THERMAL HISTORY MODELS OF ICY SATELLITES

by

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Accepted by

Chairman, Departmental Committee

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THERMAL HISTORY MODELS OF ICY SATELLITES

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Submitted to the Department of Earth and Planetary Sciences on May 6, 1975 in partial fulfillment of the requirements for the degree of Master of Science.

Time-dependent thermal histories for the icy satellites are presented. A number of starting hypotheses are used and compared, including homogeneous and inhomogeneous accretion, and variations in the amount and density of rocky material present. Phase changes in high pressure forms of water ice, and the solution chemistry of water and ammonia are considered. Application of the various models to data observed of actual icy satellites is presented.

The computer programs used to model these objects are explained in Appendix A. Appendix B presents further speculation on the nature of icy satellites, and suggests areas of further research.

Thesis Supervisor: John S. Lewis
Title: Associate Professor of Geochemistry and Chemistry
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I. Introduction

There are 33 known planetary satellites in the solar system: Earth's Moon, Phobos and Deimos, the 13 satellites of Jupiter, Saturn's 10, Uranus' 5, and Neptune's 2. Of most of these we have very little direct knowledge, except that they exist. We have reliable radius or mass measurements for only half of these bodies, and accurate density data for only 6. (see Table 1).

Earth's Moon has been visited. Phobos and Deimos have been photographed close-up, as have Ganymede and Callisto, although these pictures are of lesser quality and have yet to be published. Surface reflectance data exist for these bodies, and also for the other Galilean satellites and certain of Saturn's moons.

But from this scant data we can begin a rough classification of planetary satellites. They can be broadly divided into two groups, "rocky" and "icy". The presence of rocky satellites we know first hand, examples being the Moon, Phobos, and Deimos. In addition, we can infer from the observed density of Io (3.5 g/cm$^3$) that it too is rocky.

On the other hand, a large number of satellites we observe appear to have low densities (1-2 g/cm$^3$), high albedoes, or observed water frost spectra. We can infer that these are made largely of ices, chemical compounds not usually considered to be rock-forming minerals because of their high volatility --
they are not stable as solids at standard temperatures and pressures -- yet which most likely will be solid at the black body temperatures in the solar system beyond the asteroid belt (Lebofsky, 1975).

But the presence of ices as a large weight fraction of these satellites does not preclude the coexistence of other, more conventional rock-forming materials. In fact, the density of Ganymede (2.0 g/cm$^3$) is far higher than any common icy material, and must have a considerable amount of silicate present (or possibly, be made entirely of low-density silicates).

At this point we leave the realm of observable data, and enter that of inductive modelling. From this scant information we can hardly find a unique solution to the question of the chemical nature of the icy satellites. What we can do, however, is look at various end-members of possible ways in which these satellites may have been put together. Then, starting with each of these possibilities, we can apply the known chemical behavior of the constituents of each possibility, making assumptions where we have to, and deduce how such a possibility would have evolved over the age of the solar system. The theoretically deduced present-day properties of these bodies can then be used to assist in the design of spacecraft experiments, and in the eventual interpretation of future observational data.

Our primary interest lies in the ice phases of these
bodies, since they are the most susceptible to changes in temperature and pressure. As a result, we have presumed that the rocky portions of these objects can be described as ferro-magnesium silicates. In fact, the rock parts will likely consist of a mixture of a number of rock-forming materials, possibly with metallic and non-silicate components. But their main component will almost certainly be ferro-magnesium silicates since these have a higher cosmic abundance; and we shall refer to, and model, the rocky material as such.
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<tr>
<th>Satellite</th>
<th>Radius (km)</th>
<th>R&lt;sub&gt;max&lt;/sub&gt;</th>
<th>R&lt;sub&gt;min&lt;/sub&gt;</th>
<th>Mass (10&lt;sup&gt;23&lt;/sup&gt; g)</th>
<th>Density (g/cm&lt;sup&gt;3&lt;/sup&gt;)</th>
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<td>(8)</td>
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<td>Radius (km)</td>
<td>$R_{\text{max}}$</td>
<td>$R_{\text{min}}$</td>
<td>Mass ($10^{23}$ g)</td>
<td>Density (g/cm$^3$)</td>
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<td></td>
<td>1401±2</td>
<td>2.14±0.8$^2$</td>
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<td>Hyperion</td>
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<td>460</td>
<td>80</td>
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<td></td>
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<td>900±100</td>
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<td>1000</td>
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<td>Nereid</td>
<td></td>
<td>600</td>
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Table 1 (continued)

Masses from satellite commensurabilities, except Triton which is found from its effect on Neptune

Radii: Phobos and Deimos are from Mariner 9 imagery
Io and Ganymede are from stellar occultation
Callisto, Europa, and Titan are from ground-based measurements
Iapetus, Rhea, Dione are from radiometric/photometric techniques

$R_{\text{min}}$: Radius calculated from brightness assuming an albedo of 1.0

$R_{\text{max}}$: Radius calculated from brightness assuming an albedo of 0.03 (lowest known in solar system) or a diameter of 1/3 " (big enough to be seen as a disk)

Figures in parentheses are guesses, using albedo of 0.1 for irregular, 0.8 for regular

Notes: 1. Pioneer 10 data gives preliminary mass for Io of 900, density of 3.5
2. Lunar occultation measurements of Titan give radius of 2900, density of 1.4
Chapter 1. Chemical Composition Models

The chemical composition and structure of satellites depends on the sequence of condensation and accretion of the solar nebula. In particular, one must consider whether the condensation sequence occurred under conditions of equilibrium or disequilibrium, and whether the accretion of the material occurred differentially, with each object accreting from one component of the condensation sequence, or integrally, with all components of the sequence contributing cumulatively to objects.

The two chemical alternatives for each of the two accretion modes leads to four possible classes of models for satellites. The condensation sequence for a solar nebula with solar composition has been found for both equilibrium and disequilibrium cases by Lewis (1972). From this, the four models can be detailed as follows.

1.1 Model 1: Equilibrium condensation, integral accretion
At temperatures of 350°K, material similar to that of carbonaceous chondrites type Cl will have condensed from the solar nebula. This material is primarily iron and magnesium silicates, with carbon compounds, and hydrous silicates. The uncompressed density of such material would be \(\approx 2.4 \text{ g/cm}^3\). If the condensation temperature dropped below \(\approx 160°K\), the satellite would be formed, according to this model, of a uniform mixture roughly half water ice and have material (by
weight), with an overall density of 1.6-1.8 g/cm$^3$. Below \(\sim 100^\circ\text{K}\), we would expect the ice phase to contain a hydrate of \(\text{NH}_3\cdot\text{H}_2\text{O}\). Below \(\sim 60^\circ\text{K}\), methane clathrate with composition \(\text{CH}_4\cdot x\text{H}_2\text{O}\) (\(x\) being roughly 6.7) formed by complete reaction of \(\text{H}_2\text{O}\) ice with \(\text{CH}_4\) gas will dominate, lowering the density to roughly 1.5 g/cm$^3$. At temperatures below \(\sim 20^\circ\text{K}\), methane ice and possibly ices of argon and the other rare gases would be included, lowering the total density of the body to 1 g/cm$^3$.

Note that in any body formed by this model at temperatures below 160$^\circ\text{K}$, where ice will be an important constituent, there will be gravitational potential energy stored by the dense silicates which are distributed uniformly through the less-dense ices.

1.2 Model 2: Equilibrium condensation; differential accretion Here the condensation is the same as in the previous model, but the accretion occurs differentially, with each member of the condensation sequence forming its own type of body. Under this model, a satellite which condensed at temperatures above 160$^\circ\text{K}$ would have carbonaceous chondrite type Cl composition; one condensing between 160$^\circ\text{K}$ and 100$^\circ\text{K}$ would be made of water ice; one between 100$^\circ\text{K}$ and 60$^\circ\text{K}$ would be made of the ammonia hydrate \(\text{NH}_3\cdot\text{H}_2\text{O}\); below that, down to 30$^\circ\text{K}$, methane clathrate hydrate bodies would be formed; and at any lower temperatures, bodies of methane or rare gas ices would be formed.
The carbonaceous chondrite satellite would have a zero-pressure density \( p_0 \) of \( \approx 2.4 \text{ g/cm}^3 \), while satellites made of any of the ices except methane would have densities of about \( p_0 = 1 \text{ g/cm}^3 \). The pure methane ice body would have a density of \( p_0 \approx 0.6 \text{ g/cm}^3 \).

1.3 Model 3: Disequilibrium condensation; integral accretion  Here we assume that as each material condenses and accretes, it will cover previously condensed material, preventing it from reacting with uncondensed gases in the solar nebula. Thus, the planet is built up, layer by layer, without allowing chemical equilibrium to be achieved between the layers or between the layers and the nebular gas.

The core of such a body would consist of "dry rock", primarily metallic iron and magnesium silicates. It would have a mean density greater than \( 3.7 \text{ g/cm}^3 \). At \( \approx 160^\circ \text{K} \), a layer of water ice with up to twice the mass of the rocky core would be deposited over it, lower the density of the total body to \( \approx 1.6 \text{ g/cm}^3 \). Since this layer would contain all the water in the system, ammonia could not form a hydrate but would be free to react with \( \text{H}_2\text{S} \) gas, and at \( 140^\circ \text{K} \), \( \text{NH}_4\text{SH} \) ice would be deposited over the water ice. Assuming solar proportions of the elements in the nebula, 80% of the ammonia would remain after the \( \text{H}_2\text{S} \) has been exhausted, and at temperatures below \( 100^\circ \text{K} \) this would condense to form a layer of \( \text{NH}_3 \) ice over the \( \text{NH}_4\text{SH} \) ice. At temperatures below \( 20^\circ \text{K} \), \( \text{CH}_4 \) and argon ices would
form a top layer on the body, lowering its density to $\sim 1 \text{ g/cm}^3$.

Thus, we would have a satellite already differentiated as it was formed. Since the densest material is already in the core, and the remaining ices are deposited roughly in order of decreasing density, there would be no significant gravitational potential energy stored in this system. However, since each layer was condensed out of chemical equilibrium with the previously condensed layers, there will be stored chemical potential energy.

1.4 Model 4: Disequilibrium condensation; differential accretion In this model, each material accretes differentially into its own body, as in Model 2, but the accretion is rapid enough that the material is never allowed to equilibrate with the remaining gases in the solar nebula, so that the disequilibrium condensation sequence is followed, as in Model 3.

Thus, there would be formed dry rock bodies, with zero pressure densities greater than $3.7 \text{ g/cm}^3$; water ice bodies of $\rho_O = 1 \text{ g/cm}^3$ if condensation temperatures fell below $160^\circ\text{K}$; NH$_4$SH bodies of density $\rho_O = 1 \text{ g/cm}^3$ below $100^\circ\text{K}$; and CH$_4$ and rare gas ice bodies of density $\rho_O = 0.6 \text{ g/cm}^3$ below $20^\circ\text{K}$.

1.5 In summary, therefore we see that the chemical composition of an icy satellite will be a function both of its accretion sequence and the temperature of the accreting material. Each of the four accretion models here will provide
a series of satellite chemical composition for a sequence of formation temperatures.

1.6 We might also speculate on what sort of conditions might be necessary for the accretion of members of each of these models. The Model 1 case could occur if accretion of the nebula material were slow compared to condensation time, so that all components have the time and opportunity to equilibrate with each other and the nebula gas, and if the major force for accretion were some non-selective force such as mutual gravitation, as suggested by Goldreich and Ward (1973), Weidenschilling (1974), and others.

The Model 2 case also requires slow accretion, but assumes that chemical affinities of like material will influence accretion, i.e. ices will "stick" to each other and accrete together preferentially over rocky materials, paramagnetic crystals will be preferentially swept together by a solar magnetic field, etc.

Model 3 can be accomplished by rapid accretion; as each component is condensed, it is accreted by some "general" force as in Model 1 into the final planet. Each fresh condensate accretes so as to cover the material accreted previously, isolating it from further reactions with the solar nebula.

Model 4 also assumes rapid accretion, and some "specific" accretion force. As each body accretes, only the material on its surface (a very small fraction of its total mass) is
available for reactions with the solar gas, and so its chemistry would follow the disequilibrium condensation sequence.
2. Application of Chemical Models to Observed Satellites

As should be obvious, the formation of purely rocky satellites is possible under any of the four models of Chapter 1. The types of rocky material will vary from model to model, but differences in the models become most apparent in possible icy satellites.

Except for the rocky bodies, thermal histories of objects made by Models 2 and 4 will be uninteresting. Homogeneous single component bodies lacking internal heat sources have no endogenous means to alter their structure with time. Hence, our efforts will be concentrated on Models 1 ("homogeneous") and 3 ("heterogeneous").

2.1 Further physical considerations Water ice forms a number of phases with distinct crystal structures which are stable at various temperatures and pressures (see Figure 1). At pressures higher than 1 kilobar, the thermal histories of bodies made with water ice will be affected by the latent heats of phase changes within the ice.

The effects of such pressures on ammonia hydrate, ammonium hydrosulfide, ammonia ice, methane clathrate hydrate, or methane ice are not known, and hence cannot be modelled realistically for bodies whose central pressures exceed two kilobars.

Furthermore, the cosmic abundances of such components are by no means of such proportions to make exactly bodies of pure
Figure 1 caption: Phase diagram of water ice. Dotted lines show thermal profiles over 250 my intervals for the Callisto Model I (see Chapter 2).
ammonia hydrate (with no excess water ice), or other pure compounds. Mixtures of these compounds will occur, and the phase regions will also be a function of composition of this mixture. When melting of such a mixture occurs, it will first melt a eutectic liquid, thus changing the overall composition of the unmelted ice and the composition of the liquid will change with further melting.

Models can be made for low pressure conditions when this chemistry is known; but pressures above two kilobars may significantly shift the eutectic point, change the latent heats, and introduce high-pressure phases of ice into the mixture, almost certainly changing its chemistry. In the absence of detailed data concerning the effects of high pressure on ice mixtures, models of icy satellites where such high pressures occur will be at best highly speculative.

However, the solar system has, in its distribution of icy satellites, for once been kind to the modeller. The majority of satellites have radii below 700 km, and thus have central pressures which will not far exceed one kilobar. And the icy satellites of considerable size which we have the most information about, namely the outer Galilean satellites, are in a region of the solar system where one would expect only water ice to be stable (Lebofsky, 1975). Thus, the only satellites we might have trouble modelling are Titan, Triton, and perhaps those of the Uranian system,
about which we know so little as to make any modelling
sheerly speculative in any case.

With this in mind, we have modelled the following types
of satellites:

I. Large Icy Satellites

A. "Europa": a homogeneous mix of 10% by weight water
ice, 90% by weight silicates of density 3.7 g/cm$^3$.

B. "Ganymede", two models: i) a homogeneous mix of
20% by weight water ice, 80% by weight silicates of density 2.5;
ii) a homogeneous mix of 40% water ice, 60% silicates of
density 3.7.

C. "Callisto", three models: i) a homogeneous mix
of 60% water ice, 40% silicates of density 2.5; ii) same
proportions with the silicate density 3.7; iii) a heterogeneous
accretion model where pure water ice overlays a core of silicates
(again of density 3.7).

II. Small Icy Satellites

A. Three size classes of a homogeneous mix of 40%
by weight rocky material of density 2.5 g/cm$^3$, 60% by weight
water ice:

i) "Tethys" class, radius 500 km,

ii) "Dione" class, radius 700 km,

iii) "Iapetus" class, radius 1000 km.

B. Three size classes of a homogeneous mix of 35%
rocky material of density 2.5 g/cm$^3$, 55% water ice, and 10%
ammonia: again, "Tethys", "Dione", and "Iapetus".

III. Other possibilities

A. "Titan" clas object, 3000 km in radius, three models:
   i) homogeneous mix of 70% water ice, 30%
      silicates of density 2.5.
   ii) Similar to i, with silicate density of 3.7
   iii) Heterogeneous model, 60% by weight water ice,
      overlaying a core of silicates with density 3.7.

B. "Titania" class object, 1500 km in radius, two models:
   i) Homogeneous mix of 60% water ice, 40%
      silicates of density 2.5.
   ii) Same as i, with silicate density of 3.7

All models were run with surface temperatures of 100°K.
In addition, the "Dione", "Iapetus", and "Titania" models were re-run at an initial surface temperature of 70°K.

The results of the models are shown in Table 2, and described in further detail in the next three chapters.
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<th>mass ($10^24$ g)</th>
<th>T (°K) (surface)</th>
<th>Accretion</th>
<th>Ammonia?</th>
<th>silicates % (wt.) (g/cm$^3$)</th>
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Chapter 2. Large Icy Satellites

A. Europa

1. Data. Europa has a mass of $48 \times 10^{24}\text{g}$, as determined by the characteristics of its orbit, and a radius of 1550 km, as determined by ground-based observations, giving a total density of $3.1 \text{g/cm}^3$.

The albedo of Europa is 0.6, suggestive of an icy surface. Pilcher et al. (1972) have reported observing characteristic absorption bands of ice in the reflectance spectrum, and it is clear that ice makes up at least a small part of the satellite's chemistry.

2. Model. Since Europa's density is greater than 2.5, we assumed only the 3.7 density material for the silicates.

Assuming a density of 1.2 g/cm$^3$ for the ices, we then used the formula

$$x_{\text{sil}} = \frac{\rho_{\text{ice}}}{\rho_{\text{total}}} - \frac{\rho_{\text{sil}}}{\rho_{\text{ice}} - \rho_{\text{sil}}}$$

to determine the weight percent of ices and silicates. For Europa, this results in a mix of 10% by weight water ice, 90% silicates.

We assume a starting temperature of 100°K, and heat generation due to decay of a chondritic abundance of radioactive nuclides (see Appendix A for details of modelling). The resulting structure is illustrated in Figure 2.

We begin with a core of silicates and ice VIII,
extending to a radius of 1200 km, covered with mantles of silicates and ice VI, and silicates and ice II, with silicates and ice I as a crust on the surface.

After 250 my, substantial melting has already taken place, resulting in differentiation of the water and silicates. Melting has already proceeded completely to the surface, and a crust of pure ice I now exists. Due to the high pressures in the center, however, the melting point of ice VII has not yet been reached there and thus this region remains undifferentiated.

At 2000 my, the entire planet has been melted and differentiated. The high heat flux from the core, carried by the convecting water, has again "completely" melted the crust - any layer of ice on the surface will be too thin to be predicted by our model grid. Temperatures in the center of the core reach nearly 2000°K.

After 4500 my, a structure similar to what we expect for the present may exist. A thin crust of ice covers a convecting region of water, which is cooling off the upper layers of the silicate core. Heat production in the core has dropped as well, as the radioactive nuclides decay. However, the center is still effectively isolated from the surface, and continues to heat up, reaching temperatures of 2800°K.

The central temperature is not to be strictly believed, since here we have ignored the chemistry of the rocky material,
which will be significant long before these high temperatures are achieved. This chemistry is described briefly in discussions of the heterogeneous model of Callisto, and in Appendix B.

B. Ganymede

1. Data. Ganymede has a radius of 2635 km, known to within 10 km by stellar occultation, and a mass of $154 \times 10^{24}$ g, known as in Europa's case by orbital characteristics. This gives a mean observed density of 2.0.

The surface of Ganymede is also known to be icy, like Europa's. The surface albedo is again high, 0.4, and ice has been observed in its spectrum.

2. Model. Since Ganymede's density is less than 2.5, we can look at the possibility of low-density silicates in its makeup, as well as high-density (3.7 g/cm$^3$) material. Using the same technique as Europa, we find that the composition for Ganymede can range from 20% water, 80% silicates of density 2.5, to 40% water, 60% silicates of density 3.7.

2.1 Low-density silicate model. Starting with the 2.5 g/cm$^3$ silicate, we again set up our planet at an initial temperature of 100 K, as we did for Europa. We have at the center an ice VIII and silicate core extending to 2100 km, overlaying ice VI and silicate, and ice II and silicate mantles, and a thin ice I and silicate crust. (Figure 3.)

After 250 my, substantial melting has occurred near the
surface, resulting in differentiation from a radius of 2250 km to the surface. Under the water and silicates comes a layer of ice VI and silicates, and a core of ice VII and silicates.

By 2000 my, the planet is completely differentiated and remains so to the present. We would expect only the thinnest of icy crusts on the surface, with a thickness less than 100 km.

The central temperatures are in excess of 1000°K in the silicate core as early as 1000 million years after formation, and so we can expect any hydrous silicates will have dehydrated by this time. Such a process would require heat, resulting in a slightly cooler core than that illustrated. After dehydration, the planet with more dense silicates would probably more resemble the second model of Ganymede, using silicates of density 3.7.

2.2 High-density silicate model. This second model starts out with twice as much ice, and so needs more time to evolve. However, by 500 million years, it has reached the point illustrated in Figure 4, with a 100 km crust of undifferentiated, "dirty" ice, a 300 km region of slush (where the ice is melting), a thin mantle of water underlain by a region of silicates, then high pressure ices VI and VII with silicates.

However, by 2000 million years this too has completely differentiated, with a thicker crust than the 2.5 g/cm³ silicate model. Since this ice has been totally melted and
differentiated, it thus would look "clean". This structure would continue to the present.

