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Punishment Policies' Effect on Illicit Drug Users' Purchasing Habits

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Abstract

This paper develops a mathematical model that describes how a controlled user of an illicit drug might respond to various punishment policies. As defined here, a punishment policy specifies the expected cost of being arrested as a function of the quantity possessed.

It is shown that a "zero-tolerance" policy that assigns the same high level of punishment regardless of the quantity possessed may not minimize consumption. The set of policies that would minimize consumption is described. Because these policies may not be politically feasible, the consumption minimizing policy within a restricted class of politically plausible policies is also derived. Comparing these and two other policies suggests that reforming statutes to make the punishment increase as the quantity possessed increases may be desirable.

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1.0 Introduction

One often hears proposals to "get tough"[1] on drugs by imposing stiff sanctions for possession of even small amounts that are suitable only for personal consumption. Such "zero tolerance" policies have been criticized because they oblige enforcement agencies to allocate scarce resources to relatively less important offenders and because they violate the principle that "the punishment should fit the crime". This paper argues that, in addition, a tough "zero tolerance" policy that assigns all users the same stiff sanction regardless of the quantity possessed may actually encourage some users to increase consumption.

The reason for this counterintuitive behavior is that a significant fraction of the cost a drug user incurs is the cost of identifying and meeting with a supplier (search time costs). Roughly speaking this cost is proportional to the frequency with which the user buys. Since every time a user buys, he or she incurs some risk of being arrested, a zero tolerance policy gives the user the incentive to buy large quantities relatively infrequently[2]. This reduces search time costs which could make the user willing to incur a greater dollar cost, i.e. to buy more drugs.

This paper develops a model which makes this argument precise, but it is helpful to understand the fundamental dynamics before plunging into the mathematical details. With this in mind, the following paragraphs informally rephrase the argument using three different metaphors.

The first is the adage, "In for a penny, in for a pound". When punishment for possession does not increase with quantity, this may be good advice. Presumably the user derives some benefit from consuming the drug, and if drugs are like most consumer products "more is better". So, if the (punishment) costs are the same whether the user buys a lot or a little, why not buy a lot?

Another perspective comes from comparing the individual's decision about how much to consume to a firm's decision about how much to produce. A firm derives revenue by selling goods produced by employing a variety of factors. Likewise, the individual derives satisfaction by consuming drugs purchased by incurring a variety of costs. Firms maximize profit when marginal revenue equals

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[1]This paper uses informal terms when the benefits of being concise seem to outweigh the dangers of not being precise.

[2]Assuming, as is generally the case, that most arrests of users occur during activity related to obtaining the drugs, not because they have it in their possession at some other time.

marginal cost. Likewise, the individual maximizes utility when the marginal benefit of consuming equals the marginal cost of acquiring the drugs.

Changing punishment from an increasing function to a constant independent of the quantity consumed is like converting one of the firm's variable costs to a fixed cost. If the fixed cost is sufficiently high, the firm will cease production, but if it is profitable to produce at all, the profit maximizing quantity is determined by setting marginal revenue equal to marginal cost. Once the decision to produce has been made, fixed costs are sunk costs and do not affect the optimal production level.

Similarly, if punishment is constant for all (positive) quantities, once the individual has decided to use drugs, the level of punishment has no effect on the optimal amount of consumption[1]. Assuming that "more is better" and there are diminishing returns to consumption, reducing the marginal cost of using will increase the utility maximizing amount of consumption.

The third metaphor also compares the drug user to a firm. For a firm, when the price of an input increases the optimal mix of factor inputs changes. For example, if wages increase the firm may substitute capital for labor. Production will be lower, but depending on the relative magnitudes of the income and substitution effects, the amount of capital consumed may increase.

The user "produces" a net utility equal to the satisfaction derived from using minus the costs. If the cost of making frequent purchases increases, the user may substitute total quantity for frequency of purchase by increasing the size of each purchase. The user's net utility declines, but the total amount consumed may increase if the substitution effect is greater than the income effect.

Hopefully these comparisons have given some intuition for the argument explored in this paper, but they should not be taken too literally. Drug consumption is determined by two interdependent variables: the purchase size and the frequency of purchase, both of which affect the total punishment cost. Hence the response to a change in punishment policy is not as simple as these comparisons suggest and formalizing the argument with a mathematical model may be useful.

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[1] Actually this overstates the case. As will be seen, at least in the model developed below, the level of punishment still affects consumption even if it does not depend on the quantity possessed at the time of arrest. The effect can, however, be smaller than it would be if punishment increased with the quantity possessed.

2.0 The Basic Model

2.1 Formulation

Mathematical models are inevitably simplifications. Ideally the simplifications make it possible to draw interesting conclusions without so distorting the fundamental nature of the system modelled that the conclusion are erroneous. This section describes the modelling framework used and its underlying assumptions. The assumptions overlook much of the complexity of and uncertainty about the behavior of drug users, but it is hoped that the reader will find that they capture at least most of the first order effects.

A fundamental assumption of the modelling framework used is that drug users maximize their utility. To some it may sound ludicrous to build a model whose foundation rests on the rationality of drug users. After all, their decision to use drugs casts doubt on their foresight and their very use may cloud whatever judgement they had originally. Such a view, though commonly held, is not necessarily entirely accurate.

The fact that someone has decided to use drugs is not *prima facie* evidence that they are irrational or masochistic. Using drugs involves risks and costs, but so do most activities, and drug use is reported to bring a variety of benefits including, but not necessarily limited to, euphoria, escape, and acceptance in some social groups[1].

It is also not true that those who have begun using drugs are incapable of acting rationally[2]. First of all, few if any users are continuously "high". Even "hard core heroin addicts" come down between highs, and many if not most of their purchase decisions are probably made at these times. Secondly, many drug users, including some known as "chippers" who have used for an extended time on a regular basis, lead a "normal" life, pursuing a career, raising a family, and maintaining a circle of friends. Nevertheless, reluctance to describe a heavily "addicted" user as someone who rationally acts in their own self-interest is only reasonable.

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[1]"Usually the rush [of heroin] is described as a violent, orgasmic experience, somewhat like a sexual orgasm, "only vastly more so."", John Kaplan, The Hardest Drug: Heroin and Public Policy, (Chicago: University of Chicago Press, 1983), p.22.

[2]On the contrary, the general impression one receives from reading studies such as Johnson et al.'s Taking Care of Business: The Economics of Crime by Heroin Abusers (Lexington, MA: Lexington Books, 1985) is that even dependent users behave purposefully and thoughtfully if one remembers that their environment and objectives are not the same as those of the general public.

Hence, from the outset this model will be restricted to "controlled users". Tautologically this restriction excludes all users whose behavior is inconsistent with one or more of the models' assumptions. For example, it excludes those who do not rationally maximize their individual welfare. It is up to the reader to decide whether this restriction makes the model applicable to such a small fraction of the population that its conclusions are of no value.

Besides restricting attention to controlled users, the model focuses on users whose consumption is in "steady state". It does not apply to novice users, people whose consumption varies greatly over relatively short periods, or people who "binge". The reason for this is simple; modeling the evolution of behavior is complex. This paper only tries to describe what the ultimate changes might be by examining behavior before and after some exogenous change. In economists' lexicon, it is a comparative statics analysis.

Furthermore, as a considerable simplification, it will be assumed that the user buys a particular quantity at regular intervals. Without this assumption the analysis below would be considerably more complicated. With it, there are just two decision variables over which the user maximizes his or her utility:

q = the quantity purchased each time the user buys, and
f = the frequency with which the user buys.

Purchasing and using drugs offers a variety of advantages and disadvantages. It will be assumed that these can all be converted into some measure of utility, so the user's decision can be described as an optimization problem:

$$\text{Max } z(f,q) = B(f,q) - C(f,q)$$
$$\text{s.t.} \quad q,f \geq 0$$

where

B(f,q) = benefit per unit time the user derives from receiving q units of drugs with frequency f, and

C(f,q) = cost per unit time of purchasing q unit batches of drugs with frequency f.

There may be constraints on q and f other than that they be nonnegative. Obviously there is a minimum purchase size suppliers will sell and the frequency of purchase cannot be arbitrarily large or arbitrarily close to zero. Perhaps more significantly, the purchase size q may be limited by the amount of cash the user has[1]. It is assumed that this cash constraint is not binding. For many controlled users this is reasonable. Others may be able to borrow enough to make the desired purchase, and the borrowing costs can be counted as holding costs (discussed below). The problem can also be solved assuming the cash constraint is binding. Generally speaking, if the cash constraint is binding the user will buy as much as he or she can afford as often as possible, and the punishment policy does not affect consumption.

Limited income and wealth also threaten the steady state assumption. One might purchase a fixed quantity q with a fixed frequency f for quite some time, but not be in financial steady state. For example, such an individual might be drawing upon savings[2]. In these situations assuming an infinite horizon steady state may not be objectively realistic, but it may approximate to the user's thinking.

There is no "correct" functional form for B(f,q) or C(f,q). After all, even modelling the utility and disutility associated with licit activities that can be studied more easily is imprecise. The best one can do is make some plausible assumptions and hope that the dynamics of the system are relatively robust. The remainder of this subsection lists the assumptions made here.

Assumption A1. $B(f,q) = \alpha \sqrt{fq}$ for $f, q > 0$, where α is a positive constant.

This is the most speculative of the assumptions. For a controlled user who can maintain an inventory (i.e. one who can resist the temptation to binge), B(f,q) is probably a function of the quantity of drugs not f or q individually, i.e. $B(f,q) = \mathcal{B}(fq)$. For most consumer goods "more is better" and there are diminishing returns to consumption, so one would expect $\mathcal{B}(fq)$ to be a concave, increasing function.

The square root function is concave and increasing, but it is by no means the only such function. However, other functions, such as $\mathcal{B}(fq) = (fq)^{\varepsilon}$ and $\mathcal{B}(fq) = \ln(fq)$, lead to less tractable formulations. As will be seen, if $B(f,q) = \alpha \sqrt{fq}$ a number of results can be obtained in closed form. Since there is no obvious reason for preferring any other functional form and it is not feasible to measure the function empirically, the square root function will be used.

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[1]Some suppliers will sell on credit to familiar customers, but there are limits to the credit they will give.

[2]See, for example, Andrew H. Malcolm's two part series "The Spreading Web of Crack" in the New York Times, October 1-2, 1989, Sec. 1, p.1.

C(f,q) includes the following costs: purchase cost, the cost of the negative health effects of using drugs, search cost, the cost of keeping an inventory, and the expected cost of being arrested while buying the drugs. Quantifying these costs requires several assumptions.

Assumption A2: Purchase costs are proportional to the quantity consumed, fq. This is equivalent to assuming that price is independent of q and f for fixed fq.

Drug markets are certainly large enough that no single user's purchases can significantly affect the price. If all users doubled their purchases prices might increase, but the model focuses on an individual user's decision. However, users who buy large quantities can sometimes obtain quantity discounts[1]. Ignoring these discounts restricts the model to users whose consumption habits clearly place them in the retail market irrespective of what punishment policy is in effect.

Assumption A3: The costs of the negative health effects are proportional to qf, the quantity consumed.

Since the user is assumed to be in sufficient control of his or her habit to avoid binges, the costs of the negative health effects are probably proportional to the quantity of drugs consumed, not on q or f alone. To avoid cluttering notation, these costs are incorporated into the purchase cost term.

Assumption A4: Search costs are proportional to f.

Search costs for the first few purchases are likely to be much greater than for subsequent purchases, but after that initial transient, search costs, or at least expected search costs, are probably proportional to f for wide ranges in q. The model ignores any such transient effects and assumes search costs are proportional to f.

