MODELING ASSUMPTIONS AS A SOURCE OF ERROR IN A DYNAMICAL
1000-mb PREDICTION METHOD

by

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Signature of Co-author .............................................

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Department of Meteorology, August 1962

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Thesis Supervisor

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Chairman, Departmental Committee on Graduate Students
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Lynn Lawrence LeBlanc

Submitted to the Department of Meteorology on August 20, 1962 in partial fulfillment of the requirement for the Degree of Master of Science

ABSTRACT

During a cold outbreak into the Great Plains of the United States, 1000-mb forecasts made using a two-parameter baroclinic model showed significant error. Experience has shown that the model has poorly handled this synoptic situation in the past.

An investigation was carried out to determine the extent to which certain modeling assumptions contributed to the forecast errors.

It was found that the major source of error was contained in the temperature advection term. Further investigation revealed that the difficulty was due mainly to the assumption that the isotherms in the layer from 1000mb to 500mb do not change their orientation with height although a small part of the error could be attributed to the geostrophic assumption.

It was found that improvement could be made by dividing the layer from 1000mb to 500mb into three layers for purposes of measuring the temperature advection. This implies that 1000-mb forecasts made with a four-parameter model should show definite improvement over those made with a two-parameter model.

Thesis Supervisor: Frederick Sanders
Title: Associate Professor of Meteorology
ACKNOWLEDGEMENT

The authors are indebted to Professor Frederick Sanders for his advice and encouragement throughout the course of this study.

Acknowledgement is made to Miss Isabel Kole for the drafting of some of the maps, to Mr. Dick Royal and Miss Ann Corrigan for plotting the necessary data, and to Mrs. Jane McNabb for typing the thesis.
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A. INTRODUCTION

Reed (1957) and Estoque (1957) developed dynamic forecasting procedures for forecasting the 1000-mb level using a "two-parameter" baroclinic model. Estoque (1957) amended the basic model by including the effect of orography on vertical motion.

Although their techniques have proven to be of considerable value in making 1000-mb forecasts, several systematic errors have been noted by meteorologists who have used their model as a forecasting tool.

Before any improvement can be made in a forecasting technique, the reasons for its failure must be known. This study is an effort to determine the extent to which the basic modeling assumptions contribute to errors in the 1000-mb height forecasts.

The synoptic situation dealt with here is one which has given this model trouble in the past, namely a cold outbreak from the north into the central part of the United States. Rogers (1958) and Pappas (1961) previously conducted studies which led to a plausible explanation of some of the observed errors in the 1000-mb pattern associated with this type of synoptic situation. The investigation carried out in this paper will deal with sources of error which were discussed either not at all or only briefly in the two papers noted above.
The basic theory of the model is contained in papers by Estoque (1957) and Reed (1957). The particular form of the derivation here presented is due to Sanders (1959) and is reproduced for ease of reference and also so that all modeling assumptions will be clearly brought out.

We began by writing the simplified vorticity equation for the 1000-mb level:

\[ \frac{\partial \zeta_0}{\partial t} = -V_o \cdot \nabla (\zeta_o + f) + f \left( \frac{\partial \omega}{\partial p} \right)_0 \]

\( \zeta_0 \equiv \) relative vorticity at 1000 mb

\( V_o \equiv \) 1000-mb vector wind

\( f \equiv \) Coriolis parameter = \( 2\Omega \sin \theta \) (\( \theta \) = latitude, \( \Omega \) = angular velocity of earth's rotation)

\( \omega \equiv \) vertical velocity in \((x,y,p,t)\) coordinate system = \( \frac{\partial p}{\partial t} \)

\( p \equiv \) pressure

\( \nabla \equiv \) horizontal del operator = \( \partial/\partial x + \partial/\partial y \)

The subscript zero refers to the 1000-mb level.
In writing the vorticity equation in this form, we neglect \( \omega(\partial \zeta_o/\partial p), \zeta_o \nabla \cdot V \), the twisting terms, and the friction term. In theory this restricts the application of the equation to systems with wavelengths of the order of 2000 nautical miles or more. Use has also been made of the continuity equation, \( \nabla \cdot V = -(\partial \omega/\partial p) \).

Next assume as the fundamental modeling approximation that

\[
V(x,y,p,t) = V_{m}^{nd}(x,y,t) + B(p) V_{m,o}^{nd}(x,y,t) + V_{m}^{ir}(x,y,p,t)
\]

\[
V_{m,o}^{nd} \equiv V_{m}^{nd} - V_{o}^{nd} \quad (V_{m}^{nd} \text{ refers to the non-divergent part of the wind})
\]

\( V_{m} \equiv \) vector wind at level where \( B = 0 \) (the "mean" level).

\( B \equiv \) a scalar coefficient which is a function of pressure only.

\( V_{m}^{ir} \equiv \) irrotational part of the wind.

It follows from (2) that

\[
\zeta = \zeta_{m} + B \zeta_{m,o}
\]

Substituting from (2) and (3) into the simplified form of the vorticity equation gives

\[
\frac{\partial}{\partial t} (\zeta_{m} + B \zeta_{m,o}) = -(V_{m}^{nd} + B V_{m,o}^{nd} + V_{m}^{ir}) \cdot \nabla (\zeta_{m} + B \psi_{m,o} + f) + f(\partial \omega/\partial p)
\]
Averaging (4) with respect to pressure gives

\[ \frac{\partial \zeta^m}{\partial t} = - \nabla \cdot \nabla (\zeta^m + f) - (B^2) \nabla \zeta^m_0 \cdot \nabla \zeta^m_0 + f \frac{\omega_G}{p_G} \]

where the subscript \( G \) refers to the ground level.

In developing equation (5), the terms involving advection by \( V^m \) have been neglected and \( \omega \) at \( p = 0 \) has been set equal to zero.

In applying this model to the atmosphere, we are in effect assuming that the hodograph is a straight line, i.e. the direction of the vertical shear of \( V^m \) is the same at all pressure levels in a given air column. The magnitude of this shear varies in the same way at all times and in all air columns. The variation is given by the variation in \( B \). The values of \( B \) to be used will be discussed later.

Expanding the total time derivative of potential temperature, \( d\theta/dt \), and dividing the result by \( \theta \), we obtain

\[ \frac{1}{\theta} \frac{d\theta}{dt} = \frac{1}{\theta} \frac{\partial \theta}{\partial t} + \frac{1}{\theta} \nabla \cdot \nabla \theta + \frac{1}{\theta} \omega \frac{\partial \theta}{\partial p} \]

\[ \theta \equiv \text{potential temperature} = \frac{a}{R} \frac{1000^k}{p^{k-1}} \]

\( a \equiv \text{specific volume} \)

\( R \equiv \text{gas constant for dry air} \)

\( k \equiv \frac{R}{p} \)
\( p = \text{pressure in millibars} \)

(8) \( \ln \theta = \ln \left( \frac{\alpha}{R} \frac{1000}{p} \left( \frac{R}{k-1} \right) \right) \)

(9) \( \frac{1}{\theta} \frac{\partial \theta}{\partial t} = \frac{1}{a} \frac{\partial a}{\partial t} ; \quad \frac{1}{\theta} \nabla \theta = \frac{1}{a} \nabla a \)

Substitute (9) into (6), multiply through by \( a \) giving

(10) \( \frac{a}{\theta} \frac{d \theta}{dt} = \frac{\partial a}{\partial t} + V \cdot \nabla a + \frac{a}{\theta} \frac{\partial \theta}{\partial p} \omega \)

Substitute the hydrostatic relation,

(11) \( \alpha = -g \frac{\partial Z}{\partial p} \) (\( Z \) = height of constant-pressure surface)

into (10) yielding

(12) \( \frac{\partial}{\partial t} \left( \frac{\partial Z}{\partial p} \right) = -V \cdot \nabla \left( \frac{\partial Z}{\partial p} \right) + \frac{\alpha}{g \theta} \frac{\partial \theta}{\partial p} \omega - \frac{a}{g \theta} \frac{d \theta}{dt} \)

Equation (12) expresses the local rate of change of thickness in terms of thickness advection, adiabatic heating and diabatic heating.

