IX. COMMUNICATION RESEARCH

A. MULTIPATH TRANSMISSION

Prof. L. B. Arguimbau	W. L. Hatton	R. A. Paananen
Dr. J. Granlund	E. E. Manna	G. M. Rodgers
Dr. C. A. Stutt		R. D. Stuart

1. Speech and Music

As indicated in the last report, transatlantic frequency-modulation tests were carried out at 23.09 Mc $\pm$ 75 kc. By the time arrangements had been made for the frequency assignment the maximum usable frequency had dropped to around the assigned frequency and the tests were once more inconclusive. A further effort is being made to obtain a frequency assignment that will be usable for a larger period. It looks as though we will be unable to get the desired bandwidth at the lower frequencies and will have to be satisfied with a partial tryout.

L. B. Arguimbau, J. Granlund, C. A. Stutt

2. Television

It will be remembered from the last report that tests are under way to compare the relative performance of frequency modulation and amplitude modulation for video transmission under multipath conditions. A 10,000:1 scale model (facsimile) is being used.

The delay line has been constructed and put into operation. Owing to attenuation in excess of expectations only 40 feet of the pipe is being used, giving a delay of 40 msec, but this has proven adequate on a model basis. Some pictures have been run off undeı multipath conditions; however, many more must be taken before any definite conclusions can be drawn.

Two types of pictures are being used for test purposes. One is a test pattern having alternate black and white bars of several different widths while the other is a reproduction of a typical scene around the laboratory. Preliminary results have been obtained which will be reported later.

For comparison purposes, an A-M system is being built.

A technical report on the pre-emphasis aspect of the problem has been prepared and will be published presently.

L. B. Arguimbau, W. L. Hatton, E. E. Manna, G. M. Rodgers, R. D. Stuart

3. F-M Receiver Design

An investigation is to be made to determine the simplest and most efficient design for a receiver to reject interference. It is proposed to examine fairly carefully as many systems as time will alow in order to have qualitative and quantitative data on their relative performances, and to provide a groundwork for possible new designs.

R. A. Paananen

B. STATISTICAL THEORY OF COMMUNICATION

Prof. J. B. Wiesner	B. A. Basore	A. J. Lephakis
Prof. W. B. Davenport, Jr.	R. S. Berg	R. M. Lerner
Prof. R. M. Fano	J. J. Bussgang	H. Levick
Prof. Y. W. Lee	L. Dolansky	M. J. Levin
Prof. J. F. Reintjes	B. M. Eisenstadt	F. L. Petree
Dr. O. H. Straus	P. E. Green, Jr.	D. E. Ullery
Dr. C. A. Stutt	B. Howland	I. Uygur
	L. G. Kraft	

1. Multichannel Analog Electronic Correlator

The sampling-pulse generator, the signal-sampling circuit, and the display circuit of the five-channel correlator have been completed. The sampling-pulse generator employs a ring-chain counter to provide a half-microsecond pulse at five different output terminals. The time delay between adjacent channel pulses is variable in six steps from 1 μsec to 400 μsec. A block diagram of the unit, and waveforms, are shown in Fig. IX-1.

The signal-sampling circuit is capable of operating with a sampling-pulse width of 0.2 μsec or greater. The signal-storage element (a capacitor loaded by the input impedance of a cathode follower) shows only a 0.25 percent loss in charge over a holding time of 5 msec. The output-input characteristic of the circuit deviates from linearity by less than 1 percent of the maximum output-signal amplitude.

The display circuit uses a multiple retrace type of presentation on the screen of an electrostatic type of cathode-ray tube. Data are presented at the rate of 1000 cps. It is planned to display the information continuously as it is gathered during the process of correlation, and then to store the final value of the correlation function on the integrator capacitors during the interval it is being displayed on the cathode-ray-tube screen. This procedure has been chosen in order to eliminate the necessity for an additional storage element per channel.

Y. W. Lee, J. F. Reintjes, H. Levick, F. Petree, D. Ullery

2. Pulse Code Magnetic Recorder

Tests have been performed on the new head assembly and the driving mechanism. Upon further consideration of the compressor-expander scheme (Fig. VIII-11, p. 49, Quarterly Progress Report, April 15, 1951) it was found that due to the fact that $\beta(v)$ is a function of the voltage magnitude v and because v_b (magnitude after compression) is not the same as v_a (magnitude before compression) the suggested scheme would not restore the original voltage in general. A new arrangement is suggested in Fig. IX-2,

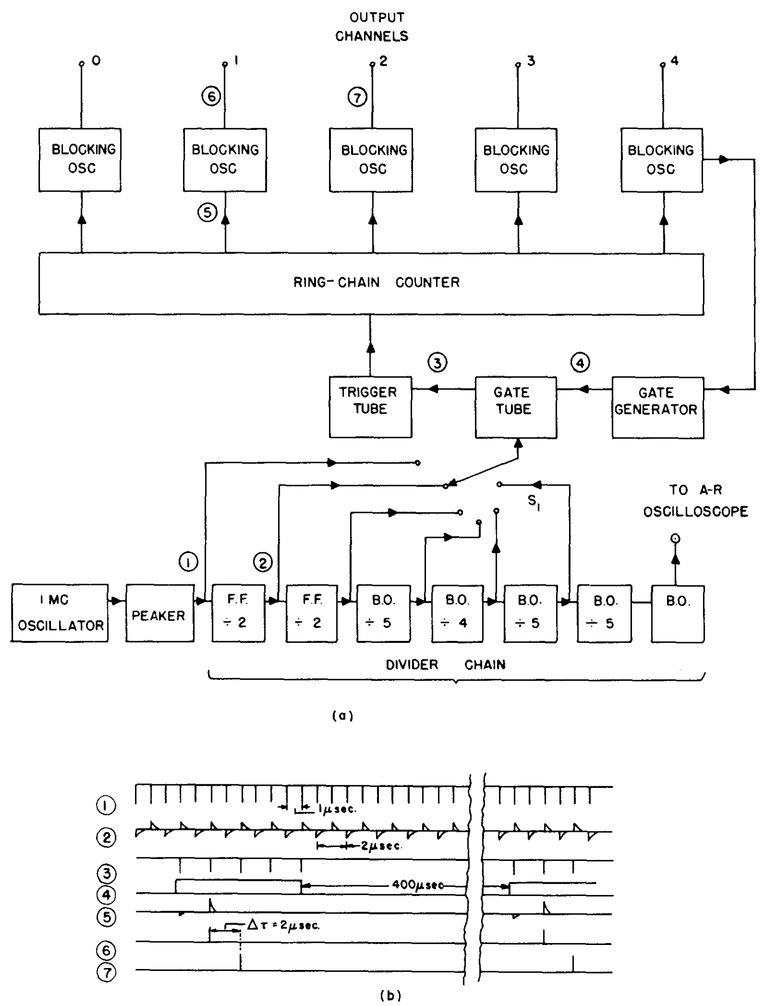

Fig. IX-1

(a) Block diagram of sampling pulse generator.
(b) Waveforms at numbered points of (a).