Assumption A5: Inventory costs are proportional to q.

Quantities held for personal use take up so little physical space one might think the inventory cost term would be negligible. In fact, inventory costs are likely to be small relative to other costs, but if they were truly zero, then it would be optimal for users to buy all

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[1]This is described in Johnson et al. for heroin users and asserted to be true for cocaine users by Peter Reuter, Gordon Crawford, and Jonathan Cave in Sealing the Borders: The Effects of Increased Military Participation in Drug Interdiction, The RAND Corporation, January 1988.

they will ever use in one purchase. Clearly this is not done in practice, so there must be disincentives to holding large inventories. First of all, storage costs per unit weight of drugs are high relative to those of many consumer products because most users make at least some effort to hide them. Second, there is the risk of having the drugs stolen or inadvertently damaged. Third, there is the opportunity cost of holding inventory; money tied up in an inventory of drugs is money that cannot be used elsewhere. Fourth, at some time most users stop using drugs, and the salvage value of an unused inventory of drugs is next to zero.

These costs are proportional to the quantity held. Suppose the user consumes at a constant rate and replenishes whenever the stock drops to some threshold. Then inventory as a function of time has a sawtooth graph and the average inventory cost per unit time is a constant, which can be ignored, plus a term proportional to q.

Assumption A6: The probability of being arrested is proportional to f, the frequency with which purchases are made.

This assumption reflects the observation that most users who are arrested are apprehended while buying or when they have the drugs in their possession soon after buying. It is much less common to be arrested while using or for possession at some other time.

If every user purchased more often, then the risk of arrest per purchase might decrease because the ratio of police power to the number of transactions would fall. But this model focuses on one individual's decision, and one individual's actions will not appreciably affect this ratio.

Factors other than the amount of police power per transaction may have an effect. Frequenting drug copping[1] areas might arouse suspicion, so the probability of arrest per purchase may actually increase with f. On the other hand, frequent purchasers may be more adept at avoiding the police. Or, it is possible that frequent purchasers may become careless. Also, infrequent purchasers may be able to take better advantage of variations in enforcement pressure. On the other hand, frequent purchasers may know more about these variations. Since the relative strength of these conflicting factors is not obvious, in this paper it will be assumed that the probability of arrest per purchase is independent of the frequency with which purchases are made.

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[1]"Copping" is a slang term used for purchasing or serving as an intermediary between a buyer and seller.

Assumption A7: The penalty the user expects to suffer if he or she is arrested while buying depends on the quantity purchased.

The expected penalty to the user if he or she is arrested will be represented by the function $c_a(q)$. To a large extent this function, referred to in the sequel as the punishment policy, can be set by policy makers.

It is reasonable to assume punishment depends on the quantity possessed at arrest (especially since the constant function is not excluded), but it may depend on other factors as well. In particular, punishment frequently depends on the offender's prior criminal record, including their record for drug offenses. This suggests that during steady state periods not only consumption patterns, but also criminal record must be constant. For example, suppose the individual is arrested for possession of narcotics but released on parole. The expected punishment for that individual upon a subsequent arrest would almost certainly increase, so a new equilibrium would be obtained.

It is possible that a dynamic purchasing strategy would be superior. For example, it may be optimal for the purchase amount to increase slowly as the amount of time since the most recent arrest increases, but such possibilities are ignored. The omission of users' criminal records is more problematic when one discusses the model's implications for optimal punishment policies.

Having made these assumptions the cost function may be modelled as

$C(f,q) = h\, q + c_s\, f + p\, c_a(q)\, f + c_p\, qf$

where

h = inventory cost coefficient,

c_s = search time cost per purchase,

$c_a(q)$ = expected cost of being arrested while buying,

p = probability of being arrested while making a purchase, and

c_p = purchase price.

2.2 Solution to the Basic Model With Linear Punishment

Given the assumptions above, if $c_a(q)$ is assumed to be linear ($c_a(q) = a + bq$ for $q > 0$) the overall problem is

$$\text{Max } z(f,q) = \alpha\sqrt{fq} - h\,q - (c_s + p\,a)\,f - (c_p + p\,b)\,fq \qquad \text{(P 1)}$$
$$\text{s.t.} \quad q,f \geq 0.$$

This is a relatively straightforward nonlinear optimization problem in two variables with nonnegativity constraints. It is solved in the Appendix by standard methods. Letting * denote an optimal value, the result is that if $\alpha \le 2\sqrt{h(c_s + pa)}$

$$z^* = f^* = q^* = 0,$$

but if $\alpha > 2\sqrt{h(c_s + pa)}$ the optimal purchase quantity and frequency are

$$q^* = \frac{\alpha}{2(c_p + pb)}\sqrt{\frac{c_s + pa}{h}} - \frac{c_s + pa}{c_p + pb}$$

$$f^* = \frac{h}{c_s + pa} q^* = \frac{\alpha}{2(c_p + pb)}\sqrt{\frac{h}{c_s + pa}} - \frac{h}{c_p + pb}$$

and the optimal rate of consumption and objective function value are

$$q^* f^* = \left(\frac{\frac{\alpha}{2} - \sqrt{h(c_s + pa)}}{c_p + pb}\right)^2, \text{ and}$$

$$z(f^*,q^*) = (c_p + pb)\, q^* f^* = \frac{\left(\frac{\alpha}{2} - \sqrt{h(c_s + pa)}\right)^2}{c_p + pb} .$$

These expressions look more complicated than they are. Consider, for example, the condition that if $\alpha \le 2\sqrt{h(c_s + pa)}$ the user is best off not purchasing or using drugs. Since α is propotional to the benefit derived from using, it makes sense that if α is small the costs of using might outweigh the benefits and so $q^* = f^* = 0$. Likewise, the holding cost parameter (h), search cost parameter (c_s), probability of arrest (p), and intercept of the punishment function (a) are all measures of cost. The higher they are, the more likely it is that α is not large enough for it to be worthwhile to consume. (The term (c_p + pb) does not appear in this condition, because it is the coefficient of a higher order term that is negligible for small q and f.)

Consider now the case when it is optimal to consume. The optimal purchase quantity q*, frequency of purchase f*, consumption rate q*f*, and utility z(f*,q*) are all increasing in α and are generally decreasing in all the cost parameters, as would be expected. The exceptions are that f* can be increasing in h, the coefficient of q in the objective function, and q* can be increasing in (c_s + pa), the coefficient of f in the objective function. This is not surprising since q and f are substitutes. They may be increasing or decreasing functions of the other's cost coefficient depending on whether the income or substitution effect dominates[1].

Note f* is proportional to q*, and the proportionality constant is the ratio of their cost coefficients. This is because everywhere else in the objective function q and f appear together as qf. Hence, to first order, reducing q by 1% can be compensated for by increasing f by q*/f* per cent, so the price of q (h) must be equal to q*/f* times the price of f (c_s + pa), i.e. $f^* = \frac{h}{c_s + p\,a} q^*$.

The expressions above have several policy implications. For instance, since the punishment policy parameters a and b always appear as pa and pb, if the probability of arrest (p) is small, increasing the expected punishment (increasing a or b or both) will have little effect.

In that case the policy maker may be forced to work with the search time cost c_s and purchase price c_p. (α and h probably cannot be influenced directly.) If the user is a "committed user" who derives great satisfaction from consuming drugs, i.e. $\alpha >> 2\sqrt{h(c_s + p\,a)}$, then increasing search time costs will not have much effect compared with increasing prices. Increasing prices will also reduce the consumption of a user who is "on the fence" ($\alpha \approx 2\sqrt{h(c_s + p\,a)}$), but it will never push them out of the market completely. Increasing search time costs enough to make $\alpha \leq 2\sqrt{h(c_s + p\,a)}$ may, however, do exactly that.

2.3 Setting the Punishment Policy

Although enforcement policy affects price and search time costs (and perhaps α and h less directly), $c_a(q)$ is all that can be controlled directly, so it is natural to ask what it should be. The answer depends, of course, on one's objectives. This paper will assume the objective is to minimize the rate of consumption (q^*f^*) which, if price is constant, is also proportional to the drug dealers' revenues.

From the solution above, if $\alpha \leq 2\sqrt{h(c_s + p\,a)}$ $q^* = f^* = 0$. Rearranging this expression shows that if $a \geq \frac{1}{p}\left(\frac{\alpha^2}{4\,h} - c_s\right)$, $q^* = f^* = 0$ for

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[1] The substitution effect dominates and thus q* and f* are increasing in the other's cost parameter if the user is a heavy user, more specifically, if $\alpha > 4\sqrt{h(c_s + p\,a)}$.

any b. Similarly, if $\alpha > 2\sqrt{h(c_s + p\,a)}$, $q^* f^* = \left(\frac{\frac{\alpha}{2} - \sqrt{h(c_s + p\,a)}}{c_p + p\,b}\right)^2$, so for any given a, q^* and f^* can be made arbitrarily small by choosing b large enough. In other words, according to this simple model, if the expected punishment (not just the threatened punishment) were made sufficiently severe, drug consumption could be eliminated.

There are limits, however, to the severity of punishment that society will tolerate[1]. Let c_a be the maximum punishment for a drug user that society will tolerate[2]. Then the constraint $c_a(q) \leq c_a$ should be added to the formulation. Society's sense of justice would probably also demand that $c_a(q)$ be a nondecreasing function of q and $c_a(0) = 0$.

3.0 Evaluating Various Punishment Policies

3.1 Policy of Maximum Punishment

One might think that imposing the harshest possible penalties ($c_a(q) = c_a$ for all $q > 0$) would minimize consumption. With such a maximum punishment policy

$$C(f,q) = h\,q + c_s\,f + p\,c_a\,f + c_p\,qf$$

and the user's optimization problem is

$$\text{Max } z(f,q) = \alpha\sqrt{fq} - h\,q - (c_s + p\,c_a)\,f - c_p\,fq$$
$$\text{s.t.} \quad q,f \geq 0.$$

This is the special case of Problem P1 with $a = c_a$ and $b = 0$. The solution can be found by substituting those values into the expressions in Section 2.2. Capital letters will be used to denote the optimal solution under a maximum punishment policy.

If $\alpha \leq 2\sqrt{h(c_s + p\,c_a)}$ then

$$F = Q = 0,$$

but if $\alpha > 2\sqrt{h(c_s + p\,c_a)}$, the optimal purchase quantity and frequency are

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[1] It is unlikely, for example, that the death penalty (imposed by slow immersion in boiling oil) will ever be instituted for the possession of trace amounts of marijuana.