This equation is now integrated with respect to pressure between the 1000-mb level and the mean level \((p_m)\). The result is:

(13) \( \frac{\partial Z_{m,o}}{\partial t} = -V_o \cdot \nabla Z_{m,o} + \int_{1000}^{p_m} \frac{\alpha}{g \theta} \frac{\partial \theta}{\partial p} \omega dp - \int_{1000}^{p_m} \frac{\alpha}{g \theta} \frac{d \theta}{dt} dp \)
The second term on the right hand side of (13) denotes the change of thickness due to adiabatic heating. The third term represents the change of thickness due to diabatic heating and will hereafter be referred to as $H$. In arriving at (13), $V^r$ is neglected and it is assumed that $V_{\text{nd}}^d \cdot \nabla (\delta Z/\delta p) = 0$ (a mild form of the thermal wind approximation) so that

$$V \cdot \nabla (\delta Z/\delta p) = V_m \cdot \nabla (\delta Z/\delta p) = V_o \cdot \nabla (\delta Z/\delta p).$$

We next assume that

$$Z_{m,o} = \frac{f}{g} (\psi_m - \psi_o)$$

where $\psi$ is the stream function.

This is a type of geostrophic assumption which implies $V_{\text{nd}}^d$ is equal to the thermal wind for the layer (variations of $f/g$ are neglected).

Since $(\partial \omega / \partial p)_o$ and $\omega$ appear as variables in the equations presented so far, some method must be obtained to relate $\omega$ and $p$. The method given by Sanders (1959) allows $\omega$ to be a maximum in the middle troposphere, and to be relatively small in the stratosphere and near the earth's surface.

First assume as a modeling approximation that

$$V^{ir} (x,y,p,t) = V^{ir}_m (x,y,t) + B(p) V^{ir}_{m,o} (x,y,t)$$
where the variables have the same meaning as previously used. It
then follows that

(17) \( \nabla \cdot V = \nabla \cdot V_m + B \nabla \cdot V_{m,o} \)

From the continuity equation

(18) \( \nabla \cdot V = -\frac{\partial \omega}{\partial p} \); \( \omega = -\int_0^p \nabla \cdot V \, dp \)

Making use of (17), (18) and the fact that \( B_o = -1; \ (m) = \frac{1}{p_G} \int_0^{P_G} (\ ) \, dp \)
we obtain

(19) \( \left( \frac{\partial \omega}{\partial p} \right)_o = \frac{\omega}{p_G} + \nabla \cdot V_{m,o} \)

and also

(20) \( \omega(p) = \frac{p}{p_G} \omega_G - C(p) \nabla \cdot V_{m,o} \)

\( C(p) \equiv \int_0^p B \, dp \)

Now substituting (20) and (15) into (13) yields

(21) \( \frac{f}{g} \frac{\partial \psi_{m,o}}{\partial t} = -\frac{f}{g} \nabla \phi \cdot \nabla \psi_{m,o} + \frac{1}{p_G} E \omega_G - D \nabla \cdot V_{m,o} + H \)

\( D \equiv \int_0^{P_m} \frac{Ca}{g^2} (\partial \phi/\partial p) \, dp \)

\( E \equiv \int_0^{P_m} \frac{pq}{g^2} (\partial \phi/\partial p) \, dp \)
D and E are parameters which ultimately will be arrived at from mean data, as will \( B(p) \) and \( C(p) \).

Substitution of (19) into (1) yields

\[
\frac{3}{\partial t} (\nabla^2 \psi_o) = -V_o \cdot \nabla (\nabla^2 \psi_o + f) + \frac{f}{P_G} \omega_G + f \cdot V_{m,o} \nabla V_{m,o}
\]

where use has been made of the definition of \( \zeta \) and \( \psi \) to arrive at \( \zeta = \nabla^2 \psi \).

Equations (21) and (22) can be combined to give

\[
\frac{3}{\partial t} (\nabla^2 \psi_o + \frac{f^2}{gD} \psi_{m,o}) = -V_o \cdot \nabla (\nabla^2 \psi_o + f + \frac{f}{gD} \psi_{m,o})
+ \left( \frac{fE}{P} \frac{f}{P_G} \right) \omega_G + \frac{f}{D} H
\]

In (23) we have assumed \( \frac{f^2}{gD} = \text{constant} \).

This is the prediction equation for the 1000-mb surface and will yield a forecast for \( \psi_o \) given \( \psi_m, V_o, D, E, \omega_G, P_G \) and \( H \). The remainder of this section will deal with the method used to evaluate these variables so that an equation in a single unknown can be obtained.

The next assumption is that the diabatic heating term involving \( H \) can be neglected. This assumption is probably not always valid, but no way has been found to handle it conveniently.

The approximation that \( \dot{V} = V_m + B \cdot V_{m,o} \) allows \( \omega_g \) to be expressed as

\[
\omega_g = V_G \cdot \nabla P_G = (V_m + B \cdot V_{m,o}) \cdot \nabla P_G
\]
We are assuming that $\omega_G$ is produced solely by an orographic effect, which is a reasonable assumption.

Although $(V_m + B V_{m,o}) \cdot \nabla P_G$ can be substituted for $\omega_G$ in equation (23), the simpler assumption is often made that

$$\omega_G = V_o \cdot \nabla P_G.$$  

Equation (23) may now be written as

$$\frac{\partial}{\partial t} \left( \frac{g}{f} \psi_o + \frac{f^2}{gD} \psi_{m,o} \right) = -V_o \cdot \nabla \left[ \frac{g}{f} \psi_o \right] + f + \frac{f^2}{gD} \psi_{m,o} - f \left( \frac{E}{D} + 1 \right) \ln P_G$$

At this point the geostrophic assumption $\nabla^2 \psi = \frac{f}{\Delta} \nabla^2 Z$ is made so that (26) can be written as

$$\frac{\partial}{\partial t} \left( \frac{g}{f} \nabla^2 Z + \frac{f}{D} Z_{m,o} \right) = -V_o^G \cdot \nabla \left[ \frac{g}{f} \nabla^2 Z_o + f + \frac{f}{D} Z_{m,o} - f \left( \frac{E}{D} + 1 \right) \ln P_G \right]$$

(Superscript $g$ on $V_o$ refers to a geostrophic wind).

Since the method of integration to be used is a graphical finite-difference method, $\nabla^2 Z$ is approximated by \[ \frac{1}{(A/m)^2} \Delta(\tilde{Z}_o - Z_o) \] where $\Delta$ is one-half the grid distance used, $m$ is the map scale factor and $\tilde{Z}_o$ is the space mean of $Z_o$. For a rectangular grid of spacing $2A$, $\tilde{Z}_o$ at a grid point is found by computing the average of the values of $Z_o$ at the four surrounding points. In finite difference notation (27) becomes

$$\frac{\partial}{\partial t} \left( \frac{g}{f} (\tilde{Z}_o - Z)(m/\Delta)^2 + \frac{f}{D} Z_{m,o} \right) = -V_o^G \cdot \nabla \left[ \frac{g}{f} (m/\Delta)^2 (\tilde{Z}_o - Z_o) \right]$$

$$+ f + \frac{f}{D} Z_{m,o} - f \left( \frac{E}{D} + 1 \right) \ln P_G$$
Multiplication by $\frac{f}{g} \left(\Delta/m\right)^2$ and replacing $\nabla \ln P_G$ by $\nabla (-\ln \frac{P_{SL}}{P_G})$

gives

$$
\frac{\partial}{\partial t} \left( \overline{Z}_o - Z_o + K \overline{Z}_{m,o} \right) = - \mathbf{v}_o^g \cdot \nabla \left( \overline{Z}_o - Z_o + K \overline{Z}_{m,o} \right)$$

$$+ \frac{f^2}{g} \left(\Delta/m\right)^2 \left( \frac{E}{D} + 1 \right) \ln \frac{P_{SL}}{P_G} - \frac{f}{g} \left(\Delta/m\right)^2 \mathbf{v}_o^g \cdot \nabla f$$

$$K \equiv \frac{f^2}{gD} \left(\Delta/m\right)^2$$

$P_{SL}$ is standard sea-level pressure and has been introduced to make the last term of (29) zero when the ground level is at sea level.

