Measurement of the $k_T$ Distribution of Particles in Jets Produced in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV


(CDF Collaboration)
We present a measurement of the transverse momentum with respect to the jet axis ($k_t$) of particles in jets produced in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV. Results are obtained for charged particles in a cone of 0.5 radians around the jet axis in events with dijet invariant masses between 66 and 737 GeV/c$^2$. The experimental data are compared to theoretical predictions obtained for fragmentation partons within the framework of resummed perturbative QCD using the modified leading log and next-to-modified leading log approximations. The comparison shows that trends in data are successfully described by the
In this analysis we measure the transverse momenta of particles in jets with respect to the jet axis \( (k_T) \), study the dependence of the \( k_T \) distribution on jet energy, and compare the results to analytical predictions of the modified leading log approximation (MLLA) [1] and next-to-modified leading log approximation (NMLLA) [2], supplemented with the hypothesis of local parton-hadron duality (LPHD) [3].

This measurement tests the applicability of perturbative QCD (pQCD) to the soft process of jet fragmentation. Detailed studies of jet fragmentation expand our understanding of the relative roles of the perturbative and non-perturbative stages of jet formation, and they probe the boundary between the parton shower and hadronization. The ultimate goal is to understand which stage of jet formation is most significant in determining the final characteristics of jets. This measurement indicates that the parton shower dominates. Moreover, we also verify how well the PYTHIA TUNE A [4,5] and HERWIG 6.5 [6] Monte Carlo generators describe jet properties in the data. This comparison is crucial for data analyses utilizing these generators, and the results can be used to tune the generators for future measurements.

Past experimental studies of the inclusive distributions of particles in jets [7–9] and the recent measurement of the two-particle momentum correlation in jets [10] agree well with theoretical predictions, suggesting that the perturbative QCD stage of jet formation is dominant. In this analysis the hypothesis of LPHD is further tested by examining whether the pQCD predictions for the transverse momentum distribution of partons can successfully reproduce the corresponding distribution of hadrons. We report a measurement of the \( k_T \) distribution \( dN/d(\ln k_T) \) of charged particles in restricted cones with an opening angle of \( \theta_c = 0.5 \) radians around the jet axis in events with dijet invariant masses in the range 66–737 GeV/c\(^2\). It has been shown in the past that the integral of the distribution is well described by MLLA predictions [9]; therefore, in this Letter, we compare only the shape of the distribution by normalizing theory to data in the region \(-0.2 < \ln[\kappa_T/(\text{GeV}/c)] < 0.0\), where both the theoretical prediction and the experimental measurement are expected to be reliable. The data are corrected for detector effects, and no additional corrections are needed for comparison to the theoretical predictions, if LPHD is assumed.

The theoretical predictions used in this analysis are formulated for dijet events. MLLA [11] is an approximation which allows one to calculate a variety of observables via a complete resummation of perturbative terms. It is an approximation in the sense that each perturbative term of order \( n \) is calculated to a precision of leading and next-to-leading logarithms:

\[
\alpha_s^2(k_T)\left[ A_n(\ln^2 E_{\text{jet}}) + B_n(\ln^{n-1} E_{\text{jet}}) + O(\ln^{n+2} E_{\text{jet}}) \right],
\]

(1)

where \( \alpha_s(k_T) \) is the strong coupling constant. The constants \( A_n \) and \( B_n \) are calculated exactly to all orders. The NMLLA calculations extend the MLLA precision by treating a number of contributions more consistently at the next-to-MLLA level, i.e., at the level of \( \alpha_s^2(k_T)\ln^{n-2} E_{\text{jet}} \). Therefore, MLLA and NMLLA both provide soft gluon resummation but at different levels of precision.

