IX. MISCELLANEOUS PROBLEMS

A. ELECTRONIC DIFFERENTIAL ANALYZER

Prof. Henry Wallman
R. Maartmann-Moe

Mr. Robert H. Cannon of the M.I.T. Mechanical Engineering Department has now completed his study of the hydrofoil speedboat. The equation solved was of the form

$$k_1 z'' + (k_2 + k_3 h_1)A_1 + (k_4 + k_5 h_2) A_2 = 1$$

$$k_6 \theta'' - (k_2 + k_3 h_1)A_1 + (k_4 + k_5 h_2) A_2 = 0$$

where z is the vertical displacement of the boat, θ the pitch angle, and h_1 and h_2 are the hydrofoil depths given by

$$h_1 = h_{10} + z - k_7 \theta + \lambda_1(t)$$

$$h_2 = h_{20} + z + k_8 \theta + \lambda_2(t)$$

where $\lambda_1(t)$ and $\lambda_2(t)$ are the ocean wave shape given by

$$\lambda_1(t) = b(1 - \cos(\omega t + \phi_1) + K_2 \cos 2(\omega t + \phi_1) - K_3 \cos 3(\omega t) + \phi_1)$$

with a similar expression for $\lambda_2(t)$. These waveforms were generated by two function generators. A_1 and A_2 represent the angles of attack of the hydrofoils given by

$$A_1 = 1 + C_1\theta + C_2 z' + C_3 \theta'$$

with a similar expression for A_2 except for the constants C.

The accuracy of the results was mainly limited by the multipliers, and also by a linearizing assumption made in the equations to avoid the necessity of building two more function generators. A large part of the study was made by varying only one parameter at the time, so that even if the absolute results were not too accurate, relative values should give fairly reliable results.

Mr. D. W. Peaceman of the Chemical Engineering Department at M.I.T. studied a problem of chemical absorption rates expressed by

$$x'' = Rxy$$

$$y'' = qxy$$

where x and y are absorption rates, R and q parameters.

Mr. R. J. Renfrow of the High-Power Magnetron Group of this Laboratory investigated the temperature distribution along a directly-heated magnetron-cathode support

$$T'' = -aT + bT^4$$

where T is the temperature along the support.

Mr. Cannon's work on the analyzer led to several minor improvements, and also pointed out the requirements on the various units of the analyzer. For instance, it is desirable to have both positive and negative outputs from all units, and the adders and integrators should be able to drive four following units without overloading and change of calibration. A change of the cathode follower circuits in the adders and integrators has reduced the output impedance by a factor of two, and at least doubled the available output signal under load.

The negative power supplies do not give sufficient stability with glow-discharge tubes when function generators and multipliers are used in the analyzer setup. Battery reference voltage is now used.

New initial condition potentiometers having a grounded center tap have improved the stability of the initial conditions so that this stability no longer is a factor in the overall stability of the analyzer.

B. ANALOG DEVICES FOR NETWORK PROBLEMS

Prof. E. A. Guillemin	H. C. Martel
Dr. R. E. Scott	R. H. Pantell
D. D. Holmes	R. P. Talambiras

1. Automatic Impedance-Function Analyzer

The construction of the automatic impedance-function analyzer has been completed and Technical Report No. 137, An Analog Device for Solving the Approximation Problem of Network Synthesis, is in preparation.

The machine is now available for solving typical network problems.

R. E. Scott

2. The Dipole Potential Analog

a. Theory

The dipole potential analog offers an attractive means for the determination of the real or the imaginary part of an impedance function from the positions of the poles of the function, and the residues in the poles. It can be shown that the potential distribution arising from a configuration of dipoles representing the magnitudes and angles of residues in the poles of an impedance function is proportional to the real part of the function. If the dipoles are rotated 90° in the clockwise direction, the potential distribution is proportional to the imaginary part of the function. Accordingly, the potential distribution along the imaginary axis of the complex frequency plane is proportional to the real or imaginary part of the impedance function at real frequencies. This fact is in itself useful, since the positive real character of the function is established if the real part of the function is positive on the real frequency axis. However, the true value of the analog lies in the fact that the impulse response of a network may be obtained from the real part of the network impedance function, and a continuous display of the impulse response may be provided on an oscilloscope. Thus, the synthesis of a network to meet a prescribed transient response may be performed.