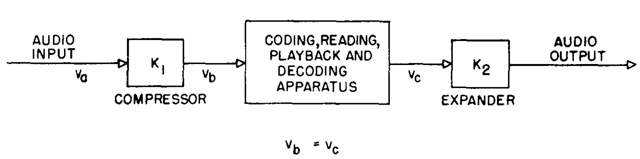

Fig. IX-2

Compressor and expander scheme.

subject to the condition that

$$K_3(v_a) = K_1(v_a) \cdot K_2\left[v_b(v_a)\right] = 1$$

for all values of v_a, while $v_b = v_c$. While there is still a possibility of using Logaten for compression, an expander having a transfer characteristic

$$K_2\left[v_b(v_a)\right] = \frac{1}{K_1(v_a)}$$

will have to be developed.

Note: In Fig. VIII-9, p. 49, Quarterly Progress Report, April 15, 1951, the two 10-k resistors at the grids of the 5687 tube should <u>not</u> be connected to the 800-ohm resistor.

J. B. Wiesner, L. Dolansky

3. Speech Probability Distributions after Filtering

Measurements of the first probability density distributions of speech after filtering were obtained with a probability analyzer. The sound pressure waves of continuous speech, obtained from a male voice recorded in an anechoic chamber, were used to obtain these measurements. The filters used in the experiments were an optical filter, i.e. a simple RLC series filter, and a pre-emphasis circuit as used in f-m transmission. The purpose of taking measurements was to observe the effects the filters had upon the first probability density distribution.

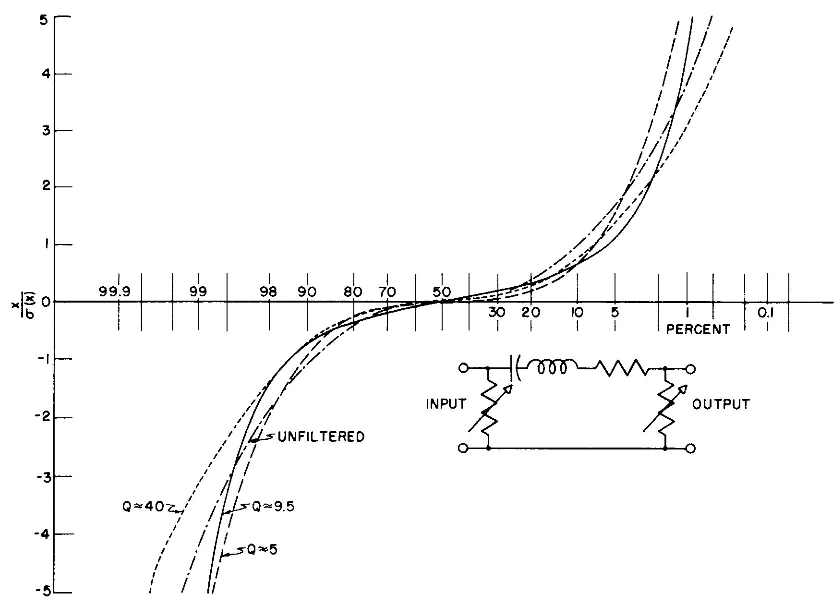

Fig. IX-3
Percentage (of time) that the value $\frac{x}{\sigma(x)}$ is exceeded (for male voice (BME) unfiltered and filtered through RLC filter tuned to 1000 cps.

The effects of varying the Q or bandwidth of the series RLC circuit upon the probability distribution functions is shown in Fig. IX-3. The first probability density distributions are plotted on arithmetic probability graph paper which has the property of showing a Gaussian distribution as a straight line. For purposes of comparison the amplitudes, x, have been normalized with respect to their respective rms values, σ. The measurements show that as the Q is increased, or the bandwidth decreased, the first probability density distribution tends toward a Gaussian distribution in the region of small amplitudes. These measurements are in theoretical agreement with the results predicted by the Central Limit Theorem of probability.

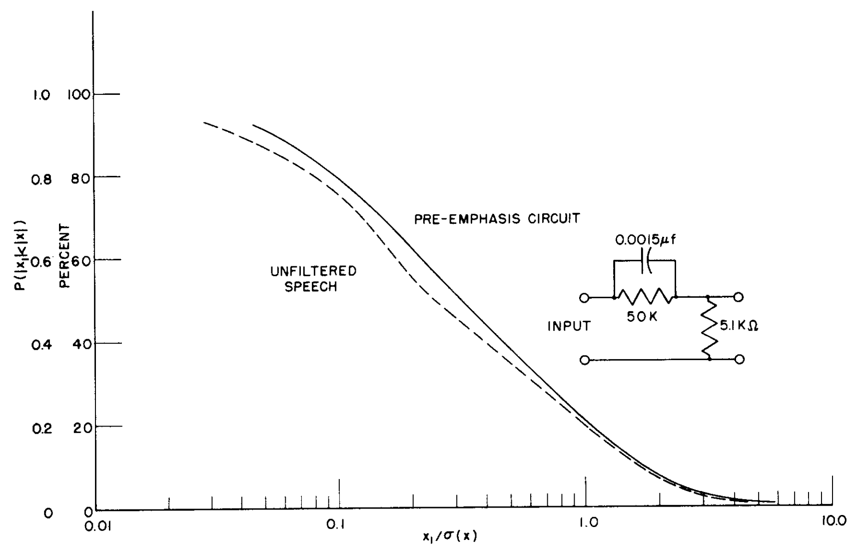

Fig. IX-4

Probability that $|x_1|$ is less than $|x|$ or percentage that $\frac{|x|}{\sigma(x)}$ exceeds $\frac{|x_1|}{\sigma(x)}$ (for male voice (BME) unfiltered and through pre-emphasis circuit).

The effect of the pre-emphasis circuit, which is essentially an RC differentiating circuit with a 75 μsec time constant, on the first probability density distribution is shown in Fig. IX-4. The curves show the probability that the filtered and unfiltered speech amplitudes exceed a given amplitude normalized with respect to the rms value of the respective speech waves. It should be noted that above 6000 cps the amplitude response of the pre-emphasis circuit used in this experiment was changed from the conventional 6-db-per-octave rise in order to minimize the effects of noise. These curves indicate that pre-emphasis has greater effects upon the smaller amplitudes than on the higher amplitudes. These results may be explained as follows. The large amplitude sounds are primarily the voiced sounds which are mainly low-frequency phenomena, while the small amplitude sounds are primarily the unvoiced sounds which are mainly high-frequency phenomena. However, at high frequencies the sound pressure spectrum of speech waves drops off more rapidly than the pre-emphasis circuit will boost these frequencies. Thus, while the high-frequency small amplitudes of speech are amplified, they do not appreciably affect the large amplitude distributions.

W. B. Davenport, Jr., B. M. Eisenstadt

4. Visual Pattern Noise

The situations in which we have a space-varying signal (pattern) in presence of a space-varying noise are of interest to the study of visual perception.

