

## V MISCELLANEOUS PROBLEMS

### A LOCKING PHENOMENA IN R-F OSCILLATORS

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Description of Project The purpose of this project is to investigate experimentally the locking phenomena between coupled microwave oscillators

Status The behavior of two c-w magnetrons operating into a single load has been investigated experimentally and the results indicated in the April 15 Progress Report In order to study a synchronized oscillator in greater detail, a constant signal from a stable source is being used to lock a 707A klystron The stable source consists of a magnetron providing a signal to the klystron system through large attenuation The operation of the klystron when locked to the synchronizing signal is being studied With synchronizing power as a parameter, the paths of operations are plotted on Rieke and Smith Charts Preliminary determinations of the variations of the bandwidth of locking with changes in synchronizing power have been made and this investigation is being continued

### B ELECTRONIC DIFFERENTIAL ANALYZER

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The past three months have been devoted to the investigation and development of an input function unit and some multiplying circuits

Input Function Generator. In an electronic differential analyzer it is necessary to provide an equivalent of the input plotting table in the mechanical differential analyzer For this purpose two separate requirements must be satisfied A voltage wave must be produced which is an accurate replica of the input function, and the voltage wave must repeat in synchronization with the repetition rate of the electronic differential analyzer These requirements can be met in several ways

- 1 The function may be synthesized by means of a number of sinusoidal harmonic generators
- 2 The function may be synthesized from a combination of sine waves, square waves, triangular waves, and pulses
- 3 The function may be filmed on a continuous strip and reproduced with a photocell as in the sound track of a movie film
- 4 The function may be drawn on the face of a cathode-ray tube and reproduced by a scanning process

Method (4) appeared to be the simplest and to offer the greatest possibilities for extension to two dimensions For this reason a device was built with essential parts as shown in Fig 1

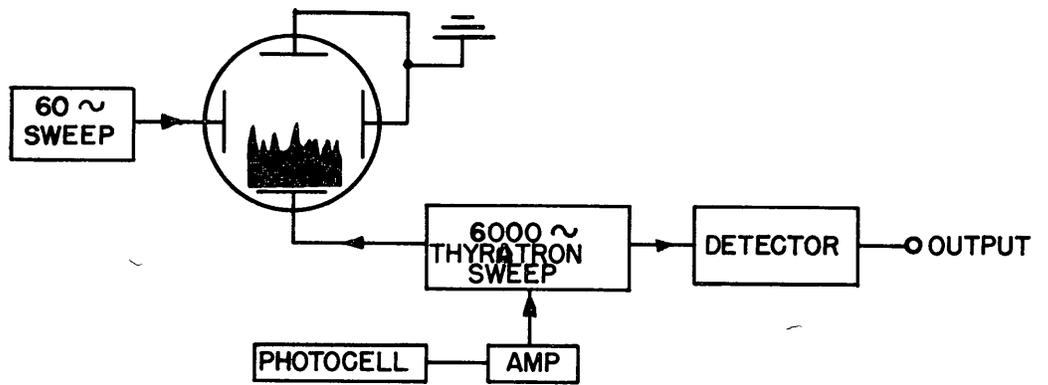


Figure 1. Block diagram of function generator

A 60-cycle sweep on the horizontal plates and a 6000-cycle sweep on the vertical plates cause the cathode-ray tube spot to scan the face of the tube. Whenever the spot emerges above the black mask representing the function, a pulse is produced by the photocell that triggers the 6000-cycle sweep and returns the spot to zero. In this way the amplitude of the 6000-cycle sweep is proportional to the ordinate of the function. Detection of this wave by either a conventional or "box-car" detector yields the desired voltage wave.

As described the device will operate for positive values of the input function only. Negative values can be handled by adding an arbitrary direct component to the original plot on the face of the cathode-ray tube and subtracting the equivalent direct voltage from the result.

