>
<td>Reference length (=radius of droplet)</td>
<td>$3.5 \times 10^{-3}$ m (typical)</td>
</tr>
<tr>
<td>$c$</td>
<td>speed of light</td>
<td>$2.9979 \times 10^8$ m/s</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>electrical permittivity. For non-polarizable materials, including metals, $= \varepsilon_o$</td>
<td>$\varepsilon_o = 8.845 \times 10^{-12}$ F/m</td>
</tr>
<tr>
<td>$\sigma_{el}$</td>
<td>electrical conductivity. Subscript omitted when there is no confusion with other properties. Range for metals:</td>
<td>$1.2 \times 10^6$ ((\Omega ) m)$^{-1}$ (typical)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>magnetic permeability. For non-magnetic materials, including liquid metals, $= \mu_o$</td>
<td>$\mu_o = 4 \pi \times 10^{-7}$ H/m</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density. Range for metals:</td>
<td>$8 \times 10^4$ kg/m$^3$ (typical)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>viscosity. Typical for transition metal at Tm: Range for metals (including metallic glass formers):</td>
<td>$4 \times 10^3$ Pa-s (typical)</td>
</tr>
<tr>
<td>$U_o$</td>
<td>Reference velocity. Typical value:</td>
<td>$5$ cm/s (typical for $\mu$)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular frequency, $= 2\pi f$. In TEMPUS:</td>
<td>$2\pi \times 160$ kHz(pos), $2\pi \times 351$ kHz(heat)</td>
</tr>
</tbody>
</table>

Table 2.1.2: Symbols and typical values.

As shown in the table, $\tau_m >> \tau_{em} >> \tau_e$. Therefore, the system may be approximated as magnetoquasistatic (MQS); that is, the electric and magnetic fields are dominated by the current flows, since for the case of liquid metals, the magnetization density is zero. Because the MQS approximations are appropriate, a simplified form of Maxwell’s equations may be employed:
\[ \nabla \times \vec{H} = \vec{J} \]
\[ \nabla \cdot \vec{B} = 0 \]
\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]  
(eq. 2.1.1)

Where \( \vec{J} \) is the current density, \( \vec{H} \) the magnetic field, \( \vec{B} \) the magnetic flux density (\( \vec{B} = \mu_0 \vec{H} \)), and \( \vec{E} \) is the electric field.

When considering a coupled magnetohydrodynamic simulation, the required complexity of the model is greatly influenced by whether or not the effect of the fluid velocity \( \vec{u} \) on the induced current density is significant. This effect is seen in equation 2.1.2.

\[ \vec{J} = \sigma_e \left( \vec{E} + \vec{u} \times \mu_0 \vec{H} \right) \]  
(eq. 2.1.2)

The parameter that determined this interaction is the magnetic Reynolds number [66] (eq.2.1.3), which for a typical levitation system is of the order of \( 10^{-4} - 10^{-3} \). Since this is much less than unity, the flow field does not significantly affect the induced current, and a semi-coupled model is appropriate, with the magnetic calculations neglecting the effects of fluid flow.

\[ \text{Re}_m = \frac{\tau_m}{\tau_{conv}} = \frac{\mu_0 \sigma U l_o}{\rho} \]  
(eq. 2.1.3)

Section 2.1.3: Selection of Calculation Method

The next task in modeling the behavior of a levitated droplet is to select an appropriate methodology for solving Maxwell’s equations. The level of fidelity and outputs required of the model determine which approach is most applicable.

The early analytical models for levitation of a metal droplet [3,5] provide a good starting point for understanding the system. These analyses are based upon consideration of the magnetization of a sphere in an alternating field. In a homogeneous field, the sphere acts much like a single magnetic dipole, and may be approximated by a single current loop. These “dipole” analyses show good agreement between experiments and theory, but only for limited conditions: a spherical sample on the axis of a system of current loops. Also, no information is provided about the fields, forces, or power distribution inside the droplet.

A more advanced analytical technique has been published by Brisley and Thornton [8] for axisymmetric, and extended by Lohöfer [10-12] to more general problems. Lohöfer’s method offers several
advantages over Okress' method, including a more realistic representation of the induced current density, support for arbitrary source current densities, and a coordinate transformation that allows easy calculation of force and power of a sample displaced in three dimensions.

Lohöfer's model is very useful for designing experiments, since it can readily give the sample heating, as well as total force and force gradient on the sample in both \( r \)- and \( z \)- directions. Also, this model is very good for evaluating levitation system designs, as it provides the forces and power for each case rather quickly. However, this model has some serious limitations for application to this work. It is limited to the case of perfect spheres, and cannot describe the distribution of the force and heating in a droplet.

Since none of the currently available analytical solutions meet all of the requirements set out in §2.1.1, numerical methods must be considered. A direct numerical solution of Maxwell's equations seems an attractive possibility, since such a method will provide distributions as well as total values for the current density, magnetic field and flux density, force, and power. It could model the effects of arbitrary 2- and 3-dimensional source currents on an arbitrarily shaped 2- or 3-dimensional body.

However, a model based on numerical solution of the PDE's has its own limitations. While such codes are available commercially, a commercial code is likely to be slow and difficult to automate and to interface to other commercial products. Writing such a program from scratch is also no small challenge, and would fall outside the scope of this work.

There is one other solution method to be considered. The "method of mutual inductances" [16-19], which lies in complexity between Lohöfer's method and a direct solution of Maxwell's equations. This method, described in §2.1.4, is a two-dimensional axisymmetric model. While this restriction limits its general applicability, a levitation melting system is well approximated as axisymmetric. The method of mutual inductances can model the effect of arbitrary axisymmetric source currents, on arbitrary axisymmetric bodies. This method provides the distributions of current density, fields, forces, and heating power, as required.

The requirement for axisymmetric sources and droplets has one important consequence: this method can not be used to calculate the radial restoring force on the sample. While calculation of the radial restoring force is important to design, the ratio of radial to axial restoring force is constant for a given coil design, and may be readily calculated by Lohöfer's method.

Since the source code and derivation are available for this method, its strengths and weaknesses are well known. Also, having the source code helps make this model more adaptable, and easily extended...
to provide input data to any commercial fluids code or postprocessor. Automation of this method has proven feasible, and, with the extensions described below, its computational efficiency is quite good.

The method of mutual inductances does have limitations. For example, the strict requirement for axisymmetry makes evaluation of the radial force on the droplet impossible. Despite these limitations, the method of mutual inductances seems best suited for modeling the magnetic effects and magnetohydrodynamics in electromagnetic levitation.

Section 2.2: Method of calculation of magnetic quantities.

The method of mutual inductances is a versatile tool for modeling magnetic effects in axisymmetric systems. The solution method is as follows: first, the physical domain (droplet) is discretized into small elements (see Fig. 2.2.1). The distribution of elements for numerical methods is somewhat of an art, but the grid distribution should follow the physics of the problem. In this case, that means the grid is distributed exponentially in the radial direction, concentrated near the outer edge of the droplet, and distributed linearly in the theta-direction.

Next, a matrix equation is constructed relating the unknown element currents to the known source currents. Upon solution of this matrix equation, the (now) known element currents are used to calculate the magnetic flux density in the droplet. Then, knowing the currents and fields, as well as the physical properties of the droplet, it is possible to calculate the Joule heating and Lorentz force experienced by the droplet.

The implementation of the method of mutual inductances which is used in this study is the end result of over fifteen years of gradual improvements made in the research group of the late Prof. Julian Szekely. The formulation presented in §2.2.1 is based on papers published by former members of the group [17-19], and extensions made by Schwartz [1], also formerly of the Szekely group.
Section 2.2.1: Calculation of Current distribution

The vector potential \( \vec{A} \) may be calculated by the Biot-Savart law:

\[
\vec{A} = \frac{\mu_0}{4\pi} \int \frac{j(\vec{r}') d\vec{r}'}{|\vec{r} - \vec{r}'|}
\]

(eq. 2.2.1)

Computational elements are chosen to be small enough that the current density in each element may be approximated by a constant current:

\[ \vec{J}_{i,f} = \vec{J}_{i,f} S_i \]

(eq. 2.2.2)

with \( S_i \) being the cross-sectional area of element \( i \) shown in Fig. 2.2.1. Since there are only currents in the \( \phi \)-direction, the vector potential has only a \( \phi \)-component:

\[
A_{\phi} = \frac{\mu_0}{4\pi} \sum_{f=1}^{N_f} \left( \sum_{i=1}^{N_i} \frac{I_{i,f} dS_i}{r_{i,f}} + \sum_{k_f=1}^{K_f} \frac{I_{k_f,f} dS_{k_f}}{r_{k_f}} \right)
\]

(eq. 2.2.3)

where the summation in \( f \) represents the superposition of the contributions from different frequencies in the source coils, and \( r_{i,f} = |\vec{r}_i - \vec{r}_f| \). The summation in \( k_f \) covers the contribution of each coil carrying current at frequency \( f \), while the summation in \( l \) represents the currents induced in different computational elements in the droplet.

Equation 2.1.1 and the constitutive relation \( \vec{B} = \nabla \times \vec{A} \) imply that the electric field \( \vec{E} \) is related to the vector potential \( \vec{A} \) by:

\[
\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla \Phi
\]

(eq. 2.2.4)

but since there are no imposed electric fields, \( -\nabla \Phi = 0 \). Since the current density \( \vec{J} = \sigma \vec{E} \), for complex current \( \vec{I} = \text{Re}\{\vec{I} e^{i\omega t}\} \), \( \vec{J} = \text{Re}\{\vec{J} e^{i\omega t}\} \), etc., where \( \text{Re}\{\} \) indicates the real part

\[
J_{\phi} = -j \sigma \omega A_{\phi}
\]

(eq. 2.2.5)

For convenience, we will drop the subscript for the \( \phi \)-component, and consider only one frequency \( f \) at a time, superposing them at the end. Substituting for \( A_{\phi} \),
\[
\frac{jI_{i,j}}{\sigma \omega_j} = \frac{\mu_0}{4\pi} \left( \sum_{l=1}^{N} \oint \frac{I_{i,f} ds}{r_{ii}} + \sum_{k_{f},=1}^{N_{k_{f}}} \oint \frac{I_{k_{f},}}{r_{k_{f}i}} ds \right)
\]  
(eq. 2.2.6)

Taking \( \oint ds \) yields
\[
\oint \frac{jI_{i,f}}{\sigma \omega_j} ds = \frac{\mu_0}{4\pi} \left( \sum_{l=1}^{N} \oint \frac{I_{i,f} ds}{r_{ii}} + \sum_{k_{f},=1}^{N_{k_{f}}} \oint \frac{I_{k_{f},}ds}{r_{k_{f}i}} ds \right)
\]  
(eq. 2.2.7)

The mutual inductance between two current loops is
\[
M_{ii} = \frac{\mu_0}{4\pi} \oint \frac{ds \cdot ds_i}{r_{ii}}
\]  
(eq. 2.2.8)

Substituting into equation 2.2.7 gives
\[
\oint \frac{jI_{i,f}}{\sigma \omega_j} ds = \sum_{l=1}^{N} I_{i,f} M_{ii} + \sum_{k_{f},=1}^{N_{k_{f}}} I_{k_{f},} M_{k_{f}i}
\]  
(eq. 2.2.9)

Since \( I_{i,f} = I_{i,f} \) and \( R_i = \oint ds_i \), equation (2.2.9) becomes
\[
\frac{j}{\omega_j} I_{i,f} R_i = \sum_{l=1}^{N} I_{i,f} M_{ii} + \sum_{k_{f},=1}^{N_{k_{f}}} I_{k_{f},} M_{k_{f}i}
\]  
(eq. 2.2.10)

Now it is convenient to separate \( I_{i,f} \) into its real and imaginary parts \( I_{i,f}^R \) and \( I_{i,f}^I \).
\[
I_{i,f}^R R_i - \omega_j \sum_{l=1}^{N} I_{i,f} M_{ii} = \omega_j \sum_{k_{f},=1}^{N_{k_{f}}} I_{k_{f},} M_{k_{f}i}
\]  
(eq. 2.2.11a)

\[
I_{i,f}^I R_i + \omega_j \sum_{l=1}^{N} I_{i,f} M_{ii} = -\omega_j \sum_{k_{f},=1}^{N_{k_{f}}} I_{k_{f},} M_{k_{f}i}
\]  
(eq. 2.2.11b)

