First Measurement of the $i t$ Differential Cross Section $\frac{d\sigma}{dM_{t\bar{t}}}$ in $pp$ Collisions at $\sqrt{s} = 1.96$ TeV


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We present a measurement of the $t\bar{t}$ differential cross section with respect to the $t\bar{t}$ invariant mass, $d\sigma/dM_{t\bar{t}}$, in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV using an integrated luminosity of 2.7 fb$^{-1}$ collected by the CDF II experiment. The $t\bar{t}$ invariant mass spectrum is sensitive to a variety of exotic particles decaying into $t\bar{t}$ pairs. The result is consistent with the standard model expectation, as modeled by PYTHIA with CTEQ5L parton distribution functions.
Because it has a mass near the electroweak symmetry breaking (EWSB) scale \[1\], the top quark plays a special role in many beyond the standard model (BSM) theories of EWSB. In the standard model (SM) the Higgs boson is responsible for EWSB and the generation of the fermion masses, but it has not yet been observed. In models with top condensation, such as technicolor and the top-color model, the role of the SM Higgs boson is filled by a composite particle that is a bound state of top quarks \[2\]. These models predict additional heavy gauge bosons that couple strongly to top quarks. The hierarchy problem, also unresolved in the SM, has recently been addressed by models with extra dimensions, such as the Randall-Sundrum (RS) \[3\] and ADD models \[4\]. In these models TeV-scale gravitons can decay, in some cases preferentially, to \(t\bar{t}\) pairs \[5\]. In all of these cases the production of \(t\bar{t}\) pairs at hadron colliders through BSM mechanisms distorts the \(t\bar{t}\) invariant mass spectrum relative to the SM expectation, as recently reviewed in \[6\].

In this Letter we report on the first measurement of the \(t\bar{t}\) differential cross section with respect to the \(t\bar{t}\) invariant mass, \(\frac{d\sigma}{dM_{t\bar{t}}}\). The analysis uses an integrated luminosity of 2.70 ± 0.16 fb\(^{-1}\) \[7\] collected with the CDF II detector between March 2002 and April 2008. Full details of the analysis presented here are given in \[8\]. Previous published studies have focused on searches for resonances in the \(M_{t\bar{t}}\) spectrum \[9\], and placed a lower limit of 720 GeV/c\(^2\) on the mass of a putative \(Z'\) boson decaying preferentially to \(t\bar{t}\). In this Letter we take a different approach in which we test the \(M_{t\bar{t}}\) spectrum, generically, for consistency with the SM. In this way we are potentially sensitive to broad enhancements of the spectrum and interference effects \[6\], as well as to narrow resonances.

The CDF II detector is described in detail elsewhere \[10\]. The components relevant to this analysis include the silicon vertex detector (SVX), the central outer tracker (COT), the central electromagnetic and hadronic calorimeters, the central muon detectors, and the luminosity counters.

We use the “lepton + jets” decay mode of the \(t\bar{t}\) pair in this study. It consists of four energetic jets, two of which originate from bottom quarks and two from the hadronic \(W\)-boson decay, a charged lepton with large transverse momentum \((P_T)\), and a large transverse momentum imbalance \((E_T)\) from the undetected neutrino from the leptonic \(W\)-boson decay \[11\]. Extra jets may appear from initial- or final-state radiation (ISR or FSR). Monte Carlo (MC) simulations of \(t\bar{t}\) production are generated using the PYTHIA MC program \[12\] with the CTEQ5L \[13\] parton distribution functions (PDFs). The decays of heavy quarks (\(b\) and \(c\)) are modeled using EVTGEN \[14\]. The HERWIG MC program \[15\] is used for studies of the systematic effects of the hadronization model. The \(t\bar{t}\) MC samples are generated with a top quark mass of 175 GeV/c\(^2\). The results presented here are insensitive to changes of the generated top quark mass of a few GeV/c\(^2\).