Now define

$$M_o \equiv \frac{f^2}{g} \left(\Delta/m\right)^2 \left( \frac{E}{D} + 1 \right) \ln \frac{P_{SL}}{P_G}$$

The term $- \frac{f}{g} \left(\Delta/m\right)^2 \mathbf{v}_o^g \cdot \nabla f$ may be written as

$$- \mathbf{v}_o^g \cdot \nabla G \quad \text{where} \quad G \equiv \int_0^\phi \frac{f^2 \Delta^2 \cotan \phi}{g \phi} \, d\phi$$

Finally equation (29) becomes

$$\frac{\partial}{\partial t} \left( \overline{Z}_o - Z + K \overline{Z}_{m,o} \right) = - \mathbf{v}_o^g \cdot \nabla \left( \overline{Z}_o - Z + G + K \overline{Z}_{m,o} + M_o \right)$$

This equation may be rewritten as
\[
\frac{d}{dt} (\bar{Z}_o - Z_o + K \bar{Z}_{\text{m},o} + G + M_o) = 0
\]

which is the basic conservation equation used in this paper. Equation (31) states that the quantity within parenthesis is conserved in the 1000-mb geostrophic flow.
C. EVALUATION OF CONSTANTS

The constants to be evaluated are:

\begin{align}
(32) \quad K &= \frac{r^2}{gD} (\Delta / \Delta m)^2; \quad C(p) = \int_0^p B(p) \, dp \\
(33) \quad D &= \int_{-1000}^{P} \frac{C(p) \delta \theta}{g \delta p} \, dp; \quad E = \int_{-1000}^{P} \frac{p \delta \theta}{g \delta p} \, dp \\
(34) \quad B(p) &= \frac{|V_p - V_m|}{|V_m - V_o|} = (A(p) - 1) \frac{V_p}{V_m - V_o}
\end{align}

where \( A(p) \equiv \frac{V_p}{V_m} \).

For the purpose of evaluating \( B(p) \), \( V \) was replaced by \( u \), the west wind component. This can be done without introducing a serious error since long-term means are involved here, and the north-south component is relatively small at mid latitudes in the long-term mean.

The basic data used in the evaluations has been taken from Buch (1954) and Petterssen (1956). Table I which contains the calculations has been extracted from Sanders (1959). In arriving at values for \( C, D \) and \( E \) a finite sum has been substituted for the integral.

It is to be noted that the "mean" level \( (B = 0) \) occurs at approximately
### TABLE I

<table>
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<td>u A</td>
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Values of u are in m sec⁻¹
Values of C are in mb

D = 6.8 X 10³ cm for values of C given above and NACA standard atmosphere temperature data.

E = 10.6 X 10³ cm for values of C given above and mean winter temperature data given in Petterssen's Fig. 6.10.1.

E = 9.1 X 10 cm for values of C given above and mean summer temperature data given in Petterssen's Fig. 6.10.2.

E = 65.9 X 10³ cm from mean winter temperature data given in Petterssen's Fig. 6.10.1.
600mb. The values of $D$ and $E$ used in this paper were $10.6 \times 10^3$ cm and $65.9 \times 10^3$ cm respectively.

In determining the value of $K$, some caution must be used. Assuming given values of $m$ (map scale factor), $D$, $f$ and $g$, $K$ is a function only of $\Delta^2$ where $\Delta$ is the grid interval used. However, the choice of the grid interval, $\Delta$, determines the minimum wavelength of the weather systems which one can treat. The optimum value of $\Delta$ therefore depends on the amount of detail desired in the forecast.

The equation which defines $K$ is

$$\frac{f^2}{gD} (\Delta/m)^2 \equiv K$$

The solution to equation (35) is given in the following table. In these calculations $f = 10^{-4}$ sec$^{-1}$.

<table>
<thead>
<tr>
<th>$K$</th>
<th>$\Delta/m$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.9</td>
<td>306</td>
</tr>
<tr>
<td>.8</td>
<td>289</td>
</tr>
<tr>
<td>.7</td>
<td>270</td>
</tr>
<tr>
<td>.6</td>
<td>250</td>
</tr>
<tr>
<td>.5</td>
<td>228</td>
</tr>
<tr>
<td>.4</td>
<td>204</td>
</tr>
<tr>
<td>.3</td>
<td>177</td>
</tr>
</tbody>
</table>

Although the mean level was found to be about 600mb, it is more convenient to use 500-mb data in making a graphical forecast. This being the case, the values of $K$ should be scaled down by the ratio $A_6/A_5$. 
which is about 5/6. Sanders (1959) has found that, using 500-mb as the mean level, a value of \((A_6/A_5)^{1/2} \approx .5\) gives good results. Assuming \(m = 1\), this gives a \(\Delta\) value of approximately 250 kilometers which is the value used in this paper.

The \(G\)-field used in this study is shown in Figure 1. The \(M_0\)-field is shown in Figure 2 and the \((G + M_0)\)-field is shown in Figure 3. In calculating the \(M_0\)-field, the values of \(P_G\) used were the NACA standard atmosphere pressures corresponding to the smoothed height of the earth's surface. The smoothed topographic chart used was that given by Berkofsky and Bertoni (1955). (We rechecked the value given by Berkofsky and Bertoni for the point 40°N, 75°E and found the point to be in error by +1000 ft. We corrected the point in computing our \(M_0\)-field. We found no further errors after recomputing the smoothed height values over the remainder of the eastern half of North America.)
D. METHOD

Using Fjortoft's (1952) technique, we may write equation (30) as:

\[ \frac{\partial}{\partial t} \left( \mathbf{Z}_o - \mathbf{Z}_o + K \mathbf{Z}_o + \mathbf{G} + \mathbf{M}_o \right) \]

\[ = - \left( \mathbf{V}_o + K \mathbf{V}_m + \mathbf{V}_G + \mathbf{V}_M \right) \cdot \nabla \left( \mathbf{Z}_o - \mathbf{Z}_o + K \mathbf{Z}_o + \mathbf{G} + \mathbf{M}_o \right) \]

where \( \mathbf{V}_o, \mathbf{V}_m, \mathbf{V}_G, \mathbf{V}_M \) are the "geostrophic" winds corresponding to the fields of \( \mathbf{Z}_o, \mathbf{Z}_m, \mathbf{G}, \) and \( \mathbf{M} \) respectively. Equation (30) is written in this form because the "steering" flow, \( \mathbf{V}_s = \mathbf{V}_o + K \mathbf{V}_m + \mathbf{V}_G + \mathbf{V}_M \), is steadier in time than \( \mathbf{V}_o^G \) while its dot product with the gradient of \( \mathbf{Z}_o - \mathbf{Z}_o + K \mathbf{Z}_o + \mathbf{G} + \mathbf{M}_o \) is equal to that of \( \mathbf{V}_o^G \). This is of great importance in obtaining graphical, finite-difference solutions to (36) when long-time steps are involved.

There are several ways to obtain a 1000-mb forecast from equation (36). One approximate method for obtaining a graphical solution, which is presently in use, follows.

First, rewrite equation (30) as:

\[ \frac{\partial}{\partial t} \left( a \mathbf{Z}_o - \mathbf{Z}_o + b \mathbf{Z}_m \right) = - \mathbf{V}_o^G \cdot \nabla \mathbf{X}^f \]
\[ a = \frac{1}{1 + K} ; \quad b = \frac{K}{1 + K} \]

\[ X' \equiv a Z_0 - Z_0 + b Z_m + a G + a M_0 \]

Again by making use of Fjortoft's technique we may write (37a) as:

\[
(37b) \quad \frac{\partial}{\partial t} (a Z_0 - Z_0 + b Z_m) = -(a V_0 + b V_5 + a V_G + a V_M) \cdot \nabla (X')
\]

The initial values of \( X' \) are then displaced in the "steering" flow in a single 24-hour time step. At any grid point, the finite difference equation then becomes

\[
(38) \quad \Delta_{24} (a Z_0 - Z_0) = X'_{\text{initial}} - X'_{\text{displaced}} - b \Delta_{24} Z_m
\]

where \( \Delta_{24} \) indicates the change at the grid point over the 24 hour period. A value of \( b \Delta_{24} Z_m \) is obtained by making an independent 24-hour forecast for the \( Z_m \) field. Since the right-hand side of (38) is now known, the forecast values of \( Z_0 \) may be obtained by application of a smoothing or a relaxation technique.