Solving eq. 2.2.11b for \( I_{i,f}^I \), and changing the index of summation from \( l \) to \( m \),
\[
I_{i,f}^I = -\omega_j \sum_{m=1}^{N} \frac{I_{m,f}^R M_{im}}{R_i} - \omega_j \sum_{k_{f},=1}^{N_{k_{f}}} \frac{I_{k_{f},} M_{k_{f}m}}{R_i}
\]  
(eq. 2.2.12)

Substituting into eq. 2.2.11a gives:
\[
I_{i,f}^R R_i - \omega_j \sum_{l=1}^{N} \left( -\omega_j \sum_{m=1}^{N} \frac{I_{m,f}^R M_{im}}{R_i} - \omega_j \sum_{k_{f},=1}^{N_{k_{f}}} \frac{I_{k_{f},} M_{k_{f}m}}{R_i} \right) M_{ii} = \omega_j \sum_{k_{f},=1}^{N_{k_{f}}} I_{k_{f},} M_{k_{f}i}
\]  
(eq. 2.2.13)

\[
I_{i,f}^R R_i + \omega_j \sum_{l=1}^{N} \sum_{m=1}^{N} \frac{I_{m,f}^R M_{im} M_{lm}}{R_i} = \omega_j \sum_{k_{f},=1}^{N_{k_{f}}} I_{k_{f},} M_{k_{f}i} - \omega_j^2 \sum_{l=1}^{N} \sum_{k_{f},=1}^{N_{k_{f}}} \frac{I_{k_{f},} M_{k_{f}m} M_{lm}}{R_i}
\]  
(eq. 2.2.14)

This equation can be written as the matrix equation
\[
[R + X]I = C
\]  
(eq. 2.2.15)

where
\[ R_y = R_\delta y \]
\[ X_y = \omega^2 \sum_{l=1}^{N} \frac{M_{ll}M_{yl}}{R_i} \]
\[ C_i = \omega \sum_{k_r=1}^{N} I_{k_r} M_{a_i} - \sum_{k_r=1}^{N} \sum_{l=1}^{N} I_{k_r} M_{l} M_{a_i} \frac{R_i}{R_l} \]

(eq. 2.2.16)

to solve for the real part of the currents \( I_{l,f} \). Then equation 2.2.12 is used to calculate the imaginary part of the current, since this operation is much more efficient computationally than solution of the matrix equation: \( O(N) \) vs. \( O(N^3) \).

### 2.2.2 Calculation of Mutual Inductances

The mutual inductance between two ring currents is given by Maxwell [67, p.339], whose derivation is followed in this section. The mutual inductance is

\[ M_{ij} = \frac{\mu_0}{4\pi} \int_{s_i}^{s_j} \frac{d\mathbf{s}_i \cdot d\mathbf{s}_j}{r_{ij}} \]

(eq. 2.2.17)

For the two concentric rings of radii \( a \) and \( \rho \) separated by vertical distance \( z \) drawn in Fig. 2.2.2,

\[ d\mathbf{s}_i = a d\phi \left( \sin \phi \frac{dz}{a} + \cos \phi \frac{d\phi}{a} \right) \]
\[ d\mathbf{s}_j = \rho d\phi' \left( \sin \phi' \frac{dz}{\rho} + \cos \phi' \frac{d\phi'}{\rho} \right) \]

(eq. 2.2.18)

Therefore the scalar product in eq. 2.2.17 is:

\[ d\mathbf{s}_i \cdot d\mathbf{s}_j = a \rho \left( \sin \phi \sin \phi' + \cos \phi \cos \phi' \right) = a \rho \cos(\phi - \phi') = a \rho \cos \epsilon \]

(eq. 2.2.19)

where \( \cos \epsilon = \cos(\phi - \phi') \), and

\[ r_{ij} = \left( a^2 + \rho^2 + z^2 - 2a \rho \cos \epsilon \right)^{1/2} \]

(eq. 2.2.20)

Then

\[ M_{ij} = \frac{\mu_0}{4\pi} \int_{a}^{b} \int_{a}^{b} \cos \epsilon d\phi' a \rho \int_{a}^{b} \frac{\cos \epsilon d\phi}{\left( a^2 + \rho^2 + z^2 - 2a \rho \cos \epsilon \right)^{1/2}} \]

(eq. 2.2.21)

The second integral may be evaluated in terms of elliptic integrals. First, since \( \cos \epsilon \) is an even function with a period of \( 2\pi \), \( \cos \epsilon = \cos(2\pi - \epsilon) \). Therefore, we may evaluate the integral only from 0 to \( \pi \) by
symmetry. Then change variables: \( \varepsilon = 2\theta + \pi \), which implies \( d\varepsilon = 2d\theta \) and \( \cos \varepsilon = 2\sin^2 \theta - 1 \).

Therefore,
\[
\int_{\varepsilon}^{2\pi} \frac{\cos \varepsilon d\varepsilon}{(a^2 + \rho^2 + z^2 - 2ap \cos \varepsilon)^{1/2}} = \int_{\pi}^{2\pi} \frac{2^2 d\theta}{\left((a + \rho)^2 + z^2 - 4ap \sin^2 \theta\right)^{1/2}} \quad \text{(eq. 2.2.22)}
\]

Let
\[
k^2 = \frac{4ap}{(a + \rho)^2 + z^2} \quad \text{(eq. 2.2.23)}
\]
and note \( 2\sin^2 \theta = \frac{1}{k^2} \left(1 - k^2 \sin^2 \theta\right) \). Therefore,
\[
\int_{\varepsilon}^{2\pi} \frac{\cos \varepsilon d\varepsilon}{(a^2 + \rho^2 + z^2 - 2ap \cos \varepsilon)^{1/2}} = \left(-\frac{2}{\sqrt{4k}}\right) \left[k \left(\frac{k - 2}{k}\right) K + \frac{2}{k} E\right] \quad \text{(eq. 2.2.24)}
\]
where \( K \) and \( E \) are the complete elliptic integrals of the first and second kind to modulus \( k^2 \), respectively.

Substituting into equation 2.2.21 gives
\[
M_{ll} = \frac{\mu_0 a \rho}{4\pi} \int_{\phi}^{\phi + \varepsilon} \left[-\frac{2}{\sqrt{4k}}\right] \left[k \left(\frac{k - 2}{k}\right) K + \frac{2}{k} E\right] \quad \text{(eq. 2.2.25)}
\]
Since \( k \), \( K \), and \( E \) are independent of \( \phi' \), this expression becomes
\[
M_{ll} = -\mu_0 \sqrt{4k} \left[k \left(\frac{k - 2}{k}\right) K + \frac{2}{k} E\right] \quad \text{(eq. 2.2.26)}
\]

In practice, the computational elements are not current loops of infinitesimal cross section, but are rectangles, often with a large aspect ratio. For such elements, there are two approaches to reaching a more accurate solution. The first method is to use “brute force”: increase the number of elements until the solution approaches the correct value. This approach has many drawbacks, the most obvious being that the time required to solve the \( N \times N \) matrix of unknown currents increases as \( O(N^3) \).

The second approach is to modify the calculation of mutual inductance to account for the shape of these elements. In these calculations we use Lyle’s method [20-21], as extended by Schwartz [1]. This method involves choosing two current loops for each rectangular element, such that the average of the four mutual inductances between pairs of loops is a better approximation of the mutual inductance between the two rectangular elements.

It is also important to make a correction to the self-inductance terms for the rectangular elements. This correction is taken from [21].

The advantages of this approach are substantial, allowing better accuracy with a 10x10 grid than was achieved with a 40x40 grid without these improvements, while running about 25 times faster [1].
Section 2.2.3: Calculation of the Magnetic Flux Density

The magnetic flux density $\mathbf{B}$ is calculated from the curl of the vector potential $\mathbf{A}$. This calculation may be carried out numerically with the existing expression, or analytically as shown in the next section. Obtaining $\mathbf{B}$ numerically has the disadvantage of being less accurate near the edges of the computational domain, where the magnitude of $I$ is greatest. However, the numerical method does have the advantage of being $O(N)$ rather than $O(N^2)$ as is the analytical method shown below, since the vector potential $\mathbf{A}$ is readily calculated from the known current distribution:

$$A_{k,j} = \frac{j I_{k,j}}{\sigma_0 S_j}$$

(eq. 2.2.27)

Also, using finite differences requires that the computational domain be discretized on a structured grid. While a finite element implementation of this calculation would reduce the required symmetry in the grid, it is still necessary to define a connectivity matrix, as well as accepting the difficulties associated with writing a finite element code.

The calculation of $\mathbf{B}$ by analytical methods requires the development of an analytical expression for the vector potential in each element from the Biot-Savart law. The development of this expression is similar to that of Maxwell's expression for mutual inductance, as seen in the derivation below, which follows Smythe [68].
Since $\tilde{A}$ is not a function of $\phi$, we may choose our axes such that the point $P$ lies in the $x-z$ plane. Also, the subscript $f$ is dropped for convenience.

$$\tilde{A} = \frac{\mu_0}{4\pi} \int \frac{\tilde{J}(\vec{r}')dV'}{|\vec{r} - \vec{r}'|}$$  \hspace{1cm} (eq. 2.2.28)

For line currents, $\tilde{J} = I_\rho d\vec{s}$

$$\tilde{A} = \frac{\mu_0}{4\pi} \int C \frac{I_\rho(\vec{r}')d\vec{s}}{|\vec{r} - \vec{r}'|}$$  \hspace{1cm} (eq. 2.2.29)

but $A_r = A_z = 0$, since $I_r = I_z = 0$, so $\tilde{A}$ has only a $\phi$-component. Evaluating (2.2.29), substitute

$$d\vec{s} = a d\phi (\sin \phi \hat{\rho} + \cos \phi \hat{\phi})$$ \hspace{1cm} (eq. 2.2.30)

But the terms in the integrand like $\frac{\sin \phi}{|\vec{r} - \vec{r}'|}$ are equal and opposite for angles $\phi$ and $2\pi - \phi$, so this part of the integral is zero. Substituting

$$|\vec{r} - \vec{r}'| = \left( a^2 + \rho^2 + z^2 - 2a\rho \cos \phi \right)^{1/2}$$  \hspace{1cm} (eq. 2.2.31)

yields

$$A_\phi = \frac{I\mu_0}{4\pi} \int_0^{\pi} C \frac{\cos \phi d\phi}{\left( a^2 + \rho^2 + z^2 - 2a\rho \cos \phi \right)^{1/2}}$$  \hspace{1cm} (eq. 2.2.32)

The integral has already been solved in section 2.2.2. Substituting equation 2.2.24 gives

$$A_\phi = \frac{I\mu_0}{k\pi} \left( \frac{a}{\rho} \right)^{1/2} \left[ \left( 1 - \frac{1}{2} k^2 \right) K - E \right]$$  \hspace{1cm} (eq. 2.2.33)

where $K$ and $E$ are the complete elliptic integrals of the first and second kind to modulus $k^2$, respectively.

