STEERING AND ROLL RATE CONTROL
OF A BOOST VEHICLE IN THE ATMOSPHERE

by

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December 1985

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Submitted to the Department of Aeronautics
and Astronautics in partial fulfillment of the
requirements for the degree of Master of Science
in Aeronautics and Astronautics

ABSTRACT

A new pitch-plane steering method and a new method of controlling
roll rate by means of a yaw steering modification are developed for the
Stage 1 atmospheric boost phase of a multi-stage solid rocket,
gimballed-engine boost vehicle. These steering methods are based on an
overall concept of a velocity-direction steering (with roll rate control
modification) system that generates the angle of attack commands used by
an angle of attack control system.

For pitch-plane steering, several zero-angle-of-attack trajectories are generated which reach a desired dynamic pressure at Stage 1
burnout. It is shown that this entire family of alternative trajectories
can be well approximated by functional relationships that give the value
of pitch-plane flight path angle corresponding to any combination of
altitude and earth-relative velocity. It is further shown that if the
velocity-direction steering is used to compute the commanded pitch-plane
velocity direction (i.e., flight path angle) from these relationships the
resulting trajectory will achieve close to the desired dynamic pressure
at engine burnout. In addition, a method to eliminate the effect of
launch attitude variations on the terminal dynamic pressure is developed.

For roll rate control, a method is developed that explicitly takes
into account the roll-axis torques that are produced by aerodynamic and
engine lateral force components as a result of vehicle asymmetry. This
method predicts the roll-torque consequences of replacing the commanded
yaw angle of attack generated by velocity-direction steering by an alter-
native command for purposes of roll rate control. Algorithms are devel-
oped which determine the magnitude and polarity of this alternative com-
mand and also determine when it is to be used, based on the sometimes
competing requirements of velocity-direction steering and roll rate
control.
Both the pitch-plane steering and roll-rate-control modification of yaw steering are tested under typical and worst case conditions.

Technical Advisor: Gilbert S. Stubbs  
Title: Staff, Charles Stark Draper Laboratory, Inc.

Thesis Advisor: Professor H. Philip Whitaker  
Title: Professor of Aeronautics and Astronautics, Emeritus; Senior Lecturer
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James Franklin Dailey

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LIST OF SYMBOLS

Math Symbols (where X and Y are generic variables)

\( \dot{X} \) \hspace{1cm} \text{Derivative of } X \text{ with respect to time; rate}

\( \ddot{X} \) \hspace{1cm} \text{Second derivative of } X \text{ with respect to time; acceleration}

\( \Delta X \) \hspace{1cm} \text{Incremental deviation in } X \text{ from a reference value}

\( |X| \) \hspace{1cm} \text{Absolute value of } X

\( \hat{X} \) \hspace{1cm} \text{Estimated value of } X

\( \overline{X} \) \hspace{1cm} \text{Vector}

\( \perp \) \hspace{1cm} \text{Perpendicular}

\( |\overline{X}| \) \hspace{1cm} \text{Magnitude of } \overline{X}

\( \overline{X} \times \overline{Y} \) \hspace{1cm} \text{Cross product of } \overline{X} \text{ and } \overline{Y}

\text{UNIT (}\overline{X}\text{)} \hspace{1cm} \text{Unit vector in the direction of } \overline{X}

\( \overline{X} \cdot \overline{Y} \) \hspace{1cm} \text{Dot product of } \overline{X} \text{ and } \overline{Y}

\( \chi \) \hspace{1cm} \text{Angle}

\( [X] \) \hspace{1cm} \text{Matrix}
Greek Symbols

\( \alpha_p \)  
Angle of attack in pitch

\( \alpha_{PC} \)  
Pitch angle of attack command

\( \alpha_{PW} \)  
Angle between the earth-relative and air-relative vehicle velocity vectors in the pitch plane

\( \alpha_{TOT} \)  
Total angle of attack

\( \alpha_{TOTC} \)  
Total angle of attack command

\( \alpha_{TOTMAX} \)  
Maximum total angle of attack

\( \alpha_{TOTMAXC} \)  
Maximum total angle of attack command

\( \alpha_Y \)  
Angle of attack in yaw

\( \alpha_{YC} \)  
Yaw angle of attack command

\( \alpha_{YRC} \)  
Yaw angle of attack command computed for roll rate control

\( \alpha_{YVC} \)  
Yaw angle of attack command computed for control of cross track velocity

\( \alpha_{YW} \)  
Angle between the earth-relative and air-relative vehicle velocity vectors in the yaw plane

\( \gamma \)  
Flight path angle

\( \gamma_{BIASC} \)  
Commanded bias to flight path angle command to adjust for launch angle
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<td>$\gamma_{C}$</td>
<td>Flight path angle command</td>
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<td>$\gamma_{\text{end}}$</td>
<td>Flight path angle at the end of Stage one</td>
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<td>$\gamma_{\text{hi}}$</td>
<td>Flight path angle for a zero-angle-of-attack trajectory that is above a nominal trajectory in altitude. (A function of earth-relative velocity.)</td>
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<td>$\gamma_{\text{lo}}$</td>
<td>Flight path angle for a zero-angle-of-attack trajectory that is below a nominal trajectory in altitude. (A function of earth-relative velocity.)</td>
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<td>$\gamma_{\text{mid}}$</td>
<td>Flight path angle for a nominal zero-angle-of-attack trajectory based on a nominal launch angle and terminating at 1200 psf dynamic pressure. (A function of earth-relative velocity.)</td>
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<td>$\delta$</td>
<td>Engine nozzle deflection angle</td>
</tr>
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<td>$\delta_{p}$</td>
<td>Nozzle deflection in the vehicle body pitch plane</td>
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<tr>
<td>$\delta_{y}$</td>
<td>Nozzle deflection in the vehicle body yaw plane</td>
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<td>$\theta$</td>
<td>Pitch angle of the vehicle. Local horizontal to body x axis.</td>
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<td>$\theta_{\text{INIT}}$</td>
<td>Pitch launch angle of vehicle</td>
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<td>$\Delta\theta_{\text{LNCH}}$</td>
<td>Deviation in launch angle from nominal launch angle</td>
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<td>$\sigma_{X}$</td>
<td>Standard deviation of X</td>
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<td>$\Sigma$</td>
<td>Summation</td>
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<td>$\phi$</td>
<td>Roll angle</td>
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\( \phi_M \)  
Measured roll angle

\( \chi_{\text{XAERO}} \)  
Angle about the vehicle x axis that the aerodynamic normal force makes in relation to the vehicle negative z axis

\( \psi \)  
Yaw angle of the vehicle

**English Symbols**

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<th>Definition</th>
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<tr>
<td>( A_{ZC} )</td>
<td>Pitch plane component of commanded acceleration normal to the vehicle x axis</td>
</tr>
<tr>
<td>( \text{AREA} )</td>
<td>Cross sectional area of vehicle</td>
</tr>
<tr>
<td>( C_A )</td>
<td>Coefficient of aerodynamic axial force</td>
</tr>
<tr>
<td>( C_{Ng} )</td>
<td>Derivative of coefficient of aerodynamic normal force with respect to ( \alpha )</td>
</tr>
<tr>
<td>( [C]_{\text{MUB+}} )</td>
<td>Measured direction cosine matrix from unrolled vehicle body axes to body axes</td>
</tr>
<tr>
<td>( C_l )</td>
<td>Coefficient of roll torque due to raceway</td>
</tr>
<tr>
<td>( C_{lC} )</td>
<td>Commanded raceway torque coefficient</td>
</tr>
<tr>
<td>( \text{CG} )</td>
<td>Center of gravity of the vehicle in body coordinates</td>
</tr>
<tr>
<td>( \text{CG}_{\text{EQ}} )</td>
<td>Equivalent cg. Position of the cg that would result in the same steady-state roll torque as the combination of center of gravity and engine hinge point offsets.</td>
</tr>
<tr>
<td>( \text{CG}_{\text{EQy}} )</td>
<td>Vehicle body y axis component of ( \text{CG}_{\text{EQ}} )</td>
</tr>
</tbody>
</table>
$CG_{EQ_Z}$  Vehicle body z axis component of $CG_{EQ}$

$CG_X$  Vehicle body x axis component of $CG$

$CG_Y$  Vehicle body y axis component of $CG$

$CG_Z$  Vehicle body z axis component of $CG$

$CGEQTORQC$  Commanded roll torque due to equivalent cg position

$CGTORQ$  Roll torque due to center of gravity offset

$CGTORQ_{AERO}$  Component of $CGTORQ$ due to aerodynamic forces

$CGTORQ_{ENG}$  Component of $CGTORQ$ due to engine forces, assuming zero hinge point offset

$F_A$  Axial aerodynamic force on the vehicle

$F_{AERO}$  Aerodynamic force normal to the vehicle x axis

$F_{AERO_{RCB}}$  Commanded aerodynamic normal force in vehicle body coordinate frame when roll rate control is used

$F_{AEROLIM}$  Limit on $F_{AERO}$ command, 4000 lbs.

$F_{AERO_{RC}}$  Commanded aerodynamic normal force in unrolled vehicle body coordinates when roll rate control is used

$F_{AEROY_B}$  Vehicle body y axis component of $F_{AERO}$

$F_{AEROY_{RC}}$  Vehicle unrolled y axis component of roll-rate-control commanded $F_{AERO}$
\begin{align*}
F_{AEROZ_B} & \quad \text{Vehicle body z axis component of } F_{AERO} \\
F_{AEROZ_C} & \quad \text{Vehicle unrolled z axis component of commanded } F_{AERO} \text{ without roll rate control} \\
F_{ENG} & \quad \text{Engine force vector normal to the vehicle x axis in unrolled body coordinates} \\
F_{ENGRC} & \quad \text{Commanded engine force vector normal to the vehicle x axis in unrolled body coordinates when roll control is used} \\
F_{ENGR_CB} & \quad \text{Commanded engine forces normal to the vehicle x axis in body coordinates when roll rate control is used} \\
F_{ENG_B} & \quad \text{Engine force vector normal to the vehicle x axis in body coordinates} \\
F_{ENGY_B} & \quad \text{Vehicle body y axis component of } F_{ENG_B} \\
F_{ENGZ_B} & \quad \text{Vehicle body z axis component of } F_{ENG_B} \\
F_{TOT} & \quad \text{Total forces on vehicle, aerodynamic and engine, in unrolled body coordinates} \\
F_{TOTYRCB} & \quad \text{Vehicle body y axis component of commanded } F_{TOT} \text{ when roll rate control is used} \\
F_{TOTZRCB} & \quad \text{Vehicle body z axis component of commanded } F_{TOT} \text{ when roll rate control is used} \\
G & \quad \text{Acceleration due to gravity} \\
H & \quad \text{Altitude} \\
H_{end} & \quad \text{Altitude at the end of Stage 1}
\end{align*}
$H_{h1}$ Altitude for a zero-angle-of-attack trajectory that is above a nominal trajectory. (A function of earth-relative velocity.)

$H_{lo}$ Altitude for a zero-angle-of-attack trajectory that is below a nominal trajectory. (A function of earth-relative velocity.)

$H_{mid}$ Altitude for a nominal zero-angle-of-attack trajectory based on a nominal launch angle and terminating at 1200 psf dynamic pressure. (A function of earth-relative velocity.)