The device which has been built will not resolve functions with rise times of less than 75  $\mu$ sec. A second model is being constructed using a hard-vacuum phototube and a hard-vacuum sweep circuit, which is expected to operate more satisfactorily.

Multipliers A difference-of-squares multiplier using push-push squarers has been built and tested. A block diagram of this circuit is given in Fig. 2.

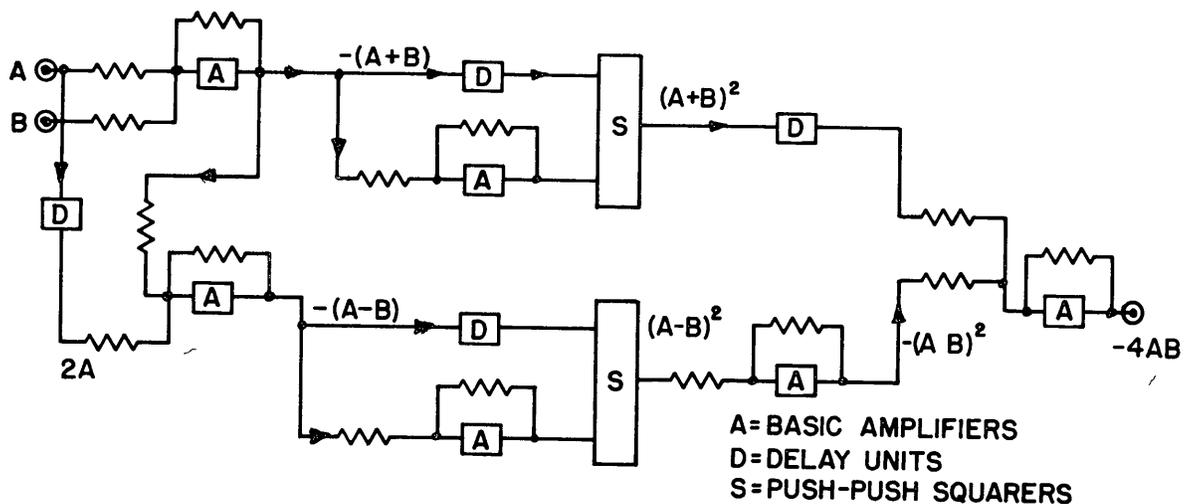


Figure 2. Difference-of-squares multiplier block diagram

This circuit has been tested with both sine-wave and pulsed inputs. A summary of the principal observed characteristics are given below

- (a) maximum output signal - - - - - 4.0v r m s
- (b) output hum level - - - - - 25 mv r m s.
- (c) output range for 1 per cent error - - - 25 db -
- (d) pulse rise time - - - - - 2.1  $\mu$ sec

The observed output range is somewhat smaller than was anticipated because of the rather high output hum level, due primarily to the tube heater supply. For the best performance it is desirable to rebalance the push-push squarers about once an hour. A self-balancing scheme has not yet been tried.

A special cathode-ray tube has been built to test the following multiplying scheme. Four collecting plates are mounted as indicated in Fig. 3 in place of the normal fluorescent screen of a cathode-ray tube.