Events from \(p\bar{p}\) collisions are selected with an inclusive lepton trigger that requires an electron (muon) candidate with \(E_T > 18\) GeV \((P_T > 18\) GeV/c\)). From the triggered events the signal sample is selected offline by requiring an isolated electron (muon) with \(E_T > 20\) GeV \((P_T > 20\) GeV/c\)). The isolation criterion requires \(I < 0.1\), where \(I\) is defined as the calorimeter transverse energy in a cone of opening radius \(\Delta R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} = 0.4\) around the lepton direction (exclusive of the lepton energy), divided by the electron (muon) \(E_T\) \((P_T)\). We further require \(E_T > 20\) GeV and at least 4 jets each with \(E_T > 20\) GeV and \(|\eta| < 2.0\). Jets are identified using a fixed-cone algorithm with a cone size of \(\Delta R = 0.4\) and are constrained to originate at the \(p\bar{p}\) collision vertex. Their energies are corrected to account for detector effects and multiple \(p\bar{p}\) interactions. For events with more than four jets with \(E_T > 20\) GeV, we use the four highest-\(E_T\) jets in the \(M_{t\bar{t}}\) reconstruction. Missing transverse energy is corrected to account for the shifts in jet energies due to the jet energy corrections described above. \(Z\)-boson candidate events are rejected by removing events containing a second isolated high-\(P_T\) lepton. We require that at least one jet in the event have an identified displaced secondary vertex, consistent with the decay of a \(B\) hadron. We label such jets, and the events that contain them, as “\(b\) tagged.” The events selected prior to the \(b\)-tag requirement are called “pretag” events. We observe 2069 pretag, and 650 \(b\)-tagged, events.

The \(t\bar{t}\) signature described above can be mimicked by several processes, including diboson (\(WW\), \(WZ\), \(ZZ\)), single-top, \(Z + \text{ jets}\), and \(W + \text{ jets}\) production, as well as processes without vector bosons to which we refer, generally, as “QCD” backgrounds. The diboson and single-top quark yields are predicted using PYTHIA and MADEVENT \[16\] MC samples, respectively, each normalized to the theoretical cross sections \[17,18\]. The residual \(Z + \text{ jets}\) background is modeled using ALPGEN \[19\], with the parton showering and underlying event model from PYTHIA. The QCD background typically has lower \(E_T\) than events with real \(W\) bosons and is evaluated by fitting the \(E_T\) distribution using templates for QCD and \(W + \text{ jets}\) sources and extrapolating the QCD fraction into the high-\(E_T\) signal region. ALPGEN is also used in the evaluation of the dominant background from \(W + \text{ jets}\) production. The \(W + \text{ jets}\) background is determined separately for events with and without heavy-flavor jets. For events with heavy-flavor jets we use the ALPGEN simulation to determine the fraction, in each jet multiplicity bin, of \(W + \text{ jets}\) events that are \(Wb\bar{b}\), \(Wc\bar{c}\), or \(Wc\). This fraction is then increased by a correction factor, determined by comparing measured and ALPGEN-predicted heavy-flavor (HF) fractions in \(W + 1\) jet data.
The number of pretag $W +$ heavy-flavor events is normalized to the total number of $W +$ jets events in each jet multiplicity bin of the data using the modified ALPGEN fractions. The background contribution from these events is given by the pretag number of events times a MC-derived tagging efficiency. Events without heavy-flavor jets can enter the signal sample if one of the jets is mistakenly $b$ tagged. Such events are called “mistags,” and they occur primarily due to tracking errors, with a smaller contribution from interactions in the material of the detector, and $K_S$ and $\Lambda$ decays. The background due to mistags in $W +$ jets events is evaluated using a measurement of the rate of mistags derived from multijet data [20]. The mistag rate is then applied to the number of pretag $W +$ jets events, with no heavy-flavor jets, in the data. This pretag number is calculated, in each jet multiplicity bin, from the total number of candidate events corrected for the contributions from non-$W +$ jets and from $W +$ heavy-flavor jets. The observed event yields and background predictions are given in Table I, where the line labeled “Other” includes dibosons, $Z +$ jets, and single top.

The precision of the measurement of $M_t$ depends on the understanding of the jet energy scale (JES). To reduce the uncertainty on the JES, we adopt an approach first used in [22] and use the measured invariant mass of the hadronically decaying $W$ boson to constrain the JES. For events with two $b$-tagged jets, the two untagged jets are chosen as the jets from the $W$ boson decay. For events with a single $b$-tagged jet, the pair with invariant mass closest to the expected mean value from $W$ boson decays is chosen. There are 503 single-tagged and 147 double-tagged events in the sample. An unbinned maximum likelihood fit, using MC templates for the dijet invariant mass distribution, for both signal and background, returns the best-fit JES and its uncertainty. The fit value of the JES is subsequently used in the analysis. The uncertainty returned by this procedure is approximately a factor of 2 lower than the nominal uncertainty on the JES.