$H_{NG}$ Engine hinge point offset of the vehicle in body coordinates

$H_{NGy}$ Vehicle body y axis component of $H_{NG}$

$H_{NGz}$ Vehicle body z axis component of $H_{NG}$

$H_{NGTORQ}$ Roll torque due to engine hinge point offset, with cg offset assumed to be zero

$hp$ Hinge point

$HP$ High pass (relating to filter)

$I_{XX}$ Roll moment of inertia

$I_{YY}$ Pitch moment of inertia

$I_{ZZ}$ Yaw moment of inertia

$K_{ΔYH}$ Steering functionalization gain

$K_{BURN}$ Ratio of actual thrust to nominal thrust
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{CG}$</td>
<td>CG estimator gain applied to the change in the cg estimate to obtain the new cg estimate</td>
</tr>
<tr>
<td>$K_{EQ}$</td>
<td>Equivalent cg gain, equal to $CG_x/L_{cp}$</td>
</tr>
<tr>
<td>$K_{STEER}$</td>
<td>Steering gain</td>
</tr>
<tr>
<td>$\bar{L}_{cp}$</td>
<td>Center of pressure location in relation to cg in vehicle body coordinate frame</td>
</tr>
<tr>
<td>$\bar{L}_{hp}$</td>
<td>Vector in the vehicle coordinate frame whose x component is $CG_x$ and whose y and z components are the distance from the engine hinge point offset to the center of gravity in the respective directions</td>
</tr>
<tr>
<td>LNCH</td>
<td>Launch</td>
</tr>
<tr>
<td>LP</td>
<td>Low pass (relating to filter)</td>
</tr>
<tr>
<td>MASS</td>
<td>Mass of vehicle</td>
</tr>
<tr>
<td>PHIXIA</td>
<td>Angle $\bar{F}_{AERO}$ makes in relation to the raceway</td>
</tr>
<tr>
<td>PHIXIA$_C$</td>
<td>Commanded value of PHIXIA</td>
</tr>
<tr>
<td>$Q$</td>
<td>Dynamic pressure</td>
</tr>
<tr>
<td>RACEWAYTORQ</td>
<td>Roll torque due to raceway</td>
</tr>
<tr>
<td>RACEWAYTORQ$_C$</td>
<td>Commanded value of RACEWAYTORQ</td>
</tr>
<tr>
<td>RADIUS</td>
<td>Cross sectional radius of vehicle</td>
</tr>
<tr>
<td>ROLLTORQ</td>
<td>Total roll torque on the vehicle, differs from $TORQUE_X$ because ROLLTORQ includes RACEWAYTORQ</td>
</tr>
<tr>
<td>ROLLTORQ$_{EST}$</td>
<td>Estimated roll torque</td>
</tr>
<tr>
<td>Variable</td>
<td>Description</td>
</tr>
<tr>
<td>----------</td>
<td>-------------</td>
</tr>
<tr>
<td>THRUST</td>
<td>Engine thrust</td>
</tr>
<tr>
<td>TORQNPC</td>
<td>Commanded value of roll torque due to a negative $\alpha_{Y_RC}$</td>
</tr>
<tr>
<td>TQRQP_C</td>
<td>Commanded value of roll torque due to a positive $\alpha_{Y_RC}$</td>
</tr>
<tr>
<td>TORQUE</td>
<td>Vector whose $y$ and $z$ components represent the total pitching and yawing torques on the vehicle</td>
</tr>
<tr>
<td>TORQUEX</td>
<td>Roll torque resulting from cg and engine hinge point offsets</td>
</tr>
<tr>
<td>TORQUEY</td>
<td>Pitching torque on vehicle</td>
</tr>
<tr>
<td>TORQUEZ</td>
<td>Yawing torque on vehicle</td>
</tr>
<tr>
<td>VEND</td>
<td>Earth-relative velocity magnitude at the end of Stage 1</td>
</tr>
<tr>
<td>VE</td>
<td>Earth-relative velocity</td>
</tr>
<tr>
<td>VNp</td>
<td>Component of earth-relative velocity normal to vehicle $x$ axis in the pitch plane</td>
</tr>
<tr>
<td>VNy</td>
<td>Component of earth-relative velocity normal to vehicle $x$ axis in the yaw plane</td>
</tr>
<tr>
<td>VW</td>
<td>Velocity of vehicle with respect to the wind</td>
</tr>
<tr>
<td>X_B, Y_B, Z_B</td>
<td>Vehicle body coordinate frame with $X_B$ along the vehicle centerline and the $Y_B - Z_B$ plane containing the engine hinge point</td>
</tr>
<tr>
<td>X_E, Y_E, Z_E</td>
<td>Earth-fixed coordinate frame with $X_E$ the local horizontal and $-Z_E$ local vertical</td>
</tr>
</tbody>
</table>
$X_{UB}, Y_{UB}, Z_{UB}$ Vehicle unrolled body coordinate frame with $X_{UB}$ along the vehicle centerline, the $Y_{UB} - Z_{UB}$ plane containing the engine hinge point, and $Y_{UB}$ parallel to the $X_E - Y_E$ plane.
CHAPTER 1

INTRODUCTION

This thesis presents a new pitch-plane steering method and a new method of controlling roll rate by means of a yaw steering modification for the Stage 1 atmospheric boost phase of a multi-stage solid rocket, gimballed-engine boost vehicle. In Stage 1, flight control is achieved through the control of the engine nozzle deflection for an engine that is gimballed so that it is moveable about both the pitch and yaw axes. The flight control system used to stabilize the inherently unstable boost vehicle uses angle of attack feedback. This method of flight control was designed by Bonnice [1] and further modified and used by Fader [2]. This thesis makes extensive use of their methods as a framework on which to investigate the new steering scheme and build the roll rate control.

1.1 Steering

The main purpose of the steering scheme is to cause the boost vehicle to follow a desired pitch-plane trajectory and control the vehicle motion in the cross-track direction. The cross-track steering used in this thesis was devised by Bonnice [1], and it is used to drive any cross-track velocity deviation to zero. The objective of the trajectory steering is to steer the vehicle in the pitch plane so as to achieve a specified dynamic pressure of 1200 psf at the end of Stage 1. Fader [2] investigated a method of trajectory steering to accomplish this objective which determined a flight path angle command, based on vehicle altitude and sensed velocity, for the vehicle to follow a single pre-computed
zero-angle-of-attack trajectory with a terminal dynamic pressure of 1200 psf. The steering scheme investigated in this thesis differs from Fader's method by re-computing a new zero-angle-of-attack trajectory based on the vehicle's current flight path angle and height versus earth-relative velocity instead of following a single pre-computed trajectory. This computation is based on storage of two pre-computed trajectories and interpolation/extrapolation between them. A second requirement of the steering and control system is that load relief be supplied to limit the forces normal to the vehicle longitudinal axis. In this thesis, the objective is to limit the product of dynamic pressure and total angle of attack to below a specified value of 10,000 lb-deg/ft². By using angle of attack as the controlled variable in the flight control system, implicit load relief is then provided as the vehicle steers to the zero-angle-of-attack trajectory. Explicit load relief is provided for by limiting the angle of attack command such that when the vehicle deviates from the desired trajectory, forces imposed on the vehicle to return it do not exceed the maximum allowable.

1.2 Roll Rate Control

The boost vehicle rolls about its longitudinal axis due to: roll torques generated by aerodynamic forces when the lateral center of gravity is offset from the center of pressure, by thrust forces when the engine hinge point is offset from the center of gravity, by aerodynamic forces on an external raceway, and also as a result of launch initial conditions of roll rate and roll angle. Roll rate control of the vehicle is required if the maximum expected roll rate exceeds the specified limit. In this thesis the limit is assumed to be 50 deg/s. This thesis will determine the maximum roll rates for various disturbance and launch conditions and then investigate a method to control the roll rate. Conventional boost vehicles, without aerodynamic control surfaces, use jets to provide roll torques to control the roll rate when required. For this
thesis, however, the vehicle is assumed to have no jets available for Stage 1. The method developed uses control of angle of attack in the yaw plane to provide these roll torques, thereby minimizing the effect of roll rate control on pitch-plane trajectory steering. In developing an effective roll rate control, it will be shown that it is necessary to estimate the effects of the center of gravity offset and engine hinge point offset, and the raceway aerodynamic torques. To complete the roll-rate-control modification and to further reduce its effects on trajectory shaping, a scheme to time share the yaw-angle-of-attack command for roll rate control with the cross-track-velocity steering is developed.
CHAPTER 2

DESCRIPTION OF BOOST SYSTEM WITHOUT ROLL RATE CONTROL

2.1 Vehicle

The boost vehicle considered in this thesis is a multistage spacecraft powered by a solid rocket engine. It is assumed that the vehicle's mass properties (mass, moments of inertia, and center of gravity location) can be estimated to specified degrees of accuracy. The thrust is assumed to be constant in flight at a level determined by the temperature of the solid rocket engine at liftoff. The specific impulse is assumed to be constant and independent of temperature, resulting in a total thrusting period that is inversely proportional to the thrust level. For the purposes of this thesis, a thrust estimator is assumed to provide a specified accuracy over the entire boost period. The time to burnout is assumed to be computed from estimated thrust. Errors in the knowledge of the thrust estimate as well as errors in the knowledge of lateral center of gravity location and engine hinge point offset were simulated as noted in the thesis.

This thesis is concerned with the first stage steering and control of the vehicle. In Stage 1, attitude control is achieved through a gim-balled engine which permits control of the thrust direction. The engine nozzle has two rotational degrees of freedom about the vehicle pitch and yaw axes. The nozzle is physically limited to a maximum total deflection of six degrees relative to the vehicle roll axis. This limit is achieved in the vehicle control system by applying a "bucket" limit that limits the resultant of commanded nozzle pitch and yaw deflections to six
degrees. A nozzle rate limit of 40 deg/sec was also imposed. The nozzle actuator was approximated by a first order lag with a time constant of 1/35 seconds.

2.1.1 Translational Dynamics

The axial force acting on the vehicle is determined by the sum of the axial components of thrust and aerodynamic force. Because the engine nozzle deflection angle is small, the axial thrust force is assumed equal in magnitude to the total thrust force, and the lateral thrust force is assumed equal in magnitude to the product of the total thrust and the nozzle deflection angle.

In simulating the lateral acceleration of the vehicle, the thrust and aerodynamic forces are resolved into components along the vehicle y and z axes, and the accelerations are computed along these axes. Later, it will be shown that in the estimation of lateral forces for control purposes it is convenient to resolve these forces into components along non-rotating y and z axes.

The forces acting on the vehicle in the pitch plane are shown in Figure 2-1. The acceleration normal to the direction of the earth-relative velocity vector is given by Equation 2.1, where a positive $v_{NP}'$ causes a positive $\alpha_p$.

$$
\begin{align*}
\dot{v}_{NP}' &= \frac{1}{\text{MASS}} \left[ \text{THRUST} \sin (\alpha_p - \alpha_{p_w} - \delta_p) \\
&- F_A \sin (\alpha_p - \alpha_{p_w}) - F_{AEROZ_B} \cos (\alpha_p - \alpha_{p_w}) \\
&- \text{MASS} \ G \cos \gamma \right] 
\end{align*}
$$

(2.1)

Similarly, for the yaw plane, the acceleration normal to the earth-relative velocity vector is given by Equation 2.2, from the forces shown in Figure 2-2.
Figure 2-1. Pitch Plane Forces.

Figure 2-2. Yaw Plane Forces.
\[ v_{NY} = \frac{1}{\text{MASS}} \left[ \text{THRUST} \sin (\alpha_y - \alpha_{yw} - \delta_y) \right. \]

\[ \left. - F_A \sin (\alpha_y - \alpha_{yw}) + F_{AERO_B} \cos (\alpha_y - \alpha_{yw}) \right] \quad (2.2) \]

2.1.2 Rotational Dynamics

To determine the vehicle angular accelerations resulting from the normal aerodynamic force and the engine thrust, it is necessary to define the effective points of application of these forces relative to the vehicle center of gravity (cg). For the engine thrust the effective point of application is called the "hinge point" (hp). For the normal aerodynamic force the effective point of application is referred to as the "center of pressure" (cp). These two points are defined relative to the cg by two vectors \( \overrightarrow{L}_{hp} \) and \( \overrightarrow{L}_{cp} \):

\[ \overrightarrow{L}_{hp} \triangleq \text{vector from the hp to the cg} \]

\[ \overrightarrow{L}_{cp} \triangleq \text{vector from the cg to the cp} \]

For the sake of convenience, the origin of the body coordinate system is assumed to be at the intersection with the x axis of a plane perpendicular to that axis which passes through the engine hinge point. In this coordinate frame the coordinates of the cg are \( CG_x, CG_y, \) and \( CG_z \). Also the coordinates of the engine hinge point are \( HNG_x (= 0, \text{by definition}), HNG_y, \) and \( HNG_z \). Accordingly, from Figure 2-3.

\[ \overrightarrow{L}_{hp} = (CG_x, CG_y - HNG_y, CG_z - HNG_z) \quad (2.3) \]
Moreover, assuming that the center of pressure is on the x axis a distance $L_{cpX}$ forward of the center of gravity, then, from Figure 2-4

$$\overline{L_{cp}} = (L_{cpX}, -CG_y, -CG_z)$$  \hspace{1cm} (2.4)

The vector moments produced by the engine thrust vector, $\overline{F_{ENG}}$, and the aerodynamic force vector, $\overline{F_{AERO}}$, are equal to the cross products of these vectors and the vectors $\overline{L_{hp}}$ and $\overline{L_{cp}}$, respectively. The total torque on the vehicle, $\overline{TORQUE}$, is given by* 

$$\overline{TORQUE} = \overline{L_{cp}} \times \overline{F_{AERO}} + \overline{F_{ENG}} \times \overline{L_{hp}}$$  \hspace{1cm} (2.5)

*The order of the cross products in the calculation of $\overline{TORQUE}$ is a result of the definitions of $\overline{L_{cp}}$ and $\overline{L_{hp}}$ and the use of the geometric center of the vehicle instead of the cg as the x axis of the coordinate frame. This is done to allow an easy examination of the individual causes of the roll torques.
The $y$ component of this vector is the torque about the vehicle pitch axis while the $z$ component is the torque about the yaw axis. The pitch and yaw angular acceleration is then

\[
\ddot{\theta} = \frac{\text{TORQUE}_y}{I_{yy}} \quad (2.6)
\]

\[
\ddot{\psi} = \frac{\text{TORQUE}_z}{I_{zz}} \quad (2.7)
\]

Normally, from the $x$ component of TORQUE we could determine the roll angular acceleration. However, there is an additional roll torque component, due to an external raceway on the vehicle, that must be added. This will be discussed in the next section in which the roll dynamics are examined in detail.
2.1.3 Roll Dynamics

There are four main factors that cause the vehicle to roll about its longitudinal (x axis) during flight:

1. Initial conditions of roll angle and roll rate as the vehicle is ejected from its launcher prior to ignition
2. Aerodynamic asymmetry
3. Center of gravity asymmetry
4. Engine hinge point offset

The effect of each of these factors on vehicle roll rate will be examined in this section.