It was determined to study such a time-space noise formed of luminous dots which appear at random points of a cathode tube screen at different times. Two methods of generation have been tried. In both, the X axis and the Y axis of a cathode ray oscilloscope were driven from two incoherent noise sources (J. C. R. Licklider, E. Dzendolet: Oscillographic Scatter Plots Illustrating Various Degrees of Correlation, Science, 107, 121-124, Jan. 30, 1948), and the luminous spot made to move along random paths. In the first method this random path was illuminated at periodic intervals of time by pulses on the Z axis, which was biased-off at other times. Difficulties arose from the fact that only low-frequency noise could be used if well-defined dots (not streaks) were to be obtained (10 μsec pulses: upper limit of noise around 500 cps). This meant that use of many dots would completely identify paths on the screen, and the picture would not have features of randomness but would exhibit space coherence.

The second scheme tried and found much more satisfactory was to sample two wide band noises and hold sampled values, so that the dot stays at a fixed position on the screen for a time. In this fashion one can obtain a satisfactory picture even with pulses of moderate width.

Experiments are planned to study pattern perception in the presence of this noise. The patterns will also be dotted. A dual beam scope will be used. The signal will be applied to one beam and the noise to the other.

O. H. Straus, J. J. Bussgang, B. Howland

C. HUMAN COMMUNICATION SYSTEMS

Prof. R. D. Luce	F. D. Barrett	D. G. Senft
J. Macy, Jr.	J. B. Flannery	P. F. Thorlakson

The work of this group will be reported at a later date.

D. SLIGHTLY LOSSY NETWORKS

Prof. E. A. Guillemin
Prof. J. G. Linvill

The design of a band-pass filter with a one-percent bandwidth centered at one megacycle has been completed. The new features of the design are (a) its use of a few high-Q elements (crystals) in combination with conventional elements to achieve substantial rejection with a small number of elements; and (b) the elimination of terminal loading. The ohmic loading of the network is done principally by the incidental dissipation of coils. As previously reported, the filter consists of three sections isolated by amplifiers. The rejection beyond 10 kc/sec from the band center exceeds 50 db. The required components include coils with Q's of 150 and two quartz crystals. Construction has been delayed by difficulty in obtaining suitable crystals. Filter crystals for a range of frequency near 100 kc are easily obtainable, but crystals cut for the 1-Mc filter by a local supplier have not been satisfactory. For this application, the critical characteristic of the crystal is a sufficient spread between its resonant and antiresonant frequencies. An insufficient frequency difference results in an over-all transmission loss in the filter and makes the alignment of elements more difficult. The difference between these frequencies is inversely proportional to the ratio of the capacitances of the crystal, C_o/C_1 (see Fig. IX-5). The minimum ratios of the capacitances for x-cut quartz is 140, and for the AT cut is about 250. However, parasitic capacitance of the crystal mounting deteriorates the capacitance ratio and this fact is apparently the reason for the difficulty encountered. At present, attention is directed toward obtaining more suitable crystals; the capacitance ratio designed for is 300. Reports of others interested in crystal filters indicate that this requirement is not excessive.

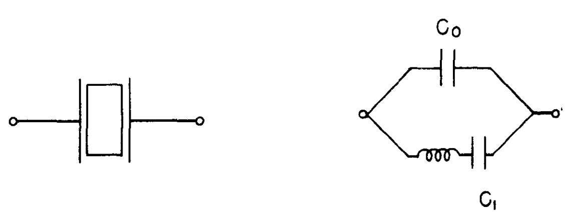

Fig. IX-5 Quartz crystal and its first-mode equivalent circuit.

J. G. Linvill

E. PARALLEL-CHAIN AMPLIFIER

Prof. E. A. Guillemin
Prof. J. G. Linvill
R. K. Bennett

The gain per stage of a conventional cascaded amplifier with a given bandwidth is theoretically limited to $(2g_m)/(\omega_o C)$, where the interstages are one-terminal-pair networks, and to $(\pi^2 g_m)/(2\omega_o C)$, where they are two-terminal-pair networks. C is the total shunt capacitance per stage and ω_o the bandwidth in radians per second. For very large bandwidths, where this limit is close to or less than unity, the Percival distributed amplifier and the parallel-chain amplifier investigated recently have been designed to circumvent this limitation. Both theoretically provide an unlimited gain over an unlimited bandwidth. (High-frequency effects are not considered.) The distributed amplifier has been successfully built, with available commercial models, and has the advantage that its frequency characteristics are little affected by tube aging or even failure. The parallel-chain amplifier is still in the development stage but it promises a better tube economy and a more flexible design technique. Its principle is to divide the band into two or more smaller parts, then amplify each of these parts separately with a stagger-tuned chain, and finally combine the outputs in a common terminus.

The construction of a parallel-chain amplifier was recently started in this Laboratory in the direction of building a 0–300 Mc, two-chain amplifier. However, the orderly investigation of the basic problems of parallel-chain amplification was hampered by encountering such high-frequency effects as shielding, cathode-lead inductance loading, and transit-time loading. It should be noted that these high-frequency effects are of a different nature from the fundamental limitation of gain-bandwidth product imposed by shunt capacitance; that limitation is independent of the frequency of the band center.

The study of wide-band amplification was isolated from the high-frequency effects by building a frequency-scaled model at 2 Mc. This model proved very successful, verifying the idea of parallel-chain amplification. Much was learned about the design and alignment of stagger-tuned amplifier chains and the conditions for their satisfactory paralleling. These results together with theoretical considerations may be found in an M.I.T. Master's thesis, 1951, by R. K. Bennett.

Having obtained satisfactory results from the 2-Mc model, work is now being resumed on the 0–300 Mc full-scale model.

R. K. Bennett

F. EXPERIMENTAL APPROXIMATION AND NETWORK ALIGNMENT

Prof. E. A. Guillemin
Prof. J. G. Linvill
W. I. Wells

The experimental solution of the approximation problem, whether one employs a potential analog (1, 2) or a model network (3), is always essentially a method of successive approximation. One makes a succession of trial solutions by setting up on the analog potential distributions corresponding to trial system functions or with a model network by obtaining frequency or time-scaled replicas of the frequency or time response function approximating the desired response function. Successive adjustments are chosen to effect an improvement in the quality of approximation. The successive adjustments are in the form of changes in pole or zero locations or, what is equivalent, changes in element values of the model network. The choice of successive adjustments is the key problem of the whole procedure. It is complicated by the fact that any pole or zero motion (or the change of any element) ordinarily affects the whole response function. Hence the prescription of a combination of adjustments to effect rapid improvement in the approximation ordinarily entails simultaneous consideration of all possible motions. One cannot simply move single poles or zeros or adjust single elements in sequence, choosing at each adjustment the single change which results in the greatest improvement.

The problem of production alignment of networks which have approximately correct element sizes plus adjustable trimmers is seen to be a simpler problem of the same sort. One adjusts element values just as in the application of a model network in the approximation problem, but here the ideal being sought is the response of a perfectly aligned network rather than a curve of specified characteristics. Here again the major obstacle is the prescription of a combination of adjustments, each of which affects the whole of the response characteristic.