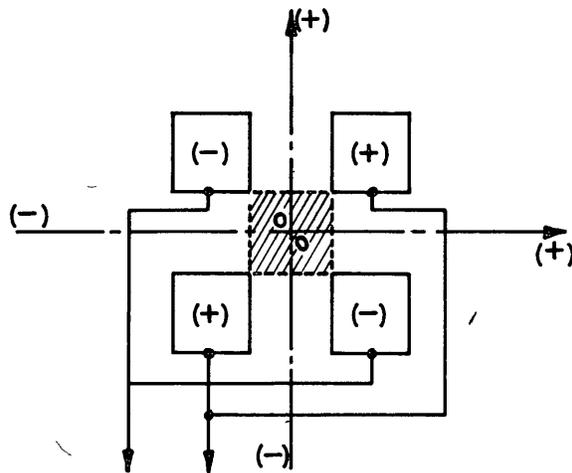


Figure 3 Target arrangement for deflection multiplier

With no external voltages applied, the electron beam is focused at the point (0,0). Sweep voltages are then applied to sweep the beam over the rectangular cross-hatched area of Fig. 3. The voltages to be multiplied are then applied to the horizontal and vertical plates, respectively. If the frequency of these voltages is low compared to the sweep frequencies, the peak currents collected by the plates marked plus and minus, respectively, are proportional to the positive and negative parts of the product of the deflecting voltages. If one of these currents is inverted and added to the other current by a suitable vacuum tube inverter and adder, the resulting sum is proportional to the complete product of the deflecting voltages.

Preliminary tests on the first tube completed indicate that this scheme may well prove superior to any of the schemes hitherto considered. Some difficulties have been experienced with secondary emission in this first tube. A second model redesigned to overcome this difficulty will be constructed after these first tests have been evaluated.

## C ELECTRONIC POTENTIAL MAPPING

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The purpose of this project is to develop pictorial displays for surface distributions of potential. It was originally undertaken in order to give a display of the distribution of potential over the area of the skull. It is believed that this will be a valuable aid in medical diagnosis, such as the location of brain tumors, and it is also believed that important information about the physiology of the brain will be disclosed. The same equipment will also be used for the display of area distributions of potential on other parts of the body, and, in particular, it is hoped that it will be useful in studies of the heart.

The present-day electroencephalograph shows the time variation of potential at a point on the skull in the form of a wave trace with time as abscissa and potential as ordinate. Except for the fact that there is no superposition of successive traces, this is similar to a radar "A" presentation. On the other hand, the purpose of this project is to show a two-dimensional display on a cathode-ray tube of the distribution of potential over the surface of the skull. This presentation is similar to that of a radar PPI, and the brightness of a point on the screen is proportional to the potential of the corresponding point on the skull.

In order to obtain the area presentation, a pick-up tube as shown in Fig 1 is used. Pick-up electrodes are located at a number (sixteen in this case) of points on the skull and these electrodes are connected to sixteen individual grids in the tube. The locations of the electrodes on the skull correspond approximately to the locations of the grids to which they are connected. The array of sixteen pick-up grids is located in front of a single anode (called anode No 2) and an electron beam is scanned across the array of grids. The potentials on the individual grids regulate the amount of beam current to anode No 2 at the time that the beam is passing through the grid in question. Consequently, if a standard type of cathode-ray tube with a phosphorescent screen is scanned in synchronism with the tube in Fig 1, and if the signal current from anode No 2 after suitable amplification is put on the intensity grid of the cathode-ray tube, the screen of this tube will show the approximate potential distributions across the skull. At the present stage of development, if different d-c potentials from a battery are placed on the sixteen pick-up grids, a good checkerboard pattern may be seen on the display tube.

The first part of the work on this project was a test to determine whether a signal grid in the path of an electron beam could be used to control the anode current in a satisfactory manner. Another purpose of the test was to find the general characteristics of tubes having the beam current controlled in this manner. A preliminary tube having only two grids was built, one grid being used as a spare or alternate while signal was introduced on the other, (see Fig 2). These tests showed that a signal as small as 35  $\mu\text{v}$  on the grid gave a satisfactory output. Microphonics of this tube made signals less than 35  $\mu\text{v}$  unusable. This type of tube also turned out to be exceptionally sensitive to stray magnetic fields and to

