ON THE ZONAL ATMOSPHERIC CIRCULATION OVER
THE PACIFIC OCEAN NEAR 10°S

by

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ABSTRACT

The dynamics of the atmospheric circulation over the
Pacific Ocean near 10°S is explored.

It is assumed that the latent heat release and the radiative heating are the dominant terms in the diabatic heating. The latent heat release is estimated from climatological precipitation fields and results obtained in numerical simulations. The radiative heating is calculated from parameterized heat fluxes at the ground and at the top of the atmosphere together with results obtained in numerical simulation. It is found that in the area under study the dominant contribution to the diabatic heating is the latent heat release.

By using scaling techniques it is found that the vertical advection of heat and the diabatic heating are the leading terms in the thermodynamic equation. This approximate balance permits us to estimate the vertical velocity field, with downward motion in the eastern part of the region and upward motion in the western part.

When the vorticity equation is scaled, a serious inconsistency is found. This inconsistency may be due to the fact that the only available actual fields are estimated from data from land stations in spite of the oceanic nature of the area of interest. Despite the presence of the inconsistency, it is still assumed that the "actual" fields are qualitatively correct. A very crude approximation to the vorticity equation yields air temperature and wind velocity fields with features similar to those in the "actual" fields.

Bjerknes (1969) described the so called Walker Circulation over the area as a zonal circulation which he postulated to be driven by the zonal gradient of sea surface tempe-
ratures. However, it is found that the condensation which drives the vertical circulation can not be due to local excess evaporation accompanying higher sea-surface temperatures. Less evaporation is found in the region of higher precipitation due to a reduction in the net surface heating of the atmosphere. This reduction is caused by the increased cloudiness accompanying the precipitation which substantially reduces the solar heating at the surface. This short wave effect dominates the tendency towards increased heating due to the local enhancement of the sea surface temperature.

Thesis Supervisor: Peter H. Stone
Title: Professor of Meteorology
TO THE MEMORY OF MY PARENTS
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I. INTRODUCTION.

The interaction between the oceans and the atmosphere is a very complex one. Exchanges of mass, momentum and energy across the interface occur on all space and time scales. The physical consequences of these interactions for both the air and the ocean can be calculated only through the use of coupled air-sea models in which each fluid is free to respond to the influence of the other (Spar and Atlas 1975). Nevertheless, it is possible to study the atmospheric response to the physical state of the sea surface using non-interactive models in which the sea surface parameters are specified rather than predicted.

A map of sea-surface temperatures for the equatorial South Pacific shows that the water from about 160°W to the South American coast is colder than the global average for these latitudes and that the Pacific waters west of 160°W are warmer than the global average. Bjerknes (1969) has described in detail, the "Walker Circulation", a circulation which he postulated, was driven by the gradient of sea temperature along and south of the equator in the Pacific Ocean. He postulated that when the cold water belt along and south of the equator is well developed, the air above it will be too cold and heavy to join the ascending motion in the Hadley circulations. Instead, the equatorial air flows westward between the Hadley circulations of the two hemispheres to the warm
west Pacific where it can take part in large-scale moist-adiabatic ascent.

The westward extent of the cold water of the equatorial South Pacific depends on upwelling of colder water from below, which depends on the distribution and strength of the easterlies along the Equator. Upwelling takes place when the frictional stress of the easterlies along the Equator on the water combined with the effect of the rotation of the earth causes water in the surface layer to move away from the Equator. It is replaced by water which "upwells" from below the surface layers. Since the water temperature decreases as depth increases in this region, the upwelling water is cooler than the original surface water and a band of low temperature water develops.

Several times in the past the normally cold equatorial eastern Pacific became anomalously warm. For instance, during most of 1972, especially after June, sea surface temperatures suggested that equatorial upwelling was unusually weak; positive anomalies dominated the equatorial region from South America to at least the international date line, extending north beyond 10°N and with values commonly greater than 3.3°C east of 120°W (Wooster and Guillen 1974).

Bjerknes (1969) showed that the great positive water temperature anomaly observed along the Equator in the central and eastern Pacific during the 1964 and 1966 calendar months
of January, was accompanied by an anomalous strength of the midlatitude westerlies over the Northeast Pacific. Bjerknes postulated that the anomalous heat supply from the equatorial ocean to the ascending branch of the atmospheric Hadley circulation would intensify that circulation and make it maintain more than the normal flux of angular momentum to the midlatitude belt of westerly winds. Bjerknes also suggested that the warmer equatorial water increases evaporation and condensation and consequently rainfall. Rowntree (1972) integrated the Geophysical Fluid Dynamics Laboratory's nine level hemispheric model over a period of 30 days with either cool or warm ($\sim +3.5^\circ C$) sea surface temperatures over the tropical eastern Pacific. His results supported Bjerknes' hypothesis. Ramage and Murakami (1973) cast serious doubts on the validity of Rowntree's results since his model incorporated a wall at the equator which could significantly distort winds and temperature in higher latitudes after eight days (Miyakoda and Umscheid 1973).

Recently, White and Walker (1975) experienced trouble correlating the intensity of the Aleutian Low ($\sim 50^\circ N, 180^\circ W$) with fluctuations in Canton Island ($3^\circ S, 170^\circ W$) rainfall. Since the rainfall is highly correlated to SSTA (sea surface temperature anomalies), the atmospheric response to these anomalies is probably a local one. Recently, Spar and Atlas (1975) carried out a two week prediction experiment with the
GISS (NASA - Goddard Institute for Space Studies) atmospheric model to evaluate the influence of sea-surface temperature (SST) variations on the behavior of the atmosphere. It was found that the local effect of the sea-surface temperature on the computed precipitation over the oceanic anomaly itself was only a local enhancement of rainfall over warm water and a suppression of rainfall over cold water. No systematic remote effect of the anomaly on the predicted rainfall was found.

Another similar experiment but with a mid-latitude sea-surface temperature anomaly was performed by Huang (1975). He used the NCAR GCM with 6 layers. The experiment had a western cold anomaly and an eastern warm anomaly in the North Pacific with extrema of $\pm 4^\circ$C. He found:

1. that the temperature at a height of 1.5 Km. from the earth's surface was about $2^\circ$C higher in the anomaly case than in the control, in the eastern North Pacific above the warm SSTA (sea-surface temperature anomaly), but did not show too much of a signal above the cold anomaly in the western Pacific.

2. that the meridional temperature gradients above and further north of the warm SSTA were strongly enhanced.

3. that there was no large temperature change at 4.5 Km.

4. the east-west component of surface wind increased more than 3 m/s over and slightly north of the warm SSTA and de-
creased in the wester North Pacific.

5. the southerly winds were much strengthened in the eastern North Pacific and in the western Pacific closer to continents.

6. the southerly winds were much reduced in the central North Pacific where the strong anomaly temperature contrast existed.

The purpose of this Thesis is to explore the dynamics of the zonal atmospheric circulation in the vicinity of the Walker Circulation during the winter of the southern hemisphere (June - July - August), to see if Bjerknes' picture of this circulation is a consistent one.
II. MATHEMATICAL FORMULATION

2.1. Equations of Motion.

We start with the time dependent Navier-Stokes equations:

\[
\begin{align*}
  u_t + u u_x + v u_y + w u_z - f v &= - \frac{1}{\rho_m} p_x + \nu u_{zz} \\
  v_t + u v_x + v v_y + w v_z + f u &= - \frac{1}{\rho_m} p_y + \nu v_{zz} \\
  p_z &= \rho_m g \frac{\Theta}{\Theta_m} \\
  \Theta_t + u \Theta_x + v \Theta_y + w \Theta_z &= \frac{\Theta}{C_p T} Q \\
  u_x + v_y + w_z &= 0
\end{align*}
\]

where \( Q \) is the diabatic heat source, \( \Theta \) the potential temperature, \( f \) is the Coriolis parameter, \( x, y, z \) the zonal, meridional, and vertical coordinates, respectively and \( u, v, \) and \( w \) the corresponding velocity components. \( \rho_m \) and \( \Theta_m \) are constants which are a representative density and potential temperature respectively. The Boussinesq approximation has been used (Spiegel and Veronis 1959). The suffixes represent de-
derivatives with respect to the indicated quantities.

First we average these equations around latitude circles from $140^\circ E$ to $110^\circ W$.

We define:

$$[\frac{\text{AV}}{2L} = \frac{1}{2L} \int_{-a}^{b} \text{d}x$$

where: $a = -55^\circ$ longitude ($140^\circ E$)

$b = 55^\circ$ longitude ($110^\circ W$)

$2L = a + b = 110^\circ$ longitude

The averaged equations are:

$$[u]_t + [u v]_y + [u w]_z - \{[v] = -\frac{1}{2\rho_m L} \left\{ \rho(x = b) - \rho(x = a) \right\} - \frac{1}{2L} \left\{ u^2(x = b) - u^2(x = a) \right\} + u [u]_{zz} \quad (2.6)$$

$$[v]_t + [v v]_y + [v w]_z + \{[u] = -\frac{1}{\rho_m} \left\{ [p]_y - \nu [v]_{zz} \right\} - \frac{1}{2L} \left\{ uv(x = b) - uv(x = a) \right\} \quad (2.7)$$

$$[p]_z = \frac{\rho_m g}{\Theta_m} [\Theta] \quad (2.8)$$
Now, we make the following substitutions in the Navier-Stokes equations:

\[
\begin{align*}
\chi &= [\chi] + \chi' \\
\theta &= [\theta] + \theta' \\
\rho &= [\rho] + \rho' \\
Q &= [Q] + Q'
\end{align*}
\]

and combine the resulting equations with the averaged equations (2.6) to (2.10) to obtain equations for the primed quantities,

\[
\begin{align*}
[u_t' + [u] u_x' + [v] [u]_y' + [v] u_y' + [u]_y v' + [w] [u]_z' + \\
+ [w] u_z' + [u]_z w'] &= \left( \frac{1}{\rho_m} \right) \frac{p'}{x} + v u_x' + [u v]_y' + [u w]_z' \\
+ \frac{1}{2L} \left( \frac{1}{\rho_m} \right) \frac{p + u^2}{x} \right)
\end{align*}
\]
\[ u_t' + [u'] u_x' + [u] u_y' + [u] u_z' + [w] v_z' + [v] w' = -\{u'\} \]

\[-\frac{1}{\beta_m} p_y' + \nu v_{zz}' + [v v]_y + [v w]_z - [v] [u]_y - [w] [v]_z + \]

\[ + \frac{1}{2 L} \left\{ u v \right\}_{x=b}^{x=a} \quad (2.12) \]

\[ p_z' = \beta_m g \frac{\theta'}{\theta_m} \quad (2.13) \]

\[ \theta_t' + [u] \theta_x' + [v] \theta_y' + [\theta]_y v_{y}' + [w] \theta_z' + [\theta]_z w' = \frac{1}{c_p} \left\{ [u \theta]_y \right\}_{x=a} \]

\[ + \left\{ w \theta \right\}_z - \left\{ [v] \left[ \theta \right]_y - [w] \left[ \theta \right]_z \right\} + \frac{1}{2 L} \left\{ u \theta \right\}_{x=b}^{x=a} \quad (2.14) \]

\[ u_x' + v_y' + w_z' - \frac{1}{2 L} \left\{ u \right\}_{x=a}^{x=b} = 0 \quad (2.14a) \]

where we have used the following approximation:

\[ \left[ \frac{\theta}{T} q \right] = [q] \quad (2.14b) \]

which is consistent with the Boussinesq approximation.

These equations can be written in other ways using the following equalities:

\[ [u v] = [u][v] - [u'v'] \]
\[ [u \ w] = [u][w] + [u'w'] \]

\[ [v \ w] = [v][w] + [v'w'] \]

\[ [v \ \theta] = [v][\theta] + [v'\theta'] \]

In this way we get:

\[ u_t^i + [u] u^i_x + [v] u^i_y + [u] v^i + [w] u^i_z + [u] z w^i - \int v^i = \]

\[ = - \frac{1}{\rho_m} \frac{p^i}{p_x} + \nu u^i_{zz} + [u'v']_y + [u'w']_z + \frac{1}{2L} \left\{ \frac{\rho}{p_m} + \nu^2 \right\}^{x=b}_{x=-a} \]

(2.15)

\[ v_t^i + [u] v^i_x + [v] v^i_y + [u] v^i_v + [w] v^i_z + [v] z w^i + \int u^i = \]

\[ = - \frac{1}{\rho_m} \frac{p^i}{p_y} + \nu v^i_{zz} + [v'v']_y + [v'w']_z + \frac{1}{2L} \left\{ \nu u^i \right\}^{x=b}_{x=-a} \]

(2.16)

\[ p^i_z = \rho_m g \frac{\theta^i}{\theta_m} \]  

(2.17)
The longitudinal averages \([ \overline{\mathbf{u}}' ]\) we will assume to be known from the climatological data and we will consider the perturbation quantities \( ( \mathbf{u}' ) \) which represent deviations from the average, as the primary unknowns.

The diabatic heating $Q'$ in the right hand side of equation (2.18) is the driving force of the circulation we will calculate. This term is made up of the following contributions:

- $Q'_L$: release of latent heat of evaporation
- $Q'_h$: turbulent sensible heat flux divergence
- $Q'_R$: radiative flux divergence
- $Q'_M$: molecular diffusion and dissipation

Newell et al. (1973) (Chapter 9) calculated that the contributions of the latent heat of evaporation and the radiative flux divergence are dominant in the region of the Walker Circulation. This result is also supported by Webster (1972) who concluded that the latent heating was responsible for most of the response near the Equator. Consequently we will neglect $Q'_h$ and $Q'_M$. In this section we will estimate the latent heat release, and in the next, the radiative flux divergence.

The latent heat perturbation $Q'_L$ is related to the perturbation of the precipitation reaching the sea surface as follows.

The total latent heat release perturbation $Q'_{TL}$ in a column of unit area and with the total height of the atmosphe-
where \( L_v \) is the latent heat of evaporation \( \rho_w \), the density of liquid water and \( P' \), the perturbation of the precipitation reaching the ground or the sea surface in cm/day.

Now, let us assume that the latent heat release perturbation per unit air volume can be written as:

\[
Q'_{Lv} = A(x, y) g(z) \tag{2.21}
\]

where \( A(x, y) \) and \( g(z) \) are unknown functions.

\( Q'_{Lv} \) and \( Q'_{TL} \) are related as follows:

\[
Q'_{TL} = \int_{Z=0}^{\infty} Q'_{Lv} \, dz \tag{2.22}
\]

Substitution of equations (2.20) and (2.21) in (2.22) yields:

\[
L_v P' \rho_w = A(x, y) \int_{Z=0}^{\infty} g(z) \, dz \tag{2.23}
\]

Although estimation of the function \( g(z) \) is difficult we can use some calculations done by Newell et al. (1973). The vertical distributions of the latent heat release were based on Northern Hemisphere cloud statistics at middle and high latitudes and on model profiles of vertical motion in the vicinity of the Equator where the cloud statistics did not appear to be as reliable. The zonal average latent heat
release is shown in their Figure 7.16. In our Figure 2.1 the corresponding distribution at 10°S is shown and for comparison in the same Figure the graph of the following approximation is also shown:

\[ Q_L = 1.1 \left[ 1 + \cos \left( \frac{\pi (z - 5.5)}{5.5} \right) \right] \text{°C/day} \quad z \leq 11 \text{ Km.} \]

\[ Q_L = 0 \quad z > 11 \text{ Km.} \quad (2.24) \]

where \( z \) is the height in kilometers. Taking into account the fact that Newell's values were based on model profiles of vertical motion we can say that the function (2.24) is a good approximation. Equation (2.24) suggests that we take the function \( g(z) \) in (2.21) as:

\[ g(z) = 1 + \cos \left( \frac{\pi (z - z_u)}{z_u} \right) \quad z \leq 2z_u \]

\[ g(z) = 0 \quad z > 2z_u \quad (2.25) \]

where \( z_u = 5.5 \text{ Km.} \).