The impulse response of a network is given by

$$h(t) = \frac{2}{\pi} \int_0^\infty H_e(\omega) \cos \omega t \, d\omega$$

where $H_e(\omega)$ is the real part of the impedance function at real frequencies. It becomes necessary to multiply the real part of the function provided by the dipole analog by cos ωt and integrate from 0 to ∞ to determine the impulse response of the network. This end may be attained by using a cosine wave of variable frequency as the driving current at the dipoles, and by performing successive integrations of the voltage along the imaginary axis. Each integration gives a point on the curve of h(t). The frequency of the driving current is varied at a much slower rate than that at which the integration is performed, so that the frequency through one cycle of integration is nearly constant. The infinite upper limit on the integral may be approximated by integrating to a very large value of ω, so that the impulse response is substantially zero.

b. Experimental work

The experimental work which has been done to date has been of a preliminary nature. Teledeltos paper has been tried as a conducting medium representing the complex frequency plane, and has been found to be satisfactory in some respects. The limitation set by the paper on the driving current of the dipoles is a decided disadvantage. The low driving currents which must be used with the conducting paper provide voltages which are difficult to detect above noise and hum. A possible remedy for this is the use of high-current, short duty-cycle pulses at the dipoles.

A circular representation of the complex-frequency plane has been investigated, utilizing a conducting ring of about 20 inches diameter as an approximate infinity. With this apparatus, the following conclusions were drawn: (1) A better infinity is required in the dipole representation than in the pole-and-zero representation for the same percentage error. (2) The dipoles must be driven from "floating" sources (no common ground) so that each dipole will assume an average potential corresponding to the potential existing at the point at which it is introduced. (3) Potential differences to be measured are of sufficient magnitude to be measurable with a fair degree of accuracy.

On the basis of the above conclusions, a semicircular double-layer device is being constructed; this makes possible the accurate representation of the point at infinity. This device will be equipped with a line of 32 fixed probes along the $j\omega$-axis to facilitate the representation of the impulse response on an oscilloscope. The construction is the same as that used in the impedance function analyzer.

D. D. Holmes

3. The Electronic Isograph

The theory and design of this device for obtaining the roots of polynomials to the sixth degree was described in the last Quarterly Progress Report.

For roots much smaller than unity, a fairly large percentage of error is introduced. Since the magnitude of error is comparatively constant for all solutions of the polynomial, it is necessary to multiply the roots by a constant to obtain a solution in the vicinity of unity so as to increase accuracy. This is a simple procedure for the isograph can be used to obtain the approximate root, whence conversion of the polynomial constants affords the necessary root translation. Since the isograph has not been completed, the percent of error cannot be determined. It is anticipated that roots in the neighborhood of unity will be accurate within several percent.

Although it is not often necessary in circuit analysis to solve polynomials higher than the sixth degree, the present isograph could be modified to include such problems. To extend solutions to the seventh degree it is necessary to have a flip-flop circuit producing an output at 14 kc. This can be accomplished by filtering the proper harmonic from the currently available 4-kc square wave; thus the sinusoidal component at a frequency of 28 kc would be the input signal to the 14-kc flip-flop multivibrator. In a similar manner eight, ninth, or tenth degree polynomials could be solved by the modified isograph.

At present, the coefficients of the various sinusoids (the powers of r referred to in the last Quarterly Progress Report) are set mechanically by use of a ganged potentiometer. Mr. Robert Talambiras is currently developing an electronic device to vary the powers of r continuously. In this manner the entire polynomial can be plotted on an oscilloscope to determine the regions of positive, real roots.

R. H. Pantell

4. A Panoramic Display for the Electronic Isograph

Some of the theoretical work being done at the Research Laboratory of Electronics by the group working on transient problems requires the determination of the values of the variable for which a polynomial has a positive real part. A direct determination is laborious since complex as well as real values of the variable must be considered. By adapting the electronic isograph the information can be obtained in a very short time as a map of the Z-plane on an oscilloscope, showing the values of Z for which the polynomial has a positive real part.