1. General Description of Suggested Technique

An experimental method has been devised through which one can prescribe the combination of element changes in a misaligned network to bring it into alignment. In a similar manner one can prescribe changes in elements of a model network to approach most rapidly a desired response characteristic. In the latter case one solves the approximation problem in a series of small steps, each leading most rapidly to the desired characteristic. The whole technique is basically founded on these facts:

(a) Changes in the magnitude or phase characteristic or in the transient response are approximately linear functions of changes in element values for small changes.

(b) For small changes the rate of change of response characteristics with any element is nearly independent of other small element changes. This means, for instance, that a given small change in element 1 effects the same change in the characteristic before or after small changes in other elements.

Fortunately the experimental technique requires negligible computation. The procedure consists of two parts. First, starting with a set of identical adjustable networks, one adjusts them in such a way that differences in their responses can be used to analyze the misadjustment of the test network. Second, one analyzes the test network using the similar networks adjusted in the first part. In doing so he determines (as will be seen) the changes in the test network to bring it into alignment or to make its response more nearly that specified. In the alignment of networks, the first part of the procedure is done only once and the analyzer so adjusted may be used on any number of networks.

In the approximation problem both steps of the procedure are used at each stage of the successive approximation. This experimental method is an experimental counterpart of an analytical method described in Technical Report No. 145.

2. Details of the Technique

To facilitate description of the suggested technique, attention will be restricted to the problem of aligning networks to give a desired transient response to impulses. The only difference occasioned if one is interested in frequency characteristics is the substitution of a sweep oscillator for a pulse generator and the insertion of detectors in the output circuits. If the approximation problem rather than the alignment problem is to be solved, one uses a function generator instead of a correctly aligned network. In the approximation problem one starts the adjustment procedure from a rough approximation of the desired complexity (desired number of elements in network or number of poles and zeros in the function) which is to be optimized.

The elements needed in the alignment procedure are represented symbolically in Fig. IX-6, where their individual operations are identified.

The elements defined in Fig. IX-6 connected as shown in Fig. IX-7 give the desired change in the response and the change in response resulting from a change in element 1. By doing the same for all elements one has all the data on which a prescription of network adjustment should be based. The prescription amounts to solving for C_i --- C_n in

$$\Delta f_{des} = C_1 \frac{\partial f}{\partial E_1} + C_2 \frac{\partial f}{\partial E_2} + \cdots C_n \frac{\partial f}{\partial E_n} \quad . \tag{1}$$

If all elements in the network being aligned are adjustable, there is an exact solution of Eq. 1. If only a fraction of the elements are adjustable, Eq. 1 cannot be solved exactly. However, C's can be chosen such that the average error squared over the period of the pulse generator is a minimum. The mathematical technique to obtain this result is well known; one forms normal-orthogonal functions linearly dependent with the derivative functions and expands Δf_{des} in terms of them (4). The normal-orthogonal functions are

$$\left.\begin{aligned}
f_{N1} &= a_{11} \frac{\partial f}{\partial E_1} \\
f_{N2} &= a_{21} \frac{\partial f}{\partial E_1} + a_{22} \frac{\partial f}{\partial E_2} = b_{21} f_{N1} + a_{22} \frac{\partial f}{\partial E_2} \\
f_{Nn} &= a_{n1} \frac{\partial f}{\partial E_1} + \cdots a_{nn} \frac{\partial f}{\partial E_n} = b_{n1} f_{N1} + \cdots + b_{nn-1} f_{N-1} + a_{nn} \frac{\partial f}{\partial E_n} \\
\text{where } \int_0^T f_{Ni} f_{Nj}\, dt &= 0 \text{ if } i \neq j \\
&= 1 \text{ if } i = j \quad .
\end{aligned}\right\} \tag{2}$$

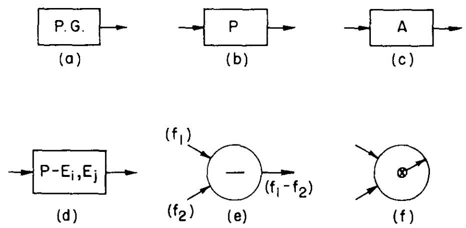

Fig. IX-6

Elements used in alignment device. (a) Pulse generator; generates periodic impulses. (b) Perfectly aligned network. (c) Network being aligned. (d) Network perfectly aligned except for elements E_i and E_j. (e) Subtractor-output; displays difference of inputs. (f) Averaging-muliplier; a wattmeter-like device indicating average product of inputs.

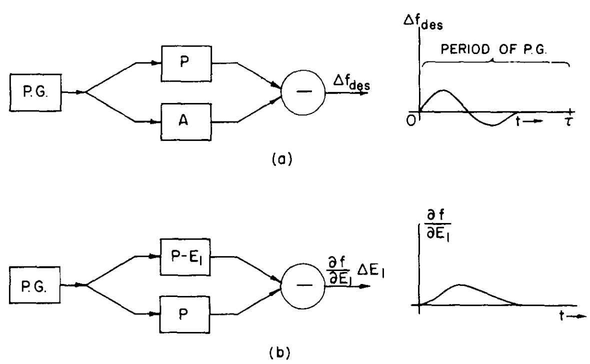

Fig. IX-7

Generation of differences to be used in alignment. (a) Generation of desired change in response of network aligned. (b) Generation of rate of change of response with change in element 1.

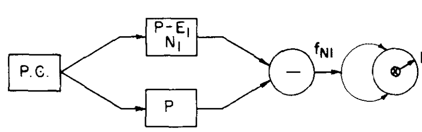

Fig. IX-8

Generation of f_{N1}.

One selects K's to approximately fulfill

$$\Delta f_{des} \cong K_1 f_{N1} + K_2 f_{N2} + \cdots K_n f_{Nn} \quad . \tag{3}$$

The optimum K's are simply

$$K_i = \int_0^T \Delta f_{des} f_{Ni} \, dt \quad . \tag{4}$$

One readily evaluates the optimum C's by

$$\left.\begin{aligned} C_1 &= a_{11}(K_1 + b_{21} K_2 + b_{31} K_3 + \cdots + b_{n1} K_n) \\ C_2 &= a_{22}(K_2 + b_{32} K_3 + \cdots b_{n2} K_n) \\ C_{n-1} &= a_{n-1\, n-1} K_{n-1} + (b_{nn-1} K_{n-1} + (b_{nn-1} a_{n-1\, n-1}) K_n \\ C_n &= a_{nn} K_n \quad . \end{aligned}\right\} \tag{5}$$

It proves to be simple to form the normal-orthogonal functions experimentally. Alignment of the network corresponds to making adjustments such that all K's become zero. This too can be done easily.

To produce the first of the set of normal-orthogonal functions one merely uses the difference in the output of a perfectly aligned network and a network perfectly aligned except for element 1, which is misaligned until the averaging-multiplier in Fig. IX-8 reads unity.