2.1.3.1 Roll Angle and Roll Rate Initial Conditions

The first factor that produces vehicle roll motion is the ejection torque acting on the vehicle prior to engine ignition. The initial roll rate produced by this torque is of obvious importance because it adds to the maximum roll rate obtainable during flight. The initial roll rate, in combination with the initial roll attitude, is of importance for still another reason: together, they influence the subsequent roll attitude of the vehicle throughout the flight. This is important because the roll torque during flight depends not only on the aerodynamic and thrust forces but on the roll attitude of the vehicle relative to these forces. The assumed allowable roll rate specification used in this thesis is a maximum initial rate of ±10 deg/sec, while the initial roll attitude is not constrained. Figure 2-5 shows the definition of the roll rate (\( \dot{\phi} \)) and roll angle (\( \phi \)) used in this thesis. The sign of the roll rate is defined in accordance with the right hand rule, while the roll angle is defined as the deflection of the vehicle negative z axis from the vehicle unrolled negative z axis. The polarity of \( \phi \) is in accordance with the right hand rule.

2.1.3.2 Aerodynamic Asymmetry

The second factor affecting the roll rate of the vehicle is aerodynamic asymmetry about the vehicle roll axis. This asymmetry is caused
by a raceway located along only one side of the vehicle that carries interstage cables. The aerodynamic effect of the raceway is to produce an offset in the center of pressure from the center of the vehicle toward the raceway, as shown in Figure 2-6. The offset increases and decreases depending on the raceway location in relation to the normal aerodynamic force. As a result of the center of pressure offset, the aerodynamic normal force produces a roll torque. The torque can be computed from the coefficient of rolling moment, $C_\phi$. This coefficient is a function of three parameters: angle of attack, Mach number, and the angle the aerodynamic force normal to the vehicle $x$ axis makes with the raceway location. The coefficient is obtained from experimentally determined tabulated values based on the above parameters. The first parameter, total angle of attack ($\alpha_{\text{tot}}$), is the angle between the vehicle $x$ axis and the wind-relative velocity, as shown in Figure 2-7. The total angle

Figure 2-5. Roll Angle and Roll Rate Definition.
of attack, by convention, is always positive. The position of the x axis in relation to $V_w$ is described by the angle of attack in yaw, $\alpha_y$, and angle of attack in pitch, $\alpha_p$. $\alpha_y$ is the angle between $V_w$ and its projection on the vehicle unrolled x-z plane, while $\alpha_p$ is the angle between the projection of $V_w$ on the vehicle unrolled x-z plane and the vehicle x axis. These angles are positive as shown in Figure 2-8. For small angles, $\alpha_p$ and $\alpha_y$, the total angle of attack can be approximated by the equation

$$\alpha_{tot} = \sqrt{\alpha_p^2 + \alpha_y^2}$$

(2.8)

The second parameter used to compute the rolling moment is the Mach number which is a function of vehicle velocity and air density. The third
Figure 2-7. Total Angle of Attack.

Figure 2-8. $\alpha_p$ and $\alpha_y$. 
parameter used to compute the rolling moment is the angle that describes the orientation of the raceway with respect to the normal aerodynamic force. This angle is determined by considering the projection of the raceway and aerodynamic forces onto the vehicle y-z plane. The angle, PHIXIA, between the aerodynamic normal force and the projected raceway orientation is described in Figure 2-9. It can be computed as the difference between the following two angles:

1. The angle from the vehicle unrolled negative z-axis, -ZUB, to the vector from the origin of the y-z plane to the projection of the raceway onto the y-z plane. This is equivalent to the vehicle roll angle, \( \phi \), because the raceway projection is along the vehicle (rolled) negative z-axis.

2. The angle, \( X_{FAERO} \), from the unrolled negative z axis, -ZUB, to the normal aerodynamic force vector \( F_{FAERO} \).

\[
PHIXIA = \phi - X_{FAERO}
\]  

(2.9)

Using these three parameters (\( q_{tot} \), Mach number, PHIXIA), the raceway torque coefficient is interpolated from tables of experimental data. The sign of \( C_\ell \) determines whether the actual raceway torque is positive or negative. The sign of \( C_\ell \) depends on the sign of PHIXIA and the Mach number (above Mach 1.5 and for PHIXIA > 90°, the sign of \( C_\ell \) may be reversed due to Mach number effects). Once \( C_\ell \) is obtained, the raceway torque (RACEWAYTORQ) is calculated from

\[
RACEWAYTORQ = C_\ell \times \frac{Q \times AREA \times RADIUS}{2}
\]  

(2.10)

where AREA is the cross-section area of the vehicle and RADIUS is the radius of the cross section. The result of this calculation is a roll torque component, due to the raceway, about the cg. This calculation and the tables of \( C_\ell \) assume that the center of gravity lies along the centerline of the vehicle. This assumption in general is not true but
results in negligible errors for the small lateral center of gravity offsets compared to the vehicle's radius considered in this thesis.

2.1.3.3 Center of Gravity Asymmetry

The third condition that affects the roll rate of the vehicle is the center of gravity asymmetry. As the solid rocket engine burns, consuming propellant mass, the center of gravity not only moves along the x axis but also moves radially across the vehicle cross section, in the y-z plane. The center of gravity position is defined as a vector, \( \vec{CG} \), in body axes, with y and z components indicating its offset relative to the vehicle centerline, as shown in Figure 2-10. For the vehicle used
Figure 2-10. Forces and Coordinates for CG Torques.

In this thesis, the lateral center of gravity offset distance from the vehicle centerline is specified as follows:

1. From engine ignition to a time halfway through Stage 1, the offset distance is negligible.

2. From the time halfway through Stage 1 to the time 3/4 of the way through Stage 1, the offset distance increases linearly with time from 0 to 0.1 inch along the Z body axis.

3. For the remainder of Stage 1, the offset remains constant at 0.1 inch.

4. In addition to the center of gravity offset behavior described in (1), (2), and (3), there is an unknown constant center of gravity offset superimposed on the known offset. This
constant offset is described statistically as having a three sigma value of 0.2 inches in any direction.

As a consequence of the center of gravity offset, a roll torque is produced about the center of gravity. This torque is caused by aerodynamic normal forces that are assumed to act through the centerline of the vehicle and by the engine force components normal to the vehicle x axis. In the description and calculation of roll torques caused by cg offsets alone, it is assumed that there is no engine hinge point offset. Thus, the engine force components also act through the center of the vehicle. Under this assumption, the roll torque caused by cg offsets is the product of the total normal force, $F_{\text{TOT}}$, and the moment arm from the geometric center of the vehicle to the center of gravity. As shown in Figure 2-10 and Equations 2.11 and 2.12, this torque can be calculated by transforming the aerodynamic and engine forces into vehicle body coordinates and then calculating the lateral center of gravity offset torque due to aerodynamic forces, $\text{CGTORQAERO}$, and the lateral center of gravity offset torque due to engine forces, $\text{CGTORQENG}$, separately; their algebraic sum being the total center of gravity torque, $\text{CGTORQ}$.

\begin{align*}
\text{CGTORQAERO} &= F_{\text{AEROY}}_{B} \cdot CG_{z} - F_{\text{AEROZ}}_{B} \cdot CG_{y} \quad (2.11) \\
\text{CGTORQENG} &= F_{\text{ENGY}}_{B} \cdot CG_{z} - F_{\text{ENGZ}}_{B} \cdot CG_{y} \quad (2.12) \\
\text{CGTORQ} &= \text{CGTORQAERO} + \text{CGTORQENG} \quad (2.13)
\end{align*}

2.1.3.4 Engine Hinge Point Offset

The fourth condition that affects the roll rate of the vehicle is the engine hinge point offset. The engine hinge point is offset from the vehicle centerline due to installation error, and its position is defined similar to the cg position as a vector $\text{HNG}$ in the vehicle body coordinate frame. (See Figure 2.11.) Although the hinge point may vary randomly from vehicle to vehicle, its value for any one vehicle may be assumed to
be constant during flight. In this thesis it is assumed to have a three sigma value of 0.16 inch. When the engine hinge point offset is different from the cg offset position in the $Y_B - Z_B$ plane, which is the usual case, the engine normal force components, acting through the hinge point, cause a roll torque about the cg. To be able to examine the effects of roll torques due to engine hinge point offsets alone, the lateral center of gravity is assumed to have no offset and to lie along the vehicle centerline. With this assumption, the hinge point offset torque, $HNGTORQ$, can be calculated in a manner similar to that used for $CGTORQ$ except that only the engine force components produce a hinge torque, as shown by Equation 2.14 and Figure 2-11.

$$HNGTORQ = F_{ENGZ_B} HNG_y - F_{ENGY_B} HNG_z$$ (2.14)
2.1.3.5 Total Roll Torque

The total roll torque on the vehicle can be calculated by algebraically adding together the three roll torque components about the center of gravity. That is, the center of gravity torque, the hinge torque, and the raceway torque, as shown by Equation 2.15.

\[
\text{ROLLTORQ} = \text{CGTORQ} + \text{HNGTORQ} + \text{RACEWAYTORQ}
\] (2.15)

As previously stated, in the determination of CGTORQ and HNGTORQ, the cg offset and hinge point offset were assumed to be in different positions. This was done so that the effects of center of gravity and engine hinge point offsets on the roll torque could be examined separately. When CGTORQ and HNGTORQ and added together in Equation 2.15, the effect of this assumption cancels out and the result, along with RACEWAYTORQ, is the true total roll torque about the center of gravity. The angular acceleration can be calculated from the total roll torque using

\[
\frac{d^2 \phi}{dt^2} = \ddot{\phi} = \frac{\text{ROLLTORQ}}{I_{xx}}
\] (2.16)

where \(I_{xx}\) is the rolling moment of inertia.

2.1.4 Atmospheric Conditions

Atmospheric disturbances in this thesis are limited to a choice between head, tail, and cross winds. These winds act horizontally (i.e., in the inertial x-y plane) with a speed versus altitude profile as shown in Figure 2-12. This profile places the maximum speed, which results from a wind spike and gust, at 30,000 feet. Since the maximum dynamic pressure on the vehicle occurs at approximately this altitude, the wind profile used represents a worst case scenario producing the maximum forces on the vehicle. The air density used in this thesis is from the U.S. Standard Atmosphere, 1962 tables. Standard day is assumed with no temperature deviations.
2.2 Measured Data

2.2.1 IMU Model

The inertial measuring unit (IMU) used in this vehicle is assumed to be equivalent to the CSDL-developed AIRS (Advanced Inertial Reference System). This instrument provides measurements of vehicle roll, pitch, and yaw attitude as well as "sensed velocity" signals resolved into the
three body axes. The sensed velocities represent the integrals of acceleration due to thrust and aerodynamic forces measured along three orthogonal, inertially fixed axes. These measurements are either used directly in the control loop or used as inputs to the estimators presented in this thesis and thus form an integral part of the overall system. The IMU model was assumed to have noise and deterministic errors equivalent to the AIRS system. In simulation runs presented in this thesis, when AIRS or IMU use is stated the associated noise and errors are also included.

An additional error source in IMU measurements is the installation misalignment or boresight error. This small misalignment of the IMU case relative to the vehicle was represented in terms of a single axis rotation of the IMU y and z axes about an axis midway between the two. This misalignment was included in simulation runs when boresight error is stated.

2.2.2 Non-IMU Error Sources

For this thesis it was assumed that the thrust could be estimated over the entire trajectory. An investigation by Whitaker [3] of a possible thrust estimator resulted in approximately a 2% estimation error. It was also assumed that the nozzle deflection could be measured and that there was accurate knowledge of aerodynamic and mass properties. Errors in the knowledge of vehicle properties were determined by Goss [4] to produce approximately a 12% one sigma error in the coefficients of the angle of attack and rate estimators. A fixed bias of 0.25 degrees was assumed in the measured nozzle deflection. This was applied in equal portions to each channel of the actuator in a worst case arrangement of +0.125 degrees in $\delta_p$ and -0.125 degrees in $\delta_y$. When so stated the above measurement errors were used in simulation runs.