[2] c_a can be drug specific if the model is interpreted as applying to just one drug and substitution is ignored.

$$Q = \frac{\alpha}{2c_p}\sqrt{\frac{c_s + p\,c_a}{h}} - \frac{c_s + p\,c_a}{c_p}$$

$$F = \frac{h}{c_s + p\,c_a} Q = \frac{\alpha}{2c_p}\sqrt{\frac{h}{c_s + p\,c_a}} - \frac{h}{c_p}$$

and the optimal rate of consumption and objective function value are

$$Q\,F = \left(\frac{\frac{\alpha}{2} - \sqrt{h(c_s + p\,c_a)}}{c_p}\right)^2, \text{ and}$$

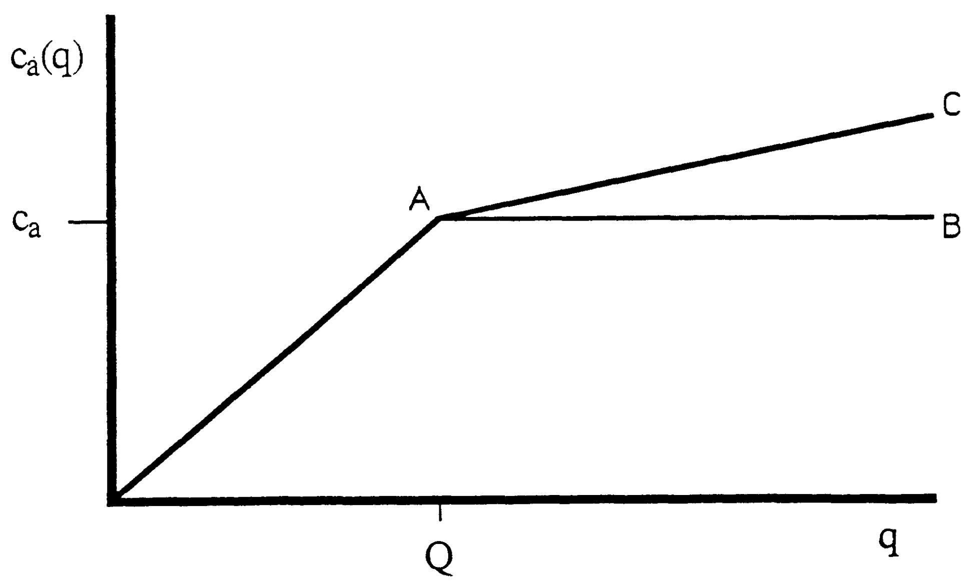
$$z(F,Q;\ c_a(q) = c_a) = c_p\,Q\,F = \frac{\left(\frac{\alpha}{2} - \sqrt{h(c_s + p\,c_a)}\right)^2}{c_p}.$$

3.2 A Policy That Leads to Less Consumption

Surprisingly, this maximum punishment policy does not minimize consumption. This section shows the rate of consumption will be lower if the expected cost of being arrested increases linearly with q from 0 to c_a for $0 \le q \le Q$ and is no less than c_a for $q \ge Q$ (See Figure 1.) i.e.,

$$c_a(q) = \frac{c_a}{Q}q \qquad \text{for } 0 \le q \le Q, \text{ and}$$
$$c_a(q) \ge c_a \qquad \text{for } q \ge Q.$$

Figure 1
A Linear Punishment Policy



The exact form of $c_a(q)$ for $q > Q$ does not matter. To see this, look again at Figure 1. The previous section showed the user prefers point A to any point on the line (A,B). But increasing the punishment cannot improve the user's utility, so the user will prefer any point on (A,B) to the corresponding point on (A,C) directly above. Since A is feasible under the new policy, this means the user will never pick any point on (A,C), i.e. $q^* \leq Q$.

More formally, let $z(f, q; \frac{c_a}{Q}q)$ denote the user's utility function with this punishment policy. For all f and all $q \geq Q$, $z(f, q; \frac{c_a}{Q}q) \leq z(f, q; c_a(q) = c_a) \leq z(F, Q; c_a(q) = c_a) = z(F, Q; \frac{c_a}{Q}q)$. This implies there is an optimal solution with $q^* \leq Q$.

Thus the exact form of $c_a(q)$ for $q > Q$ is irrelevant[1]. In particular, the same solution will be obtained if $c_a(q) = \frac{c_a}{Q} q$ for all $q \geq 0$. This is the special case of Problem P1 with $a = 0$ and $b = \frac{c_a}{Q}$ so the solution can be obtained by substituting these values into the expressions in Section 2.2. Let ~ denote quantities that are optimal with this punishment policy. If $\alpha \leq 2\sqrt{h\,c_s}$ then

$$\tilde{q} = \tilde{f} = 0,$$

but if $\alpha > 2\sqrt{h\,c_s}$ the solution is

$$\tilde{q} = \frac{\alpha}{2\left(c_p + \frac{p\,c_a}{Q}\right)}\sqrt{\frac{c_s}{h}} - \frac{c_s}{\left(c_p + \frac{p\,c_a}{Q}\right)},$$

$$\tilde{f} = \frac{\alpha}{2\left(c_p + \frac{p\,c_a}{Q}\right)}\sqrt{\frac{h}{c_s}} - \frac{h}{\left(c_p + \frac{p\,c_a}{Q}\right)},$$

$$\tilde{q}\,\tilde{f} = \left(\frac{\frac{\alpha}{2} - \sqrt{h\,c_s}}{\left(c_p + \frac{p\,c_a}{Q}\right)}\right)^2, \quad \text{and}$$

[1]The value of $c_a(q)$ for $q > Q$ will not affect the user because the user will never possess a quantity $q > Q$.

$$z(\tilde{f}, \tilde{q}; \frac{c_a}{Q} q) = \left(c_p + \frac{p\, c_a}{Q}\right) \tilde{q}\, \tilde{f} = \frac{\left(\frac{\alpha}{2} - \sqrt{h\, c_s}\right)^2}{c_p + \frac{p\, c_a}{Q}} .$$

Individuals with $2\sqrt{h\, c_s} < \alpha < 2\sqrt{h\,(c_s + p\, c_a)}$ consume a positive quantity under this punishment policy but not under a policy of maximum punishment. This suggests that switching from a maximum punishment policy to this policy might increase the number of users. This is to be expected because curious people are more likely to experiment if they know they will not be punished as severely as heavy users.

If $\alpha > 2\sqrt{h\,(c_s + p\, c_a)}$, however, switching from a maximum punishment policy has a very different result. In that case the average amount purchased is smaller with the linear punishment policy since

$$Q - \tilde{q} = \frac{\alpha\, Q}{2\sqrt{h}\,(c_p Q + p\, c_a)} \left(\sqrt{c_s + p\, c_a} - \sqrt{c_s}\right) > 0.$$

Also, it must be the case that the user is better off because he or she could have $q = Q$, $f = F$, and risk the same punishment as with the maximum punishment policy. Since the previous solution is feasible and the user maximizes utility, the new optimal solution's utility must be at least as great. In symbols,

$$z(\tilde{f}, \tilde{q}; \frac{c_a}{Q} q) \geq z(F, Q; \frac{c_a}{Q} q) = z(F, Q; c_a(q) = c_a).$$

Actually the inequality is strict because

$$z(\tilde{f}, \tilde{q}; \frac{c_a}{Q} q) - z(F, Q; c_a(q) = c_a)$$
$$= \frac{\alpha\left(\frac{\alpha}{2} - \sqrt{h\,(c_s + p\, c_a)}\right)}{c_p\,(c_p Q + p\, c_a)} \left(\sqrt{\frac{c_s + p\, c_a}{2}} - \sqrt{\frac{c_s}{2}}\right)^2 > 0.$$

Also, the user will buy more frequently since

$$\tilde{f} - F = \frac{\frac{\alpha}{2} - \sqrt{h\,(c_s + p\, c_a)}}{2\, c_p\,(c_p Q + p\, c_a)} \left[\alpha\left(\sqrt{\frac{c_s + p\, c_a}{c_s}} - 1\right) + \sqrt{\frac{h}{c_s + p\, c_a}}\; 2\, p\, c_a\right] > 0.$$

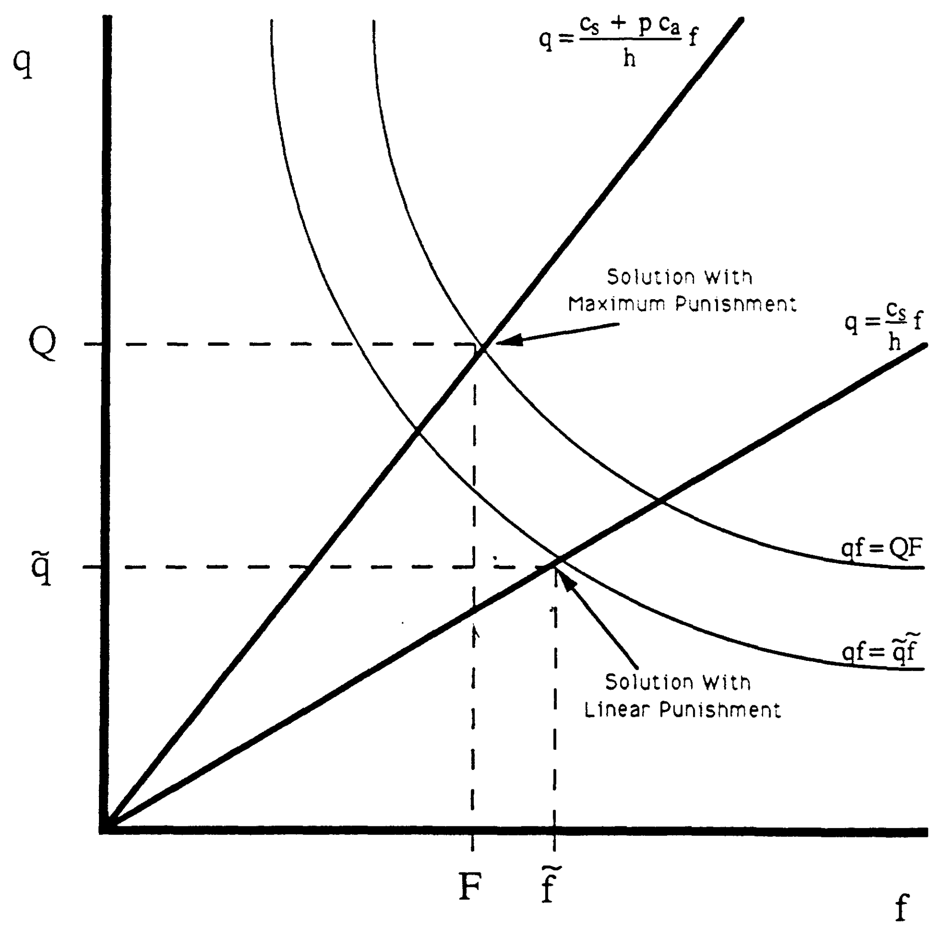
Surprisingly, the size of the average purchase decreases enough to more than counteract the increased frequency with which purchases are made, so the overall rate of consumption decreases by

$$QF - \tilde{q}\tilde{f} = \left(\frac{\frac{\alpha}{2} - \sqrt{h(c_s + p\,c_a)}}{c_p}\right)^2 - \left(\frac{\frac{\alpha}{2} - \sqrt{h\,c_s}}{\left(c_p + \frac{p\,c_a}{Q}\right)}\right)^2$$

$$= \frac{(c_p Q)^2}{\left(2\,c_p\left(c_p Q + p\,c_a\right)\right)^2}\left[\left(\alpha - 2\sqrt{\frac{c_s}{c_s + p\,c_a}}\sqrt{h\,c_s}\right)^2 - \left(\alpha - 2\sqrt{h\,c_s}\right)^2\right] > 0.$$

Figure 2 shows these changes in the q-f plane.

Figure 2
Changing from Maximum Punishment
to a Linear Policy



3.3 Consumption Minimizing Policy

The previous section showed that a maximum punishment policy need not minimize consumption. A natural question to ask is, what punishment policy will minimize consumption?

This question can also be posed as an optimization problem, but now the argument of the minimization is the function $c_a(q)$. That is, the decision maker specifies an entire function, not just a finite number of variables.

Not every function is feasible because the punishment policy must be consistent with society's sense of justice. Specifically, it seems reasonable to restrict attention to policies that do not punish people who do not have any drugs ($c_a(0) = 0$), that never threaten more than the maximum permissible punishment ($c_a(q) \leq c_a$), and that punish people possessing large quantities at least as severely as those carrying smaller amounts ($c_a(q)$ is nondecreasing).