To generate the second of the set of normal-orthogonal functions one uses a network with perfect alignment except for elements 1 and 2. Considering Eqs. 2 one observes

$$\int_0^T f_{N1} f_{N2} \, df = 0 = \int_0^T b_{21} f_{N1} f_{N1} \, dt + \int_0^T a_{22} \frac{\partial f}{\partial E_2} f_{N1} \, dt \tag{6}$$

or

$$\frac{-b_{21}}{a_{22}} = \int_0^T \frac{\partial f}{\partial E_2} f_{N1} \, dt \quad . \tag{7}$$

Hence the generation and checking of f_{N_T} is done in the three steps shown in Fig. IX-9. In step a, using a unit misalignment of element 2, one determines the correct proportion of misalignment of elements 1 and 2 in the network to form f_{N2}. In step b, the magnitude of the misalignments, keeping the correct ratio, is set by bringing the averaging-multiplier to a unit reading. Step c checks the orthogonality of the functions generated. One should observe that only one pulse generator and one perfectly-aligned

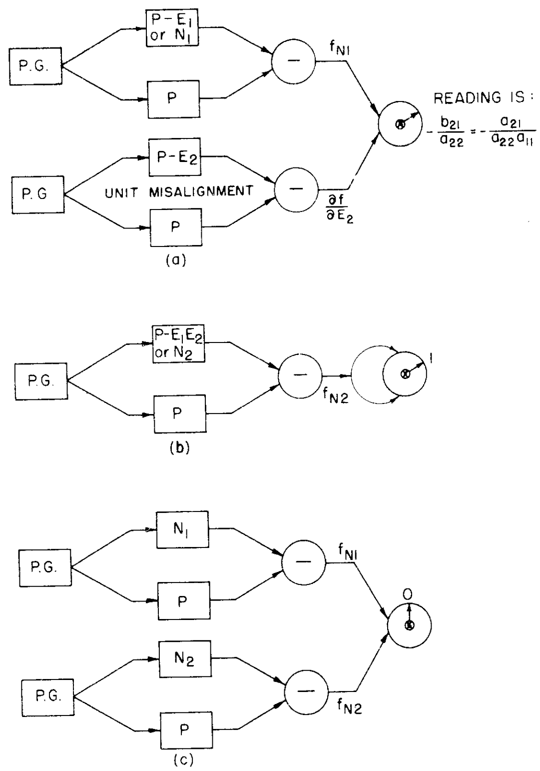

Fig. IX-9

Generation and check of f_{N2}. (a) Evaluation of A_{21}/A_{22}. (b) Generation of f_{N2}. (c) Checking orthogonality of f_{N1} and f_{N2}.

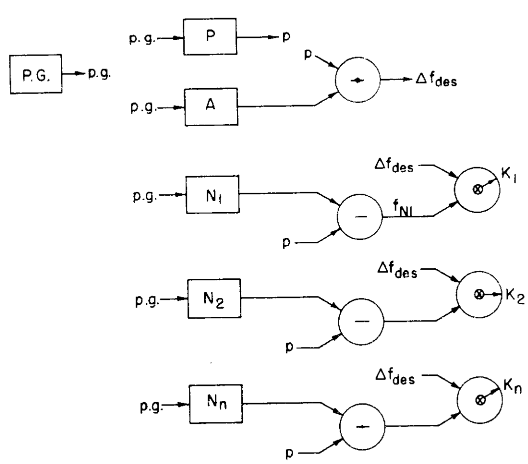

Fig. IX-10

Schematic of connections for alignment of network A.

network are really needed; their outputs may be distributed to the required points.

The generation of the remaining normal-orthogonal functions follows precisely the same pattern. The only distinction is that the step shown in Fig. IX-9a must be expanded to evaluate a larger number of ratios of element misalignments maintained in the next step of the procedure which adjusts the magnitude of the misalignments.

Once the generation of the normal-orthogonal functions is complete, the process of adjustment of the test network is simple, Fig. IX-10. One can evaluate the K's of Eq. 4 or he can adjust K_n to zero with element n in the test network, A_{n-1} to zero with element n-1 and so on. This set of n adjustments completes the network alignment.

3. Experimental Work

For alignment or approximation one requires two differencing circuits and a single averaging-multiplier. Circuits which do this kind of thing are already available in Macnee's differential analyzer. It is planned to use these first before determining if circuits should be specially made for the alignment device.

The success of such a device in solving the approximation problem is largely determined by the simplicity, convenience and accuracy of model networks. S. D. Lerner (3) designed model networks which were useful in a simpler attack on the approximation problem in the time domain. However, since differences in responses are to be measures of misalignment it seems desirable to expend effort at the outset obtaining the most convenient type of model network. Work on this aspect of the device is currently active as the thesis problem of W. I. Wells and preliminary results are promising.

References

1. J. G. Linvill: Master's Thesis, Dept. of Electrical Engineering, M.I.T. 1945
2. R. E. Scott: An Analog Device for Solving the Approximation Problem in Network Synthesis, Technical Report No. 137, Research Laboratory of Electronics, M.I.T. June, 1950
3. S. D. Lerner: Master's Thesis, Dept. of Electrical Engineering, M.I.T. 1950
4. J. G. Linvill: The Selection of Network Functions to Approximate Prescribed Frequency Characteristics, Technical Report No. 145, Research Laboratory of Electronics, March, 1950

J. G. Linvill

G. NEW METHODS OF NETWORK SYNTHESIS

Prof. E. A. Guillemin
L. Weinberg

New methods of synthesizing networks have been worked out. These methods realize a network in lattice form for the following transfer functions with terminations as indicated:

a) $K = E_2/E_1$, for an open-circuited output (see Fig. IX-11).

b) $Z_{12} = E_2/I_1$, for an open-circuited output (see Fig. IX-11).

c) $Y_{12} = I_2/E_1$, for the termination $Z = R$ or $Z = R + Ls$ (see Fig. IX-12).

d) $Z_{12} = E_2/I_1$, for the termination $Z = R$ or $Z = 1/(G + Cs)$ (see Fig. IX-12).

The advantages claimed for these RLC-lattices are that they realize an unrestricted transfer function without mutual inductance and with lossy coils, i.e. each inductance has a series resistance associated with it so that the network is physically realizable; and, furthermore, the series and shunt arms are of such a simple form that in a large number of cases the lattice can be reduced to an unbalanced form.

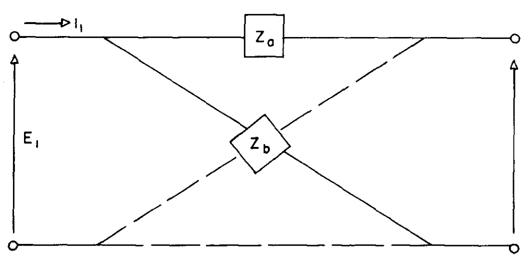

Fig. IX-11 Open-circuited lattice.