C. Callisto

1. Data. The size and density of Callisto is not quite as well known as the other Galilean satellites. Its radius is 2500±150 km, and its mass roughly $(91\pm10) \times 10^{24} \text{g}$, giving a density ranging anywhere from 1 to 2 $\text{g/cm}^3$. However, using the figures of 2500 km and $91 \times 10^{24} \text{g}$, we find a density of 1.4 $\text{g/cm}^3$.

Rather than trying to fit our models to these uncertain numbers, we have instead found the silicate to ice ratio from cosmic abundances, assuming the abundances of iron, magnesium, and silicon are applied to the silicates, and any oxygen left after the formation of the silicates to go to the formation of water, following the scheme of Lewis (1971). This gives a mix of 60% by weight water ice, 40% silicates.

Despite having the lowest density of the Galilean satellites, the surface of Callisto appears to be much darker than the others, having an albedo of 0.2, similar to that of the Moon.

2. Models. Since Callisto as we are modelling it represents the type of satellite predicted by cosmic abundances of water ice and rock, it has received the most thorough modelling. We present here three extremes in modelling: a homogeneously accreted body with silicates of density 2.5; one
with silicates of density 3.7; and a heterogeneous accretion model, with silicates of density 3.7 forming the core, and various phases of pure water ice overlaying it.

2.1 Homogeneous accretion, silicate density 2.5. The first model starts, as before, at a uniform temperature of 100°C. At the center is an ice VIII and silicate region, extending to 1300 km; next is an ice VI and silicate region, extending to 1700 km; then an ice II and silicate region, to 2300 km; and finally an ice I and silicate crust, giving the planet an initial radius of 2400 km.

Figure 1 in the previous chapter gives a detailed picture of how the ice warms up and changes phase. By 600 million years after formation, the first melting occurs, at the ice V-ice VI-water triple point. The melting region grows, mostly towards the center, until at 1250 my the planet develops the structure seen in Figure 5.

By 3000 million years, the undifferentiated crust has shrunk to 150 km in thickness, with another 100 km of partly melted and differentiated ice below it. But the crust gets no thinner, as the heat sources in the silicates decay, and the partly melted region refreezes.

Finally, it should be noted that the temperatures in the core reach 1000°C by 2000 million years; at this point, one would expect dehydration of the silicates to occur, resulting in some moderation of temperatures within the core.
2.2 Homogeneous accretion, silicate density 3.7. With the density of the silicates set upwards to 3.7, the overall picture is similar. With the more dense material, the crustal radius is less by 100 km, making cooling of the interior slightly more efficient, so that the planet is not quite as differentiated after 1250 my (Figure 6); however, the more dense silicates virtually doubles the heat generated by gravitational infall, and the smaller volume fraction occupied by the silicates lowers the conductivity of the crust, so that by 3000 my the crust is a bit thinner than in the lower density silicate case. However, it begins to refreeze by the present time, and a thick (250 km) crust results, the upper part of which remains undifferentiated.

2.3 Inhomogeneous accretion of a "dry" silicate core (density 3.7) covered by water ice. In this model we start with a core of silicates 1350 km in radius, covered by a 100 km thick region of pure ice VIII, a 200 km thick region of ice VI, a 700 km region of ice II, and a 60 km crust of ice I. The heat flux from the core slowly melts the overlying ice layers: first melting does not occur until 700 my, and by 1500 my the water region is only 250 km thick, with another 100 km above it melting. By 3000 my, the water region has nearly tripled in size, and continues to grow to the present. The present structure is roughly in steady-state, with a 100 km crust of ice I, 200 km region of ice II, and an 850 km liquid mantle
overlying the silicate core.

Central temperatures and pressures reach very high values: the model predicts a temperature of nearly 3000°K, at a pressure of 45 kbars. Naturally, at these conditions silicates will undergo considerable phase changes, and we should expect a substantially differentiation and evolution of silicate phases. The silicate phase changes might, on the one hand, absorb considerable amounts of heat which would otherwise contribute to the heat flux being convected to melt the crust; on the other hand, the differentiation of the silicates may serve to enrich the upper layers of the silicate core in uranium and thorium, thus in effect increasing the local heat flux. Such reactions in the core would start occurring early in the object's history, about the same time as melting of the ice becomes important, and would probably have a more profound effect on the ultimate picture of such a body's interior than the simple melting of ice modelled here.

D. Discussion

The only observable features of the Galilean satellites are their densities and their surface albedoes and spectra. The densities show a striking relation to distance from Jupiter, with the most rock-like satellite, Io (perhaps one should say, most ice-depleted) being the closest, and the satellite most typical of cosmic abundances of ice and silicate (the least ice-depleted) being Callisto. This description
is matched in our models of the outer three satellites. A reason for this simple decrease in density has been proposed by Pollack et al. (1974) who describe the Galilean system as a small scale solar system, with satellite-forming materials being determined by the local pressure and temperature in a proto-Jupiter nebula.

The surface albedoes are more difficult to explain, since the darkest satellites are (by the density arguments) the iciest, while the brightest (Io) should have no ice at all. However, an explanation for this has been proposed by Fanale et al. (1974) who would coat the surface of Io with salts exsolved from the interior, coat Europa with a thin crust of ice, have the crust of Ganymede be differentiated ice and that of Callisto undifferentiated ice plus rock. Our thermal models support this hitherto intuitive guess. We have demonstrated that the crusts of Europa and Ganymede must have been thoroughly melted and differentiated due to the actions of internal heat sources. Also, we show that the evolution of Callisto due to internal heat sources is not inconsistent with Fanale's suggestion.

However, the difference in the strength of our demonstrations concerning Europa and Ganymede on the one hand, and Callisto on the other, must be emphasized. If Europa and Ganymede were formed of homogeneous mixtures of silicates and ices with chondritic abundances of radioactive nuclides, they
will melt completely. But one cannot say with equal assurance that the crust of Callisto will not melt, only that internal heat sources alone might not be sufficient to melt it. Any variety of events could have occurred which could have thoroughly melted the crust: a superabundance of radioactive nuclides, rapid accretion, a period of increased solar heating, or tidal effects, just to name four.

There is the question of the stability of such a crust; ice plus silicates would presumably be denser than pure water. However, depending on the amount and density of the silicate phase, such a difference may not be too extreme: for example, ice I with 40% silicates of density 2.5 will have a mean density of 1.23 g/cm$^3$, while liquid water at high pressures but temperatures near the freezing point will have comparable density.

There is also the possibility that impacts may cause the crust to break, and this, coupled with the above mentioned density instability, would lead to a catastrophic destruction of the crust. If solar Poynting-Robertson effects are important in evolving the orbits of colliding bodies, then in the region of space where icy satellites exist, the most common form of colliding material will also be icy and thus would have to have large size or velocity to break a 100 km thick crust. But the velocity of such an object in the orbit of Jupiter is about 8 km/sec, and will have impact velocities ranging from 3 to 18 km/sec, low compared to meteorites in Earth's orbit;
and the flux of such objects, while not well known, is likely to be very low after the first billion years while the thin crust necessary for breaking will not have been formed until after that time.

On the other hand, if the important source for colliding material is cometary in origin, then its composition cannot be presumed to be only ice, and its velocity is likely to be very high, especially due to the acceleration caused by Jupiter's mass. Thus, impacts, though not necessarily common, will be of high energy and may be able to break through a 200 km thick crust.

Our models predict considerable thermal expansion, and this may produce significant cracks in the crust, leading to upwelling of the less-dense liquid material underneath and eventually to catastrophic overturn of the crustal layers. But the thermal expansion appears to be on a slow enough time scale that plastic flow of the ice should heal such cracks as they develop.

An undifferentiated crust is not necessary to explain the low albedo of Callisto. If the surface were evenly dusted by a uniform layer of an especially dark material, such as carbonaceous chondrite material, the water absorption features would be obscured and the darkness of the surface could be explained (Johnson et al., 1973). Since the observed albedo of Callisto, though lower than expected for ice, is still
fairly high, we might expect such an effect would be important even if the dusting represented only 1% of the surface area.

The Poynting-Robertson effect due to the brightness of Jupiter would allow small dark particles to drift inwards towards the planet. Such particles would first encounter Callisto, the outermost of the Galilean satellites, and provide darkening material. However, even in this case a thick, stable crust -- though not necessarily a "dirty", undifferentiated one -- would be desirable to allow this dust to accumulate over a long period of time.

In any event, the endurance of a primordial crust on Callisto cannot be said to have been demonstrated by these models, but only that its lack has not been demonstrated. Its existence is not compelled, but its possibility has been strengthened.

To accept the hypothesis that Callisto's low albedo is caused by its primordial crust would cap a triumph in planetary modelling. For as we noted in the beginning of this section, the Galilean satellites show us only two features, their increasing densities and their decreasing albedoes. The former is now well explained by Jupiter collapse-phase luminosity calculations, the latter by thermal modelling. We can take pride in our abilities to model -- as long as we don't get any more data.
Figure 2 caption: Pie sections and temperature-radius curves for four periods in the modelled history of Europa. Temperature, in degrees K, pressure, in kilobars, and weight fraction of silicates is listed down the left hand side of the pie sections; radius, in kilometers, on the right hand side. The sections labelled "ice II", ice "VI", etc., represent those regions where the indicated ice phase is stable; but note the weight percent of silicates which also exists in those regions.

Note that, with the surface temperature fixed at 100°K, the surface will always be ice I. However, when the model predicts that the top iteration layer becomes completely melted, then this thin crust is ignored. (See Appendix A for details of modelling.)
$t = 2000 \text{ my}$

$P_{\text{e,1}} = 3.7$

$t = 4500 \text{ my (present)}$
Figure 3 caption: Pie sections and temperature vs. radius curves for Ganymede, assuming 80% by weight silicates of density 2.5 g/cm³, 20% by weight water ice. See caption for Figure 2.
$t = 2000 \text{ my}$

$\rho_{sil} = 2.5$

$t = 4500 \text{ (present)}$
Ganymede
\( \rho_{\text{sil}} = 2.5 \)
**Figure 4 caption:** Pie sections and temperature vs. radius curves for Ganymede, assuming 60% by weight silicates of density $3.7 \text{ g/cm}^3$, 40% by weight water ice. See caption for Figure 2.
\[ T \ P \ x_s \]

**T P x_s**

\[ 100 \ 1.0 \]

\[ 100 \ 0.0 \]

\[ 100 \ 0.6 \]

\[ 100 \ 0.6 \]

\[ 100 \ 0.6 \]

**Ice II**

**Ice VIII**

**Ice VII**

\[ 100 \ 28.7 \]

\[ 100 \ 38.3 \]

\[ 100 \ 43.9 \]

\[ 100 \ 45.5 \]

\[ t = 0.0 \]

**Ganymede**

\[ \rho_{s_4} = 3.7 \]

\[ 351 \ 28.1 \]

\[ 351 \ 38.9 \]

\[ 351 \ 43.6 \]

\[ 351 \ 45.2 \]

\[ t = 500 \ \text{my} \]
Ganymede

$P_{sil} = 3.7$

$t = 2000 \text{ my}$

$t = 4500 \text{ my (present)}$
GANYMEDe
\(\rho_{Si} = 3.7\)
Figure 5 caption: Pie sections and temperature vs. radius curves for Callisto, assuming 40% by weight silicates of density 2.5 g/cm³, 60% by weight water ice. See caption for Figure 2.
CALLISTO
\[ P_{\text{Si}} = 2.5 \]

\[ t = 3000 \text{ my} \]

\[ t = 4500 \text{ (present)} \]
CALLISTO
$\rho_{5.1} = 2.5$
Figure 6 caption: Pie sections and temperature vs. radius curves for Callisto, assuming 40% by weight silicates of density 3.7 g/cm³; 60% by weight water ice. See caption for Figure 2.
CALLISTO

\( P_{\text{sl}} = 3.7 \)

\( t = 0.0 \text{ my} \)

\( t = 1250 \text{ my} \)
CALLISTO
$\rho_{sII} = 3.7$

$\tau = 3000 \text{ my}$

$\tau = 4500 \text{ my (present)}$
CALLISTO
$\rho_{sil} = 3.7$

---

Diagram showing temperature (°K) vs. radius (km) with three curves labeled 4500, 3000, and 1250.
Figure 7 caption: Pie sections and temperature vs. radius curves for an inhomogeneously accreted Callisto. Here the sections labelled "ice I", Ice II", etc. represent regions of pure ice of the indicated phase. Again, the temperature is indicated in degrees K, pressure in kilobars, radius in kilometers, and time in million years from formation.
Callisto
(INHOMOGENEOUS)

t = 0.0

t = 1500 my
CALLISTO (INHOMOGENEOUS)

t = 3000 my

$\text{t} = 4500 \text{ my (present)}$
CALLISTO
Inhomogeneous
Chapter 3. Small Icy Satellites

A. Water ice - silicate bodies.

We consider three size classes of satellites made of a homogeneous mixture of 60% by weight water ice, 40% by weight silicates: those of radius 500 km ("Tethys" class), those of radius 700 km ("Dione" class), and those of radius 1000 km ("Iapetus" class). The actual radii of Tethys, Dione, and Iapetus are hardly well known enough to make rigorous models of those satellites themselves; rather, we have picked these three size groups as being a typical range for small icy satellites, and have given them the names of actual satellites which may not be too different in size from the models. For the sake of comparison with the larger Galilean satellites, we continued to use a starting and surface temperature of 100°K. Dione and Iapetus were then re-run at a more Saturn-like temperature of 70°K. The results of this run are discussed in section 2.

1.1 Tethys class. Satellites of radius 500 km, or smaller, will not undergo significant changes in structure due to internal heat sources. The central pressure of a "Tethys" object is less than 0.5 kbars, so that ice I is the only ice of interest; and central temperatures reach a peak of only 170 °K, at 500 my, cooling to temperatures of 120°K at the center by the present time. Hence no melting will occur. Any evolution detected in the structure or composition of such
satellites will thus be indicative of events other than internal thermal evolution, and so the examination of such small bodies should be of great interest in the study of the history of the solar system.

1.2 Dione class. Melting of ice will occur in a 700 km object of our given composition, but such melting will be confined to the inner 300 km of the satellite, leaving the outer 400 km (which) comprises 90% of the planet's mass) untouched. Melting will begin at approximately 600 my, proceed to a maximum at about 2000 my, and at present time the entire body will be resolidified, but with substantial differentiation of ice and silicates in the inner 300 km region. There is no significant difference in the evolution of such a body with differing silicate densities, except that, of course, with weight percent of silicates constant the volume of a denser silicate core will be smaller than that of a less dense core. In any event, core temperatures never exceed 350°K and core pressures are about 1 kilobar, so evolution of the silicates is not expected to occur.

1.3 Iapetus class. Pressures in an Iapetus class body will exceed 1 kilobar at a radius of roughly 600 km, so both ice I and II will be present in the original body. As with the Dione object, melting will start at about 600 my. The melting region will have reached a radius of nearly 700 km by 750 my, and by 2000 my this entire region will be liquid. By this time
a 400 km core of silicates will have formed. But since the heat sources in the core are decaying with time the liquid region will eventually refreeze, leaving by the present time less than 100 km of water between the core and the now partly differentiated ice crust. It should be noted that the top 300 km will not be melted and hence internal heat sources will leave them unchanged to the present.

2. Effects of lowered surface temperature. The same models for the Dione and Iapetus class objects were run with initial and surface temperature of 70°K, to more accurately simulate conditions at Saturn's distance from the sun. The effect of lowering the starting temperature by 30° was, not surprisingly, that temperatures throughout the models generally ran about 30° lower than in the 100° surface model. Thus differentiation was even less pronounced in the Dione model; melting extended only out to 150 km, and the region was totally refrozen by 2250 my. The size of the core in this case was only 100 km. In the case of the 1000 km object, the lower starting temperature delayed melting by about 250 my. The melting region grew to a radius of 600 km (as opposed to 700 km for the 100° case) and ceased growing at about the same time, 2250 my. The melted region has virtually completely refrozen by the present time (4500 my), whereas the 100° model has nearly 100 km of liquid water left by this time.
B. Water ice - ammonia hydrate and silicate bodies.

The presence of ammonia as a hydrate in the ice will facilitate melting, since a mixture of ammonia hydrate and water ice will form eutectic melt at 173\(^\circ\)K, at pressures of 1 bar. The evolution of a satellite with composition of silicates and such solution of ices can be modelled (see Appendix A.4) with the following results:

2.1 Tethys Class. Even with the lower melting point of the eutectic, melting will not occur in these objects. It is possible that the eutectic melting point might be reached, but it probably could not be maintained long enough for the eutectic mixture to melt before heat loss would lower central temperatures below that point. Thus a Tethys class object would still be essentially unchanged by internal heat sources, and any evolution actually observed in such an object would be indicative of external effects.

2.2 Dione Class. The presence of ammonia as 10% of the mass of the object will result in early and substantial differentiation of a Dione class object. Melting will have reached its maximum radius by 500 my, extending to nearly 500 km, and the melting will continue until 2000 my, at which time the silicate core will have grown to over 150 km and the "slush" region will consist of about equal portions of ammonia-rich liquid and water ice. Beyond this time, cooling will set in and by the present the entire body will be frozen again.
2.3 Iapetus class. As with Dione, the presence of ammonia in Iapetus will facilitate melting and differentiation within the object. The unmelted crust will have shrunk to 150 km by 500 km after formation, with a large region of "slush", roughly 40% ammonia-rich liquid and 60% unmelted ice, between the crust and a 460 km core.

The melting continues in the slush region up to 2000 my after formation; then the top regions begin to freeze. By the present the frozen crust will be in two sections, an undifferentiated ice and silicate mix 150 km thick, overlaying a second, partly differentiated region 130 km thick. The slushy region still persists for another 300 km down, until the silicate core is reached.

C. Conclusions for Small Bodies

The point of interest for small bodies is whether melting will occur, and if so to what extent. This can best be illustrated by Figure 13, which shows the minimum thickness of the crust for various sized objects with the same starting conditions. Up to a radius of 500 km, there seem to be no circumstances where internal heat sources can possibly start melting or differentiation. Even up to 1000 km, it seems unlikely that the surfaces of such satellites will be affected by thermal evolution, although differentiation may be determined if the moments of inertia for these bodies are ever determined (which would be difficult, since presumably
these bodies are in spin-orbit resonance with the respective planets).

The fact that no internal evolution should effect these satellites means that they should be ideal recorders of any external events which may have occurred in their part of the solar system.

Beyond the Titania class, however, the crusts become thin enough that presuming the surfaces of such satellites to be untouched may be misleading. Silicate-rich crusts will be gravitationally unstable relative to the large liquid regions which did exist in these objects two billion years ago, and any number of events could be postulated which would result in catastrophic overturn of this material, with the lighter, silicate-free liquid rising to the surface and freezing.
Figure 8 caption: Temperature vs. radius curves for a Tethys-class object.
TETHYS
water ice and silicate

radius, km

T °K

0 100 200 300

0.0 4500 2000 500
Figure 9 caption: Pie sections and temperature vs. radius curves for a Dione class object (radius 700 km) made of 40% silicates of density 2.5 g/cm³; 60% water ice. See caption to Figure 2.
$\text{Dione}$

$P: I = 2.5$

water ice

$t = 0.0$

$100 \ 0$

$100 \ 0.5$

$100 \ 1.0$

$t = 750 \ \text{my}$

$195 \ 0.5$

$0.35$

slush

$0.25$

silicates
$T \cdot P$
0.4
0.9
0.0
1.0
1.0

Ice I

silicates

Water

$P_{s1} = 2.5$

$\tau = 2000 \text{ my}$

$\tau = 4500 \text{ (present)}$

Dione
Dione
water ice and silicates
Dione
water ice and silicates
starting T = 70°K
Figure 10 caption: Pie sections and temperature vs. radius curves for an Iapetus class object (radius 1000 km) made of 40% silicates of density 2.5 g/cm$^3$; 60% water ice. See caption to Figure 2.
\[ T \text{ or } P \times 10^0 \quad 0.4 \quad \text{Ice I} \]

\[ 100 \quad 1.7 \quad \text{Ice II} \]

\[ 100 \quad 2.4 \quad 0 \]

\[ t = 0.0 \]

\[ \text{TAPETUS} \]

\[ P_{s.1} = 2.5 \quad \text{water ice} \]

\[ 418 \quad 2.8 \quad 0 \]

\[ t = 750 \text{ my} \]
$\frac{TP_{x, z}}{100~0}$

- Ice I
- Water
- Silicate

$T = 2000$ my

$IAPETUS$

$P_{sil} = 2.5$

$t = 4500$ my (present)
IAPETUS
water ice and silicates

$T \, ^{\circ}K$

radius km

1000 500 0

100

2000
4500
750
**IAPETUS**
water ice and silicates only
starting $T=70^\circ K$
Figure 11 Caption: Pie sections and temperature vs. radius curves for a Dione class object (radius 700 km) made of 35% silicates of density 2.5 g/cm$^3$, 55% water ice, 10% ammonia. See caption to Figure 2.
Dione
(water-ammonia solution)
$P_{v1} = 2.5$

$t = 0.0$

$t = 500 \text{ my}$
$t = 2000 \text{ my}$

$\rho_{s_i} = 2.5$

$Dione$

(water-ammonia solution)
Dione
water-ammonia solution + silicates

radius km

T °K

0 100 200 500 2000

0 700 500 300
Figure 12 caption: Pie sections and temperature vs. radius curves for an Iapetus class object (radius 1000 km) made of 35% silicates of density 2.5 g/cm$^3$, 55% water ice, 10% ammonia. See caption to Figure 2.
\[ T \times P \times \text{X}_i \times \text{X}_{\text{him}} \]

- **TAPETUS**
  - \( P_s; 1 = 2.5 \)
  - \( t = 2000 \text{ my} \)
  - \( \text{water-ammonia hydrate solution} \)

- **Slush**
  - \( t = 4500 \text{ my (present)} \)
IAPETUS
water-ammonia solution and silicates
Figure 13 caption: Extent of differentiation as a function of radius for bodies made of 60% by weight water ice, 40% by weight silicates, for surface temperatures of 70 and 100°K. The crustal thickness represents the extremes of maximum melting (which usually occur around 2000 my after formation), not necessarily the present thickness of unmelted material.