2.3 Flight Computer Algorithms

In this section of the thesis the concepts and algorithms employed for steering, control, and estimation are presented. The original intent
of this thesis was to develop and evaluate a method for controlling the
roll rate of the vehicle. However, on the way to accomplishing this end
a new method of steering was developed. It was decided to describe and
evaluate this steering approach along with the method of roll rate con-
trol. Only a brief description is given of the angle of attack control
and estimation methods, since these are nearly identical to those used by
Fader [2] and Bonnice [1].

2.3.1 Steering
Steering of the vehicle in Stage 1 involves controlling the vehi-
cle so that it follows some desired trajectory to arrive at predetermined
terminal conditions at the end of the first stage. For this vehicle the
specified conditions are: a dynamic pressure of 1200 psf and a small
angle of attack (achieved in response to a zero-angle-of-attack command
to the control loop). The desired dynamic pressure was dictated by vehi-
cle specifications to maximize boost effectiveness, and the small angle
of attack was needed to minimize loads on the vehicle during the separa-
tion of Stage 1 and transition to Stage 2. To accomplish both of these
goals the desired trajectory was determined from a nominal zero-angle-
of-attack trajectory that terminated at 1200 psf dynamic pressure at
ingine burnout. The steering of the vehicle is divided into three
phases: (1) kick maneuver, (2) trajectory steering, (3) end of stage
steering. In addition, in phases 2 and 3 an additional steering algo-
rithm is used to control the vehicle cross-track velocity (perpendicular
to the trajectory plane).

2.3.1.1 Kick Maneuver
It is assumed that variations in the launch angle of the vehicle
in the pitch plane can be as much as ±29 degrees from the vertical (re-
sulting in launch angles of 61 degrees to 119 degrees). Since it was
found that the required vehicle attitude to begin the desired zero-angle-
of-attack trajectory, defined as the kick angle, was approximately 60
degrees, large attitude errors could initially exist. To quickly align the vehicle's attitude with that of the desired trajectory, a separate steering phase called the "kick maneuver" was devised. In this phase the vehicle control loop is designed as an attitude control loop where an attitude feedback signal is generated in the loop and compared with the attitude command signal (as opposed to the trajectory steering phase, presented in the next section, which uses angle-of-attack feedback).

During this phase, large attitude rates may result, and as a consequence, large angles of attack may be produced. These angles of attack do not result in large aerodynamic forces on the vehicle, however, because the dynamic pressure is small during this phase. The details of a kick maneuver based on commanding a constant attitude during a 5.8 second period are described by Fader [2]. In this thesis, it was decided to use a slightly different approach for the kick maneuver, one developed by James Herer of Autonetics. In this approach, the commanded attitude is varied so as to produce approximately a zero angle of attack at the end of a 12 second kick maneuver period.

2.3.1.2 Trajectory Steering

Most approaches to steering the vehicle in the boost phase involve the predetermination of a single desired trajectory for the vehicle to follow to reach the desired end conditions. One method to do this, similar to a method investigated by Fader [2], is to determine a single reference no-wind, zero-angle-of-attack trajectory of flight path angle and height as a function of vehicle sensed or earth-relative velocity (Fader used sensed velocity). This trajectory is determined after a kick maneuver from an assumed nominal launch attitude of 90 degrees (see Figure 2-13).
However, it is not in general necessary to follow one particular trajectory of $\gamma$ and $H$ versus $V_E$ to get to a desired end point. In fact, there is an infinite family of trajectories, as shown in Figure 2-14, with each trajectory having its own relationship between $\gamma$, $H$, and $V_E$, that will reach the end point. It would seem that the control problem might be eased by representing the entire family by a functional relationship,

$$\gamma = f(H, V_E) \quad (2.17)$$

and then controlling $\gamma$ based on current $H$ and $V_E$ values (rather than trying to force $\gamma$ and $H$ to conform simultaneously to a given function of $V_E$). The first question in considering this approach is whether it is feasible to implement the functional relationship of Equation 2.17.
The first step in investigating this method is to obtain a reference "family" of trajectories and then determine how, from this reference family, to determine all other possible trajectories. In Fader's method [2], the single reference trajectory is determined by launching the vehicle from a 90 degree launch attitude and then finding the attitude to kick the vehicle to such that if a zero-angle-of-attack trajectory is then flown the desired end conditions are reached. The values of $\gamma$ and $H$ as a function of $V_E$ are stored in a data file or functionalized for those launch conditions and form the reference trajectory. In this thesis, the family of zero-angle-of-attack trajectories that terminate at a common set of values of $Q$, $H$, and $V_E$ is found by reverse integration of the equations

$$\frac{d\gamma}{dt} = -\frac{G \cos \gamma}{V_E} \quad (2.18)$$
\[
\frac{dV_E}{dt} = \frac{1}{\text{MASS}} \left[ \text{THRUST} - Q \text{ AREA } C_A \right] - G \sin \gamma
\] (2.19)

\[
\frac{dH}{dt} = V_E \sin \gamma
\] (2.20)

In this reverse integration process the initial conditions for \( H \) and \( V_E \) are chosen to be the terminal conditions of the original reference trajectory. Three members of the infinite family of zero-angle-of-attack trajectories, including the original reference trajectory, were generated by specifying three different "initial" conditions for \( \gamma \): (1) the end condition, \( \gamma_{\text{end}} \), of the original trajectory, (2) \( \gamma_{\text{end}} + \Delta \gamma \), and (3) \( \gamma_{\text{end}} - \Delta \gamma \), as shown in Figure 2-15. The value of \( \Delta \gamma \) used to obtain the "high" and "low" trajectories was chosen as one degree. Values of \( \gamma \) and \( H \) versus \( V_E \) are stored for each of these reverse integrations and thus define the three trajectory members.

![Figure 2-15. Family of Trajectories from Reverse Integration.](image)

The steering algorithm, Equations 2.21-23, suggested by Stubbs, to steer the vehicle to the "nearest" zero-angle-of-attack trajectory uses
these three reference trajectories and interpolates between them to find the commanded flight path angle, \( \gamma_c \), as a function of current height and \( V_E \). This steering algorithm assumes that it is permissible to linearly interpolate between the family of reference trajectories and extrapolate beyond them if required. The steering algorithm equations are

\[
\gamma_c = \gamma_{\text{mid}} + K_{\Delta\gamma H} (H - H_{\text{mid}}) \tag{2.21}
\]

where

\[
K_{\Delta\gamma H} = \frac{\gamma_{\text{lo}} - \gamma_{\text{mid}}}{H_{\text{lo}} - H_{\text{mid}}} \quad \text{for } H < H_{\text{mid}} \tag{2.22}
\]

or

\[
K_{\Delta\gamma H} = \frac{\gamma_{\text{hi}} - \gamma_{\text{mid}}}{H_{\text{hi}} - H_{\text{mid}}} \quad \text{for } H > H_{\text{mid}} \tag{2.23}
\]

where the subscripts "hi," "mid," and "lo" refer to the three trajectory members with end conditions \( \gamma_{\text{end}} - \Delta\gamma \), \( \gamma_{\text{end}} \), \( \gamma_{\text{end}} + \Delta\gamma \), respectively.

Note: \( \gamma_{\text{hi}} \), \( \gamma_{\text{lo}} \), \( \gamma_{\text{mid}} \), \( H_{\text{hi}} \), \( H_{\text{lo}} \), \( H_{\text{mid}} \) are functions of \( V_E \).

The quantities \( \gamma_{\text{mid}}', H_{\text{mid}}', (\gamma_{\text{lo}} - \gamma_{\text{mid}}), \) and \(-(H_{\text{lo}} - H_{\text{mid}})\) in Equations (2.21) and (2.22) are plotted in Figure 2-16 as a function of the magnitude of earth-relative velocity. A similar set of plots is shown in Figure 2-17 for the case of an off-nominal high thrust (1.15 times the nominal thrust) and in Figure 2-18 for the case of an off-nominal low thrust (0.85 times the nominal thrust). (Off-nominal thrust can be expressed by the parameter, \( K_{\text{BURN}} \), where \( K_{\text{BURN}} = \text{THRUST}/(\text{NOMINAL THRUST}) \).) To test the linearity assumption for the steering algorithm another set of hi and lo trajectories was determined using \( \Delta\gamma \) of \( \pm 2 \) degrees in the reverse integration. The results show that the assumption was valid. Both the numerator and denominator of \( K_{\Delta\gamma H} \) increased by a factor of approximately 2 corresponding to the increase in
Δγ by a factor of ±2. This is demonstrated in Figure 2-19 in which the numerator and denominator are plotted as functions of V_E for the cases Δγ = +1, +2, -1 and -2. A similar set of plots is shown in Figure 2-20 for the off-nominal high thrust case and Figure 2-21 for the off-nominal low thrust case. Also it was found that in addition to the curves being linearly related they were mirror images about the middle reference trajectory. For implementation, then, only the mid and hi trajectories are needed as the lo trajectory is merely the negative or mirror image of the hi trajectory. Specific values supporting this hypothesis are given in Table 2-1.
Figure 2-16. Variables for Nominal Thrust Trajectories with $\Delta \gamma = \pm 1^\circ$. 
Figure 2-17. Variables for High Thrust Trajectories with $\Delta \gamma = \pm 1^\circ$. 
Figure 2-18. Variables for Low Thrust Trajectories with $\Delta \gamma = \pm 1^\circ$. 
Figure 2-19. Comparison of $K$ Variables for $\Delta \gamma = \pm 1^\circ$ and $\pm 2^\circ$, Nominal Thrust $\Delta \gamma_H$.
Figure 2-20. Comparison of $K_{\Delta\gamma H}$ Variables for $\Delta\gamma = \pm 1^\circ$ and $\pm 2^\circ$, High Thrust.
Figure 2-21. Comparison of KH Variables for $\Delta \gamma = \pm 1^\circ$ and $\pm 2^\circ$, Low Thrust.
Table 2-1. Numerical Comparison of Mirror Image and Linear Relationship of $\Delta\gamma_H$ Variables for Nominal, High and Low Thrust.

<table>
<thead>
<tr>
<th>$v_E$ (ft/s)</th>
<th>$T_{BURN}$ (ft)</th>
<th>$H_{lo} - H_{mid}$ (ft)</th>
<th>$\gamma_{lo} - \gamma_{mid}$ (deg)</th>
<th>$H_{hi} - H_{mid}$ (ft)</th>
<th>$\gamma_{hi} - \gamma_{mid}$ (deg)</th>
<th>$H_{lo} - H_{mid}$ (ft)</th>
<th>$\gamma_{lo} - \gamma_{mid}$ (deg)</th>
<th>$H_{hi} - H_{mid}$ (ft)</th>
<th>$\gamma_{hi} - \gamma_{mid}$ (deg)</th>
<th>$H_{lo} - H_{mid}$ (ft)</th>
<th>$\gamma_{lo} - \gamma_{mid}$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>-2400</td>
<td>2425</td>
<td>0.965</td>
<td>-0.965</td>
<td>-4800</td>
<td>4850</td>
<td>4750</td>
<td>1.930</td>
<td>1.932</td>
<td>-1.930</td>
<td>-1.930</td>
</tr>
<tr>
<td>0.85</td>
<td>-3075</td>
<td>2975</td>
<td>1.08</td>
<td>-1.07</td>
<td>-6150</td>
<td>5950</td>
<td>6200</td>
<td>2.160</td>
<td>2.175</td>
<td>-2.140</td>
<td>-2.140</td>
</tr>
<tr>
<td>1.15</td>
<td>-1900</td>
<td>1875</td>
<td>0.898</td>
<td>-0.898</td>
<td>-3800</td>
<td>3750</td>
<td>3775</td>
<td>1.796</td>
<td>1.772</td>
<td>-1.996</td>
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</tr>
<tr>
<td>1.0</td>
<td>-1675</td>
<td>1675</td>
<td>0.965</td>
<td>-0.965</td>
<td>-3350</td>
<td>3350</td>
<td>3320</td>
<td>1.930</td>
<td>1.928</td>
<td>-1.930</td>
<td>-1.930</td>
</tr>
<tr>
<td>0.85</td>
<td>-2025</td>
<td>2000</td>
<td>0.999</td>
<td>-0.992</td>
<td>-4050</td>
<td>4000</td>
<td>3980</td>
<td>1.998</td>
<td>1.990</td>
<td>-1.994</td>
<td>-1.990</td>
</tr>
<tr>
<td>1.15</td>
<td>-1350</td>
<td>1350</td>
<td>0.942</td>
<td>-0.945</td>
<td>-2700</td>
<td>2700</td>
<td>2700</td>
<td>1.884</td>
<td>1.875</td>
<td>-1.890</td>
<td>-1.880</td>
</tr>
<tr>
<td>1.0</td>
<td>-680</td>
<td>680</td>
<td>0.983</td>
<td>-0.983</td>
<td>-1360</td>
<td>1360</td>
<td>1350</td>
<td>1.966</td>
<td>1.968</td>
<td>-1.966</td>
<td>-1.965</td>
</tr>
<tr>
<td>0.85</td>
<td>-780</td>
<td>780</td>
<td>0.990</td>
<td>-0.985</td>
<td>-1500</td>
<td>1550</td>
<td>1560</td>
<td>1.985</td>
<td>1.975</td>
<td>-1.970</td>
<td>-1.980</td>
</tr>
<tr>
<td>1.15</td>
<td>-560</td>
<td>575</td>
<td>0.980</td>
<td>-0.980</td>
<td>-1125</td>
<td>1150</td>
<td>1150</td>
<td>1.960</td>
<td>1.955</td>
<td>-1.960</td>
<td>-1.958</td>
</tr>
</tbody>
</table>
Figure 2-22. Comparison of $\gamma_{\text{mid}}$ and $H_{\text{mid}}$ for Nominal, High and Low Thrust.
The next step was to determine the feasibility of interpolating between the families of trajectories for varying values of off-nominal thrust or $K_{\text{BURN}}$. From the previously generated data the values of $H_{\text{mid}}$ and $Y_{\text{mid}}$ as a function of $V_E$ were plotted, Figure 2-22, and revealed that a linear interpolation between the families of trajectories based on $K_{\text{BURN}}$ of 1.0, 1.15, and 0.85 was valid. This observation is also supported by Table 2-1.