The MLLA + LPHD and NMLLA + LPHD approaches view jet fragmentation as a predominantly perturbative QCD process. The MLLA and NMLLA predictions predict the average number of partons \( N \) and the transverse momentum distribution of partons with respect to the direction of the initial parton. The predictions are valid for partons in a small cone with opening angle \( \theta_c \) around the direction of the initial parton, and they assume that the parton momentum is much smaller than the jet energy (soft approximation). The predictions are functions of \( Y = \ln(Q/Q_{\text{eff}}) \), where \( Q = E_{\text{jet}}\theta_c \) is the so-called jet hardness and \( Q_{\text{eff}} \) is the lowest allowed transverse momentum of partons. The LPHD hypothesis states that the hadronization process takes place locally and, therefore, properties of partons and hadrons are closely related. For instance, the parton and hadron \( k_T \) distributions are assumed to be related via a constant factor \( K_{\text{LPHD}} \), which is independent of the jet energy and whether the jet originates from a quark or a gluon [12]. Past studies have shown that \( K_{\text{LPHD}} \approx 1 \) [9].

Theoretical NMLLA predictions for the \( k_T \) distribution are shown in Fig. 1. The direction of the initial parton is used as the jet axis. The lower boundary of the range of validity of the predictions is determined by \( Q_{\text{eff}} \) and is \( k_T > Q_{\text{eff}} \); however, in this measurement, we consider only particles with \( k_T > 0.5 \) GeV/c (\( \ln[k_T/(\text{GeV}/c)] > -0.6 \)), motivated by the poor reconstruction quality of tracks with low \( k_T \). The upper boundary is determined by the soft approximation requirement \( k_T/E_{\text{jet}} \ll 1 \), the requirement that the double differential distribution \( \frac{d^2N}{dk_T^2} \) be positive over the perturbative region [1]; \( k \) here is the absolute value of parton momentum. This translates into \( \ln[k_T/(\text{GeV}/c)] \leq \ln(Q/\text{GeV}) - 2.5 \) for MLLA and \( \ln[k_T/(\text{GeV}/c)] \leq \ln(Q/\text{GeV}) - 1.6 \) for NMLLA [13].

The range of validity extends to higher \( k_T \) regions for...
increasing jet energy. The shape of the distribution shows a weak dependence on the value of $Q_{\text{eff}}$.

Jets originating from gluons are expected to have more particles with large $k_T$, on average, than jets originating from quarks. In theory, the $k_T$ distribution is calculated for quark and gluon jets separately. Dijet events at the Tevatron consist of both quark and gluon jets. In order to compare data to theory, we rewrite the formula for the predictions as follows:

$$\frac{dN}{d\ln(k_T)} = f_g \left( \frac{dN}{d\ln(k_T)} \right)_g + \left( 1 - f_g \right) \left( \frac{dN}{d\ln(k_T)} \right)_q,$$  

where $\left( \frac{dN}{d\ln(k_T)} \right)_q$ and $\left( \frac{dN}{d\ln(k_T)} \right)_g$ are the predictions for quark and gluon jets, respectively, and $f_g$ is the theoretical fraction of gluon jets in the data.

The measurement is based on events produced at the Tevatron collider in $p\bar{p}$ collisions at a center of mass energy of 1.96 TeV and recorded by the CDF II detector. The total integrated luminosity is 775 pb$^{-1}$. A detailed description of the detector can be found in [14] and references therein. Here we briefly describe the components of the detector that are relevant to this analysis. The silicon microstrip detector is used to reconstruct event vertices and to measure the distance of closest approach $d_0$ of charged particles to the beam line in the plane transverse to the beam direction. The silicon detector is surrounded by the central outer tracker, an open-cell drift chamber providing up to 96 measurements of a charged particle track over the radial region from 40 to 137 cm. The entire CDF II tracking system is located inside a 1.4 T solenoidal magnet and is surrounded by calorimeters used to measure the energy of charged and neutral particles. The central electromagnetic, central hadronic, and wall hadronic calorimeters are made of lead (electromagnetic) and iron (hadronic) layers interspersed with plastic scintillator. The CDF II trigger system is a three-level filter with calorimeter information available at the first level [15].