Combining (2.23) and (2.25) we obtain:

\[ A(x, y) = \frac{I_y \rho_w}{2z_u} p' \quad (2.26) \]
FIGURE 2.1

Latent Heat Release at 10°S

x Newell et al. (1973)

Approximation (see text)
Substituting (2.26) in (2.21) we obtain:

\[ Q'_{L_v} = \frac{L_v \rho_w}{2 z_u} P' (x, y) g(z) \]  

(2.27)

Then, the latent heat release perturbation per unit mass is:

\[ Q'_{L} = \frac{L_v \rho_w}{2 z_u \rho_m} P' (x, y) g(z) \]  

(2.28)

Precipitation values will be taken from Schutz and Gates (1972). That field was obtained from visual interpolation of Moller's (1951) seasonal analysis. A very similar field for the tropical Pacific Ocean can be found in Taylor (1970). He constructed a revised chart of rainfall for the Pacific using conventional data available from about 100 land stations and total cloud cover between 30°S and 30°N summarized for 2.5° squares. Although that last kind of data is obviously not a substitute for rainfall observations it was used to make a general assessment of likely gradients in rainfall and probable regions of minimum and maximum values. So the precipitation field we will use is based upon direct measurement at a few widely spaced atoll and islands throughout the tropical Pacific. We will use these values because we have not been able to find a field based on better considerations.

The precipitation perturbation field \( P' = P - [P] \)
is shown in Figure 2.2. In the same Figure the graph of the following approximation is shown for comparison:

\[ P' (x, y) = -A_p (y) \sin \left( \frac{\pi x}{L} + \gamma (y) \right) \]  \quad (2.29)

where:
- \( L = 55^\circ \) longitude
- \( A_p (y) = a + b y \)
- \( a = 2.3 \text{ mm/day} \)
- \( b = 0.14 \text{ mm/} (\text{day}^1 \text{latitude}) \)
- \( \gamma (y) = c + d y \)
- \( c = -0.52 \text{ rad.} \) \quad (2.29 a)
- \( d = 0.1 \text{ rad/}^1 \text{latitude} \)

\(-55^\circ \text{ long.} \ (140^\circ \text{E}) \leq x \leq 55^\circ \text{long.} \ (110^\circ \text{W})\)

\(-5^\circ \text{ lat.} \ (15^\circ \text{S}) \leq y \leq 5^\circ \text{lat.} \ (5^\circ \text{S})\)

In this way the latent heat release perturbation \( Q_L' \) as given in equation (2.28) is completely defined. The precipitation \( P' \) and the \( z \) - distribution are given in equations (2.29) and (2.25) respectively.
FIGURE 2.2: Precipitation Perturbation Function ($P'$).
2.3. Diabatic Heating Perturbation due to Radiative Flux Divergence.

According to results calculated with the GISS general circulation model, the diabatic heating due to the radiative flux divergence (mainly long wave) is approximately constant from the sea surface up to the 500 mb level in July near 10°S (Stone et al. 1976). This means that for a Boussinesq fluid, the radiative flux is directly proportional to height.

Let us call \( F'_g \) the net upward long wave perturbation flux at the sea surface level, \( F'_T \) the upward infrared perturbation flux at the top of the atmosphere and \( F'_R \) the upward long wave perturbation flux at any height.

A linear interpolation between the 500 mb level and the sea surface yield:

\[
F'_R (z) = F'_T - \left( 1 - \frac{z}{z_5} \right) (F'_T - F'_g) \quad z \leq z_5
\]

(2.30)

where \( z_5 \) is the mean height corresponding to the 500 mb level. The net upward long wave flux at the sea surface is:

\[
F'_g = F'_b - F'_a
\]

(2.31)

where \( F'_b \) is the heat lost by the sea as long wave radiation to the atmosphere and space and \( F'_a \) is the flux of long wave radiation from the atmosphere into the sea. Following Held
and Suarez (1974) we approximate:

\[ F_b = A_s + B_s T_s \]  
\[ F_a = A \ \downarrow + B \ \downarrow T_a \]  

where \( T_s \) and \( T_a \) are the sea surface and 500 mb air temperatures respectively and \( A_s, B_s, A \ \downarrow \) and \( B \ \downarrow \) are constants.

From (2.32) we can write:

\[ F'_b = B_s T'_s \]  
\[ F'_a = B \ \downarrow T'_a \]  

where \( B_s \) and \( B \ \downarrow \) are constants given by Held and Suarez,

\[ B_s = 9.5 \text{ ly} / (\text{day } ^\circ \text{C}) \]
\[ B \ \downarrow = 11.2 \text{ ly} / (\text{day } ^\circ \text{C}) \]

Substitution of (2.33) in (2.31) yields:

\[ F'_g = B_s T'_s - B \downarrow T'_a \]  

We also follow Held and Suarez in approximating

\[ F'_T = B \downarrow T'_a \]  

where: \( B \downarrow = 7 \text{ ly} / (\text{day } ^\circ \text{C}) \)

The diabatic heating perturbation \( Q'_R \) due to the radiative flux is given by:
\[ Q_R^i = - \frac{t}{\rho_m} \frac{\partial F_R^i}{\partial z} \]  \hspace{3cm} (2.36)

Combining (2.30) and (2.36) we obtain:

\[ Q_R^i = \frac{1}{z_5 \rho_m} (F_{g}^i - F_T^i) \] \hspace{3cm} (2.37)

Substitution of (2.34) and (2.35) in (2.37) yields:

\[ Q_R^i = \frac{1}{\rho_m z_5} \left[ B_S T_S^i - T_a^i (B_T^i + B_L^i) \right] \quad z \leq z_5 \] \hspace{3cm} (2.38)

In Figure 2.3, the sea surface temperature perturbation field \( T_S^i \) as taken from Schutz and Gates (1972) is shown. For comparison, in the same figure the graph of the following approximation is also shown:

\[ T_S^i = A_T \cos \left( \pi \frac{x}{L} + k y \right) \] \hspace{3cm} (2.39)

where: \( A_T = 1.6 \) °C
\[ k = 0.15 \text{ rad} / \circ \text{latitude} \] \hspace{3cm} (2.39 a)
\[ L = 55 \circ \text{longitude} \]

The 500 mb temperature perturbation \( T_a^i \) in equation (2.38) will be calculated later as one of the parameters this model will predict.
FIGURE 2.3: Sea Surface Temperature Perturbation Field ($T'_S$)
2.4. Summary

In this Chapter we deduced the set of equations we will use in the present calculations.

The driving force is the diabatic heating $Q'$ in the thermodynamic equation (2.18). As we said before the main contributions to $Q'$ are the latent heat release ($Q'_{L}$) and the heating due to radiative processes ($Q'_{R}$). The latent heat release contribution was estimated in equation (2.28) and the corresponding heating due to radiation in equation (2.38). We can see that $Q'_{L}$ is proportional to the precipitation $P'$ and that $Q'_{R}$ depends on the sea surface temperature $T'_{s}$. Both $P'$ and $T'_{s}$, as given by equations (2.29) and (2.39), are periodic functions of $x$ with a wavelength equal to the length of the domain of $x$ (longitude). This fact indicates that the perturbation parameters will also be periodic. Hence all the boundary terms in equations (2.15) to (2.19) cancel out.

Summarizing, we specify from the climatological data the sea surface temperature and the precipitation perturbations. These perturbations give rise to perturbations in the atmosphere temperature and wind fields. The zonal average fields $[ ]$, also have to be known, since they condition the atmosphere's response to $P'$ and $T'_{s}$. 
III. ORDER OF MAGNITUDE ESTIMATES AND NON-DIMENSIONALIZATION.


The equations we are going to use in this paper have been derived in Chapter II. Let us write them again:

\[
\begin{align*}
\frac{d}{dt} + \left[ u \right] u'_x + \left[ v \right] u'_y + \left[ w \right] u'_z + \left[ u \right] w'_z &= \int u' - \frac{1}{\rho_m} \left( p'_x + \nu u'_{zz} + \left[ u' v' \right] \gamma + \left[ u' w' \right] \gamma \right)_{x = b} + \frac{1}{2L} \left\{ \frac{p}{\rho_m} + u u' \right\}_{x = -a} \\
\frac{d}{dt} + \left[ u \right] u'_x + \left[ v \right] u'_y + \left[ w \right] u'_z + \left[ v \right] w'_z &= \int u' - \frac{1}{\rho_m} \left( p'_y + \nu v'_{zz} + \left[ v' v' \right] \gamma + \left[ v' w' \right] \gamma \right)_{x = b} + \frac{1}{2L} \left\{ u v' \right\}_{x = -a} \\
p'_z &= \frac{\rho_m g}{\theta_m} \\
\frac{d}{dt} + \left[ u \right] \theta'_x + \left[ v \right] \theta'_y + \left[ w \right] \theta'_z + \left[ \theta \right] \theta'_z + \left[ \theta \right] w'_z &= \frac{1}{c_p} \int \theta' + \left[ u' \theta' \right] \gamma + \left[ w' \theta' \right] \gamma + \frac{1}{2L} \left\{ u \theta' \right\}_{x = b} + \frac{1}{2L} \left\{ u \right\}_{x = -a} \\
u'_{xx} + v'_{yy} + w'_{zz} &= \frac{1}{2L} \left\{ u \right\}_{x = -a} = 0
\end{align*}
\]
Although actual data is very scarce in the studied area, we use the fields published by Newell et al. (1971) and Oort and Rasmussen (1971) in order to estimate the magnitude of the different terms in the equations. All the fields except the vertical velocity were deduced from land stations data. The vertical velocity field was obtained from the continuity equation. Due to the lack of data we have to use longitudinal averages around the earth instead of the averages for the area from $140^\circ E$ to $110^\circ W$.

The following values, taken from the above sources, will be used in the estimation of the magnitude of the terms in equation (2.15).

\[
[u] \sim 3 \text{ m/s}
\]
\[
[u'_x] \sim 0.08 \times 10^{-5} \text{ s}^{-1}
\]
\[
[u'_y] \sim 0.16 \times 10^{-5} \text{ s}^{-1}
\]
\[
[u'_z] \sim 0.3 \text{ cm/s}
\]
\[
[w] \sim 2 \text{ m/s}
\]
\[
[u'_y] \sim 1.3 \times 10^{-5} \text{ s}^{-1}
\]
\[
f \sim 2.5 \times 10^{-5} \text{ s}^{-1}
\]
\[
[u'_w] \sim 0.1 \times 10^{-5} \text{ m/s}^2
\]
\[
[u'_v'] \sim 0.2 \times 10^{-5} \text{ m/s}^2
\]
\[
\nu \sim 0.01 \times 10^{-5} \text{ m/s}
\]
\[
\frac{1}{\rho m} p'_x \sim 8 \times 10^{-5} \text{ m/s}^2
\]

where we have taken $\nu$ as:
\[
\nu \sim 10^4 \text{ cm}^2/\text{s}
\]
We obtain the following estimates in $10^{-5}$ m/s² units for the $u$-momentum equation (2.15):

$$u_t' + [u] u_x' + [v] u_y' + [u] v' + [w] u_z' + [u] w' =$$

$$= \left\{ \begin{array}{l}
\frac{1}{\rho_m} \{ p' + u u' \} \quad \text{x=b} \\
\rho_m v_x' + \nu v_{zz}' + [u'u']_y + [u'w']_z + \frac{1}{2\rho_m} \left\{ \frac{p'}{\rho_m} + u w' \right\} \quad \text{x=-a}
\end{array} \right.$$  

For the estimation of the terms in the $v$-equation the following values will be used:

$$v_t' \sim 0.04 \times 10^{-5} \text{ m/s}^2$$
$$v_x' \sim 0.13 \times 10^{-5} \text{ s}^{-1}$$
$$v_y' \sim 0.2 \times 10^{-5} \text{ s}^{-1}$$
$$[v]_y' \sim 0.07 \times 10^{-5} \text{ s}^{-1}$$
$$v_z' \sim 3 \times 10^{-5} \text{ s}^{-1}$$
$$[v]_z' \sim 20 \times 10^{-5} \text{ s}^{-1}$$
$$u' \sim 3 \text{ m/s}$$

Again we have taken $\nu$ as:

$$\nu \sim 10^4 \text{ cm}^2/\text{s}$$
The results in $10^{-5} \text{ m} / \text{s}^2$ are:

$$v'_t + [u] v'_x + [v] v'_y + [v] v'_y + [w] v'_z + [v]_z w' = - f u'$$

(0.04) (0.4) (0.4) (0.15) (0.01) (0.3) (7.5)

$$- \frac{1}{\rho_m} p'_y + \nu v'_{zz} + [v'v']_y + [v'w']_z + \frac{1}{2L} \left\{ u v' \right\}_x = b$$

(3.2)

(5) (0.01) (0.1) (0.03) (0.06)

For the thermodynamic equation we will use the following values:

$$\theta'_t \sim 0.001 \times 10^{-5} \degree \text{K/s}$$

$$\theta'_z \sim 4 \times 10^{-4} \degree \text{K/m}$$

$$\theta'_x \sim 0.05 \times 10^{-5} \degree \text{K/m}$$

$$[\theta]_z \sim 3 \times 10^{-3} \degree \text{K/m}$$

$$\theta'_y \sim 0.13 \times 10^{-5} \degree \text{K/m}$$

$$[v'\theta']_y \sim 0.04 \times 10^{-5} \degree \text{K/s}$$

$$[\theta]_y \sim 0.2 \times 10^{-5} \degree \text{K/m}$$

$$[w'\theta']_z \sim 0.01 \times 10^{-5} \degree \text{K/s}$$

The results in $10^{-5} \degree \text{K/s}$ are:

$$\theta'_t + [u] \theta'_x + [v] \theta'_y + [\theta] y v' + [w] \theta'_z + [\theta]_z w' =$$

(0.001) (0.15) (0.26) (0.4) (0.1) (5)

$$= \frac{1}{C_p} Q' + [v' \theta']_y + [w' \theta']_z + \frac{1}{2L} \left\{ u \theta' \right\}_x = b$$

(3.3)

(0.04) (0.01) (0.02)
For the continuity equation, the estimates in $10^{-5} \text{s}^{-1}$ are:

$$
\frac{U_x^1 + V_y^1 + W_z^1 - \frac{1}{2L} \left\{ u \right\}_{x=-a}^{x=b}}{2L} = 0 \tag{3.4}
$$

We could not get an estimate of $w_z^1$ because we could not find enough values in the literature.

If we retain only leading terms and the diabatic heating term ($Q'$), the equations become:

$$
(f - [u]_y) v' - \frac{1}{\rho_m} p_x^1 = 0 \tag{3.5 a}
$$

$$
f u' + \frac{1}{\rho_m} p_y^1 = 0 \tag{3.5 b}
$$

$$
u_x^1 + v_y^1 + w_z^1 = 0 \tag{3.5 c}
$$

$$
[\Theta]_z w' = \frac{1}{c_p} Q' \tag{3.6 a}
$$

$$
p_z' = \rho_m g \frac{\Theta'}{\Theta_m} \tag{3.6 b}
$$

We have assumed in equations (3.5) and (3.6) that the drives for the perturbations are cyclic so that the boundary terms are zero.
3.2. Non-dimensionalization.

In this section the equations of the model will be written in terms on non-dimensional parameters using proper scales. Then the equations will be simplified taking into account the magnitudes of the coefficients of the non-dimensional terms and these results will be compared with those obtained in Section 3.1.