To check the feasibility of implementing this method of representing the family of reference trajectories in the flight computer, an attempt was made to functionalize the data used for the reference trajectories. As previously stated, due to symmetry only the mid and hi trajectories are needed for this purpose. Functionalization of this data was relatively straightforward and required two cubic polynomials for $Y_{\text{mid}}$ and $H_{\text{mid}}$ for each of the three values of $K_{\text{BURN}}$ (0.85, 1.0, 1.15). The cubic spline method described by Fader [2] was used for this functionalization. Additionally, the quantities $Y_{\text{hi}} - Y_{\text{mid}}$ and $H_{\text{hi}} - H_{\text{mid}}$ were functionalized using one linear function for each of the quantities for each value of $K_{\text{BURN}}$.

The basic purpose of the steering loop is to control $\dot{\gamma}$, where $\dot{\gamma}$ is given by

$$
\dot{\gamma} = \frac{A_{\text{NV}}}{V_E} - \frac{G \cos \gamma}{V_E}
$$

and $A_{\text{NV}}$ is the total, nonfield acceleration normal to the vehicle velocity vector. It is convenient to control $\dot{\gamma}$, by controlling $A_{\text{NV}}$. Under "no wind" conditions $A_{\text{NV}}$ is proportional to the acceleration, $A_z$, produced by the aerodynamic force normal to the vehicle x axis. This acceleration is proportional to the vehicle angle of attack, which is a quantity that can be estimated and used for feedback in the control
system. Thus, to complete the steering loop, the $\gamma_C$ determined by the steering algorithm given in Equation 2.21 is converted to a pitch plane angle of attack command, $\alpha_{PC}$, as shown below, which is the input to the flight control system.

In Equation 2.21 an expression is given for the commanded flight path angle, $\gamma_C$, as a function of altitude, $H$, and the functionalized parameters $Y_{mid}$, $H_{mid}$, $(Y_{hi} - Y_{mid})$, and $(H_{hi} - H_{mid})$. The steering loop can generate a desired value of the commanded flight path angle rate, $\dot{\gamma}_C$, proportional to the error signal $(\gamma_C - \gamma)$ as follows

$$\dot{\gamma}_C = K_{STEER} (\gamma_C - \gamma) \quad (2.25)$$

This command is achieved by the appropriate application of aerodynamic and thrust forces normal to the vehicle velocity vector. It does not require compensating for the effects of gravity since these effects were accounted for in determining the functionalized parameters used to compute $\gamma_C$. Therefore, if the pitch plane acceleration (exclusive of gravity) normal to the vehicle velocity vector is given by $A_{NV}$, Equation 2.24 can be written as

$$\dot{\gamma}_C = \frac{A_{NV}}{V_E} = K_{STEER} (\gamma_C - \gamma) \quad (2.26)$$

Now, under "no-wind" conditions and steady-state conditions of zero pitch moment, $A_{NV}$ is proportional to the aerodynamic force normal to the vehicle x axis, $A_z$. Therefore,

$$A_z = K' A_{NV} = V_E K' K_{STEER} (\gamma_C - \gamma) = V_E K_{STEER} (\gamma_C - \gamma) \quad (2.27)$$
Several values of the steering gain, $K_{\text{STEER}}$, were tried in boost simulations and a constant value of $K_{\text{STEER}} = 0.2 \text{ sec}^{-1}$ was found to provide satisfactory performance for all trajectories considered. The acceleration $A_z$ is proportional to vehicle angle of attack in pitch, $\alpha_p$, according to the relationship

$$ A_z = \frac{Q \text{ AREA} C_{Na} \alpha_p}{\text{MASS}} \quad (2.28) $$

Therefore, Equation 2.27 can be rewritten as an angle-of-attack command by solving for $\alpha_{NV_C}$ and substituting $A_z$ as given by Equation 2.28 for $\alpha_{NV_C}$:

$$ \frac{Q \text{ AREA} C_{Na}}{\text{MASS}} \alpha_{PC} = V E K_{\text{STEER}} (\gamma - \gamma) \quad (2.29) $$

Then, solving for $\alpha_{PC}$,

$$ \alpha_{PC} = \frac{\text{MASS}}{Q \text{ AREA} C_{Na}} V E K_{\text{STEER}} (\gamma - \gamma) \quad (2.30) $$

This command is supplied as the input to the angle-of-attack control system whose feedback signal is provided by an angle-of-attack estimator. Although the assumption of equality between $A_z$ and $\alpha_{NV}$ is strictly true only for no-wind conditions, simulation results show that the angle of attack command based on this assumption produces satisfactory performance in the presence of winds.

Two additional features were incorporated into the steering loop design: the inclusion of a 7 deg/sec rate limit applied to the change in $\alpha_{PC}$ divided by the sampling time and the use of integral control of $\alpha_{PC}$ to reduce steady-state errors. The functional block diagram showing the pitch-plane-steering method is shown in Figure 2-23.
Figure 2-23. Pitch Plane Steering Block Diagram.
2.3.1.3 Cross Track Velocity Steering

Cross-track-velocity steering is provided to drive the component of the earth-relative velocity in the \( Y_E \) direction, \( V_Y \), to zero. A method to do this was developed and analyzed in a thesis by Bonnice [1]. His method involved using \( V_Y \) as the feedback quantity, with suitable gains, to determine a yaw angle of attack command for velocity control, \( \alpha_{YVC} \). Thus as \( V_Y \) goes to zero, \( \alpha_{YVC} \) does also. His algorithm for determining \( \alpha_{YVC} \) is

\[
\alpha_{YVC} = K_C K_{FN} \frac{\text{MASS}}{V_Y} \tag{2.31}
\]

where

\[
K_C = \frac{1}{\text{THRUST} - Q \text{ AREA} \left( C_A + C_{N\alpha} \left( 1 - \frac{L_{cp}}{CG} \right) \right)} \tag{2.32}
\]

and

\[
K_{FN} = K_{\gamma 1} + K_{\gamma 2} K_{CT} \tag{2.33}
\]

\( K_{\gamma 1} \) and \( K_{\gamma 2} \) are gains that are varied at selected time intervals (see Bonnice [1] and Fader [2]). \( K_{CT} \) is an integer that is incremented at the steering loop sampling frequency, between the selected time intervals, to provide linear interpolation of \( K_{FN} \) between gain changes.

2.3.1.4 Load Relief

In order to ensure that the commanded angles of attack, \( \alpha_{PC} \) and \( \alpha_{YVC} \), do not lead to excessive loads on the vehicle, the magnitudes of these commands are limited so that the resulting normal aerodynamic force does not exceed a specified value, \( F_{AEROLIM} \). The value of \( F_{AEROLIM} \) was chosen as 4000 lb which results in a maximum value of the product of dynamic pressure times angle of attack being less than
10,000 lb-deg/ft\(^2\). Using \( F_{\text{AEROLIM}} \), the maximum total angle of attack command is determined from

\[
\alpha_{\text{TOTMAX}} = \frac{F_{\text{AEROLIM}}}{(\text{AREA} \cdot \text{C}_n)}
\]  

Equation (2.34)

The total commanded angle of attack, \( \alpha_{\text{TOTC}} \), calculated using Equation 2.8, is compared with \( \alpha_{\text{TOTMAX}} \) and if it is greater, both \( \alpha_p \) and \( \alpha_y_{VC} \) are reduced using the following equations

\[
\alpha_p = \frac{\alpha_p \cdot \alpha_{\text{TOTMAX}}}{\alpha_{\text{TOTC}}}
\]  

Equation (2.35)

\[
\alpha_y_{VC} = \frac{\alpha_{y_{VC}} \cdot \alpha_{\text{TOTMAX}}}{\alpha_{\text{TOTC}}}
\]  

Equation (2.36)

2.3.1.5 End of Stage Steering

In the trajectory steering method developed, \( \gamma_C \) has a tendency to diverge as the end of Stage 1 is reached. This is caused by the fact that \( K_{\Delta \gamma H} \) in the steering algorithm (Equations 2.21 through 2.23) approaches infinity at the end of Stage 1. In the equation for \( K_{\Delta \gamma H} \), \( \gamma_{hi} - \gamma_{mid} \) is always a finite quantity, remaining close to the original value of \( \Delta \gamma = 1^\circ \) used to generate the initial family of trajectories. The denominator of \( K_{\Delta \gamma H} \), \( H_{hi} - H_{mid} \), goes to zero, as shown in Figures 2-16 through 2-18. This occurs because the initial condition of \( H \) used for the reverse integration (end condition of the trajectory) is the same, by definition, for all three curves generated. However, it turns out that this divergence becomes appreciable only in the last few seconds of the trajectory and during this period it is no longer necessary to use the algorithms that cause the divergence. The primary concern near the end of Stage 1 is to minimize the angle of attack. To help ensure this, both trajectory plane steering and cross-track velocity steering are discontinued three seconds before the end of Stage 1, and a constant zero-angle-of-attack command is generated in both
pitch and yaw. Additionally, it has been found that, for the trajectory plane steering, the transition to a constant zero-angle-of-attack command is smoother if the trajectory plane steering algorithms are replaced with a "gravity turn" three seconds prior to issuing the zero-angle-of-attack command. Thus, six seconds before the predicted burnout of Stage 1 the commanded flight path angle, $\gamma_C$, is updated every steering cycle from the gravity turn relationship of Equation 2.37.

$$\gamma_C = \gamma_{C_{\text{LAST}}} - \text{TIME}_{\text{STEER}} \cdot \gamma_{\text{gravity}}$$

where $\text{TIME}_{\text{STEER}} = 0.1$ sec (steering loop period) and

$$\cos \gamma_{\text{LAST}} \gamma_{\text{gravity}} = G \frac{\text{cos} \gamma_{\text{LAST}}}{V_E}$$

2.3.1.6 Nominal Launch Angle

The initial results of this steering method were good but revealed an undesirable asymmetry in the variation in burnout dynamic pressure. This asymmetry was caused by extreme variations from the assumed nominal launch attitude of 90° pitch. For the purposes of this thesis the launch attitude (at engine ignition) is assumed to vary in the pitch plane by as much as ±29° from the vertical. Table 2-2 shows the values of dynamic pressure at burnout obtained using stored reference curves based on a nominal launch angle of 90°. (Here it might be noted that the dynamic pressure at burnout for the nominal no-wind launch is 1213 psf rather than the desired 1200 psf. This deviation is due to a combination of differences between the simple single-plane simulation used to generate the steering reference curves and the six degree-of-freedom simulation used to generate Table 2-2. A small adjustment in the kick angle used to generate the steering reference curves could have eliminated this deviation.)
Table 2-2. Burnout Dynamic Pressure for Different Launch Angles and Wind Conditions

<table>
<thead>
<tr>
<th>Wind</th>
<th>Launch Angle</th>
<th>61°</th>
<th>90°</th>
<th>104.5°</th>
<th>119°</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>Dynamic Pressure+</td>
<td>1207</td>
<td>1213</td>
<td>1238</td>
<td>1287</td>
</tr>
<tr>
<td>Head</td>
<td>Dynamic Pressure+</td>
<td>1247</td>
<td>1253</td>
<td>1278</td>
<td>1327</td>
</tr>
<tr>
<td>Tail</td>
<td>Dynamic Pressure+</td>
<td>1167</td>
<td>1173</td>
<td>1198</td>
<td>1247</td>
</tr>
</tbody>
</table>

It was decided that the deviation in dynamic pressure from 1200 psf caused by launch angle variations could be minimized by choosing a launch angle for trajectory functionalization that results in a symmetric variation in dynamic pressure. In other words, it was decided that a new "nominal" launch angle was needed that would result in the 80 psf spread in dynamic pressure at burnout, between the launch angle extremes of 61° and 119°, being centered about 1200 psf. In an empirical search for a new "nominal" launch angle, the six degree-of-freedom simulation was used to determine the kick angles that would result in a 1200 psf burnout dynamic pressure for various launch angles between 61° and 119°. The results shown in Table 2-3 indicate that the terminal velocities for extreme variations in launch angle are symmetrical about a launch angle of 108°. It was found that using this launch angle as a new "nominal" launch angle for trajectory functionalization resulted in a symmetrical variation in dynamic pressure for extreme variations from this launch angle. Thus, this launch angle, and its associated kick angle of 58.435°, was used to generate a new family of zero-angle-of-attack reference trajectories. This family of three trajectories, stored as steering data, was used to generate the no-wind results presented in Table 2-4. This table shows that the dynamic pressure variation with launch angle is now approximately symmetric about the new nominal launch angle, and also that the maximum deviation from the burnout dynamic pressure corresponding to the nominal launch angle has been reduced from 74 psf (in
Table 2-3. Burnout $Q = 1200$ psf Zero $\alpha_p$ Trajectories.