The dotted lines represent minimum crustal thickness for bodies of 55% water ice, 35% silicates, and 10% ammonia with surface temperature of 100°K.
Chapter 4. Other Possible Size Classes

In addition to the Galilean satellites and the small icy satellites of Saturn, there exist the large satellites Titan and Triton, and it is possible some of the Uranian satellites may be in the size region of 1000-2000 km radius. To get an idea of how these objects may evolve with time, we have made two more size class models: the "Titan" class, with a mass of $140 \times 10^{24}$ g and radius roughly 3000 km, and the "Titania" class, with radius of 1500 km.

A. Titan class.

1.1 Model i: homogeneous mixture of 30% by weight silicates of density 2.5, 60% by weight water ice. In this model, Titan starts with a core of ice VIII and silicates 1700 km in radius, covered with a mantle of ice VI and silicates 300 km thick, one of ice II and silicates 700 km thick, and a 60 km thick crust of ice I and silicates. With the reduced amount of silicates, melting occurs over 1000 my after formation, much later than for Callisto, but by 1500 my a substantial region (≈800 km thick) has begun to melt, with silicates beginning to gather in the lower part of the melting region. By 3000 my, melting has almost reached its maximum extent, with a 300 km thick crust of ice I and ice II, a 100 km region where melting is still taking place, 1200 km of water and over 1000 km of silicates gathered towards the center. But even at this time a small kernel of ice VII remains.
unmelted, since the high central pressures (~30 kilobars) have raised the melting point to above 400°K.

By the present time, 4500 my, this kernel has melted and been replaced by silicates. However, at the top of the liquid region melting has ceased and ice is beginning to reform at the bottom of the crust. The central temperatures in the core reach 1000°K, so it is possible that dehydration of the silicates may have begun.

1.2 Model ii: A similar mixture as model i, but with silicate density of 3.7. The higher density silicates increase the starting pressures somewhat, so the higher pressure forms of ice make up more of the volume of the satellite. Melting again does not begin until after 1000 my, and by 1500 my the partly melted region is slightly smaller than for model i. By 3000 my, the planet looks substantially like the lower silicate density model, but a larger region of ice VII and silicates persists at the center of the satellite, due to the higher central pressures. In fact, model ii predicts that such a kernel of undifferentiated ice VII and silicates might persist even to the present. The effect of this kernel is to act as a sink for heat, lowering temperatures considerably in the silicate region and thus lowering the heat flux from this region. This, in turn, results in a less differentiated crust than was found by model i.
1.3 Model iii. Inhomogeneously accreted body, with silicates 40% of the total mass making up a core, and pure water ice forming an overlying mantle. For Titan, the core of "dry" silicates (density 3.7) would be roughly 1600 km in diameter, and start out covered by an ice VIII region 300 km thick, an ice VI region 250 km thick, an ice II region 350 km thick, and an ice I crust less than 100 km thick. Melting proceeds very slowly (as in the Callisto inhomogeneous case) and after 1500 my the liquid region extends for only 300 km. After 3000 my it has doubled its size, and continues to grow to the present, extending for over 700 km under a 200 km crust of ice I and II.

As was the case with the inhomogeneous model described for Callisto, the changes in the silicate phase would likely equal in importance those in the icy phase, and in fact would be likely to determine the time scale in which the melting of the icy layer occurs. Again as in Callisto, we would expect that differentiation within the silicate phase would result in enrichment of uranium and thorium in the upper regions of the core, thus enhancing the melting of ice, but also will result in overall lower temperatures within the core. Therefore, a true picture of the evolution of an inhomogeneously accreted satellite would require accurate information on the nature of the silicates present.
B. "Titania"

"Titania" class objects have radius of 1500 km, and were run, as usual, with two densities for the silicates, 2.5 and 3.7 g/cm$^3$. In addition to the standard 100°K model, a low surface temperature (70°K) model was also run as was done for the small satellites, to more accurately model satellites farther from the sun.

1.1 Model i: 100°K, silicate density of 2.5. There are only two phases of ice present in this model, as it starts: an inner core 1250 km thick of ice II and an outer shell of ice I. Melting starts after 600 my, and reaches its maximum extent by 1000 my, reaching a radius of 1300 km. The melting within this region continues until 2500 my; after this time, the top of the liquid region begins to refreeze, and by 4500 my about 60 km has been added to the thickness of the crust.

1.2 Model ii: 100°K, silicate density of 3.7. This case is almost exactly identical with Model i in timescale and size of the melting region.

1.3 Model iii: 70°K surface temperature, silicate density of 2.5. The lower temperatures cause this model to differentiate much less than model i; the minimum thickness of the crust is about 150 km thicker than the Model i case. However, the time scale for melting, differentiating, and refreezing are in general closely similar.
Figure 14 caption: Pie sections and temperature vs. radius curves for a Titan class object, of composition 30% silicates of density 2.5 g/cm$^3$, 70% water ice. See caption to Figure 2.
\[
\text{Titan} \\
\rho_{s,1} = 2.5
\]

\[t = 0.0 \text{ my} \]

\[t = 1500 \text{ my} \]
t = 3000 my
TITAN
P_{s,1} = 2.5

t = 4500 my (present)
TITAN
ρ_{ii} = 2.5
Figure 15 caption: Pie sections and temperature vs. radius curves for a Titan class object, of composition 30% silicates of density 3.7 g/cm$^3$; 70% water ice. See caption to Figure 2.
$t = 3000 \text{ my}$

$TITAN$

$P_{3.1}=3.7$

$T = 4500 \text{ my (present)}$
TITAN
$\rho_{Si} = 3.7$
Figure 16 caption: Pie sections and temperature vs. radius curves for an inhomogeneously accreted Titan class object. See caption to Figure 7.
Titan (inhomogeneous)

$t = 3000 \text{ my}$

$t = 4500 \text{ my}$
TITAN
in homogeneous

radius km

T °K

3000
2000
1000
0.0

4500
3000
1500
Figure 17 caption: Pie sections and temperature vs. radius curves for a Titania class object, of composition 40% by weight rock of density 2.5 g/cm$^3$; 60% by weight water ice. See caption to Figure 2.
$t = 2500 \text{ my}$

$\rho_{\text{sil}} = 2.5$

$t = 4500 \text{ my (present)}$
Figure 18 caption: Pie sections and temperature vs. radius curves for a Titania class object, of composition 40% by weight rock of density 3.7 g/cm$^3$, 60% by weight water ice. See caption to Figure 2.
$T = 0.0 \text{ my}$

$\rho_{3.1} = 3.7$

$T = 1000 \text{ my}$

Titania
t = 2500 my

TITANIA
Psill = 3.7

$TP_{100 00}$
$123 0.4$
$253 2.2$
$256 3.5$
$266 5.0$
$682 9.8$
$1175 13.1$
$1400 14.1$

$TP_{1500}$
$100 0$
$117 .4$
$240 2.3$
$256 3.6$
$266 5.1$
$562 9.9$
$1015 13.2$
$1308 14.1$

$TP_{110}$

$t = 4500 \text{my (present)}$
TITANIA
\[ \rho_s \approx 3.7 \]
TITANIA
water ice and silicate
starting T = 70°
Chapter 5. Summary and Conclusions

The idea of melting and differentiation within an icy satellite due to internal heat sources was first proposed by Lewis (1971). This work has been an attempt to quantify his suggestions by rigorous modelling of heat generation and transport with time, the effects of high pressures upon the crystal structure of ice, and the type of melting which would be expected for a mixture of ammonia hydrate and water ices.

We have found that, for the Galilean satellites, considerable evolution of the interiors is inevitable from internal heat sources alone, and may substantially affect the surface properties of these bodies. Large liquid mantles which form early in their histories and persist to the present are found for all starting conditions considered, and the results of different postulations of starting conditions can be seen only in how thick these melting regions actually are. As should be intuitively obvious, the more silicate material that is present, the more rapidly and extensively the ice will melt. Heterogeneously accreted bodies will not melt as thoroughly as homogeneously accreted bodies. The extent of melting is not a strong function of silicate density.

For smaller bodies, at least partial melting can be expected for any body over 500 km in radius. If ammonia is present in cosmic abundances, this melting will be substantial, leaving in general only the outer 200 km of the satellite
undifferentiated. The greatest extent of melting occurred about 2500 my ago and these bodies will be totally refrozen today. If the rings of Saturn are the result of the breakup of such an icy satellite, as suggested by Reiffenstein (1968), then such a breakup could not have occurred within the past billion years.

Our models are quite general and can be applied to many various hypothetical cases. Although no ice-rich satellites have been proved to have radii between 1000 and 2500 km, it is obvious that any such body would be substantially differentiated. Titan, even with a decreased amount of silicates, melts its interior thoroughly; however, with the reduced amount of heat source, it would maintain a thick crust. Since the presence of a methane atmosphere indicates that pure water ice is not the only light component of Titan, it is certain that Titan will be more easily melted than we have assumed, and will have a much thinner crust. Also, the appropriate boundary temperature for Titan may be substantially above the grey body equilibrium temperature, due to a possible large greenhouse effect produced by Titan's massive methane-rich atmosphere.

Study of the icy satellites of all sizes will help uncover the answers to many of the questions of planetary science. The smaller bodies cannot alter their surfaces by endogenous means, so any alteration which is discovered will be a clear indication
of extra-planetary events. The exact way in which the larger satellites evolve can indicate whether their accretion was homogeneous or heterogeneous, and lend support to whichever set of starting hypotheses model into an object closest to what we observe.
Appendix A

A.1 Program ICEPULSE

The purpose of ICEPULSE is to determine the interior structure of an icy satellite at uniform temperature (here set at 100°K, though this too can be varied) given only the mass, the weight percent of silicates, the density of the silicates, and the assumption that water ice is the only important ice present. A starting "first guess" total radius is also used to get the program started.

The question at hand is where the boundaries are between regions of ice I-and-silicates mix, ice II-and-silicate mix, ice VI-and-silicate, and ice VIII-and-silicate. These boundaries are pressure-dependent, and the pressure in turn depends on the density distribution within the satellite, that is to say on the sizes of each of the regions.

In particular, the local equation of hydrostatic equilibrium is

\[ dP = -\rho g dr \]

where \( g = \frac{GM(r)}{r^2} \). Given a body with regions 1-4 and boundaries A, B, C, and D (see Figure A) we can see that for some radius \( r \) between B and D:

\[ dP = -\frac{GM}{r^2} dr - \rho G_{12} \pi \rho_2 \left(\frac{r^3 - C_3^3}{r^2}\right) dr \]

\[ = -\frac{GM}{r^2} dr + \rho G_{12} \pi \rho_2 C_3^3 \frac{r^2 dr}{r^2} - \rho G_{12} \pi \rho_2 rdr \]

where \( M = M_3 + M_4 \).
Figure A caption: Iteration layers.
Integrating from $B$ to $r$, we get

$$\int_{B}^{r} dp = - \int_{B}^{r} \frac{\rho_2 GM}{r^2} dr + \rho_2 \int_{B}^{r} \frac{G \pi \rho_2 C^3}{r^2} dr - \rho_2 \int_{B}^{r} \frac{G \pi \rho_2}{r^2} dr$$

$$P_r - P_B = \rho_2 G\{(M - \frac{4}{3}\rho_2 C^3)\left(\frac{1}{r} - \frac{1}{B}\right) + \frac{2}{3}\rho_2 \pi (B^2 - r^2)\}$$

This can be rearranged into a 3rd degree polynomial in $r$:

$$r^3 + \left(\frac{3}{2} \frac{(P_r - P_B)}{\rho_2 G \pi} + \frac{3}{2} \frac{M}{\rho_2 \pi B} - \frac{2C^3}{B} - B^2\right) r - \left(\frac{3}{2} \frac{M}{\rho_2 \pi} - 2C^3\right) = 0$$

Thus, if we know $B$ and we know the pressure at which the boundary occurs (namely the pressure at which the ice of region 2 becomes the ice of region 3, which is a function of temperature) we can solve for $C$.

In ICEPULSE the coefficient of the first order term is called $A$, the zeroth order term $B$. A subroutine THROOT is called which solves for the three roots of this equation. If any root is imaginary, it is given the value of $10^{60}$ (an arbitrary "very large number"). Then a function RTRANS is called to find the one solution of the three which will be between zero and the total radius (which we have made a first guess at).

For our first cut, we assume a constant density within the planet ($P_B$ is set to 0) and the boundary radii are found for the pressures of the given ice phase transitions (which are found to $T = 100^\circ K$). From these boundaries, the first cuts at $M_A$, $M_B$, $M_C$, $M_D$ are calculated; from these a new set
of transition radii are found, and from these at new set of $M_A', M_B', M_C', M_D'$ calculated. These masses are then added together and compared against the known mass. If they do not agree within $5 \times 10^{-23}$ g (about 5% for icy satellites, which is better than the masses are actually known for some of them) then the total radius is reset by the formula

$$r_{\text{total}} = \left( \frac{\text{mass known}}{\text{mass calculated}} \right)^{1/3} r_{\text{total}}^{\text{old}}$$

and the procedure is repeated. Convergence usually occurs after two iterations, if the first guess at the total radius is good.

A.2 Program 'ONIONKIN'

A2.0 Introduction. 'ONIONKIN attempts to determine the thermal history of an icy satellite which was accreted in "onionskin" fashion, that is to say, with an already formed core of silicates covered by layers of pure ice VIII, ice VI, ice II, and ice I. The heat source is assumed to be long-lived radioactive nuclides in the silicates. (One version also considers heat released due to phase changes in the ice as if it were compressed adiabatically during accretion. See Appendix B for details). Heat is conducted away through the ice, and the ice is thus heated until a phase boundary is encountered. At that point, its temperature is held constant until enough heat has been absorbed to change phase. While changing phase, its density changes; this in turn causes its volume to change, changing the radii of the points above it.
and in turn the local gravitational acceleration at those points and thus the pressure there which may in turn bring about further phase changes.

When the melting point is reached, the phase change proceeds as before. Upon completion of melting, heat transport by convection becomes important. This intuitively obvious result can be proved by looking at the Rayleigh number of the water under these conditions,

\[ Ra = \frac{g \alpha \Delta T d^3}{\kappa v} \]

which must be greater than 1708 for convection to occur. For terms typical of the core-mantle boundary in an icy satellite and a minimum T equal to the change in melting temperature over distance due to increasing pressure, it can be shown that Ra is greater than 1708 for any convecting cell more than 2 cm thick. In general, over the entire convecting region the value of Ra will be many orders of magnitude greater than 1708.

A2.1 Modelling of Conduction. The basic equation for conduction of heat in a spherically symmetric body is

\[ C_p \rho \frac{\partial T}{\partial t} = \frac{1}{r^2} \left( r^2 \frac{\partial}{\partial r} \left( K \frac{\partial T}{\partial r} \right) \right) + A \]

which cannot be solved analytically when K, C_p, \rho, and A are all functions of position and time. However, this can be solved in finite difference form with a computer. In the case of ONIONKIN (and CALLISTO) the planet is first broken into 25 layers, each roughly 100 km apart. Care must be taken to
understand the difference between the layers and the layer boundaries (of which there are 26). In addition, an imaginary layer is created at the center to satisfy one of our boundary conditions, bringing the total number of boundaries to 27.

Given a section of our layered planet (as seen in Figure A) the temperature at point 2 will be determined by the difference in temperatures between points 1 and 2, and 2 and 3; the conductivities along lines b and c; the radial distances to and between these points; and the density and heat capacity in the region containing point 2. In particular, since

$$\frac{C_p \rho}{\partial T} \frac{\partial T}{\partial t} = \frac{1}{t_2} \gamma (\frac{Kr^2}{\partial T} \frac{\partial T}{\partial r} + A(t,r))$$

we get

$$C_p \rho \frac{\partial T}{\partial t} = \frac{1}{R_2} \left( \frac{K_b R_b (T_1 - T_2)}{R_1 - R_2} - \frac{K_c R_c (T_2 - T_3)}{R_2 - R_3} \right) + A(t,r)$$

Over a finite time $\Delta t$, the temperature at 2 will change by

$$\Delta T = \left[ \frac{1}{R_2} \frac{K_b R_b (T_1 - T_2)}{R_1 - R_2} - \frac{K_c R_c (T_2 - T_3)}{R_2 - R_3} \right] \frac{\Delta t}{C_p T_2}$$

This equation is similar to that used by Toksöz et al. (1972), but is more flexible since it allows for changes in $\Delta r$. The volumes of the various regions are allowed to change with changing phases; and since the temperature is found for a point within a region, not a boundary between regions, $1/R_2^2$ is always finite and the problem of using this equation at the center of the planet (as $r \to 0$) is avoided. Toksöz et al.
(1972) provide an analysis suggesting that this equation will converge when the condition

\[ \frac{K \Delta t}{\rho C_p \Delta r^2} < \frac{1}{2} \]

is maintained.

To use this equation, two boundary conditions are needed; the surface temperature is held constant, and the temperature gradient at the center is set equal to zero. To accomplish this last condition, an imaginary layer is created whose temperature is always equal to the temperature of the bottom layer.

In addition, it is necessary to know completely the temperature profile at one given instant of time. For ONIONKIN we assume \( T = T_{\text{surface}} \) everywhere at time \( t_0 = 0 \) (except for the case of compressional heating described in Appendix B).

A2.2 Heat Sources

Heat sources are assumed to be radioactive nuclides in the silicate core, particularly \( ^{40}\text{K}, ^{232}\text{Th}, ^{235}\text{U}, ^{238}\text{U} \). The heat generated from each of these nuclides is determined from the known energy of decay, the known half life, and an assumed present day abundance (from which the original abundance can be determined). The heat of the nuclides are summed together and added to the conduction equation as term A. This term is a function both of time (since the energy decays exponentially with time) and of radius, with no heat being
generated in the icy parts of the planet, only in the core.

For our model, we assumed chondritic abundances for the heat sources in the silicates (after Kaula) with the present day abundance of K being 815.0 ppm, U being 0.012 ppm, and Th being 0.04 ppm. These are variables in the program and could be reset to other values as desired. The equations for heat production are thus

\[
\begin{align*}
A_{40K} &= (\text{ppm})_K (0.01148) \exp(0.000521t) \\
A_{235U} &= (\text{ppm})_U (115.396) \exp(0.000927t) \\
A_{238U} &= (\text{ppm})_U (60.304) \exp(0.000154t) \\
A_{232Th} &= (\text{ppm})_Th (10.527) \exp(0.00005t) \\
A_{\text{total}} &= (A_{40K} + A_{235U} + A_{238U} + A_{232Th}) \times 10^{5.7}
\end{align*}
\]

where the numerical constant for each term includes both a factor for the original abundance of the isotope of interest 4.5 by ago and the heat production in ergs per million years per gram due to decay of the isotope. (In fact, all these constants are low by $10^6$, hence the $10^5$ factor in the sum). The $t$ in the exponential is in units of million years, the unit of time used in the program.

Since the $A$ term in the heat equation is heat production per unit volume, the $A_{\text{total}}$ calculated here must also be multiplied by $\rho$ before it is used there.
A2.3 Heat capacity, conductivity, and density data

The heat capacity and conductivity of all phases of ice are assumed to be equal to that known for ice I. Both are functions of temperature; for the temperature range 100-300°K, the following formulae match the data in Fletcher (1970) nicely (within 2%)

\[ K = -1.8298 \times 10^{18} + 2.438 \times 10^{21}/T \]
\[ C_p = 7.0 \times 10^4 T + 0.169 \times 10^7 \]

(K is in units of ergs/cm°K million years, \( C_p \) in ergs/g°K).

For water, the terms (based on Clark, 1966) are

\[ K = 0.185 \times 10^{19} \] (assumed constant with \( T \))
\[ C_p = 1.1936 \times 10^4 T + 3.6188 \times 10^7 \]

and for silicates, data from Diment (1964) for serpentine

\[ K = 0.7462 \times 10^{19} \]
\[ C_p = 0.75 \times 10^7 \]

For the silicates, both terms are assumed constant with temperature. Although the silicate phase goes through the most extreme in temperature, the conductivity is most easily described as a constant since at high temperatures the effectiveness of radiative transport of heat increases while the lattice conductivity is dropping. Furthermore, while some data on conductivity and heat capacity of silicates are available for high temperatures, such materials have not been studied in the region we are most interested in, between 100 and 300°K.
The density of ice depends radically on its crystal phase. The densities we have used are those listed by Fletcher as those found for each phase, measured at room pressure (1 bar):

<table>
<thead>
<tr>
<th>ice phase</th>
<th>density (g/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.93</td>
</tr>
<tr>
<td>II</td>
<td>1.17</td>
</tr>
<tr>
<td>III</td>
<td>1.14</td>
</tr>
<tr>
<td>V</td>
<td>1.23</td>
</tr>
<tr>
<td>VI</td>
<td>1.31</td>
</tr>
<tr>
<td>VII</td>
<td>1.50</td>
</tr>
<tr>
<td>VIII</td>
<td>1.50</td>
</tr>
</tbody>
</table>

Obviously at the pressures of natural occurrence these densities may be too low. However, the density of water at high pressures is known well (as listed in Clark, 1966) and the volume difference of the various phases at the triple points is also well documented; by calculating densities from these figures, we have found that the figures given by Fletcher in general agree to within a few hundredths of a g/cm$^3$.