The policy maker would like to find the punishment policy $c_a(q)$ satisfying these constraints that minimizes the rate of consumption, qf. But q and f both depend on the punishment policy. They are determined when the user maximizes his or her utility, and as the preceding discussion showed, the user's optimal purchasing pattern is affected by the punishment policy.

Mathematically this can be represented as a nested optimization problem:

$$\underset{c_a(q) \in F}{\text{Min}} \left\{ f^* q^* \mid (f^*, q^*) = \arg \underset{f,q \geq 0}{\text{Max}} \left\{ z(f,q) = \alpha\sqrt{fq} - hq - (c_s + p\, c_a(q))\, f - c_p\, f\, q \right\} \right\} \quad \text{(P 2)}$$

where $F = \{f: \Re \rightarrow \Re \mid f(0) = 0, f(q) \leq c_a\ \forall q, \text{ and } f() \text{ is nondecreasing}\}$ is the set of feasible punishment policies.

The internal maximization represents the user's problem of maximizing utility subject to the punishment policy $c_a(q)$. The outer minimization represents the policy maker's problem of choosing a punishment policy that minimizes the rate of consumption, qf.

The derivation of the solution is more technical than the rest of the paper, so readers may want to skip the introductory lemmas and proceed directly to the theorem at the end of this section.

Setting the partial derivative of z(f,q) with respect to f equal to zero

$$\frac{\partial z(f,q)}{\partial f} = \frac{\alpha}{2}\sqrt{\frac{q}{f}} - \left(c_s + pc_a(q) + c_p q\right) = 0$$

shows that

$$f^* = \frac{\frac{1}{4}\alpha^2 q^*}{\left(c_s + pc_a^*(q^*) + c_p q^*\right)^2}.$$

Substituting this into $z(f,q) = \alpha\sqrt{fq} - hq - (c_s + p\,c_a(q))\,f - c_p f q$ implies that

$$z\left(f^*, q^*\right) = \frac{\alpha^2 q^*}{4\left(c_s + p\,c_a^*(q^*) + c_p q^*\right)} - h q^*.$$

<u>Lemma 1</u>: $q^* \le Q = \frac{\alpha}{2c_p}\sqrt{\frac{c_s + p\,c_a}{h}} - \frac{c_s + p\,c_a}{c_p}$, the optimal quantity when $c_a(q) = c_a$ for all $q > 0$.

<u>Proof:</u> Suppose to the contrary that $q^* > Q$. Since $c_a(q) = c_a$ is in the set of feasible punishment policies

$$q^* f^* = \frac{\alpha^2 q^{*2}}{4\left(c_s + p\,c_a^*(q^*) + c_p q^*\right)^2} \le \frac{\alpha^2 Q^2}{4\left(c_s + p\,c_a + c_p Q\right)^2} = QF.$$

Since $c_a^*(q)$ must also be feasible, $c_a^*(q^*) \le c_a$. So

$$\frac{\alpha^2 Q^2}{4\left(c_s + p\,c_a + c_p Q\right)^2} \le \frac{\alpha^2 Q^2}{4\left(c_s + p\,c_a^*(q^*) + c_p Q\right)^2},$$

which implies

$$\frac{q^*}{c_s + p\,c_a^*(q^*) + c_p\hat{q}} \le \frac{Q}{c_s + p\,c_a^*(q^*) + c_p Q}.$$

Thus since $c_s + pc_a^*(q^*) > 0$ and $c_p > 0$, $q^* \le Q$. Contradiction. QED.

<u>Lemma 2</u>: There is an optimal punishment policy with $c_a^*(Q) = c_a$.

<u>Proof:</u> By Lemma 1, $q^* \le Q$. If $q^* = Q$ then $c_a^*(Q) = c_a$ minimizes f^* and thus q^*f^*.

Suppose $q^* < Q$ and $c_a^{**}(q)$ is an optimal punishment policy with $c_a^{**}(Q) < c_a$. Then the punishment policy

$$c_a^*(q) = \begin{cases} c_a^{**}(q) & q < Q \\ c_a & q \ge Q \end{cases}$$

yields the same solution as $c_a^{**}(q)$ does because $z(f, q; c_a^*(q)) = z(f, q; c_a^{**}(q))$ for all $q < Q$ and $z(f, q; c_a^*(q)) \le z(f, q; c_a^{**}(q)) \le z(f^*, q^*; c_a^{**}(q^*)) = z(f^*, q^*; c_a^*(q))$ for all $q \ge Q$. QED.

<u>Lemma 3</u>: There is an optimal punishment policy $c_a^*(q)$ and quantity q^* such that $c_a^*(q^*) \le U(q^*)$ where

$$pU(q) = \frac{\frac{1}{4}\alpha^2 q\, c_p}{\left(\frac{\alpha}{2} - \sqrt{h\left(c_s + pc_a\right)}\right)^2 + h\,q\,c_p} - c_s - c_p q.$$

Proof: By Lemma 2 there exists an optimal punishment policy with $c_a^*(Q) = c_a$. Since (F,Q) is feasible for the user, $z(f^*, q^*; c_a^*(q)) \geq z(F, Q; c_a^*(q)) = z(F, Q; c_a(q) = c_a)$, i.e.

$$z\left(f^*, q^*; c_a^*(q)\right) = \frac{\alpha^2 q^*}{4\left(c_s + p\, c_a^*(q^*) + c_p q^*\right)} - h q^* \geq \frac{\left(\frac{\alpha}{2} - \sqrt{h\left(c_s + p c_a\right)}\right)^2}{c_p}.$$

Solving for $pc_a^*(q^*)$ gives the desired inequality. QED.

Theorem 1: Any punishment policy that satisfies the following conditions will minimize consumption.

1) $c_a^*(q) = 0$ for $0 \leq q \leq \bar{q}$,
2) $c_a^*(q) \geq U(q)$ for $\bar{q} \leq q \leq Q$, and
3) $c_a^*(q) = c_a$ for $q \geq Q$

where

$$U(q) = \frac{1}{p}\left[\frac{\frac{1}{4}\alpha^2 q\, c_p}{\left(\frac{\alpha}{2} - \sqrt{h\left(c_s + pc_a\right)}\right)^2 + h q\, c_p} - c_s - c_p q\right] \text{ and}$$

$Q = \frac{\alpha}{2c_p}\sqrt{\frac{c_s + p\,c_a}{h}} - \frac{c_s + p\,c_a}{c_p}$ is the optimal purchase quantity when $c_a(q) = c_a$. With any consumption minimizing policy:

$$\bar{q} = \frac{1}{c_p}\left[\frac{\alpha}{2}\sqrt{\frac{c_s + p\,c_a}{h}} - c_s + \frac{p\,c_a}{2} - \frac{\sqrt{p\,c_a}}{2}\sqrt{p\,c_a + \frac{\alpha}{h}\left(\alpha - 2\sqrt{h\left(c_s + p\,c_a\right)}\right)}\right],$$

$$\bar{f} = \frac{\alpha^2 \bar{q}}{4\left(c_s + c_p\bar{q}\right)^2},$$

$$\bar{q}\bar{f} = \frac{\alpha^2 \bar{q}^2}{4\left(c_s + c_p\bar{q}\right)^2} = \left(\frac{\alpha\bar{q}}{2\left(c_s + c_p\bar{q}\right)}\right)^2, \text{ and}$$

$$z\left(\bar{f}, \bar{q}; c_a^*(q)\right) = \frac{\left(\frac{\alpha}{2} - \sqrt{h\left(c_s + p\,c_a\right)}\right)^2}{c_p}.$$

Proof: Note that U(q) is an indifference curve for the user since $z(f, q; c_a(q) = U(q)) = z(F, Q; c_a(q) = c_a)$. It has the following properties: $U(Q) = c_a$, $\frac{d\left(U(q=0)\right)}{dq} > 0$, $\frac{d\left(U(q=Q)\right)}{dq} = 0$, and $\frac{d^2\left(U(q)\right)}{dq^2} < 0$ so U(q) is increasing for $0 \leq q \leq Q$. Also U(q) = 0 for

$$q = Q + \frac{p\,c_a}{2\,c_p} \pm \frac{\sqrt{p\,c_a}}{2\,c_p}\sqrt{p\,c_a + \frac{\alpha}{h}\left(\alpha - 2\sqrt{h}\left(c_s + p\,c_a\right)\right)}\,.$$

Let $\bar{q}$ denote the smaller of the two zeros. Then

$$c_p\bar{q} = \frac{\alpha}{2}\sqrt{\frac{c_s + p\,c_a}{h}} - c_s + \frac{p\,c_a}{2} - \frac{\sqrt{p\,c_a}}{2}\sqrt{p\,c_a + \frac{\alpha}{h}\left(\alpha - 2\sqrt{h}\left(c_s + p\,c_a\right)\right)}$$

and $0 < \bar{q} < Q$.

Now Lemma 3 implies $q^* \geq \bar{q}$; otherwise, $c_a^*(q^*)$ would be negative. Think of the problem in the quantity-punishment plane. The solution $(q^*, c_a^*(q^*))$ lies in the region bounded by $\bar{q} \leq q^* \leq Q$ and $0 \leq c_a^*(q^*) \leq U(q^*)$. Along the curve $U(q^*)$ for $\bar{q} \leq q^* \leq Q$,

$$q^* f^* = \frac{\alpha^2 q^{*2}}{4\left(c_s + p\,U(q^*) + c_p q^*\right)^2} = \frac{\frac{1}{4}\alpha^2 q^{*2}}{\left(\dfrac{\frac{1}{4}\alpha^2 c_p q^*}{\left(\frac{\alpha}{2} - \sqrt{h\left(c_s + p\,c_a\right)}\right)^2 + hc_p q^*}\right)^2} = \frac{\left(\left(\frac{\alpha}{2} - \sqrt{h\left(c_s + p\,c_a\right)}\right)^2 + hc_p q^*\right)^2}{\frac{1}{4}\alpha^2 c_p^2}\,.$$

This is strictly increasing in q^*, so it is minimized along this curve by taking $q^* = \bar{q}$.