A discussion of the principles of the electronic isograph was given in the last Quarterly Progress Report. Briefly, a polynomial

$$a_o + a_1 Z + a_2 Z^2 + \ldots + a_n Z^n = 0$$

where $Z = x + jy = R\epsilon^{j\beta}$, may be written in the polar form as

$$(a_k R^k \cos k\beta + j a_k R^k \sin k\beta) = 0 \quad .$$

In the present isograph the multiplications and summations are performed electrically and the sums of the cosine and sine terms applied respectively to the horizontal and vertical plates of an oscilloscope. A Nyquist plot of the polynomial is obtained on the oscilloscope, and R is varied manually until the plot passes through the origin, indicating a root of the polynomial.

For mapping purposes R must vary continuously at a frequency easily viewed on an oscilloscope, e.g. twenty cps. If the variation in R is made linear, i.e. R is represented by a sawtooth waveform, the power of R can be obtained by passing the sawtooth through successive integrating circuits. R^k and the appropriate cosinusoidal wave are then multiplied in a linear modulator, passed through a coefficient potentiometer, and shifted 90° in phase to give the sine terms. If the voltages representing $R \cos \beta$ and $R \sin \beta$ are applied respectively to the horizontal and vertical plates of an oscilloscope a closely wound spiral will be traced out twenty times per second as R and β vary. If now the voltage representing the sum of the cosine terms (the real part of the polynomial) is amplified and clipped and applied to the intensity grid of the oscilloscope so that the beam is on only when the voltage is positive, a display of the Z-plane will be obtained on the oscilloscope, with the regions of Z giving rise to a positive real part of the polynomial appearing bright on the screen. Similarly the regions of Z where the imaginary part is positive may also be represented on the oscilloscope. Roots are indicated by pulsing the intensity grid of the oscilloscope whenever the voltages representing the real and imaginary parts simultaneously pass through zero.

Work at present is concentrated on the linear modulator used to obtain the term $R^k \cos k\beta$. The requirements on the modulator are rather exacting since both the carrier and the modulating envelope must have very low distortion.

R. P. Talambiras

5. Electronic Commutator

A survey was made of the field of electronic commutators in the hope of finding a type which could replace the motor-driven mechanical unit which is used on the impedance function analyzer. Among the types investigated were the multivibrator string (unsynchronized and synchronized), multivibrator ring, matrix (resistor, crystal, and crystal-resistor), and some special tube types (radial beam tubes and the Cyclophon). All of these types had their own advantages and disadvantages, but among them the matrix is both the newest and perhaps the most promising. None of them has been actually built and operated with more than 32 channels, and although in theory it appears that they could be expanded to handle 100 lines or more, in practice this is not so easy. The circuitry types require close tolerance circuitry to give reliable operation, and the special tube types are only moderately stable in operation in their present form.

In the network synthesizer a great many voltage readings (on the order of 100) must be taken and recorded. If these voltages are commutated and placed on an oscilloscope, thereby effectively taking all the readings at once, an immeasurable time saving is effected. The commutator called for in this case is one capable of handling over 100 input lines of low value d-c and sampling these lines at least 60 times per second. The high number of lines and the low value d-c which must be handled are the factors which eliminate previously developed types and necessitate a special investigation.

Work is now in progress on a solution to the problem. Two possible methods are being investigated. One consists of a tree of single-pole double-throw electronic switches each column of the tree operating at half the frequency of its neighbor. An individual switch unit consists of two triodes whose grids are pulsed to drive the tube either fully conducting or cut-off. The other method utilizes diodes, again in a tree, to short out the undesired inputs effectively. Both methods are well suited for d-c, should handle fairly low values of the input, and show promise of being expandable for a large number of lines.

H. C. Martel

C. OFFSET WAVEGUIDE JUNCTION

L. D. Smullin
W. Glass

The desirability of using capacitive coupling irises for tunable cavities was pointed out in R.L.E. Technical Report No. 105, "Design of Tunable Resonant Cavities with Constant Bandwidth". The mechanical problem of making a conventional waveguide capacitive iris with a 0.015-in. gap with reasonable tolerances seemed formidable. What appeared to be a good solution was the offset waveguide junction, Fig. IX-la. This could be made with a pair of flat flanges on the guides to be joined, setting the gap by means of a feeler-gage, and soldering. Thus, there are no delicate inserts within the waveguide, and making gaps as small as 0.005 in. becomes a rather easy operation. Electrically, the offset capacitive junction is a pure shunt capacitance, or the equivalent of an infinitely thin iris.