The density of water we used was 1.0 g/cm$^3$. In retrospect, this figure may have been a bit too small, and 1.1 or 1.2 may have been better to account for the effects of high pressure on the liquid. However, this error is of importance only in the most massive satellites modelled (for example, Ganymede, which by our calculations ends up with a radius nearly 100 km
larger than that observed when it becomes completely melted). In addition, using a density of water equal to 1.0 simplified many equations in the program, a not insignificant factor for a program which requires over 100,000 iterations.

The density of the silicate cannot be pinned down to any one figure. The densities of chondrites are measured at room pressure, and may not represent their densities in situ. The density of serpentine varies from sample to sample. An even the use of serpentine is useful only where we assume the silicates are hydrous. What we have done, therefore, is to pick two extremes, densities of 2.5 and 3.7, which represent respectively hydrous and anhydrous silicates.

A2.4 Modelling Convective Heat transport

The pressure temperature profile followed in the convecting system is assumed to be the same as the melting curve profile, wherever a simple adiabatic increase with depth would intersect this profile.

The simplest way to model convective heat transport is to substitute an "effective conductivity" into a heat conduction equation, where this conductivity is found by multiplying the lattice conductivity $K$ by the Nusselt number, $\text{Nu}$, which is found empiracally to be proportional to the cube root of the Rayleigh number. For our situation, one iteration layer on the liquid-ice phase boundary with the temperature gradient set only by the rise in melting temperature with
pressure will have $Ra$ on the order of $10^{24}$, so that $Nu$ will be of order $10^8$. To increase conductivity by eight orders of magnitude while maintaining our condition for the stability of the finite difference equation, that

$$\frac{K\Delta t}{C_p \rho \Delta r^2} < \frac{1}{2}$$

would mean either raising the size of $\Delta r$ by four orders of magnitude (thus making $\Delta r$ much larger than the satellite itself) or by dropping $\Delta t$ by 8 orders of magnitude, (thus increasing the computer time needed by a like amount; iterating 4.6 billion years in 10 year increments would take about 10 years of CPU time. Since both of these alternatives are impractical, one looks to other means for modelling convection.

This high Nusselt number is telling us what we already know intuitively, namely that convection here will be extremely efficient in transporting heat; for all practical purposes perfectly efficient. Thus, we can assume perfect transfer of heat from the core to the top of the convecting region. The flux of heat from the core is found by the formula

$$\dot{Q} = -K \frac{\partial T}{\partial r}$$

The temperature of the top layer of the core is set equal to that of the overlying liquid layer, the difference in temperature between here and the underlying core layer is found, multiplied by the conductivity, and divided by the distance
between the two layers. This gives us $Q$, the heat flux per unit area per unit time. This number is multiplied by $\Delta t$, the size of the time iteration, and by the area of the core-mantle interface, $4\pi R_{\text{core}}^2$.

In addition, the mantle still maintains a gradient which can gain or lose heat; the $\Delta H$ found for the mantle region ($= \Delta T C_{\text{mass}}^{\text{mantle}}$) is added to the flux. This total heat is then added to the first unmelted layer overlaying the liquid region, by increasing that region's $\Delta T$ by the amount $\frac{\text{flux}}{C_{\text{mass}}^{\text{region}}}$. In the case that the mantle requires more heat to maintain itself than the flux from the core provides, this net negative number is added to the top liquid region, and the unmelted regions are left untouched.

A2.5 Modelling Phase Changes in the Ice

Ice exists in seven solid crystal structures, and the phase diagram is fairly well known (see Figure 1, in Chapter 1). However, some extrapolation is needed for boundaries at low temperatures; for this purpose, all boundaries not determined by experiment are straight lines, and the boundary between ice II and ice VI is found by writing the II-V and V-VI boundaries as equations for temperature as a function of pressure, $T = a + bP$, then averaging the values for $a$ and $b$.

This gives 15 lines to be considered. Equations for pressure as a function of temperature can be derived from the experimental data (Bridgman, 1912; 1937; Brown et al, 1966; Kell et al., 1968; Nisibata, 1972; Pistorius, 1963, 1968). Linear equations
were rewritten as \( T = a + bP \). All non-linear equations were written as \( T = a + bP + cP^2 \).

For each iteration layer, the temperature and pressure are known. Through a series of logical statements, the program determines in which phase the ice in this layer will be stable. If this phase is the same as the current phase of the ice for this layer, no action is needed. However, if the phase is different, a phase change must occur. The \( \Delta H \) of phase change is known at each triple point. Where the phase boundary runs between two triple points, we can find two values for \( \Delta H \); in this case, we set up \( \Delta H \) as a linear function of \( T \), and solve for \( \Delta H \) at whatever temperature exists on the phase boundary at the given pressure of the iteration layer. If we only know one triple point, we either make \( \Delta H \) a constant, or find it as a function of \( T \) by assuming \( \Delta S \) to be constant, so that \( \Delta H = T\Delta S \).

We compare the temperature on the phase boundary line for our given pressure with the temperature of the iteration layer during the previous time iteration. If the difference of these two temperatures is less than the \( \Delta T \) found by the heat conduction equation (the difference between \( T \)'s of the previous and the current time iteration), then we set the temperature of the layer equal to the phase-boundary temperature and the remaining heat \( \left[ \frac{C_p}{\Delta T - (T_{phase bound} - T_{last time})} \right] \), is used towards changing phase. If the difference \( T_{phase bound} - T_{last time} \)
is greater than $\Delta T$, then $T_{\text{new}} = T_{\text{old}}$ and all of $C_p \Delta T$ goes to changing phase. This avoids unrealistic jumps in temperature as phase changes across a strongly pressure-dependent boundary (i.e. where $dP/dT$ is very small) due to changes in pressure, which otherwise would be difficult to describe in a finite difference scheme, and allows us to use the change in temperature as a standard for how much heat has gone into the phase change.

During the phase change, the two phases of ice (or, ice and water) are assumed to coexist in the iteration layer in proportions equal to the fraction of necessary latent heat which has been accumulated. From these proportions, new densities are calculated, and in the case of the ice-water change, new values for $C_p$ and $K$ as well, averaging by weight fraction for the former and volume fraction for the latter.

If phase change is achieved, the new values for $\rho$ (and $C_p$ and $K$, if relevant) are set. Any heat in excess of the latent heat needed which has been accumulated is used to change the temperature appropriately.

If, during the phase change, the direction of the phase change changes, the accumulated heat is released as needed to maintain the region at the phase boundary until it is exhausted.

The routines work equally well for positive and negative latent heats, or positive and negative $\Delta T$'s.

With the change in phase (and hence density) there comes
a change in the volume of the region, and hence a change in the radial distance of each iteration layer. Consequently, there is a change in the local gravitational acceleration, and thus a change in pressure. After each time iteration, new radial distances, \( r \), are found; the local gravitational acceleration is found, by the formula

\[
g = 6.67 \times 10^8 \times \frac{\text{mass below this level}}{r^2},
\]

and the pressure locally is found by

\[
P_i = P_{i-1} + \rho_i g_i (r_{i-1} - r_i).
\]

(The surface is \( i = 1 \), and \( P_1 = 0 \).)

A3 CALLISTO

Program CALLISTO deals with the evolution of an icy body which is originally a homogeneous conglomeration of ices and silicates. It is very similar to ONIONKIN, and treats conduction, convection, and phase changes in the same manner.

The weight percent of silicates in each region is always known, and the heat production in that region is simply the heat production of the silicates times their mass fraction.

The density and conductivity of each region is the sum of the values of each component times its volume fraction; the heat capacity is the sum of the values of each component times its mass fraction.

Program CALLISTO models the additional complication that silicates will fall from molten regions to form a core. It is these additional sections which need to described here.
A3.1 Infall of silicates from the melting region. As the ice melts, silicates will be released and fall inward as far as the ice region is melted, displacing water upwards. The amount of silicates released per time iteration is found by determining the fraction of ice melted in that iteration, times the weight fraction of silicates in the ice. The weight fraction of the silicates in the liquid region is, of course, zero.

Heat will be released as gravitational potential energy is lost. This heat will be equal to $\Delta H = gm\Delta h$, where $g$ is the local gravitational acceleration, $m$ is the effective mass, and $\Delta h$ is the distance of fall. The effective mass is equal to the mass of the silicates, minus the mass of the water it is displacing, found as the mass of the silicates times the factor $(\rho_{\text{silicates}} - \rho_{\text{water}})/\rho_{\text{silicates}}$. This heat is calculated for each layer the mud passes through, and applied to melting the still unmelted ice in this layer, or to maintaining the thermal gradient in convecting water: the heat of infall through completely melted layers is transported by the convection to the first unmelted layer above the convecting region. The silicates are collected in the level above the first unmelted iteration layer they reach, until the volume of that layer is filled with silicates. If more silicates remain after that volume is filled, these are collected in the next layer upwards.
The continual shifting of silicates will result in changing weight fractions throughout the melting region, with corresponding changes in density, conductivity, heat capacity, and heat generation.

A3.2 Settling of silicate layers through melting ice regions. Since the melting point of all phases of ice except ice I increases with pressure, the first melting may occur well away from the center of the planet, resulting in a structure where slush or water overlays a silicate layer, which itself then overlays still unmelted ice.

As this ice melts, silicates from the overlaying regions will displace the mobile water, resulting in percolation of the water to the region above the silicate layers, and a general settling of the silicates. As before, this also releases heat due to loss of gravitational potential energy.

So as to avoid dealing with very small numbers, the program waits until the ice layer below is 10% melted before settling begins. The heat released in settling is evenly distributed over the two iteration layers involved, and their temperatures are found by determining the heat content of the various components, reassigning the distribution of this heat to match the new distribution of these components, and redetermining the temperatures from this new heat distribution by dividing by the heat capacity of each layer, itself newly derived for the new distribution of components.
AMMONIAD is a variation of CALLISTO which eliminates those sections dealing with phase changes, and instead considers the melting history of a planet whose ice phase is a solid solution of ammonia hydrate and water ice. The ice is assumed to be less than 33% ammonia by weight.

AMMONIAD handles conduction and convection, determination of heat capacities, conductivities, densities, and heat production, and motion of silicates from melting regions in the same fashion as CALLISTO. The only new section is that dealing with the melting of the ice solid solution.

A4.1 Melting of the eutectic liquid. The ammonia–water system has a complex phase diagram. The section of interest involves the melting of ammonia hydrate, $\text{NH}_3\cdot\text{H}_2\text{O}$ ($x_{\text{NH}_3} = 0.5$) and water ice ($x_{\text{NH}_3} = 0$). The eutectic point for this pair is at 173°K and composition $x_{\text{NH}_3} = 0.333$: the first liquid formed in melting appears at this temperature with this composition. We assume the melting point is depressed by pressure to an extent proportional to the extent equal pressure would depress the melting point of pure water ice.

We assume the total weight percent of ammonia is at cosmic abundance, 0.1, for the whole planet including silicates; this works out to 0.17 for the icy component. The silicate weight percent is dropped to 0.36, and its density is assumed to be 2.5. Therefore, all the ammonia will go into the liquid
phase. As more energy is applied to the system, beyond that necessary to melt the eutectic melt, the ice will melt into the liquid, changing its overall composition, until the original composition is reached and the ice totally melted.

Temperature-concentration and enthalpy-concentration curves are known for the ammonia-water system (see Perry, The Chemical Engineering Handbook). By finding the change in enthalpy, one can determine the change in concentration; and from this, the temperature of the liquid-ice mixture can be determined.

With the eutectic liquid melted, we assume convection will occur throughout the "slushy" region. Therefore, to model the heat flow in this region, we find the $\Delta T$ with time expected by the conduction equation for each iteration layer, and multiply this by the heat capacity and mass of the layer, to find the total heat flow in this layer. The heat figures for all the slushy layers are then added together; in addition, the flux from the core below the slushy region is found as in CALLISTO and added in. This total figure is divided by the mass of the whole slushy region, and the resulting heat per unit mass is added uniformly as the increase in enthalpy for each region.

Silicates fall from the slushy regions, as in CALLISTO, in proportion to the amount of melting which has occurred, and gather in the center to form a core.
**List of Variables: CALLISTO and ONIONKIN**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Heat production total, ergs/gram-million years</td>
</tr>
<tr>
<td>AGE</td>
<td>Age of the body to which the program will operate, million years</td>
</tr>
<tr>
<td>AK</td>
<td>Heat production due to decay of $^{40}\text{K}$</td>
</tr>
<tr>
<td>ATH</td>
<td>Heat production due to decay of $^{232}\text{Th}$</td>
</tr>
<tr>
<td>AU235</td>
<td>Heat production due to decay of $^{235}\text{U}$</td>
</tr>
<tr>
<td>AU238</td>
<td>Heat production due to decay of $^{238}\text{U}$</td>
</tr>
<tr>
<td>BH20</td>
<td>First term, conductivity of water</td>
</tr>
<tr>
<td>BICE</td>
<td>First term, conductivity of ice</td>
</tr>
<tr>
<td>BMUD</td>
<td>First term, conductivity of silicates</td>
</tr>
<tr>
<td>*BSTU</td>
<td>First term, conductivity of non-silicates</td>
</tr>
<tr>
<td>CH20</td>
<td>Second term, conductivity of water</td>
</tr>
<tr>
<td>CICE</td>
<td>Second term, conductivity of ice</td>
</tr>
<tr>
<td>CMUD</td>
<td>Second term, conductivity of silicates</td>
</tr>
<tr>
<td>*CSTU</td>
<td>Second term, conductivity of non-silicates</td>
</tr>
<tr>
<td>CP</td>
<td>Heat capacity</td>
</tr>
<tr>
<td>CPA(i)</td>
<td>First term, heat capacity of iteration region i</td>
</tr>
<tr>
<td>CPB(i)</td>
<td>Second term, heat capacity of iteration region i</td>
</tr>
<tr>
<td>DH20</td>
<td>First term, heat capacity of water</td>
</tr>
<tr>
<td>DICE</td>
<td>First term, heat capacity of ice</td>
</tr>
<tr>
<td>DMUD</td>
<td>First term, heat capacity of silicates</td>
</tr>
<tr>
<td>DH(i)</td>
<td>The amount of energy stored towards phase change in region i</td>
</tr>
<tr>
<td>DR1</td>
<td>The distance between $R_i$ and $R_{i+1}$</td>
</tr>
<tr>
<td>DR2</td>
<td>The distance between $R_{i-1}$ and $R_i$</td>
</tr>
</tbody>
</table>
DR1H  Half of DR1
DR2H  Half of DR2
DR3H  Half the distance between $R_{i-2}$ and $R_{i-1}$
DT    Change in temperature over a time iteration
*DTDR Local temperature gradient
DTIM  Change in time per iteration, million years
DHTIM Half of DTIM
EH20  Second term, heat capacity of water
EICE  Second term, heat capacity of ice
EMUD  Second term, heat capacity of silicates
F(i)  Weight fraction of silicates in region i
*FMUD Overall weight fraction of silicates in satellite
*F2   Weight fraction of silicates after partial melting of region
FLUX  Heat flux from surface of core transferred through convection layers
G(i)  Local acceleration of gravity in region i
*HEAT Heat due to settling of silicates through ice region
HEATUP Increase in heat due to phase change reversal
*HEATA Heat content of silicate region plus displaced water
*HEATB Heat content of ice region plus displacing silicates
*HOTH20 Heat content of water component
*HOTICE Heat content of icy component
*HOTMUD Heat content of silicate component
HLATN(i) The latent heat needed for phase change in region i
I, II to II5, II Various indices used in program
IFAZE(i)  The ice phase of region i
IFAZ(i)  The phase to which region i is changing
IFA = IFAZ(i)
IF = IFAZE(i)
ITL  The number of region i (= i)
ITOP  The ITL of the topmost liquid region
L, L1-L15,  Various logical variables used in phase changing routine
*N  Index, used to store ITL when necessary
P(i)  Pressure at region i
PCT(i)  Percent of region i which has changed phase (or turned to water)
PPMK  Amount of K in silicates, parts per million
PPMTH  Amount of Th in silicates, parts per million
PPMU  Amount of U in silicates, parts per million
PHCNT  Percent to which phase change has proceeded
Q  Heat production per unit volume; also time at which heat production is calculated
R(i)  Distance of region i from center of satellite, in cm
RTOTAL  Assumed first total radius of satellite
RTR1  Radius at which first phase boundary occurs
RTR2  Radius at which second phase boundary occurs
RTR3  Radius at which third phase boundary occurs
RO(i)  Density of region i
*ROTIO  Ratio of $(\rho_{silicate} - \rho_{water})/\rho_{silicate}$
ROICE(x)  Density of ice phase, ice x; ice 4 is silicates, ice 9 is water
ROW Old density of region
ROMUD Density of silicates
*SSAM Mass of liquid, convecting region
T(i) Temperature of region i
TT(i) Old temperature of region i
T1-T15 Temperatures at different phase boundaries for given pressure
TSUR Surface temperature
TCON Temperature at region ITOP, used to find TVECT
TVECT Adiabatic temperature in convecting region
TEMPUP Temperature found rom HEATUP
THIRD 1/3
TIME Elapsed time, in million years
UHEAT Heat gained or lost in maintaining convecting region
UHOT Heat gained or lost in given iteration region in convecting region
VOL(i) Volume of region i
*VOLWAT Volume of wate to be displaced by silicates
X Index used in setup
XC Constant to convert latent heats to proper units, times weight percent of ice in given region
XK Conductivity
XKP Conductivity of previous layer
XKN Conductivity of next layer
XKA(i) First term, conductivity of region i
XKB(i) Second term, conductivity of region i
XMAS(i) Mass of region i
*XM  Old mass of region i (or i-l)
*XMA  Old mass of region i-l (or i)
XMBLO(i)  Mass of satellite below region i
*XMOUT  Effective mass of silicates leaving given region
*XMOVE  Total effective mass of silicates leaving all regions
*XMUD  Mass of silicates settling from silicate region to melting ice
XRO  Density of non-silicates in given region

Additional variables in AMMONIAD
HLA  Latent heat needed to melt the eutectic liquid
UJOT  Heat change in slushy layer
UJEAT  Total heat change in slushy region
SSAMY  Total mass of slushy region
X  Weight fraction of ammonia in liquid
FNH3  Weight fraction of ammonia in planet
XHH3  Weight fraction of ammonia in water ice
XLOLO  Slope of line on a plot of composition of liquid vs. percent of ice melted, through points of first melt and final melt
TMI  Zero-pressure eutectic melting point
TM  Eutectic melting point at given pressure

* not in ONIONKIN
ICE PULSE

1. SET INITIAL CONDITIONS

2. FIND INITIAL TRANSITION RADII (ASSUMING HOMOGENEOUS PLANET)

3. FIND MASS IN EACH OF THE REGIONS

4. DO MASSES SUM TO THE TOTAL MASS?
   - YES
   - NO
     a. FIND TRANSITION RADII FOR THE NEW MASS DISTRIBUTION

5. FIND MASS IN EACH OF THE REGIONS

6. DO MASSES SUM TO THE TOTAL MASS?
   - YES
   - NO
     a. CHANGE $R_{\text{TOTAL}}$ BY CUBE ROOT OF % DIFFERENCE BETWEEN CALCULATED MASS SUM AND TOTAL

7. PRINT RESULTS

8. END
SUBROUTINE PULSE (FMEC, FICE, FRCU, XMAS, RTOTAL)
DIMENSION RCICE(8), PTR(3), RTR(3), RTRP(3)

F=F0
I=100
PTR(2)=11.5
RCICE(1)=0.92*FMEC/(FMEC*1.2+FICE*FRCU)
RCICE(2)=1.17*FMEC/(FMEC*1.14+FICE*FRCU)
RCICE(3)=1.14*FMEC/(FMEC*1.14+FICE*FRCU)
RCICE(4)=1.24*FMEC/(FMEC*1.24+FICE*FRCU)
RCICE(5)=1.31*FMEC/(FMEC*1.31+FICE*FRCU)
RCICE(6)=1.31*FMEC/(FMEC*1.31+FICE*FRCU)
RCICE(7)=1.53*FMEC/(FMEC*1.53+FICE*FRCU)
RCICE(8)=3.01*FMEC

CL=MAS3.23E73/(RTOTAL*RTOTAL*RTOTAL)
PTR(1)=(-39.749-0.041) / 1.0
DL=1

A=2*PTR(1) /(RTICE*4.56*45L-17)-RTOTAL*RTOTAL
B=C

CALL TROOT(A, P, E, F, G)

2 RTR(1)=RTTRAN(F, F, E, F, RTOTAL)

XMAS1=4.1883*RCICE(1) *(RTOTAL*RTOTAL*RTOTAL-RTR(1)*RTR(1)*RTR(1))
XMAS2=4.1883*RCICE(2) *(RTR(1)*RTR(1)*RTR(1))-RTR(1)*RTR(2)*RTR(2)
XMAS3=4.1883*RCICE(3) *(RTR(2)*RTR(2)*RTR(2))-RTR(3)*RTR(3)
XMAS4=4.1883*RCICE(4) *(RTR(3)*RTR(3)*RTR(3))