In this measurement, jets are reconstructed based on calorimeter information using a cone algorithm with cone radius $R = \sqrt{(\Delta \phi)^2 + (\Delta \eta)^2} = 1.0$ [16]. The energy of each jet is then corrected to compensate for the nonlinearity and nonuniformity of the energy response of the calorimeter, the energy deposited inside the jet cone from sources other than the leading parton, and the leading parton energy deposited outside the jet cone. A detailed description of this procedure can be found in [17]. The overall uncertainty on the jet energy scale is 3%.

In this Letter, we give a brief overview of the event and track selection; a detailed description of the procedure and the evaluation of the systematic uncertainties can be found in [10]. Events were collected using a single calorimeter tower trigger with a transverse energy ($E_T$) [18] threshold of 5 GeV and with single jet triggers with $E_T$ thresholds of 20, 50, 70, and 100 GeV. To reject events with poorly measured jets, we require the two leading jets to be well balanced in $E_T$. We allow up to two extra jets, but their energy is required to be small: $E^{\text{extra}}_T < 5.5$ GeV + 0.065($E^1_T + E^2_T$), where $E^1_T$, $E^2_T$, and $E^{\text{extra}}_T$ are the transverse energies of two leading jets and an extra jet, respectively. The final sample consists of approximately 250 000 events and is further divided into eight bins according to the dijet mass as measured by the calorimeters.

FIG. 1. NMLLA predictions [2] of the $k_T$ distribution in jets. The figures show how the distribution depends on (a) the jet hardness (shown for a gluon jet), (b) the origin of the jet (quark or gluon), and (c) the parton shower cutoff $Q_{\text{eff}}$. $N'$ is the number of partons in the region $-0.2 < \ln(k_T/(\text{GeV}/c)) < 0.0$. 

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and defined as $M_{jj} = \sqrt{(E_1 + E_2)^2/c^4 - (P_1 + P_2)^2/c^2}$, where $E$ and $P$ are the energies and momenta of the two leading jets, respectively. Measurements are performed in the dijet center of mass frame where $Q = E_{\text{jet}, \theta} = M_{jj} \theta / 2$. All particles are treated as pions for Lorentz boosts. To evaluate possible biases that may originate from the particular choice of jet reconstruction algorithm, we compare results of the measurement using three different values of the parameter $R$ in the jet reconstruction algorithm (0.4, 0.7, and 1.0). The resulting systematic uncertainty is 1%.

We use full three-dimensional track reconstruction [19,20]. Poorly reconstructed and spurious tracks are removed by a requirement on the quality of the track fit in the drift chamber $\chi^2_{\text{fit}} < 6.0$ [20]. Charged particles are required to have $p_T > 0.3$ GeV/c. Requirements on the track impact parameter $d_0$, radius of conversion $R_{\text{conv}}$, and $|\Delta z| = |z_{\text{track}} - z_{\text{vertex}}|$ are also applied (see [10] for details). These requirements are designed to ensure that the tracks originate at the primary vertex and are not produced by cosmic rays, multiple $p\bar{p}$ interactions within the same bunch crossing, $\gamma$ conversions, and $K^0$ and $\Lambda$ decays. The correction for the remaining fraction of secondary tracks is estimated by comparing the $k_T$ distribution in PYTHIA TUNE A at the charged hadron level and at the level of the detector simulation. It is found to be $\sim 3\%$ and is assigned as the systematic uncertainty associated with the remaining fraction of secondary tracks. In order to correct for tracks from the underlying event, we apply the following procedure. On an event-by-event basis, two complementary cones are positioned at the same polar angle with respect to the beam line as the original dijet axis but in the plane perpendicular to the dijet axis. We assume that cones formed in such a fashion collect statistically the same amount of background from the underlying event as the cones around the jet axis [9], and we subtract the $k_T$ distribution in complementary cones from the distribution in jet cones. The effect is $\sim 5\%$ with 2% systematic uncertainty associated with the method.