We reconstruct $M_t$, the $t\bar{t}$ invariant mass, using the four-vectors of the $b$-tagged jet and the three remaining leading jets in the event, the lepton and the transverse components of the neutrino momentum, given by $\not{E}_T$. We divide the $M_t$ distribution into nine bins between 0 and 1400 GeV/$c^2$, with bin widths ranging from 50 GeV/$c^2$ for bins for which a large number of events are expected to 600 GeV/$c^2$ for the highest bin. The resolution in $M_t$ is somewhat smaller than the bin width, ranging from 11% near twice the top mass, to 15% at high mass. We subtract from the bulk $M_t$ distribution the expected contribution from the backgrounds listed in Table I, which is modeled using the Monte Carlo samples described above. The resulting $M_t$ signal distribution suffers from resolution smearing and is corrected using a regularized unfolding technique, described below, which also accounts for the longitudinal component of the neutrino momentum.

In order to extract the true underlying $M_t$ distribution from the background-subtracted reconstructed distribution, we use the MC program to create a response matrix $\hat{A}$, such that $\hat{A}x = d$ where $x$ is the true, binned distribution and $d$ is the measured, binned distribution. Because of statistical fluctuations in bins with small numbers of events, inverting the response matrix $\hat{A}$ to solve for $x$ given $d$ often yields spurious results. Instead we use singular-value decomposition (SVD) unfolding, as described in [23], where the solution is regularized by populating the response matrix with event multiplicities instead of probabilities. The application to this analysis is described in detail in [8].

From the unfolded $M_t$ distribution, we calculate the differential cross section according to

$$
\frac{d\sigma}{dM_t} = \frac{N_i}{\mathcal{A}_i \int L dt \cdot \Delta_{M_t}},
$$

where $N_i$ is the background-subtracted, unfolded, number of events observed in each bin; $\mathcal{A}_i$ is the acceptance in bin $i$; $\Delta_{M_t}$ is the width of bin $i$; and $\int L dt$ is the integrated luminosity measured from a mixture of data and MC simulations. We use PYTHIA with a GEANT-based [20] detector simulation to measure the geometric and kinematic acceptance. The lepton trigger and identification efficiencies are measured in data using $Z \to \ell\ell$ decays. We account for the difference in efficiency for identifying an isolated high-$P_T$ lepton in data and MC with a scale factor. Similarly we use a scale factor to correct for the difference in efficiency in data and MC for tagging a $b$ jet. The efficiency in data is determined in a heavy-flavor-enriched data sample of low-$P_T$ electrons, from the semi-leptonic decay of $B$ hadrons.

Our systematic uncertainties arise from MC modeling of the acceptance, true and reconstructed $M_t$ distributions, and background distributions. In addition, the uncertainties of our efficiency of lepton identification, $b$-tagging efficiency, and integrated luminosity affect the measurement. The lepton identification uncertainty arises due to the extrapolation from $Z \to \ell\ell$ events, where the efficiency is measured in data, to the higher multiplicity $t\bar{t}$ environment. The uncertainty on the $b$-tagging efficiency is largely due to the limited number of events in the data sample that is used. These uncertainties, together with a small uncertainty due to the finite size of the MC simulation used to

<table>
<thead>
<tr>
<th>Process</th>
<th>4 jets</th>
<th>$\geq 5$ jets</th>
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<tbody>
<tr>
<td>$W +$ HF</td>
<td>$58.0 \pm 12.2$</td>
<td>$11.6 \pm 2.9$</td>
</tr>
<tr>
<td>Mistags</td>
<td>$18.9 \pm 4.8$</td>
<td>$3.5 \pm 1.5$</td>
</tr>
<tr>
<td>QCD</td>
<td>$20.9 \pm 18.5$</td>
<td>$15.6 \pm 6.0$</td>
</tr>
<tr>
<td>Other</td>
<td>$13.9 \pm 0.8$</td>
<td>$3.1 \pm 0.2$</td>
</tr>
<tr>
<td>$t\bar{t}(6.7$ pb)</td>
<td>$358.6 \pm 49.7$</td>
<td>$121.5 \pm 16.8$</td>
</tr>
<tr>
<td>Total prediction</td>
<td>$470 \pm 57$</td>
<td>$146 \pm 19$</td>
</tr>
<tr>
<td>Observed</td>
<td>$494$</td>
<td>$156$</td>
</tr>
</tbody>
</table>

TABLE I. Summary of sample composition [20,21].
calculate the acceptance, comprise the acceptance uncertainty in Table II.