In Chapter II we discussed the different diabatic heating sources we will consider. The total diabatic heating $Q'$ is given by:

$$Q' = Q_L^I + Q_R^I$$

(3.7)

where:

$$Q_L^I = -\frac{L_v \rho_w A_p(y)}{2 \rho_m z_u} \left[ 1 + \cos \frac{z - z_u}{z_u} \right] \sin \left[ \pi \frac{x}{L} + \gamma(y) \right]$$

$$z \leq 2 z_u$$

(3.7 a)

and

$$Q_R^I = \frac{1}{\rho_m z_5} \left[ B_s T_s - T_a (B + B') \right]$$

$$z \leq z_5$$

(3.7 b)

The different constants and functions in equations (3.7 a) and (3.7 b) have already been defined in that Chapter.
Let us now write equation (2.18) again but taking into account that the boundary term is zero as we discussed in Section 2.4:

\[
\frac{\theta'_t + [u] \theta'_x + [v] \theta'_y + [\theta] \nu' + [w] \theta'_z + [\theta] w'}{C_p} = \frac{1}{C_p} \left( Q'_L + Q'_R \right) + [\nu' \theta']_y + [w' \theta']_z \tag{3.8}
\]

Let us write:

\[
\begin{align*}
  u &= U u^* & \quad x &= L_1 x^* & \quad u' &= U_1 u_1^* & \quad \theta &= \Theta \Theta^* \\
  v &= V v^* & \quad y &= A y^* & \quad v' &= V_1 v_1^* & \quad \theta &= \Theta \Theta_1^* \\
  w &= W w^* & \quad z &= H z^* & \quad w' &= W_1 w_1^* \\
  f &= f'_o f^* & \quad p' &= P p_1^*
\end{align*}
\]

then assuming no time dependence equation (3.8) becomes:

\[
\begin{align*}
  \frac{U \Theta}{L_1} \left[ u^* \right] \frac{\partial \Theta}{\partial x^*} + \frac{V \Theta}{A} \left[ v^* \right] \frac{\partial \Theta}{\partial y^*} + \frac{\Theta}{A} \left[ \Theta^* \right] \frac{\partial \nu^*}{\partial y^*} + \\
  + \frac{W \Theta}{H} \left[ w^* \right] \frac{\partial \Theta}{\partial z^*} + \frac{\Theta W_1}{H} \left[ \Theta^* \right] \frac{\partial w^*}{\partial z^*} = \frac{1}{C_p} \left( Q'_L + Q'_R \right) + \\
  + \frac{V_1 \Theta}{A} \left[ \nu'_1 \Theta^*_1 \right] \frac{\partial \nu^*_1}{\partial y^*} + \frac{W_1 \Theta}{H} \left[ w'_1 \Theta^*_1 \right] \frac{\partial w^*_1}{\partial z^*} \tag{3.9}
\end{align*}
\]
Taking into account what we said in Section 2.4 about the boundary terms, the hydrostatic and continuity equations (2.17), (2.19) and (2.10) become:

\[
\frac{P}{H} \frac{\partial p^*}{\partial z^*} = \frac{\rho_m g}{\Theta} \frac{\Theta_i}{\Theta_m} \tag{3.10}
\]

\[
\frac{U_1}{L_1} \frac{\partial u_1^*}{\partial x^*} + \frac{V_1}{A} \frac{\partial u_1^*}{\partial y^*} + \frac{W_1}{H} \frac{\partial w_1^*}{\partial z^*} = 0 \tag{3.11}
\]

\[
\frac{V}{A} [u^*]_{y^*} + \frac{W}{H} [w^*]_{z^*} = 0 \tag{3.12}
\]

In these equations we will assume:

\[
\frac{V}{A} \sim \frac{W}{H} \tag{3.13}
\]

\[
\frac{U_1}{L_1} \sim \frac{W_1}{H} \tag{3.14}
\]

\[
\frac{P}{H} \sim \frac{\rho_m g}{\Theta} \frac{\Theta_i}{\Theta_m} \tag{3.15}
\]

From Newell et al. (1971) we write the following order of magnitudes:

\[ U \sim 2 \text{ m/s} \]
$L_1 \sim 50^\circ$ longitude $= 5 \times 10^6$ m.

$A \sim 10^\circ$ latitude $= 10^6$ m.

$\Theta_1 \sim 1^\circ K$

$\Theta \sim 50^\circ K$

$H \sim 12$ Km. $= 12 \times 10^3$ m.  

$V_1 \sim 2$ m/s

$W \sim 1.4 \times 10^{-4}$ mb/s $= 0.32$ cm/s

$\theta_m \sim 320^\circ K$

$\rho_m \sim 10^{-3}$ grm/cm$^3$

Then from equation (3.13):

$V \sim \frac{W_A}{H} = 0.27$ m/s  

(3.17)

After dividing equation (3.9) by $\Theta W_1 / H$ we obtain:

$$\frac{U}{W_1} \frac{H}{L_1} \left[ u^* \right] \frac{\partial \theta^*}{\partial x'^*} + \frac{V}{\omega_{\Lambda}} \frac{H}{A} \left[ V^* \right] \frac{\partial e_i^*}{\partial y'^*} +$$

$$\frac{W}{W_1} \left[ w_i^* \right] \frac{\partial e_i^*}{\partial z'^*} + \left[ e_i^* \right]_{z'^*} \omega_{i}^* = \frac{H}{\Theta W_1 C_p} \left( Q_l + Q_c \right) +$$

$$\frac{V_1}{W_1} \frac{H}{A} \left[ v_i^* \theta_i^* \right]_{y'} + \Theta \left[ w_i^* \theta_i^* \right]_{z'^*}$$

(3.18)
In equation (3.18) we have not considered the term containing $[\Theta_y]$ because its magnitude is very small at these latitudes as it is possible to deduce from data in Talijard et al. (1969) or Schutz and Gates (1972) (see Figure 4.1).

Now let us scale the diabatic heating source in equation (3.9). Taking into account that $z_u \sim z_5$ we can write equation (3.7) in the following form:

$$Q' = -\frac{L_v \rho_w A_p(y)}{2 \rho_m z_u} [1 + \cos \left(\frac{z - z_u}{z_u}\right) \sin \left(\frac{\eta}{L} + \gamma(y)\right) +$$

$$+ \frac{1}{\rho_m z_5} [B_s T_s'(B\uparrow + B\downarrow)]] \quad z \leq z_5 \quad (3.7)$$

We will see in Chapter IV that the leading term in $Q'$ is the latent heat contribution. The function $A_p(y)$ has already been defined in (2.29 a) by the following expression:

$$A_p(y) = a + b y$$

where:

$$a = 2.3 \text{ mm/day}$$

$$b = 0.14 \text{ mm/(day }^0\text{ latitude)}$$

Let us write the last equation in another way:

$$A_p(y) = a \left(1 + \frac{b}{a} y^*\right) \quad (3.19)$$
where \( A - \frac{b}{a} \) < 1 \hfill (3.19a)

Substitution of equation (3.19) in (3.7) yields:

\[
Q' = - \frac{L_v \rho_w a}{2 \rho_m z_u} \left( 1 + \frac{b}{a} y^* \right) \left[ 1 + \cos \left( \frac{z - z_u}{z_u} \right) \right] \sin \left[ \frac{x}{L} + \gamma (y) \right] + \frac{1}{\rho_m z_u} \left[ B_s T_s' - T_a' (B + B') \right] z \leq z_u \hfill (3.20)
\]

Notice that this expression takes its maximum value at \( z = z_u \).

Approximately:

\[
Q'_{\text{max}} = - \frac{L_v \rho_w a}{\rho_m z_u} \left( 1 + \frac{b}{a} y^* \right) \sin \left[ \frac{x}{L} + \gamma (y) \right] \hfill (3.20a)
\]

Equation (3.3) suggests that the only term in equation (3.8) capable of balancing the diabatic heating source term \( Q' \) is \([\Theta^*]_{z^*} \) \( W_1^* \), so taking into account equations (3.18), (3.20) and (3.20a) we choose the following scale:

\[
W_1 \sim \frac{H}{c_p} \frac{L_v \rho_w a}{\rho_m z_u} \hfill (3.21)
\]

or using (3.16):

\[
W_1 \sim 0.28 \text{ cm/s} \hfill (3.22)
\]
From (3.14):
\[ \frac{W_1 L_1}{U_1} = \frac{1.17 \text{ m/s}}{H} \quad (3.23) \]

If we use (3.16), (3.22) and (3.23) in equations (3.10) and (3.11) we can find the order of magnitude of the different terms. They are:

\[ \frac{\partial p^*_i}{\partial z^*} = \theta^*_i \quad (3.24) \]

\[ \frac{\partial u^*_i}{\partial x^*} + \frac{V_1 L_1}{U_1 A} \frac{\partial v^*_i}{\partial y^*} + \frac{\partial w^*_i}{\partial z^*} = 0 \quad (3.25) \]

Now let us scale the horizontal momentum equations (2.15) and (2.16). After dividing the scaled \( u \) - equation by \( f_0 V_1 \) and taking into account what we said in Section 2.4 about the cancellation of the boundary term, we obtain:
After dividing the scaled \( v \) - equation by \( f_0 U_1 \) and as in (3.26) neglecting the boundary term we obtain:

\[
\frac{V_i}{U_i} \frac{U}{f_0 L_1} \left[ u^* \right] \frac{\partial u_i^*}{\partial x^*} + \frac{V}{V_i} \frac{U}{f_0 A} \left[ u^* \right] \frac{\partial u_i^*}{\partial y^*} - \frac{U_i}{f_0 A} \left[ u_i^* u_i^* \right]_{y^*} +
\]

\[
\frac{V}{U_i} \frac{W}{f_0 H} \left[ w^* \right] \frac{\partial u_i^*}{\partial z^*} + \frac{V}{V_i} \frac{W_i}{f_0 H} \left[ u_i^* \right]_{z^*} w_i^* = - \int u_i^* -
\]

\[
\frac{Y H}{A \int u_i \Theta_m} \frac{\partial p_i^*}{\partial y^*} + \frac{V_i}{V_i} \frac{U}{f_0 H^2} \frac{\partial^2 u_i^*}{\partial z^2} + \frac{V_i}{U_i} \frac{V_i}{f_0 A} \left[ u_i^* u_i^* \right]_{y^*} +
\]

\[
+ \frac{V_i}{U_i} \frac{W_i}{f_0 H} \left[ u_i^* w_i^* \right]_{z^*}
\]

(3.27)
Let us now define the non-dimensional number $\epsilon_e$

$$\epsilon_e = \frac{W_1 \Theta_1}{W_1 \Theta} \sim 3 \times 10^{-2}$$

then equation (3.18) becomes:

$$\frac{U}{H} \left[ u^* \right] \frac{\partial \theta^*_i}{\partial x^*} + \epsilon_e \left[ u^* \right] \frac{\partial \theta^*_i}{\partial y^*} + \epsilon_e \left[ u^* \right] \frac{\partial \theta^*_i}{\partial z^*} +$$

$$+ \left[ \theta^*_i \right]_{x^*} \frac{\epsilon_e}{W_1 \epsilon_\theta} \left( \Theta_L + \Theta_R \right) + \frac{V}{V} \epsilon_e \left[ u^* \theta^*_i \right]_{y^*} +$$

$$+ \frac{W_1}{W} \epsilon_e \left[ u^* \theta^*_i \right]_{z^*} \quad (3.28)$$

where we have made use of (3.13).

Let us now expand $\theta^*_i$ and $W^*_1$ in the following series:

$$\theta^*_i = \theta^0 + \epsilon_e \theta^{(1)} + \epsilon_e^2 \theta^{(2)} + \ldots \quad (3.29)$$

$$W^*_1 = W^0 + \epsilon_e W^{(1)} + \epsilon_e^2 W^{(2)} + \ldots$$

since $\epsilon_e \sim 3 \times 10^{-2}$
In equation (3.28) we have the following coefficients:

\[
\frac{U}{W} \frac{H}{L_1} \varepsilon_0 \sim 4.5 \times 10^{-2} \sim \varepsilon_v
\]  
\[
(3.30)
\]

\[
\frac{W_1}{W} \varepsilon_0 \sim 3 \times 10^{-2} \sim \varepsilon_v
\]

the coefficient

\[
\frac{V_1}{V} \varepsilon_0 \sim 2 \times 10^{-1} \sim \varepsilon_v^{1/2}
\]

but taking into account that the term

\[
\frac{V_1}{V} \varepsilon_0 \left[ v_i^* \quad \Theta_i^* \right] y^*
\]

depends on the correlation between \( v_i^* \) and \( \Theta_i^* \) we will assume that this term is of the order \( \varepsilon_v \) at the most.

Substituting (3.29) in (3.28) and taking into account (3.30) we obtain:

(3.31)
\[ O(\varepsilon_0) : \]

\[
\frac{U}{W L'_1} \left[ u^* \right] \Theta^0_x^* + \left[ v^* \right] \Theta^0_y^* + \left[ w^* \right] \Theta^0_z^* + \left[ \Theta^* \right] z^* \cdot w(1) =
\]

\[
\frac{V_1}{V} \left[ v^* \right] \Theta^*_y^* + \frac{W_1}{W} \left[ w^* \right] \Theta^*_z^*
\]

(3.32)

Let us rewrite (3.31) in a more complete form using the following scales:

\[
T'_s = \Theta_s T'_s
\]

\[
T'_a = \Theta_a T'_a
\]

Substitution of these scales in equation (3.20) yields:

\[
Q' = \frac{L_v \rho_w a}{\rho_m z_u} \left\{ \frac{1}{2} \left( 1 + \frac{b}{a} \right) \left[ 1 + \cos \frac{\pi}{2} \left( \frac{z - z_u}{z_u} \right) \right] \right\}
\]

\[
\cdot \sin \left[ \frac{\pi x}{L_v} + \delta(y) \right] - \frac{z_u B_s}{z_s L_v a \rho_m} \left[ \Theta_s T'_s \right.
\]

\[
- \left( B_s + B_s \right) \frac{\Theta_a T'_a}{\Theta_s T'_s} \left\} \right.
\]

(3.33)
Combining (3.33), (3.31) and (3.21) we obtain:

\[
[\Theta^*_x]_x \tilde{w}_i^* = -\frac{1}{2} \left( 1 + \frac{b}{a} A \gamma^* \right) \left[ 1 + \cos \pi \left( \frac{z - z_u}{z_u} \right) \right] \sin \left( \frac{\pi X}{L} + \gamma(y) \right)
\]

\[
- \frac{z_u B_S \Theta_s}{z_S L_v \alpha_p} \left[ \frac{T_{1s}^*}{B_S} \Theta_a^* - \frac{(B_{\uparrow} + B_{\downarrow}) \Theta_a^*}{B_S} \right] T_{1a}^*
\]

(3.34)

Now consider the horizontal momentum equations. Let us define the Rossby Number,

\[
\epsilon_v = \frac{U}{f_o A} \sim 7 \times 10^{-2}
\]

then equation (3.26) can be written in the following way:

\[
\frac{U_i}{V_i} A \frac{\epsilon_v}{L_1} \left[ u^* \right] \frac{\partial u_i^*}{\partial x^*} + \frac{U_i}{V_i} \frac{\epsilon_v}{U V_i} \left[ v^* \right] \frac{\partial u_i^*}{\partial y^*} + \epsilon_v \left[ u^* \right] \gamma v_i^* +
\]

\[
\frac{A}{H} \frac{U_i}{V_i} w^* \epsilon_v \left[ w^* \right] \frac{\partial u_i^*}{\partial z^*} + \frac{W_i}{V_i} \frac{A}{H} \epsilon_v \left[ u^* \right] \frac{\partial u_i^*}{\partial z^*} \tilde{w}_i^* = f^* v_i^* -
\]

\[
\frac{y H}{\Theta} \left[ \Theta_l^* \right] + \frac{U_i}{V_i} \nu \frac{\partial^2 u_i^*}{\partial x^*} + \frac{U_i}{V_i} \epsilon_v \left[ u_i^* \right] \gamma^* +
\]

\[
\frac{U_i}{V_i} \frac{W_i}{f_o H} \left[ u_i^* \right] \frac{\partial u_i^*}{\partial y^*} + \frac{U_i}{V_i} \frac{W_i}{f_o H^2} \frac{\partial z^2}{\partial x^*} u
\]

(3.35)
where:

\[
\frac{U_1 A}{V_1 L_1} \varepsilon_v \sim 7 \times 10^{-3} \sim \varepsilon_v^2
\]

\[
\frac{U_1 V}{U_1 V_1} \varepsilon_v \sim 5 \times 10^{-3} \sim \varepsilon_v^2
\]

\[
\frac{A U_1 W}{H U V_1} \varepsilon_v \sim 4.5 \times 10^{-3} \sim \varepsilon_v^2
\]

\[
\frac{W_1 A}{V_1 H} \varepsilon_v \sim 9 \times 10^{-3} \sim \varepsilon_v^2
\]

\[
\frac{g H}{\Theta_m f_o L_1 V_1} \sim 1.5
\]

\[
\frac{U_1 \varepsilon_v}{U} \sim 4.2 \times 10^{-2} \sim \varepsilon_v
\]

\[
\frac{U_1 W_1}{V_1 f_o H} \sim 6 \times 10^{-3} \sim \varepsilon_v^2
\]