To find the magnetic flux density $\tilde{B}$, we take the curl of the vector potential $\tilde{A}$. The components of $\tilde{B}$ in cylindrical coordinates are:
\[ B_{\rho} = -\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_{\rho}) + \frac{1}{\rho} \frac{\partial}{\partial \rho} (A_{\rho}) = -\frac{\partial A_{\rho}}{\partial \rho} \]  
\[ B_{\phi} = \frac{\partial}{\partial \rho} (A_{\phi}) - \frac{\partial}{\partial \rho} (A_{\rho}) = 0 \]  
\[ B_{z} = -\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_{\rho}) + \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_{\phi}) = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_{\phi}) \]  
(eq. 2.2.34)

So evaluating \( B_{\rho} \) gives

\[ B_{\rho} = -\frac{\partial A_{\rho}}{\partial \rho} = -\frac{\mu_{0}}{k} \left( -\frac{1}{k^2} \frac{\partial K}{\partial \rho} \left( \frac{a}{\rho} \right)^{1/2} \right) \left[ \left( 1 - \frac{1}{2} k^2 \right) K - \frac{1}{2} E \right] \]
\[ - \frac{\mu_{0}}{k} \left( \frac{a}{\rho} \right)^{1/2} \left[ -\frac{1}{2} (2k) \frac{\partial K}{\partial \rho} K + \left( 1 - \frac{1}{2} k^2 \right) \frac{\partial K}{\partial \rho} - \frac{\partial E}{\partial \rho} \frac{\partial K}{\partial \rho} \right] \]  
(eq. 2.2.35)

the derivatives are [70]:

\[ \frac{\partial K}{\partial \rho} = \frac{E}{k(1-k^2)} \frac{K}{k} \]
\[ \frac{\partial E}{\partial \rho} = \frac{E}{k} \frac{K}{k} \]
\[ \frac{\partial K}{\partial \rho} = \frac{2k}{k} \frac{E}{k} \]
\[ \frac{\partial E}{\partial \rho} = \frac{2k}{k} \frac{E}{k} \]
\[ \frac{\partial K}{\partial \rho} = \frac{k}{2 \rho} \frac{k}{4 \rho} \frac{k}{4} \]
\[ \frac{\partial E}{\partial \rho} = \frac{k}{2 \rho} \frac{k}{4 \rho} \frac{k}{4} \]

Substituting and combining terms gives

\[ B_{\rho} = \frac{\mu_{0} I}{2 \pi} \frac{z}{\rho \left( (a + \rho)^2 + z^2 \right)^{1/2}} \left[ -K + \frac{a^2 + \rho^2 + z^2}{\left( a - \rho \right)^2 + z^2} \frac{E}{E} \right] \]  
(eq. 2.2.37)

For the \( z \)-component,

\[ B_{z} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_{\rho}) = \frac{1}{\rho} A_{\rho} + \frac{\partial}{\partial \rho} A_{\phi} \]  
(eq. 2.2.38)

\[ B_{z} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_{\rho}) = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_{\phi}) \]
\[ + \frac{\mu_{0}}{\pi} \left( \frac{a}{\rho} \right)^{1/2} \left[ \left( 1 - \frac{1}{2} k^2 \right) K - \frac{1}{2} E \right] \]
\[ + \frac{\mu_{0}}{k} \left( \frac{a}{\rho} \right)^{1/2} \left[ -\frac{1}{2} (2k) \frac{\partial K}{\partial \rho} K + \left( 1 - \frac{1}{2} k^2 \right) \frac{\partial K}{\partial \rho} - \frac{\partial E}{\partial \rho} \frac{\partial K}{\partial \rho} \right] \]  
(eq. 2.2.39)
combining terms yields

\[ B_z = \frac{\mu_0 I}{2\pi} \frac{1}{(a + \rho)^2 + z^2} \left[ K + \frac{a^2 - \rho^2 - z^2}{(a - \rho)^2 + z^2} E \right] \]  

(eq. 2.2.40)

**Section 2.2.4: Calculation of the Magnetic Force and Induced Heating**

Now that calculation of current and magnetic flux passing through each element has been completed, the current \( I_i \) by equations 2.2.15 and 2.2.12, and \( B_i \) by numerical differentiation of \( A_i \) or by equations 2.2.37 and 2.2.40. Hence, the volumetric Lorentz force and volumetric Joule heating of the sample are readily calculated.

Since the frequency of the oscillating current is much greater than the mechanical response frequency of the droplet, a time average force is taken. The force per unit volume \( \bar{F} \) is:

\[ \bar{F} = \sum_{f=0}^{\infty} \frac{1}{\sigma} \int_0^1 \mathbf{J} \times \mathbf{H} dt = \frac{1}{2} \sum_{f>0} \text{Re} \left( \mathbf{j} \times \mathbf{h}^* \right) \]  

(eq. 2.2.41)

where again, \( \mathbf{j} = \text{Re} \left( \mathbf{J} e^{j\omega t} \right) \), etc. The summation in \( f \) covers the different discrete driving frequencies.

Similarly, the thermal response of the droplet is much slower than the oscillating current, so again a time average is taken. The volumetric heat input into the droplet is:

\[ \bar{F} = \sum_{f=0}^{\infty} \frac{1}{\sigma} \int_0^1 \mathbf{j} \cdot \mathbf{J} dt = \frac{1}{2\sigma} \sum_{f>0} \mathbf{j} \cdot \mathbf{J} \]  

(eq. 2.2.42)

Note that \( \mathbf{j} \cdot \mathbf{J}^* \) is always real.

**Section 2.3: Verification**

An essential part of any modeling effort is validation of the model. The model must be compared to experiments, test cases, and asymptotic expressions. These comparisons are made in the following sections.

**Section 2.3.1: Comparison with Experiments**

As a part of the TEMPUS redesign for MSL-1, experimental measurements of the levitation force were made by and in collaboration with Georg Lohöfer of the DLR in Cologne, Germany. These
measurements were made by measuring changes in weight of a copper sphere suspended in the TEMPUS coils, at various operating currents. A sketch of the experimental apparatus appears in Fig. 2.3.1.

Fig 2.3.1: Apparatus for measuring electromagnetic force on a sphere. Any net electromagnetic force changes the sphere's apparent weight. The force may be measured along the $x$- and $y$- axes of the coil by rotating the coil assembly, as shown.

The results of the comparison are presented in Fig. 2.3.2. The points represent measurements, and the lines are the model's predictions. The experiments and predictions agree within the error of the experiments, which provides some confidence in the predictive capabilities of the model.
Section 2.3.2: Standard Test Case

One standard test case is that of the "fluxball". This test takes advantage of the fact that current passing through a coil wrapped with uniform spacing in the \( z \)-direction on the surface of a sphere produces a uniform magnetic field inside, directed along the axis of the windings. With \( N \) turns in total and a current \( i \), the magnetic field is:

\[
\vec{H} = \frac{Ni}{3R} \hat{z},
\]

(eq. 2.3.1)

for a spherical winding of radius \( R \) [70, p. 336.]

For DC or low-frequency fields, this relation should hold true even for the fields inside a metal sphere. The calculated field is shown in Fig. 2.3.3, calculated for \( N=100 \), \( i=1.5A \), and \( R=1 \) cm. The
numerical value is $|\mathbf{H}| \approx 5000 \text{ A/m}$, which corresponds for a non-magnetic sphere to a magnetic flux density of $6.2832 \times 10^{-3}$ Tesla. The calculations agree with theory within 0.01%.

Fig 2.3.3: Calculated magnetic flux density in "fluxball" spherical coil. Calculated flux density agrees with analytic value is $6.2832 \times 10^{-3}$ T within ±0.01%.

Section 2.4: Asymptotic Calculations

Section 2.4.1: Dependence on Frequency

An expression for the power induced in the sphere in a uniform applied field is: [3, 68, p.378]

$$ P = \frac{3\pi R}{\sigma \mu_0} F(x) B^2 \quad \text{(eq. 2.4.1)} $$

$$ F(x) = \frac{1}{2} \left[ \frac{v^* I_{1/2}(\nu^*) I_{1/2}(\nu) + v I_{1/2}(\nu) I_{1/2}(\nu^*)}{I_{1/2}(\nu) I_{1/2}(\nu^*)} \right] = x \frac{\sinh 2x + \sin 2x}{\cosh 2x - \cos 2x - 1} $$

where $v = (j\mu_0 \omega \sigma)^{1/2}$ and $x = \frac{R}{\delta} = R\sqrt{\mu_0 \sigma}$

Both forms of the function $F(x)$ are exact for the uniform field. The first form comes from evaluating the integral given in Smythe[68], equation 10.07(1). The second form, cited by [3,6] comes from using the recurrence relations for the modified Bessel functions $I_n(\nu)$ to express the terms like $I_{1/2}(\nu)$.
in terms of $I_{\pm\frac{1}{2}}(v)$, and then employing the hyperbolic substitutions for $I_{\pm\frac{1}{2}}(v)$, and is simplified from Smythe's equation in §10.07 [68, p.378]. This form in terms of hyperbolic and trigonometric functions is still an exact solution, but in much more manageable form.

A simple analytical expression for the force on a levitated sphere has been derived by many authors [3,4,6] for various combinations of circular coils, but all of these may be written in the form suggested by Rony [6]:

$$\bar{F} = -\frac{\pi R^3}{\mu_0} G(x) V \bar{B}^2$$

$$G(x) = 1 - \frac{3}{2x} \frac{\sinh 2x - \sin 2x}{\cosh 2x - \cos 2x}$$

(eq. 2.4.2)

This expression for force uses a form for the magnetization of a sphere in a uniform magnetic field, but requires a field gradient to generate any net force. This seeming contradiction implies that this formulation is valid only when the change in the field gradient is small over the extent of the sphere. Nevertheless, this expression has the correct asymptotic behavior and represents a reasonable approximation of the response of the droplet.

The asymptotic behavior of the functions $F(x)$ and $G(x)$ is apparent in Fig. 2.3.4. At low frequency or conductivity ($x < 1$), $F(x), G(x) \propto x^4$. At high frequency or conductivity ($x > 10$), $G(x)$ approaches a constant value, and $F(x) \propto x$.

Now that the asymptotes of the functions are known, the asymptotic behavior of the force and power on the sphere can be evaluated, as in table 2.4.1. The agreement of our magnetic model to these asymptotes may be verified by comparison to the computed results presented in Chapter 3.
Fig 2.4.1: Functions $G(x)$ and $F(x)$, and their ratio. $G(x)$ and $F(x)$ both increase as $x^4$ for small $x$, but $G(x)$ approaches a constant value and $F(x)$ is proportional to $x$ for large $x$.

<table>
<thead>
<tr>
<th>Low Frequency</th>
<th>High Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F \propto I^2 \omega^3 \sigma^2$</td>
<td>$F \propto I^2$, independent of $\sigma, \omega$</td>
</tr>
<tr>
<td>$P \propto I^2 \omega^3 \sigma$</td>
<td>$P \propto I^2 \omega^3 \sigma^{-\frac{1}{2}}$</td>
</tr>
<tr>
<td>$\frac{F}{P}$ independent of $I, \omega$</td>
<td>$\frac{F}{P}$ independent of $I$</td>
</tr>
<tr>
<td>$\frac{F}{P} \propto \sigma$</td>
<td>$\frac{F}{P} \propto \sigma^{-\frac{1}{2}} \omega^{-\frac{1}{2}}$</td>
</tr>
</tbody>
</table>

Table 2.4.1: Asymptotic behavior of levitation force, induced power, and force to power ratio for levitated spheres.
Section 2.4.2: Dependence on Sample Size

The dependence on sample size of the positioning force, absorbed power, and ratio of force to power is easily determined by a similar method. In fact, it is the force per unit volume and power per unit surface area that are relevant, since the containment of a sample depends on the force divided by the mass, and the temperature depends on the power divided by the surface area.

Re-arranging equations 2.4.1 and 2.4.2 gives:

\[
\frac{F}{V} \propto G(x) \quad \text{(eq. 2.4.3)}
\]

\[
\frac{P}{A} \propto \frac{F(x)}{R}
\]

Our prior expansion of the functions \( F(x) \) and \( G(x) \) give the following relations:

<table>
<thead>
<tr>
<th>Low Frequency</th>
<th>High Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{F}{V} \propto R^4 )</td>
<td>( \frac{F}{V}, \frac{P}{A} ) independent of ( R )</td>
</tr>
<tr>
<td>( \frac{P}{A} \propto R^3 )</td>
<td></td>
</tr>
<tr>
<td>( \frac{F}{V} \propto R )</td>
<td>( \frac{P}{A} ) independent of ( R )</td>
</tr>
<tr>
<td>( \frac{P}{A} )</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.4.2: Asymptotic dependence on sample radius of levitation force, induced power, and force to power ratio for levitated spheres.
Chapter 3: Magnetic Results

This chapter presents some of the results of electromagnetic calculations undertaken in the support of the design of a new levitator, the Auburn EML. The purpose of these calculations is:

- To reexamine the design of past and present levitators from a theoretical standpoint, and determine which features may be improved.
- To provide specific results and recommendations for the Auburn EML design effort.

Section 3.1: Introduction

In this chapter, results of some electromagnetic calculations performed for the design of Auburn University’s new microgravity levitator are presented. These calculations illustrate the capabilities of the magnetic model described in the previous chapter by the use of parametric studies for a wide range of operating conditions. This work was performed in support of the experimental facility under construction by Prof. Tony Overfelt of Auburn University’s Space Power Institute.

The Auburn EML is designed for performing materials science experiments that require short-duration microgravity conditions, such as are provided by parabolic flights. These experiments may involve topics such as nucleation and growth, phase selection, and thermophysical property measurements. The coil design currently used in the development of the levitator is shown in Fig. 3.1.1.