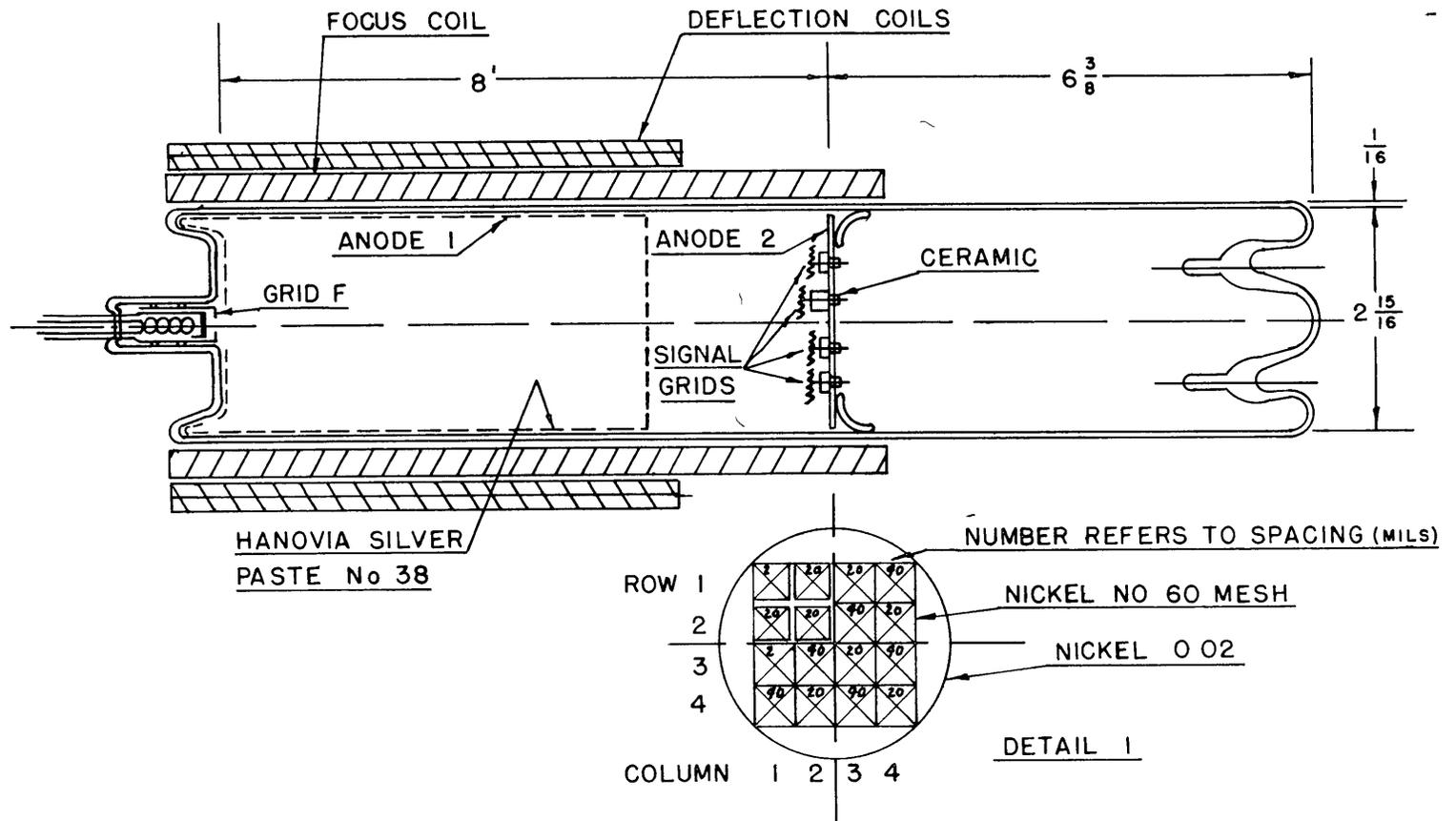


Figure 1 First complete pick-up tube for electronic potential mapping

