V MISCELLANEOUS PROBLEMS

A LOCKING PHENOMENA IN R-F OSCILLATORS

Staff  Professor J B Wiesner
       E E David Jr

Investigation of a microwave oscillator, synchronized by an invariant external signal has been completed. A graphical method has been developed which accurately describes the operation of the locked oscillator. The analysis is applied to the Rieke diagram of the tube. A report on this work is now in progress. It is hoped that this type of analysis may be extended to include more generalized synchronizing systems.

B ELECTRONIC DIFFERENTIAL ANALYZER

Staff  Professor H Wallman
       A B Macnee
       R E Scott

The past three months have been devoted to further investigations of function generators and multiplying circuits. As explained in previous progress reports, these components (or their equivalents) are prime essentials for an electronic differential analyzer.

Function Generators. It is anticipated that function generators will play two distinct roles in the electronic differential analyzer:

(1) They will serve to introduce arbitrary functions into the machine, (e.g., plotted data);

(2) They will generate specified functions of an arbitrary applied voltage, (e.g., reciprocals, squares, etc.)

The circuits which are described below generate an arbitrary function (given as an opaque mask) of the X-plate voltage of a cathode-ray tube. When a linear sweep is applied to the X-plates, a time-varying voltage with the same waveform as the mask is produced. If an arbitrary voltage is applied to the X-plates of the cathode-ray tube, the output will be the same function of the arbitrary voltage. For example, if the function mask is a rectangular hyperbola, the output will be the reciprocal of any voltage appearing on the X-plates. The flexibility of the function generator may be further increased by the use of feedback from the Y-plates to the X-plates through suitable elements.

Mark II Double-Sweep Function Generator. A second model of the function generator which was described in the Quarterly Progress Report of July 15, 1947, has been constructed with several improvements. An 829 hard-vacuum phototube replaces the gas tube used in the original model, and a Dumont 248 oscilloscope with a 5JP5 screen replaces the 208 oscilloscope with a 5LP5 screen. In the new circuit it has been possible to use a sweep frequency of 50,000 cycles per second. Figure 1 shows the essential components of the apparatus.

A 50,000-cycle sweep is applied to the Y-plates and an arbitrary voltage to the X-plates. Whenever the cathode-ray tube spot appears above the function mask, a pulse is
produced by the photocell which triggers the Y-sweep and returns it to zero. Thus the envelope of the sweep wave follows the mask for any X-voltage. When the output is detected it is the 'mask' function of the X-voltage.

The Mark II circuit has a resolving time of about 50 µsec when the detector output is filtered to give about 1 per cent ripple. This device is probably sufficiently good to be used as the input plotting table of the electronic differential analyzer. It is probably too slow however, to be useful for more general applications including the generation of functions which has been discussed above. A function generator system similar to the above has been tried. It entails the use of d-c amplifiers however and they lead to stability difficulties. For this reason this system has been abandoned in favor of the almost identical system employing a pilot-carrier frequency which is described below.

A Continuous-Tracking Function Generator

A second type of function generator is shown in Fig. 2.

The no-signal position of the cathode-ray tube electron beam is above the highest point of the function mask which is placed on the tube face. The beam is intensity-modulated at a frequency of 200-500 km/sec. The modulated light from this beam is detected...
by the photocell, the photocell output is amplified detected, and fed back to the vertical deflecting plates with a polarity which drives the electron beam downward. The beam is thus forced downward to an equilibrium position of partial obscurity behind the opaque function mask. When a sweep voltage is applied to the horizontal deflecting plates, the beam is constrained to follow the function mask. Because of the linear relationship between the beam position and the vertical deflecting voltage, this feedback voltage becomes the desired output function. This scheme provides d-c amplifier performance without the hum and drift problems characteristic of high gain d-c amplifiers.

An experimental model of this system employing a 1-Mc/sec bandpass amplifier having a 25 kc/sec bandwidth, has been tested. This model operates successfully, but is limited in its resolving time because of the narrowness of the amplifier passband. A second model which should be capable of better resolving times is now being built.

**Target Multiplier** The second model of the target multiplier described in the last quarterly report has not yet been received from the tube shop.

**Electron-Velocity Multiplier** Another multiplier which depends on the deflection of an electron beam for its operation, is shown in Fig. 3.