![Fig 3.1.1: Auburn EML Coil Design. The outer four coils generate the quadrupole positioning field, and the inner two coils, the dipole heating field.](image)

In a parabolic flight, the aircraft follows a free-fall trajectory, where the acceleration of gravity is exactly canceled by centrifugal acceleration. Thrust is adjusted to balance drag and keep the total acceleration inside the aircraft near zero. In order to maximize the time of reduced acceleration, the aircraft begins a parabola with a steep, high-g climb at high speed. Then it is flown along the parabola, passing through the point of maximum altitude at greatly reduced speed and continues until it is in a steep, high-speed dive. Finally, the aircraft is pulled out of the dive, again at high acceleration, leaving it ready to
climb again. For NASA's KC-135 parabolic flight aircraft, the high acceleration level is about 1.8g and the duration of reduced acceleration is about 25 sec.

In order to support parabolic flight experiments, the levitator must provide many special capabilities. Although parabolic flights do provide reduced acceleration, the acceleration environment is very "noisy", with typical average value of several milli-g, but the typical variation is also of this order. Therefore, the levitator must provide very good containment of the sample. Additionally, it may be desirable to contain the sample through the high-g phase between parabolas, when the acceleration may reach approximately 2g.

Also, because the time of reduced acceleration available in parabolic flights is small (about 25 sec), the levitator must provide for rapid heating of the sample. Because of problems with samples adhering to the sample holder, it is not feasible to preheat a sample near its melting point, compounding this problem. The levitator must be able to heat a sample from room temperature to processing temperature in less than 5 seconds to allow time for processing and re-solidification before the next high-g phase.

In the following sections, results are presented for calculations made with the use of the magnetic model described in Chapter 2 and compared to the asymptotic relations presented in §2.3.3. The calculations are made for 6.35 mm (0.250") diameter samples.

Section 3.2: Results for Auburn Univ. Microgravity Levitator

The following figures present the results of calculations on the Auburn EML. One important feature of a levitation system is positioning force gradient (N/m) on the sample, which is estimated by the total force on a droplet displaced 1 mm from the midplane of the coils in the z-direction. The force on the sample at 1mm displacement is studied as a function of applied frequency and sample conductivity. Also, the heating of the sample is presented as a function of the same variables. Both of these quantities are presented in units normalized with respect to the oscillating current: (N/m)/A^2 for force gradient and W/ A^2 for heating power.

Finally, the ratio of force gradient to power is presented, again versus applied frequency and sample conductivity. For an optimal design, this ratio should be:
- maximized for positioning coils so as to reduce the minimum sample temperature.
minimized for the heating coils to minimize the disturbance of the sample positioning on changes in heating power, such as modulations and pulses for thermophysical property measurements.

Of course the range of currents and frequencies that may be employed are subject to constraints including power supply cost and availability and total power consumption.

It is convenient to express the levitation force and induced heating normalized by the square of the oscillating current. The normalized force and heating are measures of the efficiency of positioning and heating the sample, respectively, and are characteristic of the combination of a particular coil geometry, oscillating current frequency, and sample size and material.

The relative positioning efficiency of the positioning and heating coils is shown in Fig. 3.2.1. For the case of the Auburn EML, the heating field is destabilizing; i.e., a sample slightly displaced in the +z-direction is forced farther in that direction by the heater field. This tendency must be overcome by a larger stabilizing force from the positioning field so that there is a net restoring force on the sample. As Fig 3.2.1 shows, the relative efficiencies of the heating and positioning coils are comparable, so in practice it will be necessary to use a larger current in the positioning coils than in the heating coils, unless the frequency of one or both is changed. The effect of frequency on positioning force is discussed later in this section.
Fig 3.2.1: Relative positioning efficiency (N/A²) of positioning and heating coils for Auburn EML, with 100kHz positioning and 100kHz heating. Note stable force from positioning coils is opposed by destabilizing force from the heater coils.

The dependence of positioning and heating efficiency, and their ratio, on frequency is presented in Figs. 3.2.2, and on electrical conductivity in 3.2.3. These figures demonstrate that the dependence is as predicted by the dipole model in both high- and low- frequency regimes. In the plots vs. frequency, the sample’s electrical conductivity was held constant at 2.3x10⁶ S/m, the value for liquid zirconium at its melting point [34]. Similarly, in the plots vs. electrical conductivity, the frequency was held constant at 215 kHz, which is near the average for the levitators considered.

A more complete series of plots, which may be used in the design of a levitation system, is available in the Appendix. Those plots show that the dependence is the same for other frequencies and conductivities, and provide values of the positioning efficiency, heating efficiency, and their ratio for both coils, over the range of 1 kHz to 10 MHz, and an electrical conductivity from 10⁴ to 10⁸ S/m.

In Figure 3.2.2, the frequency dependence of positioning efficiency, heating efficiency, and their ratio is presented for the Auburn EML for the case of a 6.35 mm diameter zirconium samples at its melting point. The positioning efficiency of the positioning coils (open squares) is shown to be proportional to the square of frequency for low frequencies, and independent of frequency at high frequency. The heating
efficiency of the heating coils (open circles) is also proportional to the square of frequency for low frequencies, but increases as the square root of frequency at high frequency.

As previously stated, the ratio of force gradient to absorbed power (the ratio of positioning efficiency to heating efficiency) is an important factor in the design of levitators. In order to permit processing of low-melting materials under a given acceleration, this ratio should be maximized. But as shown in Fig. 3.2.2 (solid squares), this ratio is independent of frequency for low frequencies, and declines with increasing frequency (the ratio is proportional to the reciprocal of the square root of the frequency) at high frequencies. Since the force per unit current increases over the range of frequencies where the force per power is constant, the optimum frequency for processing low-melting materials is near the transition region in force per power shown in Fig. 3.2.2; i.e., near the point where force per power begins to fall.

For the heating coils, the goal is to minimize the ratio of force gradient to absorbed power. Reducing this ratio means that there is less disturbance of the sample on sudden changes in the heating current, as in pulses or modulations for thermophysical property measurements. Also, since the heating field provides a destabilizing force, having a high ratio of force gradient to power can require restrictions on the operation of the levitator so as to avoid compromising sample containment by overpowering the positioning field. So for the heating coils, the frequency should be maximized subject to other constraints, with the ratio of force gradient to absorbed power declining as the square root of frequency.

These asymptotes are consistent with the results of the "dipole" theory presented in §2.3.3.
Fig 3.2.2: Frequency dependence of positioning efficiency of the positioning coils, heating efficiency for the heating coils, and the ratio of positioning efficiency to heating efficiency for the positioning coils of the Auburn EML, for a 6.35 mm diameter zirconium sample.

In Figure 3.2.3, the dependence of positioning efficiency, heating efficiency, and their ratio on the electrical conductivity of the sample is presented for the Auburn EML for the case of a 6.35 mm diameter sample levitated with currents oscillating at 215 kHz. The positioning efficiency of the positioning coils (open squares) is shown to be proportional to the square of conductivity for low conductivity, and independent of conductivity at high conductivity. The heating efficiency of the heating coils (open circles) is linear in conductivity for low values of conductivity due to the $\sigma^{-1}$ term in the expression for power in equation 2.3.3. At high conductivity, the heating efficiency is proportional to the reciprocal square root of conductivity.

The ratio of force gradient to absorbed power is proportional to the sample’s electrical conductivity at low conductivity, and proportional to the square root of conductivity at high conductivity, again as predicted by the dipole theory presented in §2.3.3.
Fig 3.2.3: Dependence of positioning efficiency of the positioning coils, heating efficiency for the heating coils, and the ratio of positioning efficiency to heating efficiency for the positioning coils of the Auburn EML on the sample’s electrical conductivity. These calculations use a fixed frequency of 215 kHz.

Section 3.3: Discussion and Recommendations

Section 3.3.1: Recommendations for Positioning Coils

Based on the results shown in the previous section, it is possible to narrow the range of frequencies that should be considered for the positioning coils. However, in comparing the effectiveness of different frequencies, there are two important metrics that must be considered: minimum sample temperature and positioning efficiency. In an ideal system, a low minimum sample temperature is desirable to allow either processing of low-melting materials at moderate accelerations, or high-melting materials at high accelerations. But it is also desirable to have a high positioning efficiency to avoid requiring an excessive positioning current. A large positioning current requires added expense and complexity in the power supply, and may exceed the available total supply power available to a flight facility.
As can be seen in Fig. 3.3.1, the minimum sample temperature for a sample with a given degree of containment is constant at low frequencies, and increases at higher frequencies. For positioning efficiency, however, the situation is opposite: constant at high frequencies and lower at lower frequencies. Because of these relations, the optimal value of force per unit power is at the high-frequency end of the plateau, about 10 kHz. Similarly, the optimal value of the positioning efficiency occurs at the low-frequency end of its plateau, about 1 MHz.

![Graph showing force per unit power and force gradient vs. frequency](image)

**Fig. 3.3.1:** Positioning efficiency (N/m-A²) and force per unit power ((N/m)/W) of Auburn coils with 6.35 mm Zr sample. Optimal frequencies are 21 kHz for force/power and 215 kHz for positioning efficiency.

In order to choose between the two possible optima for the positioner frequency, it is convenient to restate the data of Fig. 3.3.1 in terms of temperature rather than force per unit power, and current instead of positioning efficiency. These data are presented in Fig. 3.3.2 (a) and (b).

It is also useful to consider the two “85%” cases: the data point nearest 85% of maximum force per unit power, and the one nearest 85% of maximum positioning efficiency. These values, shown in Table 3.3.1, demonstrate very clearly why a choice between the two metrics of performance is necessary: the optimal frequencies differ by an order of magnitude. Furthermore, the levitation current is 2.5 times greater for the low temperature case, but that increase in current yields a remarkable temperature difference of 290K at 0.03g and almost 700K at 1g.
<table>
<thead>
<tr>
<th>Frequency</th>
<th>% of max. F</th>
<th>% of max. F/P</th>
<th>Current to levitate at 0.03 g</th>
<th>Temperature at 0.03 g</th>
<th>Current to levitate at 1 g</th>
<th>Temperature at 1 g</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.5 kHz</td>
<td>15 %</td>
<td>87 %</td>
<td>75 A</td>
<td>584 K</td>
<td>428 A</td>
<td>1402 K</td>
</tr>
<tr>
<td>216 kHz</td>
<td>83 %</td>
<td>17 %</td>
<td>31 A</td>
<td>874 K</td>
<td>182 A</td>
<td>2099 K</td>
</tr>
</tbody>
</table>

In order to choose between these two cases, or some frequency in between, it is necessary to know the operating limitations of the levitator. If the supply power is available, then a positioning frequency near 20 kHz, the best frequency for minimum temperature, should be used. However, there is the possibility of increasing the positioning force for a given current by more than six times by raising the frequency toward 200 kHz at the expense of a higher minimum sample temperature.

At first glance, this problem of minimum sample temperature seems not to be important for a high-melting material like zirconium. However, Fig. 3.3.2 does give a good reason to consider this effect: it is possible to levitate a zirconium sample in the solid state through the high-g portion of the parabola! For a positioning frequency of about 20 kHz, the equilibrium temperature in vacuum is about 1670 K at 2g acceleration and 1mm vertical displacement. This is 450 K below zirconium's melting point, and is slightly below the melting points of other important metals as well, including nickel-based superalloys and stainless steels.

In fact, levitation through the high-g portion of the parabola is more feasible than it seems for these superalloys and steels, since such materials are usually processed under a reducing gas environment to inhibit evaporation and remove any oxide that may be present on the sample. The gas also cools the sample by conduction to the water-cooled coils, and can cause a significantly reduced temperature as compared to a sample processed in vacuum, often hundreds of degrees less.
There are, however, limitations that may prevent the realization of this potential. Most importantly, the positioning current required to generate these large forces is quite large: 605 A for the case under discussion. For comparison, maximum coil currents in the low hundreds of amperes are typical for a microgravity levitator, while the range of ground-based levitators is from many tens to many thousands of amps [3,4].
Also, in order to take advantage of levitating the sample through the high-g portion of a parabola, some form of feedback control would be necessary to allow reduction of the positioning force as the acceleration is reduced. This is because the sample’s kinetic energy as it translates through the center of the field is proportional to the square of the field strength, so large changes in the positioning field strength require damping of the sample translations.

**Section 3.3.2: Recommendations for Heating Coils**

Recommendations for the heating coil frequency are much more straightforward. As the heating efficiency increases with the square root of frequency, the heating frequency should be as high as possible. Again, practical limitations such as the availability of generators and restrictions on the use of certain frequencies may limit the frequency of the heater, but higher frequencies are more efficient in heating the sample.