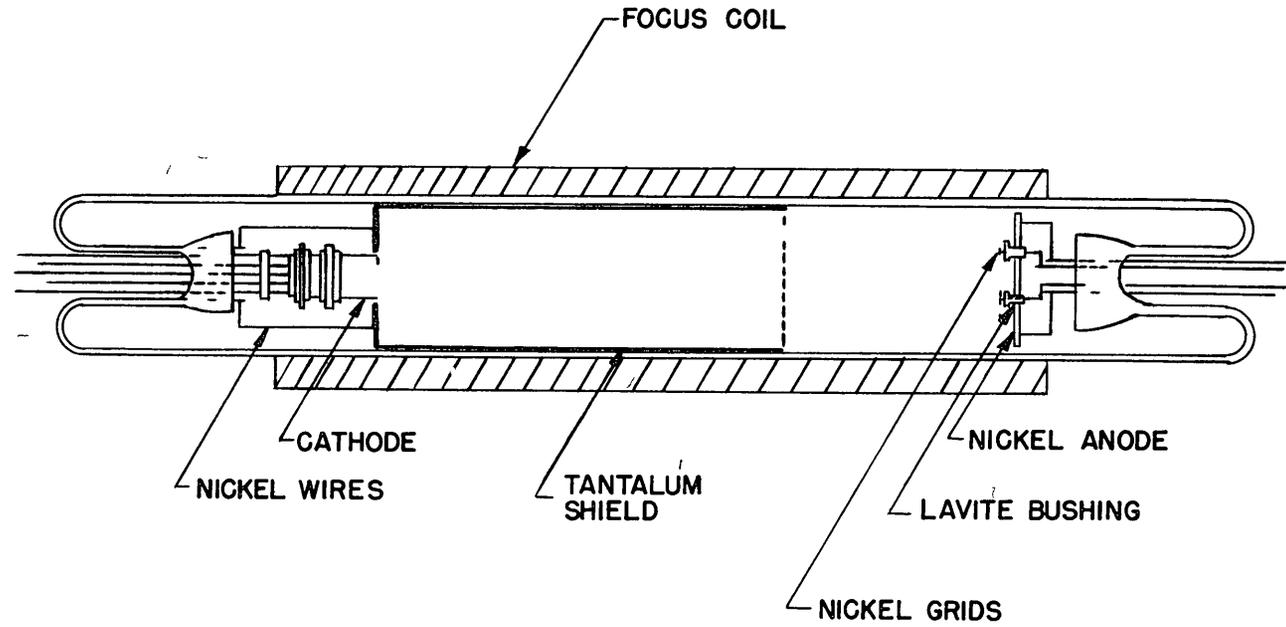


Figure 2 Preliminary tube for testing control of the beam current by a signal grid remote from the cathode