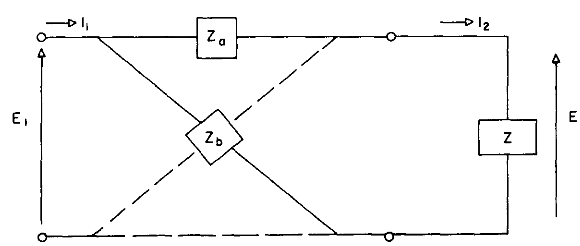

Fig. IX-12 Lattice with termination Z.

In addition, methods have been worked out for directly synthesizing some of the unbalanced forms of network (discussed in Quarterly Progress Report, April 15, 1951) without proceeding through the intermediary of the lattice structure.

These procedures, as well as the lattice synthesis procedure, will be explained in detail in forthcoming Technical Report No. 201.

L. Weinberg

H. TRANSIENT PROBLEMS

Prof. E. A. Guillemin W. H. Kautz
Dr. M. V. Cerrillo H. M. Lucal

1. Basic Existence Theorems

In the last Quarterly Progress Report, April 15, 1951, we reported an existence theorem on the electrical reproduction of real bounded functions. This theorem states that there can be found a set of finite sinusoidal currents whose sum reproduces the given bounded function in an arbitrary interval and inside a set of tolerances and mode of convergence. Alternately, the theorem also states that it is possible to find a passive electrical network, having a finite number of elements, such that the output reproduces the given function, under a given set of tolerances and mode of convergence, when the network is excited by a step function. A similar situation can be worked out for impulse excitation. The basic mathematical ideas involved rest upon "Window Functions".

A group of other basic existence theorems has been developed. Among them, one plays a particular role because it deals with the existence of solutions for the following important problem:

Let $f(t)$ be a given bounded single-value real function. Let us consider $f(t)$ as the driving force of a four-terminal network. Let $g(t)$ be the corresponding network output. Now, the theorem goes along the following general lines:

" Let $f(t)$ and $h(t)$ be given, respectively, as the input and desired output function of a four-terminal network. Then, it is possible to find a passive network such that its output function $g(t)$ approaches the required $h(t)$ under a given set of tolerances, for almost every value of t, except in an enumerable set of open disjointed small intervals, whose width may be as small as desired, but which can never be removed."

A suitable comprehensive explanation and illustration of the exact meaning of the theorem cannot be given in the compact form required in the progress report. It will appear in forthcoming technical reports (No. 55 group).

During the last three months all work has been devoted to the preparation of manuscripts of several reports.

M. V. Cerrillo

2. Network Synthesis for Prescribed Transient Behavior

The transient synthesis problem (1) has now been completely solved in the general case. The given input, desired output, and the tolerances on the output may be prescribed analytically, graphically, or as sets of equispaced ordinates as functions of time. The tolerances may be specified precisely as an envelope of error versus time, or merely qualitatively, or may remain unspecified over one or more intervals of time; any combination of these criteria is also acceptable. The amount of computational effort required for the solution of any problem falling in this category is small enough to encourage its direct application to engineering problems. The only type of signal which must be excluded from consideration is a rapidly oscillating aperiodic signal with changing frequency. (Although the method is applicable here, the amount of work required for a solution is too great to make the technique practical.)

The principal advantage of the proposed solution, other than its generality, is felt to lie along the following lines. First, the economy of network elements is good; that is, the number of elements is close to the minimum number that is theoretically required by a given input, output, and set of output tolerances. Second, the final time-domain approximation to the desired output signal need not be computed and plotted to find the approximation error. Improved means of error estimation allow the error to be determined to any desired degree of accuracy in the process of finding the approximation. This improvement renders practical the design of complicated networks (ten to twenty poles, say), should these be necessary; in most cases, in fact, the determination of the error becomes easier as the network complexity is increased. The third advantage is that it is possible to introduce into the method of solution a number of constraints which are likely to arise in practice. Chief among these are

1. The system may be constrained to be an RC-type network (passive or, if active, without feedback).

2. Any number of zeros and poles of the system may be fixed in advance; for example, part of the system may be fixed, or, for wideband amplifiers, any number of zeros may be moved to infinity.

3. The transient output, the transient response to any test signal, the frequency and phase characteristics of the system, and/or any of the derivatives of these may be forced to have specified values at arbitrary values of time and frequency. For example, for wideband amplifiers, the zero-frequency gain and fixed g_m/C-ratio may be prescribed for a given number of stages by fixing the final value and initial value and derivatives of the step response.

4. An arbitrary delay may be introduced to get the best output response, or any fixed delay may be imposed.

The signals applied to and obtained from the system are assumed to be aperiodic, although the technique of solution proposed is directly applicable to problems involving random and periodic signals. The approach used by Wiener and extended by Lee and by Costas has been generalized somewhat to permit the introduction of constraints and the estimation of the approximation error. For random signals, the only measurement of the goodness of the system is, as with Wiener's method, the final mean squared error; output signal tolerances cannot be specified directly. The practical value of this criterion is questionable, but there seems to be no alternative at hand.

Previous solutions to the transient synthesis problem have been characterized primarily by their lack of generality, but also by the fact that they are intricate and do not always lead to practical, usable networks. In overcoming these disadvantages, several elegant and otherwise desirable mathematical solutions to the approximation problem were discarded on the grounds that they were either too intricate or did not lend themselves to any means of error estimation. This was particularly true of some frequency-domain approximation methods investigated previously for particular problems by various authors.

While the method of solution cannot be described in detail here, it may be outlined briefly. For analytically given signals of the aperiodic type, a set of operations called "preliminary simplifications" is first performed on the signals. This step includes normalization, extraction of carriers and multiplying and additive exponentials from the desired response (these may be reinserted after the approximation is made without violating the realizability criteria), and the introduction of the desired error weighting function. The second step consists of a representation of the resulting transient in terms of one or more of a set of "constitutent transients"; the transients of this set have been chosen so that their frequency transforms are capable of being developed in rapidly convergent expansions over certain portions of the frequency plane. These constituent transients are, therefore, easily approximatable "building-block" functions out of which an arbitrary transient may be composed. They offer excellent control over the approximation error in the time domain. The third step in the procedure is that of redistributing the error by time-domain operations, and of introducing whichever of the above-listed constraints are prescribed. The last step is the formation of the over-all system-function and the synthesis of the network.

Fortunately, practically all of these operations with analytically representable signals have their analogs in the mathematics of empirical functions. Differential equations, integral equations, and the Laplace transform, for example, now become difference equations, simultaneous linear equations, and the Mellin transform. Consequently, the above-described techniques for manipulating analytically given signals may be extended (in theory, at least) to signals given graphically or as a set of discrete values (and, of course, to any analytical signal too complex to undergo the necessary operations above). The art of the interpolation of empirical functions is well developed, and many techniques are directly applicable to the present problem.