We consider several variations to the MC model of the signal and background. For the signal MC simulation we compare the results using HERWIG to the default PYTHIA generator. The uncertainty due to the limited knowledge of ISR is constrained by studies of radiation in Drell-Yan events in the data. We vary both ISR and FSR within these limits and add the deviations from the nominal value in quadrature. The uncertainty due to possible differences in the PDFs from the nominal CTEQ5L PDF is evaluated by varying the PDF using the 20 CTEQ5L eigenvectors, which represent 90% C.L. variations. The deviations from the nominal values are added in quadrature with deviations measured using the MRST PDF [24] with two alternate choices for the strong coupling constant. The uncertainty on the background prediction consists of two pieces: the uncertainty on the background normalization, given in Table I, and a background shape systematic for the MC modeling of the shapes. The systematic uncertainty due to the JES includes a generic energy-correction systematic uncertainty as well as a contribution from the modeling of the $b$-jet energy scale. Pseudoexperiments are performed for each variation described above and the difference between the mean \( \frac{d\sigma}{dM_{\tilde{t}\tilde{t}}} \) in each bin with the shifted parameters and the default model is taken as the systematic uncertainty in that bin. The results are presented in Table II. The 6% uncertainty on the luminosity measurement in each bin [7] is not included in the total in Table II. Two effects cause the uncertainty in the bins smaller than outside of that range. One is the turn-on threshold of the \( M_{\tilde{t}\tilde{t}} \) spectrum, which is insensitive to systematic variations because we fix the top quark mass at 175 GeV/c\(^2\). The second is the PDF uncertainty, which is much greater at large \( M_{\tilde{t}\tilde{t}} \) than at small \( M_{\tilde{t}\tilde{t}} \).

The measured \( \frac{d\sigma}{dM_{\tilde{t}\tilde{t}}} \) is shown in Fig. 1 and tabulated in Table III.

We check consistency with the SM prediction using the Anderson-Darling (AD) statistic [25], which places an emphasis on potential discrepancies in the tail of the \( M_{\tilde{t}\tilde{t}} \) distribution. The distribution of the AD statistic for this analysis is rapidly falling, with small values corresponding to more likely results. Using MC simulations, we optimize the bin range of the AD statistic for maximum sensitivity to new physics and a minimum of false positives and find \( M_{\tilde{t}\tilde{t}} \approx 450 \text{ GeV/c}^2 \) to be the most sensitive region of \( M_{\tilde{t}\tilde{t}} \). We perform pseudoexperiments using the SM MC distributions of \( M_{\tilde{t}\tilde{t}} \) with the sample composition given in Table I. We calculate a \( p \) value by taking the fraction of

![](image)

FIG. 1 (color online). \( \frac{d\sigma}{dM_{\tilde{t}\tilde{t}}} \) measured with 2.7 fb\(^{-1}\) of integrated luminosity.

<table>
<thead>
<tr>
<th>TABLE III.</th>
<th>The acceptance and measured differential cross section in each bin. The uncertainties on the cross-section values are, respectively, statistical + JES, systematic, and luminosity.</th>
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<tbody>
<tr>
<td>( M_{\tilde{t}\tilde{t}} \text{[GeV/c}^2\text{]} )</td>
<td>( A_i )</td>
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<td>( \leq 350 )</td>
<td>0.016 ± 0.001</td>
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<td>350–400</td>
<td>0.023 ± 0.001</td>
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<td>400–450</td>
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<td>0.029 ± 0.001</td>
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<td>550–600</td>
<td>0.030 ± 0.001</td>
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<td>700–800</td>
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<td>800–1400</td>
<td>0.023 ± 0.001</td>
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Integrated cross section [pb] 6.9 ± 1.0 (stat + JES)
pseudoexperiments with a larger observed AD statistic than that in data. The observed $p$ value is 0.28. We conclude that there is no evidence of non-SM physics in the $M_{H_2}$ distribution.

We thank Rikkert Frederix, Fabio Maltoni, and Tim Stelzer for stimulating discussions, advice, and help with MADEVENT generation. We thank the Fermilab staff and the technical staffs of the participating institutions for their vital contributions. This work was supported by the U.S. Department of Energy and National Science Foundation; the Italian Istituto Nazionale di Fisica Nucleare; the Ministry of Education, Culture, Sports, Science and Technology of Japan; the Natural Sciences and Engineering Research Council of Canada; the National Science Council of the Republic of China; the Swiss National Science Foundation; the A.P. Sloan Foundation; the Bundesministerium für Bildung und Forschung, Germany; the Korean Science and Engineering Research Council of Canada; the National Science Foundation; the A. P. Sloan Foundation; the Korean Research Foundation; the Institut National de Physique Nucleaire et Physique des Particules/CNRS; the Russian Foundation for Basic Research; the Ministerio de Ciencia e Innovación, and Programa Consolider-Ingenio, Spain; the Slovak R&D Agency; and the Academy of Finland.

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