(3.36)

The friction term will be estimated assuming:

\[\nu \sim 10^4 \text{ cm}^2/\text{s}\]
In this way we obtain:

\[
\frac{U_i}{V_i} \frac{\nu}{f_0 H_2} \sim 1.4 \times 10^{-4} \sim \epsilon_3^3 \tag{3.36}
\]

Let us now make the following expansions in series:

\[
(u_i^*, v_i^*, p_i^*) = (u_i^0, v_i^0, p_i^0) + \epsilon_v (u_i^{(1)}, v_i^{(1)}, p_i^{(1)}) +
+ \epsilon_v^2 (u_i^{(2)}, v_i^{(2)}, p_i^{(2)}) + ... \tag{3.37}
\]

Substitution of (3.37) and (3.29) in (3.35) but taking into account (3.36) and that \(\epsilon_0 \sim 3 \times 10^{-2} \sim \epsilon_v\) we obtain:

0 (\(\epsilon_v^0\)):

\[
f^* v^0 = \frac{g H}{f_0 L_1 V_1} \frac{\Theta_1}{\Theta_m} \frac{\partial p^0}{\partial x^2} = 0 \tag{3.38}
\]

0 (\(\epsilon_v\)):

\[
[u^x]_y^* v^0 = \int^x u^{(i)} - \frac{g H}{f_0 L_1 V_1} \frac{\Theta_1}{\Theta_m} \frac{\partial p^{(i)}}{\partial x^*} +
+ \frac{U_i}{\nu} [u_i^* v_i^*]_y^* \tag{3.39}
\]
Now consider the $v$ - momentum equation (3.27). If we take into account the definition:

$$\epsilon_v = \frac{U}{f_0 A}$$

then equation (3.27) becomes:

$$\frac{V_i}{U_i} A \frac{\epsilon_v \left[ u^* \right]}{L_i} \frac{\partial u^*_l}{\partial x^*} + \frac{V_i}{U_i} \frac{\epsilon_v \left[ v^* \right]}{U} \frac{\partial v^*_l}{\partial y^*} + \frac{V_i}{U_i} \frac{\epsilon_v \left[ v^* \right]}{U} v^*_{l, v} = - \int u^*_l$$

$$\frac{V_i}{U_i} \frac{\epsilon_v \left[ u^* \right]}{U} \frac{\partial u^*_l}{\partial y^*} + \frac{A}{U_i} \frac{\epsilon_v \left[ v^* \right]}{U} \frac{\partial u^*_l}{\partial z^*} + \frac{V_i}{U_i} \frac{\epsilon_v \left[ v^* v^*_l \right]}{U} +$$

$$\frac{\rho}{f_0 A U_i \theta_m} \frac{\partial \rho^*_l}{\partial y^*} + \frac{V_i}{U_i} \frac{\mu}{f_0 H^2} \frac{\partial^2 v^*_l}{\partial z^*} + \frac{V_i}{U_i} \frac{\epsilon_v \left[ v^* v^*_l \right]}{U}$$

$$\frac{A}{L_i} \frac{V_i}{U} \frac{\epsilon_v \left[ u^* w^*_l \right]}{z^*}$$

(3.40)

The following magnitudes can be calculated for the coefficients in (3.40):

$$\epsilon_v \frac{V_i}{U_i} \frac{A}{L_i} \sim 2.8 \times 10^{-2} \sim \epsilon_v$$

$$\epsilon_v \frac{V_i}{U} \frac{V}{U} \sim 1.9 \times 10^{-2} \sim \epsilon_v$$
\[
\frac{A}{L_1} \frac{V}{U} \varepsilon_V \sim 1.9 \times 10^{-3} \sim \varepsilon_V^2
\]

\[
\frac{g_H \Theta_i}{f_0 A U_1 \Theta_m} \sim 15
\]

\[
\frac{V_1}{U_1} \frac{V}{U} \sim 6 \times 10^{-4} \sim \varepsilon_V^3 \quad (3.41)
\]

\[
\frac{V_1}{U_1} \frac{V_1}{U} \varepsilon_V \sim 14 \times 10^{-2} \sim \varepsilon_V^{1/2}
\]

\[
\frac{V_1}{U} \frac{A}{L_1} \varepsilon_V \sim 1.4 \times 10^{-2} \sim \varepsilon_V
\]

Substitution of (3.37) and (3.29) in (3.40) but taking into account (3.41) we obtain

\[
0 \left( \varepsilon_V^0 \right):
\]

\[
-f^* u^0 - \frac{g_H \Theta_i}{f_0 A U_1 \Theta_m} \frac{\partial p^0}{\partial y^*} = 0 \quad (3.42)
\]

\[
0 \left( \varepsilon_V \right):\]
Now consider the hydrostatic equation (3.24). Substitution of (3.29) and (3.37) in it yield:

\[ \frac{\partial p^{(i)}}{\partial z^*} = \frac{\epsilon_v}{\epsilon_v^0} \phi^0 \]  
\[ \frac{\partial p^{(1)}}{\partial z^*} = \frac{\epsilon_v}{\epsilon_v^0} \phi^{(1)} \]  

Finally consider the continuity equation (3.25). Again, substituting (3.29) and (3.37) in it, we obtain:

\[ \frac{\partial u^{*}}{\partial x^*} + \frac{V_1 L_1}{U_1 A} \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0 \]
3.3. Summary

In this Chapter we have obtained two simplified sets of equations using two different approaches. In Section 3.1 the order of magnitude of the terms in the equations was estimated from data published by Newell et al. (1971). In Section 3.2 the equations were written in terms of non-dimensional parameters and after some formal expansions in series were made, we obtained a new simplified system of equations. The series expansions (3.29) and (3.37) are power series of the following non-dimensional numbers. The Rossby Number:

\[ \xi_v = \frac{U}{f_0 A} \]

and

\[ \xi_\theta = \frac{W \Theta_1}{W_1 \Theta} \]

As long as the Rossby Number is small we have geostrophic balance in the momentum equations (see (3.38) and (3.42)). The \( \xi_\theta \) number can also be written in the following form:

\[ \xi_\theta = \frac{\Theta / \Theta_1}{(W_1 / W)} \]

indicating that it compares the relative temperature perturba-
tion with the relative vertical velocity perturbation. As long as that number is small, the vertical advection and the diabatic heating are in balance (see (3.31)).

Both sets of equations are very similar. The only difference is in the $u$-momentum equation. In Section 3.1 we obtained the following equation:

$$
(\frac{1}{f - [u]_y}) \nu' = \frac{p'_x}{\rho_m} \quad (3.5 \text{a})
$$

and in Section 3.2, the geostrophic approximation was obtained for the $v'$-field (equation 3.38). We will discuss in more detail these approximations later. Meanwhile we will assume that equation (3.5 a) is the correct one, a fact that is supported by the discussion in Section 4.2.

Notice that in the derivation of Section 3.1, the error was of the order of approximately 10% and in Section 3.2 the error was of order $\epsilon_v$ or $\epsilon_g$ ($\sim 14\%$).
IV. DISCUSSION OF THE DYNAMICS LIMITATIONS.

4.1. Introduction.

In Chapter III we have deduced the set of equations which is the system we will try to solve in this Chapter.

For convenience let us write again that system.

\[
[\Theta]_{z} w' = - \frac{L_v \rho_w a}{2 \rho_m z u c_p} (1 + \frac{b}{a}) \left[ 1 + \cos \Theta \left( \frac{z - z_u}{z_u} \right) \right] \\
\cdot \sin \left( \frac{x}{L} + \Theta(y) \right) + \frac{1}{\rho_m z 5 c_p} [B_s T_s - (B_\|^t + B_{\|^t}) T_{a}^t] \quad z \leq z_5
\]

(4.1)

\[
(f - [u]_y) v' = \frac{1}{\rho_m} p_x'
\]

(4.2)

\[
u' = - \frac{1}{\rho_m} p_y'
\]

(4.3)

\[p_z' = \frac{\rho_m g}{\theta_m} \theta'
\]

(4.4)

\[u'_x + v'_y + w'_z = 0
\]

(4.5)

where the constants have already been defined in Chapter II.

Now let us try to solve for the unknows of this system of e-
equations. They are $T'_a$, $w'$, $v'$, $p'$ and $\theta$. Actually $\theta'$ and $T'_a$ are related so we have five equations and five unknowns.

The function $[\theta]_z$ can be obtained from the data published by Talijard et al. (1969) or Schutz and Gates (1972). That function is shown in Figure 4.1 where we can see that it is approximately independent of latitude and directly proportional to height. From that Figure we can obtain:

$$[\theta]_z \approx 5^\circ K/Km. \quad (4.6)$$

In order to solve the system of equations (4.1) to (4.5) we can first obtain the vorticity equation from (4.2) and (4.3). Using (4.5) we obtain:

$$\left( \beta - [u]_{yy} \right) v' - [u]_y v'_y = f w'_z \quad (4.7)$$

Now, from equations (4.2), (4.7) and (4.1) we can obtain an equation where the unknowns are $p'$ in the left hand side and $T'_a$ in the right hand side. In principle this last equation can be combined with equation (4.4) to obtain $T'_a$. Then we can solve for the other unknowns.

Equation (4.7) is an approximation of the complete vorticity equation obtained from (2.15) and (2.16). This equation is:
FIGURE 4.1 Zonal Average Potential Temperature Field \([\Theta]\)
\[ [u] \frac{\partial \zeta_x}{\partial x} + [v] \frac{\partial \zeta_y}{\partial y} + [w] \frac{\partial \zeta_z}{\partial z} + [u] \frac{\partial \zeta_y}{\partial y} + (\rho - [u]_{yy}) \frac{\partial u'}{\partial y} \frac{\partial \zeta_z}{\partial z} = \]

\[ = (f - [u]_{y}) w_{z}' - [v]_{z} w_{x}' + [u]_{z} w_{y}' + [w]_{y} u_{z}' + [u]_{zy} w_{y}' \\
- [u'u']_{yy} - [u'w']_{zy} \]  

(4.8)

where: \( \zeta' = v_{x}' - u_{y}' \) and where we have neglected the boundary terms as we discussed in Section 2.4 and considered steady state conditions.

Due to the way the approximate vorticity equation (4.7) was obtained, that approximation may not be a good one for the following reason. When we deduced equation (4.7) by cross differentiation of equations (4.2) and (4.3) we eliminated the pressure. Now, the pressure gradient components are leading terms balancing the Coriolis components in the approximations (4.2) and (4.3). When the pressure is eliminated by cross differentiation, the derivatives of the Coriolis force components are no longer balanced by the corresponding derivatives of the pressure. Thus, the derivatives of the other terms we neglected in the approximations (4.2) and (4.3) may become important in balancing the corresponding derivatives of the Coriolis terms. Then an estimation of the different terms in the complete vorticity equation (4.8) is in order at this point.
4.2. Vorticity Equation Order of Magnitude Estimates at 500 mb.

As before, the magnitude of the different terms in the vorticity equation (4.8) is very difficult to estimate due to the lack of enough data in the studied area. However, the fields published by Newell et al. (1971) and (1973) suggest that we take the following values:

\[
[v] \approx 0
\]

\[
[w] \approx 0
\]

Then, the vorticity equation (4.8) becomes:

\[
[u] \frac{\partial u'}{\partial x} + (\beta - [u]_{yy}) u' = (f - [u]_y) w'_z + [u]_z \omega' + [u]_{zy} w'_z + \\
+ \nu \frac{\partial^2 u'}{\partial z^2} - [u w']_{yy} - [u' w']_{zy}
\]

In order to estimate the magnitude of the different terms in the vorticity equation we have to know first the zonal average field \([u]\). From visual interpolation of Newell's analysis of the zonal velocity field we obtain Figures 4.2 and 4.3. In Figure 4.2, we have the zonal velocity average \([u]\) as a function of height at three different latitudes. In Figure 4.3 the \([u]_y\) field obtained using finite differences is shown.

The following approximation can be obtained for the \([u]\) field near 500 mb \((\sim 5.8\text{ Km.})\):
FIGURE 4.2. Zonal Average of the Zonal Velocity Component (\[u\])
FIGURE 4.3 Meridional Derivative of the zonal average of the Zonal Wind.
\[ [u] = m_u \, z^2 + (\frac{1}{2} \, c_u \, y^2 + d_u \, y + n_u) \, z + (\frac{1}{2} \, e_u \, y^2 + f_u \, y + r_u) \]

(4.10)

where: \( m_u = 6 \times 10^{-1} \, \text{m/ (s Km}^2) \)

\( n_u = -5.9 \, \text{m/ (s Km)} \)

\( r_u = 9.86 \, \text{m/s} \)

\( c_u = 7.2 \times 10^{-3} \, \text{s}^{-1} \, \text{Km}^{-1} \, \text{o} \, \text{lat}^{-1} \)

(4.11)

\( d_u = -1.22 \times 10^{-6} \, \text{s}^{-1} \, \text{Km}^{-1} \)

\( e_u = 0.86 \times 10^{-6} \, \text{s}^{-1} \, \text{o} \, \text{lat}^{-1} \)

\( f_u = -3.04 \times 10^{-6} \, \text{s}^{-1} \)

The vertical velocity perturbation field \( w' \) is going to be estimated from equation (4.1). That equation can be written in the following way:

\[ w' = F_1 (y, z) \sin \left( \hat{\Pi} \frac{x}{L} + \hat{\mathcal{G}}(y) \right) + G_1 \cos \left( \hat{\Pi} \frac{x}{L} + k \, y \right) + \]

\[ + H_1 \, T' \]

(4.12)
where:

\[ F_1(y,z) = -\frac{L_p \rho_m a}{2 \rho_m \mu c_p [\Theta]_z} \left(1 + \frac{b}{y}\right) \left[1 + \cos \pi \frac{(z - z_u)}{z_u}\right] \]

\[ (4.13) \]

\[ G_1 = \frac{B_s A_T}{\rho_m c_p z_5 [\Theta]_z} \]

\[ (4.14) \]

\[ H_1 = -\frac{(B_1 + B_1)}{\rho_m c_p z_5 [\Theta]_z} \]

\[ (4.15) \]

The following \( w' \)-derivatives are needed in equation (4.9):

\[ w'_z = F_1 z(y, z) \sin \left(\frac{x}{L} + \gamma(y)\right) \]

\[ (4.16) \]

\[ w'_y = F_1 y(y, z) \sin \left(\frac{x}{L} + \gamma(y)\right) + G_3(y, z) \cos \left(\frac{x}{L} + \gamma(y)\right) - k G_1 \sin \left(\frac{x}{L} + k y\right) + H_1 T'_ay \]

\[ (4.17) \]

where: \( G_3(y, z) = \gamma'(y) F_1(y, z) \)

Now, let us estimate the magnitude of the terms in the vorticity equation (4.9) at 500 mb. Using equations (4.13), (4.14), (4.15), (4.16) and (4.17) we can find the following
values we will need.

d = day  
y = 0 = 10^0 S

\[ z_5 = 5.8 \text{ Km ( } \sim 500 \text{ mb) } \]
\[ \beta = 2.07 \times 10^{-6} \text{ m}^{-1} \text{ d}^{-1} \]
\[ F_1 (0, z_5) = -204 \text{ m d}^{-1} \]
\[ F_1 z (0, z_5) = 10^{-2} \text{ d}^{-1} \]
\[ F_1 y (0, z_5) = -1.24 \times 10^{-4} \text{ d}^{-1} \]

\[ [u]_z = 97.7 \text{ d}^{-1} \]
\[ G_1 = 21.84 \text{ m d}^{-1} \]
\[ [u]_{zy} = -1.05 \times 10^{-4} \text{ m}^{-1} \text{ d}^{-1} \]
\[ [u]_{yy} = 1.1 \times 10^{-6} \text{ m}^{-1} \text{ d}^{-1} \]
\[ H_1 = -26.15 \text{ m} \text{ } ^{\circ}C^{-1} \text{ d}^{-1} \]
\[ G_3 (0, z_5) = -2.04 \times 10^{-4} \text{ d}^{-1} \]
\[ f (10^0 S) = -2.16 \text{ d}^{-1} \]