The oscillating currents required to hold a 6.35 mm zirconium sample at different temperatures, and to melt the sample from room temperature in 5 seconds, are shown in Fig. 3.3.3 for a range of different frequencies.
Fig. 3.3.3: Oscillating current required to hold a 6.35 mm Zr sample at various temperatures vs. oscillating current frequency. Also plotted is the current required to melt the sample from room temperature in 5 seconds.

Section 3.3: Summary and Conclusions

In summation, it is possible to use this methodology to analyze and improve existing levitator designs to optimize their performance. The optimal design parameters depend on the available power and on the samples to be processed.

The analysis must consider the optimum values of current and frequency both for the levitation and the heating of the sample. The force per unit power due to the positioning coils must be as high as practical in order to allow the processing of low-melting samples. The force per unit power in the heating coils, however, must be as low as possible in order to minimize the disturbance of the sample’s position on changes in the heat input, such as occur at the end of melting or during thermophysical property measurements.

For the case of the Auburn levitator, the design must be optimized for processing zirconium under conditions of both parabolic flight and space experiments, while providing the capability of processing the largest practical range of materials. These constraints lead to the assertion that the positioning frequency
should be as close as practical to 20 kHz, with higher frequencies allowing better positioning at the expense of the minimum achievable sample temperature. The heating coils should be operated at the greatest practical frequency, with the heating efficiency increasing with the square root of the frequency.

It is interesting to compare these frequencies to those employed in some existing levitators, as shown in Table 3.3.2. From the asymptotic analysis in §2.4.1, the best ratio of force to power occurs when the parameter \( R/\delta \) is between 1 and 2. The lowest coil current for positioning is at a frequency such that \( R/\delta \) is about 10, and the best heating efficiency occurs as \( R/\delta \) approaches infinity. It can be seen from the table that the different existing levitators meet these criteria with varying degrees of success, but none uses a positioning frequency as low as recommended by this analysis, nor a heating frequency that is as high as desirable.

<table>
<thead>
<tr>
<th>System</th>
<th>Frequency</th>
<th>Material</th>
<th>Electrical Conductivity</th>
<th>Diameter</th>
<th>( R/\delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flemings</td>
<td>208 kHz</td>
<td>Ni</td>
<td>2.0( \times 10^6 ) S/m</td>
<td>6 mm</td>
<td>3.8</td>
</tr>
<tr>
<td>TEMPUS Positioning</td>
<td>160 kHz</td>
<td>Zr</td>
<td>2.3( \times 10^6 ) S/m</td>
<td>7 mm</td>
<td>4.2</td>
</tr>
<tr>
<td>TEMPUS Positioning</td>
<td>160 kHz</td>
<td>Met. Glass</td>
<td>5.0( \times 10^5 ) S/m</td>
<td>8 mm</td>
<td>2.2</td>
</tr>
<tr>
<td>TEMPUS Heating</td>
<td>351 kHz</td>
<td>Zr</td>
<td>2.3( \times 10^6 ) S/m</td>
<td>7 mm</td>
<td>6.2</td>
</tr>
<tr>
<td>MEL Positioning</td>
<td>300 kHz</td>
<td>Zr</td>
<td>2.3( \times 10^6 ) S/m</td>
<td>5 mm</td>
<td>4.1</td>
</tr>
<tr>
<td>MEL Heating</td>
<td>600 kHz</td>
<td>Zr</td>
<td>2.3( \times 10^6 ) S/m</td>
<td>5 mm</td>
<td>5.8</td>
</tr>
<tr>
<td>Auburn Positioning</td>
<td>21 kHz</td>
<td>Zr</td>
<td>2.3( \times 10^6 ) S/m</td>
<td>6.35 mm</td>
<td>1.4</td>
</tr>
<tr>
<td>Auburn Heating</td>
<td>1 MHz</td>
<td>Zr</td>
<td>2.3( \times 10^6 ) S/m</td>
<td>6.35 mm</td>
<td>9.6</td>
</tr>
<tr>
<td>Auburn Heating</td>
<td>10 MHz</td>
<td>Zr</td>
<td>2.3( \times 10^6 ) S/m</td>
<td>6.35 mm</td>
<td>30.3</td>
</tr>
<tr>
<td>Lowest Min T Positioning</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1-2</td>
</tr>
<tr>
<td>Lowest Positioning Current</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>~10</td>
</tr>
<tr>
<td>Optimal Heating</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \rightarrow \infty )</td>
</tr>
</tbody>
</table>

Table 3.3.2: Processing parameters for various levitators. Values for the Auburn levitator are the recommendations of this study. Optimum values are the results of the asymptotic analysis in §2.4.1.

The values given in Table 3.3.2 for electrical conductivity are at the melting point for the pure metals, taken from ref. [34], and estimated for the class of zirconium-based glass formers. Since the temperature gradient in samples of this size are of the order of 0.1-1°C, and a typical value of the fractional rate of change of electrical conductivity \( \frac{1}{\sigma} \frac{d\sigma}{dT} \) is about 0.03% per degree [34, pp. 232-3] for transition metals, the temperature dependence of conductivity is not relevant to these static calculations. Temperature dependence of properties is important to the design of experiments, however, since the operating range in temperature may be almost 1000°C in a single cycle.
Chapter 4: Fluid Flow Calculations

This chapter presents some results of magnetohydrodynamic calculations for levitated droplets. The main goals of these calculations are:

- To describe the droplet’s internal flow patterns, and the dependence of these patterns on experimental parameters.
- To estimate the maximum fluid velocity as a function of design and operational parameters.
- To determine which design changes may reduce the flow velocity driven by positioning forces.

The results of these calculations are relevant to the design of both levitators and experiments. For example, the frequency of the positioning current has an important impact on the flow in the sample, but must be fixed in the design phase for current designs. Also, once the design of the levitation system is fixed, many experiment types, including property measurements and experiments on nucleation and phase selection are sensitive to fluid flow.

Section 4.1: Introduction

The flow model described in this chapter uses a semi-coupled method for calculating the magnetohydrodynamic flows in the droplet. In this method, the magnetic field determines the flow field, but is not affected by the flow in the droplet. The justification for this approach is discussed in §2.1.2, and is based on the magnetic Reynolds number. The magnetic Reynolds number allows an easy estimate of the importance of fluid flow in the calculation of the electromagnetic quantities; for $Re_m$ much less than unity, as is the case for the conditions described in this work, the effect of flow on the induced current may be neglected.

The flow in the droplet is governed by the equation of continuity and the Navier-Stokes equation (eq.4.1.1), with an added term to include the magnetic force $\mathbf{F}$. Since liquid metals are incompressible, the density $\rho$ is constant. The other symbols are velocity $\mathbf{u}$, viscosity $\mu$, and pressure $P$.

$$\nabla \cdot \mathbf{u} = 0$$
$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = \mu \nabla^2 \mathbf{u} - \nabla P + \mathbf{F}$$

The results presented here are for the case of a fixed, spherical droplet. The boundary conditions used for the “free” surface are “free slip”, i.e. zero shear stress on the surface, and that the normal component of velocity is zero on the surface. These conditions are:
\[ \frac{\partial \mathbf{u}}{\partial t} \bigg|_{r=R_0} = 0 \]
\[ \mathbf{u} \cdot \mathbf{\hat{r}} \bigg|_{r=R_0} = 0 \] (eq. 4.1.2)

The symmetry condition on the axis is similar: no velocity perpendicular to the axis at the axis, and the derivative of velocity perpendicular to the axis is also zero. These conditions may be stated as:

\[ \frac{\partial \mathbf{u}_z}{\partial r_{cyl}} \bigg|_{r_{cyl}=0} = 0 \]
\[ \mathbf{u} \cdot \mathbf{\hat{r}}_{cyl} \bigg|_{r_{cyl}=0} = 0 \] (eq. 4.1.3)

The fluids calculations were performed using FIDAP, a commercial finite-element fluids package. The magnetic force field is calculated by the method of mutual inductances as described in chapter 2. In order to allow the use of different computational grids for the magnetic and fluid flow calculations, the magnetic forces are interpolated onto the fluid flow grid.

The combined magnetohydrodynamic model can be used for quantitative prediction of the droplet's internal flows, as long as those flows are laminar. These results are useful for many different types of experiments, including the thermophysical property measurements described in Chapter 5, as well as recalescence and nucleation experiments.

The models may also be used to compare flow conditions for different operational parameters and even different sample materials. This benefit has the potential of generalizing the results of turbulent transition experiments to all levitated samples. Finally, this model lays the groundwork for a comprehensive review of turbulence models, which might enable quantitative evaluation of the flow inside the droplets even in the transitional or turbulent regime.

The results discussed in this chapter involve calculations for spherical droplets levitated in the TEMPUS coils. These results show the principal characteristics of the flow in levitated droplets, and also the response of the flows to different processing parameters and materials properties.
Section 4.2: Fluid Flow Results

Section 4.2.1: Typical Fluid Flow Patterns for Levitated Droplets

The internal recirculations in a levitated droplet depend on a number of factors, including sample size, density, and viscosity, and the coil geometry, frequency, and current. However, for a wide range of variation in the other variables, the pattern of the flow is qualitatively unchanged for a particular coil geometry.

A typical flow pattern for a sample levitated in TEMPUS at moderate positioning current and no heating current is shown in Fig. 4.2.1. This pattern is observed for low Reynolds number flow, and is qualitatively unchanged over many orders of magnitude in Reynolds number. This flow pattern is observed so long as two conditions are observed: the positioning forces are dominant over the force from the heating coil, and the Reynolds number of the flow is below about 100. The cases of positioning-dominated flows with higher Reynolds number and the transition to heating-dominated flow are discussed in later sections.

The low Reynolds number positioning-dominated flow pattern is distinguished by four recirculating loops. The flow is mirror-symmetric about the equator, as is the Lorentz force (Fig. 4.2.2). The flow at the droplet's equator is outward, because the force due to the quadrupole positioning field reaches a maximum about 45 degrees latitude on the sphere, and is zero at the equator due to symmetry. The flow is directed inward near the force maximum until the inertia of the fluid becomes more important at higher Reynolds number, as explained in the following section.

The maximum velocity (0.1 cm/s in this case) occurs near 30 degrees latitude on the sphere, in the loops nearest the equator of the sample. The maximum velocity of the flow is, however, a strong function of the sample viscosity and the applied force (which is proportional to the square of the current), and a weak function of the oscillating current frequency, as shown in subsequent sections.
Fig 4.2.1: Typical low Reynolds number flow pattern for positioner-dominated flow. 7mm diameter PdSi sample in TEMPUS, viscosity = 35 mPa·s, positioner current = 211 A, max. velocity = 0.1 cm/s. Re = 1.0. Note the four recirculation loops, with the flow directed outward at the equator.
Fig 4.2.2: Electromagnetic force contours (left) and vectors (right) for 7mm diameter PdSi sample in TEMPUS, positioner current = 211 A, Heating current = 0 A. Note the force field is mirror symmetric, with the magnitude going to zero at the equator of the sphere.
The pattern of heating-dominated fluid flow in the droplet is quite different. In this case, the force from the dipole heating field (fig. 4.2.4) is maximum on the equator, and still directed inward. This force drives two recirculating loops, with the maximum velocity at about 30 degrees latitude, as shown in Fig. 4.2.3. Unlike the case of positioner-dominated flow, the heater-dominated flow pattern is not noticeably affected by the Reynolds number for the range of conditions studied here.

The flow pattern shown in Fig. 4.2.3 is for the hypothetical case of 46.4A in the heating coils and zero A in the positioning coils. In fact, since the net force acting on the droplet due to the heating coils would tend to expel the droplet from the coils, this case is not a real operating condition, but is presented to illustrate the effect of the heating field alone on the droplet’s internal flow.

The flow due to the heating coils is much stronger than that of the positioning coils. For most real operating conditions except free cooling of the droplet, the heating field will dominate the fluid flow in the droplet. This effect is more thoroughly examined in the following section.