<table>
<thead>
<tr>
<th>Launch Angle (degrees)</th>
<th>61</th>
<th>90</th>
<th>104.5</th>
<th>108</th>
<th>110</th>
<th>119</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terminal Velocity (ft/sec)</td>
<td>5723</td>
<td>5723</td>
<td>5692</td>
<td>5676</td>
<td>5668</td>
<td>5626</td>
</tr>
</tbody>
</table>

Table 2-4. 108°, No-Wind, Nominal Launch Angle Results

<table>
<thead>
<tr>
<th>Launch Angle</th>
<th>Burnout $Q$</th>
<th>Terminal Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>119</td>
<td>1240.1</td>
<td>5629.4</td>
</tr>
<tr>
<td>108</td>
<td>1202.7</td>
<td>5676.5</td>
</tr>
<tr>
<td>90</td>
<td>1169.9</td>
<td>5719.8</td>
</tr>
<tr>
<td>61</td>
<td>1165.2</td>
<td>5726.0</td>
</tr>
</tbody>
</table>

Table 2-2) down to 37.5 psf (in Table 2-4). This same nominal launch angle was tried using Fader's steering method with similar results.

2.3.1.7 Corrections for Launch Angle Deviations from Nominal

Since the launch angles are known at the beginning of the trajectory, it would seem desirable to seek some method of compensating for the predicted effects of launch angle variations from nominal. The reasoning that led to choosing a constant added bias in the commanded flight path angle as a method for this compensation is described below.

The original method of generating the family of trajectories involved perturbing the flight path angle of the mid trajectory at the burnout point by a selected $\Delta \gamma$ and then performing reverse integration to
determine the resulting $\Delta \gamma$ and $\Delta H$ over the entire trajectory. This resulted in a $\Delta \gamma$ that remained nearly constant at its burnout value over the entire trajectory (within 15%). It was felt that if the off-nominal launch-angle trajectories could be biased by some constant $\gamma_{\text{BIAS}}$, then there was a reasonable chance of achieving a desired dynamic pressure at burnout of 1200 psf. It was noted that for a 119 degree launch the dynamic pressure was higher than desired, 1240 psf from table 2-4; this indicated a depressed trajectory. Thus, the $\gamma_{\text{BIAS}}$ needed to be positive, raising the trajectory to reduce the dynamic pressure. The converse was true for the 61 degree launch angle. Through trial and error, it was found that using a $\gamma_{\text{BIAS}}$ of $+1.3$ degrees, added to the $\gamma_C$ normally used for the 119 degree launch-angle no-wind trajectory, resulted in the same dynamic pressure (approximately 1200 psf) as the 108 degree nominal launch angle. Noting the normal symmetry of the dynamic pressure and terminal velocity about the nominal launch angle, a $\gamma_{\text{BIAS}}$ of $-1.3$ degrees was tried for the 61 degree launch angle. This also resulted in the same dynamic pressure at staging as the 108 degree nominal launch angle. Encouraged by the symmetry of the problem, it was found that the $\gamma_{\text{BIAS}}$ could be predicted for any given launch angle by interpolation based on the unbiased terminal velocity of the launch angle in question. For example, the $\gamma_{\text{BIAS}}$ for any launch angle ($\text{LNCH}_x$) between 108° and 119° can be calculated from

$$
\gamma_{\text{BIAS}}_{\text{LNCH}_x} = \gamma_{\text{BIAS}}_{119° \text{ LNCH}_x} \left( \frac{\text{V}_{\text{END UNBIASED}} - \text{V}_{\text{END UNBIASED}}} {\text{108° NOM LNCH}_x - \text{108° NOM LNCH}_x} \right) \left( \frac{\text{V}_{\text{END UNBIASED}} - \text{V}_{\text{END UNBIASED}}} {\text{119° NOM LNCH}_x - \text{119° NOM LNCH}_x} \right)
$$

(2.39)

The results of the above calculation were tested and proved accurate as shown in Table 2-5. Additionally, the last two entries in Table 2-5 indicate that the effect of winds on burnout dynamic pressure is the same
as in the case where no $\gamma_{BIASC}$ term was included. That is, for example, using $\gamma_{BIASC}$, a headwind increases the terminal value of dynamic pressure over that obtained in the no-wind case by approximately 40 psf. As an aside it should be noted that the nominal dynamic pressure for the 108 degree launch angle and the dynamic pressure with $\gamma_{BIASC}$ included is approximately 1202.7 for all launch angles. The runs done in Table 2-5 used an ideal angle-of-attack estimator to avoid biasing the results. Using the realistic angle-of-attack estimator in the simulations of Table 2-5 causes a slight deviation in the trajectory and results in the no-wind dynamic pressure, using $\gamma_{BIASC}$, for all launch angles of approximately 1200 psf.

To implement the use of $\gamma_{BIASC}$, the data points in Table 2-5 of $\gamma_{BIASC}$ versus the deviation in launch angle from the nominal 108 degrees were functionalized using a cubic spline fit. The method required one cubic polynomial for launch angles greater than 108 degrees and one for launch angles less than 108 degrees. These cubics deviated from the actual data points by less than 0.24% above and 0.015% below 108 degrees. The functionalization of the bias, $\gamma_{BIASC}$ (in radians), in terms of the launch angle, $\theta_{init}$ (in degrees), and its deviation from the 108° nominal launch angle, $\Delta\theta_{LNCH}$, is described by the following equations:

If $\theta_{init} > 108^\circ$,

$$\Delta\theta_{LNCH} \text{(radians)} = \frac{\theta_{init} \text{(degrees)} - 108}{57.3}$$ (2.40)

$$\gamma_{BIASC} \text{(radians)} = -2.5346 \times 10^{-6} + 9.5977 \times 10^{-2} \Delta\theta_{LNCH}$$

$$+ 1.1004 \times 10^{-1} \Delta\theta_{LNCH}^2 + 2.9380 \times 10^{-2} \Delta\theta_{LNCH}^3$$ (2.41)
Table 2-5. Comparison of Terminal Dynamic Pressure Using Calculated $\gamma_{\text{BIAS}_C}$

<table>
<thead>
<tr>
<th>Wind</th>
<th>Launch &lt; (deg)</th>
<th>$w/o$ $\gamma_{\text{BIAS}_C}$ Q (psf)</th>
<th>$V_{\text{End}}$ (ft/s)</th>
<th>Calculated $\gamma_{\text{BIAS}_C}$ (deg)</th>
<th>Using $\gamma_{\text{BIAS}_C}$ Q (psf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>61</td>
<td>1165.2</td>
<td>5726.0</td>
<td>-1.3000</td>
<td>1202.7</td>
</tr>
<tr>
<td>None</td>
<td>90</td>
<td>1169.9</td>
<td>5719.8</td>
<td>-1.1364</td>
<td>1202.4</td>
</tr>
<tr>
<td>None</td>
<td>96</td>
<td>1177.5</td>
<td>5709.5</td>
<td>-0.8678</td>
<td>1202.3</td>
</tr>
<tr>
<td>None</td>
<td>102</td>
<td>1188.3</td>
<td>5695.2</td>
<td>-0.4926</td>
<td>1202.4</td>
</tr>
<tr>
<td>None</td>
<td>108</td>
<td>1202.7</td>
<td>5676.5</td>
<td>0</td>
<td>1202.4</td>
</tr>
<tr>
<td>None</td>
<td>110</td>
<td>1208.3</td>
<td>5669.5</td>
<td>0.1991</td>
<td>1202.7</td>
</tr>
<tr>
<td>None</td>
<td>112</td>
<td>1214.4</td>
<td>5661.5</td>
<td>0.4155</td>
<td>1202.3</td>
</tr>
<tr>
<td>None</td>
<td>116</td>
<td>1228.2</td>
<td>5644.1</td>
<td>0.8950</td>
<td>1202.9</td>
</tr>
<tr>
<td>None</td>
<td>119</td>
<td>1240.1</td>
<td>5629.4</td>
<td>1.3000</td>
<td>1202.7</td>
</tr>
<tr>
<td>Head</td>
<td>119</td>
<td>1281.8</td>
<td>5613.1</td>
<td>1.3000</td>
<td>1243.1</td>
</tr>
<tr>
<td>Head</td>
<td>90</td>
<td>1208.8</td>
<td>5703.7</td>
<td>-1.1364</td>
<td>1243.0</td>
</tr>
</tbody>
</table>

Simulation Conditions: AIRS Noise/Errors and Ideal Angle-of-Attack Estimation

If $\theta_{\text{init}} < 108^\circ$,

$$\Delta \theta_{\text{LNCH}} (\text{radians}) = \frac{108 - \theta_{\text{init}} (\text{degrees})}{57.3}$$  \hspace{1cm} (2.42)

$$\gamma_{\text{BIAS}_C} (\text{radians}) = 2.9841 \times 10^{-7} - 9.2548 \times 10^{-2} \Delta \theta_{\text{LNCH}}$$

$$+ 1.0263 \times 10^{-1} \Delta \theta_{\text{LNCH}}^2 - 2.8686 \times 10^{-2} \Delta \theta_{\text{LNCH}}^3$$  \hspace{1cm} (2.43)
To incorporate $\gamma_{BIASC}$ into the trajectory steering, the steering algorithm of Equation 2.21 was changed to

$$\gamma_C = \gamma_{mid} + \gamma_{BIASC} + K_{\Delta H} (H - H_{mid}) \quad (2.44)$$

The performance obtained by using the functionalized $\gamma_{BIASC}$ was nearly identical to that obtained by using Equation 2.39 to calculate $\gamma_{BIASC}$. Therefore, the use of $\gamma_{BIASC}$ effectively compensates for variations in burnout dynamic pressure due to launch angle variations. This was achieved by employing two simple functions and a measurement of launch angle. The same bias in flight path angle was added to the commanded flight path angle of Fader's steering method and produced essentially the same improvement in performance.

2.3.2 Control

The flight control system used in this thesis was designed by Bonnice [1] and modified by Fader [2]. Since the flight control system was not developed or modified in this thesis, it is only briefly described here. A complete description can be found in the above-mentioned publications.

The purpose of the flight control system is to stabilize the vehicle's inherently unstable attitude response while satisfying the commanded input from the steering loop. This is done by using two configurations, one for the kick maneuver and one for the remainder of the boost phase. Both configurations use two feedback loops that are sampled at 100 Hz as opposed to the steering of the vehicle which is sampled at 10 Hz. The inner loop of both configurations uses attitude rate feedback. The outer loop of the kick maneuver configuration uses attitude feedback while the outer loop of the second configuration uses angle of attack feedback.
2.3.3 Estimation

The estimators needed for the boost steering and control system without roll control are (1) an angular rate estimator to estimate the vehicle pitch and yaw rates, and (2) an angle-of-attack estimator to estimate the vehicle pitch plane angle of attack, \( \alpha_p \), and the vehicle yaw plane angle of attack, \( \alpha_y \). These estimators were designed by Bonnice [1].

This thesis uses Bonnice's estimators with only slight modification. In the case of both estimators it was found that improved performance could be attained by doing estimation in the unrolled body coordinate frame where the variables are more slowly varying than in the rolled body frame.

Three measured quantities are required as inputs to the rate estimator: nozzle deflection \((\delta_p \text{ and } \delta_y)\), the change in the integral of IMU-sensed acceleration \((\Delta V_y \text{ and } \Delta V_Z)\), and the attitude change measured by the IMU \((\Delta \theta \text{ and } \Delta \psi)\). These quantities are processed through an estimator function and the appropriate high or low pass complementary filter combination to produce a rate estimate as shown in Figure 2-24. The time constant of the filters as determined by Fader [2] is 0.07833 sec.

