V MISCELLANEOUS PROBLEMS

A LOCKING PHENOMENA IN R-F OSCILLATORS

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Description of Project The purpose of this project is to investigate the locking phenomena between coupled microwave oscillators

Status The behavior of two c-w magnetrons operating into a single load has been investigated experimentally Results agreed well with theoretical expectations both in the locked and unlocked conditions The predicted locking spectrum and power transfer between tubes were observed No power exchange was apparent when the tubes were tuned to identical frequencies and, in this condition, each magnetron was operating into a matched load Line lengths between tubes was found not to be critical so long as the system remained symmetrical

In order to study locking phenomena in greater detail, the behavior of a klystron locked to an attenuated magnetron signal will be studied By this arrangement, mutual effects between tubes are eliminated Further study of the closely coupled oscillators will be attempted at a later date

B ELECTRONIC DIFFERENTIAL ANALYZER

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Introduction This project is concerned with the development of an all-electronic, high speed differential analyzer The work in progress may be divided into two general problems

(1) The investigation of multiplying circuits
(2) The investigation of the problems of initial conditions and stability for some simple ordinary differential equations with constant coefficients

Multipliers Two multiplying circuits have been built, based on the difference-of-squares scheme described in the last Progress Report These circuits differ only in the square law devices used

The first circuit uses crystal diodes type Sylvania 1N34, to give the necessary square law characteristic This multiplier has a range of two decades in the output and a percentage error of from 5 to 10 per cent depending upon the amplitude of the output signal The primary limitations is found to be the characteristic of the crystal diodes If a more satisfactory two-terminal square law element can be found, having a larger square law range and better uniformity this circuit would be usable The special high vacuum square-law diodes have been abandoned because of the large number of floating power supplies required for their use Thyrite is being considered as an alternative square-law element

The second circuit tested uses a push-push triode circuit to obtain the necessary square-law characteristic This multiplier has a range in the output of two decades with a percentage error of less than 5 per cent The range limitation in this case is in the phase inverters rather than the square-law circuit A new unit is being made up to overcome this difficulty The circuit however suffers from drift in the balancing adjustment of the push-push squarers The use of a self-balancing scheme to overcome this difficulty is being considered

Differential Equations A number of time integrators and differentiators have been built up and used to solve three simple equations given below

$$\frac{dy}{dt} = -Ay + F(t), \quad (1)$$

$$\frac{d^2y}{dt^2} = -Ay - B\frac{dy}{dt} + F(t), \quad (2)$$

$$\frac{dy}{dt} = +Ay \quad (3)$$

The units were set up to solve these equations at a repetition rate of 60 cps, and the solutions were displayed on the face of a cathode-ray tube Only simple functions such as sine waves, square waves, and impulse functions were used for $F(t)$ It was found that although either integrating or differentiating units could be used to solve Eqs (1) and (2), Eq (3) could be solved only with integrators, for the following reason Any available differentiator will have a high frequency root because of the finiteness of its amplifier passband If such a differentiator is used to solve Eq (3), the solution is dominated by this high frequency root rather than the desired root of the equation itself For this reason it is now felt that differentiators will have only a limited application in a general electronic differential analyzer

Initial Conditions There are two aspects to the problem of inserting inital conditions in an electronic differential analyzer The first is to obtain the correct initial voltages for the variables, and the second is to return the circuit to its original condition so that a repetitive solution may be obtained

(1) Methods of Obtaining Initial Voltages Fundamentally the initial values of the variables must be in the form of charges stored upon condensers in the circuit In practice this gives two possibilities Either the condensers in the integrators may be charged initially, or a battery may be placed in series with each uncharged integrator at the initial instant These two methods are illustrated in Fig 1