The forces due to the heating field are shown in Fig. 4.2.4. These forces have the expected mirror symmetry, reflecting the symmetric coil arrangement. The force is maximum at the equator, and everywhere directed inward.
Fig 4.2.3: Typical flow pattern for heater-dominated flow. 7mm diameter Zr sample, viscosity = 4.0 mPa-s, heater current = 46.4 A, max. velocity = 12 cm/s. The Reynolds number is 620. Note the two recirculation loops, with the flow directed inward at the equator.
Fig 4.2.4: Electromagnetic force contours (left) and vectors (right) for 7mm diameter PdSi sample in TEMPUS, positioner current = 0A, heater current=46.2A. Note the force field is mirror symmetric, with the maximum value occurring on the equator of the sphere.
Section 4.2.2: Transition Between Positioning and Heating-Dominated Flow

In figure 4.2.5, the flow pattern for a 7mm Zr sample in TEMPUS is presented for the case of 100A positioner and various heating currents, in order to illustrate the transition from positioning-dominated to heating-dominated flow. The first plot, which is similar to the typical low Reynolds number case, represents 100A positioner and 0A heater. In the following frames, the heating current is increased, while the positioning current remains constant.

As can be seen in Fig 4.2.5 (b), (c), and (d), the forces from the heating field oppose the flow of the positioner’s equatorial loops and augment the flow of the positioner’s polar loops. This behavior is expected from Fig 4.2.5, which shows that the force due to the heating field is directed inward at the equator. Therefore, this force should augment loops which flow inward near the equator and out at the poles, and suppress circulations with the opposite orientation.

In §4.2.1, it was asserted that the internal flow due to the heating field is much stronger than the flow due to the positioning field. This fact is illustrated in Fig 4.2.5. The droplet’s internal flow is dominated by the positioner in the first two frames, Fig 4.2.5 (a) and (b), which represent 0A and 16.2A in the heating coils.

The droplet’s flow is in a mixed-control regime in Fig 4.2.5 (c), and (d), which correspond to 22.4A and 26.3A in the heating coils, respectively. In these two frames, the equatorial circulation loop is slowed to extinction. Meanwhile, the polar loops, which were very slow in the positioner-dominated case, are flowing faster, and the maximum velocity now occurs in these loops. Note that the maximum velocity in the droplet has actually decreased slightly due to the opposition of the heating and positioning forces.

For heating currents above 30A, the flow is clearly dominated by the heating field, as is shown in the last two frames, Fig 4.2.5 (e) and (f). The maximum flow velocity has resumed its predicted increase, and the flow pattern is stable with respect to increasing heater current.

These currents compare to typical operating conditions of about 200A for positioning and about 250 A heating during melting, although this type of flow transition might be seen when maintaining a sample in the liquid state with a low heating current.
Fig 4.2.5: Development of flow pattern for increasing heating current at constant positioning current of 100A for 7mm Zr sample. The heating forces oppose the rotation of the faster positioner loop, so the increasing heating forces cause the faster loop of the positioner flow to slow and disappear, while the slower positioner loop becomes faster.
Section 4.2.3: Changing Fluid Flow Patterns for Different Reynolds Numbers

As mentioned before, the pattern of the positioning-dominated flow in a levitated droplet is qualitatively independent of the velocity for small Reynolds numbers. In this section, results are presented over the range of transition from this low Reynolds number regime to a different pattern that is only seen at higher flow velocities. It may be expected that there are further laminar flow patterns which are stable at still higher Reynolds numbers, only this one transition is detailed here.

In this transition, the viscous-dominated four-loop pattern described in §4.2.1 gradually gives way to a pattern in which the inertia of the fluid plays a more important role, as shown in Fig. 4.2.6. As the Reynolds number increases, either due to decreasing viscosity (i.e., increasing temperature of the sample), or increasing force on the sample, the two equatorial loops become faster and larger. The two polar loops, which contain a lesser mass of fluid due to the spherical shape of the droplet, must become smaller. As these polar loops become smaller, they move farther from the point of maximum force and become slower, eventually becoming stagnant. The droplet's flow pattern starts to deviate from the low Reynolds number pattern at a Reynolds number of about 100 (Fig. 4.2.6 (c)); the transition is complete by a Reynolds number about 400.

It is worthy of note, however, that despite the qualitative changes in the pattern of the internal circulation, the dependence of maximum velocity on force and viscosity is constant, as will be shown in §4.2.4.
Fig 4.2.6: Development of flow pattern at increasing Reynolds number Re. The figures are for a 7mm diameter PdSi droplet at various viscosities in TEMPUS at a containment acceleration of 4x10^{-3} g. The pattern is independent of Re below about Re=100, and the polar loops are still present at Re=246, but not at higher Reynolds number.
Section 4.2.4: Effect of Positioning Forces on Internal Circulation Velocity

A parametric study was performed to determine the response of the predicted flow velocity to changes in materials and operational parameters. The materials property chosen was viscosity, since this property changes over at least two orders of magnitude over the range of investigation in the experiments on PdSi discussed in Chapter 5. The operational parameters for this and the following section are oscillating current frequency and containment acceleration. The results presented in this section are presented for a Pd$_{82}$Si$_{18}$ sample.

Since the net force on the droplet is identically zero for zero displacement, some other measure of force is required. In this section, containment acceleration is used as a measure of force. The containment acceleration is that required to push the droplet to 1 mm displacement against the positioning force of the levitator; i.e. for a 1-gram droplet, an initial force gradient of 10 N/m is required to contain the sample at 1 mm displacement at 1 g acceleration.

The following plots present the results of approximately 450 fluids calculations. In Fig. 4.2.7 (a), the maximum velocity for each case is plotted against the sample’s viscosity, for several different acceleration levels. As can be seen, the velocity decreases linearly as the viscosity increases for all acceleration levels studied (parallel lines in the figure), and lower velocities are produced for lower accelerations. Also note that dividing by the corresponding acceleration level brings all of the different lines into convergence (on the upper line in the figure), demonstrating that the predicted velocity is directly proportional to the containment acceleration. The predicted velocities range from about 1 $\mu$m/s to about 30 cm/s.

In Fig 4.2.7 (b), the same data are plotted against acceleration for various viscosities. In this case, multiplying by the corresponding viscosity causes all of the lines to coincide on the lower line of the plot; velocity is inversely proportional to viscosity in the range covered by these calculations.

This behavior may be explained by an examination of the equation of motion, which governs the fluid flow:

$$\rho \frac{\partial \vec{u}}{\partial t} + \rho (\vec{u} \cdot \nabla) \vec{u} = \mu \nabla^2 \vec{u} - \nabla P + \vec{F}$$  (eq. 4.2.1)

where the applied pressure gradient $\nabla P$ is zero, and in steady-state flow, the $\frac{\partial}{\partial t}$ term is also zero. In the low Reynolds number limit, the convective term $\rho (\vec{u} \cdot \nabla) \vec{u}$ may also be neglected. If the velocity
distribution may be separated into its magnitude \( U_o \) and spatial variation \( \hat{u} \), then the dependence of velocity \( U_o \) on viscosity may also be simplified:

\[
\mu \nabla^2 \hat{u} = \mu U_o \left( \nabla^2 \hat{u} \right)
\]  

(eq. 4.2.2)

The force is similarly separated into magnitude \( F \) and variation \( \hat{F} \).

Combining the remaining terms yields the dependence of the magnitude of velocity on the viscosity and force:

\[
U_o \propto \frac{F}{\mu}
\]  

(eq. 4.2.2)

which matches the dependence seen in Fig. 4.2.7.
Fig 4.2.7: Dependence of maximum velocity on acceleration and viscosity. Note that the lines all collapse onto the indicated line when divided by acceleration in (a) or multiplied by the appropriate viscosity in (b); the plotted velocity is inversely proportional to viscosity and directly proportional to acceleration.
Section 4.2.5: Effect of Frequency on Internal Circulation Velocity

A similar series of calculations were performed, but with frequency instead of containment acceleration as the second variable. In these calculations, the containment acceleration was held constant by adjusting the coil current as required. These calculations were also performed for the Pd$_{82}$Si$_{18}$ alloy. Because of the low electrical conductivity of this alloy, the quantitative results presented here will differ for other alloys; however, the trends should still be similar.

The results of these calculations are shown in Fig 4.2.8. As can be seen in the figure, the flow is unaffected by the frequency for frequencies below about 100 kHz. At this frequency, the ratio of radius to skin depth is about 1.3. For higher frequencies, the velocity increases, up to a maximum at about 2 MHz of about 3 times the low frequency velocity, and then falls back near the low-frequency limit for higher frequencies.

In terms of the fluid flow velocity, TEMPUS at 160 kHz is above the highest frequency giving minimum velocity, but this would only decrease the flow velocity by about 15%, while MEL at 300 kHz could have its flow reduced by more than 30% by decreasing the positioner frequency. The recommendation for the Auburn levitator to run about 20 kHz, made on the basis of minimum sample temperature, also insures minimum fluid flow in the samples.
Fig 4.2.8: Dependence of maximum velocity on oscillating current frequency at constant containment acceleration, for typical transition metals (μ about 4mPa-s) and for Pd-18Si at Tm (μ about 40mPa-s). The flow velocity is independent of frequency below about 100 kHz, with a maximum of value 3 times the plateau value at about 2 MHz.

Section 4.4: Conclusion

In this chapter, results were presented for fluid flow in levitated droplets. The general flow patterns were discussed, along with some special cases involving qualitative changes in the flow patterns. Quantitative results were presented for a wide range of sample conditions and operating parameters, with the results providing verification for the model, as well as guidance for the effect of levitator design on fluid flow. While these results are only a small part of what has been and can be done with this model, they do demonstrate part of its utility.

The main findings of this chapter may be summarized as follows:

- A transition in flow pattern from positioning-dominated to heating-dominated flows was predicted for 7mm diameter zirconium samples to occur about 27A heating current for a positioner current of 100A. This transition results in the maximum velocity being approximately 3.5 cm/s for the range of 15-35 A heating current.
In this system, the heating current has a more significant effect in driving internal flow than the positioning current. For example, a heating current of 50A results in an internal flow velocity about 12 cm/s, while a similar positioning current results in only about 1 cm/s for the same 7mm diameter zirconium sample.

A transition in the positioning-dominated flow pattern is observed as a result of changing Reynolds number, for example by changing the temperature and therefore the viscosity of the sample. The pattern changes from four to two loops as the flow goes from a viscous to an inertial regime, respectively. However, this change in pattern does not significantly affect the relation between viscosity and maximum velocity observed.

The maximum velocity is directly proportional to containment acceleration and inversely proportional to viscosity for the range studied here.

The maximum flow velocity at constant containment acceleration is independent of frequency up to about 100 kHz. Then the velocity increases with increasing frequency up to a maximum of 3 times the low-frequency value at about 2 MHz for the PdSi sample studied. Thereafter the velocity falls with increasing frequency.

The fluid flow calculations presented here can specify the operational conditions under which low velocities may be expected. Reducing the internal flow velocity is critical to the measurement of viscosity, and is a parameter under investigation in other studies, including nucleation and phase selection.
Chapter 5: Measurements of Surface Tension and Viscosity in Microgravity

The theory and practice of measuring surface tension and viscosity by a containerless microgravity technique are presented in this chapter. These measurements have three main purposes:

- To demonstrate the measurement of viscosity by this technique for the first time.
- To determine the limits of applicability of this technique, in terms of materials parameters.
- To measure properties of materials of scientific and commercial interest.

These experiments are reliant on thermal, magnetic, and fluid flow models for their planning. The sample must be processed with a well-controlled thermal profile and well contained without extensive calibration on-orbit. Furthermore, the viscosity experiments cannot be performed without laminar flow conditions.

Section 5.1: Introduction

A chronology and history of the development of the oscillating droplet technique for measuring surface tension and viscosity has been presented in §1.3.5. The basis of these measurements is the oscillatory response of a liquid sphere when a deforming force is released. The frequency of these oscillations is determined by the surface tension of the droplet, and the damping by its viscosity, subject to certain limitations.