![Figure 2-24. Angular Rate Estimator.](image-url)
The angle-of-attack estimator is implemented in a similar arrangement, as shown in Figure 2-25. Measured nozzle deflection and IMU-measured ΔV are used as the inputs to an angle-of-attack estimation function to produce a low frequency angle-of-attack estimate while the input of IMU-measured attitude change is used as the input to the high frequency estimate of the change in angle of attack. The filter of the angle-of-attack estimator used this thesis was to work satisfactorily with a time constant of 0.5 seconds. The outputs of the angle-of-attack estimator in the two control channels are the components of estimated angle of attack in pitch, \( \hat{\alpha}_p \), and yaw, \( \hat{\alpha}_y \).

Figure 2-25. Angle of Attack Estimator.
CHAPTER 3

INVESTIGATION OF ROLL DYNAMICS WITHOUT ROLL RATE CONTROL

The first step in the investigation of the roll rate control problem is to determine whether or not roll rate control is necessary to maintain the vehicle roll rate within the specified limits. For this thesis, the maximum permissable roll rate is assumed to be 50 deg/sec. It is also desirable to reduce the roll rate as much as practical to improve the IMU performance. What must be determined, then, is the maximum uncontrolled roll rate for various initial conditions and disturbance profiles. As discussed in Section 2.1.3, the roll torque of the vehicle depends on the orientation of the raceway, cg offset, and hinge point offset to the aerodynamic and engine forces. This orientation will be affected by the initial conditions at launch. To vary all these parameters and conditions in a six degree-of-freedom simulation to determine the worst case roll rate would be very expensive in terms of computer time. To reduce the cost, a single degree-of-freedom simulation was developed as described in the following section.

3.1 Simulation Method

The first step in the worst case roll rate analysis is to determine the relative orientation of the cg offset and hinge point offset to the raceway location that results in the maximum possible roll torque. Under steady-state conditions the engine force and aerodynamic force normal to the vehicle x axis are parallel. This is due the the requirement that in steady state, the pitching and yawing moments must be balanced.
With the steady-state assumption, Equations 2.10 through 2.15 show that when the raceway, cg offset location, and hinge point are in line, with the cg on the opposite side of the vehicle center from the raceway and the hinge point on the same side as the raceway (Figure 3-1), their associated roll torque components are in phase. With all the roll torque components in phase, their combined effect is maximized for any given condition. The sign and magnitude of the roll torque at any time are determined by the vehicle body components of the forces.

![Figure 3-1. Raceway, CG, HNG Orientation for Max Uncontrolled Roll Rate.](image)

The search for conditions that will result in a maximum roll rate from these in-phase torque components may be facilitated by taking advantage of the fact that the use of unrolled axes for estimation and control results in the same angle-of-attack components, normal aerodynamic force components, and normal engine force components, independent of roll motion, if there is no roll-rate control and if the effects of nozzle
actuator lags can be neglected. As a consequence, it is possible to generate and store in a data file the time histories of these roll-torque-determining variables by performing a single six-degree-of-freedom simulation in which the vehicle roll rate and roll torque are set equal to zero. These time histories can be translated into the corresponding roll torque components by transforming the variables into body axes, and the computed torques can be used to generate the total roll acceleration, \( \ddot{\phi} \), in a simple single-axis simulation of vehicle rotation about the roll axis, where

\[
\ddot{\phi} = \frac{1}{I_{xx}} \text{[Sum of Roll Torque Components]} \tag{3.1}
\]

The simple single-axis simulation is designed to perform a set of trial simulation runs using the same unrolled-axis time histories for a sequence of initial roll angles ranging from -180 degrees to +180 degrees and an accompanying sequence of initial roll rates ranging from -10 deg/sec to +10 deg/sec. Comparing the results of these trial simulations, the maximum roll rate can be determined, as well as the initial condition roll angle and roll rate from which it results, for any given launch angle and wind condition. Additionally, the launch angle can be eliminated as a variable in determining the maximum uncontrolled roll rate by realizing that the larger the launch angle the larger the pitch plane forces are during the kick maneuver. The largest pitch plane forces are capable of generating the largest roll torques. This implies that the worst case launch angle for the maximum uncontrolled roll rate is the largest launch angle, in this case 119 degrees.

3.2 Maximum Uncontrolled Roll Rate Results

Using the procedure described in the last section, with the random* \( HNZ = -0.0133 \text{ ft} \) and the random \( CGZ = 0.0167 \text{ ft} \) superimposed

*The use of the word "random" here refers to a random variation from flight to flight. For a given flight, the quantities are constant.
over the deterministic CG location, maximum roll rates were determined. The results are shown in Table 3-1. Plots and further data on runs 1 through 3 are shown in Section 7.1, Figures 7-2 through 7-4, and Section 7.2, Table 7-4 runs 1 through 3 respectively. Since the maximum roll rate exceeds the assumed design limit, some method of reducing the roll rate is needed.

Table 3-1. Maximum Roll Rates

<table>
<thead>
<tr>
<th>Run #</th>
<th>Wind</th>
<th>Initial</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\phi$ (deg)</td>
<td>$\dot{\phi}$ (deg/sec)</td>
</tr>
<tr>
<td>1</td>
<td>Head</td>
<td>-10</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>Tail</td>
<td>30</td>
<td>-10</td>
</tr>
<tr>
<td>3</td>
<td>Cross</td>
<td>10</td>
<td>-10</td>
</tr>
</tbody>
</table>

Simulation Conditions: 119° launch angle, nominal thrust, IMU measurement noise and deterministic errors, angle-of-attack estimation
Chapter 4

Estimation Requirements for Roll Rate Control

4.1 Description of Roll Rate Control Method

The method of roll rate control developed in this thesis attempts to decouple the control of the roll rate from the pitch plane steering problem. By doing this, the roll rate control has the minimum impact on the trajectory and range of the vehicle while simplifying the problem of design. In this method, forces in the vehicle unrolled yaw plane are commanded through the control of the yaw angle of attack such that roll torque components opposite to the existing roll rate are generated. These torque components are caused by the raceway, cg offset, and hinge point offset as previously described. In this method of control, the major item that must be determined is whether a positive or negative commanded yaw angle of attack will result in a torque component opposite to the existing roll rate. As it turns out, this decision is more difficult than it first appears to be.

4.2 Analysis of Raceway Control

The first method investigated for determining the polarity of the commanded angle of attack was one in which the sign of the commanded yaw angle of attack is based on raceway position alone. Since the magnitude and sign of the raceway torque resulting from an angle of attack are known, the determination of the polarity of the commanded yaw angle of attack for roll control is a simple process. In vehicles where the raceway torque that can be generated is larger than that of the cg offset and hinge point offset this method is feasible and thus should be a con-
sideration in the design of the raceway. However, for the vehicle assumed in this thesis, the cg and hinge torques in combination can be roughly twice that of the raceway. Therefore, there are orientations where the determination of roll rate command based on raceway position alone will result in increasing the roll rate with each control input. To show this, the method of roll rate control through raceway position is briefly developed.

For raceway control, the magnitude of the commanded yaw angle of attack for roll rate control, $\alpha_{RC}'$, was selected as a constant 2 degrees (except when limited by load relief). The polarity of $\alpha_{RC}'$ is determined based on the roll angle, roll rate direction, and torque coefficients to attempt to provide a roll torque component in opposition to the existing roll rate. Figure 4-1 shows the total force component due to roll control, $F_{TOTRC}'$, that results from $\alpha_{RC}'$ and the engine side force required to balance the yawing moment resulting from $\alpha_{RC}'$. The direction (polarity) of $F_{TOTRC}'$ required to reduce the roll rate is determined by the quadrant location of the raceway as shown in Figure 4-1.

Figure 4-1. Required Direction of the Commanded Force for Raceway Position and Roll Rate.

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As shown in Figure 4-1, this method of control can provide roll rate control in only two quadrants of roll angle for a given direction of roll rate. Based on the raceway torque coefficients, control inputs in the other possible quadrants result in either no raceway torque generated (for less than 1.5 Mach) or a raceway torque opposite to that desired (for greater than 1.5 Mach). This method of control was implemented and analyzed. Under the same conditions as were assumed for the maximum uncontrolled roll rate analysis, that is the raceway, cg, and hinge torques in phase (see Figure 3-1), this method of roll rate control based on raceway position worked effectively, as indicated in Table 4-1.

Table 4-1. Raceway Control Analysis.

<table>
<thead>
<tr>
<th>Conditions: crosswind, 119° launch angle, initial roll angle +10°, initial roll rate -10°/s</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Raceway and hinge point on same side of vehicle center, CG on opposite side</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Raceway and CG on same side of vehicle center, hinge point on opposite side.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maximum Roll Rate</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orient.</td>
<td>Uncontrolled</td>
<td>Raceway Control</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>-60.90°/s @ 37.4 s</td>
<td>-39.66°/s @ 14.2 s</td>
</tr>
<tr>
<td>2</td>
<td>20.29°/s @ 29.4 s</td>
<td>62.99°/s @ 48.6 s</td>
</tr>
</tbody>
</table>

However, when the raceway, cg, and hinge are in a relative orientation such that the raceway torque is out of phase with the other two, this method does not work. Raceway control was tried with the orientation shown in Figure 4-2. In this orientation, whenever a roll-rate-control input is applied based on the raceway position alone, a torque opposite to the desired direction is applied due to the large magnitudes of the unknown cg and hinge torques compared to those of the raceway.
torque. The result is a rapidly increasing roll rate that far exceeds that which would have occurred had no roll rate control been used. (See plots, Figure 4-4 and Table 4-1.) A full worst case analysis was not done but Table 4-1 shows a comparison of uncontrolled roll rate, raceway control, and the roll rate control method to be developed in Chapter 6 for the orientation of raceway, cg, and hinge position used in the maximum uncontrolled roll rate analysis and for the worst case orientation described in Figure 4-2. Plots for comparison of the uncontrolled roll rate case and the roll rate control method developed in Chapter 6 with the raceway control for orientation 2 of Table 4-1 are given in Figures 4-3 and 4-5 respectively.
Simulation conditions of Table 4-1, orientation #2

Figure 4-3. Uncontrolled Roll Rate for Raceway Control Comparison.
Simulation conditions of Table 4-1, orientation #2

Figure 4-3. Uncontrolled Roll Rate for Raceway Control Comparison (cont.).
Simulation conditions of Table 4-1, orientation #2

Figure 4-4. Raceway Control.
Simulation conditions of Table 4-1, orientation #2

Figure 4-4. Raceway Control (cont.).
Simulation conditions of Table 4-1, orientation #2

Figure 4-4. Raceway Control (cont.).
Simulation conditions of Table 4-1, orientation #2

Figure 4-5. Roll Rate Control for Raceway Control Comparison.
Simulation conditions of Table 4-1, orientation #2

Figure 4-5. Roll Rate Control for Raceway Control Comparison (cont.).
Simulation conditions of Table 4-1, orientation #2

Figure 4-5. Roll Rate Control for Raceway Control Comparison (cont.).
4.3 Need for CG Estimation

To be able to provide effective roll rate control an estimate must be made of all roll torque components which can affect the polarity of the net roll torque resulting from either a positive or a negative yaw angle-of-attack command. A comparison of the two possible net roll torques can then be made to permit the selection of the appropriate polarity of the yaw angle-of-attack command for the roll-rate-control input. It has already been stated that the raceway torque component can be determined as a function of roll angle. It remains, therefore, to estimate the cg offset torque and the hinge point offset torque. To estimate these last two components, an estimate of the random portions of the cg offset and engine hinge point offset must be made.
CHAPTER 5

DEVELOPMENT OF A ROLL RATE CONTROL ESTIMATOR

To estimate the cg offset and hinge point offset, it is first necessary to estimate the roll torques produced by these offsets at any one time. The torque resulting from the combination of cg and hinge point offsets is obtained by subtracting the raceway torque from the total roll torque on the vehicle. To be able to estimate the total roll torque on the vehicle, knowledge is required of both the roll angular acceleration, $\dot{\phi}$, and the roll moment of inertia, $I_{XX}$.