Figure 2 shows the distributions in data corresponding to the dijet mass bins with $Q = 27$ GeV ($95 < M_{jj} < 132$ GeV/$c^2$), 68 GeV ($243 < M_{jj} < 323$ GeV/$c^2$), and 119 GeV ($428 < M_{jj} < 563$ GeV/$c^2$). The distributions in the other five dijet mass bins are similar. The fraction of gluon jets in the sample $f_g$, which is used to mix the theoretical prediction for quark and gluon jets, is obtained using PYTHIA TUNE A with the CTEQ5L parton distribution functions (PDFs) [21]. $f_g$ decreases from 0.7 for $Q = 19$ GeV to 0.2 for $Q = 155$ GeV. The error bars correspond to the statistical uncertainty only, while the shaded area corresponds to the statistical and systematic uncertainties added in quadrature. The total systematic uncertainty is dominated by the uncertainty of the normalization bin and ranges from $\sim 20\%$ at low $k_T$ to $\sim 100\%$ at high $k_T$.

The systematic uncertainty due to the PDFs is evaluated by comparing results for the fraction of gluon jets obtained using the CTEQ5L and CTEQ6.1 [22] PDF sets and is found to be negligible (<1%). The individual systematic uncertainties include contributions from the dependence of predicted $k_T$ distributions on PDFs and uncertainties in the ratios of parton branching fractions.
uncertainties for results with different jet hardresses are strongly correlated.

The solid line corresponds to the NMLLA theoretical curve [2] for \( Q_{\text{eff}} = 230 \text{ MeV} \), extracted from fits of inclusive momentum distributions [9]. The dashed line corresponds to the MLLA theoretical curve calculated according to [1] for the same value of \( Q_{\text{eff}} \). The NMLLA predictions generally have a wider range of validity than the MLLA predictions. The NMLLA results for \( Q_{\text{eff}} = 230 \text{ MeV} \) provide an excellent description of the data over the entire range of particle \( k_T \) and the dijet masses used in this measurement. The overall qualitative agreement between the data and the MLLA calculation [1] for \( Q_{\text{eff}} = 230 \text{ MeV} \) is very good within its range of validity. The extrapolation beyond the range (to higher \( k_T \)) fails to reproduce the data, predicting more particles than observed. Theoretical predictions do not include contributions from jets originating from heavy quarks; however, this effect is found to be well within the uncertainty of the measurement.

We also compare the \( k_T \) distribution of charged particles in data to predictions of the PYTHIA TUNE A and HERWIG 6.5 Monte Carlo generators. Predictions of the Monte Carlo generators for final stable particles are in agreement with each other and with results obtained in data. Figure 2 shows distributions in data compared to PYTHIA TUNE A at the parton and the final stable particle levels. The distribution for partons is obtained using a parton shower cutoff value of 500 MeV, the lowest possible setting in the generator. The qualitative agreement between the NMLLA predictions and charged hadrons from PYTHIA TUNE A is found to be fairly good and is due to the tunings of the hadronization parameters in PYTHIA TUNE A, while the distribution at the parton level shows significant deviations. The HERWIG 6.5 predictions at the level of final stable particles are similar to those of PYTHIA TUNE A.

In summary, we have measured the transverse momenta of particles with respect to the jet axis for a wide range of dijet invariant masses 66–737 GeV/\( c^2 \). The data are compared to calculations using the modified leading log and next-to-modified leading log approximations. Within the range of their validity, the next-to-modified leading log approximation calculations provide an excellent description of trends seen in the data over the entire range of dijet masses. This agreement indicates that hadronization effects are small and provides further support for the hypothesis of local parton-hadron duality. The modified leading log approximation predictions qualitatively show the same trends; however, the quantitative disagreement with the data is significant in this case, indicating the importance of the next-to-modified leading log approximation corrections.

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[18] $E_T$ here is defined as energy multiplied by $\sin(\theta)$, where $\theta$ is the polar angle with respect to the beam axis.