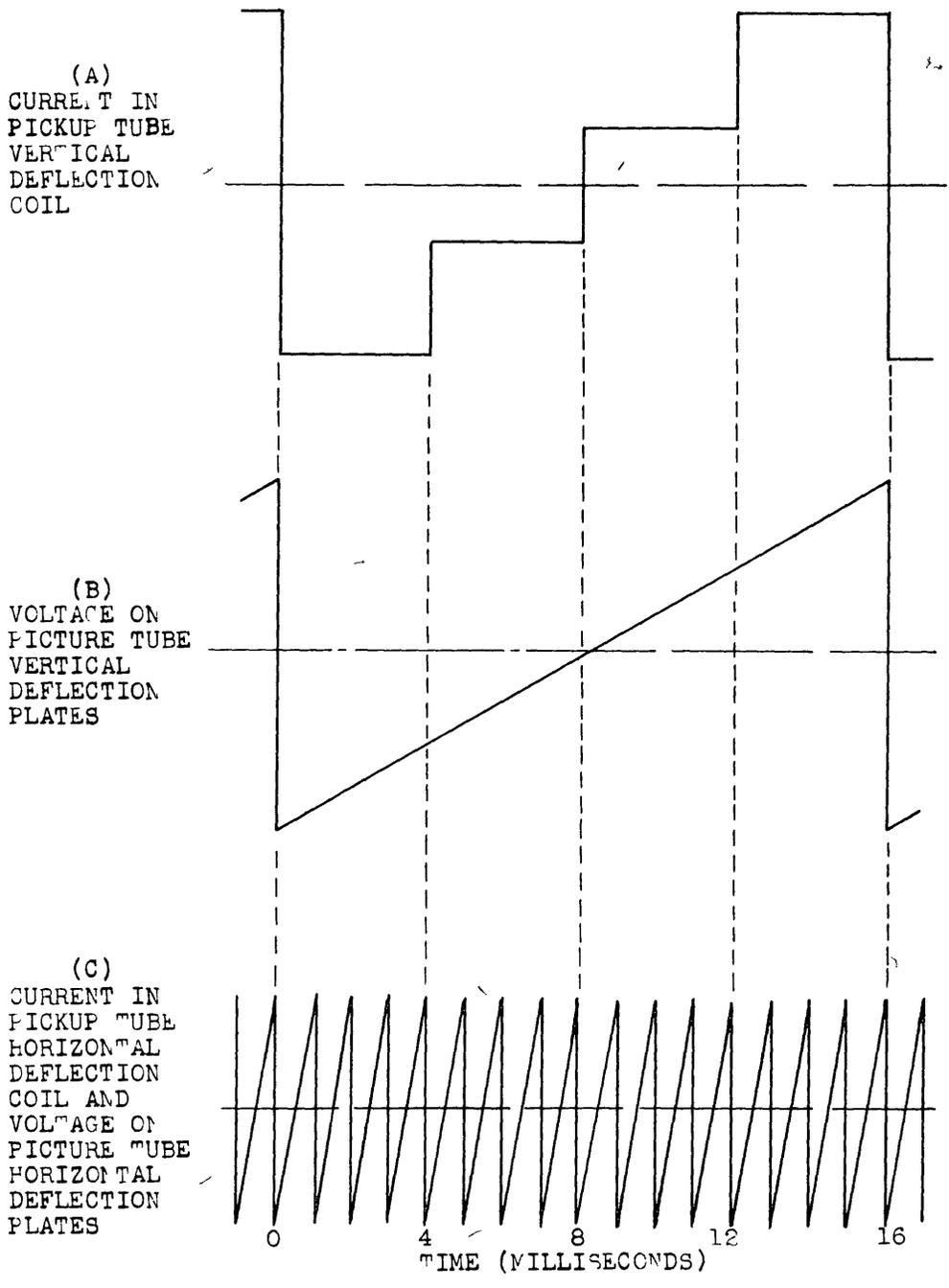


Figure 3 Wave shapes of the scanning signals of the pick-up and display tubes

electrical pick-up. However, after elaborate magnetic shielding, and thorough filtering, the sensitivity limit was finally determined by microphonics. Precautions were consequently used in all succeeding tubes to reduce microphonics.

The ultimate limiting sensitivity of the method is, of course, imposed by random noise. This limit was never reached. Large beam currents were used, however, in order to decrease the random noise, since an elementary analysis indicates that the noise rises only as the square root of the beam current while the signal rises linearly.

After this first tube showed that the method of signal grid pick-up was satisfactory, a second tube was made to study the effects of scanning. This is the tube shown in Fig 1. The scanning signals used in this case are shown in Fig 3. It will be noted that each horizontal line of grids in the pick-up tube is scanned four times before proceeding to the next line. At the same time, the vertical motion of the beam in the display tube is uniform. This method improves the area display as illustrated in Fig 4. Because of the large spot size of the pick-up tube, it was decided to scan the centers of the grids each time. This is the reason for the step scanning signal in the pick-up tube.