Since

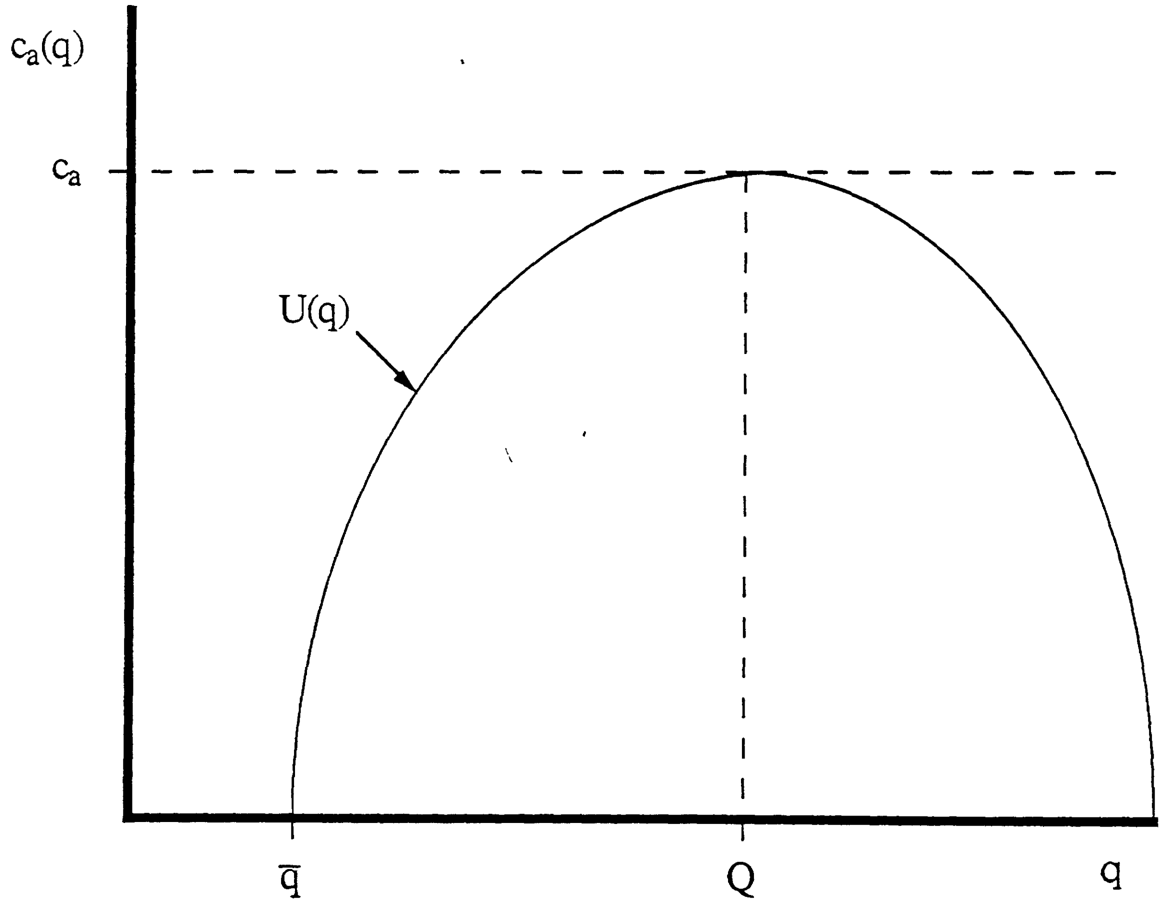
$$q^* f^* = \frac{\frac{1}{4}\alpha^2 q^*}{\left(c_s + p c_a^*(q^*) + c_p q^*\right)^2}$$

no point $(q^*, c_a^*(q^*))$ in the region can be better than the corresponding point $(q^*, U(q^*))$ on the curve $U(q^*)$. Hence, a policy that forces $q^* = \bar{q}$ is optimal, and any $c_a^*(q)$ such that $c_a^*(q) = 0$ for $0 \leq q \leq \bar{q}$, $c_a^*(q) \geq U(q)$ for $\bar{q} \leq q \leq Q$, and $c_a^*(Q) = c_a$ for $q \geq Q$ will do. Actually the optimal policies will be perturbations of these policies that ensure the user strictly prefers $q \approx \bar{q}$ to any larger q. QED.

$U(q)$ is an indifference curve. It is the set of all points such that when the user makes purchases of size q, the frequency of purchase f is related to q by the necessary conditions for optimality, and the expected punishment for possessing q is $U(q)$, then the user's utility is the same as it is at the best possible point under a maximum punishment policy (i.e., $q = Q$ and $f = F$ when $c_a(q) = c_a$.). In other words, the user is equally happy at any point along the curve $U(q)$.

Figure 3 shows the general shape of U(q). It is positively sloped for $q < Q$, concave, reaches a maximum of c_a at $q = Q$, and equals zero for $q = Q + \frac{p\,c_a}{2\,c_p} \pm \frac{\sqrt{p\,c_a}}{2\,c_p}\sqrt{p\,c_a + \frac{\alpha}{h}\left(\alpha - 2\sqrt{h}\left(c_s + p\,c_a\right)\right)}$. The smaller of these two zeros is $\bar{q}$ and $0 < \bar{q} < Q$.

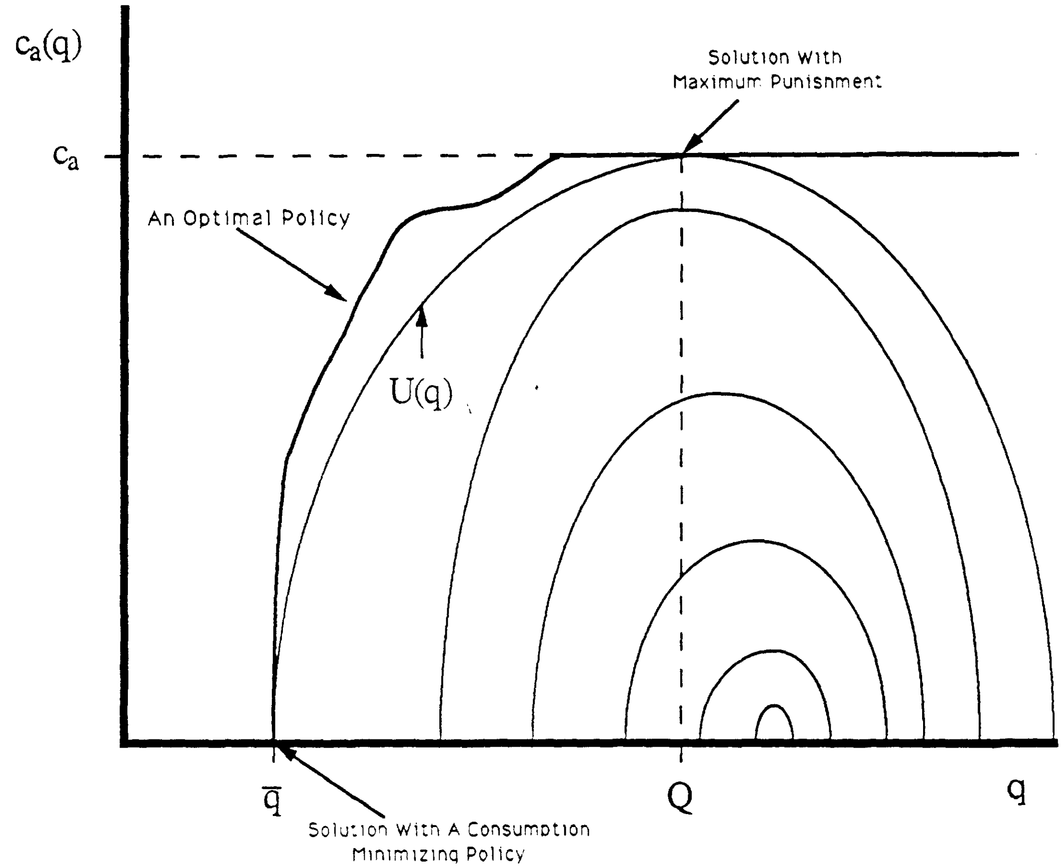
Figure 3
The Indifference Curve U(q)



The theorem states that any punishment policy $c_a(q)$ that is zero for quantities less than $\bar{q}$, is at least as great as U(q) for $\bar{q} \le q \le Q$, and equals c_a for $q \ge Q$ minimizes consumption. Note the consumption minimizing policy is not unique because the value for $\bar{q} < q < Q$ is not uniquely specified.

Figure 4 shows one such policy. (The curves beneath U(q) are also indifference curves; each represents a lower utility for the user.) It coaxes the user to reduce q from Q to $\bar{q}$ by reducing the punishment for smaller quantities. Since the rate of consumption qf is increasing in q, this also reduces consumption. As long as the punishment $c_a(q)$ must be nonnegative though, there is a limit to how much one can reduce consumption because $U(q) < 0$ for $q < \bar{q}$.

Figure 4
Indifference Curves and
A Consumption Minimizing Policy



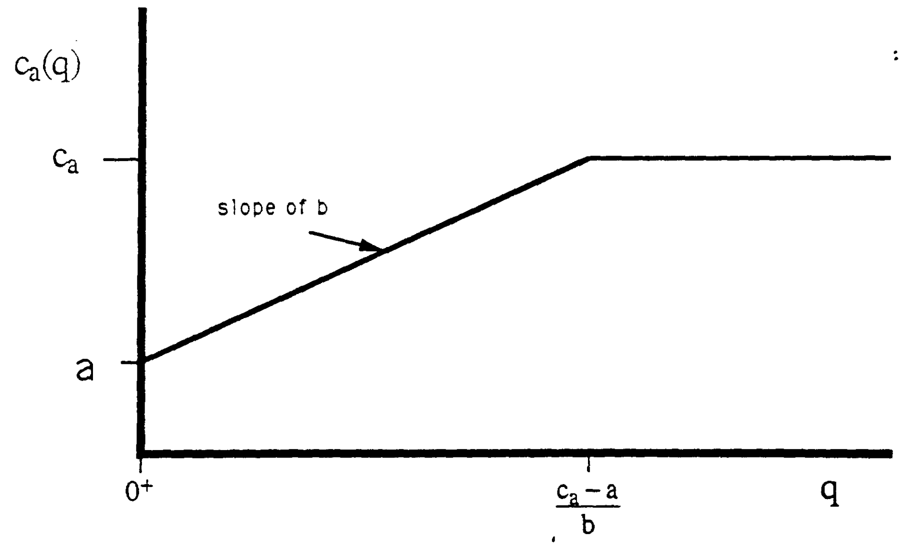
It might appear that the consumption-minimizing policy is too complicated to implement, but there is a class of policies that minimize consumption, and not all of them are complicated. For example, imposing no penalty for $q \leq \bar{q}$ and the maximum penalty c_a for $q > \bar{q}$ minimizes consumption. Nevertheless, it may be that none of these are politically feasible because they all impose no punishment for possession of quantities up to the threshold quantity $\bar{q} > 0$.

3.4 Consumption Minimizing Linear Policy

It has been shown that a maximum punishment policy does not minimize consumption. The previous section described the policies that do minimize consumption, but noted that they may not be politically feasible because they impose no punishment for quantities less than the threshold $\bar{q}$. This section finds the consumption minimizing policy within a restricted class of policies that might realistically be implemented.

One can only guess at the kinds of punishment policies that are politically acceptable, but given public sentiment at the moment, one restriction might be to consider only those policies that assign some punishment for possession of any positive amount, no matter how small[1]. Also, legislators might quite reasonably object if the punishment were calculated by some complicated mathematical expression that the public could not understand. This suggests restricting attention to linear punishment policies. Then legislators would just have to specify the minimum punishment, the maximum punishment, and the smallest quantity that merits the maximum punishment, and then draw a straight line connecting these points. Figure 5 shows a typical linear punishment policy.

Figure 5
A Typical Linear Punishment Policy



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[1]This sentiment is reflected in the words of the recently released National Drug Control Strategy, "we need a national enforcment strategy that casts a wide net and seeks to ensure that all drug use -- whatever its scale -- faces the risk of criminal sanction.", (Washington, D.C.: U.S. Government Printing Office, September, 1989), p.18.

Mathematically the problem is the same as the one in the previous section except the punishment policy must be selected from a smaller set. Specifically, it is

$$\underset{c_a(q)\in G}{\text{Min}} \left\{ f^* q^* \mid (f^*, q^*) = \arg \underset{f,q \geq 0}{\text{Max}} \left\{ z(f,q) = \alpha\sqrt{fq} - hq - (c_s + p\, c_a(q))\, f - c_p\, f\, q \right\} \right\} \quad \text{(P 3)}$$

$$G = \left\{ g: \Re \to \Re \mid g(0) = 0,\ g(q) = a + bq \text{ with } a,b \geq 0 \text{ for } 0 < q \leq \frac{c_a - a}{b},\ g(q) = c_a \text{ for } q \geq \frac{c_a - a}{b} \right\}$$

Theorem 2 gives the solution. The superscript ^ is used to denote the optimal quantities for this problem. Again, the proof is rather technical and can be skipped.