21 CONTINUE

A=3*PTR(1) /(RCICE(1)*RCICE(1)*RCICE(1)*RCICE(1)*RCICE(1)*RCICE(1)*RCICE(1)*RCICE(1))

B=2*RTR(1)*RTR(1)*RTR(1)*RTR(1)-3*(XMASS+XMASS+XMASS+XMASS+XMASS+XMASS+XMASS)

CALL TROOT(A, B, C)
RTTRP(1)=RTTRAN(F, F, E, F, RTOTAL)

A=(PTR(2)-PTR(1))/RCICE(2)*RCICE(2)*RCICE(2)*RCICE(2)*RCICE(2)*RCICE(2)*RCICE(2)*RCICE(2)

B=2*RTR(2)*RTR(2)*RTR(2)*RTR(2)*RTR(2)*RTR(2)*RTR(2)*RTR(2)

C=(RCICE(2)*RCICE(2)*RCICE(2)*RCICE(2)*RCICE(2)*RCICE(2)*RCICE(2)*RCICE(2))
CALL THROUT(A,B,C,E,F,G)
RTRP(2)=RTRANS(E,F,G,RTRP(1))
Z=(PTR(1)-P(2))/RCICE(6)=RCICE(6)*13.469615E-17+3*(XMASS8
2)/(RCICE(6)*6.2831=2*RTR(2)-2*RTR(2)
G=2*RTR(2)*RTR(3)-3*(XMASS8
1)
CALL THROUT(A,B,C,E,F,G)
RTRP(3)=RTRANS(F,G,H,RTRP(2))
RL=RTR(3)
LG 3: I, 3
3 RTR(1)=RTRP(1)
XMASS1=4.1446*RCICE(1)-(RTOTAL+RTR(1)+RTR(1)*RTR(1))
XMASS2=4.1996*RCICE(2)*(RTR(1)+RTR(1)-RTR(2)*RTR(2))
XMASS3=4.1996*RCICE(3)*(RTR(2)+RTR(2)-RTR(3)*RTR(3))
XMASS4=4.1996*RCICE(4)*(RTR(3)+RTR(3)-RTR(4))
XCTL=XMASS1+XMASS2+XMASS3+XMASS4
XDIF=XMAS-XTOT
1 LIF-2*XCLF/1.625
IF(1:IF)4, 41, 4
41 IF=(IF-3), 41, 4
4 FR=(XMAS/XICT)*.3333333333
II=II+1
IF(1-1C)30, 31, 31
30 G0 TC 5
39 CONTINUE
RTRP=FR+RTOTAL
G0 TC 21
5 PCENT=PTR(3)+ROICE(3)+RCICE(8)*13.47E-17*RTR(3)*RTR(3)
PRINT 500,XMAS,T,FICE,RCICE
500 FORMAT('1*A PLANET WITH MASS:',G12.2,'* GRAMS AT A CONSTANT T =',F6.2
1,'* MAIE EF',2*XIF,'* PER CENT WATER ICE, THE REMAINDER BEING',/2,'* SILICATES OF DENSITY',F6.2,'* G/CC WILL HAVE THE FOLLOWING PRESS
URE AND ICE PHASE STRUCTURE ---',/,'*CRADIUS PRESSURE ICE P
4 PHASE
XIF)
PRINT 501,RTRP
PRINT 502, RTR(1), PTR(1), XMASS1
PRINT 503, RTR(2), PTR(2), XMASS2
SUBROUTINE THROU(C1, C2, C3, C4, C5)  
THIRD=1.0/3.0  
IF(C2)<5.4  
5 IF(C1)>2,52,51  
51 L1=1.0E69  
L2=1.0E69  
L3=0.0  
GO TO 55  
52 L1=0.0  
L2=0.0  
L3=0.0  
GO TO 55  
53 F1=SCRT(-C1)  
F2=-F1  
F3=0.0  
GO TO 55  
4 CONTINUE:  
C3=C2*C2*0.25*C1*C1/2.0  
IF(C1)<3,2,1  
1 F1=SCRT(C3)-C2*0.5  
F2=-1.0*SCRT(C3)-C2*0.5  
I1=1  
I2=1  
IF(I1)11,12,12
11 C1=-C1
L1=-1
12 IF(C2)13,14,14
13 C2=-C2
L2=-1
14 C1=11*(L1**THIRD)
C2=12*(C2**THIRD)
E1=LI+12
F2=1.0E0
T3=3.2
GO TO 55
2 F1=(-C2*0.5)**THIRD
E1=-2*F1
E2=F1
E3=L2
CC 1C 55
3 E1=2*(C2*0.25*C2-C3)**(1.0/6.0)*COS(THIRD*ATAN(SCRTH-C3)/(C2*0.5))
F2=1.732 *((C2*C2*0.25-C3)**(1.0/6.0))*SIN(THIRD*ATAN(SC
1RTH-C3)/(C2*0.5))
L2=-1*0.5+F2
L3=-L1*0.5+F2
55 CONTINUE
PRINT 101, E1
PRINT 101, C2
PRINT 1G1, E3
101 FORMAT(*0X=*,-5PF12.2)
RETURN
END
FUNCTION RTRANS(E, F, G, H, AA)
IF(E-AA)12?, 110, 110
110 IF(F-AA)121, 111, 111
111 IF(G-AA)127, 112, 112
112 RTRANS=G
RETURN
120 IF(H)110, 110, 130
121 IF(F)111, 111, 131
122 IF(G)112, 112, 132
130 RTRANS=E
RETURN
131 RTRANS=F
RETURN
132 RTRANS=G
RETURN
END
BEGIN

DEFINE DATA;
SET INITIAL CONDITIONS

DOES TIME = 4.5 B.Yrs ??

YES

END

NO

INCREMENT TIME BY \( \Delta t \)

FIND VOLUME, RADIUS, GRAVITY FOR EACH LAYER, GIVEN \( \rho \), MASS FOR LAYER

FIND HEAT PRODUCED BY RADIOACTIVE NUCLIDES IN SILICATES

FIND \( K, C_p \) FROM THESE, FIND \( \Delta T \) FOR THIS TIME STEP, DUE TO CONDUCTION

A

G

H
A

WHAT SIGN IS NET FLUX FROM CONVECTING ZONE?

POSITIVE OR ZERO

ARE WE AT BOTTOM OF CRUST?

NO

ADD FLUX TO THIS LAYER

NO

ARE WE AT TOP OF THE CONVECTING ZONE?

YES

ADD FLUX TO THIS LAYER

NO

ARE WE AT TOP OF THE CORE?

YES

SET T TO T OF BOTTOM LAYER OF CONVECTING ZONE

NO

FIND NET FLUX OF HEAT OUT OF CORE PLUS HEAT GAINED IN THE CONVECTING ZONE

B
B

**ARE WE IN THE SILICATE CORE?**

No

1 (AMMONIAD ONLY)

**ARE WE CHANGING PHASE?**

No

**GIVEN T, P, FIND THE PHASE OF ICE WE OUGHT TO BE IN**

E

**SHOULD WE BE IN A NEW PHASE OF ICE?**

No

**ARE WE IN THE LIQUID REGION?**

No

**SET T TO THE ADIABATIC T, OR THE LOCAL MELTING T, WHICHEVER IS HIGHER**

C

No

**FIND OH, T OF PHASE CHANGE**

D
KEEP T CONSTANT; ADD (ΔTΔCp) TO HEAT STORED FOR PHASE CHANGE

FIND % OF PHASE CHANGE ACHIEVED

WHAT IS 0%?

% > 100%

CHANGE PHASE; FIND NEW AT FROM HEAT LEFTOVER

% < 0%

CHANGE TO LIQUID?

IS ΔH POSITIVE OR NEGATIVE?

YEAHS

FIND NEW Cp, K

NO

REVERT TO OLD PHASE

LET T INCREASE BY ΔH/Cp; CHANGE PHASE

TO 2 (CALLISTO ONLY)
PRINT OUT T, P, etc. (once every N iterations)

ARE WE AT THE BOTTOM OF THE PLANET?

MOVE DOWN ONE ITERATION LAYER

NO

YES

G

H
WHAT SORT OF LAYER ARE WE IN?

IS $T$ ABOVE THE EUTECTIC POINT?

YES

SET $T = \text{EUTECTIC TEMPERATURE}$

NO

ADD $\Delta T \cdot c_p$ TO HEAT STORED FOR MELTING

IS MELTING OF THE EUTECTIC MIX FINISHED?

YES

NO

IS ALL THE ICE NOW LIQUID?

YES

NO

SET $T = \text{adiabatic } T \ (\text{from topmost all-liquid region})$

INCREASE STORED ENTHALPY BY AMOUNT $\Delta T \cdot c_p$

FIND $X_j$ mole fraction of ammonia in the liquid, from the change in enthalpy

FIND $T$ FROM NEW VALUE OF $X$

FIND HOW MUCH SILICATE IS RELEASED BY MOST RECENT MELTING

FIND HEAT OF FUSION OF SILICATES FROM ABOVE; ADD TO HEAT STORED FOR MELTING

CHANGE WT. FRACT. OF SILICATE PRESENT, $c_p$, $K$, etc. APPROPRIATELY
**THIS IS PROJECT ONIONSkin, CALLED PROGRAM ONIONKIN. YOU DECIDE WHY.**

**IMPLICIT LOGICAL (L)**

**DIMENSION** R0ICE(9), R(27), XMAS(27), XMBO(27), G(27), RO(27), VOL(27), P1(27), T(27), TT(27), CPA(27), CPE(27), XKA(27), XKB(27), F(27), DH(27), IFA2(27), IFA3(27), HLATN(27)

**THESE ARE JUST INITIAL CONDITIONS**

```
DTIM=1.0
II=27
I3=500
TSUR=1.0
I4=II-1
RTOTAL=2600.0F5
TIME=0.0
AGE=4501
WHTIM=DTIM/2.0
PPK=15.0
PPMU=.012
PPMTH=.04
ROMUD=3.7
R0ICE(1)=0.92
R0ICE(2)=1.17
R0ICE(3)=1.14
R0ICE(4)=ROMUD
R0ICE(5)=1.23
R0ICE(6)=1.31
R0ICE(7)=1.5
R0ICE(8)=1.5
R0ICE(9)=1.0
TT(1)=TSUR
BICE=-1.829BE13
CICE=2.438E21
DICE=7.9F4
EICE=0.169E7
BH2O=0.145F19
CH2O=0.0
```
D4H2O=1.1936E4
E3H2O=3.6183E7
LMUN=6.7462E19
C'\mu N=0.0
D\mu N=0.0
L\mu N=0.75E7
I?=13
THIRD=1.0/3.0
IFAVE(1)=1
P(1)=0.0
VOL(1)=0.0
XMAS(1)=0.0
XMBLD(1)=0.0
F(1)=0.0
RTR1=2000.0E5
RTR2=1800.0E5
RTR3=1550.0E5
UHOT=0.0
HEATUP=0.0
FLUX=0.0

C THIS SETS THE INITIAL BOUNDARIES OF PHASE CHANGES
C

II=2
AF=0.0
X=4TR1
R(1)=RTOTL
DR?=?PTOTAL/(II-2)
DO 10 ITL=2,II
R(ITL)=R(ITL-1)-DR2
IF(X-R(ITL),1,11)
11 IF (RTR2-X)12,13,14
12 II=6
X=RTR2
GO TO 1
13 II=8
X=RTR3
GO TO 1
14 II=4
15 AF=1.0
16 IF A ZE(ITL)=II
17 F(ITL)=AF
18 PRINT 1000
19 FORMAT('TIME RADIUS TEMP PRESSURE* PX* RO MUD CP'

20 DO 15 ITL=1,II
21 T(ITL)=TSUR
22 DH(ITL)=0.0
23 HLAFN(ITL)=0.0
24 RO(ITL)=ROICE(IF A ZE(ITL))
25 IF A ZE(ITL)=IF A ZE(ITL)
26 X< ATL)=ROCUM*F(ITL)/ROMUN+BI CE*(1-F(ITL))/ROICE(IF A ZE(ITL)))*RO(ITL)
27 XKP(ITL)=(CMUD)*F(ITL)/ROMUN+C ICE*(1-F(ITL))/ROICE(IF A ZE(ITL)))*RO(ITL)
28 CPA(ITL) =CMUD*F(ITL)+ICE*(1-F(ITL))
29 CPR(ITL) =EMUD*F(ITL)+ICE*(1-F(ITL))
30 CONTINUE
31 DO 16 ITL=2,II
32 VOL (ITL)=4.18892*(R(ITL-1)+R(ITL-1)+R(ITL-1)-R(ITL)*R(ITL)*R(ITL))
33 XM AS (ITL)=RO(ITL)*VOL (ITL)
34 XM BLO(1)=XM BLO(1)-X M AS (ITL)
35 CONTINUE
36 G(1)=6.67E-9*(XM BLO(1)/(R TOTAL*R TOTAL))
37 I5=I4-1
38 DO 17 ITL=2,II
39 XM BLO (ITL)=XM BLO (ITL-1)-X M AS (ITL)
40 G(ITL)=6.67E-9*(XM BLO (ITL)/(R (ITL)*R (ITL)))
41 P(ITL)=P(ITL-1)+RO(ITL)*G(ITL)*OR2*1.0E-9
42 PRINT 1001,TIME,R(ITL),T(ITL),P(ITL),IF A ZE(ITL),RO(ITL),F(ITL),VOL
43 L1(ITL),XM AS(ITL),DH(ITL),DH(ITL),G(ITL)
44 1001 FORMAT('F7.3,-SPF9.0,OPF9.2,F7.2,'ICE*,I2,F5.2,2PF5.0,0PE10.3,
45 1*F11.3)
46 17 CONTINUE
HERE WE START THE TIME ITERATION

2 TIME=TIME+DTIM
   UHEAT=UHOT
   UHOT=0.0
   I=AGE-TIME
   IF(I)99,21,21
   21 I?22,22,23
   22 I=I+1
   23 I2=12-1
   R(I1-1)=0.0
   DO 221 I=1,15
   ITL=I-1
   VOL(ITL)=XMAS(ITL)/RJ(ITL)
   221 R(ITL-1)=(VOL(ITL)/4.1888+R(ITL)*R(ITL)*R(ITL))*THIRD
   DO 222 I=2,15
   XMBLO(I)=XMBLO(I-1)-XMAS(I)
   G(I)=6.67E-8*XMBLO(I)/R(I)*R(I)
   222 P(I)=P(I-1)+R(J(1)*G(I)*(R(I-1)-R(I)))*1.0E-9
   P(I4)=P(15)
   T(I1)=T(I1-1)
   DO 24 ITL=2,11
   24 TT(ITL)=T(ITL)
   IF(I2)25,25,26
   25 PRINT 1002,TIME,R(1),TSUR
   IF(MEANWHILE, BACK AT THE PAST...%/ F7.0,-5PF9.0,0PF9.2,6X,
   1'SURFACE')
   26 ITL=2
   DR?H=(R(I)-R(1))/2.0
   DR?H=DR?H
   XK=XKA(1)+XKB(1)/TSUR
   XKN=XKA(2)+XKB(2)/TT(2)
   Q=DHTIM-TIME
HERE WE GO THROUGH THE RADIUS ITERATIONS

3  XP=XP
  XP=XPN
  XPN=XPX(ITL+1)*XPX(ITL+1)/TT(ITL+1)
  CP=CPX(ITL)*TT(ITL)+CPX(ITL)
  Q=^*RTX(ITL)*FX(ITL)
  DR1=RX(ITL)-RZ(ITL+1)
  DR2=RZ(ITL)-RZ(ITL)
  DR3H=DR2H
  DR2H=DR1H
  DR1H=DR1/2.0
  L=TRUE.

  DT=(XP*XP(RX(ITL-1)-RZ(ITL-1))^2/(DR3H+DR2H)-XP*RX(ITL)
    *RZ(ITL-1)^2/(DR2H+DR1H))/((RZ(ITL)+DR2H)*(RZ(ITL)+
    2DR2H)*RX2)+Q^2*XN*TX(XRX(ITL))

  IF (FLUX*GT*0.0) GO TO 302
  IF (IFAZF(ITL)=EQ.9.0 AND IFAZE(ITL-1)=NE.9) DT=DT+FLUX/(CP*XMAS(ITL))
  IF (IFAZF(ITL)=EQ.9.0 AND IFAZE(ITL-1)=NE.9) L=FALSE.
  GO TO 303

302 CONTINUE
  IF (IFAZF(ITL)=EQ.9.0 AND IFAZE(ITL+1)=EQ.9.0 OR (IFAZF(ITL)=EQ.9.0 AND
    1.IFAZE(ITL)=NE.9 AND IFAZE(ITL+1)=EQ.9) ) DT=DT+FLUX/(CP*XMAS(ITL))
  GO TO 303

303 CONTINUE
  T(ITL)=TT(ITL)+DT
  IF (.NOT. (IFAZF(ITL)=EQ.4.0 AND IFAZE(ITL-1)=EQ.9)) GO TO 301
  T(ITL)=T(ITL-1)
  FLUX=-12.56636*XP*RZ(RX(ITL)-RZ(ITL)+DTIM*(TT(ITL)-TT(ITL+1))/(DR1H+DR2H)
    +UHFAT
  1H=0
  IF (IFAZE(ITL)=EQ.4.0) GO TO 32
  IF (IFAZE(ITL)=NE.9 AND IFAZE(ITL)) GO TO 35
THIS IS OUR PHASE FINDER ROUTINE

CONSIDER THE SYMBOL "TX" WHERE X IS SOME NUMBER BETWEEN ONE AND FIFTEEN, REPRESENTING ONE OF THE FIFTEEN PHASE BOUNDARIES IN THE PHASE DIAGRAM OF ICE. TX WILL BE THE TEMPERATURE WHICH MUST BE PRESENT, GIVEN THE PRESSURE, FOR THE LOCUS OF INTEREST TO FALL UPON LINE X. LX IS THE LOGICAL CONDITION THAT THE REAL TEMPERATURE IS LESS THAN TX, DEPENDING ON WHETHER LX IS TRUE OR FALSE WE CAN SEE WHETHER THE LOCUS OF INTEREST FALLS WITHIN A REGION BOUNDED BY VARIOUS LINES X.

THAT'S WHAT IT SAYS HERE, I'M ONLY READING IT.

5 IFAZ(ITL)=8
   T14=24.29*P(ITL)-236.923
   T15=270.15
   L14=TXL(T(I TL),LT,T14)
   L15=TXL(T(I TL),LT,T15)
   IF(L14.AND.L15)GO TO 6
   IFAZ(ITL)=6
   T9=24518-2337.9476*P(ITL)+56.355*P(ITL)*P(ITL)
   T4=220.66+9.265*P(ITL)-0.1424*P(ITL)*P(ITL)
   T13=40.4-30.395*P(ITL)
   T3=234.1-341.9297*P(ITL)
   L9=TXL(T(I TL),LT,T9)
   L4=TXL(T(I TL),LT,T4)
   L13=TXL(T(I TL),LT,T13)
   L3=TXL(T(I TL),LT,T3)
   IFAZ(ITL)=2
   T12=271.9-14.577*P(ITL)
   T11=205.32+20.486*P(ITL)-2.986*P(ITL)*P(ITL)
   T10=4.271+116.7536*P(ITL)
   L12=TXL(T(I TL),LT,T12)
   L11=TXL(T(I TL),LT,T11)
   L10=TXL(T(I TL),LT,T10)
   IF(L10.AND.L11.AND.L12.AND.L13)GO TO 6
   IFAZ(ITL)=1
   T6=1028.94-371.747*P(ITL)
   TL=273.16+6.932*P(ITL)-1.837*P(ITL)*P(ITL)
   L6=TXL(T(I TL),LT,T6)
L1 = T(ITL) * LT * T1
IF (L6 .AND. L1 .AND. (NOT L10)) GO TO 6
IFAZ (ITL) = 5
T3 = 223.49 + 12.934 * P(ITL) - 0.489 * P(ITL) * P(ITL)
T7 = 332.226 * P(ITL) - 914.6
L3 = T(ITL) * LT * T3
L7 = T(ITL) * LT * T7
IF (L7 .AND. L7 .AND. L8 .AND. (NOT L12)) GO TO 6
IFAZ (ITL) = 3
T2 = 230.12 + 13.6717 * P(ITL) - 1.7835 * P(ITL) * P(ITL)
T7 = T(ITL) * LT * T2
IF (L2 .AND. (NOT L7) .AND. (NOT L11) .AND. (NOT L6)) GO TO 6
IFAZ (ITL) = 7
T5 = 139.49 + 11.015 * P(ITL) - 0.08931 * P(ITL) * P(ITL)
L5 = T(ITL) * LT * T5
IF ((NOT L15) .AND. L9 .AND. L5) GO TO 6
IFAZ (ITL) = 9
IF (IFAZE (ITL) .EQ. 9 .AND. L) GO TO 4

THIS IS OUR PHASE CHANGER ROUTINE.