If the junction is offset as shown in Fig. IX-lb, an inductive iris is produced; and the diagonal offset produces a resonant iris, Fig. IX-lc. The inductive iris is not a pure shunt inductance, but must be represented as an equivalent T or π network. This arises from the asymmetry of the incident magnetic fields on the two sides of the junction which

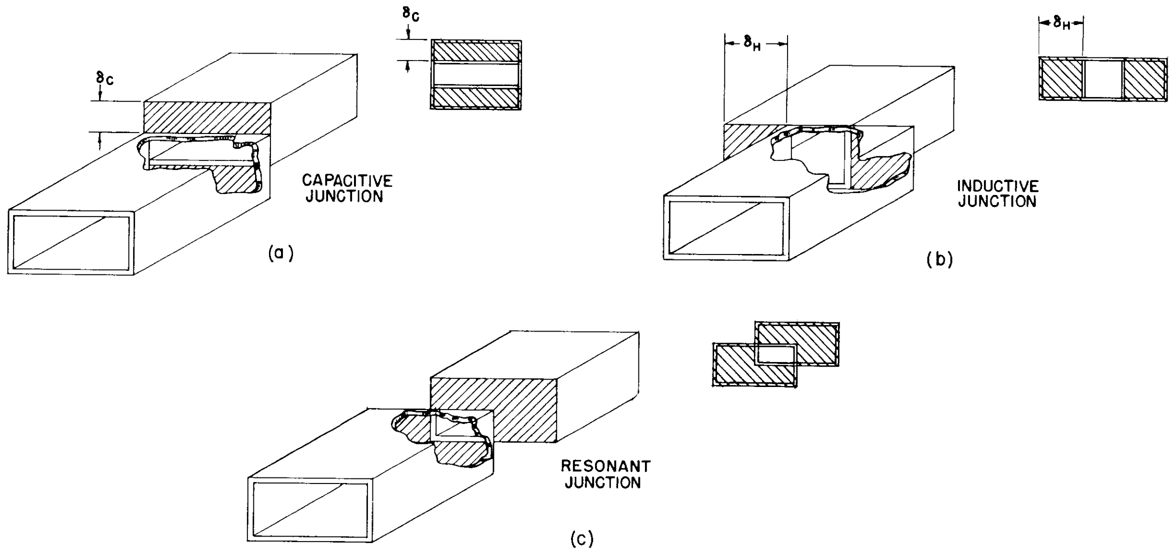

Fig. IX-1 Offset waveguide junctions.

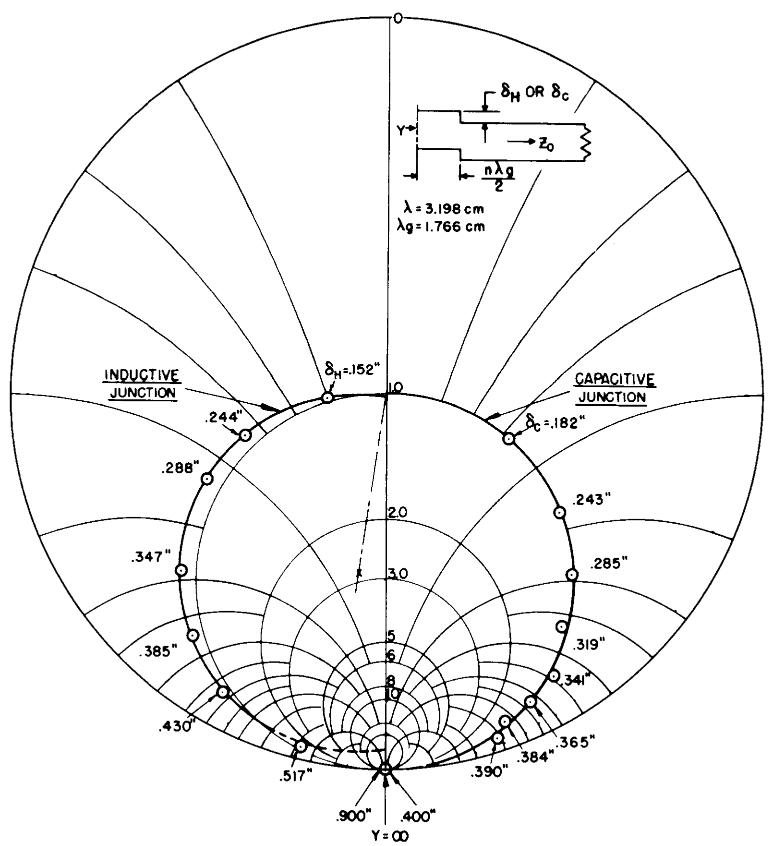

Fig. IX-2 Admittance of capacitive or inductive offset junctions with matched terminations. Measurements made at $\lambda = 3.198$ cm for various values of the offset δ_H or δ_c.

makes it impossible for one to cancel the other completely.