![Diagram](image)

**Figure 3** Electron-velocity multiplier

Electrons leave the cathode-ray tube gun with a velocity along the z-axis of \( v_0 \). Passing through the first pair of deflecting plates, they are given a velocity component in the x-direction

\[
v_x = \frac{(e/m)}{d_1}(E_x/v_0)
\]

The beam next passes through an axial field \( B_y \), which is proportional to \( E_y \). This gives the beam a velocity component in the y-direction which is approximately.
\[ v_y = \left(\frac{\varepsilon}{m}\right) d_2 \left(\frac{v_0}{v_x}\right)^2 \]  

(2)

Substituting from Eq (1) one has

\[ v_y = \left(\frac{\varepsilon}{m}\right) d_2 \left(\frac{v_0}{v_x}\right)^2 (E_x E_y) \]  

(3)

This velocity which is proportional to the product \((E_x E_y)\), can be measured by a feedback scheme as follows. The fluorescent screen of the cathode-ray tube is divided down the line \(y = 0\) by an opaque partition and photocells are placed on either side. The electron beam is intensity-modulated at a carrier frequency and the difference of the outputs of the two photocells is amplified by a carrier amplifier. The output of this amplifier is detected by a phase-detector and fed back to a pair of horizontal deflecting plates in such a manner as to keep the net \(y\)-deflection zero. This feedback voltage is thus proportional to the product \((E_x E_y)\). Equations (2) and (3) are accurate for \((\varepsilon B/m)(d_2/v_0)\ll\pi/9\).

Modifications of this scheme using all-electrostatic deflections are possible by replacing the axial coil with a pair of specially shaped deflecting plates. Preliminary tests of this scheme using a Dumont 5LP5 cathode-ray tube and a low-pass amplifier feedback system indicate further work is warranted.

C ELECTRONIC POTENTIAL MAPPING

Staff Dr Stanford Goldman

An extended discussion of this project was given in the Quarterly Progress Report of July 15 1947.

During the summer, work was started on the set of tunable uni-control low-frequency filters for the preamplifier of the pickup tube. The new scanning pickup tube has also just about been completed.

D BROADBANDING OF ARBITRARY IMPEDANCES

Staff Professor R M Fano

A paper "Theoretical Limitations on the Broadband Matching of Arbitrary Impedances" to be presented at the National Electronics Conference in Chicago in November has been sent to the Program Chairman.

The manuscript of the technical report (No 41) on the same subject has been completed.
E ACTIVE NETWORKS

Staff Professor E A. Guillemin
Manuel Cerrillo

The present state of the theory of the synthesis of active networks is almost reduced to a limited field in which not much constructive work has been done. Most of the solutions are restricted to very particular and specific cases and structures.

It is intended now to go somewhat deeper into the problem, in order to develop a more general and suitable theory of these systems.

The first steps have already been started, using the Lagrange problem in the Calculus of Variations as the fundamental mathematical tool. Active networks are considered here as a properly extended passive network in which some of the variables are subjected to a specific set of constraints. The effect of vacuum tubes and other characteristics are introduced in this fashion. In passive networks, the currents must be such as to keep a minimum the power dissipated in the system.

Special attention will be paid to the "sufficient condition" for the existence of an extremum and an effort is now being made to find the proper electrical interpretation.

F MATHEMATICAL PROBLEMS

Staff R M Redheffer

The last quarterly period has been spent on miscellaneous problems. The following was suggested by Professor P Franklin.

Given a set of linear equations suppose the unknowns are computed approximately and upon substitution back into the equations are found to satisfy them within an error $\varepsilon_i$ for the $i$-th equation. It is required to find an estimate for the error in the unknowns in terms of the substitution errors $\varepsilon_i$. If the matrix of the system is $A$ with $|A| \neq 0$, let $\lambda$ be the smallest characteristic value of $AA^T$. It has been proved that the length of the unknown error vector is not greater than $M/\sqrt{\lambda}$, where $M = \sqrt{\sum \varepsilon_i^2}$ is the length of the known error vector, and that a value this large can actually always be attained. If $T$ is the sum of the squares of the coefficients in the equations and if there are $n + 1$ equations, then we have $M/\sqrt{\lambda} < (|M|A|)(T/n)^{1/2}$. This form is suitable for numerical calculation.

The following problem was suggested by R L Kyh. If a transverse partition in a rectangular waveguide is connected to a source of potential, the resulting d-c field is easily calculated. It has been shown that the dot product of this field and the microwave field of suitable mode, will give the same value, when integrated over the cross section, as would be found with a uniform d-c field.

A report has been written on the interaction of antennas and dielectric sheets, preparatory to a journal article.
SYNTHESIS OF OPTIMUM LINEAR SYSTEMS

Staff Professor Y W Lee

In order that Professor N Wiener's new communication engineering theory be made available to engineers, an exposition of this theory has been under preparation. This is in great need since the theory is based upon new ideas and certain mathematical techniques which are unfamiliar to most engineers. A part of the expository writing has been finished.

It should be mentioned here that Wiener's classical work covers not only the theory and design of optimum linear systems but also other problems of which the rigorous calculation of the rate of transfer of information is outstanding on account of its fundamental importance in communication engineering.

Some experimental work will be started shortly.