It was anticipated that a spurious signal would be introduced as the scanning beam crossed over from one signal grid to another, and it was also anticipated that this spurious signal would be a serious limiting factor in the operation of

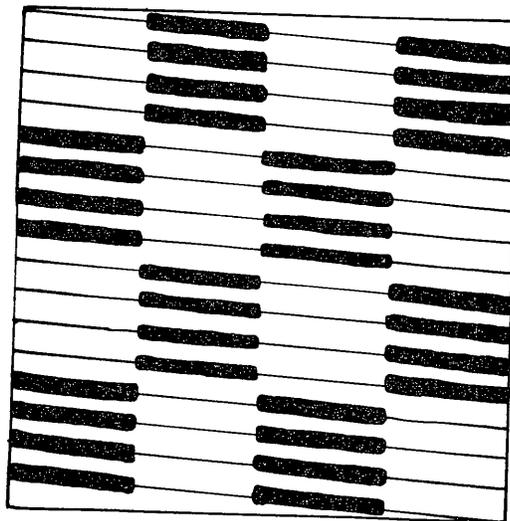


Figure 4 Sketch of observed display pattern when signal grids of pick-up tube are connected to d-c potentials arranged in a checkerboard pattern.

the system. For this reason two arrangements of grids were used in the tube shown in Fig 1 to see which arrangement would give the least amount of this spurious signal (hereinafter to be called "edge effect"). The four grids in the upper left-hand section of the array were all at the same distance (20 mils) from the anode, but there was a slight open space between them. The other 12 grids had alternating spacings of 20 and 40 mils from the anode, but their projected edges were as nearly in line as possible. The results of the experiments indicated that the latter arrangement was much better.

A new tube is now being built incorporating all the improvements suggested by the previous work. It is estimated that this tube will operate with signals of the size of one mv, the limiting factor being edge effect. Since edge effect limited the sensitivity of the previous tube at a much higher level than random noise, a tube with fine spot size and low beam current will be used.

While continued tube development could probably improve the sensitivity of the pick-up tube so that it could operate directly at an encephalograph level of about 10  $\mu$ v, present plans call for bypassing this development by building 16 small pre-amplifiers, one for each grid. These amplifiers will also be necessary for tuning purposes as will be explained below.

Four different methods of display are planned and it is expected that one or more of these will give information that can be interpreted and will be useful in studies of the brain and the heart. The four methods are

- (1) Direct display of the instantaneous potentials just as they are picked up
- (2) Motion picture photography of displays of type (1) and redisplay at slow motion

- (3) A "spectroheliograph" type of display in which each pre-amplifier preceding the sixteen grids is tuned to the same frequency with a bandwidth of one or two cycles. Present encephalograph information indicates that such area displays of the rectified output signal in a narrow frequency band will be especially valuable.

It is planned to tune the frequency range from one to fifty cycles by using the signals picked up to modulate a carrier of about 100 cycles. In this way, the 16 amplifiers can be fixed tuned around 100 cycles, and all 16 pre-amplifiers can be tuned through the frequency range between zero and 50 cycles by varying the oscillator frequency through the range from 100 to 150 cycles. Using the same oscillator to supply a carrier for all 16 pre-amplifiers will then allow all the tuning to be done with a single control dial.

- (4) A "stroboscopic" type of display in which the frame frequency (frequency of scanning an entire picture) is continuously varied from 1 to 100 per second. This will pick out the activity which occurs near the frame frequency or its harmonics. Preliminary tests with a signal generator supplying the grid signal give very encouraging results with this method.

#### V. D BROADBANDING OF ARBITRARY IMPEDANCES

Staff Professor R M Fano

A technical report (No 41) entitled, "Theoretical Limitations of the Broadband Matching of Arbitrary Impedances", is being prepared for publication. This paper deals with the general problem of matching an arbitrary load impedance to a pure resistance by means of a reactive network. It consists primarily of a systematic study of the origin and nature of the theoretical limitations on the tolerance and bandwidth of match and of their dependence on the characteristics of the given load impedance.

The necessary and sufficient conditions are derived for the physical realizability of a function of frequency representing the input reflection coefficient of a matching network terminated in a prescribed load impedance. These conditions of physical realizability are then transformed into a set of integral relations involving the logarithm of the magnitude of the reflection coefficient. Such relations are particularly suitable for the study of the limitations on the bandwidth and tolerance of match. Definite expressions for these quantities are obtained in special cases.