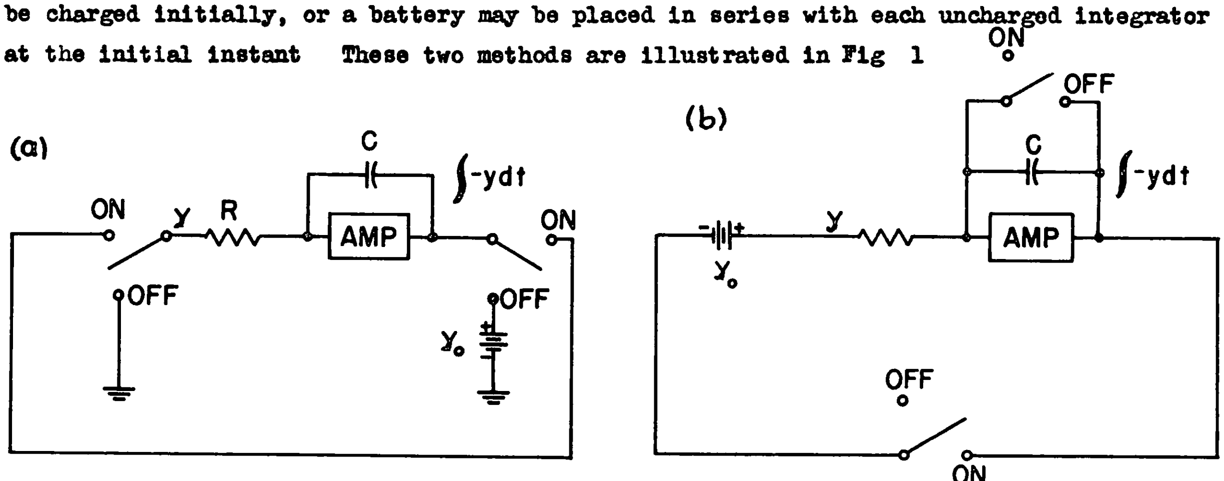


Figure 1 Solution of the equation $\frac{dy}{dt} + y = 0$ (a) By means of an initially charged condenser of voltage $V=y_o$, (b) by means of a series battery of voltage $V=y_o$

Both of these methods have been tried The time constant for charging the condenser C in Fig 1a is RC

(2) <u>Methods of Returning the Solution to the Initial State</u> In a repetitive system it is necessary to return the circuit to its initial condition before a second solution may start This has been done by

a Switching methods which force the variables to assume their initial values

b Reversing the independent variable

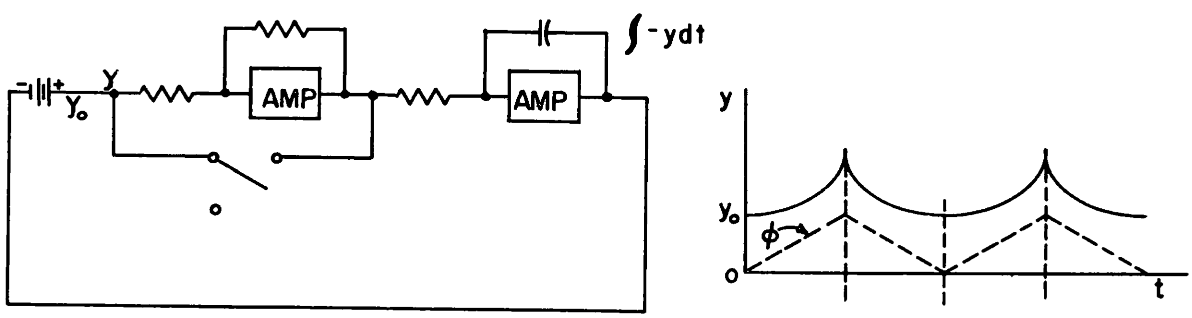
If the initial conditions are applied by charging the condenser, as in Fig 1a , the circuit will return to its initial condition when the switches are in the OFF position The disadvantages of this method are that the charging time constant must be long compared to the solution time and that a separate switch is required for each integrator

If the initial conditions are applied by means of series batteries, as in Fig 1b it is necessary to open the loop at the adder and to discharge the condensers in order to return the circuit to its initial condition

The second method by which the circuit may be returned to its initial state is by reversing the independent variable If an integrator were available which would integrate with respect to an arbitrary voltage, this method could be used If a satisfactory multiplier were available, such an integrator could be developed from a time integrator according to the following equation

$$\int y \times d\phi \quad = \int \left[y \times \frac{d\phi}{dt} \right] dt$$

At the present time a satisfactory multiplier is not available, but if a function ϕ is chosen, as in Fig 2b whose derivative is alternately plus or minus one, the multiplication by $\frac{d\phi}{dt}$ can be performed by switching an inverter in and out of the circuit Because the independent variable is now a cyclic function, the solution automatically returns to its initial values once every cycle The circuit for solving a simple equation by this method is shown in Fig 2a