<u>Theorem 2.</u> If $\alpha \leq 2\sqrt{h(c_s + p\, c_a)}$ then setting $c_a(q) = c_a$ gives

$$\hat{q} = \hat{f} = 0$$

If $\alpha > 2\sqrt{h(c_s + p\, c_a)}$ the consumption minimizing policy has an intercept of 0 and slope

$$\frac{c_p}{p} \frac{\alpha\left(\sqrt{h(c_s + p\, c_a)} - \sqrt{h\, c_s}\right) - h\, p\, c_a}{\left(\frac{\alpha}{2} - \sqrt{h(c_s + p\, c_a)}\right)^2}$$

which gives

$$\hat{q} = \frac{\left(\frac{\alpha}{2} - \sqrt{h(c_s + p\, c_a)}\right)^2}{\left(\frac{\alpha}{2} - \sqrt{h\, c_s}\right) c_p} \sqrt{\frac{c_s}{h}},$$

$$\hat{f} = \frac{\left(\frac{\alpha}{2} - \sqrt{h(c_s + p\, c_a)}\right)^2}{\left(\frac{\alpha}{2} - \sqrt{h\, c_s}\right) c_p} \sqrt{\frac{h}{c_s}},$$

$$\hat{q}\hat{f} = \left(\frac{\left(\frac{\alpha}{2} - \sqrt{h(c_s + p\, c_a)}\right)^2}{\left(\frac{\alpha}{2} - \sqrt{h\, c_s}\right) c_p} \right)^2, \text{ and}$$

$$z(\hat{f}, \hat{q};\, c_a(q) = b^* q) = \frac{\left(\frac{\alpha}{2} - \sqrt{h(c_s + p\, c_a)}\right)^2}{c_p}.$$

<u>Proof:</u> If $\alpha \leq 2\sqrt{h(c_s + p\, c_a)}$ then setting $c_a(q) = c_a$ for all $q > 0$ is optimal because it yields the trivial solution.

Suppose $\alpha > 2\sqrt{h(c_s + p\, c_a)}$. It is difficult to solve over G directly, but if the feasible set G is replaced by

$$G_2 = \left\{ g: \Re \to \Re \mid g(0) = 0,\ g(q) = a + bq \text{ with } a,b \geq 0 \text{ for } q > 0 \right\}$$

then the Appendix gives the solution to the inner optimization problem in closed form. This can be taken advantage of by solving the problem with $G = G_2$ subject to the additional constraint that the user's utility must be at least as great as that obtained when $c_a(q) = c_a$. For convenience, that value will be denoted by κ. Thus,

$$z(f^*, q^*, c_a(q) = a + bq) \geq z(F, Q, c_a(q) = c_a) = \frac{\left(\frac{\alpha}{2} - \sqrt{h(c_s + p\,c_a)}\right)^2}{c_p} = \kappa.$$

This constraint eliminates all solutions admitted when $G = G_2$ that should not be allowed since, if the solution has $a + bq^* > c_a$, then $z(f^*, q^*, c_a(q) = a + bq) \leq z(f^*, q^*, c_a(q) = c_a) \leq z(F, Q, c_a(q) = c_a)$. Hence the problem reduces to

$$\underset{a,b \geq 0}{\text{Min}} \left(\frac{\left(\frac{\alpha}{2} - \sqrt{h(c_s + p\,a)}\right)^2}{c_p + p\,b} \right)$$

$$\text{subject to} \quad g(a,b) = \kappa - \frac{\left(\frac{\alpha}{2} - \sqrt{h(c_s + p\,a)}\right)^2}{c_p + p\,b} \leq 0.$$

If $\alpha > 2\sqrt{h(c_s + p\,a)}$ the objective function is decreasing in both a and b and g(a,b) is increasing. This means the solution will satisfy the inequality as an equality and thus the problem reduces to

$$\underset{a,b \geq 0}{\text{Min}} \quad \frac{\kappa}{c_p + p\,b}$$

$$\text{subject to} \quad b(a) = \frac{c_p}{p} \frac{\alpha\left(\sqrt{h(c_s + p\,c_a)} - \sqrt{h(c_s + p\,a)}\right) + h\,p\,(a - c_a)}{\left(\frac{\alpha}{2} - \sqrt{h(c_s + p\,c_a)}\right)^2}.$$

Since the objective function is decreasing in b and $\frac{db(a)}{da} < 0$ when $\alpha > 2\sqrt{h(c_s + p\,a)}$, the optimal linear policy is to choose

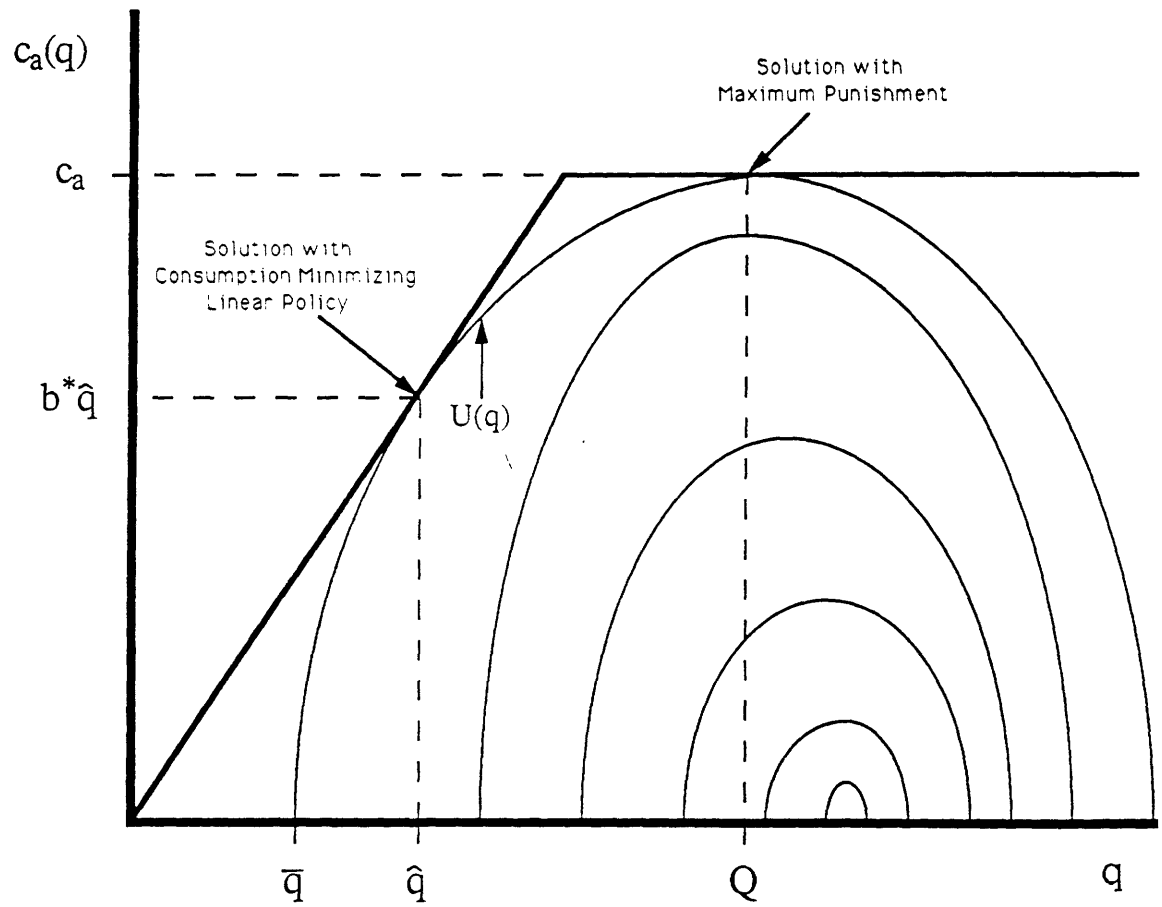
$a^* = 0$ and

$$b^* = \frac{c_p}{p} \frac{\alpha\left(\sqrt{h(c_s + p\,c_a)} - \sqrt{h\,c_s}\right) - h\,p\,c_a}{\left(\frac{\alpha}{2} - \sqrt{h(c_s + p\,c_a)}\right)^2} \geq 0.$$

Note, $b^* > c_a/Q$, so the optimal linear policy has a steeper slope than the proportional policy considered above, and thus $a^* + b^*Q = b^*Q > c_a$. (See Figure 6.)

Substituting these values of a^* and b^* into the expressions in Section 2.2 gives the desired result. QED.

Figure 6
The Consumption Minimizing Linear Policy



The expression for the optimal slope is complicated, but some algebraic manipulation shows it is greater than c_a/Q, and so it is steeper than the slope of the linear policy considered in Section 3.2. (See Figure 6.)

Note the consumption minimizing policy's intercept is zero. It imposes the lightest possible sanctions for possession of trace amounts. This is consistent with the previous section's finding that the overall consumption minimizing policy calls for no punishment whatsoever for amounts up to some positive threshold.

Looking at Figure 6 shows why this is so. The policy maker must pick a straight line connecting the vertical axis with the horizontal line $c_a(q) = c_a$. The new equilibrium point will be at the place where this line is tangent to an indifference curve, either U(q) or one below U(q). The new equilibrium point will be farthest to the left (yielding the lowest consumption) if the line is tangent to U(q) itself and the slope is as steep as possible. This is accomplished by making the line's vertical intercept zero.

3.5 Policy of Not Punishing Users

Before comparing the policies discussed above, it is useful to consider what would happen if users were not punished, i.e. if $c_a(q) = 0$ for all q. Note, this is not a model of legalization because the search cost and price would almost certainly fall substantially if the drug were made legal. It is closer to a "decriminalization" policy that leaves antidrug laws on the books, but does not enforce them against users.

The solution can be obtained directly from the expression in Section 2.2 because this is the special case of Problem P1 with a = b = 0. Italics are used to denote optimal values when there is no punishment. If $\alpha \le 2\sqrt{h\,c_s}$,

$$q = f = 0,$$

but if $\alpha > 2\sqrt{h\,c_s}$ the soluton is

$$q = \left[\frac{\alpha}{2}\sqrt{\frac{c_s}{h}} - c_s\right]\frac{1}{c_p},$$

$$f = \left[\frac{\alpha}{2}\sqrt{\frac{h}{c_s}} - h\right]\frac{1}{c_p},$$

$$qf = \left(\frac{\frac{\alpha}{2} - \sqrt{h\,c_s}}{c_p}\right)^2, \text{ and}$$

$$z(f, q; c_a(q) = 0) = \frac{\left(\frac{\alpha}{2} - \sqrt{h\,c_s}\right)^2}{c_p}.$$

4.0 Discussion

It is assumed here that the objective is to minimize consumption, so in this section the adjective "optimal" is used in place of the term "consumption minimizing".

4.1 Comparison of Policies

The five policies considered above are compared below. Table 1 reviews the notation used for each policy.