From data in Talijard et al. (1969) we can find the \( T'_a \) (500 mb temperature) field. The following values are typical:

\[ T'_a \sim 1.5^{\circ}C \]
\[ T'_{ay} \sim 1 \times 10^{-6} \text{ } ^{\circ}C/\text{m} \]

From (4.18) and (4.19) we have:
From (4.18) and (4.19 a) we can also write:

\[ H_1 T'_a \sim 0.19 F_1 (0, z_5) \]
\[ H_1 T'_y \sim 0.13 G_3 (0, z_5) \]
\[ H_1 T'_y \sim 0.21 F_{1y} (0, z_5) \]
\[ H_1 T'_a \sim G_1 \]

Thus, at 500 mb level, equations (4.12) and (4.17) can be approximated as follows:

\[ w' \sim F_1 (y, z) \sin \left( \frac{\pi x}{L} + \gamma (y) \right) \quad (4.20) \]
\[ w'_y \sim F_{1y} (y, z) \sin \left( \frac{\pi x}{L} + \gamma (y) \right) + G_3 (y, z) \cos \left( \frac{\pi x}{L} + \gamma (y) \right) - k G_1 \sin \left( \frac{\pi x}{L} + k y \right) \quad (4.21) \]

We can now estimate the magnitude of the terms in the
verticity equation (4.9). Using Newell's maps for the zonal and meridional fields and the values in (4.18) we obtain:

\[
\begin{align*}
[u] \xi_x' & \sim \begin{cases} 
[u] v_x' & \sim + 1.4 \times 10^{-3} d^{-2} \\
[u] u_y' & \sim + 8.37 \times 10^{-4} d^{-2}
\end{cases} \\
\beta v' & \sim 9 \times 10^{-2} d^{-2} \\
f w_z' & \sim 2.2 \times 10^{-2} d^{-2} \\
[u] w_y' & \sim - 2 \times 10^{-2} d^{-2} \\
[u] z_y & \sim 2 \times 10^{-2}
\end{align*}
\]

\[\mu \xi_z' \begin{cases} 
\mu v_z' & \sim 1.5 \times 10^{-4} d^{-2} \\
\mu u_y' & \sim - 2.1 \times 10^{-3} d^{-2}
\end{cases} \quad (4.22)
\]

We could not estimate the last two terms in equation (4.9) for lack of data. However, zonal averages around the earth are available from Oort and Rasmusson (1971). From that data we have:

\[ [u' v']_{yy} \sim 0 \]

\[ [u' w']_{zy} \sim - 7.5 \times 10^{-3} d^{-2} \]
All these values suggest that we approximate the vorticity equation (4.9) as follows:

\[
(\beta - [u]_y) v' = (f - [u]_y) w' + w'_y + [u]_{zy} w' \quad (4.23)
\]

4.3. Dynamics at 500 mb.

We will try to find \( T'_a \) using the simplified vorticity equation (4.23).

Combining (4.2), (4.23), (4.16), (4.20) and (4.21) we obtain:

\[
\begin{align*}
\frac{G_y(y, z)}{\rho_m G(y, z)} \frac{\partial}{\partial x} & = K(y, z) \sin \left( \pi \frac{x}{L} + \theta(y) \right) + M(y, z) \cos \left( \pi \frac{x}{L} + \theta(y) \right) \\
& + N(y, z) \sin \left( \pi \frac{x}{L} + \kappa y \right) \\
\end{align*}
\quad (4.24)
\]

where:

\[
\begin{align*}
G(y, z) & = f(y) - [u]_y \\
K(y, z) & = G(y, z) F_{1z}(y, z) + [u]_y F_{1y}(y, z) + [u]_{zy} F_1 \\
M(y, z) & = [u]_y G_3(y, z) \\
N(y, z) & = -k G_1 [u]_z
\end{align*}
\quad (4.25)
\]
From (4.24) we can obtain:

\[
\frac{1}{P_m} \frac{G_v(y, z)}{G(y, z)} \rho' = \frac{L}{\pi} \left\{ -k(y, z) \cos \left( \frac{X}{L} + \gamma(y) \right) +
\right.
\]
\[
+ M(y, z) \sin \left( \frac{X}{L} + \gamma(y) \right) - N(y, z) \cos \left( \frac{X}{L} + \xi(y) \right) \right\}
\]

(4.26)

Combination of (4.26) and (4.4) yields:

\[
\Theta' = \frac{\Theta_m L}{\pi g} \oint \frac{G(y, z)}{G_Y(z)} \left\{ -k(y, z) \cos \left( \frac{X}{L} + \gamma(y) \right) +
\right.
\]
\[
+ M(y, z) \sin \left( \frac{X}{L} + \gamma(y) \right) - N(y, z) \cos \left( \frac{X}{L} + \xi(y) \right) \right\}
\]

or:

\[
\Theta' = \frac{L \Theta_m}{\pi g} \left\{ k_1(y, z) \cos \left( \frac{X}{L} + \gamma(y) \right) +
\right.
\]
\[
+ M_1(y, z) \sin \left( \frac{X}{L} + \gamma(y) \right) + N_1(y, z) \cos \left( \frac{X}{L} + \xi(y) \right) \right\}
\]

(4.27)
where:

\[
K_1 (y, z) = - \left[ K_z \frac{G}{G_y} + K \frac{G_z}{G_y} - K \frac{G_y}{G_y^2} \right]
\]

\[
M_1 (y, z) = \left[ M_z \frac{G}{G_y} + M \frac{G_z}{G_y} - M \frac{G_y}{G_y^2} \right] \quad (4.28)
\]

\[
N_1 (y, z) = - \left[ N_z \frac{G}{G_y} + N \frac{G_z}{G_y} - N \frac{G_y}{G_y^2} \right]
\]

where for simplicity, \(G, K, M\) and \(N\) represent the functions \(G (y, z), K (y, z), M (y, z)\) and \(N (y, z)\) respectively which were defined in equations (4.25).

We can evaluate equation (4.27) at the average height of the 500 mb level \((\sim 5.8\text{ Km})\) and obtain the 500 mb potential temperature perturbation field.

Consistent with the Boussinesq approximation we take:

\[\Theta' \sim \Theta'\]
In order to test our assumptions (4.19) we will calculate the \( y \) - derivative of the temperature. From (4.27) we can write:

\[
\theta'_y = \frac{\Theta_m}{g} \left\{ \frac{K_{1M}(y, z) \cos \left( \frac{\pi x}{L} + \gamma(y) \right)}{\nu} + 
\right.
\]

\[
+ \frac{M_{1K}(y, z) \sin \left( \frac{\pi x}{L} + \gamma(y) \right)}{\nu} + N_{1y}(y, z) \cos \left( \frac{\pi x}{L} + k y \right) - 
\]

\[
- k N_{1}(y, z) \sin \left( \frac{\pi x}{L} + k y \right) \right\} 
\]

\[ (4.29) \]

where:

\[
K_{1M}(y, z) = K_{1y}(y, z) + \gamma'(y) M_{1}(y, z) 
\]

\[ (4.30) \]

\[
M_{1K}(y, z) = M_{1y}(y, z) - \gamma'(y) K_{1}(y, z) 
\]

Using all the information we have discussed in this chapter we can now calculate the 500 mb temperature and its \( y \) - derivative at \( y = 0 \) (10°S). The results are:

\[
T' = -0.55 \cos \left( \frac{\pi x}{L} + c \right) + 0.20 \sin \left( \frac{\pi x}{L} + c \right) - 
\]

\[
- 0.03 \cos \frac{\pi x}{L} o_K 
\]

\[ (4.31) \]
$$T_{ay} = 6.66 \times 10^{-7} \cos \left( \pi \frac{x}{L} + c \right) + 5.3 \times 10^{-7} \sin \left( \pi \frac{x}{L} + c \right)$$

$$+ 2.52 \times 10^{-8} \cos \left( \pi \frac{x}{L} \right) + 5.04 \times 10^{-8} \sin \left( \pi \frac{x}{L} \right) \text{K/m.}$$

(4.32)

These results are in agreement with our earlier assumptions (4.19). We will also test the hypothesis that (4.23) is a good approximation to the vorticity equation (4.9) as the data in Newell (1971) had suggested.

In order to do that, let us calculate the first term in the left hand side of (4.9) which had been estimated as relatively small. We have:

$$[u] \dot{\zeta}^i_x = [u] (v^i_{xx} - u^i_{xy})$$

(4.33)

Now, from (4.23), (4.25), (4.20), (4.21) and (4.16):

$$v^i_{xx} = - \frac{\Omega^2}{L^2 G_y(z)} \left\{ K(y,z) \sin \left( \pi \frac{x}{L} + \gamma(y) \right) + 

+ M(y,z) \cos \left( \pi \frac{x}{L} + \gamma(y) \right) + N(y,z) \sin \left( \pi \frac{x}{L} + k y \right) \right\}$$

(4.34)
The second term in the right hand side of (4.33) will be calculated as follows. From the continuity equation we can write:

\[ u'_x = -v'_y - w'_z \]

or:

\[ u'_{xy} = -(v'_{yy} + w'_{zy}) \]  \hspace{1cm} (4.35)

The vertical velocity perturbation term in (4.35) can be derived from (4.16). We have:

\[ w'_{zy} = F_{zy}(y,z) \sin \left( \frac{X}{L} + \gamma(y) \right) + \gamma'(y) F_{1z} \cos \left( \frac{X}{L} + \gamma(y) \right) \]  \hspace{1cm} (4.36)

The first term in the right hand side in (4.35) can be obtained from (4.23), (4.25), (4.20), (4.21) and (4.16). We have:

\[ v'_{yy} = \frac{1}{G_y(z)} \left\{ K_{MK}(y,z) \sin \left( \frac{X}{L} + \gamma(y) \right) + \right. \]

\[ + M_{KM}(y,z) \cos \left( \frac{X}{L} + \gamma(y) \right) + N_2(y,z) \sin \left( \frac{X}{L} + k y \right) + \]

\[ + S(y,z) \cos \left( \frac{X}{L} + k y \right) \left\} \right. \]  \hspace{1cm} (4.37)
where:

\[ K_{MK}(y, z) = K_{yy}(y, z) - 2 \mathcal{V}'(y) M_y(y, z) - [\mathcal{V}'(y)]^2 K(y, z) \]

\[ M_{KM}(y, z) = M_{yy}(y, z) + 2 \mathcal{V}'(y) K_y(y, z) - [\mathcal{V}'(y)]^2 M(y, z) \]

\[ N_2(y, z) = N_{yy}(y, z) - k^2 N(y, z) \] \hspace{1cm} (4.38)

\[ S(y, z) = 2k N_y(y, z) \]

Having found analytical expressions for \( \mathcal{V}'_{yy} \) and \( \mathcal{W}'_{zy} \) in (4.37) and (4.36) respectively, the term \( u'_{xy} \) in (4.33) can now be calculated using (4.35).

Due to the fact that the \( y \)-derivatives of the \([u]\) field are known only near \( 10^0 \)S (see Figures 4.2 and 4.3), we can only make calculations near that latitude.

With the information we have so far we can obtain the following results at \( y = 0 \) (10°S) and \( z = z_5 \) (5.8 Km.):

\[ [u] v'_{xx} = -4.18 \times 10^{-4} \sin \left( \frac{\pi}{L} + c \right) - 2.3 \times 10^{-3} \cos \left( \frac{\pi}{L} + c \right) \]

\[ - 3.71 \times 10^{-4} \sin \frac{\pi}{L} \text{ day}^{-2} \] \hspace{1cm} (4.39)
\[ u_{xy}^\prime = -7.2 \sin \left( \frac{X}{L} + c \right) + 26.2 \cos \left( \frac{Y}{L} + c \right) \]

\[ + 1.8 \sin \frac{Y}{L} + 3.7 \cos \frac{X}{L} \left( 10^{-3} \text{ day}^{-2} \right) (4.40) \]

The last result is not in agreement with what we have estimated in (4.22).

Two possible reasons may explain this lack of agreement. The first is that we should have included the last two terms in the right hand side of equation (4.9) in the approximate vorticity equation (4.23). However, as we said before, even their order of magnitude is difficult to estimate due to the fact that almost all the studied area is oceanic and the data considered in Newell et al. (1971) comes from land stations only. This last fact suggests the second reason, namely, that the fields published by Newell et al. are not reliable in the studied area. At this point we emphasize that Newell's data was the basis for the estimation of the magnitude of some derivatives of the perturbation quantities in (4.8) as well as the average fields (4.8 a and 4.10).

This second reason is supported by Zipser (1974). Among the several shortcomings he found in Newell's book there is one that concern us. He pointed out the fact that errors or biases in key Southern Hemisphere stations are permitted to destroy the analysis over large areas. For example, the wind maximum over Australia in June - August should extend far into
the Pacific. Due to the inconsistencies found by Zipser, he concluded that the analysis in Chapter 3 of Newell's book should be viewed with skepticism and should be used with the greatest selectivity. Then the present calculations, suggest that instead of 4.23, a good approximation to the vorticity equation (4.9) may be the following:

\[
[u] (v_{xx} - u_{xy}) + G_y(z) v' = K(y, z) \sin \left( \frac{\pi x}{L} + \gamma(y) \right) + \\
+ M(y, z) \cos \left( \frac{\pi x}{L} + \gamma(y) \right) + N(y, z) \sin \left( \frac{\pi x}{L} + k y \right) \tag{4.41}
\]

Equation (4.41) suggests the following:

\[
v'_{xx} = - \frac{\pi^2}{L^2} v'
\tag{4.42}
\]

From (4.35) and (4.36) we can also write:

\[
u'_{xy} = - v'_{yy} - F_1 z y (y, z) \sin \left( \frac{\pi x}{L} + \gamma(y) \right) - \\
- \gamma'(y) F_1 z (y, z) \cos \left( \frac{\pi x}{L} + \gamma(y) \right) \tag{4.43}
\]

Substitution of (4.42) and (4.43) in (4.41) yields:
\[
[u] v''_{yy} + \left\{ G_y(z) - \frac{\pi^2}{L^2} [u] \right\} v' = K_2(y,z) \sin \left( \frac{\pi x}{L} + \gamma(y) \right) + \\
+ M_2(y,z) \cos \left( \frac{\pi x}{L} + \delta(y) \right) + N(y,z) \sin \left( \frac{\pi x}{L} + k y \right) \quad (4.44)
\]

where:

\[
K_2 (y, z) = K (y, z) - [u] F_{1y}
\]

\[
M_2 (y, z) = M (y, z) - [u] \delta' (y) F_{1z} (y, z)
\]

In principle, equation (4.44) may be solved for \(v'\), then equation (4.3) would give the pressure field. Having the pressure field we may find the zonal velocity and the temperature fields from (4.2) and (4.4) respectively.

The main difficulty in trying to solve equation (4.44) is the estimation of the zonal average field ([u]). As we have discussed earlier in this Section, the serious shortcomings of Newell's data which is the only source for winds we have been able to find, do not permit reliable estimates of the zonal average fields to be obtained.

The same troubles found in the discussion of the dynamics at 500 mb occur at lower heights.

As a result of the discussion in this Section the only field that has been obtained in a satisfactory form is the
vertical velocity perturbation field $w'$ which, assuming (4.19) is satisfied by the unknown $T_a$ field, is given by equation (4.20). For convenience let us write again this result:

$$w' = F_1(y, z) \sin \left( \pi \frac{x}{L} + \varphi(y) \right) \quad (4.20)$$

The results obtained for this parameter are discussed in the next Section.

4.4. Vertical Velocity Perturbation Field.

This field as estimated from equation (4.20) is shown in Table 4.1.