6 IF (IFAZE (ITL) .EQ. IFAZ (ITL)) 60, 32, 60
60 IF (IFAZF (ITL) .EQ. 9 .AND. L) GO TO 4
XC = 0.232444F7
IFA = IFAZ (ITL)
IF = IFAZE (ITL)
GO TO (61, 62, 63, 64, 65, 66, 67, 68, 69), IFA
61 GO TO (64, 612, 613, 64, 64, 64, 64, 64, 64, 64, 619), IF
612 HLATN (ITL) = XC * (180)
T(ITL) = T10
GO TO 59
613 HLATN (ITL) = XC * (973.89 - 4.252 * T6)
T(ITL) = T6
GO TO 59
619 HLATN (ITL) = XC * (3782.5 - 19.08 * T1)
T(ITL) = T1
GO TO 59
62  XC=-XC
   GO TO (612,64,632,64,652,662,64,64,629),IF
629  II=5
   IF(P(ITL).LT.3.5) II=3
   IF\$2(ITL)=11
   GO TO (64,64,639,64,659),II
63  GO TO (631,632,64,64,635,64,64,64,639),IF
631  HLATN(ITL)=XC*(4.252*T6-973.89)
     T(ITL)=T6
   GO TO 59
632  HLATN(ITL)=XC*(8.077*T11-1705.96)
     T(ITL)=T11
   GO TO 59
635  HLATN(ITL)=XC*(0.137*T7-18.09)
     T(ITL)=T7
   GO TO 59
639  HLATN(ITL)=XC*(R477.01-37.4*T2)
     T(ITL)=T2
   GO TO 59
64  TIME=TIME+1000.00,0
   I?=0.0
   GO TO ??
65  GO TO (64,652,653,64,64,656,64,64,659),IF
652  HLATN(ITL)=XC*(1.1525*T12)
     T(ITL)=T12
   GO TO 59
653  HLATN(ITL)=XC*(19.09-0.137*T7)
     T(ITL)=T7
   GO TO 59
656  HLATN(ITL)=XC*4.0
     T(ITL)=T8
   GO TO 59
659  HLATN(ITL)=XC*(969.8-8.1565*T3)
     T(ITL)=T3
   GO TO 59
66  GO TO (64,662,64,64,665,64,667,668,669),IF
662  HLATN(ITL)=XC*(1.14758*T13)
     T(ITL)=T13
   GO TO 59
665   HLATN(ITL)=XC*(-4.0)
          T(ITL)=T5
          GO TO 59
667   HLATN(ITL)=XC*(10.29*T9-102.98)
          T(ITL)=T9
          GO TO 59
668   HLATN(ITL)=XC*785.0
          T(ITL)=T14
          GO TO 59
669   HLATN(ITL)=XC*(-404.88-3.1434*T4)
          T(ITL)=T4
          GO TO 59
670   GO TO (64,64,64,64,64,64,64,678,679), IF
671   HLATN(ITL)=XC*(102.88-0.29*T9)
          T(ITL)=T9
          GO TO 59
672   HLATN(ITL)=XC*264.0
          T(ITL)=T15
          GO TO 59
673   HLATN(ITL)=XC*(-404.88-3.1434*T5)
          T(ITL)=T5
          GO TO 59
674   XC=-XC
          GO TO (64,64,64,64,64,64,668,678,64,64), IF
675   XC=-XC
          GO TO (619,672,639,64,659,669,679,64,64), IF
676   II=5
          IF(P(ITL).LT.3.5) II=3
          IF(AZL(ITL)=II
          GO TO (64,64,64,632,64,652), II
677   IF(HEATUP.NE.0.0) GO TO 351
          IF(DT*DT.GT.(T(ITL)-TT(ITL))*(T(ITL)-TT(ITL))) GO TO 351
          IF(II)351,35,351

C
C AFTER WE HAVE VISITED THE PHASE CHANGER, WE COME BACK HERE TO TAKE
C WHATEVER ACTION IS APPROPRIATE. IF NO PHASE CHANGE IS REQUIRED, WE GO
C BACK TO 3 TO ITERATE AGAIN AT THE NEXT RADIUS POINT.
C
35   T(ITL)=TT(ITL)
351  DH(ITL)=DH(ITL)+DT*CP
352 PHCNT=DIH(IL) / (HATN(IL) + 1.0)
   IF (PHCNT .GT. 1.0) GO TO 37
   IF (PHCNT .LT. 0.0) GO TO 38
   C
   WE COME HERF IF WE NEED HEAT TO CHANGE PHASE.
   C
36  R(IIL)=ROICE(IFA/IL) .ROICE(IFA/IL) / (PHCNT .ROICE(IFA/IL)
   1) + (1.0 - PHCNT) .ROICE(IFA/IL))
   IF (9 - IFA/IL) .GE. 32, 43, 32
   C
   WE COME HERE IF WE HAVE ENOUGH HEAT TO HAVE FINISHED CHANGING PHASE.
   C
37  IFA/IL=IFA/IL
       R(IIL)=ROICE(IFA/IL)
       T(IIL)=T(IIL) + (DH(IIL) - HATN(IIL))/CP
       DH(IIL)=0.0
       IF (9 - IFAZ(IIL)) .GE. 32, 47, 32
   C
   WE COME HERE IF WE ONCE STARTED TO CHANGE PHASE, BUT HAVE SINCE
   C
   CHANGED OUR MINDS INSTEAD... FOR EXAMPLE, IF THIS POINT WERE ONCE
   C
   HEATING UP AND CHANGING PHASE, BUT NOW THE CONDITIONS IN THE PLANET
   C
   HAVE CHANGED SO THAT THIS POINT NOW WANTS TO COOL DOWN.
   C
38  IF (HATN(IIL)) .GE. 382, 381, 381
   381  IFAZ(IIL)=IFA/IL
        R(IIL)=ROICE(IFA/IL)
        T(IIL)=T(IIL) + DH(IIL)/CP
        DH(IIL)=0.0
       GO TO 32
C
   I FORGET WHAT THE HECK THIS PART DOES.
C
382  IFA/IL=IFA/IL
       R(IIL)=ROICE(IFA/IL)
       TEMPU/T=T(IIL)
       HEATUP=-HATN(IIL)
       T(IIL)=TEMPUP+HEATUP/CP
       GO TO 5
\[ DH(\text{ITL}) = \text{HEATUP} - (T(\text{ITL}) - \text{TEMPUP}) \cdot \text{CP} + DH(\text{ITL}) \]

\[ \text{HEATUP} = 0.0 \]

GO TO 352

C
C THIS IS THE PART THAT CONVECTS THE LIQUID WATER
C

4 IF(IFAZE(\text{ITL}-1)-9)41, 42, 41

41 \text{ITOP} = \text{ITL}

\text{TCON} = \text{T(\text{ITL})}

42 \text{X} = 1.3 \times \exp\left(\frac{\text{P(\text{ITL})} - \text{P(\text{ITOP})}}{(\text{CP} \times \text{RO(\text{ITL})})}\right)

\text{TVECT} = \text{TCON} \times \exp(\text{X})

\text{UNIT} = \text{UNIT} + \text{IT} \times \text{CP} \times \text{XMAS(\text{ITL})}

\text{T(\text{ITL})} = \text{TVECT}

\text{IF(T2.GT.T(\text{ITL}))} \text{T(\text{ITL})} = T2

\text{IF(T3.GT.T(\text{ITL}))} \text{T(\text{ITL})} = T3

\text{IF(T4.GT.T(\text{ITL}))} \text{T(\text{ITL})} = T4

\text{IF(T5.GT.T(\text{ITL}))} \text{T(\text{ITL})} = T5

\text{T(\text{ITL})} = T(\text{ITL}) + 0.001

\text{IFAZ(\text{ITL})} = \text{IFAZE(\text{ITL})}

GO TO 32

43 \text{CONTINUE}

\text{XKA(\text{ITL})} = (1.0 - \text{PHCNT}) \times \text{ROICE(IFAZE(\text{ITL})) + DH2O \times \text{PHCNT}} \times \text{RO(\text{ITL})}

\text{XKR(\text{ITL})} = (1.0 - \text{PHCNT}) \times \text{ROICE(IFAZE(\text{ITL})) + CH2O \times \text{PHCNT}} \times \text{RO(\text{ITL})}

\text{CPA(\text{ITL})} = (1.0 - \text{PHCNT}) \times \text{DH2O \times \text{PHCNT}}

\text{CPB(\text{ITL})} = (1.0 - \text{PHCNT}) \times \text{CH2O \times \text{PHCNT}}

GO TO 32

47 \text{XKA(\text{ITL})} = \text{DH}2O

\text{XKR(\text{ITL})} = \text{CH}2O

\text{CPA(\text{ITL})} = \text{DH}2O

\text{CPB(\text{ITL})} = \text{CH}2O

32 \text{IF(T2.GT.3, 33, 34)

33 \text{PRINT 1001, TIME, R(\text{ITL}), T(\text{ITL}), P(\text{ITL}), IFAZE(\text{ITL}), RO(\text{ITL}), F(\text{ITL}), CP,} 

\text{XK, 0, DH(\text{ITL}), G(\text{ITL})}

\text{PRINT 1005, PHCNT, DT, IFAZE(\text{ITL}), TT(\text{ITL}), FLUX, UHEAT, XMAS(\text{ITL}), ITL}

1005 \text{FORMAT(8G14.4,/)}

34 \text{ITL = ITL + 1}

\text{IF(ITL-ITL)2, 2, 3}

99 \text{CALL EXIT}

END
SUBROUTINE WORLD(RTOTAL, RTR1, RTR2, RTR3, FMUD, ROMUD)
IMPLICIT LOGICAL (L)
DIMENSION ROICE(9), R(27), XMAS(27), XMDLO(27), G(27), RD(27), VOL(27), P
I(27), T(27), TT(27), CPA(27), CPB(27), XKA(27), XKB(27), F(27), DI(27), IFA
27E(27), IFAZ(27), HLATN(27), PCT(27)

C These are just initial conditions

I1 = 27
I3 = 250
TSUR = 100.0
I4 = I1 - 1
TIMF = 0.0
AGE = 4501
DTIM = 1.0
DHTIM = DTIM * 0.5
PPMK = 815.0
PPMU = 0.012
PPMTH = 0.04
ROICE(1) = 0.92
ROICE(2) = 1.17
ROICE(3) = 1.14
ROICE(4) = ROMUD
ROICE(5) = 1.23
ROICE(6) = 1.31
ROICE(7) = 1.5
ROICE(8) = 1.5
ROICE(9) = 1.0
TT(1) = TSUR
BICE = -1.8298E18
CICE = 2.438E21
DICE = 7.0E4
FICE = 0.169E7
BH20 = 0.185E19
CH20 = 0.0
DH20 = 1.1936E4
EH20 = 3.6188E7
BMUD = 0.7462E19
CMUD = 0.0
DMUD=0.0
EMUD=0.75E7
I2=13
THIRD=1.0/3.0
IFAZE(1)=1
P(1)=0.0
VOL(1)=0.0
XMAS(1)=0.0
XMBLO(1)=0.0
XMOVE=0.0
FLUX=0.0
STEAT=0.0
UHOT=0.0
SSAM=1.0
ROTIO=1.0-ROICE(9)/ROMUD
HEATUP=0.0

C THIS SETS THE INITIAL BOUNDARIES OF PHASE CHANGES
C

II=1
X=RTR1
R(1)=RTOTAL
DR2=RTOTAL/(II-2)
DO 1 ITL=2,II
   R(ITL)=R(ITL-1)-DR2
   IF(X-R(ITL))1,1,11
11 IF (RTR2-X)12,13,14
II=2
X=RTR2
GO TO 1
II=6
X=RTR3
GO TO 1
II=8
1 IFAZE(ITL)=II
PRINT 1000
1000 FORMAT(' TIME RADIUS TEMP PRESSURE DT',7X,'DT/DR PHASE %MUD
1D RO MASS LITTLE G PERCENT MELTED OLD T')
HERE WE CALCULATE THE INITIAL CONDITIONS AT EACH POINT WITHIN THE PLANET

DO 15 ITL=1,11
T(ITL)=TSUR
F(ITL)=FMUD
DH(ITL)=0.0
HLATN(ITL)=0.0
PCT(ITL)=0.0
RO(ITL)=ROICE(IFAZE(ITL))*ROMUD/(F(ITL)*ROICE(IFAZE(ITL))+(1-F(ITL)))*RCMUD)
IFAZ(ITL)=IFAZE(ITL)
XKA(ITL)=(DMUD*F(ITL)/ROMUD+BICE*(1-F(ITL)))/ROICE(IFAZE(ITL)))*RO(1ITL)
XKB(ITL)=(CMUD*F(ITL)/ROMUD+CICE*(1-F(ITL)))/ROICE(IFAZE(ITL)))*RO(1ITL)
CPA(ITL)=DMUD*F(ITL)+DICE*(1-F(ITL))
CPB(ITL)=EMUD*F(ITL)+EICE*(1-F(ITL))
15 CONTINUE
DO 16 ITL=2,14
VOL(ITL)=4.1888*(R(ITL-1)*R(ITL-1)-R(ITL)*R(ITL))*ROMUD
XMAS(ITL)=RO(ITL)*VOL(ITL)
XMBLO(I)=XMBLO(1)+XMAS(ITL)
16 CONTINUE
G(1)=6.67E-8*(XMBLO(1)/(RTOTAL*RTOTAL))
I5=14+1
DO 17 ITL=2,15
XMBLO(ITL)=XMBLO(ITL-1)-XMAS(ITL)
G(ITL)=6.67E-8*(XMBLO(ITL)/(R(ITL)*R(ITL)))
P(ITL)=P(ITL-1)+RO(ITL)*G(ITL)*DR2*1.0E-9
17 CONTINUE
XMBLO(I4)=0.0
G(I4)=0.0
P(I4)=P(I5)

HERE WE START THE TIME ITERATION

2 TIME=TIME+DTIM
I=AGE-TIME
IF(I) = 99, 21, 21
21 IF(I2) = 22, 22, 23
22 I2 = 13
   PRINT 1003, FLUX, UHEAT, STEAT
1003 FORMAT('OFUX OF HEAT:', E12.4, ' \n HEAT TO MAINTAIN WATER:', E12.4, '
 HEAT OF CORE FORMATION:', E12.4)
   PRINT 1000
23 I2 = I2 - 1
   R(I1 - 1) = 0.0
   UHEAT = UHUT
   UHOT = 0.0
   DO 221 I = 1, 15
      ITL = I1 - 1
      VOL(ITL) = XMAS(ITL)/RO(ITL)
220 R(I2) = T(TL) = T(1T) = T(I1 - 1)
   CONTINUE
   DO 222 I = 2, 15
      XMBLO(I) = XMBLO(I - 1) - XMAS(I)
      G(I) = 6.67E-8 * XMBLO(I)/R(I) * R(I)
222 P(I) = P(I - 1) + RO(I) * G(I) * (R(I - 1) - R(I)) * 1.0E-9
      P(I4) = P(I5)
      T(I) = T(I1 - 1)
   DO 24 ITL = 2, 11
24 TT(ITL) = T(ITL)
25 IF(I2) = 25, 25, 26
1002 FORMAT('MEANWHIL\' , BACK AT THE PAST---//'F7.0, -5PF9.0, 0PF9.2, 6X, 1'SURFACE')
26 ITL = 2
   DR2H = (R(I1) - R(I2)) * 0.5
   DR1H = CR2H
   XK = XKA(I) + XKB(I)/TSUR
   XKN = XKA(I) + XKB(I)/TT(I)
   Q = DHTIME - TIME
   AK = PPMK * 0.01148 * EXP(Q * 0.000521)
   AU235 = PPMU * 115.396 * EXP(Q * 0.000927)
   AU238 = PPMU * 70.846 * EXP(Q * 0.000154)
   ATH = PPMTH * 10.527 * EXP(0.00005)
   A = (AK + AU235 + AU238 + ATH) * 1.0E6
HERE WE GO THROUGH THE RADIUS ITERATIONS

3 XKP=XK
   XK=XKN
   XKN=XKA(ITL+1)+XKC(ITL+1)/TT(ITL+1)
   CP=CPA(ITL)*TT(ITL)+CPB(ITL)
   Q=A*RO(ITL)*F(ITL)
   DR1=R(ITL)-R(ITL+1)
   DR2=R(ITL-1)-R(ITL)
   DR3H=DR2H
   DR2H=DR1H
   DR1H=DR1*0.5
   L=.TRUE.
   DT=((XKP*R(ITL-1)*R(ITL-1)=TT(ITL-1)-TT(ITL))/(DR3H+DR2H)-XK*R(ITL)
   *(TT(ITL)-TT(ITL+1))/(DR2H+DR1H))/((R(ITL)+DR2H)*(R(ITL)+
   2*DR2H)*DR2H*Q)*DTIM/(CP*RO(ITL))
   IF (FLUX.GT.0.0) GO TO 302
   IF(IFAZE(ITL).EQ.9.AND.IFAZE(ITL-1).NE.9)DT=DT+FLUX/(CP*XMAS(ITL))
   IF(IFAZE(ITL).EQ.9.AND.IFAZE(ITL-1).NE.9) L=.FALSE.
   GO TO 303
302 CONTINUE
       IFAZE(ITL).NE.9.AND.IFAZE(ITL+1).EQ.4) DT=DT+FLUX/(CP*XMAS(ITL))
303 CONTINUE
   T(ITL)=TT(ITL)+DT
   IF(.NOT.((IFAZE(ITL).EQ.4.AND.IFAZE(ITL-1).EQ.9))GO TO 301
   T(ITL)=T(ITL-1)
   FLUX=-12.56636*XK*R(ITL)*R(ITL)*DTIM*(TT(ITL)-TT(ITL+1))/(DR1H+DR2H+1)
   UHEAT
301 CONTINUE
   DT=DT*(T(ITL)-T(ITL-1))/(DR2H+CR3H)
   II=0
   IF(IFAZE(ITL).EQ.4) GO TO 32
   IF(IFAZE(ITL).NE.IFAZ(ITL)) GO TO 35
CONSIDER THE SYMBOL "TX" WHERE X IS SOME NUMBER BETWEEN ONE AND FIFTEEN, REPRESENTING ONE OF THE FIFTEEN PHASE BOUNDARIES IN THE PHASE DIAGRAM OF ICE. TX WILL BE THE TEMPERATURE WHICH MUST BE PRESENT, GIVEN THE PRESSURE, FOR THE LOcus OF INTEREST TO FALL UPON LINE X. LX IS THE LOGICAL CONDITION THAT THE REAL TEMPERATURE IS LESS THAN TX. DEPENDING ON WHETHER LX IS TRUE OR FALSE WE CAN SEE WHETHER THE LOcus OF INTEREST FALLS WITHIN A REGION BOUNDED BY VARIOUS LINES X.

THAT'S WHAT IT SAYS HERE, I'M ONLY READING IT.

5

IFAZ(ITL)=8
T14=24.39*P(ITL)-236.923
T15=270.15
L14=T(ITL).LT.T14
L15=T(ITL).LT.T15
IF(L14.AND.L15)GO TO 6
IFAZ(ITL)=6
T9=24518-2337.9476*P(ITL)+56.355*P(ITL)*P(ITL)
T4=220.66+9.2065*P(ITL)-0.14124*P(ITL)*P(ITL)
T13=403.4-30.395*P(ITL)
T8=2430.1-341.297*P(ITL)
L9=T(ITL).LT.T9
L4=T(ITL).LT.T4
L13=T(ITL).LT.T13
L8=T(ITL).LT.T8
IF(L4.AND.(.NOT.L3).AND.(.NOT.L9).OR.(P(ITL).LT.20.79)).AND.(.NOT.L14.OR.(T(ITL).GT.270.335)).AND.(.NOT.L13))GO TO 6
IFAZ(ITL)=2
T12=299.9-14.577*P(ITL)
T11=205.32+20.48*P(ITL)-2.2986*P(ITL)*P(ITL)
T10=4.271+110.2536*P(ITL)
L12=T(ITL).LT.T12
L11=T(ITL).LT.T11
L10=T(ITL).LT.T10
IF(L10.AND.L11.AND.L12.AND.L13)GO TO 6
IFAZ(ITL)=1
T6=1028.04-371.747*P(ITL)
T1=273.16-6.832*P(ITL)-1.827*P(ITL)*P(ITL)
L6=T(ITL).LT.T6
L1=T(ITL).LT.T1
IF(L6.AND.L1.AND.(.NOT.L10))GO TO 6
AFTER WE HAVE VISITED THE PHASE CHANGER, WE COME BACK HERE TO TAKE
WHATEVER ACTION IS APPROPRIATE. IF NO PHASE CHANGE IS REQUIRED, WE GO
BACK TO 3 TO ITERATE AGAIN AT THE NEXT RADIUS POINT.
IFAZ (ITL) = 5
T3 = 223.69 + 10.934 * P (ITL) - 0.489 * P (ITL) * P (ITL)
T7 = 332.226 * P (ITL) - 914.6
L3 = T (ITL) . LT . T3
L7 = T (ITL) . LT . T7
IF (L7 . AND . L3 . AND . L8 . AND . (. NOT . L12)) GO TO 6
IFAZ (ITL) = 3
T2 = 230.12 + 13.6717 * P (ITL) - 1.7835 * P (ITL) * P (ITL)
L2 = T (ITL) . LT . T2
IF (L2 . AND . (. NOT . L7) . AND . (. NOT . L11) . AND . (. NOT . L6)) GO TO 6
IFAZ (ITL) = 7
T5 = 139.49 + 11.015 * P (ITL) - 0.08931 * P (ITL) * P (ITL)
L5 = T (ITL) . LT . T5
IF (. NOT . L15) . AND . L9 . AND . L5) GO TO 6
IFAZ (ITL) = 9
IF (IFAZ (ITL) . EQ . 9 . AND . L) GO TO 4

THIS IS OUR PHASE CHANGER ROUTINE.