Table 1
Summary of Notation Used for Different Policies

Capital	Q	Section 3.1: Maximum Punishment	$c_a(q) = c_a$	
Tilda	$\tilde{q}$	Section 3.2: First Linear Policy	$c_a(q) = \frac{c_a}{Q} q$	$0 \le q \le Q$
			$c_a(q) \ge c_a$	$q \ge Q$
Bar	$\bar{q}$	Section 3.3: Optimal Policy	$c_a(q) = 0$	$0 \le q \le \bar{q}$
			$c_a(q) = c_a$	$q \ge \bar{q}$
Hat	$\hat{q}$	Section 3.4: Optimal Linear Policy	$c_a(q) = b^* q$	$0 \le q \le \frac{c_a}{b^*}$
			$c_a(q) = c_a$	$q \ge \frac{c_a}{b^*}$
Italics	q	Section 3.5: No Punishment	$c_a(q) = 0$	

If $\alpha \le 2\sqrt{h(c_s + p\,c_a)}$ the optimal linear policy, overall optimal policy, and maximum punishment policy all give the same solution, so for purposes of comparison, it is assumed that $\alpha \ge 2\sqrt{h(c_s + p\,c_a)}$.

Some tedious algebra allows one to rank the policies in terms of their ability to reduce consumption.

1) Optimal Policy
2) Optimal Linear Policy
3) First Linear Policy
4) No Punishment
5) Maximum Punishment

The ranking for the purchase size, q, and the frequency of purchase, f, depends on the parameter values. Specifically, if $\alpha > 2\sqrt{h}\left(\sqrt{c_s + p\,c_a} + \sqrt{c_s}\right)$ then in order of increasing purchase size the ranking is

1) Optimal Policy
2) Optimal Linear Policy
3) First Linear Policy
4) No Punishment
5) Maximum Punishment

and in order of increasing purchase frequency it is

1) Maximum Punishment
2) Optimal Linear Policy
3) First Linear Policy
4) No Punishment.

If $\alpha < 2\sqrt{h}\left(\sqrt{c_s + p\,c_a} + \sqrt{c_s}\right)$, however, then in order of increasing purchase size the ranking is

1) Optimal Policy
2) Optimal Linear Policy
3) First Linear Policy
4) Maximum Punishment
5) No Punishment

and order of increasing purchase frequency it is

1) Optimal Linear Policy
2) Maximum Punishment
3) First Linear Policy
4) No Punishment

The user's utility at equilibrium can be similarly ranked in decreasing order:

1) No Punishment
2) First Linear Policy
3) Maximum Punishment (3 way tie)
   Optimal Policy
   Optimal Linear Policy.

Figures 7 and 8 on the following pages show these comparisons graphically. When two quantities appear to be equal in the figures it indicates that their relative size depends on the parameter values.

Figure 7
Comparison of Five Punishment Policies

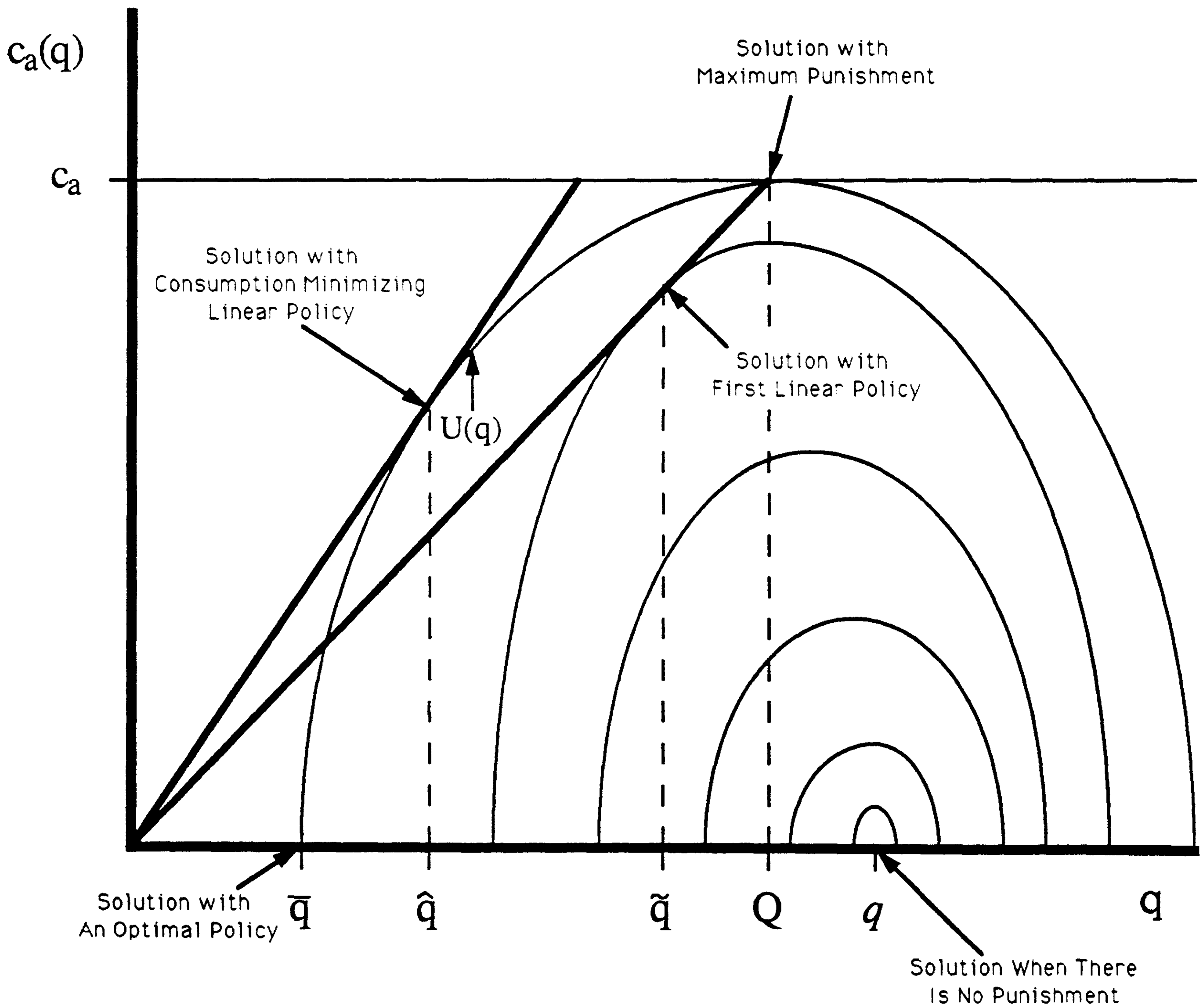
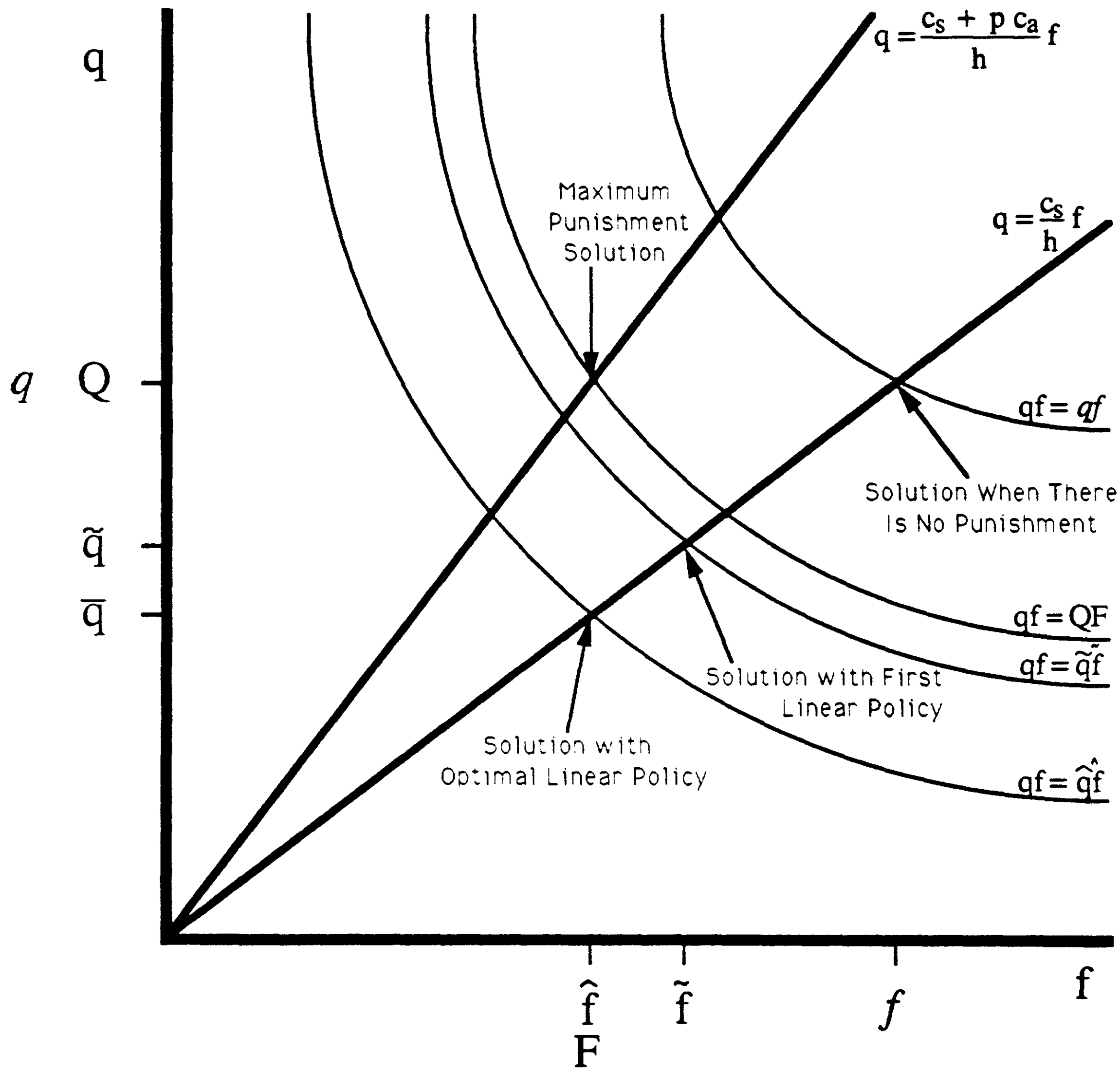


Figure 8
Comparison of Policies in the q-f Plane



4.2 Potential for Reducing Consumption by Changing Policy

Simply signing quantities is not always very helpful. For example, Figure 8 shows that switching from a maximum punishment policy to the optimal linear policy will reduce consumption, but by how much? A 50% reduction might justify overcoming political and bureaucratic obstacles that a 5% reduction might not.

In general it is difficult to do more than determine relative magnitudes because the parameter values are not known, but some algebra shows that

$$\frac{\widehat{qf}}{QF} = \frac{QF}{qf}.$$

That is, the ratio of consumption under the optimal linear policy to that under a maximum punishment policy is the same as the ratio of consumption under maximum punishment to consumption with no punishment. Thus, one can (theoretically) achieve the same percentage reduction in consumption going from a maximum punishment policy to a proportional punishment policy as one can going from no punishment to maximum punishment!

The United States essentially has a maximum punishment policy. While penalties may increase with quantity, generally the upper limit of the lowest punishment category is more than is usually held for personal consumption[1]. Thus, the punishment facing an individual user is essentially independent of quantity[2]. (Actually there may be some quantity dependence because larger quantities may encourage police to collect evidence more carefully and the district attorney to prosecute more vigorously, but this effect is probably relatively minor.)

Also, most people concede that there is at least a reasonable chance that consumption would increase appreciably if users were not threatened with punishment. This paper suggests that if that is the case, one could hope for further appreciable reductions by reforming laws to make punishment proportional to the quantity possessed. Such a reform would be in keeping with the national drug strategy which states that "Punishment should be flexible -- let the penalty fit the nature of the crime."[3]

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[1] Controlled Substances Act, Chapter 13, Subchapter I, Part D, Section 841.