Present work is concentrated along three lines. First, some examples are being worked out to illustrate the techniques developed. The most practical of these is the pulse amplifier problem, which is concerned with obtaining the minimum possible rise time in the step response of a cascade amplifier of given gain, g_m/C-ratio, number of stages, and overshoot. Preliminary results look promising. Second, an attempt is being made to introduce some new constraints in addition to those itemized above. Most important of these is the restriction that the system be a passive two-terminal-pair

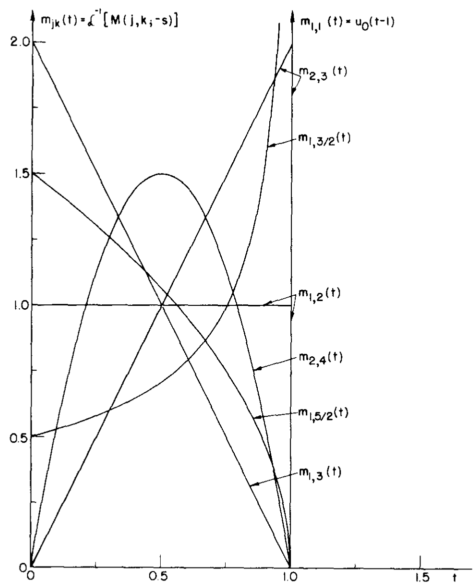

Fig. IX-13
A few of the rapid-variation group of constituent transients.

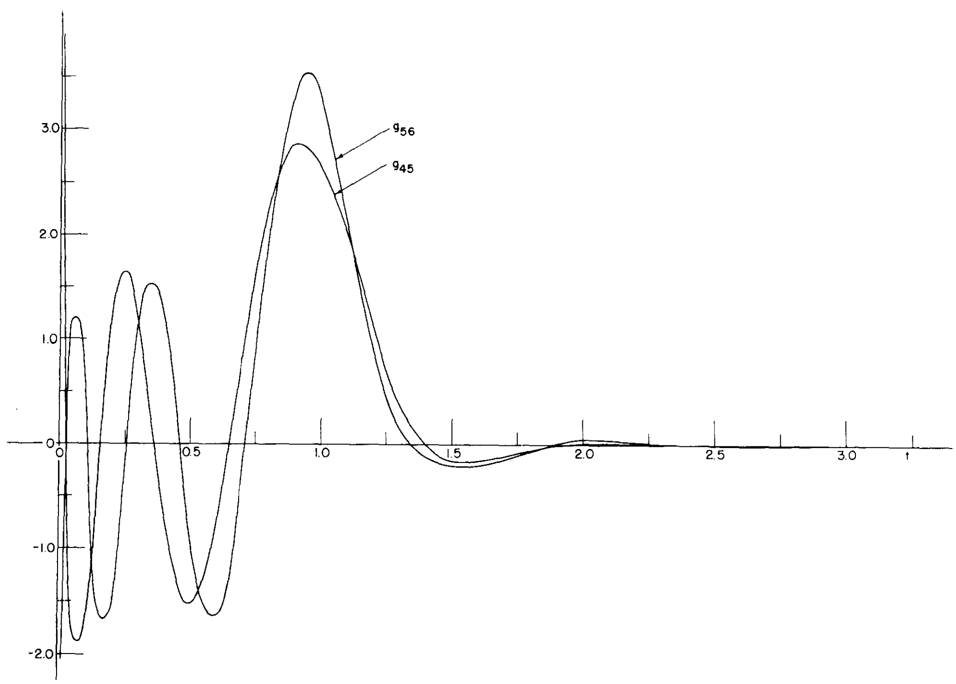

Fig. IX-14
Realizable approximations to a delayed impulse
$\left[m_{1,1}(t) = u_o(t-1)\right]$, valid for large t.

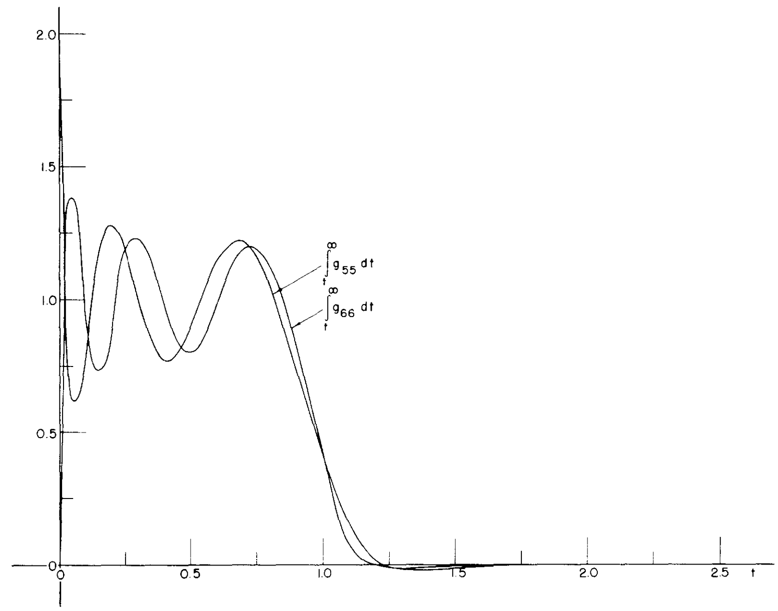

Fig. IX-15

Realizable approximations to the unit pulse function $\left[m_{1,2}(t) = u_{-1}(t) - u_{-1}(t-1)\right]$, valid for large t.

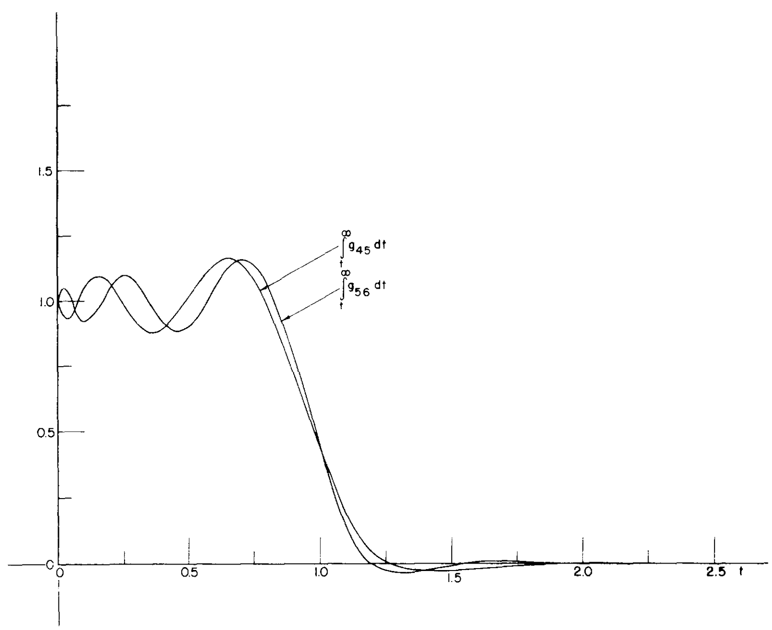

Fig. IX-16

Realizable approximations to the unit pulse function $\left[m_{1,2}(t) = u_{-1}(t) - u_{-1}(t-1)\right]$, valid for large t.