(a) Circuit for solving the equation

$$y = \int_0^t -y\,dt + y_0$$

(b) The solution obtained as displayed in real time

Figure 2

The trouble with all switching methods is in obtaining a reliable switch Mechanical and electronic switches both have their disadvantages If a general integrator were available, it would be possible to use other voltages (e.g. sin waves) as the independent variable and to eliminate the need for any switches at all This is one reason why attention is still being directed to the development of a satisfactory multiplier

C ELECTRONIC POTENTIAL MAPPING

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Description of Project The purpose of this project is to develop a pictorial display for surface distributions of potentials It is expected that when developed, this equipment will be an aid in medical diagnosis

Status Deflection coils and synchronizing and sweep circuits for the 16-element pickup and scanning tube mentioned in the last Progress Report have been built The over-all system, both pickup and display, is now in operation When different d-c potentials are put on the different pickup grids, a checkerboard pattern is obtained on the display tube Studies of the scanning process in this type of system are now in progress

D TRANSIENT PHENOMENA IN WAVEGUIDES

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For the narrow-band application of waveguides, the propagation of signals is characterized by the group velocity, and there is very little distortion of the signal When a pulse or other wide-band signal is transmitted through a considerable length of waveguide, it is deformed on account of the non-linear variation of the phase velocity with respect to the frequency A theoretical study of the transient phenomena in waveguides was started a year ago by Cerrillo and considerable progress has been made toward the understanding and prediction of such phenomena The experimental work was started by Winter recently using a special hard tube pulser to modulate the magnetron, a 336-foot section of S-band waveguide and the Fast Sweep Synchroscope

Consider a waveguide of a simple cross section extending from $x_3 = 0$ to $x_3 = \infty$ An arbitrary signal is applied at $x_3 = 0$ through a probe, an iris or from an adjoining section of waveguide The propagation phenomena of the signal can be described as a linear superposition of an infinite number of modes of which, only the dominant one, or in the case of over-size waveguides, the few lowest ones are of importance The theoretical study can therefore be confined to one mode initially

If the six components of field were known as functions of time at the input point $x_3 = 0$, we could determine the frequency spectrum of the signal by Fourier analysis At any one frequency, these six components are not independent To find the frequency spectrum of the signal at x_3, we have to multiply the components at $x_3 = 0$ in the frequency domain by an exponential to take care of the phase shift In practice the applied signal is usually quite complicated and unknown To make the calculation feasible, we must make reasonable assumptions about the input signal The theoretical limitation of the input signal is that with $p \geq 2$ the field components in the frequency domain behave as ω^{-p} in amplitude when the frequency approaches infinity where $\omega = 2\pi \times$ frequency

Any one of the field components in the frequency domain at x_3 can be expressed as

$$F(s,\sqrt{s^2+1})\, e^{-K\sqrt{s^2+1}} \tag{1}$$

where $F(s,\sqrt{s^2+1})$ is usually the ratio of two polynomials in s and $\sqrt{s^2+1}$,

K is a linear function of x_3 and the cutoff frequency

s = frequency/cutoff frequency

To find the inverse transforms of (1), we first use the complex transformation $z = s-\sqrt{s^2+1}$ Substitute the above expression in $F(s,\sqrt{s^2+1})$ and obtain

$$G(z) = \frac{1}{2}(1+\frac{1}{z^2})F(z)$$

which is the ratio of two polynomials in z Compute the roots of the denominator of G(z) and expand G(z) in partial fractions If G(z) has simple poles only it can be expressed as

$$G(z) = \sum_{k=1}^{n} \frac{R_k}{z-z_k}$$

where z_k is the kth root and R_k is the residue at the pole z_k

The inverse transform of (1) for simple poles consists of two parts

$$\mathcal{L}^{-1}\left\{F(s,\sqrt{s^2+1})e^{-K\sqrt{s^2+1}}\right\} \begin{cases} = 0, \qquad \text{for } \tau < K \\ \left.\begin{aligned} &= \sum R_k\left\{e^{\frac{\tau}{2}(z_k-\frac{1}{z_k})+\frac{K}{2}(z_k+\frac{1}{z_k})} - V_0(\Omega_k,T) - jV_1(\Omega_k,T)\right\} - R_0J_0(T) \\ &= -\sum R_k\left\{U_2(\Omega_k,T)+jU_1(\Omega_k,T)\right. - R_0J_0(T) \end{aligned}\right\} \tau > K \end{cases}$$