In manipulating equation (36) in this manner, several additional assumptions and/or approximations are introduced which may lead to error in the forecast field of \( Z_0 \). Some errors to be expected are:

1. Errors resulting from the use of relaxation or smoothing techniques.
(2) Errors resulting from non-steadiness of the steering flow over the forecast period.

(3) Errors resulting from the forecast error in the field of $Z_m$.

These errors can become very important under certain circumstances (see Rodgers, 1958), but none of them are inherent in equation (36). They appear only when the equation is solved in a certain fashion as noted above.

The question of the validity of equation (36) should be answered first as it is the basic equation of the method. The most straightforward way to do this is to use equation (36) for forecasting the quantity $X \equiv \tilde{Z}_0 - Z_0 + K Z_{m,o} + G + M_o$. The forecast values of $X$ may be compared with the observed values and the resulting error chart will show to what extent the supposedly conserved quantity, $X$, is actually conserved. This was done as described below.

a) $X$ was computed over the forecast area at 00 GCT on day 1 and at 00 GCT on day 2.

b) The steering flow, $V_s$, was computed for the same area at 00 GCT on day 1, 1200 GCT on day 1, and 00 GCT on day 2.

c) The quantity, $X$, for 00 GCT on day 1 was advected in the steering flow of 00 GCT day 1 for 6 hours, the steering flow of 1200 GCT day 1 for the next 12 hours, and the steering flow of 00 GCT
day 2 for the last 6 hours to give a 24-hour forecast of X at each grid point.

d) The observed value of X at 00 GCT day 2 was subtracted from the forecast value at each grid point to give an error chart.

e) The resulting error map was analyzed to delineate areas of positive and negative error, i.e. where \( X_{fcst} - X_{obs} \) was positive or negative.

NOTE: [The qualitative relationship between an error in X and an error in \( Z_0 \) can be seen. We have

\[
\Delta_{24}(\tilde{Z}_o - Z_o + K Z_m, 0) = \Delta_{24} X
\]

Since the change in \( \tilde{Z}_o \) is small compared to the change in \( Z_o \), the following approximation can be made (\( K = 1/2 \)):

\[
-\Delta_{24} 1.5 Z_o + \frac{1}{2} \Delta_{24} Z_m = \Delta_{24} X
\]

Our procedure allows no error in \( \frac{1}{2} \Delta_{24} Z_m \). For a value of \( \Delta_{24} X \) which is too large (\( X_{fcst} - X_{obs} \) is positive), there will be a negative error in the \( Z_o \) forecast.]

If we use the above method, the effects of non-steadiness in the steering flow are greatly reduced, although not completely eliminated.

Moreover, the steering flow is computed from observed fields of \( Z_o \) and \( Z_m \).
so there is no possibility of error in the steering flow except for observational and analysis error.

In the above manner, forecasts of $X$ and error charts were made for each of the days 23 through 27 February 1962, valid at 00 GCT. The value of $K$ used was 0.5, $\Delta$ was 250 km, and the mean level was taken as 500 mb. Additional techniques were used to identify the error sources and these will be discussed as they are introduced.
E. RESULTS AND DISCUSSION – Part I

Twenty-four hour forecasts of the $X$ field were made for the days (1962) 23 through 27 February, all valid at 00 GCT. The method described in the preceding section was used to make these forecasts. The forecast area was the area covered by the grid points shown in Figure 4. The map projection used was a polar stereographic projection, true at $60^\circ$ latitude with a scale of 1:30,000,000.

1) **Synoptic Situation:** Surface and 500-mb maps for this period are shown in Figures 5 and 6. At the beginning of this series there existed a quasi-stationary front along the southern part of the United States with a low-pressure area in the Great Lakes region. The frontal line extended westward and northward along the eastern slopes of the Continental Divide into a developing cyclone in southwestern Canada. A cold high pressure area was in evidence over northwestern Canada and Alaska. Subsequently the low-pressure area in the Lakes region moved northeast and a high-pressure ridge began building rapidly in its wake so that by 00 GCT on the 24th there was a large area of high pressure over southern Canada orientated east-west, with the quasi-stationary east-west front continuing over the south Central part of the United
States. The pattern over the next three days showed the development of a cyclone center over the Central Mississippi Valley and a reinforcing of the cold air over the plains states as the cyclone center moved to the northeast. By 00 GCT on the 27th, there was again a cyclone centered just northeast of the Great Lakes, a front extending southwestward into Texas, and a ridge of high pressure extending northward from central Texas into Canada.

The 500-mb flow during this whole period was characterized in general by an almost stationary trough extending from the Saskatchewan-Manitoba provinces into the southwestern part of the United States. The winds at 500mb during this period were generally from a WSW to SW direction in the area extending from the east slopes of the Continental Divide to the eastern coast of the United States.

2) Results: Forecast and observed fields of X, as well as error charts (forecast–observed) are shown in Figures 7–9. A few general remarks should be made here before considering the maps in detail.

a) A comparison of forecast and observed charts shows that in general there was quite good agreement in the types of features occurring, however, the locations and intensities did not agree very well in some instances.

b) The error charts show that appreciable errors were made in every forecast, with the average maximum error on any one chart
being near 400 ft. The error patterns exhibited a certain amount of organization as opposed to a random distribution, since errors of the same sign were grouped together.

c) The error charts exhibit a certain amount of conservativeness, i.e., certain features may be identified from one day to the next.

The above points indicate that most of the errors contained in these forecasts result from deficiencies in the model. If this were not so, and the errors were due entirely to mistakes made in drawing trajectories, measuring gradients etc., then we would not have expected to find organization or conservativeness among the error patterns.

Past experience has shown that this model does not handle cold out-breaks into the Plains states of the United States very well. This deficiency in the model has been noted by other investigators in the field. In particular, R. J. Reed, currently with the Numerical Weather Prediction Unit, has brought to our attention the fact that the model did not handle this particular series very well in tests being conducted at NWP. (Reed, R. J., Personal Communication, 1962). In some cases the forecast model "overbuilds" the resulting high pressure area, and in other cases the associated front is not forecast to move rapidly enough. The reason for the different error in the two cases is generally thought to be associated with the character of the flow above the cold air mass.
The particular sequence that is dealt with in this study is one in which a cold outbreak occurred. It is a case where the model apparently did not move the cold ridge far enough southward. The exact error in the Z-field is not shown but is implicit in the X-error charts.

The remaining discussion will be confined almost entirely to the area bounded by the east slopes of the Rocky Mountains to the west, the Canada-U.S. border to the north, the Mississippi River to the east, and the southern U.S. border to the south. In this area serious error occurred in the model during the cold outbreak.

The most logical place to begin in attempting to locate the error sources is the surface map itself. A comparison of the error chart for 00 GCT 23 February with the 0600 GCT 23 February surface chart shows the center of positive error over Montana is associated with the leading edge of a cold, high pressure ridge. The negative errors occur where the pressures are relative low, e.g., the Minnesota-Iowa and Colorado-Utah areas.

The error chart for 00 GCT 24 February shows a large area of positive error covering the entire area of interest. The surface map for 0600 GCT 24 February shows a large cold high pressure ridge covering approximately the same area.

The error chart for 00 GCT 25 February shows an area of negative error extending from northeast Texas into Minnesota and an area of
positive error extending from Idaho into the Texas-Oklahoma Panhandle. The surface map shows an inverted trough extending from northeastern Oklahoma into Minnesota with a weak ridge and extremely cold temperatures to the west of this trough.

The error chart for 00 GCT 26 February shows a weak area of negative error over Illinois and a large area of positive error centered over northern Wyoming. The surface chart for 0600 GCT 26 February shows a low-pressure area centered in western Illinois and a strong ridge over the plains states with a high pressure center over Montana.

The error chart for 00 GCT 27 February shows an area of positive error centered over the Arkansas-Oklahoma-Kansas area, with negative error along the northern United States border. The surface map shows a ridge of high pressure extending from southern Texas northward into Canada. There is a weak high pressure center in South Dakota.