The static deformation of a liquid droplet is described by an equation of the form:

\[ R(\theta, \varphi) = R_0 \left[ 1 + \varepsilon Y_l^m(\theta, \varphi) \right] \]  
(eq. 5.1.1)

where \( \varepsilon \) is the relative magnitude of oscillations of mode \( l, m \), \( Y_l^m(\theta, \varphi) \) are the spherical harmonics of mode \( l, m \), \( |m| < l \), and \( \varphi \) is the azimuthal angle \( \phi - \phi_s \) along which the oscillations are oriented. This equation leads to the form of oscillations of a viscous droplet:

\[ R(t, \theta, \varphi) = R_0 \left[ 1 + \sum_{l,m} \varepsilon_{l,m} Y_l^m(\theta, \varphi) e^{i(l_m \cdot r_0)} \right] \]  
(eq. 5.1.2)
<table>
<thead>
<tr>
<th>Mode $l$</th>
<th>Possible values of $m$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l = 0$</td>
<td>$m = 0$</td>
<td>Spherical Symmetry, $R(t, \theta, \varphi) = R(t)$. Not observed.</td>
</tr>
<tr>
<td></td>
<td>$m = 0$ $m = \pm 1$</td>
<td>Pure translation along z-axis</td>
</tr>
<tr>
<td></td>
<td>$m = \pm 2$</td>
<td>Pure translation in x-y plane</td>
</tr>
<tr>
<td>$l = 2$</td>
<td>$m = 0$ $m = \pm 1$</td>
<td>axisymmetric, “football” mode. See Fig. 5.1.1</td>
</tr>
<tr>
<td></td>
<td>$m = \pm 2$</td>
<td>mirror symmetric “peanut” mode. See Fig. 5.1.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>twofold mirror symmetric “pinch” mode. See Fig 5.1.3.</td>
</tr>
</tbody>
</table>

Table 5.1.1: Summary of natural oscillation modes of a liquid droplet, to $l = 2$.

In order to determine which oscillation modes are to be observed, consider the first oscillation mode in Table 5.1.1: $l = 0$. For this mode, only $m = 0$ is permitted. Since [73]

$$Y_0^0(\theta, \varphi) = \sqrt{\frac{1}{4\pi}}$$  \hspace{1cm} (eq. 5.1.3)

This mode involves only uniform changes in the radius of the droplet. Since metal droplets are incompressible, this mode is not observed.

For mode $l = 1$, there are three possibilities: $m = 0$ and $m = \pm 1$. [73]:

$$Y_1^0(\theta, \varphi) = \sqrt{\frac{3}{8\pi}} \sin \theta$$ \hspace{1cm} (eq. 5.1.4)

$$Y_1^1(\theta, \varphi) = \frac{\sqrt{3}}{4\pi} \sin \varphi$$

$$Y_1^{-1}(\theta, \varphi) = \frac{3}{8\pi} \sin \varphi$$

The two cases $m = \pm 1$ are translations in the x-y plane, along oscillation direction $\varphi$, and the $m = 0$ case is a translation along the z-axis. While these modes are observed, they are not related to the fluid properties of the sample.

It is the oscillations of mode $l = 2$ that can reveal the fluid properties of the sample. These oscillations are proportional to [73]:

$$Y_2^0(\theta, \varphi) = \sqrt{\frac{5}{96\pi}} \sin^2 \theta \cos^2 \varphi$$

$$Y_2^1(\theta, \varphi) = \sqrt{\frac{5}{24\pi}} \sin \theta \cos \varphi$$

$$Y_2^{-1}(\theta, \varphi) = \sqrt{\frac{5}{24\pi}} \sin \varphi \cos \theta$$ \hspace{1cm} (eq. 5.1.5)

$$Y_2^2(\theta, \varphi) = \sqrt{\frac{5}{96\pi}} \sin^2 \theta \cos^2 \varphi$$

$$Y_2^{-2}(\theta, \varphi) = \sqrt{\frac{5}{96\pi}} \sin^2 \theta \sin^2 \varphi$$
The $l = 2$, $m = 0$ mode is symmetric about the $z$-axis, as can be seen in fig. 5.1.1. This mode is the only one observed from a well-behaved symmetric drop excited by a symmetric force, as is the case for microgravity oscillation experiments with viscous samples.

The $l = 2$, $m = \pm 1$ modes shown in fig. 5.1.2 have mirror symmetry about the $z - \varphi$ plane, but are not rotationally symmetric. These oscillations require either a shear excitation or an asymmetric droplet.

The $l = 2$, $m = \pm 2$ modes shown in fig. 5.1.3 are the final type of surface oscillations possible for the fundamental mode. These oscillations are symmetric about the $x - y$ plane, the $z - \varphi$ and the plane perpendicular to these two. These oscillations may be excited directly by a force that is not rotationally symmetric.
Fig. 5.1.1: Oscillations of "football" mode $l = 2, m = 0$ of a liquid sphere, with

$$R(t, \theta, \varphi) = R_0 \left[ 1 + \varepsilon_{2,0} Y_2^0(\theta, \varphi) e^{i\omega_0 t} \right]$$

for different values of time $t$ presented left to right. These deformations are axisymmetric.
Fig. 5.1.2: Oscillations of mode $l = 2, m = \pm 1$ of a liquid sphere, with

$$R(t, \theta, \varphi) = R_0 \left[ 1 + e_{z, l_2} Y_{l_2}^{m_2}(\theta, \varphi) e^{i\omega_{l,m} t} \right]$$

for different values of time $t$ presented left to right. These deformations have one plane of mirror symmetry.
Fig. 5.1.3: Oscillations of mode $l = 2$, $m = \pm 2$ of a liquid sphere, with

$$R(t, \theta, \varphi) = R_0 \left[1 + \varepsilon_2 t e^{i \omega t} \right]$$

for different values of time $t$ presented left to right. These deformations are mirror symmetric about the $x$-$y$ plane and one other plane.
Section 5.2: Theory of Measurements

The relation between surface tension and oscillation frequency for small deformations of an inviscid sphere was given by Lord Rayleigh [46]:

\[
\omega_l^2 = \frac{l(l-1)(l+2)\gamma}{\rho R_o^3} \tag{eq. 5.2.1}
\]

where \( \omega_l \) is the angular frequency of oscillation mode \( l \), for a droplet of surface tension \( \gamma \), density \( \rho \), and radius \( R_o \). This equation is also commonly written in terms of the cycle frequency \( f_l \) and mass \( M \) of the droplet:

\[
f_l^2 = \frac{l(l-1)(l+2)\gamma}{3\pi M} \tag{eq. 5.2.2}
\]

The viscosity \( \mu \) of the droplet determines the damping of the oscillations, as described by Lamb [47]:

\[
r_l = \frac{\rho R_o^2}{(l-1)(2l+1)\mu} \tag{eq. 5.2.3}
\]

This relation will give the molecular viscosity of the droplet if the flow is laminar, but will only give an effective viscosity if the flow is turbulent or transitional. The flow state of the droplet is investigated by comparing the measured viscosity at different levels of background flow; a measured value that is independent of the induced flow velocity is a valid molecular viscosity.

Reid[72] gives the limitations on the ratio of surface tension to viscosity for this technique, as well as a correction to the oscillation frequency for finite viscosity. According to analysis, and the nonlinear analysis by Suryanarayana and Bayazitoglu [63], the surface oscillations of a drop depend on the parameter \( \alpha^2 \), defined as:

\[
\alpha^2 = \frac{\omega_R \rho R_o^2}{\mu} = \sqrt{l(l-1)(l+2)\gamma \rho R_o^3 \mu} \tag{eq. 5.2.4}
\]

where \( \omega_R \) is the Rayleigh frequency defined in eq. 5.2.1. For values of \( \alpha^2 > 3.69 \), the droplet will oscillate; below this value, a non-periodic response will be observed[72].

The difference between the droplet's observed oscillation frequency and its Rayleigh frequency is tabulated by Suryanarayana and Bayazitoglu [63], whose paper also quantified the viscous effect on the frequency of oscillation. The deviation from Rayleigh's theory due to viscous effects is about 10% for \( \alpha^2 = 10 \), and about 1% for \( \alpha^2 = 59 \). For the conditions of the data presented in §5.4, the range in \( \alpha^2 \) is from about 200 to 2500, so the viscous effects on the measured frequency are negligible. However some metallic glass-
forming samples processed in TEMPUS have values of $\alpha^2$ as low as 1.8; these droplets do not even oscillate, much less oscillate linearly.

For ground-based measurements of surface tension, Cummings and Blackburn have given a theoretical correction to account for the effects of gravity and the magnetic field on the droplet oscillations [56]. For a droplet levitated in magnetic field whose intensity varies linearly in the $z$-direction, the correction for mode $l = 2$ is:

$$\omega^2 = \frac{1}{5} \left( \omega_{m=0}^2 + 2 \omega_{m=\pm1}^2 + 2 \omega_{m=\pm2}^2 \right) - \omega_t^2 \left( 1.9 - 1.2 \left( \frac{z_o}{R_o} \right) \right)$$

(eq. 5.2.5)

where $\omega_R$ is again the Rayleigh frequency of the droplet for mode $l = 2$. $\omega_{m=0}$, $\omega_{m=\pm1}$, and $\omega_{m=\pm2}$ are the frequencies of the various values of $m$, and $\omega_t$ the translational frequency. The vertical displacement $z_o$ of the droplet is determined by the requirement that the force on the droplet exactly balance the acceleration $g$ of gravity:

$$z_o = \frac{g}{2 \omega_t^2}$$

(eq. 5.2.6)

Section 5.3: Experimental Method

In these experiments a levitated liquid metal droplet is positioned and deformed. When the deforming force is released, the droplet oscillates, and the oscillations are captured by high-speed video. The apparent area of the droplet is calculated for each frame, and the oscillations are fit to determine the frequency and damping constant, which are determined by the surface tension and viscosity of the droplet, respectively.

These experiments were performed in TEMPUS, a microgravity electromagnetic levitator. TEMPUS provides cameras which view the top and side of the sample, and different pyrometers allow accurate temperature measurement for the wide range of materials processed.

Results are presented for the alloy Pd$_{14}$Si$_{82}$. This alloy was chosen for several reasons. Its good undercoolability, combined with its low evaporation rate, makes a very large range of temperature and viscosity accessible to these measurements. Because of its high viscosity at the melting point, there is also a large range in viscosity values accessible. Finally, this material is well-studied and important to the theory of glass-forming alloys [74-87], although it is itself rather a poor glass former.
A typical thermal cycle for these experiments is shown in Fig. 5.3.1. First the sample is stably positioned in the coils, then melted. The sample is heated above its melting point, with the superheat being determined by the sample material and the planned experiment. For the PdSi alloys, a superheat of about 700K was used to permit measurements to be made over a wide temperature range. The sample is then allowed to cool and undercool. During cooling, surface oscillations are excited in the sample by short pulses of the heating field. Eventually, the sample recalesces, and the solid sample is cooled until it may be safely touched to damp any rotations and prepare for the next cycle.

Fig 5.3.1: Typical thermal profile for droplet oscillation experiments in TEMPUS. Sample is melted and superheated, then allowed to cool. As the droplet cools, the heating field is pulsed to excite surface oscillations. The sample recalesces and is fully solidified, then mechanically damped in preparation for the next cycle.

The heater pulses excite oscillations of mode \( l = 2, \ m = 0 \), the axisymmetric "football" mode. These oscillations are recorded on video at 120 fps (frames per second) on the top view and 60 fps on the side view. The video signal is processed by a realtime analysis system developed by the author, and the resulting area-time signal is stored for later analysis. This system for realtime data collection and analysis was successfully demonstrated during the MSL-1 mission, with some viscosity data being available within minutes after the experiment, even before the sample had solidified.
In the present state of the analysis system, what is actually stored is a spectrum for each frame: a table of the number of pixels at each of the 256 intensities measured. Then a threshold algorithm is applied to determine the apparent area of the droplet in pixels (see Fig. 5.3.2(a)). More advanced measurement methods are planned for future work, including edge detection and fitting the droplet’s shape to spherical harmonics directly.

The measured area-time signal is then processed to determine the surface tension and viscosity. First, a Fourier power spectrum is used to find the frequency of the surface oscillations (Fig. 5.3.2(b)). Then the data are band-pass filtered using a flat-topped Hamming window centered about this oscillation frequency. An example of the filtered oscillations is shown in Fig. 5.3.2(c).

Finally, the filtered oscillations are fit to the functional form of a damped oscillation:
\[ A(t) = A_0 \sin(\omega(t - t_0))e^{-\frac{t-t_0}{\tau}} \]  
(eq. 5.3.1)

The frequency of the surface oscillations may be determined either from the power spectrum, or by fitting to the filtered oscillations in the time domain. The damping constant \( \tau \) is determined by a least-squares fit.