Both summations are over roots z_k for $|z_k| < 1$ The Lommel's functions, V_0, V_1, U_2, and U_1, are defined in Watson's "Theory of Bessel Functions" p 537 Also

$K = 2\pi x_3$/cutoff wavelength

$\tau = 2\pi t \times$ cutoff frequency

$T = \sqrt{\tau^2 - K^2}$

and $\Omega_k = (\tau - K)/jz_k$

The available tables for Lommel's functions are unfortunately not of any use in these transient expressions It was necessary to obtain the appropriate expansions for numerical computation

Asymptotic expansions and a simple graphical method were developed to construct the envelopes and phase functions of the waves corresponding to the typical inverse transforms This method will be discussed in a report on this subject

The above theory was applied to the TE_{01} wave in the standard S-band rectangular waveguide, and some of the results are shown in Fig 1 The column to the left gives

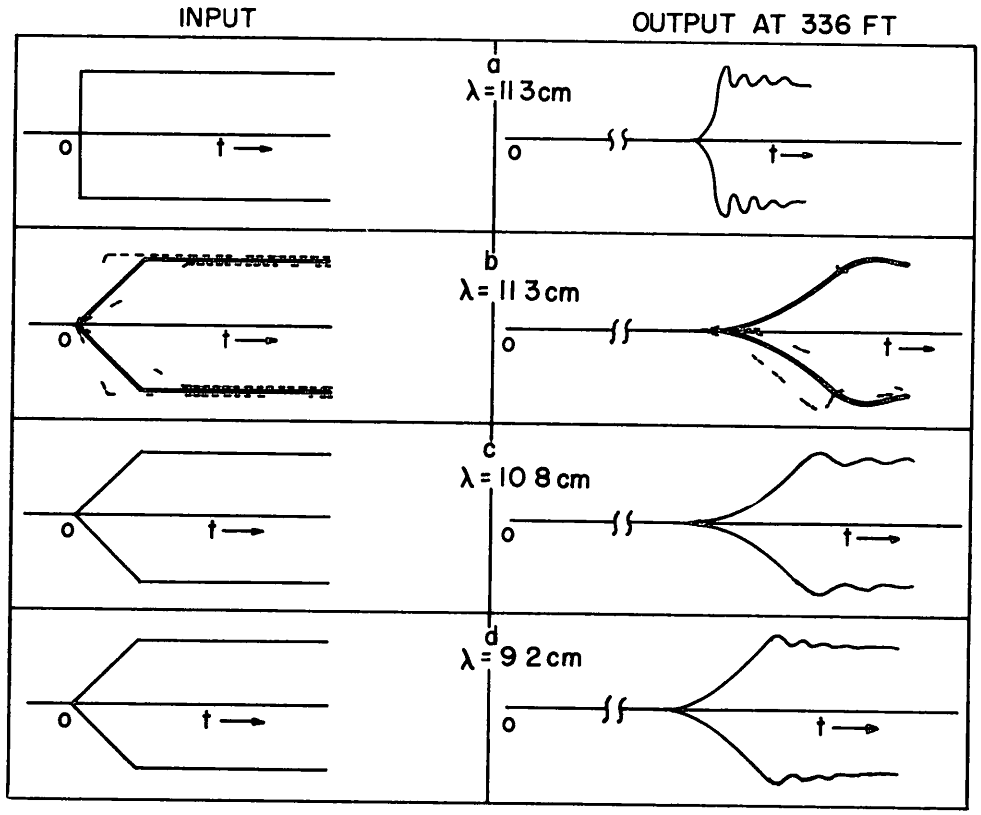

Figure 1 Theoretical wave forms of the input and output signals

the envelopes of the input functions at the beginning of the waveguide and the column to the right gives the envelopes of the corresponding output function at the end of 336 feet of waveguide The time scale is approximately 2 5 x 10^{-8}sec/inch For an input represented by a step function, the distortion of the signal at the output is shown in Fig 1a For the same input waveform the distortion varies with the wavelength (Fig 1b 1c 1d), the distortion is more prominent at wavelengths near cutoff At shorter wavelengths, as the initial slope of the input envelope increases (Fig 1a, 1b) there appears a number of oscillations in the envelope at the output The same phenomenon exists as the applied wavelength is reduced