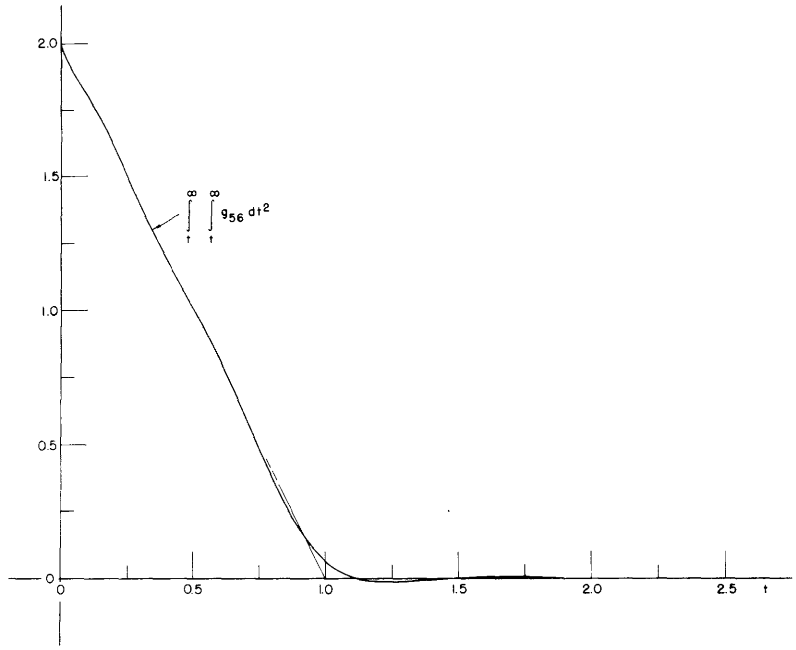

Fig. IX-17

Realizable approximation to $m_{1,3}(t)$, valid for large t.

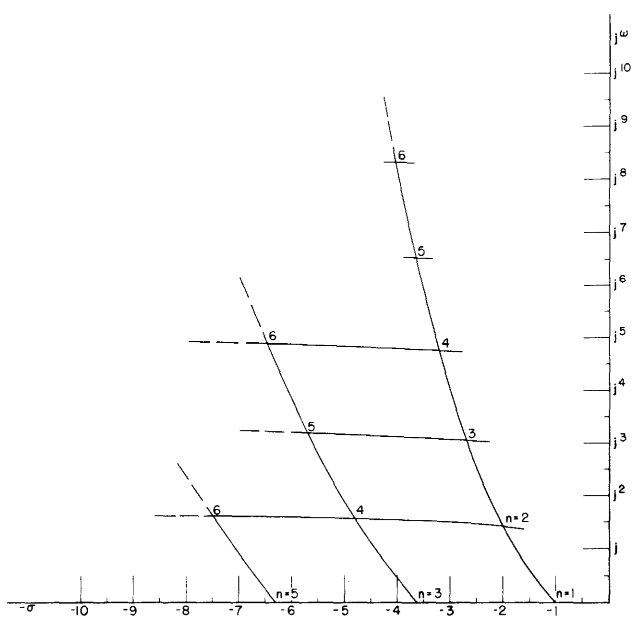

Fig. IX-18

Pole locations for $G_{n-1,\,n}(s)$ entries in the Padé table of e^{-s} about $s = 0$. Only the second quadrant of the s plane is shown, since pole is accompanied by its conjugate.

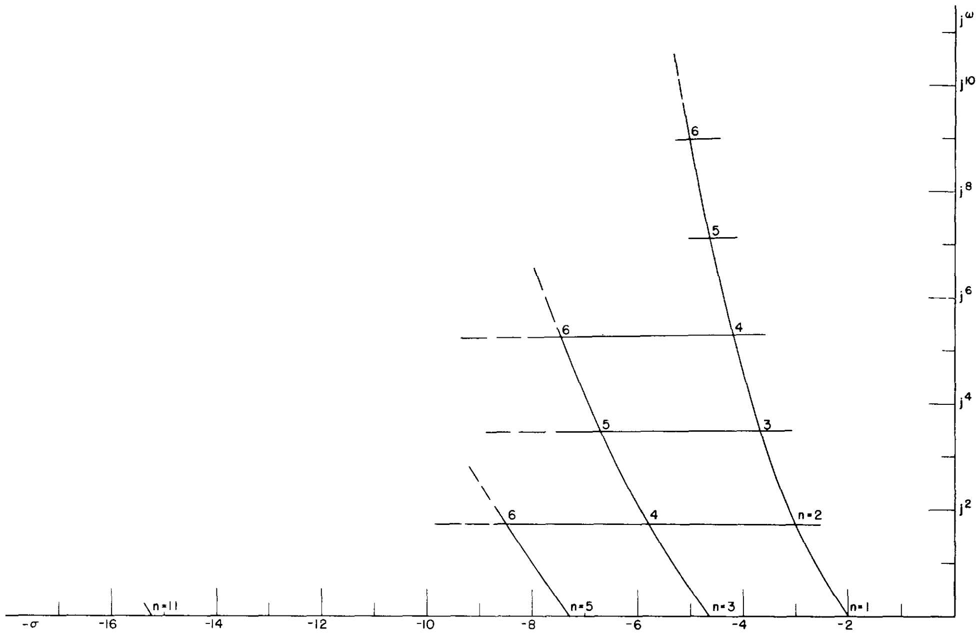

Fig. IX-19

Pole locations for $G_{n,n}(s)$ entries in the Padé table of e^{-s} about $s = 0$. The zeros occur at corresponding points in the right half-plane.

network. Third, preparations are being made for an experiment in which networks generating certain of the constituent transients will be constructed. The purposes of this experiment are (1) to verify that the actual network has the calculated theoretical response, and (2) to determine whether the approximation error can be further reduced by manipulation of the physical network in some way.

Complete details of the methods referred to will be presented in the forthcoming Technical Report No. 209.

Some of the more important constituent transients with a few of their time-domain approximations are shown in the accompanying figures. Figure IX-13 illustrates the most important of the "rapid transition" group of constituent transients—those used to represent impulses and discontinuities in signals and their derivatives. Figures IX-14 through IX-17 show a few of the realizable approximations to some transients of this group, all valid for large values of time. The extremely rapid convergence for large t should be noted. Similar expansions may be made for small or for intermediate values of time, or for the entire time scale simultaneously. The indices μ and ν in the approximation $g_{\mu\nu}(t)$ indicate the number of finite zeros and poles, respectively, in the approximate transform, $G_{\mu\nu}(s)$. The intersections of the curves in Figs. IX-18 and IX-19 indicate the positions of these poles for $G_{n-1,n}$ and $G_{n,n}$, respectively. All of these approximations for large time have been derived from frequency domain expansions about $s = 0$, and represent entries in the Padé table (2) for e^{-s}, the transform of a delayed impulse.

References

1. Quarterly Progress Report, Research Laboratory of Electronics, M.I.T. p. 75, April 15, 1951

2. O. Perron: Die Lehre von den Kettenbrüchen, pp. 420-67, Teubner, Leipzig, 1929

W. H. Kautz