X. ANALOG COMPUTER RESEARCH

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A. THE OPERATION OF PRESENT COMPUTERS

1. Integral Equation Solver

During the past quarter, the machine has been essentially completed and a number of test problems have been run.

The central mathematical problem is that of solving for α in the operational equation $P\alpha + \beta = 0$, where P is a given linear operator defined in function space, β is a given functional element, and α is the unknown functional element. This equation characterizes integral equations and boundary value problems in ordinary and partial differential equations.

The machine operates on the principle of successive approximations. This method is significant for finding particular numerical solutions for a vector-space interpolation of the above equation. The variational principle is that of associating with the equation $P\alpha + \beta = 0$ a scalar-valued functional $W(\alpha)$ that attains its extreme when the equation $P\alpha + \beta = 0$ is satisfied. If the equation has a unique solution, the functional $W(\alpha)$ has a unique extremum that is either a maximum or a minimum.

The function $W(\alpha)$ defines a surface in function space. Hence the problem of solving the equation $P\alpha + \beta = 0$ may be regarded as the problem of constructing trajectories $\alpha(\tau)$ in function space which define paths $W\left[\alpha(\tau)\right]$ on the W surface leading to the appropriate mountain peak or valley: The manifold possibilities for effecting these ascents or descents correspond to the many different methods of successive approximations. Relaxation methods and many classical iterative procedures are described by particular types of trajectories.

The basic method which is implemented by the machine is the method of vector step descent. The function $W(\alpha)$ is intimately related to the physical complex described by the equation $P\alpha + \beta = 0$. It is in the nature of a performance index and the solution of the equation $P\alpha + \beta = 0$ by successive approximations corresponds to adjusting this index to an extremal value. An initial guess α_0 is made for the solution. A sequence of approximate solutions $\alpha_0, \alpha_1, \ldots, \alpha_2, \ldots$ is then constructed with the aim of finding an extremal of $W(\alpha)$. The experience and intuition of the particular person for whom the problem exists can be and ought to be exploited in choosing the trajectory of successive approximations. The writer concludes that the successive approximation method is only at its maximum effectiveness when the person for whom the problem exists remains immersed in the procedure of solution. This conclusion provides the justification for designing a special purpose machine to implement these methods.

The computer uses digital inputs and outputs in the form of paper tapes punched in binary code. The internal operations are in part digital, in part analog. The whole machine is an experiment in hybrid digital-analog design. An example of the mathematical function performed by the machine is the transformation of a 40-vector by a 40×40 matrix. A single operation of this type requires 40 sec.

The machine is useful for implementing the method of vector step descent. In addition it is effective for evaluating integral transformations such as Fourier and convolution integrals. It is hoped that it will focus some engineering attention on methods of successive approximation.

A technical report is in preparation at the present time covering the appropriate theory, the design of the machine, and several examples of its use.

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B. THE USE OF COMPUTING ELEMENTS FOR NONCOMPUTATIONAL PROBLEMS

1. The Use of Computing Elements in Nonlinear Time-Domain Filters and Multiplexers

The first phase of the program to use differentiation, clipping, and reintegration in time-domain filters has been completed and a journal article is being prepared to describe the work done.

2. A Nonlinear Filter to Remove 60-Cycle Hum from Low-Frequency Transients

During the past quarter the nonlinear filter has been completed and tested on a number of signals. The complete instrument consists of two channels, one to filter out the noise voltage from the input signal, and one to pass the desired signal.

The noise is separated by a process of differentiation, clipping, and filtering. Reintegration is not used here because the noise signal is a sine wave and two differentiations yield the signal in the proper phase. At the moment when the noise passes through zero a pulse is obtained which is used to operate a gate in the signal channel. The advantage of the nonlinear filter is that the desired signal is obtained without introducing the undesired transient response of a linear filter.

The signal channel consists of a dc amplifier followed by a high-quality sampling circuit which is actuated by the pulse from the blocking oscillator. The net result is that the signal plus noise is sampled once per cycle at the point where the noise is passing through zero. The output is held constant between samples and therefore a step approximation to the desired transient is obtained.

Some idea of the effectiveness of the filter can be obtained from the fact that a transient of 10-mv amplitude can be obtained in the presence of 5 volts to 10 volts of noise. The noise completely saturates the amplifier on the peaks, but the sampling occurs while the noise is passing through zero, and the amplifier is linear in this region.

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C. THE DESIGN OF NEW COMPUTING ELEMENTS

1. Transistor Multiplier (of Analog Type)

If a transistor (the investigation concerns point-contact transistors, type Bell 1698) is supplied on the emitter side with a voltage instead of a current, one observes the property that the collector resistance is a constant throughout a fairly large variation range of the collector current (or voltage), only dependent on the emitter voltage. Moreover the collector resistance is a linear function of the emitter voltage within a certain range. The physical explanation of this property will appear in a technical report.

This property makes it possible to use the transistor as a multiplier. If the two multiplying voltages V_x and V_y are made to control the emitter voltage V_e and the collector current i_c, respectively, as in the circuit shown in Fig. X-1, the collector voltage V_c will be proportional to the product $V_x \cdot V_y$. The input V_x is transmitted to the transistor through a cathode-follower tube V_1, with a voltage divider R_1 to R_2 as cathode resistor (R_2 is of the order of 20 ohms). V_y is made to control the collector current via the tube V_2.

Figure X-2 shows a typical display of the collector voltage V_c against V_y with V_x as a parameter. The variation range of V_c is 4 volts. In the display of Fig. X-2 it is possible to substitute the lines with straight lines so that the maximum errors do not exceed ± 2 percent. The slopes of these lines determine the collector resistance r_c, which in Fig. X-3 has been plotted against V_x. The maximum error in treating r_c as a linear function of V_x (or V_e) is not greater than ± 2 percent. This means, however, that the maximum error in the multiplier is the sum of the maximum values of these two errors, i.e. ± 4 percent.

The error due to an increase in the temperature of about 20°C from room temperature lies well below the above indicated figure, ± 4 percent, as does the error due to aging of the transistor during the 500 hours the multiplier has been tested. The aging effect appears as a slow decrease in the collector resistance r_c.

It is possible to obtain better accuracy if the variation multiplication range is diminished. However, the above indicated range seems to be well chosen with respect to the aging and temperature errors.

The frequency response of the multiplier is good. Without any special arrangements it works to approximately 100 kc with the indicated accuracy.

In the multiplier of Fig. X-1 the input V_y is allowed to vary from zero to a positive value; V_x, however, cannot be reduced to zero. Of course, it is possible to use the transistor as a multiplying element in a multiplier covering all four quadrants, by adding constant voltages to the inputs and by subtracting (or adding) the inputs from the output.

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