

## X. ANALOG COMPUTER RESEARCH

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The analog computer group has three main interests in the field of specialized computing devices:

- A. The operation of computers already in existence
- B. The design of new computing elements
- C. The analysis of computers as elements of systems.

### A. THE OPERATION OF PRESENT COMPUTERS

The computers under the direct supervision of the group include:

1. The original Macnee differential analyzer
2. The Scott impedance function analyzer
3. The Pantell electronic isograph
4. The Holmes dipole potential analog.

During the past quarter attention has been focused on the differential analyzer in the hope that a satisfactory pattern could be derived for the operation of machines of this type. In pursuit of this purpose the machine has been placed in excellent operating condition and demonstrated to numerous groups, including Prof. Morse's seminar on computing methods and the Electrical Engineering staff colloquium. A descriptive manual explaining the operation of the machine and discussing its applications and its limitations has been prepared for distribution to those who are interested in the analyzer.

At the present time the machine is being used by Mr. L. Weinberg to take some Fourier transforms, and also by Prof. Kopal to solve a modified Schrödinger equation of the form  $\ddot{y} + [\lambda f_1(t) + f_2(t)]y = 0$  where  $f_1(t)$  and  $f_2(t)$  are arbitrary functions.

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### B. THE DESIGN OF NEW COMPUTING ELEMENTS

#### 1. Integral Equation Computer

A digital-analog computer (1) is being designed to perform certain integral transformations, solve certain types of integral equations and sets of linear equations. The computer is intended to provide solutions with an accuracy of about 1 percent and thereby to serve not only as a research tool but also as a source of approximate mathematical solutions for iteration to higher accuracy with other computing facilities.

The computer will receive information for the most part numerically in binary code on punched paper tapes, will operate on this information, in part digitally, in part by analog measurement, and will yield results in the form of a paper tape punched in binary or decimal code. The technical aspects of the design differ notably from the published

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proposals of Prof. Wallman (2) for a similar computer.

The integral transformations that may be performed include the Fourier and Hilbert transforms and the expansion of a function in an orthonormal set of functions.

The type of integral equation best suited to solution with the proposed equipment is the second kind (3) of the form

$$\phi(x) = f(x) + \lambda \int_a^b K(x, u) \phi(u) du \quad (1)$$

where  $\phi(x)$  is unknown. Solution is obtained by iteration from an approximate solution  $\phi_0(x)$ .

To the extent that finite sets of linear equations may be used to approximate linear integral equations of the first kind (3), equations of the first kind may be solved with this equipment. For example, in the Wiener-Hopf equation (2), where  $h(\sigma)$  is

$$\phi_{im}(\tau + a) = \int_0^{\infty} h(\sigma) \phi_{ii}(\tau - \sigma) d\sigma \quad (2)$$

an unknown impulse response,  $h(\sigma)$  might be determined at  $n$  values of  $\sigma$  using Weddle's rule to approximate the integral.

The resulting set of  $n$  simultaneous linear equations is solved by a modified Gauss-Seidel (4) procedure in which the matrix of the equations is inserted repeatedly from a punched tape while the unknowns are inserted as analog variables and are manually adjusted for least-square error.

A study is being made of certain problems, particularly in electrical engineering, whose integral formulation would render them suitable for solution with the equipment being designed. Some of the work of Schwinger (5) may be cited as an excellent example of the method.

### References

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