[2] Tracy Thompson's "Can Justice Be A Little Too Blind?" (The Washington Post National Weekly Edition, September 4-10, 1989, pp.31-32) gives examples when stiff mandatory penalties have been imposed on people who might have been treated more leniently until recently.

[3] National Drug Control Strategy, p.19.

There are at least two significant qualifications to this conclusion, however. They are discussed in the next section.

4.3 Qualifications to the Conclusion Above

The discussion above is based on the observation that

$$\frac{\hat{qf}}{QF} = \frac{QF}{qf}.$$

However, the fact that the ratios are exactly equal depends on the assumption that the satisfaction derived from using increases as the square root of the quantity consumed, and clearly that is somewhat of an arbitrary assumption. One would expect the function to be concave because of diminishing returns, but its exact form is unknown and unlikely to be so simple. It is quite possible, however, that the basic conclusion is more robust than the exact equality of the ratios. That conclusion is that if moving from no punishment to maximum punishment can significantly reduce consumption, it is likely that moving to a proportional policy will lead to further significant reductions.

The second and more serious qualification is that the slope of the optimal linear policy depends on the individual user's utility parameters. It is difficult to imagine how one might measure the utility parameters accurately enough to quantitatively compute the optimal slope. Furthermore, the optimal slope varies from individual to individual.

The problem with not knowing the users' utility parameters is most easily illustrated by considering an attempt to implement a policy of no punishment when $q \le \bar{q}$ and maximum punishment if $q > \bar{q}$. Since $\bar{q}$ is unknown, the policy would actually be

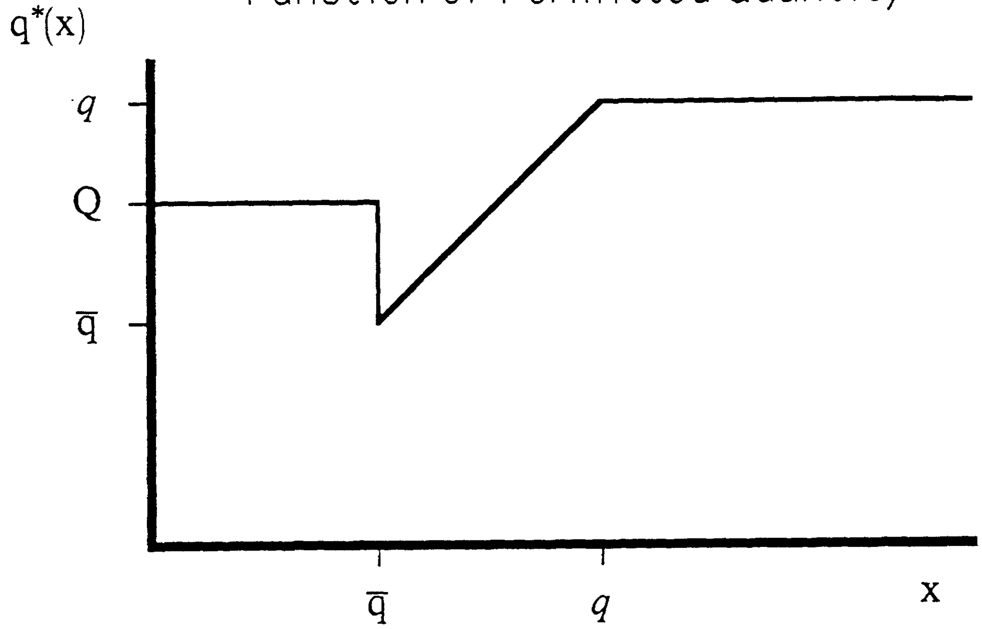
$$c_a(q) = 0 \qquad 0 \le q \le x$$
$$c_a(q) = c_a \qquad q \ge x$$

where x is some estimate of $\bar{q}$. Figure 9 shows how the resulting purchase size depends on x. If $x < \bar{q}$ (the government underestimates $\bar{q}$), the purchase size and rate of consumption will be the same as they would be under a policy of maximum punishment. If $x = \bar{q}$ the new policy will work as desired. As x increases beyond $\bar{q}$ (the government overestimates $\bar{q}$), the purchase size increases linearly. Solving for the optimal frequency, one finds that for $\bar{q} < x < q$,

$$qf = \left(\frac{\alpha x}{2(c_s + c_p x)}\right)^2,$$

so the rate of consumption rises with x until $x = q$. At that point, consumption levels off at qf, the rate of consumption when there is

Figure 9
Purchase Size As a
Function of Permitted Quantity



no punishment. That rate may be significantly higher than the original rate QF.

Hence, if the government overestimates $\bar{q}$, changing the punishment policy will have no effect on consumption, and if it underestimates $\bar{q}$ badly enough, consumption might actually increase!

Attempting to implement the optimal linear punishment policy poses a similar dilemma. If the slope is too steep, changing from a policy of maximum punishment will have no effect, but if the slope is too shallow changing the punishment policy may increase rather than decrease consumption.

4.4 Further Work

This section discusses some possible directions for further work. The first follows from the preceding discussion. Since the user's utility parameters are not known, a sensitivity analysis would be desirable. One could postulate a probability density function for the utility parameters and try to find a policy that minimizes the expected rate of consumption. If the probability density function represented the empirical distribution of parameters across the entire population, then the resulting policy would minimize aggregate consumption.

It is difficult to predict the results of such an analysis. One may discover that the optimal policy is insensitive to certain parameters. At the other extreme, uncertainty about the parameters might favor maintaining a maximum punishment policy that avoids the risk of inadvertently increasing consumption in the manner discussed above.

Viewing the probability density function as the statistical distribution of users' parameters suggests considering objectives other than minimizing consumption. For example, policy makers might not be indifferent between having a heavy user increase consumption by two units per day and having two non-users begin using one unit per day.

Other directions for additional work include assessing the sensitivity of the model as a whole to certain of its assumptions. For example, what happens if the satisfaction derived from using does not grow as the square root of the quantity consumed or the probability of arrest per purchase is not independent of the frequency of purchase? Similarly, one could ask about the effects of quantity discounts. Changes of this nature jeopardize the model's analytical tractability, so much of the evaluation would probably be numerical. But it would be valuable to know if the main conclusion about the potential for proportional punishment to reduce consumption continues to hold.

Finally, the model currently assumes search time costs, the probability of arrest, and price are all fixed constants. Although policy makers do not set these parameters directly, they clearly are not exogenous constants. Interesting optimization questions can be posed by assuming policy makers can affect these parameters (within limits) at some cost.

4.5 Limitations of the Model

The model discussed in this paper completely ignores several important issues. Perhaps foremost among these are questions about substitution between different kinds of drugs, both licit and illicit. A classic example of this is the possibility that clamping down on marijuana might lead to increased use of potentially more dangerous substances such as cocaine. Less direct substitution effects may exist as well. For example, severe punishment for possession of heroin may have enhanced the popularity of cocaine, and widespread use of cocaine may now be fueling demand for heroin for "speedballing"[1]. The current model says nothing about coordinating punishment policies for different drugs.

The model is also static. A variety of parameters may change over time. In particular, the model does not keep track of the

[1] "Speedballing" is the slang term for using heroin and cocaine together.

individual's arrest history even though punishment (and perception of punishment) generally depend on past arrests. Hence, the model cannot even consider questions about special policies for repeat offenders.

Finally, the model uses point estimates for q and f. Clearly consumption is not perfectly regular. But allowing for some distribution of purchase quantities and inter-purchase intervals would probably make the analysis much more complicated without producing significant insight. Nevertheless, it is important to remember that this simplification has been made.

5.0 Summary

This paper argues that a "zero tolerance" policy of imposing a maximum level of punishment irrespective of the quantity possessed may not minimize consumption. Switching from such a policy to one in which punishment is proportional to quantity could increase some users' welfare while simultaneously bringing reductions in consumption comparable to those achieved by moving from no punishment to a policy of maximum punishment.

However, for a variety of reasons the model is not suitable for quantitatively computing the optimal punishment policy. First of all, the model makes many assumptions, for example that search time costs and the probability of arrest per purchase are independent of the frequency of purchase, that may not hold exactly. It also ignores important issues such as substitution between drugs, the treatment of repeat offenders, and the ability of policy makers to control parameters such as price, search time costs, and the probability of arrest. Most significantly though, the policies prescribed are functions of the user's utility parameters. These parameters cannot be measured, and even if they could be, they vary from user to user.

These points, however, do not negate the paper's principal conclusion that a "zero tolerance" policy for users may not minimize consumption. There are other compelling reasons for not following a "zero tolerance" policy. It commits scarce enforcement resources to relatively minor offenders, crowds courts and prisons, and violates the principle that "the punishment should fit the crime". Nevertheless, perhaps in response to calls to "get tough" on drugs, the United States currently has essentially a "zero tolerance" policy for users. This paper suggests that the desire to be "tough on drugs", at least in as much as that represents a desire to minimize consumption, should not preclude consideration of a policy that makes punishment proportional to the quantity possessed.

Appendix

The general form of this problem is

$$\text{Max } z(f,q) = \alpha\sqrt{fq} - \beta q - \gamma f - \delta fq$$
$$\text{s.t.} \quad q,f \geq 0.$$

The first order conditions are

$$\nabla z(f,q) = \begin{bmatrix} \frac{\partial z(f,q)}{\partial f} \\ \frac{\partial z(f,q)}{\partial q} \end{bmatrix} = \begin{bmatrix} \frac{\alpha}{2}\sqrt{\frac{q}{f}} - \gamma - \delta q \\ \frac{\alpha}{2}\sqrt{\frac{f}{q}} - \beta - \delta f \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

The Hessian

$$\nabla^2 z(f,q) = \begin{bmatrix} -\frac{\alpha}{4}q^{\frac{1}{2}}f^{\frac{-3}{2}} & \frac{\alpha}{4}q^{\frac{-1}{2}}f^{\frac{-1}{2}} - \delta \\ \frac{\alpha}{4}q^{\frac{-1}{2}}f^{\frac{-1}{2}} - \delta & -\frac{\alpha}{4}f^{\frac{1}{2}}q^{\frac{-3}{2}} \end{bmatrix}$$

is negative definite because its first principal minor is less than zero,

$$P_1 = -\frac{\alpha}{4}q^{\frac{-1}{2}}f^{\frac{-3}{2}} < 0$$

and its second principal minor

$$P_2 = \left(\frac{\alpha}{4\sqrt{qf}}\right)^2 - \left(\frac{\alpha}{4\sqrt{qf}} - \delta\right)^2 > 0$$

is positive. Hence, z(f,q) is concave, and solving the first order conditions gives the unique global maximum at

$$q^* = \frac{\alpha}{2\delta}\sqrt{\frac{\gamma}{\beta}} - \frac{\gamma}{\delta}$$
$$f^* = \frac{\beta}{\gamma}q^* = \frac{\alpha}{2\delta}\sqrt{\frac{\beta}{\gamma}} - \frac{\beta}{\delta}.$$

If $\alpha \leq 2\sqrt{\beta\gamma}$ these expressions are negative so

$$z^* = f^* = q^* = 0,$$

but if $\alpha > 2\sqrt{\beta\gamma}$, q^* and f^* yield a nontrivial solution with

$$q^* f^* = \left(\frac{\frac{\alpha}{2} - \sqrt{\beta\gamma}}{\delta}\right)^2, \text{ and}$$

$$z(f^*, q^*) = \delta\, q^* f^* = \frac{\left(\frac{\alpha}{2} - \sqrt{\beta\gamma}\right)^2}{\delta}.$$