Although some inconsistencies have been found in the maps in Newell et al. (1971) and (1973), we think that the main large scale features are present in those fields. Local errors may invalidate the use of those fields in detailed quantitative calculations as in Section 4.3 but some qualitative conclusions may still be obtained from them.

In Newell et al. (1973) we can find the vertical velocity field estimated using two different approaches. In the first, they used the thermodynamic equation with the same balance as we considered in our equations (3.6a) or (3.31), namely, the vertical motion advection term and the diabatic
**TABLE 4.1**

*Vertical Velocity Perturbation Field at 500 mb (z ~ 5.8 km)*

*Deduced from Equation (4.20) (10^{-4} mb/s)*

<table>
<thead>
<tr>
<th></th>
<th>140°E</th>
<th>150</th>
<th>160</th>
<th>170</th>
<th>180</th>
<th>170</th>
<th>160</th>
<th>150</th>
<th>140</th>
<th>130</th>
<th>120</th>
<th>110°W</th>
</tr>
</thead>
<tbody>
<tr>
<td>5°S</td>
<td>0.3</td>
<td>-0.8</td>
<td>-1.7</td>
<td>-2.1</td>
<td>-1.8</td>
<td>-0.9</td>
<td>0.2</td>
<td>1.3</td>
<td>1.9</td>
<td>2.0</td>
<td>1.4</td>
<td>0.3</td>
</tr>
<tr>
<td>10°S</td>
<td>1.0</td>
<td>0.1</td>
<td>-0.8</td>
<td>-0.9</td>
<td>-1.7</td>
<td>-1.4</td>
<td>-0.6</td>
<td>0.3</td>
<td>1.2</td>
<td>1.6</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>15°S</td>
<td>1.1</td>
<td>0.7</td>
<td>0.0</td>
<td>-0.6</td>
<td>-1.1</td>
<td>-1.2</td>
<td>-0.9</td>
<td>-0.4</td>
<td>0.3</td>
<td>0.9</td>
<td>1.2</td>
<td>1.1</td>
</tr>
</tbody>
</table>
heating are in balance. The diabatic heating rate involves contributions from latent heat and radiative processes as we did in Chapter II. They also used the climatological precipitation rates in order to estimate the total latent heat release in an atmospheric column and the same vertical distribution as we did in Chapter II. With respect to the radiative heating rates, due to lack of enough data in the Southern Hemisphere they used Northern Hemisphere data for the appropriate season and we used Held and Suarez's (1974) parameterizations of the long wave fluxes at the sea level and at the top of the atmosphere as well as the result obtained by Stone et al. (1976) that the radiative flux divergence is approximately constant from the sea surface up to the 500 mb level in July near 10°S (see Section 2.3). Thus the only difference between Newell's procedure and ours is in the way of estimate the radiative heating contribution. However, as we will see later in this Section, our estimates indicate that the diabatic heating perturbation corresponding to the latent heat release \( Q_L' \) is larger than the corresponding to the radiative heating \( Q_R' \). Newell et al. (1973) only give magnitudes of the zonal averages of the latent heat release and radiative heating contributions but no zonal distributions with which we could compare our estimates. What we can do is compare our vertical velocity field with Newell's, which is shown in our Figure 4.4. This com-
FIGURE 4.4 Vertical Velocity Field at 500 mb obtained from the Thermodynamic Equation by Newell et al. (1973).
parison will indicate the reliability of our estimates of the
diabatic heating contributions, particularly the radiative
one, since the latent heat release has been estimated using
the same data as Newell et al. Figure 4.5 shows the vertical
velocity field as obtained by Newell et al. (1973) using the
thermodynamic equation (see Figure 4.4) at 10°S and 500 mb
level. From that Figure the zonal average can be estimated.
We obtain:

\[ [w] = -0.65 \times 10^{-4} \text{ mb/s} \]

Having estimated the zonal average we can obtain the verti-
cal velocity perturbation field. The results are shown in
the following Table 4.2

<table>
<thead>
<tr>
<th></th>
<th>140E</th>
<th>155E</th>
<th>176E</th>
<th>176W</th>
<th>124W</th>
<th>110W</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°S</td>
<td>1.65</td>
<td>0.65</td>
<td>-1.35</td>
<td>-1.35</td>
<td>0.65</td>
<td>1.65</td>
</tr>
</tbody>
</table>

*TABLE 4.2*

*Vertical Velocity Perturbation Field at 500 mb in $10^{-4}$ mb/s*

*Deduced from Newell et al (1973)*

*(see Figures 4.4 and 4.5)*
FIGURE 4.5: Vertical Velocity at 10°S (ω in 10^{-4} mb/s) deduced from Fig. 4.4
Now we are in position of compare our estimate of the vertical velocity perturbation field (Table 4.1) with Newell's (Table 4.2) at $10^\circ$S. We can see that both fields are very similar. This fact indicates the reliability of our estimate of the radiative heating contribution, namely, this contribution is smaller than the corresponding to the latent heat release.

In the second approach, Newell et al. used the continuity equation to find the vertical velocity field. Their result is shown in our Figure 4.6. Both Newell's calculations (Figure 4.4 and 4.6) have similar features but there are also significant differences in amplitude and in variations of the patterns with longitude particularly in the studied area.

As we said before in this Section, we will compare our estimates of the latent heat release perturbation with the radiative heating perturbation field and see what diabatic heating contribution is the main driving force of the circulation we are studying. From (2.28), (2.25), (2.29) and (4.13):

$$Q_L' = c_p \left[ \theta \right]_z F_1 (y, z) \sin \left( \pi \frac{x}{L} + \phi(y) \right) \quad (4.46)$$

Besides, from (2.38), (2.39), (4.14) and (4.15):

$$Q_R' = c_p \left[ \theta \right]_z \left\{ G_1 \cos \left( \pi \frac{x}{L} + k y \right) + H_1 T_a \right\} \quad (4.47)$$
FIGURE 4.6 Vertical Velocity Field at 500 mb obtained from the Continuity Equation by Newell et al. (1973).
Now, the $z$-dependence of the function $F_1(y, z)$ in (4.46) is given by the function $g(z)$ in (2.25) and Figure 2.1. Then we can see that $F_1(y, z)$ has its maximum value at a height $z = z_u = 5.5 \text{ Km}$. At this height we have:

$$F_1(0, z_u) = -205.5 \text{ m d}^{-1} \quad (4.48)$$

From (4.14), $G_1$ is constant and its value is:

$$G_1 = 21.8 \text{ m d}^{-1} \quad (4.49)$$

With respect to the term $H_1 T'_a$ in equation (4.47), we have found in (4.19 a) the following typical value:

$$H_1 T'_a \approx 39 \text{ m d}^{-1} \quad (4.19a)$$

This value is based on a typical value of $T'_a \sim 1.5^\circ C$ found in the literature (Talijard et al. (1969)). Now, both contributions in the right hand side of equation (4.47) are out of phase so that considering their amplitudes given in (4.49) and (4.19 a), it seems that $Q'_R$ is much smaller than $Q'_L$. According to what we have just seen, we can write:

$$|Q'_R| \sim 0.1 |Q'_L|$$

where $| |$ means amplitude.
V. QUALITATIVE CONSIDERATIONS.

We shall not go further into quantitative calculations due to the limitations we have discussed in last Chapter. However, a qualitative physical picture of the atmospheric circulation at these latitudes can be obtained assuming that the true climatic fields are similar to those published by Newell (see Section 4.4). As we will see later, fields with several main features similar to those in Newell's book at low heights, can be obtained by the following approximate balance in the vorticity equation (4.9):

\[ \beta v' = f w'_z \]  

indicating that the horizontal convergence is due only to the \( \beta \) effect. Notice that this balance is independent of the zonal average fields whose reliable estimation has not been obtained.

From (5.1) and (4.16) we can obtain:

\[ v' = \frac{f}{\beta} F_{1z} (y, z) \sin (\Pi \frac{x}{L} + \gamma (y)) \]  

Assuming that \([u]_y \ll f\) in (4.2) we can obtain

\[ p' = -\rho \frac{f^2}{\beta \Pi} F_{1z} (y, z) L \cos (\Pi \frac{x}{L} + \gamma (y)) \]
\[ \theta' = - \frac{\Theta_m f^2 L}{\beta \eta L} F_{1zz} (y,z) \cos \left( \eta \frac{x}{L} + \gamma (y) \right) \] (5.4)

\[ u' = \frac{L}{\eta \beta} \left\{ [2 \beta F_{1z} (y,z) + \eta F_{1yz} (y,z)] \cos \left( \eta \frac{x}{L} + \gamma (y) \right) - \right. \]

\[ \left. - \eta F_{1z} (y,z) \gamma' (y) \sin \left( \eta \frac{x}{L} + \gamma (y) \right) \right\} \] (5.5)

For convenience let us write again the functions and constants involved in the estimation of the different parameters in this Section:

\[ F_{1z} (y,z) = \frac{L_v \rho_w \ a \ \eta}{2 \rho_m \ z_u^2 \ c_p [\Theta]_z} \left( 1 + \frac{b}{a} y \right) \sin \eta \left( \frac{z - z_u}{z_u} \right) \]

\[ a = 2.3 \ mm / \ day \]
\[ b = 0.14 \ mm / (day \ \degree \ latitude) \]
\[ \beta = 2.4 \times 10^{-11} \ m^{-1} \ s^{-1} \]
\[ \gamma (y) = c + d \ y \]
\[ z_u = 5.5 \ Km \]
\[ c_p = 0.24 \ cal / (gr \ \degree \ K) \]
\[ [\Theta]_z = 5 \ \degree \ K / \ Km \]
\[ \rho_w = 1 \ gr / \ cm^3 \]
\[ \Theta_m = 320 \ \degree \ K \]
\[ L_v = 590 \ cal / gr \]
\[ L = 55^\circ \ longitude \]
\[ c = -0.52 \ rad \]
\[ d = 0.1 \ rad / \degree \ lat. \]
\[ \rho_m = 10^{-3} \text{ gr/cm}^3 \]

In the following Section we will compare the fields calculated in base of (5.1) with "actual" values in the literature.

5.1. Potential Temperature Perturbation Field.

Tables 5.1 and 5.2 show the Potential Temperature Perturbation Fields estimated using (5.4) and the actual field deduced from data in Talijard (1969) respectively.

Both fields have similar features. Negative values near the western boundary and positive in the center. Aside from all the limitations of the approximation (5.1) we are using, let us remember that near the boundaries we have large errors in using equation (2.29) for the perturbation of the precipitation field as we can see in Figure 2.2. This fact may explain the large differences between the values in Tables 5.1 and 5.2 near the boundaries.

5.2. Zonal Velocity Perturbation Field.

Tables 5.3 and 5.4 show the u' field estimated using equation (5.5) and the "actual" field deduced from visual interpolation of Newell's data which is shown in Figure 5.1.
### TABLE 5.1

Potential Temperature Perturbation Field at $z = 1.5$ Km (850 mb) from equation (5.4) ($^\circ$K)

<table>
<thead>
<tr>
<th></th>
<th>140E</th>
<th>150</th>
<th>160</th>
<th>170</th>
<th>180</th>
<th>170</th>
<th>160</th>
<th>150</th>
<th>140</th>
<th>130</th>
<th>120</th>
<th>110W</th>
</tr>
</thead>
<tbody>
<tr>
<td>5S</td>
<td>-0.1</td>
<td>-0.1</td>
<td>-0.1</td>
<td>0.0</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.0</td>
<td>-0.1</td>
<td>-0.1</td>
<td>-0.1</td>
<td></td>
</tr>
<tr>
<td>10S</td>
<td>-0.3</td>
<td>-0.4</td>
<td>-0.3</td>
<td>-0.1</td>
<td>0.1</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.0</td>
<td>-0.2</td>
<td>-0.3</td>
</tr>
<tr>
<td>15S</td>
<td>-0.3</td>
<td>-0.5</td>
<td>-0.6</td>
<td>-0.4</td>
<td>-0.2</td>
<td>0.1</td>
<td>0.4</td>
<td>0.6</td>
<td>0.5</td>
<td>0.3</td>
<td>0.0</td>
<td>-0.3</td>
</tr>
</tbody>
</table>

### TABLE 5.2

Potential Temperature Perturbation Field at 850 mb from Taliard et al. (1969) ($^\circ$K)

<table>
<thead>
<tr>
<th></th>
<th>140E</th>
<th>150</th>
<th>160</th>
<th>170</th>
<th>180</th>
<th>170</th>
<th>160</th>
<th>150</th>
<th>140</th>
<th>130</th>
<th>120</th>
<th>110W</th>
</tr>
</thead>
<tbody>
<tr>
<td>5S</td>
<td>-1.5</td>
<td>-1.3</td>
<td>-0.1</td>
<td>0.4</td>
<td>0.3</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>10S</td>
<td>-2.2</td>
<td>-2.2</td>
<td>-0.6</td>
<td>0.5</td>
<td>0.8</td>
<td>0.8</td>
<td>0.7</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>15S</td>
<td>-2.0</td>
<td>-2.4</td>
<td>-1.4</td>
<td>0.0</td>
<td>0.7</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Comparing the fields we can see several differences and some similarities, although some displacement exists between the similar features. For instance Table 5.3 shows a maximum in eastward velocity located near $160^\circ W$ and Table 5.4 shows the maximum at $180^\circ W$.

5.3 Meridional Velocity Perturbation.

Table 5.5 shows this field as estimated from equation (5.2). We will not attempt to test this field due to the lack of data. Newell's maps do not permit us estimate values for this field.

5.4. Summary

In this Chapter, we assumed that the fields in Newell et al. (1971) and (1973), although having some inconsistencies, are qualitatively correct. Then, a very crude approximation to the vorticity equation (4.9) have been tested. That approximation is equation (5.1) which has produced fields in the lower atmosphere that have several features similar to those in Newell's, but also notable discrepancies.
FIGURE 5.1  Zonal Velocity Field at 850 mb (after Newell et al. (1971)).
<table>
<thead>
<tr>
<th></th>
<th>140E</th>
<th>150</th>
<th>160</th>
<th>170</th>
<th>180</th>
<th>170</th>
<th>160</th>
<th>150</th>
<th>140</th>
<th>130</th>
<th>120</th>
<th>110W</th>
</tr>
</thead>
<tbody>
<tr>
<td>5S</td>
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<td>2.0</td>
<td>1.6</td>
<td>0.7</td>
<td>-0.5</td>
<td>-1.5</td>
<td>-2.0</td>
<td>-1.9</td>
<td>-1.2</td>
<td>-0.1</td>
<td>1.0</td>
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<tr>
<td>10S</td>
<td>0.6</td>
<td>1.3</td>
<td>1.6</td>
<td>1.4</td>
<td>0.7</td>
<td>-0.2</td>
<td>-1.0</td>
<td>-1.5</td>
<td>-1.5</td>
<td>-1.1</td>
<td>-0.3</td>
<td>0.6</td>
</tr>
<tr>
<td>15S</td>
<td>-0.5</td>
<td>0.2</td>
<td>0.8</td>
<td>1.1</td>
<td>1.1</td>
<td>0.7</td>
<td>0.1</td>
<td>-0.5</td>
<td>-1.0</td>
<td>-1.2</td>
<td>-1.0</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

**Table 5.3**

Zonal Velocity Perturbation Field Estimated from equation (5.5) at 1.5 Km (850 mb) (m/s)

<table>
<thead>
<tr>
<th></th>
<th>140E</th>
<th>150</th>
<th>160</th>
<th>170</th>
<th>180</th>
<th>170</th>
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<th>150</th>
<th>140</th>
<th>130</th>
<th>120</th>
<th>110W</th>
</tr>
</thead>
<tbody>
<tr>
<td>5S</td>
<td>2.1</td>
<td>2.1</td>
<td>-0.4</td>
<td>-1.6</td>
<td>-2.9</td>
<td>-2.1</td>
<td>-1.2</td>
<td>-0.4</td>
<td>0.2</td>
<td>0.8</td>
<td>1.4</td>
<td>2.1</td>
</tr>
<tr>
<td>10S</td>
<td>-1.0</td>
<td>-1.5</td>
<td>-2.0</td>
<td>-1.7</td>
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<td>-1.1</td>
<td>-0.7</td>
<td>-0.1</td>
<td>0.4</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
</tr>
<tr>
<td>15S</td>
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<td>-2.6</td>
<td>-1.8</td>
<td>-0.9</td>
<td>-0.1</td>
<td>0.2</td>
<td>0.4</td>
<td>0.7</td>
<td>1.0</td>
<td>1.3</td>
<td>1.6</td>
<td>1.8</td>
</tr>
</tbody>
</table>

**Table 5.4**

Zonal Velocity Perturbation Field deduced from Newell et al. (1971) at 850 mb (m/s)
Meridional Velocity Perturbation Field as Estimated from Equation (5.2) (m/s)

<table>
<thead>
<tr>
<th></th>
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<th>170</th>
<th>180</th>
<th>170</th>
<th>160</th>
<th>150</th>
<th>140</th>
<th>130</th>
<th>120</th>
<th>110W</th>
</tr>
</thead>
<tbody>
<tr>
<td>5S</td>
<td>0.0</td>
<td>-0.2</td>
<td>-0.3</td>
<td>-0.4</td>
<td>-0.3</td>
<td>-0.1</td>
<td>0.1</td>
<td>0.3</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.0</td>
</tr>
<tr>
<td>10S</td>
<td>0.3</td>
<td>-0.0</td>
<td>-0.3</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.4</td>
<td>-0.1</td>
<td>0.2</td>
<td>0.4</td>
<td>0.5</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>15S</td>
<td>0.5</td>
<td>0.2</td>
<td>-0.1</td>
<td>-0.3</td>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.4</td>
<td>-0.1</td>
<td>0.2</td>
<td>0.5</td>
<td>0.5</td>
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</tr>
</tbody>
</table>
VI. OBSERVATIONS AND CONCLUSIONS.