From the preceding description of the error charts and the surface maps, it appears that there is a tendency for areas of positive error in the forecast of X to coincide with areas of relatively high pressure at 1000mb. Since the comparison was subjective in nature and applies only to a specific area and time, the preceding statement is not to be interpreted as a generalization. However, the association was strong in this particular series, and as has been noted, other investigators...
have also found that significant errors occur in the high-pressure areas associated with cold outbreaks.

It seems apparent then, that one or more assumptions in the model are not valid in these cold ridges. Further, making these assumptions leads to positive error in forecasting $X$.

One of the major assumptions in this model is that the actual winds may be replaced by the geostrophic winds. Without this assumption, it would have been impossible to arrive at the basic conservation equation presented in equation (30).

In spite of this fact, however, it may well be that better forecasts could be made if the actual 1000-mb wind was used as the advecting flow instead of the geostrophic wind. This would be the case if the advective property of the wind was of major importance in this model.

In order to test this hypothesis new forecasts were made using the same method as before except that the "actual" 1000-mb winds were used as the advecting flow instead of the geostrophic winds. The word "actual" is in quotation marks because it was impossible to obtain actual 1000-mb winds when the 1000-mb surface was below the ground. In these cases the wind report closest to 1000mb was used, and this meant using winds as high as 850mb in the mountainous regions. Generally speaking the wind reports used were between 900 and 1000mb in the Plain states and 950 to
1000mb east of the Mississippi.

These actual wind forecasts, as well as all subsequent investigations were confined to the 25th, 26 and 27th of February because it was felt that the main points could be adequately illustrated without using the whole five-day series.

The error charts resulting from the actual wind forecasts are shown in Figure 10. The most striking feature of these charts is that the errors in the area of interest are almost exclusively negative, i.e. \( X_{\text{fcast}} - X_{\text{obs}} \) is negative. The errors using the geostrophic winds were predominately positive in this area. The absolute values of the errors are approximately the same in both cases. The important result is that no improvement was made by merely using the actual wind as the advecting velocity instead of the geostrophic wind.

In order that a more direct comparison might be made between forecasts using geostrophic trajectories and forecasts using actual wind trajectories, maps were constructed of the difference between forecast values using geostrophic winds and forecast values using actual winds. These appear in Figure 11. The maps indicate that forecast X values made using the geostrophic system were consistently higher than those made with actual wind trajectories in the area under consideration.

Since no essential improvement was obtained by using actual winds
as the advecting flow, the major error sources must be contained in the individual terms of X which are being advected. This quantity may be conveniently divided into three terms. They are:

a) Vorticity \( \equiv \overline{Z_0} - Z_0 + G \)

b) Orographic \( \equiv M_0 \)

c) Thickness (temperature) \( \equiv \frac{1}{2} Z_{5,0} \)

It is seen by inspecting equation (30) that the local rate of change of X is given by the sum of the vorticity advection, the orographic advection and the temperature advection. Moreover, it was theorized that in this particular situation, the temperature advection term was of greater importance than the vorticity and the orographic term combined.

To test this hypothesis instantaneous advection rates of \( \overline{Z_0} - Z_0 + G + M_0 \), and \( \frac{1}{2} Z_{5,0} \) were computed at 12-hour intervals beginning 00 GCT 24 February and ending 00 GCT 27 February using both actual and geostrophic 1000-mb winds. That is, the instantaneous values of the following four quantities were computed:

1) \(- V_o \cdot \nabla (\overline{Z_0} - Z_0 + G + M_0)\)

2) \(- V_o \cdot \nabla (\frac{1}{2} Z_{5,0})\)

3) \(- V_o^g \cdot \nabla (\overline{Z_0} - Z_0 + G + M_0)\)

4) \(- V_o^g \cdot \nabla (\frac{1}{2} Z_{5,0})\)
Next, a weighted average of the instantaneous advection rates of each quantity was computed for each 24-hour forecast period. For example,

\[
\frac{1}{4} \left[ -V \cdot \nabla \left( \bar{Z}_0 - Z_o + G + M_o \right) \right]_{00 \text{ GCT 24 Feb}} \\
+ \frac{1}{4} \left[ -V \cdot \nabla \left( \bar{Z}_0 - Z_o + G + M_o \right) \right]_{12 \text{ GCT 24 Feb}} \\
+ \frac{1}{4} \left[ -V \cdot \nabla \left( \bar{Z}_0 - Z_o + G + M_o \right) \right]_{00 \text{ GCT 25 Feb}}
\]

was computed to give an average value of quantity (1) over the forecast period ending 00 GCT 25 February. Weighted averages are denoted with a bar over the term, e.g.,

\[ -V \cdot \nabla \left( \bar{Z}_0 - Z_o + G + M_o \right) \].

At this point maps of

\[ -V \cdot \nabla \left( \bar{Z}_0 - Z_o + G + M_o \right), \quad -V \cdot \nabla \left( \bar{Z}_0 - Z_o + G + M_o \right), \]

\[ -V \cdot \nabla ^{\frac{1}{2}} Z_{5,0}, \quad V \cdot \nabla ^{\frac{1}{2}} Z_{5,0} \]

were available for 00 GCT 25 February, 00 GCT 26 February and 00 GCT 27 February.

A comparison of these charts showed that the advection of
temperature was, on the average, approximately twice as strong as the advection of vorticity and the orographic term combined. This was true for both the actual wind case and the geostrophic wind case, as is shown in the following table

<table>
<thead>
<tr>
<th>Advected Quantity</th>
<th>Mean Advection Rate (magnitude ft/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25 Feb</td>
</tr>
<tr>
<td>Vorticity and orographic, geos</td>
<td>4</td>
</tr>
<tr>
<td>Thickness, geos</td>
<td>7</td>
</tr>
<tr>
<td>Vorticity and orographic, actual</td>
<td>2</td>
</tr>
<tr>
<td>Thickness, actual</td>
<td>5</td>
</tr>
</tbody>
</table>

It was decided at this point to investigate the temperature advection term further. In developing the prediction equation, the term

\[ V_0 \cdot \nabla Z_{5,0} \]

arises from an attempt to approximate

\[ \int_{1000}^{500} V_0 \cdot \nabla(\delta Z/\delta p) \, dp. \]

This approximation occurs in equation (13). In making this substitution it has been assumed that the winds are geostrophic and that the isotherms do not change their orientation with height. These assumptions are generally reasonable ones to make, but they have not been justified as yet for the particular synoptic situation of this study.
A better approximation to the integral above would be obtained by dividing the layer from 1000mb to 500mb into N parts and writing

\[
\int_{1000}^{500} V \cdot \nabla \left( \frac{\partial Z}{\partial p} \right) \, dp \approx \sum_{i=1}^{N} V_i \cdot \nabla (\Delta Z)_i
\]

\( V_i \) is the average wind in the \( i \)th layer.

The summation above could be evaluated as follows:

\[
\sum_{i=1}^{3} V_i \cdot \nabla (\Delta Z)_i = \frac{1}{2} \left( V_{1000} \cdot \nabla (Z_{850} - Z_{1000}) + V_{850} \cdot \nabla (Z_{850} - Z_{1000}) \right) \\
+ \frac{1}{2} \left( V_{700} \cdot \nabla (Z_{700} - Z_{850}) + V_{700} \cdot \nabla (Z_{700} - Z_{850}) \right) \\
+ \frac{1}{2} \left( V_{500} \cdot \nabla (Z_{500} - Z_{700}) + V_{500} \cdot \nabla (Z_{500} - Z_{700}) \right)
\]

This was done for the 24th, 25th, 26th and 27th of February at 00 GCT and 1200 GCT and taken as a standard against which the values of \( V \cdot \nabla \frac{1}{2} Z_{5,0} \) and \( V^g \cdot \nabla \frac{1}{2} Z_{5,0} \) were compared. (The values computed by the summing process were first multiplied by one half because of the factor \( K \) which appears later in the derivation.)