6 IF (IFAZ (ITL) = IFAZ (ITL)) GO TO 60, 32, 60
60 IF (IFAZ (ITL) . EQ . 9 . AND . L) GO TO 4
XC = 0.232 + 44E7 * (1 - F (ITL))
IFA = IFAZ (ITL)
IF = IFAZ (ITL)
GO TO (61, 62, 63, 64, 65, 66, 67, 68, 69), IFA
61 GO TO (64, 612, 613, 64, 64, 64, 64, 64, 64, 64, 64, 619), IF
612 HLATN (ITL) = XC * (18)
T (ITL) = T10
GO TO 59
613 HLATN (ITL) = XC * (973.89 - 4.252 * T6)
T (ITL) = T6
GO TO 59
619 HLATN (ITL) = XC * (3792.5 - 19.08 * T1)
T (ITL) = T1
GO TO 59
62 XC = - XC
GO TO (612, 64, 632, 64, 652, 662, 64, 64, 64, 629), IF
629 II = 5
IF (P (ITL) . LT . 3.5) II = 3
IF (ROMUD - RO(ITAL)) < 81,481,483

481 XMA = ROMUD * XMAS(ITAL) / RO(ITAL)
XM = XMAS(ITAL) - XMA
XMAS(ITAL) = XMA
XMAS(ITAL - 1) = XMAS(ITAL - 1) + XM
IFAZE(ITAL) = 4
IFAZ(ITAL) = 4
F(ITAL) = 1.0
DH(ITAL) = 0.0
RO(ITAL) = ROMUD
F(ITAL - 1) = (F(ITAL - 1) * (XMAS(ITAL - 1) - XM) * ROTIO + XM) / (XMAS(ITAL - 1) * ROTIO)
ROW = RO(ITAL - 1)
RO(ITAL - 1) = XMAS(ITAL - 1) + RO(ITAL - 1) / (XMAS(ITAL - 1) - XM)
XKA(ITAL) = BMUD
XKB(ITAL) = CMUD
CPA(ITAL) = EMUD
CPB(ITAL) = DMUD
F2 = XM / (XMAS(ITAL - 1) * ROTIO)
XKA(ITAL - 1) = (BMUD * F2 / ROMUD + XKA(ITAL) * (1.0 - F2) / ROW) * RO(ITAL - 1)
XKB(ITAL - 1) = (CMUD * F2 / ROMUD + XKB(ITAL) * (1.0 - F2) / ROW) * RO(ITAL - 1)
CPA(ITAL - 1) = DMUD * F2 + CPA(ITAL) * (1.0 - F2)
CPB(ITAL - 1) = EMUD * F2 + CPB(ITAL) * (1.0 - F2)
ITAL = ITAL - 1
IF (ROMUD - RO(ITAL)) < 81,481,482

483 F(ITAL) = (F(ITAL) * (XMAS(ITAL) - XMOVE) * ROTIO + XMOVE) / (XMAS(ITAL) * ROTIO)
F2 = XMOVE / (XMAS(ITAL) * ROTIO)
XKA(ITAL) = (BMUD * F2 / ROMUD + XKA(ITAL) * (1.0 - F2) / ROW) * RO(ITAL)
XKB(ITAL) = (CMUD * F2 / ROMUD + XKB(ITAL) * (1.0 - F2) / ROW) * RO(ITAL)
CPA(ITAL) = DMUD * F2 + CPA(ITAL) * (1.0 - F2)
CPB(ITAL) = EMUD * F2 + CPB(ITAL) * (1.0 - F2)

482 XMOVE = 0.0
ITAL = N
GO TO 341

C THIS MOVES WATER UP THROUGH SLUSH, AND MUD DOWN THROUGH SLUSH.
C

390 N = ITL
391 VOLWAT = XMAS(ITAL) * PCT(ITAL) * (1.0 - F(ITAL))
XMUD = ROMUD * VOLWAT
XM = XMAS(ITAL)
IFAZ(ITL)= 11
GO TO (64,64,639,64,659), 11
63 GO TO (631,632,64,64,635,64,64,64,639), IF
631 HLATN(ITL)=X*(4.252*76-973.89)
T(ITL)=T6
GO TO 59
632 HLATN(ITL)=X*(8.077*T11-1705.96)
T(ITL)=T11
GO TO 59
635 HLATN(ITL)=X*(0.137*T7-18.09)
T(ITL)=T7
GO TO 59
639 HLATN(ITL)=X*(8477.01-37.4*T2)
T(ITL)=T2
GO TO 59
64 TIME=TIME+10000.0
I2=0
GO TO 32
65 GO TO (64,652,653,64,64,656,64,64,659), IF
652 HLATN(ITL)=X*(1.1525*T12)
T(ITL)=T12
GO TO 59
653 HLATN(ITL)=X*(18.09-0.137*T7)
T(ITL)=T7
GO TO 59
656 HLATN(ITL)=X*4.0
T(ITL)=T8
GO TO 59
659 HLATN(ITL)=X*(969.8-8.1585*T3)
T(ITL)=T3
GO TO 59
66 GO TO (64,662,64,64,665,64,667,668,669), IF
662 HLATN(ITL)=X*(1.14758*T13)
T(ITL)=T13
GO TO 59
665 HLATN(ITL)=X*(-4.0)
T(ITL)=T8
GO TO 59
\[ F(\text{ITL}) = F_2 \times \frac{(X\text{MAS}(\text{ITL}) + \text{XMOUT})}{X\text{MAS}(\text{ITL})} \]
\[ \text{XMOVE} = \text{XMOVE} + \text{XMOUT} \]
\[ \text{BSTU} = \text{BICE} \times \frac{(1.0 - \text{PHCNT})}{\text{ROICE}(\text{IFAZ}(\text{ITL})) + \text{BH2O} \times \text{PHCNT} \times \text{XRO}} \]
\[ \text{CSTU} = \text{CICE} \times \frac{(1.0 - \text{PHCNT})}{\text{ROICE}(\text{IFAZ}(\text{ITL})) + \text{CH2O} \times \text{PHCNT} \times \text{XRO}} \]
\[ X\text{KA}(\text{ITL}) = \text{BIMUD} \times F(\text{ITL}) \times \text{ROMU} \times \text{BSTU} \times (1 - F(\text{ITL})) / \text{XRO} \times \text{RO}(\text{ITL}) \]
\[ X\text{KB}(\text{ITL}) = \text{CIMUD} \times F(\text{ITL}) \times \text{ROMU} \times \text{CSTU} \times (1 - F(\text{ITL})) / \text{XRO} \times \text{RO}(\text{ITL}) \]
\[ \text{CPA}(\text{ITL}) = \text{DMUD} \times F(\text{ITL}) \times \text{CIMUD} + \text{PHCNT} + \text{DH2O} \times \text{PHCNT} \]
\[ \text{CPR}(\text{ITL}) = \text{EMUD} \times F(\text{ITL}) \times \text{CIMUD} + \text{PHCNT} + \text{EH2O} \times \text{PHCNT} \]
\[ \text{GO TO 32} \]

47 \[ \text{PCT}(\text{ITL}) = 1.0 \]
\[ \text{XMOUT} = F(\text{ITL}) \times X\text{MAS}(\text{ITL}) \times \text{ROTIO} \]
\[ X\text{MAS}(\text{ITL}) = X\text{MAS}(\text{ITL}) - \text{XMOUT} \]
\[ \text{XMOVE} = \text{XMOVE} + \text{XMOUT} \]
\[ \text{UHEAT} = \text{UHEAT} + G(\text{ITL}) \times \text{XMOUT} \times \text{DR}^2 \]
\[ \text{STEAT} = \text{STEAT} + \text{XMOUT} \times G(\text{ITL}) \times \text{DR}^2 \]
\[ \text{RO}(\text{ITL}) = \text{ROICE}(9) \]
\[ F(\text{ITL}) = 0.0 \]
\[ X\text{KA}(\text{ITL}) = \text{BH2O} \]
\[ X\text{KB}(\text{ITL}) = \text{CH2O} \]
\[ \text{CPA}(\text{ITL}) = \text{DH2O} \]
\[ \text{CPB}(\text{ITL}) = \text{EH2O} \]
\[ \text{GO TO 4} \]

48 \[ \text{XMOUT} = F(\text{ITL}) \times X\text{MAS}(\text{ITL}) \times \text{ROTIO} \]
\[ \text{XMOVE} = \text{XMOVE} + \text{XMOUT} \]
\[ \text{RO}(\text{ITL}) = \text{ROICE}(9) \]
\[ F(\text{ITL}) = 0.0 \]
\[ \text{UHEAT} = \text{UHEAT} + G(\text{ITL}) \times \text{XMOUT} \times \text{DR}^2 \]
\[ X\text{MAS}(\text{ITL}) = X\text{MAS}(\text{ITL}) - \text{XMOUT} \]
\[ \text{GO TO 32} \]

484 \[ X\text{MAS}(\text{ITL}) = X\text{MAS}(\text{ITL}) \times \text{XMOVE} \]
\[ \text{ROW} = \text{RO}(\text{ITL}) \]
\[ \text{RO}(\text{ITL}) = X\text{MAS}(\text{ITL}) \times \text{RO}(\text{ITL}) / (X\text{MAS}(\text{ITL}) - \text{XMOVE}) \]
\[ \text{N} = \text{ITL} \]

C. THIS TAKES ALL THE MUD THAT HAS FALLEN OUT OF THE SLUSH AND PILES IT UP AT THE BOTTOM OF THE SLUSH REGION.
667 HLATN(ITL)=XC*(0.29*T9-102.88)  
T(ITL)=T9  
GO TO 59
668 HLATN(ITL)=XC*286.0  
T(ITL)=T14  
GO TO 59
669 HLATN(ITL)=XC*(-404.88-3.1434*T4)  
T(ITL)=T4  
GO TO 59
67 GO TO (64,64,64,64,64,66,64,67,64,66,67,64,67,69), IF
676 HLATN(ITL)=XC*(102.88-0.29*T9)  
T(ITL)=T9  
GO TO 59
678 HLATN(ITL)=XC*264.0  
T(ITL)=T15  
GO TO 59
679 HLATN(ITL)=XC*(-404.88-3.1434*T5)  
T(ITL)=T5  
GO TO 59
68 XC=-XC  
GO TO (64,64,64,64,64,66,64,67,64,64), IF
69 XC=-XC  
GO TO (619,692,639,64,659,669,679,64,64), IF
692 II=5  
IF (P(ITL).LT.3.5) II=3  
IFA/(ITL) = II  
GO TO (64,64,64,64,64,66,64,64,66,64,64), II
59 IF (HEATUP.NE.0.0) GO TO 343  
IF(DT*DT.GT.(T(ITL)-TT(ITL))*(T(ITL)-TT(ITL)))* GO TO 351  
IF (II) 351,35,351
35 T(ITL)=TT(ITL)
351 DH(ITL)=DH(ITL)+DT*CP  
352 PHCNT=DH(ITL)/(HLATN(ITL)+1.0)  
IF(PHCNT.GT.1.0) GO TO 37  
IF (PHCNT.LT.0.0) GO TO 38
C
C WE COME HERE IF WE NEED HEAT TO CHANGE PHASE.
C
36 XRO=ROICE(IFAZE(ITL))-ROICE(IFAZ(ITL)) /(PHCNT -ROICE(IFAZE(ITL)))+
4 IF(IFAZE(ITL-1)-9)41,42,41
41 ITOP=ITL
TCON=TT(ITL)
42 X=1.3 EA*(P(ITL)-P(ITOP))/(CP*RO(ITL))
TVECT=TCON*EXP(X)
UHOT=UHOT+DT*CP*X*AS(ITL)
T2=230.12+13.6717*P(ITL)-1.7835*P(ITL)*P(ITL)
T3=223.69+10.934*P(ITL)-0.489*P(ITL)*P(ITL)
T4=220.66+9.2065*P(ITL)-0.14124*P(ITL)*P(ITL)
T5=139.49+11.4015*P(ITL)-0.06931*P(ITL)*P(ITL)
T(ITL)=TVECT
IF(T2.GT.T(ITL)) T(ITL)=T2
IF(T3.GT.T(ITL)) T(ITL)=T3
IF(T4.GT.T(ITL)) T(ITL)=T4
IF(T5.GT.T(ITL)) T(ITL)=T5
T(ITL)=T(ITL)+0.001
IFAZ(ITL)=IFAZE(ITL)
PCT(ITL)=1.0
IF(F(ITL).NE.0.0.AND.IFAZE(ITL+1).EQ.9) GO TO 48
GO TO 32
43 XKA(ITL)=(BICE-PHCNT+BH20*(1.0-PHCNT)/ROICE(IFAZE(ITL)))*RO(ITL)
XKB(ITL)=(CICE-PHCNT+CH20*(1.0-PHCNT)/ROICE(IFAZE(ITL)))*RO(ITL)
CPA(ITL)=DICE-PHCNT+DH20*(1.0-PHCNT)
CPB(ITL)=EICE-PHCNT+EH20*(1.0-PHCNT)
PCT(ITL)=1.0-PHCNT
GO TO 32

C THIS FIGURES OUT HOW MUCH MUD WILL FALL OUT OF THE SLUSH, HOW MUCH HEAT
C THIS WILL GENERATE, AND WHAT THE NEW PROPERTIES OF THIS REGION, SANS MUD,
C WILL BE. VERY CLEVER PIECE OF PROGRAMMING, Isn'T IT?

46 F2=F(ITL)*(1.0-PHCNT)/(1.0-PCT(ITL))
IF(F2.GT.F(ITL))F2=F(ITL)
PCT(ITL)=PHCNT
XMOUT=(F(ITL)-F2)*XMAS(ITL)*ROTIO
DH(ITL)=DH(ITL)+XMOVE*G(ITL)*(R(ITL-1)-R(ITL))/XMAS(ITL)
STEAT=STEAT+XMOOE*G(ITL)*DR2
XMAS(ITL)=XMAS(ITL)-XMOUT
RO(ITL)=RO(ITL)*XMAS(ITL)/(XMAS(ITL)+XMOUT)
1(1.0-PHNT)*ROICE([FAZ([ITL]))
RO([ITL])=XRO*ROMUD/(F([ITL])*XRO+(1-F([ITL]))*ROMUD)
IF([FAZ([ITL))EQ.9. AND, [FAZ([ITL)),NF.9] GO TO 43
IF (9-[FAZ([ITL)])37,46,32
C
C WE COME HERE IF WE HAVE ENOUGH HEAT TO HAVE FINISHED CHANGING PHASE.

37 [FAZ([ITL])=[FAZ([ITL])
XRO=ROICE([FAZ([ITL]))
RO([ITL])=XRO*ROMUD/(F([ITL])*XRO+(1-F([ITL]))*ROMUD)
T([ITL])=T([ITL])+(DH([ITL])-HLATN([ITL]))/CP
DH([ITL])=0.0
IF(9-[FAZ([ITL)])32,47,32
C
C WE COME HERE IF WE ONCE STARTED TO CHANGE PHASE, BUT HAVE SINCE CHANGED OUR MINDS INSTEAD...FOR EXAMPLE, IF THIS POINT WERE ONCE HEATING UP AND CHANGING PHASE, BUT NOW THE CONDITIONS IN THE PLANET HAVE CHANGED SO THAT THIS POINT NOW WANTS TO COOL DOWN,

38 IF(HLATN([ITL)])382,381,381
381 [FAZ([ITL])=[FAZ([ITL])
PCT([ITL])=0.0
XRO=ROICE([FAZ([ITL]))
RO([ITL])=XRO*ROMUD/(F([ITL])*XRO+(1-F([ITL]))*ROMUD)
T([ITL])=T([ITL])+DH([ITL])/CP
DH([ITL])=0.0
GO TO 32
382 [FAZ([ITL])=[FAZ([ITL])
RO([ITL])=ROICE([FAZ([ITL]))*ROMUD/(F([ITL])=ROICE([FAZ([ITL]))+(1.0-F([ITL]))*ROMUD)
TEMPUP=T([ITL])
HEATUP=-HLATN([ITL])
T([ITL])=TEMPUP+HEATUP/CP
GO TO 5
383 DH([ITL])=HEATUP-(T([ITL])-TEMPUP)*CP+DH([ITL])
HEATUP=0.0
GO TO 352
C
C THIS IS THE PART THAT CONVECTS THE LIQUID WATER
THESE ARE JUST INITIAL CONDITIONS

II=27
I3=100
FNHG=0.1
FMUD=0.36
RTOTAL=700.66
ROMUD=2.5
TSUR=100.9
I4=I1-1
TIME=0.0
AGE=450.1
DTIM=1.0
DHTIM=DTIM*0.5
PPMK=815.0
PPMU=0.012
PPMT=0.04

PROGRAM
AMMONIA
ROICE(1)=6.92
ROICE(2)=1.17
ROICE(3)=1.14
ROICE(4)=RMUD
ROICE(5)=1.23
ROICE(6)=1.31
ROICE(7)=1.5
ROICE(8)=1.5
ROICE(9)=1.5
T(1)=TSUR
BICE=-1.3298E18
GICE=2.4338E21
DICE=7.0E4
FICF=0.169E7
EH2O=0.185E19
CH2U=0.0
DH2O=1.936E4
EH2U=3.6185E7
LMUD=0.462E19
CMUD=0.0
LMUD=0.0
LMUD=0.75E7
T2=13
THIRC=1.0/3.0
IFAZE(1)=1
P(1)=0.0
SSA^Y=1.0
VOL(1)=0.0
XMAS(1)=0.0
XMALC(1)=0.0
<MOVLE=0.0
FLUX=0.0
STEAT=0.0
UHUT=0.0
SSA^=0.0
XNH3=FNH3/(1.0-FMUD-FNH3)
XLOPE=(1.0-3.0*FNH3/(1.0-FMUD))/XNH3-THIRD
TFM=273.0-900.0*FNH3*FNH3
TMI=173.0
HLA=3.147E-9*2.0*FMH3
ROVIC=1.0-ROICE(9)/ROMUD
HEATR=0.0
II=1
R(I) =RTOTAL
DR2 =RTOTAL/(II-2)
DO 1 ITL=2,II
R(I) =R(I+1)-DR2
1 IFIZE(I) =II
PRINT 1600
1600 FORMAT(' TIME RADIUS TEMP PRESSURE DT,DT/DR PHASE XMH
1 RC RC MASS LITTLE G PERCENT MELTED OLD T')
C
HERE WE CALCULATE THE INITIAL CONDITIONS AT EACH POINT WITHIN THE PLANET
C
DO 15 ITL=1,II
T(I) =TSUR
F(I) =FMUD
DHI(I) =0.0
HLATN(I) =0.0
PCT(I) =0.0
RO(I) =XOICF(IFAZE(I))**ROMUD/(F(I)**ROICE(IFAZE(I))+(1-F(I))**ROMUD)
IFAZ(I) =IFAZE(I)
XKAI(I) =(FMUD+F(I))/ROMUD+XICF*(1-F(I))/ROICE(IFAZE(I))**ROIC
XKPI(I) =(CMUD+F(I))/ROMUD+XICF*(1-F(I))/ROICE(IFAZE(I))**ROIC
CPA(I) =DMUD+F(I)**XICF*(1-F(I))
CPRI(I) =EMUD+F(I)**XICF*(1-F(I))
15 CONTINUE
DO 16 ITL=2,II
VOL(I) =4.1388*(R(I-1)**2.0-R(I)**2.0-R(I-1)**R(I)**R(I))
XMAS(I) =R(I)**R(I) *VOL(I)
XMBCO(I) =XMBCN(I)*XMAS(I)
16 CONTINUE
G(I) =6.67E-8*(XMBCO(I)/(RTOTAL**2.0))
II=II+1
DO 17 ITL=2,15  
XMBL0(ITL)=XMBL0(ITL-1)-XMAS(ITL)  
G(ITL)=6.67E-11*(XMBL0(ITL)/(R(ITL)*R(ITL)))  
P(ITL)=P(ITL-1)+HC(ITL)*G(ITL)-3.241.0E-9  
CONTINUE  
XMBL0(14)=0.0  
G(14)=0.0  
P(14)=P(15)

C C HERE WE START THE TIME ITERATION  
C

2 TIME=TIME+DT  
I=AGE-TIME  
IF(I)99,21,21  
21 IF(I2)22,22,23  
22 I2=13  
PRINT 1003,FLUX,UIJFAT,STEAT  
1003 FORMAT(1,D)FLUX OF HEAT:*,E12.4,*, HEAT TO MAINTAIN WATER:*,E12.4,*,  
HEAT OF CORE FORMATION:*,E12.4)  
PRINT 1000