![Graphs showing unfiltered oscillations, power spectrum, and filtered oscillations.](image)

Fig 5.3.2: MSL-1(STS-83) experimental data Pd-18Si sample, excited oscillations at 1215C. (a) unfiltered oscillations, (b) Power Spectrum of the unfiltered oscillation shown in (a) with surface tension peak visible at 25.0 Hz, which corresponds to a surface tension of 1.7 N/m and (c) filtered oscillations with exponential damping constant of 3.42s, giving viscosity 9.0 mPa-s.
Section 5.4: Experimental Results

As a part of the experiments performed in TEMPUS in the MSL-1 (STS-83) and MSL-1R (STS-94) Space Shuttle missions, measurements of the surface tension and viscosity of Pd-18at%Si were performed. This alloy was chosen for its high viscosity at the melting point, its moderate melting point (820°C), and its very low evaporation rate. The low evaporation rate allows processing over a wide range of temperatures. Data are presented from the STS-83 flight for temperatures ranging from 725-1425°C (100°C undercooled to 650°C superheated).

![Surface Tension of Pd-18Si STS 83 OVCR 10](image)

Fig 5.4.1: Uncorrected surface tension data for Pd-18Si.

The uncorrected surface tension data for Pd-18Si are presented in fig. 5.4.1. For these data, the standard error relative to a linear fit is 2.0%. It should be pointed out that this precision is quite good for surface tension measurements. However, the average standard error of each series is only 0.13-0.53%.

A major part of this additional scatter is due to the magnetic force on the surface of the sample, which increases the oscillation frequency. The trend can be seen that the higher surface tension values correspond to a higher positioner control voltage. These data may, however, be corrected for this effect by the Cummings and Blackburn formula, with appropriate values for microgravity.
For the case of microgravity droplets with symmetric deformations, only the $m = 0$ mode is observed. Also, since the acceleration $g$ approaches zero, the term in the displacement $z_o$ also approaches zero, yielding the appropriate correction for microgravity from equation 5.2.5:

$$\omega^2 = \omega_{obs}^2 - 1.9 \omega_r^2$$

(eq. 5.4.1)

It has always been assumed that the correction due to the magnetic force on the sample would be negligible in microgravity. However, for these results, the corrective term of the order of 1-3% is important to the overall precision of the measurements. The corrected data are presented in fig. 5.4.2.

![Surface Tension of Pd-18Si STS 83 OVCR 10](image)

Fig 5.4.2: Corrected surface tension data for Pd-18Si.

As is apparent from comparison of figures 5.4.1 and 5.4.2, applying this correction to the data greatly reduces the scatter among the different series. The corrected data have a linear correlation with a standard error of only 0.95%.

The viscosity data are presented below in fig. 5.4.3, along with some data and correlations reported by other authors (Table 5.4.1) for this alloy and similar glass-forming alloys. These data are in good agreement with Nishi [75], Lee [86], and Egry [88], but are different by two orders of magnitude from the results reported by Steinberg, et al. [77-8]. This controversy is still under investigation.
It is not possible to determine from the existing data whether the viscosity of Pd-18Si follows an Arrhenius, power-law, or Vogel-Fulcher relation, but data from other experiment runs may provide more insight into the behavior in the undercooled regime.

Table 5.4.1: Correlations for the viscosity of Pd-Si alloys from experimental data, reported by various authors.

<table>
<thead>
<tr>
<th>Alloy/Reference</th>
<th>Correlation</th>
<th>Melting Point Viscosity (mPa-s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pd 77.5 Cu 6 Si 16.5 Egry et al [88]</td>
<td>$\eta = 0.114 \exp\left(\frac{6260}{T}\right)$</td>
<td>49</td>
</tr>
<tr>
<td>Pd 78 Cu 6 Si 16 Nishi et al [75]</td>
<td>$\eta = 0.0589 \exp\left(\frac{8082.1}{T}\right)$</td>
<td>151</td>
</tr>
<tr>
<td>Pd 77 Cu 6.5 Si 16.5 Lee et al [86]</td>
<td>$\eta = 2.32 \exp\left(\frac{938.5430}{T - 726}\right)$</td>
<td>63</td>
</tr>
<tr>
<td>Pd 84 Si 16 Nishi et al [86]</td>
<td>$\eta = 0.062 \exp\left(\frac{5825.5}{T}\right)$</td>
<td>13</td>
</tr>
<tr>
<td>Pd 82 Si 18 Szekely et al [this work]</td>
<td>$\eta = 0.1917 \exp\left(\frac{5754.4}{T}\right)$</td>
<td>37</td>
</tr>
</tbody>
</table>
Fig 5.4.3: Viscosity data for Pd-18Si, (a) linear vs. T, and (b) log vs. 1/T. Other relations from [75,86,88].
Section 5.5: Conclusion

The data presented here, while only a small part of that gained in the MSL-1 and -1R shuttle flights, do prove the oscillating drop technique is capable of providing containerless measurements of surface tension and viscosity, both in the superheated and undercooled regimes. Additional data were taken for Pd-Cu-Si, Zr, Fe-Cr-Ni steels, and several Zr-based glass forming alloys, due to collaborations with numerous other TEMPUS PI’s.

The findings of this chapter may be summarized as follows:

- Experimental measurements show that this technique does provide viscosity data
- The viscosity of Pd$_{49}$Si$_{18}$ was found to be $0.1917 \exp(5754.4/T)$ in mPa-s, with $T$ in K.
- The surface tension of this same alloy was measured to a precision of better that 1% to be equal to $1.949 \times 1.91 \times 10^{-4} T$ in N/m for $T$ in K.
- The Cummings and Blackburn correction, while only about 1-3% for microgravity measurements, is still important.

Section 5.6: Acknowledgment

The author gratefully acknowledges Dr. Gerardo Trapaga’s contribution of the correlations for the Pd-Si alloys, in addition to all of the help previously mentioned.
Chapter 6: Conclusions

Mathematical modeling is critical to the understanding of a system as complex as an electromagnetic levitator. Methods and results have been presented which are useful both for the design of new levitation systems and also the design of experiments for a particular levitation system. These results are separated into two parts: electromagnetic force and heating, and fluid flow. Finally, results of experimental measurements of surface tension and viscosity of a liquid metal droplet, carried out on the Space Shuttle by a containerless technique using the TEMPUS EML are presented.

Magnetic Modeling:
The results of the magnetic modeling presented in chapters 2 and 3 indicate that many improvements to current levitation systems are possible. Calculations show that:

1. The use of a two-frequency levitator is essential to processing both high- and low-melting point materials in vacuum and gas environments, since this approach allows independent control of heating and positioning.

2. The heating coils should operate at as high a frequency as is practical, since heating efficiency increases as the square root of frequency.

The frequencies used in existing levitation devices (150kHz-1.2 MHz positioning, 300-600 kHz heating) are far from optimal for the example employed here of 6mm diameter zirconium samples.

3. In the case of the positioning coils, a compromise must be made between the best minimum sample temperature and most efficient positioning (force/current).

4. The lowest minimum sample temperature is provided by a positioning frequency about 20 kHz for the 6mm diameter zirconium samples considered here.

5. The best positioning efficiency, however, requires a frequency about 200 kHz for the 6mm diameter zirconium samples considered here.

Fluid Flow Characteristics:
The fluid flow velocity in levitated droplets is a critical factor in a number of the experiments performed with electromagnetic levitation, whether as a variable in nucleation and phase selection experiments, or as an impediment to measurements such as viscosity measurements. The flow patterns in a droplet levitated in TEMPUS are presented, along with a more detailed examination of transition regions where the flow pattern changes. Calculations show that:

1. The maximum flow velocity is proportional to the ratio of force to viscosity for the range considered.
(2) Also, the flow velocity is independent of frequency below about 150 kHz for the case presented, and is higher than this minimum for higher frequencies.

**Measurement of Surface Tension and Viscosity:**

The containerless measurement technique using the Space Shuttle’s microgravity environment opens a new regime of temperature and materials to the measurement of surface tension and viscosity. The ability to measure the properties of undercooled and refractory melts without the interference of a container may provide the precision and range necessary to improve the understanding of the liquid state, as well as providing better properties for process modeling.

The surface tension measurements presented cover the range of about 100°C undercooling to over 600°C superheat. Viscosity measurements consistent with literature values are also presented for a similar range of temperature. These experiments show that:

1. The Cummings and Blackburn correction to the measured surface tension, while only about 1-3% for the conditions studied, is still important.
2. The corrected surface tension data have a precision better than ±1%.
3. These experiments prove for the first time the possibility of an electromagnetic levitation technique for the measurement of viscosity.
Chapter 7: Future Work

Some of the areas for future work include:

- Detailed error analysis of oscillation experiments. A better understanding of the sources of uncertainty in these measurements will improve both precision and accuracy.

- Analysis of turbulent transition. Experiments performed during MSL-1 were intended to capture a flow transition in the viscosity experiments. Further analysis will show whether we were successful. This data may be extrapolated to any material of interest, giving the required conditions for measurement of viscosity, and allow the evaluation of the possibility to conduct EML-based viscosity measurements on the ground.

- A survey of turbulence models. The internal flows of levitated droplets in the turbulent regime are of interest to many researchers, and it is not known which, if any, of the present turbulence models can describe this system well.

- Experimental verification of the frequency recommendations for the Auburn EML, and application of similar calculations to ground-based research levitators.

- Application of the magnetic and MHD modeling techniques of this work to industrial problems such as magnetic containment, continuous casting, and induction heat treatment.
Appendix: Magnetic Results for Auburn EML

The figures created for the design of the Auburn EML are presented here. Some of these calculations are presented in Figures 3.2.2-3.3.1. These calculations show the dependence of positioning efficiency, heating efficiency, and their ratio for the Auburn EML's heating and positioning coils on oscillating current frequency and electrical conductivity of the sample, for 6.35 mm diameter samples. These results are presented in the following 12 figures, A.1-A.6 (a) and (b).

These figures should be interpreted based on the explanation in Chapter 3.
Fig A.1: (a) and (b): Positioning Efficiency of Heating Coils
Fig A.2: (a) and (b): Positioning Efficiency of Positioning Coils
Dependence of Heating Efficiency on Conductivity and Frequency

6.35mm (0.250in) sample, Auburn Heating Coils

Fig A.3: (a) and (b): Heating Efficiency of Heating Coils
Fig A.4: (a) and (b): Heating Efficiency of Positioning Coils
(a) Dependence of Positioning Efficiency on Conductivity and Frequency

6.35mm (0.250in) sample, Auburn Heating Coils

(b) Dependence of Positioning Efficiency on Conductivity and Frequency

6.35mm (0.250in) sample, Auburn Heating Coils

Fig A.5: (a) and (b): Ratio of Positioning to Heating for Heating Coils
(a) Dependence of Positioning Efficiency on Conductivity and Frequency

6.35mm (0.250in) sample, Auburn Positioning Coils

![Graph showing the relationship between positioning force per unit power and electrical conductivity for different frequencies.]

(b) Dependence of Positioning Efficiency on Conductivity and Frequency

6.35mm (0.250in) sample, Auburn Positioning Coils

![Graph showing the relationship between positioning force per unit power and frequency for different conductivities.]

Fig A.6: (a) and (b): Ratio of Positioning to Heating for Positioning Coils
References


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60) J. Watkins, priv. comm.


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Biographical Note

Robert Wyatt Hyers was born on July 26, 1971, in Decatur, Georgia, to Robert C. and Ruth P. Hyers. He attended Tucker High School in Tucker, Georgia, where he was salutatorian and STAR student of the class of 1989. He also received second place in the state of Georgia in the VICA (Vocational and Industrial Clubs of America) skill olympics in auto body repair in 1989. He was proprietor of a small, independent auto body repair firm, doing contract repairs for a local construction company in 1989.

He received an S.B. degree in Materials Science and Engineering from the Massachusetts Institute of Technology in September 1992, through the Course IIIB co-op program. His co-op work was done at Allied-Signal, Inc., in Morristown, New Jersey, on a melt-spun high-temperature aluminum alloy. During his time as an undergraduate, he served as a tutor and blacksmithing workshop assistant for the Integrated Studies Program, and did research through the Undergraduate Research Opportunities Program (UROP) with Prof. Regis Pelloux on the fatigue of 2024 aluminum, and for Prof. J. Szekely on the time-temperature profiles to be expected in the IML-2 TEMPUS experiments.

He entered graduate school at MIT in Materials Science and Engineering in September 1992, as a teaching assistant for the undergraduate course “Transport Phenomena in Materials Engineering” under Prof. J. Szekely, and continued doing research in his group with fellowships from the Mining and Minerals Resources Research Institute (MMRRI) and NASA Graduate Student Researchers Program.