The experimental arrangement follows A special hard tube pulser (see the Quarterly Progress Report, October 15, 1946 pp 36-40) was used to modulate a magnetron The magnetron output was fed into a 336-foot length of S-band rectangular waveguide terminated in a matched load The standing-wave ratio in the guide was less than 1 1 for the frequency band that could be covered by the magnetrons available The r-f pulse was observed on the Fast Sweep Synchroscope (see RLE Report No 27) A probe was inserted in the waveguide at the input end The voltage picked up by this probe was connected to the deflecting plates of the cathode-ray tube through a 25-foot length of cable (6 db loss one way). The same probe was used to observe the r-f pulse after it had traveled 336 feet of waveguide.

The pictures shown in Fig. 2 were obtained from the experimental arrangement described above. It will be observed that there is a definite trend in the direction indicated in the curves of Fig. 1. Within the experimental limitations of this experiment it may be said that there is substantial agreement between experiment and theory.

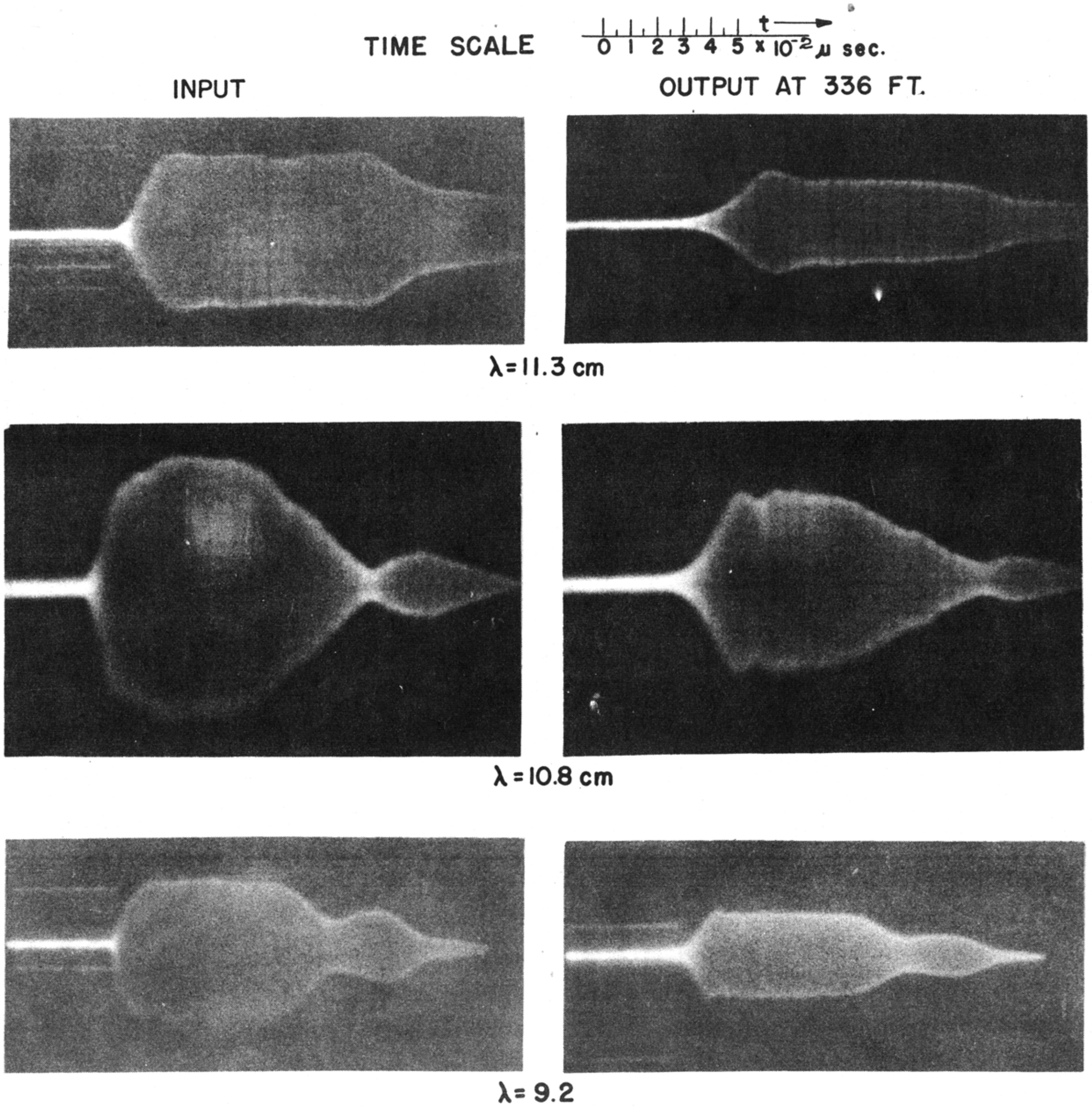

Figure 2. Observed waveforms of the input and output signals.

V E BROADBANDING OF ARBITRARY IMPEDANCES

Staff R M Fano

The theoretical limitations on the broadband matching of arbitrary impedances have been investigated The results of this work will be presented in a forthcoming RLE technical report

F ELEMENTARY PROPERTIES OF THE PARTITION FUNCTION IN THE ORDER-DISORDER PROBLEM

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In connection with the order-disorder problem in binary alloys, one attempts to solve the problem rigorously for special models Examples of these are the square and cubic arrays Here the only difficulties in calculating the partition function of the system lie in finding the number of arrangements of A+B atoms so that there is a definite number of A-A and B-B pairs of nearest neighbors Such pairs may be called "bonds"

If $W(n\ x)$ represents the number of arrangements in an n-point lattice, each arrangement having x bonds, we form the functions $f(\xi) = \sum_x W(n,x)\,\xi^x$ and $F(\xi) = \lim \frac{1}{n} \log f(\xi)$ In the two-dimensional case it was shown that any boundary may be slightly deformed, in such a way as to make $W(x,n)=0$ for all odd x If the number of arrangements is not zero for some odd