Figure IX-2 shows the behavior of the inductive and capacitive junctions of the 1/2 in. $\times$ 1 in. waveguide at one frequency as functions of the offset distance δ_H or δ_c. The capacitive junction and the matched load beyond it behave as an admittance 1 + jb. The inductive junction must be represented by a T of inductances, terminated in Z_o. The series inductance has a reactance $x \simeq 0.05$, when the parallel inductance has a susceptance $b \simeq 0.8$.

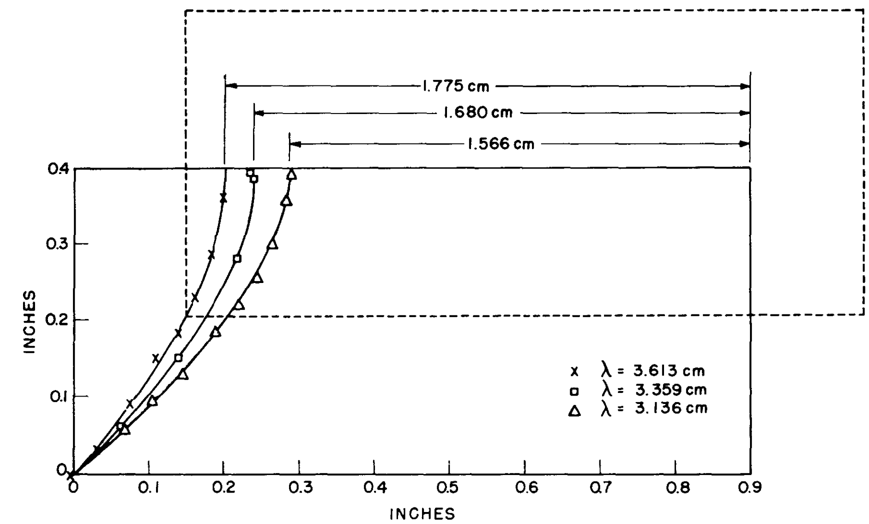

Fig. IX-3 Curves of constant resonant frequency for resonant offset junctions.

The frequency characteristic of the capacitive junction was measured and it was found that $b_c \sim 1/\lambda_g$ where b_c is the susceptance of the junction and λ_g is the guide wavelength. This is in agreement with the theory.

Figure IX-3 shows the tuning curves for the resonant junction; each line represents the locus of constant resonant frequency, but varying Q_L ($Q_L = 0$ at $\delta_c = \delta_H = 0$). The length of the inductive gap at the point where the capacitive gap vanishes is equal to one-half the air wavelength, within experimental error.

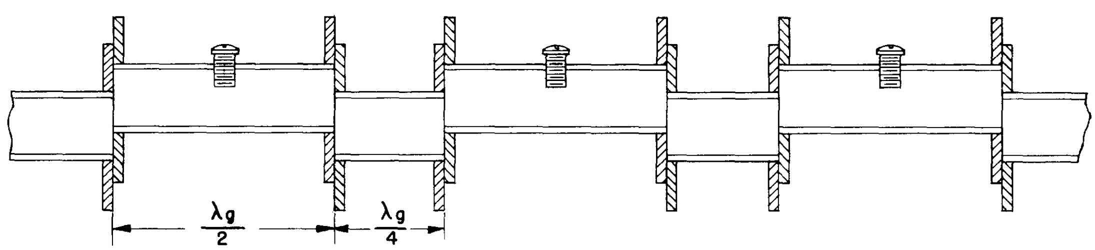

Fig. IX-4 Schema of filter assembly made with offset junctions.

Figure IX-4 illustrates how a multiple-cavity filter could be assembled from standard lengths of waveguide.

Future work will be concerned with the application of these junctions to filter problems.