6.1. Relationship Between Evaporation and Precipitation.

In this Section we will see that the pattern of the surface evaporation field is different from the precipitation pattern, suggesting that at low levels advection plays an important role in transporting moisture from one place to another.

From Schutz and Gates (1972) we can obtain the surface evaporation perturbation field. The results are shown in Table 6.1. The precipitation perturbation field can also be obtained from Schutz and Gates. The results are shown in Table 6.2.

The evaporation field (Table 6.1) shows that in the eastern part the evaporation is larger than in the western. Besides, our results (Table 4.1) as well as Newell's (Figure 4.5) indicate that the vertical motion is downwards in the eastern part so that the large evaporation there may flow westward to the west Pacific where it can take part in large-scale ascent. This hypothesis is supported by the precipitation field (Table 6.2) which shows larger values in the west Pacific. The condensation which drives the vertical circulations can not be due to local excess evaporation accompanying higher sea surface temperatures, because a deficit accompanies the higher SST.
### TABLE 6.1

**Actual Surface Evaporation Perturbation Field from Schutz and Gates (1972)**

( mm / day )

<table>
<thead>
<tr>
<th></th>
<th>140E</th>
<th>150</th>
<th>160</th>
<th>170</th>
<th>180</th>
<th>170</th>
<th>160</th>
<th>150</th>
<th>140</th>
<th>130</th>
<th>120</th>
<th>110W</th>
</tr>
</thead>
<tbody>
<tr>
<td>6S</td>
<td>-1.6</td>
<td>-1.7</td>
<td>-0.0</td>
<td>0.2</td>
<td>0.8</td>
<td>0.8</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9</td>
<td>1.0</td>
<td>1.5</td>
<td>1.1</td>
</tr>
<tr>
<td>10S</td>
<td>-0.2</td>
<td>-0.5</td>
<td>-0.3</td>
<td>-0.2</td>
<td>-0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>14S</td>
<td>-0.1</td>
<td>-0.2</td>
<td>-0.2</td>
<td>0.0</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.5</td>
<td>0.5</td>
<td>0.3</td>
</tr>
</tbody>
</table>

### TABLE 6.2

**Precipitation Perturbation Field from Schutz and Gates (1972)**

( mm / day )

<table>
<thead>
<tr>
<th></th>
<th>140E</th>
<th>150</th>
<th>160</th>
<th>170</th>
<th>180</th>
<th>170</th>
<th>160</th>
<th>150</th>
<th>140</th>
<th>130</th>
<th>120</th>
<th>110W</th>
</tr>
</thead>
<tbody>
<tr>
<td>6S</td>
<td>5.8</td>
<td>1.3</td>
<td>2.4</td>
<td>1.5</td>
<td>1.5</td>
<td>0.6</td>
<td>-0.8</td>
<td>-1.7</td>
<td>-1.9</td>
<td>-2.1</td>
<td>-2.6</td>
<td>-2.8</td>
</tr>
<tr>
<td>10S</td>
<td>-2.7</td>
<td>1.2</td>
<td>2.5</td>
<td>2.5</td>
<td>2.3</td>
<td>1.8</td>
<td>0.6</td>
<td>-1.0</td>
<td>-1.5</td>
<td>-1.8</td>
<td>-2.4</td>
<td>-2.7</td>
</tr>
<tr>
<td>14S</td>
<td>-2.6</td>
<td>-0.6</td>
<td>1.1</td>
<td>1.7</td>
<td>1.7</td>
<td>1.5</td>
<td>1.0</td>
<td>0.2</td>
<td>-0.6</td>
<td>-1.0</td>
<td>-1.6</td>
<td>-2.1</td>
</tr>
</tbody>
</table>
As we will see later in this Section, the lower evaporation in the region of higher precipitation, is due to a reduction in the net heating of the atmosphere by the surface. This reduction is caused by the increased cloudiness accompanying the precipitation which substantially reduces the solar heating at the surface. This is shown in Table 6.3 which gives the surface solar radiation perturbation field deduced from Schutz and Gates (1972). That effect dominates the tendency towards increased heating due to the local enhancement of the sea surface temperature.

In order to lend credence to the hypothesis we stated in the last paragraph, let us estimate the heat balance at the surface. The main components in the heat budget are:

- $S$: flux of solar energy reaching the sea surface
- $F_b$: heat lost by the sea as long wave radiation to the atmosphere and space
- $F_{ho}$: flux of sensible heat into the atmosphere
- $LE_o$: heat lost by evaporation. Eventually the evaporated water will condense in the atmosphere at a certain level over the sea and the latent heat of transformation will heat the atmosphere
- $F_a$: flux of long wave radiation from the atmosphere into the sea.

where $L$ is the latent heat of vaporization and $E_o$ the moistu-
<table>
<thead>
<tr>
<th></th>
<th>140E</th>
<th>150</th>
<th>160</th>
<th>170</th>
<th>180</th>
<th>170</th>
<th>160</th>
<th>150</th>
<th>140</th>
<th>130</th>
<th>120</th>
<th>110°W</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-37.7</td>
<td>-35.7</td>
<td>-28.7</td>
<td>-28.7</td>
<td>-24.7</td>
<td>-8.7</td>
<td>48.3</td>
<td>57.3</td>
<td>49.3</td>
<td>22.3</td>
<td>-0.7</td>
</tr>
<tr>
<td>10°S</td>
<td>25.6</td>
<td>-11.4</td>
<td>-27.4</td>
<td>-31.4</td>
<td>-34.4</td>
<td>-27.4</td>
<td>-14.4</td>
<td>38.6</td>
<td>48.6</td>
<td>22.6</td>
<td>-1.4</td>
<td></td>
</tr>
<tr>
<td>14°S</td>
<td>55.9</td>
<td>10.9</td>
<td>-11.1</td>
<td>-25.1</td>
<td>-28.1</td>
<td>-27.1</td>
<td>-19.1</td>
<td>-10.1</td>
<td>17.9</td>
<td>33.9</td>
<td>22.9</td>
<td>-8.1</td>
</tr>
</tbody>
</table>

**Table 6.3**

Surface Solar Radiation Perturbation Field (S')

*From Schutz and Gates (1971)*

(ly / day)
re flux at the sea surface level

\[ S + F_a - F_b - F_{ho} - LE_o = 0 \]

Assuming a steady state condition, e.g. the temperature of the body of the water is not changing, we have:

\[ F_{ho} + LE_o = S + F_a - F_b \] (6.1)

Following Held and Suarez (1974) we write:

\[ F_b = A_s + B_s T_s \]
\[ F_a = A_s + B_t T_a \] (6.2)

where \( T_s \) and \( T_a \) are the sea-surface and 500 mb air temperatures respectively and \( A_s, B_s, A_t \) and \( B_t \) are constants:

\[ B_s = 9.5 \ \text{ly}/(\text{day} \ ^\circ\text{C}) \] (6.3)
\[ B_t = 11.2 \ \text{ly}/(\text{day} \ ^\circ\text{C}) \]

The solar radiation \( S \) received at the sea surface will be taken from Schutz and Gates (1972).
From equations (6.1) and (6.2) we can write:

\[ F_{ho} + LE_0 = S + A \frac{\partial}{\partial t} + B \frac{\partial}{\partial t} T_a - A_s - B_s T_s \]  

(6.4)

For zonal average conditions we have:

\[ [F_{ho} + LE_0]_{AV} = [S] + A \frac{\partial}{\partial t} + B \frac{\partial}{\partial t} [T_a] - A_s - B_s [T_s] \]  

(6.5)

Then the perturbation balance is:

\[ (F_{ho} + LE'_0) = S' + B \frac{\partial}{\partial t} T'_a - B_s T'_s \]  

(6.6)

where:

\[ (F_{ho} + LE'_0) = (F_{ho} + LE_0) - [F_{ho} + LE_0]_{AV} \]

\[ S' = S - [S] \]  

(6.7)

\[ T'_a = T_a - [T_a] \]

\[ T'_s = T_s - [T_s] \]

are the perturbations of sensible plus latent heat fluxes, of surface solar radiation, of 500 mb temperature and of sea surface respectively.

Equation (6.6) gives the heating of the atmosphere by energy flow from the surface in terms of solar radiation, and sea surface and 500 mb temperatures. The surface solar radiation perturbation field \((S')\) has already been obtained from Schutz and Gates (1972). The sea surface temperature
perturbation field \( (T_s') \) can also be obtained from Schutz and Gates and the 500 mb temperature perturbation field can be deduced from data in Talijard et al. (1969). Besides, according to Roll (1965), Budyko (1974), the flux of sensible heat \( F_{ho} \) is much less than the latent heat flux \( LE_o \). The relative contributions of the different terms in the right hand side of equation (6.6) are given in Tables 6.3, 6.4 and 6.5. Table 6.3 will be written again for convenience. Values corresponding to \( 10^0 \)S are shown in Figure 6.1.

Before we draw some conclusions from equation (6.6), let us test it. Neglecting the flux of sensible heat \( (F_{ho}) \) in the left hand side we can calculate the evaporation field \( (E_o') \). We take the latent heat of vaporization \( (L) \) as 590 cal/grm. The results are shown in Table 6.6.

Notice that the surface evaporation field in Table 6.6 is very similar to the same field taken from Schutz and Gates (1972) shown in Table 6.1 as it should be because both fields have been calculated from the same equation, namely the heat balance at the surface.

The following conclusions can be obtained from equation (6.6) neglecting the flux of sensible flux, and Figure 6.1 or Tables 6.3, 6.4 and 6.5:

1. The signals corresponding to the sea-surface and 500 mb temperature contributions are of similar amplitude and out of phase so that their combined contribution tends to be
### TABLE 6.3

Surface Solar Radiation Perturbation Field ($S'$) (from Schutz and Gates (1972))

(ly / day)

<table>
<thead>
<tr>
<th></th>
<th>140°E</th>
<th>150</th>
<th>160</th>
<th>170</th>
<th>180</th>
<th>170</th>
<th>160</th>
<th>150</th>
<th>140</th>
<th>130</th>
<th>120</th>
<th>110°W</th>
</tr>
</thead>
<tbody>
<tr>
<td>6S</td>
<td>-16.7</td>
<td>-37.7</td>
<td>-25.7</td>
<td>-28.7</td>
<td>-24.7</td>
<td>-8.7</td>
<td>48.3</td>
<td>57.3</td>
<td>49.3</td>
<td>22.3</td>
<td>-0.7</td>
<td></td>
</tr>
<tr>
<td>10S</td>
<td>25.6</td>
<td>-11.4</td>
<td>-27.4</td>
<td>-31.4</td>
<td>-34.4</td>
<td>-14.4</td>
<td>18.6</td>
<td>38.6</td>
<td>48.6</td>
<td>22.6</td>
<td>-1.4</td>
<td></td>
</tr>
<tr>
<td>14S</td>
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<td>10.9</td>
<td>-11.1</td>
<td>-25.1</td>
<td>-28.1</td>
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<td>-8.1</td>
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### TABLE 6.5

$B_s T'_s$ (ly / day) (see Equation 6.6)

<table>
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<th>170</th>
<th>180</th>
<th>170</th>
<th>160</th>
<th>150</th>
<th>140</th>
<th>130</th>
<th>120</th>
<th>110°W</th>
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<td>11.4</td>
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<td>5.7</td>
<td>3.8</td>
<td>-2.8</td>
<td>-5.7</td>
<td>-15.2</td>
<td>-24.7</td>
</tr>
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<td>-6.9</td>
<td>0.7</td>
<td>5.4</td>
<td>7.3</td>
<td>11.1</td>
<td>9.2</td>
<td>8.3</td>
<td>1.6</td>
<td>0.7</td>
<td>-11.7</td>
<td>-19.3</td>
</tr>
<tr>
<td>14S</td>
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<td>-12.3</td>
<td>-3.8</td>
<td>3.8</td>
<td>3.8</td>
<td>11.4</td>
<td>14.2</td>
<td>12.3</td>
<td>7.6</td>
<td>1.9</td>
<td>-8.5</td>
<td>-16.1</td>
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</tbody>
</table>
### TABLE 6.4

<table>
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<th>B</th>
<th>T, (ly/day)</th>
<th>(see Equation 6.6)</th>
</tr>
</thead>
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<td>140E</td>
<td>150 160 170</td>
<td>180 170 160 150 140</td>
</tr>
<tr>
<td>6S</td>
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<td>-3.4 -9.0 -15.7</td>
</tr>
<tr>
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<td>-1.1 -9.0 -17.9 -25.8</td>
</tr>
<tr>
<td>14S</td>
<td>2.2 4.5 9.0 13.4 14.6 13.4 11.1 3.4</td>
<td>-5.6 -15.7 -26.9 -34.7</td>
</tr>
</tbody>
</table>

### TABLE 6.6

Surface Evaporation Perturbation Field Obtained from Equation (6.6) (mm/day)

<table>
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<th>B</th>
<th>T, (mm/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>140E</td>
<td>150 160 170</td>
</tr>
<tr>
<td>6S</td>
<td>-0.2 -0.8 -0.8 -0.6 -0.5 -0.4 -0.1 0.9 1.0</td>
</tr>
<tr>
<td>10S</td>
<td>0.6 -0.1 -0.5 -0.4 -0.5 -0.4 -0.2 0.3 0.6</td>
</tr>
<tr>
<td>14S</td>
<td>1.3 0.5 0.0 -0.3 -0.3 -0.4 -0.4 -0.3 0.1</td>
</tr>
</tbody>
</table>
FIGURE 6.1 Magnitudes of the terms in the right hand side of equation (6.6) at 10°S.
small

2. Then, the form of the evaporation perturbation field \( E' \) is given mainly by the surface solar radiation perturbation field \( S' \). The influence of the sea surface temperature \( (T'_s) \) on the evaporation is dominated by the solar radiation effect.

Thus, as we said before in this Section, the lower evaporation in the region of higher precipitation (western part) is due to a reduction in the net heating of the atmosphere by the surface. This reduction is caused by the increased cloudiness accompanying the precipitation which substantially reduces the solar heating at the surface as Figure 6.1 or Table 6.3 shows.


In this Section we will deduce the analytic expressions for the transport of moisture by the zonal and meridional wind in order to test further the relationship between evaporation and precipitation suggested in Section 6.1.