x, however, there will then be the same number for ever and odd x The latter situation always prevails in three dimensions These results the first of which was obtained previously by van der Waerden show that $f(\xi)$ does not behave in a simple way as $n \to \infty$ in two dimensions Nevertheless we have $W(n,x/4) \geqslant W(n\ x)$ if $x < n/2$ with the reverse inequality when $x > 3n/2$, for suitable boundaries

The first three derivatives of $F(\xi)$ are zero at the origin and the fourth is 4!, in two dimensions, in three, the first five derivatives are zero and the sixth is 6! We have $F'(1) = 1$ or 3/2 in two or three dimensions, with $F(1) = \log 2$ in either case In two dimensions we have $0 \leqslant F(\xi) \leqslant \log(1+\xi^2)$ on the real axis and $0 < F'(\xi) \leqslant 2/\xi$ for $\xi > 0$, more generally one may write $f^{(r)}(\xi) \leqslant (\sqrt{2})^n (n/4) \frac{d^r}{d\xi^r} (\sqrt{1+\xi^2})^n$ In three dimensions we have $0 < F(\xi) \leqslant \log(1+\xi^3)$ and $0 < F'(\xi) \leqslant 3/\xi$ for $\xi > -1$ and $\xi > 0$, respectively. For all ξ, in the two cases, $F(\xi)$ satisfies the equation $F(\xi) - F(1/\xi) = \log \xi^2$ or $\log \xi^3$

G REMARKS ON APPROXIMATION OF A SPECIFIED AMPLITUDE AND PHASE BY A LINEAR NETWORK

Staff R M Redheffer

If the real or imaginary part of an impedance is specified as a function of frequency then the other part is completely determined Hence it is generally impossible to specify both the real and imaginary parts exactly over any interval, no matter how small In connection with such problems as feedback amplifiers, for example, the question arises, "How shall we adjust the constants of the network to approximate a prescribed real and imaginary part simultaneously, and what is the minimum error for the optimum network?" This question was suggested by Guillemin To minimize the sum of the mean square errors, one finds that the constants must be solutions of a certain set of linear equations If there are n constants, there are n equations, and the value of the minimum error may be expressed without finding the constants as a determinant of order n+1 The conditions have been proved sufficient as well as necessary provided a certain n^{th} order determinant does not vanish This determinant depends only on the range over which the functions are to be approximated, not on the functions themselves, and its zeros if there are any, are isolated