23 I2=I2-1  
R(I1-1)=0.0  
SAMY=SSAM  
SSAW=1.0  
UJFAT=UJUT  
UJUT=0.0  
UHFLAT=UHOT  
UHOT=0.0  
DO 221 I=1,15  
ITL=11-I  
VOL(ITL)=XMAS(ITL)/RC(ITL)  
220 R(ITL-1)=(VOL(ITL)/4.18888*(ITL)*R(ITL)*R(ITL))*3THIRD  
221 CONTINUE  
DO 222 I=2,15  
XMBL0(I)=XMBL0(I-1)-XMAS(I)  
G(I)=6.67E-11*XMBL0(I)/(R(I)*R(I))  
P(I)=P(I-1)+RO(I)*G(I)*(P(I-1)-R(I))*1.0E-9
\[
\begin{align*}
P(14) &= P(15) \\
T(11) &= T(10) \\
& \text{DO 24 ITL = 1, 1} \\
24 & \text{ TT(ITL) = T(ITL) } \\
& \text{IF(12) = 25, 25, 26} \\
25 & \text{ PRINT 100?, TIME, R(1), TSUR} \\
& \text{FORMAT(9*FANWHILE, BACK AT THE PAST---'// F7.0, -5PF9.0, 0PF9.2, 6X, } \\
& \text{1*SURFACE')} \\
26 & \text{ ITL = 2} \\
& \text{ UR21 = (R(1) - R(2)) * 0.5} \\
& \text{ DR11 = DR2H} \\
& \text{ XK = XKA(1) + XKB(1) / TSUR} \\
& \text{ XKN = XKA(2) + XKL(2) / TT(2)} \\
& \text{ O = DHT1M - TIME} \\
& \text{ AK = PPMK * 0.01143 * EXP(Q * 0.00521)} \\
& \text{ AU235 = PPMU1 * 15.376 * EXP(Q * 0.00927)} \\
& \text{ AU238 = PPMU2 * 6.304 * EXP(Q * 0.00054)} \\
& \text{ ATH = PPMTH * 10.527 * EXP(Q * 0.0005)} \\
& \text{ A = (AK + AU235 + AU238 + ATH) * 1.056} \\
\end{align*}
\]

C HERE WE GO THROUGH THE RADIUS ITERATIONS

3 XKP = XK \\
XK = XKN \\
XKN = XKA(ITL+1) + XKP(ITL+1) / TT(ITL+1) \\
CP = CPA(ITL) * TT(ITL) + CP3(ITL) \\
L = A * QD(ITL) * F(ITL) \\
DR1 = R(ITL) - R(ITL+1) \\
DR2 = R(ITL-1) - R(ITL) \\
DR3 = DPE2H \\
DR4H = DR1H \\
DR1H = DR1H * 0.5 \\
DT = ((XKP * R(ITL-1) - R(ITL-1) * (TT(ITL-1) - TT(ITL))) / (DR3H + DR2H) - XK * R(ITL) \\
L) * R(ITL) / (TT(ITL) - TT(ITL+1)) / (DR2H + DR1H) / (Q * (ITL) + DR2H) * P(ITL) + \\
2(DR2H * DR1H + C) * DTM / (CP * RG(ITL)) \\
IF (FLUX * GE * 0.9) GC TO 302 \\
IF (IFAZE(ITL) * EQ * 9. AND. IFAZE(ITL-1) * NE * 9) DT = DT + FLUX / (CP * XAS(ITL)) \\
IF (IFAZE(ITL) * EQ * 9. AND. IFAZE(ITL-1) * NE * 9. AND. DT * LT * 0.0) IFAZE (ITL)
1=7
GO TO 303
302 CONTINUE
1.IFAZE(ITL).NE.9.AND.IFAZE(ITL+1).EQ.4)) DT = DT + FLUX/(CP*XMAS(ITL))
303 CONTINUE
T(I TL) = T(I TL) + DT
IF(.NOT.(IFAZE(ITL).EQ.4.AND.IFAZE(ITL-1).EQ.9)) GO TO 31
T(I TL) = T(I TL - 1)
FLUX = -12.56636*XK*R(I TL)*R(I TL)*DT
301 CONTINUE
DTDR = (T(I TL) - T(I TL - 1))/(DR2*H+DR3*H)
II = 0
IF(IFAZE(ITL).EQ.4) GO TO 32
IF(IFAZE(ITL))
Y2 = -2.61E-2*(P(I TL) - 0.001)
TM = TM * EXP(Y2)
GO TO (31.5,9,32.9,9,6,9,4), IF
9 TIME = TIME + 10000000
12 = 0
GO TO 32
4 IF(IFAZE(ITL-1) .EQ. 9) 41, 42, 41
41 ITOP = I TL
TCON = TT(I TL)
42 X = 1.3 E5*(P(I TL) - P(I TOP))/(CP*R0(I TL))
TVFCT = TCON*EXP(X)
UHOT = UHOT + DT - CP*XMAS(I TL)
T(I TL) = TVFCT
IFAZE(I TL) = IFAZE(I TL)
PCT(I TL) = 1.0
IF(F(I TL).NE.0.0.AND.IFAZE(I TL + 1).EQ.9) GO TO 4E
GO TO 32
48 XMOUT = F(I TL)*XMAS(I TL)*RCTIO
XMOVE = XMOVE + XMOUT
RDI(I TL) = RDI*E(9)
F(I TL) = 0.0
UHFAT = UHEAT + G(I TL)*XMOUT*DR?
XMAS(I TL) = XMAS(I TL) - XMOUT
GO TO 32
50 IF AZE(ITL)=2
 5 DH(ITL)=DH(ITL)+CP*DT
   T(ITL)=TM
   IF(HLA-DH(ITL))60,60,51
51 IF(DH(ITL) .GE. 0.0) GO TO 72
   T(ITL)=T(ITL)+DH(ITL)/CP
   IF AZE(ITL)=1
   DH(ITL)=9.0
   GO TO 32
60 IF AZE(ITL)=7
   IF AZE(ITL)=9
   PCT(ITL)=THIRD
   DT=(DH(ITL)-HLA)/CP
   DH(ITL)=9.0
6 UJOT=UJOT+DT*CP*XMAS(ITL)
   SSAM=SSAM+XMAS(ITL)
   DH(ITL)=DH(ITL)-HEAT/SSAM
   DT=THIRD-0.157E-10*DH(ITL)-0.6275E-2*DH(ITL)-DH(ITL)
   T(ITL)=273.0-300.0*E*X
   PHCNT=1.0-YNH3*YLCPE+YLCPE*X
46 F2=F(ITL)*(1.0-PHCNT)/(1.0-PCT(ITL))
   IF(F2 .GT. F(ITL)) F2=F(ITL)
   PCT(ITL)=PHCMT
   XMOUT=(F(ITL)-F2)*XMAS(ITL)*ROTIC
   DH(ITL)=DH(ITL)+XMOVE*G(ITL)*(R(ITL-2)-R(ITL))/XMAS(ITL)
   STEAT=STEAT+XMOVE*G(ITL).DR2
   XMAS(ITL)=XMAS(ITL)-XMOUT
   XD(ITL)=XD(ITL)+XMAS(ITL)/(XMAS(ITL)+XMOUT)
   F(ITL)=F2*(XMAS(ITL)+XMOUT)/XMAS(ITL)
   XMOVE=XMOVE+XMOUT
   IF(DH(ITL).LT.5.0) GO TO 61
   IF(T(ITL).LT.TFM) GO TO 32
   IF AZE(ITL)=9
   KD(ITL)=KDICT(9)
47 PCT(ITL)=1.0
   XMOUT=F(ITL)*XMAS(ITL).ROTIC
   XMAS(ITL)=XMAS(ITL)-XMOUT
   XMOVE=XMOVE+XMOUT
   UHEAT=UHEAT+G(ITL)*XMOUT.DR2
   STEAT=STEAT+XMOUT*G(ITL).DR2
THK TAKES ALL THE MUD THAT HAS FALLEN OUT OF THE SLUSH AND PILES IT UP AT THE BOTTOM OF THE SLUSH REGION.

484 \[ X'AS(ITL) = X'AS(ITL) \times X'MOVE \]
\[ ROW = RO(ITL) \]
\[ RO(ITL) = X'AS(ITL) \times RO(ITL)/(X'AS(ITL) - X'MOVE) \]
\[ V = ITL \]
\[ IF(ROMTD - RO(ITL)) 491, 481, 483 \]

481 \[ X'M = RMTD \times X'AS(ITL)/R(C(ITL)) \]
\[ X'MAS(ITL) = X'M \]
\[ X'AS(ITL) = X'M \]
\[ X'MAS(ITL-1) = X'MAS(ITL-1) + X'M \]
\[ IF(AE(ITL) = 4 \]
\[ IF(AE(ITL) = 4 \]
\[ F(IFL) = 1, 0 \]
\[ DH(ITL) = 0, 0 \]
\[ RO(ITL) = RMTD \]
\[ F(IFL-1) = (F(IFL-1) \times X'MAS(ITL-1) - X'M) \times ROTIO + X'M)/(X'MAS(ITL-1) - ROTIO) \]
\[ ROW = R0(IFL-1) \]
\[ RO(IFL-1) = X'MAS(ITL-1) \times RO(IFL-1)/(X'MAS(ITL-1) - X'M) \]
\[ X'AS(ITL) = RMTD \]
\[ X'AS(ITL) = RMTD \]
\[ CPA(ITL) = RMTD \]
\[ CPA(ITL) = EMUD \]
\[ CPA(ITL) = EMUD \]
\[ CPA(ITL) = EMUD \]

\[ CPA(ITL) = EMUD \]
$t_2 = \frac{XY}{XMAS(\text{ITL}-1) \times \text{ROTIC}}$

$XKA(\text{ITL}-1) = (RMUD \times F_2 / RMUC + XKA(\text{ITL}) \times (1.0 - F_2) / RO) \times \text{ROTIC}$

$XKP(\text{ITL}-1) = (CMUD \times F_2 / CMUC + XK(\text{ITL}) \times (1.0 - F_2) / RO) \times \text{ROTIC}$

$CPA(\text{ITL}-1) = DMUD \times F_2 + CPA(\text{ITL}-1) \times (1.0 - F_2)$

$CPB(\text{ITL}-1) = EMUD \times F_2 + CP\times(\text{ITL}-1) \times (1.0 - F_2)$

$\text{ITL} = \text{ITL}-1$

$\text{IF}(R \text{CMUD} - RO(\text{ITL})) \leq 481, 482, 483$

$F(\text{ITL}) = (F(\text{ITL}) \times (XMAS(\text{ITL}) - MOVE) \times \text{ROTIC} + MOVE) / (XMAS(\text{ITL}) \times \text{ROTIC})$

$F_2 = \frac{\text{MOVE}}{(XMAS(\text{ITL}) \times \text{ROTIC})}$

$XKA(\text{ITL}) = (RMUD \times F_2 / RMUC + XKA(\text{ITL}) \times (1.0 - F_2) / RO) \times \text{ROTIC}$

$XKB(\text{ITL}) = (CMUD \times F_2 / CMUC + XK(\text{ITL}) \times (1.0 - F_2) / RO) \times \text{ROTIC}$

$CPA(\text{ITL}) = DMUD \times F_2 + CPA(\text{ITL}) \times (1.0 - F_2)$

$CPB(\text{ITL}) = EMUD \times F_2 + CP\times(\text{ITL}) \times (1.0 - F_2)$

$XMOVE = 0.0$

$\text{ITL} = N$

$\text{GO TO 341}$

$31 \text{ IF}(T(\text{ITL}) \geq 481) \text{ GO TO 356}$

$32 \text{ IF}((\text{IFAZ}(\text{ITL}+1) > 9.9) \text{ AND } (\text{IFAZ}(\text{ITL}), F_6, 9)) \text{ GO TO 424}$

$341 \text{ CONTINUE}$

$342 \text{ IF}(12) = 33, 33, 331$

$33 \text{ CONTINUE}$

$\text{PRINT 1001, TIME, R(\text{ITL}), T(\text{ITL}), P(\text{ITL}), CT, ITDR, IFAZE(\text{ITL}), IFAZ(\text{ITL}), IF(\text{ITL}), RO(\text{ITL}), XMAS(\text{ITL}), G(\text{ITL}), PCT(\text{ITL}), T(\text{ITL})}$

$1001 \text{ FORMTAF(F7.0, -5PF7.2, OPE8.2, F7.2, 2E10), 3, 12, 13, 2PF6.9, CF7.2, 4312.4}$

$331 \text{ ITL} = \text{ITL} + 1$

$332 \text{ IF}(I1 - \text{ITL}) \leq 2, 2, 3$

$99 \text{ CONTINUE}$

$\text{CALL EXIT}$

$\text{END}$
Appendix B

This appendix is meant to serve as a broom closet in which the skeletons of various half-thought-out ideas and unproved intuitions concerning icy satellites are to be stored. The first section deals with speculations about the nature of the interior of Titan, including the effect of considering heat released in compression of the ice which formed Titan on its thermal history; a model of Titan assuming an ammonia hydrate mixture like that used for the smaller satellites; and the possibility of Titan as a satellite formed by Chemical Model 2 or 4, namely made of pure ice or pure methane clathrate. The second part includes some brief comments on the silicate phase of icy satellites, considering its possible evolution under increased temperatures, and how it may chemically react with the liquid water.

The points brought up here should serve as starting points for further studies into the evolution of icy bodies.

B.1 Effect of compression on temperatures. In modelling the evolution of icy satellites, we assumed they started with an initial temperature uniform through all ice phases. However, a closer look at how these bodies were formed shows that this is only a simplifying assumption. The bodies must have grown from small particles, formed at very low solar nebula pressures, and so would obviously start out as ice I. Only as the satellite grew in size would pressures in the
interior force phase changes. An these phase changes would result in a change in enthalpy in the ice, which could be reflected in the temperatures of the regions involved.

In particular, as ice I was compressed to form ice II, $41.8 \times 10^7$ ergs/gram would be released. If all this heat were used to raise the temperature of the ice II region, it would increase from the starting temperature, say $100^\circ$K, to $141^\circ$K. The transition from ice II to ice VI takes $26.67 \times 10^7$ ergs/g, so this temperature would be lowered to $116^\circ$K in the ice VI region. The ice VI to ice VIII change releases $66.5 \times 10^7$ ergs/g, giving a temperature of $171^\circ$K. Note that here we are assuming pure ice is being compressed. For an ice and silicate mixture such as the homogeneous accretion model predicts would have only a fraction of its total mass as ice, and so its temperature change would be proportionately smaller (plus a small correction for the different heat capacity of the ice and silicate mixture as compared to the pure ice). So these temperatures are the maximum we would expect, and apply only to the inhomogeneous case where pure ice actually is accreted.

We ran a version of ONIONKIN with these temperatures assumed, for the case of a Titan-sized object. The effect of compression should be most pronounced in this model, since it is the largest considered and thus has the largest amounts of high pressure ice. We found that the results did not differ
significantly from the case where compression was ignored. The first melting did not occur until after 700 my, and the water region was never more than 50 km greater than that predicted by the uniform temperature model for any given time. Since in this, the most extreme case of compressional heating, the effects were not pronounced, we assumed we could ignore this added complication in our models.

It is worth pointing out that, with or without this assumption about compressional heating, the initial starting temperature profile we use is bound to be very arbitrary. The conditions in the nebula around any of the major planets while the satellites were being formed are likely to have been quite complicated, and changing with time. The amount of heat gained by these satellites due to energy of accretion is another question we have deliberately ignored. The purpose of these thermal models is to demonstrate what the effect of internal heat sources alone will have on the evolution of icy satellites; any further evolution which may be discovered as these satellites are explored can then be attributed to other causes. Therefore, to treat this additional complication of compressional heating, while ignoring other, possibly more important early heat sources, could be misleading.
B.2 Titan as an ammonia-rich planet. It is obvious from study of our models for a water ice - silicate composition Titan-sized object that a thin, easily destroyed crust is not formed. However, the addition of other ices, such as ammonia hydrate or methane clathrate hydrate, will serve to lower the melting point of the ice and almost certainly produce a thinner crust. Since a thin crust is needed if we expect the observed methane atmosphere to have been outgassed (as would be necessary for a Model 1 chemistry), one can easily come to the obvious insight that, say, the presence of methane is necessary for methane to be outgassed.

A more constructive approach to forming a thin crust is to analyze quantitatively how thin a crust a substance like methane or ammonia in the ice can cause. Since the ammonia hydrate - water ice system is better understood and the ammonia modelling system has already been developed to deal with the smaller ammonia-rich satellites, we decided to model Titan assuming it was nothing more than a 3000 km sized ammonia-rich object, completely ignoring the effects of high pressure on the ice.

The result is that melting occurs very early in the history of the planet (roughly 150 my) and substantial differentiation takes place by 500 my, with the crust reaching a thickness of 100 km soon thereafter. From then to the present, not much more happens. The core slowly grows
as more of the ice melts into the eutectic region, but no area ever becomes completely melted as the think crust cannot insulate the slushy region from the surface.

B.3 A Model 2 Hypothesis for Titan. Chemical Model 2 postulates that the products of equilibrium condensation may accrete themselves discretely into single component planets. Since the densities of those satellites whose densities we can measure are generally higher than predicted for single-component icy bodies, this model does not seem to hold much promise. However, we did consider how the high-pressure phases of ice might raise the density of an exceptionally large icy satellite, Titan, for which the latest observations give a quite low density.

By applying program ICEPULSE (see Appendix A) to a pure one-component H₂O body of mass 1400 x 10⁻²⁴ g, we discovered that the total radius would be just under 3000 km (2989), with the ice I-II transition occurring at 2915 km, the ice II-VI transition at 1885 km, and the ice VI-VIII transition at 1593 km. The central pressure of such a body would be 21 kilobars, and its density just over 1.2 g/cm³, slightly less than the observed density.

Naturally such a body would not evolve with time, since there are no heat sources within it. However, it is likely that if the single component were methane clathrate hydrate, it would still of necessity develop a methane atmosphere.
Recall that methane forms a clathrate with water by having methane molecules occupy spaces in the cage-like ice I lattice. However, the high pressure forms of ice are possible precisely because the ice reforms its crystal structure to occupy space as efficiently as possible, destroying such spaces. Therefore, the clathrate might accrete into a Titan-like planet, only to be destroyed as more matter accretes on top of it. The increasing pressure would tend to "squeeze out" the methane, which would then have no choice but to migrate to the surface.

Thus it is possible for a Titan-like object with no heat sources at all to still form an atmosphere. This sort of procedure would only be hastened by the presence of silicates, which would tend to build up central pressures and temperatures all the faster; and thus the atmosphere of Titan is more likely due to pressure effects than thermal effects, regardless of its original composition.

B.4 Silicate dehydration. Perhaps the most ignored component of the icy satellites in the preceding analysis has been the silicate phase. This is simply because the exact nature of the silicates present is not known. To say that they are "chondritic" is hardly to pin them down; while it seems reasonable to assume that they would be made of the same material as meteorites, one cannot prove what sort of meteorites would make a most reasonable guess.

The silicates predicted by the equilibrium model are a
mixture of feldspars and hydrous minerals, most notably serpentine. Serpentine is found in large quantities in carbonaceous chondrites, as are various hydrated sulfate salts which are not predicted by any model. These salts (gypsum, epsomite, etc.) will use up a considerable amount of the water predicted by cosmic abundances and have fairly low densities (1.75 for epsomite). One can even demonstrate that a body made of epsomite and the remaining cosmic abundance of water and other silicates will have a density of about 2.0, the same as that observed for Ganymede. However, epsomite is not stable at temperatures much above 350°C, and we have ignored its presence in the icy satellites. (It may conceivably be important for the very small ones, however.)

The most important of the hydrous silicates must certainly be serpentine, since its formation is not limited by the abundances of any element less common than silicon (as opposed to, say, tremolite which has necessary calcium).

The dehydration of hydrous minerals has been studied at low pressures, and thermodynamic principles can be used to extrapolate the results to high pressures. The temperature for dehydration of serpentine at 1 kilobar is roughly 800°C; by 10 kilobars, it will be in the region of 1000°C. Therefore, in the larger satellites where core temperatures may exceed 1000°C, dehydration may be important. On the other hand, the high pressures of such regions will serve to delay
Figure B caption: Pressure-temperature space for ice and silicates. The ice phase diagram is after Fletcher (1970). The serpentine dehydration line and the peridotite solidus (with excess water) are extrapolated from Wyllie (1971, p. 121).
Peridotite solidus (with excess water)

5 serpentine $\rightarrow$ 6 forsterite + talc + 9 H₂O

Pressure, kilobars

Temperature, $^\circ$K
dehydration. The curve shown in Figure B thus must be considered plausible, but very speculative.

The presence of water will also serve to lower the melting points of many silicates. This may turn out to be the more important change in the silicates in the larger satellites' interiors.

B.5. Even where water is not being dehydrated, it is still intimately associated with the silicates - especially in those cases where it must "percolate" up through an overlaying mantle of silicates - and there will certainly result some non-trivial low temperature solution chemistry. Even on the Earth, the exact nature and extent of chemical reaction with the Earth's oceans and land masses is not well understood; icy satellites with both an excess of water and larger amounts of soluble minerals (assuming the silicates resemble carbonaceous chondritic material) should provide interesting challenges and opportunities for chemical oceanographers. Most recently, Fanale et al. have studied the dissolution of salts in carbonaceous chondrites, in connection with the surface of Io; such work should be extended to consider its effects on the interiors of the other Galilean satellites. Additional complications, and opportunities, arise with the addition of ammonia and methane ices to the mix. Substitution of ammonia in feldspars for potassium is likely, and may result in the transport of $^{40}$K
radiogenic heat sources within the convecting mantle. Given the temperatures of the interiors, and especially of the silicate layers through which liquid will be percolating, the possibility exists of simple organic chemistry taking place, involving either methane from the ice or carbon in the silicate phase. However, we stop short of postulating life forms in these mantles; we leave such to others more experienced than ourselves in such speculations.
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References