These standard values were then averaged in the same manner as the \( V \cdot \nabla \frac{1}{2} Z_{5,0} \) and \( V^g \cdot \nabla \frac{1}{2} Z_{5,0} \) values to produce a weighted average over the 24-hour forecast period. Denoting
as \( S \) and the weighted average of \( S \) over the forecast period as \( \bar{S} \),
maps were constructed of

\[
\bar{S} = (-V_o \cdot \nabla \frac{1}{2} Z_{5,0}) \quad \text{and} \quad \bar{S} = (-V_o^e \cdot \nabla \frac{1}{2} Z_{5,0}).
\]

These maps were drawn for the 25th, 26th and 27th of February and represent conditions during the preceding 24 hours leading up to the forecast valid time. These maps appear in Figures 12 and 13.

Assuming that \( \bar{S} \) is an adequate representation of the integrated temperature advection, we see from Figure 12 that the use of

\[-V_o \cdot \nabla \frac{1}{2} Z_{5,0}\]

as the temperature advection term gives too much cold advection in the area of interest. This is true for all three days.

The values of \( \bar{S} - (-V_o \cdot \nabla \frac{1}{2} Z_{5,0}) \) may be considered as correction terms to the error charts of Figure 10. The numbers can be compared quantitatively if the values of \( \bar{S} - (-V_o \cdot \nabla \frac{1}{2} Z_{5,0}) \) are first multiplied by 24 (hours). A comparison of this nature shows that the correction terms and the errors are of comparable magnitude but opposite sign. This indicates that the temperature advection term is the predominant source of error in the forecasts of \( X \) made using actual winds.

The same argument applies to Figure 13 and a comparison of Figures
13 and 9. The only difference is that in this case the use of the term $-V_o \cdot \nabla \frac{1}{2} Z_{5,0}$ gave too little cold advection. Again the correction term is of the right sign and magnitude. This is further evidence that the temperature advection term is the predominant source of error in these forecasts.

It can be seen by referring to the derivation of equation (30) that the term $-V_o \cdot \nabla \frac{1}{2} Z_{5,0}$ is obtained with the aid of two major assumptions. The first is that the winds are geostrophic everywhere. The second assumption is that the isotherms do not change their orientation with height. It is possible to obtain some insight as to which of these assumptions is responsible for the error introduced by use of the term $-V_o \cdot \nabla \frac{1}{2} Z_{5,0}$. The logical way to proceed is to recompute the values of $\bar{S}$ using geostrophic winds instead of actual winds. These "geostrophic" values of $\bar{S}$ will be denoted $\bar{S}_g$. If the values of $\bar{S}_g$ are approximately equal to those of $\bar{S}$, then it may be said that the geostrophic assumption is acceptable. This would also logically imply that the second assumption noted above is a bad assumption in this case. If, however, the values of $\bar{S}_g$ are drastically different from those of $\bar{S}$, then it can be said that the geostrophic assumption is inadequate in this particular circumstance.

A test was therefore made to determine if the geostrophic winds could be used to give an integrated thickness advection which would
compare favorably with that given by actual winds. To state it more concisely, the following hypothesis was tested:

\[
S \neq S_g
\]

\[
S_g \equiv - \left[ v_o^g \cdot \nabla \frac{1}{2}(Z_{850} - Z_{1000}) + v^g_{850} \cdot \nabla \frac{1}{2}(Z_{700} - Z_{850}) + v^g_{700} \cdot \nabla \frac{1}{2}(Z_{500} - Z_{700}) \right]
\]

It was not necessary to include all the terms that went into computing \( S \) because

\[
-v^g_{850} \cdot \nabla \frac{1}{2}(Z_{850} - Z_{1000}) = -v_o^g \cdot \nabla \frac{1}{2}(Z_{850} - Z_{1000})
\]

and the same holds true for the other two layers.

The test was made for all three days.

Figure 14 shows that the difference between \( S \) and \( S_g \) is fairly small. The average values of \( \bar{S} - \bar{S}_g \) for the three days are contained in the following table.

<table>
<thead>
<tr>
<th>Valid Time</th>
<th>( \bar{S} - \bar{S}_g ) (average magnitude in ft/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 Feb 00 GCT</td>
<td>1.8</td>
</tr>
<tr>
<td>26 Feb 00 GCT</td>
<td>2.0</td>
</tr>
<tr>
<td>27 Feb 00 GCT</td>
<td>2.3</td>
</tr>
</tbody>
</table>

A value of 2.0 is equivalent to an error in the forecast of \( X \) of
approximately 50 feet. The average values of \( \bar{S} - (-V_o \cdot \nabla Z_{5,0}) \) 
and \( \bar{S} - (-V_o \cdot \nabla Z_{5,0}) \) computed over the same grid points are shown 
below.

<table>
<thead>
<tr>
<th>Valid Time</th>
<th>( \bar{S} - (-V_o \cdot \nabla Z_{5,0}) ) [ft/hr]</th>
<th>( \bar{S} - (-V_o \cdot \nabla Z_{5,0}) ) [ft/hr]</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 Feb 00 GCT</td>
<td>4.3</td>
<td>6.4</td>
</tr>
<tr>
<td>26 Feb 00 GCT</td>
<td>5.3</td>
<td>8.4</td>
</tr>
<tr>
<td>27 Feb 00 GCT</td>
<td>6.2</td>
<td>9.4</td>
</tr>
</tbody>
</table>

It is readily apparent from these two tables that most of the 

improvement afforded by the use of \( \bar{S} \) as the temperature advection term 
can be obtained by use of \( \bar{S} \). It is of further interest to note that 
even though the differences between \( \bar{S} \) and \( \bar{S} \) are comparatively small, 
the geostrophic term \( \bar{S} \) still exhibits a tendency to give too little 
cold advection in the area under consideration.

Since \( \bar{S} \) corresponds closely with \( \bar{S} \), we can conclude that the 
error in the X forecast is due mainly to the inadequacy of the assumption that the isotherms do not change their orientation with height.

To demonstrate how this error works, we drew the hodograph of the 
geostrophic winds for a point just east of North Platte, Nebraska 
(Figure 15). Note that if the layer between 500mb and 1000mb had
been divided into three layers, namely, 500mb to 700mb, 700mb to 850mb, 850mb to 1000mb, the contribution of the temperature advection term \(S_g\) would have been slight cold advection. This is shown in the top half of Figure 15. However, if the 500mb to 1000mb layer is treated as a whole the contribution of the term \(\nabla \cdot \nabla Z_{5,0}\) is strong warm advection. This is shown in the bottom half of Figure 15. An inspection of the forecast X-observed X chart for 00 GCT 27 February (Figure 9e) shows that this point had a large positive error which agrees very well with the conclusion above.
F. RESULTS AND DISCUSSION - Part II

Despite the many assumptions and approximations which are found in the 1000-mb forecasting model under discussion, the model produces surprisingly good results. Nevertheless there are some basic problems and some of these will be hard to overcome. Two of the most serious problems arise simply because the 1000-mb level is near the ground.

a) In the basic model, the winds are assumed geostrophic at all levels and in all terms in the equation. This is a poor assumption at 1000mb.

b) Even if the winds were geostrophic, the presence of the Rocky Mountains and the way in which pressure reductions are made can cause the 1000-mb geostrophic winds computed from reported pressures to be quite different from the actual geostrophic winds (see Sangster, 1960). The fact that the 1000-mb "surface" is generally beneath the ground over half of the United States makes the basic parameters of wind, temperature and pressure rather meaningless in this area.

It would be a difficult, if not impossible, task to determine how bad these problems really are. However, since this model can be developed for any pressure level, one indirect method would be to develop the model
for 850mb. At this level and above, the ageostrophic part of the wind is much less, and the 850-mb surface is above the ground over a much larger area.

Two important questions to be answered with the aid of the 850-mb forecasts are:

1) Is there a significant reduction of error in forecasts of the field of the "conservative quantity", over the errors in the forecasts of X?

2) Do the errors in the forecasts of the new "conservative quantity" correlate positively with the errors in X (on a geographical area basis)?

If the answer to (1) is yes, then there would conceivably be some merit in making 850-mb forecasts and extrapolating downward to arrive at a 1000-mb forecast.

If the answer to (2) is yes, this would indicate that the basic model deficiency during these cold outbreaks is not wholly confined to the very lowest levels of the atmosphere.