Consider an air volume as indicated in Figure 6.2. Let \( u'_E \) and \( u'_W \) be the zonal velocities at the eastern and western boundaries respectively and \( v'_N \) and \( v'_S \) the meridional velocities at the northern and southern boundaries respectively. Let us approximate that these velocities are constants. Later we will discuss the reasons for this approximation. Let
\( r_E, r_w, r_N \) and \( r_S \) the corresponding air water content in grams of water per \( \text{cm}^3 \). \( q_E, q_w, q_N \) and \( q_S \) will be the specific humidity in grams of water per kilogram of air near the respective boundaries.

FIGURE 6.2 Moisture Transports (see text).

The input of moisture by the zonal wind at the eastern boundary (\( R_E \)) is given by:

\[
R_E = \int_0^H r_E(z) u_E \, \text{d}z
\]  \hspace{1cm} (6.8)

The output of moisture by the zonal wind at the western boundary (\( R_w \)) is given by:

\[
R_w = \int_0^H r_w(z) u_w \, \text{d}z
\]  \hspace{1cm} (6.9)

The convergence of moisture is given by:

\[
C_M = R_E - R_w
\]
\[ C_M = \rho_m \left[ u_E \int_0^H r_E(z) \, dz - u_W \int_0^H r_W(z) \, dz \right] \]  \hspace{1cm} (6.10)

What we can get easily are not the \( r \)'s but the \( q \)'s. Both are related as follows:

\[ r = \rho_m q \]  \hspace{1cm} (6.11)

where \( \rho_m \) is the air density in Kg/cm\(^3\).

Substitution of (6.11) in (6.10) yields:

\[ C_M = \rho_m \left[ u_E \int_0^H q_E(z) \, dz - u_W \int_0^H q_W(z) \, dz \right] \]  \hspace{1cm} (6.12)

In the next Section we will show that approximately:

\[ q(z) = q_0 \exp \left( - \frac{z}{h} \right) \]  \hspace{1cm} (6.13)

where \( h \) is a height scale which will be estimated in the next Section and \( q_0 \) is the specific humidity at the surface.

Substituting (6.13) in (6.12) and letting \( H \to \infty \) we obtain:

\[ C_M = \frac{\rho_m}{\rho_w} \left[ u_E q_{0E} h_E - u_W q_{0W} h_W \right] \]  \hspace{1cm} (6.14)

where \( h_E \) and \( h_W \) are the scale heights at the boundaries.

If all this moisture falls as rain the precipitation would be:

\[ P_M = \frac{C_M}{\rho_w} \frac{1}{L} = \frac{\rho_m}{\rho_w} \left[ u_E q_{0E} h_E - u_W q_{0W} h_W \right] \]  \hspace{1cm} (6.15)
where \( \rho_w \) is the water density.

In a similar way the precipitation due to the convergence by the meridional wind is given by:

\[
P_{M_\gamma} = \frac{\rho_m}{\rho_w 1} [v_S q_{0S} h_S - v_N q_{0N} h_N]
\]  

(6.16)

where \( q_{0S} \) and \( q_{0N} \) are the surface specific humidities at the southern and northern boundaries respectively.

If the convergence in (6.14) is negative, then that divergence must be compensated by evaporation from the surface.

6.3. Height Scale of Water Vapor in the Atmosphere.

In equation (6.13) the height scale \( h \) of the specific humidity is needed. We are going to approximate that \( h \) is the same as the scale height of the saturated water vapor content of the atmosphere, i.e. the specific humidity is not much less than the corresponding to the saturation.

The Clausius-Clapeyron equation gives the latent heat \( L_{12} \) necessary to change reversibly one unit mass of substance from phase 1 at temperature \( T \) and pressure \( e_s \) to phase 2 at the same value of \( T \) and \( e_s \). Mathematically:

\[
L_{12} = T(v_2 - v_1) \frac{de_s}{dT}
\]

where \( v_1 \) and \( v_2 \) are the specific volumes of phase 1 and 2
respectively.

For the liquid-vapor transition, the Clausius-Clapeyron equation is usually approximated by ignoring the specific volume of the liquid phase \( v_1 \) and using the perfect gas law for \( v_2 \). Thus:

\[
\frac{d e_s}{dT} = \frac{e_s L_v}{R_v T^2}
\]

Assuming that \( L_v \) is constant we can integrate the Clausius-Clapeyron equation:

\[
e_s = e_o \exp \left[ -\frac{L_v}{R_v} \left( \frac{1}{T} - \frac{1}{T_0} \right) \right]
\]

where \( T_0 \) is the triple point temperature (273 °K) and \( e_o \) the corresponding pressure. \( L_v \) and \( R_v \) are the latent heat of vaporization and gas constant for water vapor respectively.

The relationship between \( e_s \) and the corresponding specific humidity \( q_s \) is:

\[
e_s = \frac{R_v}{R} p_d q_s
\]

where \( p_d \) and \( R \) are the dry air pressure and gas constant for air respectively.

Combining all these equations we get:
\[ q \leq q_s = \frac{R}{R_v} \frac{e_0}{p_d} \exp \left[ -\frac{L_v}{R_v \frac{1}{T} - \frac{1}{T_o}} \right] \]

The following values are given in standard Tables:

\[ \xi = \frac{R}{R_v} = 0.621 \]

\[ e_0 = 6.11 \text{ mb} \]

\[ T_o = 273 \, ^\circ\text{K} \]

\[ L_v = 586 \text{ cal./gr.} = 2.501 \times 10^6 \text{ kj ton}^{-1} \]

\[ R_v = 461.5 \text{ kj ton}^{-1} \, ^\circ\text{K}^{-1} \]

then:

\[ q_s = \xi \frac{e_0}{p_d} \exp \left[ -5420 \, ^\circ\text{K} \left( \frac{1}{T} - \frac{1}{T_o} \right) \right] \]

Let us define \( T = T_o + T' \)

where \( T' \leq T_o \)

then:

\[ q_s = \xi \frac{e_0}{p_d} \exp \left[ \frac{5420 \, ^\circ\text{K}}{T_o^2} T' \right] \]
or:

\[ q_s = \varepsilon \frac{e_o}{p_d} \exp \left[ 0.073 T' \right] \]

The longitudinal averages of temperature can be found in Newell et al. (1971). For these temperatures we can calculate the saturation specific humidity \( q_s \). The results at \( 10^\circ S \) are:

<table>
<thead>
<tr>
<th>( p ) (mb)</th>
<th>( T' ) ((^{\circ}K))</th>
<th>( q_s ) (gr./kr.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>26.1</td>
<td>25.5</td>
</tr>
<tr>
<td>950</td>
<td>22.8</td>
<td>21.1</td>
</tr>
<tr>
<td>900</td>
<td>19.6</td>
<td>17.6</td>
</tr>
<tr>
<td>850</td>
<td>16.9</td>
<td>15.3</td>
</tr>
<tr>
<td>700</td>
<td>8.6</td>
<td>10.2</td>
</tr>
<tr>
<td>500</td>
<td>-6.3</td>
<td>4.8</td>
</tr>
</tbody>
</table>

From these values we can say that the scale height for the saturated water vapor pressure at this latitude is about 3 Km. or 300 mb. Then we take \( h = 3 \) Km. in equation (6.13). Besides, the approximations we have made in this Section permit us to take: \( h_B = h_w = h_N = h_S = h \).

In Section 6.2 we considered the horizontal velocities as constants in height in calculating the moisture transports. That approximation is justified taking into account that under the approximations considered in the present Section most of the moisture is at low heights.
6.4. **Moisture Transports. Results.**

In this Section, we shall calculate the convergence (divergence) of moisture in different parts of the studied area and compare the results with the precipitation (evaporation) values.

As we discussed in the last Section, the horizontal velocity we will use in calculating these transports shall be taken from a characteristic low height. In the present calculations we take the velocities at the 850 mb level.

The horizontal velocity we will consider is that we obtained in Chapter V. In that Chapter we only obtained the perturbation fields while in this Chapter we need total fields. Due to the lack of better data we will take the zonal average of the zonal velocity \([u]\) as given by Newell et al. (1971) and which is show in Figure 4.2. Then at 850 mb \((z \sim 1.5 \text{ Km})\) and 10°S, we have:

\[ [u] \approx -4.5 \text{ m/s} \]

The zonal average of the meridional velocity is more difficult to estimate. However, data in Newell et al. (1971) suggests that the value at 850 mb is near to zero. Now, as the perturbation field values are also near to zero (see Table 5.5), the error in the estimation of the total meridional velocity field is very high. Thus, no calculation will be made with this velocity component.
In addition, we will approximate that the air is saturated at the surface so that a knowledge of its temperature means knowledge of its specific humidity.

The first air volume we will considered is that with the following boundaries: 5°N, 15°S, 140°W and 160°E. The following values can be found:

<table>
<thead>
<tr>
<th></th>
<th>E (140°W)</th>
<th>W (160°E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>5.7 m/s</td>
<td>3.2 m/s (easterlies)</td>
</tr>
<tr>
<td>T</td>
<td>26.7°C</td>
<td>27.0°C</td>
</tr>
<tr>
<td>q_o</td>
<td>22.7 gr/Kg</td>
<td>23.1 gr/Kg</td>
</tr>
<tr>
<td>h</td>
<td>3 Km</td>
<td>3 Km</td>
</tr>
</tbody>
</table>

There is convergence in this area. According to (6.15) the precipitation we would obtain amounts to:

$$P_{MX} = 2.5 \text{ mm/day} \quad (6.17)$$

From Schutz and Gates (1972) we can obtain an average value for the precipitation in that area. The result is:

$$P = 3.85 \text{ mm/day} \quad (6.18)$$

Then, the zonal convergence of moisture is comparable to the observed precipitation. The meridional transport may also contribute.

Now, let us consider the area between 110°W and 140°W. The following values can be found:
E (110°W)  W (140°W)

u  3.9 m / s  5.7 m / s (easterlies)
T  24.0°C  26.7°C
g_0  19.2 gr / Kg  22.7
h  3 Km  3 Km

Divergence is found in this area. Its value, according to (6.15), is:

\[ P_{Mx} = -4.8 \text{ mm / day} \]

The average evaporation from the surface in the area can be found from Schutz and Gates (1972). We find:

\[ E = 5.55 \text{ mm / day} \]

Thus, the surface evaporation can compensate the zonal divergence of moisture in this area.
CONCLUSIONS

1. The problems we found in Chapter IV in estimating the zonal average fields which are needed to calculate the perturbation fields show that more information is needed in the studied area to obtain reliable fields for use in quantitative studies.

As the studied area is essentially of oceanic nature, that information is difficult to obtain. However, Meteorological Satellites data may be helpful in the deduction of air temperatures (Staelin et al. (1969)) and horizontal velocities (Ramage (1975)).

The same is true for the determination of the distribution of the precipitation, which is the main drive for the zonal circulations.

2. We have checked a position that the balance assumed by Newell et al. in the thermodynamic equation holds. The resulting approximation equation, (3.31) permits us estimate the vertical velocity (see Table 4.1).

3. An essentially geostrophic balance has been obtained for the horizontal momentum equations. However, the term \([u_y v']\) may be important in the \(u\) - momentum equation. That decision depends on a more reliable determination of the zonal average fields (see 1 of these conclusions).
4. With respect to the vorticity equation, it seems that the $y$-dependence is important (see Section 4.2). A finer conclusion will again depend on a more reliable estimate of the average fields. However, fields similar to those in Newell et al. (1971) and (1973), which we assume are qualitatively correct, were obtained under the approximation (5.1).

5. As we saw in Section 4.4, the main contribution to the diabatic heating source comes from the latent heat release. The horizontal distribution of the latent heat release perturbation function $Q_L^1$ was estimated from the climatological rainfall data and the vertical distribution, from numerical simulations (see Chapter II).

The condensation which is the primary drive for the vertical circulations can not be due to local excess evaporation because the area of lower evaporation, e.g. the western part, is that of higher precipitation. (see Tables 6.1 and 6.2). The lower evaporation in the region of higher precipitation is due to a reduction in the net heating of the atmosphere by the surface caused by the increased cloudiness accompanying the precipitation which substantially reduces the solar heating at the surface. This effect dominates the tendency towards increased heating due to the local enhancement of the sea surface tem-
perature (see Section 6.1). As the eastern area has relatively higher evaporation, lower precipitation and downward motion (see Section 4.4), the excess evaporation there has to flow westward to the west Pacific in order to take part in large-scale ascent. Estimates of the zonal convergence support this affirmation (see Section 6.4). Eventually the moisture will condense forming clouds and releasing latent heat. This physical picture points out the fact that once the Walker Circulation has started, the sea-surface temperature plays a secondary role. The tendency towards increased heating due to the local enhancement of the sea surface temperature is dominated by the heating (cooling) of the atmosphere by the surface due to short wave radiation (see Section 6.1). This statement is different from Bjerknes's hypothesis (see Chapter I) who postulated that the heating (cooling) of the atmosphere by the warm (cool) sea surface is the driving force of the circulation.

6. The sea-surface temperature perturbation field as taken from Schutz and Gates (1972) is shown in Table 6.7. This pattern may be important in initiating the circulation according to the following picture.

The excess evaporation which would accompany higher sea surface temperatures initially would produce local
### TABLE 6.7

**Sea Surface Temperature Perturbation Field (°C)**

*(From Schutz and Gates (1972))*

<table>
<thead>
<tr>
<th></th>
<th>140°E</th>
<th>150</th>
<th>160</th>
<th>170</th>
<th>180</th>
<th>170</th>
<th>160</th>
<th>150</th>
<th>140</th>
<th>130</th>
<th>120</th>
<th>110°W</th>
</tr>
</thead>
<tbody>
<tr>
<td>6S</td>
<td>-1.1</td>
<td>0.1</td>
<td>0.7</td>
<td>1.2</td>
<td>1.2</td>
<td>0.9</td>
<td>0.6</td>
<td>0.4</td>
<td>-0.3</td>
<td>-0.6</td>
<td>-1.6</td>
<td>-2.6</td>
</tr>
<tr>
<td>10S</td>
<td>-1.9</td>
<td>-0.7</td>
<td>0.1</td>
<td>0.6</td>
<td>0.8</td>
<td>1.2</td>
<td>1.0</td>
<td>0.9</td>
<td>0.2</td>
<td>0.1</td>
<td>-1.2</td>
<td>-2.0</td>
</tr>
<tr>
<td>14S</td>
<td>-2.2</td>
<td>-1.3</td>
<td>-0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>1.2</td>
<td>1.5</td>
<td>1.3</td>
<td>0.8</td>
<td>0.2</td>
<td>-0.9</td>
<td>-1.7</td>
</tr>
</tbody>
</table>
excess condensation, precipitation and cloudiness. The latent heat release would drive a vertical circulation similar to that discussed in paragraph 5 of these conclusions, with relatively upward motion in the center and downward motion near the boundaries. The higher precipitation and increased cloudiness in the center would reduce evaporation there. The excess evaporation near the eastern boundary would be transported to the west by the zonal wind and would take part in large-scale ascent. Again condensation would take place and so on.

7. The scaling of the continuity equation (3.25) or the estimates of its terms (3.4), indicate that the Walker Circulation is not a planar one in the longitude-height plane but a three dimensional Circulation where variations along the longitudinal, latitudinal and vertical directions are important.

8. As we saw in Chapter I, Rowntree (1972) has used a version of the nine-level hemispheric model of the atmosphere developed at the Geophysical Fluid Dynamics Laboratory described by Miyakoda et al. (1969), to test Bjerknes' hypothesis that fluctuations of ocean temperatures in the tropical east Pacific are responsible for major variations in the atmosphere both locally and in middle latitudes. In
the model, shortwave and longwave radiation is calculated using climatological cloud coverage which is only a function of latitude and height (see Appendix I of Miyakoda et al. (1969)). In light of our result that zonal dependence of cloudiness and short wave radiation are essential features of the Walker Circulation, Rowntree's model has a serious limitation for simulating the atmospheric response to sea surface temperature anomalies in tropical latitudes.
REFERENCES


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