The practical problem of approaching the optimum theoretical tolerance by means of a network with a finite number of elements is also considered. Design curves are provided for a particularly simple but very important type of load impedance. In addition, a very convenient method is presented for computing the values of the elements of the resulting matching network.

#### E APPROXIMATION OF A SPECIFIED AMPLITUDE AND PHASE BY A LINEAR NETWORK

Staff: R M Redheffer

The problem of designing a circuit to approximate a specified amplitude and phase characteristic over a finite frequency range has been further investigated. It has been shown that the fundamental determinant  $\frac{\sin(k-j)a}{k-j}$  vanishes at the origin only, so that the procedures developed will be operable in all cases. The effect of a constraint, closely related to a specification on the maximum loss in gain, has been likewise investigated. If the approximation is to hold over the infinite band there are three sources of error, viz, error due to the fact that the functions are not conjugate, error due to use of only a finite number of terms in the approximate series, and error due to the constraint. A simple form has been obtained for the total error, which exhibits the role played by each of these errors separately. A method of determining the optimum coefficients for approximation over a specified frequency range, subject to the constraint, has also been found.

## V F SYNTHESIS OF OPTIMUM LINEAR SYSTEMS

Staff Professor Y W Lee

The methods of analysis and design of communication systems in present-day practice are based upon the assumption that the message for transmission is either a steady-state or a transient phenomenon. Thus the terms "steady-state approach" and "transient approach" are frequently used in transmission problems, especially those of servomechanisms. Engineers who have been investigating problems of a more exacting nature such as those arising in television, radar, and servomechanisms are aware of the fact that either "approach" is unsatisfactory. An underlying difficulty has been the insufficient understanding of the fundamental nature of the problems and especially the nature of a message. The message, embodied in electrical or mechanical form, is neither in a steady state nor in a transient state. A design based upon the steady-state viewpoint, which in fact, is in contradiction with the objective since a steady-state current carries no information, is necessarily poor if the input is nearly of a transient character. Likewise, a system designed on the transient basis behaves badly with a nearly steady input. A series of adjustments may bring about a suitable compromise. But there is definitely a need for the development of a new technique founded on sound principles.

Professor Norbert Wiener in his NDRC Report entitled "The Extrapolation, Interpolation and Smoothing of Stationary Time Series" (1942) discloses a completely new theory of communication engineering which is a radical departure from present ideas. In this theory the message is conceived to have definite statistical attributes which are invariant under a shift in time. The mathematical formulation of the filter problem, for instance, is one of minimization of mean-square error between the result of operation by a linear system on the past of the message and noise (or unwanted messages and noise), and the message. This formulation leads to an integral equation. The solution of the equation depends upon a factorization which results in an operator (system function) of a closed form. This is termed the optimum system. The essential data for the physical realization of the system function are the mean power spectra of both the message and the noise. Outstanding features of this method are (1) that it yields a linear system for optimum performance on the average, (2) that a preassigned lag or lead (prediction) may be incorporated into the design, (3) that message and noise are correctly characterized by their statistical parameters, and (4) that the resulting system function is always realizable and stable.

For most engineers the original paper of Wiener is difficult to read as it contains a considerable amount of rigorous mathematics. However, it has been simplified. Furthermore, a number of applications of the theory, besides those given in Wiener's NDRC Report, has recently been investigated so that available now are, among the more important cases, the theory of (a) the optimum predictor, (b) the optimum filter, (c) the optimum compensator, (d) the optimum differentiator, and (e) the optimum inverse operator. The mathematical development covering the theory and applications will be presented in a future report.

Design procedures using the networks of Wiener and Lee are being organized.

The method of design by minimization of average error finds an important application in the solution of problems by means of digital computing machines. Here data (message) fed into the machine are corrupted by an error (noise) due to the necessity of rounding off numbers. This is statistical in nature. The design of the setting of the machine for a minimum of average error in the result falls under Wiener's theory. Some work in this direction is planned. The theory of the optimum inverse operator mentioned above is directly applicable here.