The development of the 850-mb model follows that of the 1000-mb model. The only change is in the values of the empirically determined constants. The final equation has exactly the same form; therefore, the derivation will be omitted and only the calculation of the constants will be presented.
1) $B(p)$ is now approximated by

$$\frac{u_p - u_m}{u_m - u_{850}}$$

(subscript 850 on $u$ refers to 850-mb level)

where the variables have the same meaning as previously. The values of $B$ are as follows:

<table>
<thead>
<tr>
<th>Level mb</th>
<th>60$^\circ$N</th>
<th>50$^\circ$N</th>
<th>40$^\circ$N</th>
<th>30$^\circ$N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.67</td>
<td>-1.97</td>
<td>-1.87</td>
<td>-1.31</td>
</tr>
<tr>
<td>100</td>
<td>.64</td>
<td>.71</td>
<td>.64</td>
<td>.76</td>
</tr>
<tr>
<td>200</td>
<td>1.12</td>
<td>1.26</td>
<td>1.37</td>
<td>1.25</td>
</tr>
<tr>
<td>300</td>
<td>1.15</td>
<td>.97</td>
<td>1.06</td>
<td>1.08</td>
</tr>
<tr>
<td>400</td>
<td>.78</td>
<td>.72</td>
<td>.73</td>
<td>.70</td>
</tr>
<tr>
<td>500</td>
<td>.39</td>
<td>.46</td>
<td>.38</td>
<td>.31</td>
</tr>
<tr>
<td>600</td>
<td>-.02</td>
<td>.03</td>
<td>-.02</td>
<td>-.12</td>
</tr>
<tr>
<td>700</td>
<td>-.45</td>
<td>-.36</td>
<td>-.43</td>
<td>-.55</td>
</tr>
<tr>
<td>800</td>
<td>-.81</td>
<td>-.79</td>
<td>-.81</td>
<td>-.86</td>
</tr>
<tr>
<td>850</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
<td>-1.00</td>
</tr>
<tr>
<td>900</td>
<td>-1.19</td>
<td>-1.21</td>
<td>-1.20</td>
<td>-1.15</td>
</tr>
<tr>
<td>1000</td>
<td>-1.56</td>
<td>-1.64</td>
<td>-1.59</td>
<td>-1.47</td>
</tr>
</tbody>
</table>

$$(B)_{m} = 0 \quad 0 \quad 0 \quad 0 \quad 0$$

2) $C(p) = \int_{0}^{p} B(p) dp$

Using the method of a finite sum to approximate the integral, the following values are obtained:

<table>
<thead>
<tr>
<th>$p$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>850</td>
<td>187</td>
</tr>
<tr>
<td>750</td>
<td>276</td>
</tr>
<tr>
<td>650</td>
<td>316</td>
</tr>
<tr>
<td>550</td>
<td>318</td>
</tr>
</tbody>
</table>
3) \[ D = \int \frac{P_m}{1000} \frac{C \Delta \theta}{g \Delta \rho} \, dp \approx \frac{R}{g} \sum_{i=1}^{3} \left( \frac{\bar{C} \bar{T} \Delta \theta}{\bar{p} \bar{\theta}} \right) \]

\[ E = \int \frac{P_m}{1000} \frac{p a \Delta \theta}{g \Delta \rho} \, dp \approx \frac{R}{g} \sum_{i=1}^{3} \left( \frac{\bar{T} \Delta \theta}{\bar{\theta}} \right) \]

\[ P_m = 600 \text{ mb} \]

The layer \( i = 1 \) refers to layer from 850mb to 800mb, \( i = 2 \) refers to layer from 800mb to 700mb, and \( i = 3 \) refers to layer from 700mb to 600mb.

The basic data was again taken from Petterssen (1956) and the results are as follows:

<table>
<thead>
<tr>
<th>Layer</th>
<th>( \left( \frac{\bar{C} \bar{T} \Delta \theta}{\bar{p} \bar{\theta}} \right) ) (cm)</th>
<th>( \left( \frac{\bar{T} \Delta \theta}{\bar{\theta}} \right) ) (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>850-800</td>
<td>.97</td>
<td>3.76</td>
</tr>
<tr>
<td>800-700</td>
<td>1.63</td>
<td>4.56</td>
</tr>
<tr>
<td>700-600</td>
<td>1.93</td>
<td>3.98</td>
</tr>
<tr>
<td><strong>Sum</strong></td>
<td><strong>4.53</strong></td>
<td><strong>12.30</strong></td>
</tr>
</tbody>
</table>

\[ D = (R/g) \times 4.53 = 2.93 \times 10^3 \times 4.53 \text{ cm} = 13.3 \times 10^5 \text{ cm} \]

\[ E = (R/g) \times 12.30 = 2.93 \times 10^3 \times 12.30 \text{ cm} = 36.2 \times 10^5 \text{ cm} \]

\[ E/D = \frac{36.2}{13.3} = 2.72 \]
4) The value of $K$ was found from solving the following equation:

\[
\frac{f^2}{gD}(\Delta/m)^2 = K
\]

With the new values of $D$ and the same values as previously used of the other parameters, the following table is obtained:

<table>
<thead>
<tr>
<th>$K$</th>
<th>$\Delta/m$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.9</td>
<td>343</td>
</tr>
<tr>
<td>.8</td>
<td>324</td>
</tr>
<tr>
<td>.7</td>
<td>303</td>
</tr>
<tr>
<td>.6</td>
<td>280</td>
</tr>
<tr>
<td>.5</td>
<td>255</td>
</tr>
<tr>
<td>.4</td>
<td>229</td>
</tr>
<tr>
<td>.3</td>
<td>198</td>
</tr>
</tbody>
</table>

Since these values are not significantly different from those obtained for the 1000-mb model, it is felt that a value of $K = .5$ and a $\Delta$ of 250 km can be used for this model as well, if 500mb is used as the mean level instead of 600mb. ($f/m$ is taken as $10^{-4} \text{ sec}^{-1}$.)

5) \[
m_{850} = \frac{f^2}{g} (\Delta/m)^2 \left( \frac{E}{D} + 1 \right) \ln \left( \frac{P_{SL}}{P_G} \right)
\]

In this equation the only quantities which have values different from those used in the 1000-mb model are $D$ and $E$. For this model \[
\frac{E}{D} + 1 = 2.72 + 1 = 3.72. \text{ The value of } \frac{E}{D} + 1 \text{ in the 1000-mb model was } 6.2 + 1 = 7.2. \text{ Therefore we can write the equation}
\]
In this model, the $M_{850}$ field was taken as $\frac{3.72}{7.2} M_0 \approx 0.5 M_0$.

6) The $G$ field remains unchanged.

The 850-mb equivalent of equation (36) is

$$\begin{align*}
\frac{\partial}{\partial t} (\overline{Z}_{850} - Z_{850} + 0.5 \overline{Z}_{5,850} + G + M_{850}) &= \\
- (\overline{V}_{850} + 0.5 V_{5,850} + V_G + V_M_{850}) \\
\cdot \nabla (\overline{Z}_{850} - Z_{850} + 0.5 \overline{Z}_{5,850} + G + M_{850})
\end{align*}$$

The subscript 850 refers to the 850-mb level and the other symbols retain their previous meaning. For convenience,

$$Y \equiv (\overline{Z}_{850} - Z_{850} + 0.5 \overline{Z}_{5,850} + G + M_{850}) .$$

Using the identical procedure described in section D, forecasts of $Y$ were made for 00 GCT on the 25th, 26th and 27 of February.

The forecasts were made using geostrophic winds only. Error charts were made showing forecast - observed values of $Y$. The error charts are shown in Figure 16.

A comparison of Figures 16 and 9 shows that, in general, the errors in forecast $Y$ values are less than the errors in the forecast $X$ values.
The average error in Y is roughly two-thirds of the average error in X. It is felt that this is not a significant reduction since the Y values themselves are approximately two-thirds the X values.

There is a weak tendency for both X and Y forecasts to show error of the same sign in the previously defined area of interest. This indicates that even at 850mb, this model makes a systematic error when applied to a shallow cold outbreak.

The following table contains a more objective comparison. The values in the table were computed using values from grid points for which Y forecasts were made. This means that the values quoted under X forecasts were not obtained from all of the X values available. The actual number of grid points on which the table is based is 49.

