The Research Laboratory of Electronics recently undertook to investigate certain problems associated with air navigation. This work is to supplement other programs initiated or supervised by the Air Navigation Development Board. The objective of the current program is to determine, under restricted conditions, the quantitative relation between the degree of control of planes enroute to an airport and the resulting congestion and delay.

It is probable that in the future there will be provided more than one landing strip or airport in the vicinity of a large city. During the initial phase of this research program, each such strip will be treated as an independent unit. Under normal traffic conditions, it can be expected that each landing strip will be characterized by some maximum traffic capacity or rate of acceptance for incoming planes. These planes will converge toward the landing strip from all directions. It is desirable that the planes arrive at a uniform rate equal to the maximum acceptance rate of the landing strip, whenever such scheduling is necessary for optimum utilization of the ground facilities. There will be factors, however, which will contribute to an arbitrary delay for planes enroute and hence to congestion at the airport. It is obvious that some rigid scheduling of planes before take-off, and some control enroute, will be required for the minimization of enroute delay and congestion in the neighborhood of the landing terminal.

The control of planes during the trip is necessary for two principal reasons. The first has to do with safety in the air. The second is concerned with the effective utilization of the landing strip and the reduction of trip time as mentioned above. In the present approach to the problem, considerations of safety will be set aside. It will be assumed that anticollision measures will be available enroute as well as during the landing procedure. Under these conditions the problem may be formulated as follows.

Planes are scheduled for takeoff from various airports at such times that, under ideal flying conditions, they will land at a uniform rate equal to or less than the maximum acceptance rate of the terminal strip. As each plane proceeds towards its destination its actual speed deviates from that scheduled by a random amount which follows some reasonable statistical law. Corrective control of the flight will be exercised on the basis of information directly available in the plane, or data transmitted from the ground, or both. The degree of corrective control may be classified as follows:

Case (1) No control. Each plane proceeds toward the terminal according to the original flight plan and is subject to random deviations from that plan caused by weather, wind, etc. No attempt is made to adjust the speed or course in order to realize the scheduled time of arrival.

Case (2) Continuous control. Each plane, throughout its flight, is subjected to
continuous corrective control which opposes any deviations from the scheduled flight plan and results in a landing time which coincides with that originally scheduled. Cases (1) and (2) represent extremes of the degree of control.

Case (3) Discrete control. For each flight path, a definite number of check points are established along the route. An actual arrival at the first check point is compared with that expected on the basis of the particular scheduling for that plane. Attempts are then made to reduce the deviation before the plane reaches the second check point. These attempts will not, of course, be completely successful so that a deviation will exist when the plane arrives at the second check point. The discrete control process is repeated at the second check point and again at each of the others along the route.

Case (4) Discrete control with rescheduling. As in case (3), a number of check points are established between the point of takeoff and the landing strip. When a plane arrives at the first check point, the ground control at the landing strip reassigns to that plane a scheduled landing time based upon the available information concerning other incoming planes, which may differ from the originally scheduled landing time. The plane is instructed to follow a new flight plan consistent with the reassigned landing time. The process is repeated at each check point.

For convenience, the prescribed minimum time separation between consecutive landings is taken as the time unit and is given the symbol \( t_0 \). The deviation of the actual arrival time at a point from the scheduled time is measured in terms of \( t_0 \). The traffic parameter \( \epsilon \) is defined as the ratio of average arrival rate at the landing strip to the prescribed maximum acceptance rate. The enroute delay of a plane is assumed random, subject to a certain probability distribution. In order to approximate the practical situation, it is assumed further that the enroute deviations are limited to some finite spread, \( S \). In other words, the probability density spectrum of enroute deviation vanishes outside a certain region of width \( S \). Three different forms of the deviation probability curve are assumed: rectangular, triangular, and parabolic. In order to simplify the problem the deviation is quantized to integral multiples of \( t_0 \).

When the control is incomplete, some planes arriving at the landing strip must wait in the air until the strip is available. This waiting time, called stack delay, results in an additional difference between the actual and scheduled landing time of certain planes.

Unlike the problem treated by Pearcey, wherein a Poisson distribution is assumed, the problem stated above is not easily susceptible to mathematical analysis. After some sample computation, arrangements have been made to carry out the numerical work on the I.B.M. punched-card machines available in the M.I.T. Center of Analysis.

Case (2), complete control, is a trivial one requiring no computation. Case (1), no control, has been programmed in detail for the machines. A sequence of 1,000 planes is scheduled for landing at unit intervals, corresponding to \( \epsilon = 1 \). Smaller values of the
traffic parameter $\epsilon$ are accommodated by withholding a fraction $(1-\epsilon)$ of the thousand planes; those withheld are chosen at random. The enroute delay for each plane is determined from tables of random numbers which are arranged to yield rectangular, triangular, or parabolic distributions of the delay. Planes are landed in order of their arrival at the airport, after whatever waiting period is necessary. The machine computations will provide data on the basis of which the statistics of enroute and stack delay may be analyzed.

Work will begin shortly on the formulation of cases (3) and (4), preparatory to machine computation.

References


The work reported in this section is supported in part by the Air Navigation Development Board of the Department of Commerce.