<table>
<thead>
<tr>
<th>Average error per grid point in feet (absolute magnitude)</th>
<th>Y forecast</th>
<th>X forecast</th>
</tr>
</thead>
<tbody>
<tr>
<td>00 GCT 25 Feb</td>
<td>77</td>
<td>132</td>
</tr>
<tr>
<td>00 GCT 26 Feb</td>
<td>69</td>
<td>122</td>
</tr>
<tr>
<td>00 GCT 27 Feb</td>
<td>92</td>
<td>115</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Maximum error in feet (absolute magnitude)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>00 GCT 25 Feb</td>
<td>210</td>
<td>.380</td>
</tr>
<tr>
<td>00 GCT 26 Feb</td>
<td>220</td>
<td>340</td>
</tr>
<tr>
<td>00 GCT 27 Feb</td>
<td>220</td>
<td>380</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Range of error (feet)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>00 GCT 25 Feb</td>
<td>-170 to +210 = 380</td>
<td>-330 to +380 = 710</td>
</tr>
<tr>
<td>00 GCT 26 Feb</td>
<td>-100 to +220 = 320</td>
<td>-300 to +340 = 640</td>
</tr>
<tr>
<td>00 GCT 27 Feb</td>
<td>-210 to +220 = 430</td>
<td>-230 to +380 = 610</td>
</tr>
</tbody>
</table>
G. CONCLUSIONS

It was found that the quantity $(\bar{Z} - Z_o + \frac{1}{2} Z_o + G + M_o)$ is not truly conserved in the 1000-mb flow. Errors were found in the forecast values of the conservative quantity which were large enough to be quite significant, regardless of whether geostrophic or actual 1000-mb winds were used as the advecting velocity. Detailed investigations of the errors in the midwestern United States during a cold outbreak revealed that a major part of the error resulted from the use of a poor approximation of the mean temperature advection in the layer from 1000mb to 500mb. A much better approximation is obtained if the layer is split into three layers as was done in arriving at the expression

$$S \equiv - \sum_{i=1}^{3} V_i \cdot \nabla \frac{1}{2} (\Delta Z)_i.$$  

If $V_i$ is taken as the geostrophic wind the results are somewhat less satisfactory, although a significant reduction in the final error is still achieved.

Further investigations revealed that no significant improvement in forecast values of the conservative quantity was achieved by adapting the model for use at the 850-mb level.
H. RECOMMENDATIONS

The results of this study show that a definite improvement can be made in the 1000-mb forecasts obtained from this model by using the term $S$ as the thickness advection term.

A straightforward way of applying the knowledge gained here would be to use a four-level geostrophic model instead of the two-level model. A four-level model using initial data at 1000mb, 850mb, 700mb, and 500mb would give better 1000-mb forecasts than those made using the two-level model. With the aid of a modern high-speed computer, this method is practical and its possibilities should be explored.
BIBLIOGRAPHY


----------, 1957: Graphical integration of a two-level model. J. Meteor., 14, 38–42


APPENDIX
Fig. 2. $M_0$ field—Contours in 10's of feet

$\Delta = 250 \text{ km}$

$\frac{E}{D} = 6.21 \quad \frac{f}{f_m} = 10^{-4} \text{ sec}^{-1}$
Fig. 3 $G \times M_0$ field Contours are 10's of feet
Fig. 4. Grid point map
Fig. 5a. 23 Feb. 1962
SURFACE WEATHER MAP AND STATION WEATHER AT 1:00 A.M. E.S.T.

Fig. 5c. 25 Feb. 1962
Fig. 6b. 500mb
25 Feb. 1962, 0000CT
Fig. 6c. 500mb
26 Feb. 1962, 0000 UTC
Fig. 6d. 500mb
27 Feb. 1962, 0000Z
Fig. 7a. Forecast X(100's of feet) (Geostrophic)
23 Feb. 1962, 0000 GMT
Fig. 7b. Forecast X (100's of feet)
(geostrophic)
24 Feb. 1962, 0000 GMT
Fig. 7c. Forecast X(100's of feet) (Geostrophic)
25 Feb. 1962, 00GCT
Fig. 7d. Forecast X (100's of feet) (Geostrophic)
26 Feb. 1962, 00GCT
Fig. 7e. Forecast X (100's of feet)
(Geostrophic)
27 Feb. 1962, 00GCT
Fig. 8a. Observed X (100's of feet)
22 Feb. 1962, 0000Z
Fig. 8b. Observed X (100's of feet)
23 Feb. 1962, 0000CT
Fig. 8c. Observed X (100's of feet)
24 Feb. 1962, 0000 GMT
Fig. 8d. Observed X (100's of feet)
25 Feb. 1962, 00GCT
Fig. 8e. Observed X (100's of feet)
26 Feb. 1962, 0000CT
Fig. 8f. Observed X (100's of feet)
27 Feb. 1962, OOGCT
Fig. 9a. Forecast X - Observed X (10's of feet)  
(Geostrophic Case)  
23 Feb. 1962, 0000 UTC
Fig. 9b. Forecast $X$ - Observed $X$ (10's of feet)
(Geostrophic Case)
24 Feb. 1962, 00GCT
Fig. 9c. Forecast X – Observed X (10's of feet)
(Geostrophic Case)
25 Feb. 1962, 00GCT
Fig. 9d. Forecast X - Observed X (10's of feet)
(Geostrophic Case)
26 Feb. 1962, 00GCT
Fig. 9e. Forecast X - Observed X (10's of feet) (Geostrophic Case)
27 Feb. 1962, 00GCT
Fig. 10a. Forecast X - Observed X (10's of feet)
(Actual wind case)
25 Feb. 1962, 0000CT
Fig. 10b. Forecast X - Observed X (10's of feet)
(Actual wind case)
- 26 Feb. 1962, 0000 GMT
Fig. 10c. Forecast X – Observed X (10’s of feet)
(Actual wind case)
27 Feb. 1962, 0000 GMT
Fig. 11a. Forecast X (Geostrophic) - Forecast X (Actual wind)
(10's of feet)
25 Feb. 1962, 00GCT
Fig. 11b. Forecast X (Geostrophic) - Forecast X (Actual wind)
(10's of feet)
26 Feb. 1962, 00GCT
Fig. 11c. Forecast X (Geostrophic) – Forecast X (Actual wind)
(10's of feet)
27 Feb. 1962, 0000CT
Fig. 12a. $\bar{S} = (V_0 \cdot V 1/2, \bar{Z}_0)$ (ft/hr)
25 Feb. 1962, 00GCT
Fig. 12c. \( S = \left(-v_0 \cdot V_{1/2} Z_0\right) \) (ft/hr)
27 Feb. 1962, 0000CT
Fig. 13b. \( \vec{S} - \left( \frac{\vec{V}^2}{\rho} - \frac{1}{2} \vec{V} \cdot \vec{S} \right) \) (ft/hr)
26 Feb. 1962, 0000UT
Fig. 13c. $\bar{S} = (-\bar{V}_g \cdot V \bar{1}/2 \bar{z})_0$ (ft/hr)
27 Feb. 1962, 0000CT
Fig. 14a. $\vec{S} - \vec{S}_g$ (ft/hr)
25 Feb. 1962, OOGCT
Fig. 1.1b. $S - S_g$ (ft/hr)
26 Feb. 1962, 0000 CT
Fig. 1hc. $S - S_g \ (ft/hr)$
27 Feb. 1962, 0000 UTC
Geostrophic Winds at Levels Indicated

COLD

500mb

700mb

1000mb

850mb

WARM

Assumed (Modeled) Hodograph

COLD

500mb

1000mb

WARM

Fig. 15. Sample hodograph using geostrophic winds. Approximately those of North Platte, 0000Z 27 Feb. 1962
Fig. 16a. Forecast Y - Observed Y (10's of feet)
(Geostrophic winds)
25 Feb. 1962, 0000 GMT
Fig. 16b. Forecast Y - Observed Y (10's of feet)
(Geostrophic winds)
26 Feb. 1962, 00GCT
Fig. 16c. Forecast Y - Observed Y (10's offeast) (Geostrophic winds)
27 Feb. 1962, OOGCT