5.1 Roll Angular Acceleration Estimation

Roll angular acceleration is not directly available from the IMU so it must be estimated from the IMU-measured roll angle. To do this, a relationship derived by Goss [4] has been selected to estimate the average roll angular acceleration over a period of time in which the effects of random errors in attitude measurements can be appropriately minimized by statistical averaging. The Goss relationship is derived as a solution to the following problem: Given a sequence of measurements, $\phi_m(0)$, $\phi_m(T)$, ..., $\phi_m(nT)$, where T is a fixed period between measurements, find the parabola $\phi(t)$ which produces the best fit (i.e., minimum least square error) to the measurements. A relationship is derived for the estimated average angular acceleration over nT by taking the second derivative, $\ddot{\phi}$, of this function $\phi(t)$. The resulting expression for estimated roll angular acceleration, $\hat{\phi}$, is

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\[ \dot{\phi} = \left( \frac{60}{T^2(n-1)n(n+1)(n+2)(n+3)} \right) \]
\[ \left\{ \sum_{i=0}^{n} \left[ (n^2 - n - 6ni + 6i^2) \phi_m (iT) \right] \right\} \] \hspace{1cm} (5.1)

or
\[ \dot{\phi} = \left( \frac{60}{T^2(n-1)n(n+1)(n+2)(n+3)} \right) \left\{ n(n-1) \sum_{i=0}^{n} \phi_m (iT) \right\} \]
\[ -6n \sum_{i=0}^{n} i \phi_m (iT) + 6 \sum_{i=0}^{n} i^2 \phi_m (iT) \} \] \hspace{1cm} (5.2)

If the standard deviation of the measurement error is given by \( \sigma_\phi \), then the standard deviation of the acceleration estimation error due to measurement noise is
\[ \sigma_\varphi = \frac{12\sqrt{5}}{T^2(n-1)n(n+1)(n+2)(n+3)} \sigma_\phi \] \hspace{1cm} (5.3)

In the ideal case where the roll angular acceleration is constant throughout the estimation period of \( nT \) seconds, acceleration estimation error is produced only by measurement errors. In general, however, the acceleration estimate will also be in error due to the fact that the acceleration varies during the estimation time interval. Therefore, dynamically, for a given measurement time interval, \( T \), of 0.01 sec, the smaller the number of measurements the more constant the roll angular acceleration would be and thus the more accurate the estimate. However, based on Equation 5.3, a larger number of estimates is needed in the presence of IMU measurement noise. For example, from Equation 5.3 it can be seen that for 100 measurements \( \sigma_\varphi = 2.684 \sigma_\phi \), while for the minimum measurement number of 3, \( \sigma_\varphi = 24,494 \sigma_\phi \). Even as many measurements as 50 still results in magnifying the angular measurement error by roughly 15 times. It was determined experimentally that for the vehicle
model being considered in this thesis, a measurement number of 100 provided a good compromise. In this case an estimate of $\dot{\phi}$ is obtained every second. With this number of measurements it was found that the magnitude and sign of $\dot{\phi}$ closely followed $\dot{\phi}$ in the presence of measurement noise and deterministic errors (see plots Figure 5-1). The percentage error in the estimate was generally within 10-15\% if the absolute value of $\dot{\phi}$ was greater than 1 deg/sec$^2$. For smaller values the percentage error could possibly get quite high. (See Figure 5-1.)

With the estimate of roll angular acceleration it is possible to determine the total roll torque on the vehicle. The raceway torque is then subtracted from this total torque to determine the torque resulting from the cg and hinge point offsets.
Figure 5-1. Roll Acceleration Estimate.
5.2 Raceway Torque Coefficient Functionalization

The raceway torque coefficient is available from a table of experimental values as a function of Mach number, estimated angle of attack, and the angle the estimated normal aerodynamic force makes with the raceway. The force and angle of attack are available from the angle-of-attack estimation as described in Section 2.3.3, where the relative angle of the estimated force and the measured raceway position is then calculated as described in Section 2.1.2.2, Equation 2.10, and shown in Figure 2-9. A raceway torque coefficient table, obtained for use in this thesis, itself has 2268 elements, making it too large for inclusion in the vehicle's flight computer. By considering the symmetry of the vehicle and by finding interpolation constants for the table that allow one line of the table, based on angle of attack, to be useable for many other Mach number, angle-of-attack combinations, the size of the table needed for inflight estimation of the raceway torque was reduced to 152 elements.

5.3 Center of Gravity Estimation

To estimate the cg and hinge point offsets it is assumed that the hinge offset is constant and that the cg offset varies slowly enough to be considered essentially constant during the measurement period. This is consistent with the original specifications on the cg and hinge offset in Sections 2.1.2.3 and 2.1.2.4. The first approach that was considered for estimating the cg and hinge point offsets was to take four measurements and then solve four simultaneous torque equations for the two sets of y and z vehicle body coordinates for $\overline{CG}$ and $\overline{HNG}$. This worked well under ideal measurement conditions but did not produce accurate results when noise and deterministic errors were included, and estimated angular acceleration was used. This problem of performance degradation was solved by an alternative approach that (a) does not attempt to separate the effects of the hinge point and cg offsets, (b) does not perform estimation when a low roll acceleration results in poor data, and (c)
minimizes errors by an averaging process that takes into account the effects of the time-varying angles between forces and offsets.

The alternative estimation method may be derived by first considering the case where there is a cg offset but no hinge point offset. In this case the roll torque produced by aerodynamic and engine forces depends only on the resultant of these forces in combination with the cg offset. The cg offset is defined as a radial vector CG from the centerline of the vehicle to the cg position. The estimation of this offset is then carried out in terms of estimated time averages of the raceway torque and the total-normal-force torque over the period in which the average roll angular acceleration is estimated from IMU measurements. The geometry of the average normal aerodynamic force, $\bar{F}_{\text{AERO}}$, the average normal engine force, $\bar{F}_{\text{ENG}}$, the average total normal force, $\bar{F}_{\text{TOT}}$, and the vector CG is shown in Figure 5-2.

![Figure 5-2. Force and Center of Gravity Vectors.](image-url)
From Figure 5-2, the cg vector has components along $F_{TOT}$ and perpendicular to $F_{TOT}$. The only portion of $CG$ that is responsible for the cg roll torque is that portion that is perpendicular to $F_{TOT}$, representing the moment arm. In this method an attempt is made to estimate only that component of $CG$ that is perpendicular to $F_{TOT}$, thereby disregarding the component of $CG$ along $F_{TOT}$ which does not contribute to the roll torque. The estimation procedure is derived as follows. First it is noted that the average total roll torque on the vehicle, can be estimated from the average estimated roll angular acceleration, $\dot{\phi}$, using

$$ROLLTORQ_{EST} = \dot{\phi} \ I_{XX}$$ (5.4)

The difference between the estimated average total roll torque and the average raceway torque (estimated), over the measurement period, is the torque due to all other causes, in this case the cg torque. This cg torque is due only to the component of the cg vector that is perpendicular to the total force vector, as previously mentioned. The perpendicular component of $CG$ is given by the equation

$$CG_{component} \downarrow F_{TOT} = \frac{CGTORQ \ * \ UNIT (F_{TOT})}{|F_{TOT}|}$$ (5.5)

where

$$CGTORQ = (ROLLTORQ_{EST} - RACEWAYTORQ_{EST}) \ UNIT (X_B)$$ (5.6)
Define

\[ \overline{U}_\perp = \text{UNIT}(\overline{x}_B) \ast \text{UNIT}(\overline{F}_\text{TOT}) \]  
(5.7)

Then,

\[ \frac{(\text{ROLLTORQ}_\text{EST} - \text{RACEWAYTORQ}_\text{EST})}{|\overline{F}_\text{TOT}|} \overline{U}_\perp \]  
(5.8)

Using the above relationships, the following procedure may be used to update the cg estimate after each angular acceleration estimation period:

1. Compute the component of the previous updated cg estimate, \( \overline{CG}_{\text{LAST}} \), along the present vector, \( \overline{U}_\perp \), and subtract it from the present value of \( \overline{CG}_{\text{component} \perp \overline{F}_\text{TOT}} \), \( \overline{CG}_{\text{NOW}} \), to obtain the change in the estimate:

\[ \Delta \overline{CG} = \overline{CG}_{\text{NOW}} - (\overline{CG}_{\text{LAST}} \ast \overline{U}_\perp) \overline{U}_\perp \]  
(5.9)

2. Update the present cg estimate according to the equation

\[ \overline{CG}_{\text{NEW}} = \overline{CG}_{\text{LAST}} + K_{CG} \Delta \overline{CG} \]  
(5.10)

where \( K_{CG} < 1 \). (Note: \( \overline{CG}_{\text{NEW}} \) becomes the value of \( \overline{CG}_{\text{LAST}} \) to be used for the next estimation.)

For a constant value of \( K_{CG} \), the estimate of \( \overline{CG}_{\text{component} \perp \overline{F}_\text{TOT}} \) will approach the true value of \( \overline{CG}_{\text{component} \perp \overline{F}_\text{TOT}} \) as \((1 - K_{CG})^n + 0\), where \( n \) is the number of estimation periods. It should be noted that the
component of \( \bar{CG} \) along \( \bar{F}_{TOT} \) is not estimated. This is not a problem, however, since \( \bar{F}_{TOT} \) rotates in relation to the vehicle body, ultimately permitting estimation of both lateral components of \( \bar{CG}, \) \( \bar{CG}_y \) and \( \bar{CG}_z \).

An attractive feature of this method of estimation is the use of the gain, \( K_{CG} \). This gain factor controls the weight applied to any one estimate and thus controls the speed at which the estimated cg approaches the true cg. Simulation studies have shown that the accuracy of any particular update of the cg estimate obtained in one average period is usually satisfactory only if the angular acceleration in this averaging period exceeds some minimum level. Accordingly, the value of \( K_{CG} \) was made either 0.3 or 0 depending on whether the estimated average roll angular acceleration was greater or less than a simulation-based value of 1 deg/sec^2:

\[
K_{CG} = 0.30 \text{ for } |\hat{\phi}| \geq 1 \text{ deg/sec}^2 \quad (5.11)
\]

\[
K_{CG} = 0 \text{ for } |\hat{\phi}| < 1 \text{ deg/sec}^2 \quad (5.12)
\]

The use of these gains has resulted in estimates of \( \bar{CG} \) with less than approximately 15% error in the presence of the error sources described in section 2.2 (see plots, Figure 5-3).

It is shown below that in the case where there is a nonzero offset of the engine hinge point the estimated \( \bar{CG} \) produced by the above procedure automatically includes the effect of the engine hinge point offset.
Figure 5-3. CG Estimation.
5.4 Derivation of CG Equivalent

As mentioned previously, it is convenient to represent the effect of the engine hinge point offset in terms of a roll torque component resulting from the engine side force acting through the moment arm of the hinge point relative to the vehicle centerline in the absence of a cg offset. This torque component, HNGTORQ, is then added to the engine-produced torque resulting from the cg offset alone with no hinge point offset, CGTORQENG, to obtain the total torque resulting from the engine side force. This total engine torque, in turn, is added to the torque resulting from the normal aerodynamic force acting through the cg offset moment arm, CGTORQ_AERO, to obtain the roll torque resulting from offsets

\[
\text{TORQUE}_X = \text{HNGTORQ} + \text{CGTORQ}_\text{ENG} + \text{CGTORQ}_\text{AERO} \tag{5.13}
\]

The following analysis shows that the combined effect of the cg and engine hinge point offsets can be represented by an equivalent cg offset in the steady-state case where the engine side force and the aerodynamic force are parallel to produce zero pitch and yaw torques. Simulation results have shown that this steady-state condition where the forces are parallel is a good approximation of the actual vehicle state in spite of transient deviations from this condition. The geometrical relationships between forces and moment arms resulting from this steady-state condition are shown in Figure 5-4. The following derivation of an expression for the equivalent cg offset vector, \( \text{CG}_E \), is based on Figure 5-4.

As a result of the fact that the pitch and yaw torques are zero in the steady-state condition, the engine and aerodynamic side forces are not only parallel but proportional as well. That is

\[
\text{F}_\text{AERO} = K_{EQ} \text{F}_\text{ENG} \tag{5.14}
\]

where

\[
K_{EQ} = \frac{CG_X}{L_CP} \tag{5.15}
\]
As a consequence of this proportionality, the sum of these forces

$$\vec{F}_{TOT} = \vec{F}_{AERO} + \vec{F}_{ENG} \quad (5.16)$$

is proportional to the engine side force:

$$\vec{F}_{TOT} = (1 + K_{EQ})\vec{F}_{ENG} \quad (5.17)$$

This proportionality between $\vec{F}_{TOT}$ and $\vec{F}_{ENG}$ is key to the derivation of

$\overline{CG}_{EQ}$, as shown below.

The torque resulting from the combination of engine and aerodynamic forces may be expressed as follows in terms of the force vectors and offset vectors:
TORQUE = \vec{F}_{AERO} \cdot \vec{CG} + \vec{F}_{ENG} \cdot (\vec{CG} - \vec{HNG}) \quad (5.18)

or

TORQUE = (\vec{F}_{AERO} + \vec{F}_{ENG}) \cdot \vec{CG} - \vec{F}_{ENG} \cdot \vec{HNG} \quad (5.19)

Substituting Equation 5.14 into Equation 5.19, the offset torque can be expressed in terms of the engine force alone:

TORQUE = (1 + \kappa_{EQ}) \vec{F}_{ENG} \cdot \vec{CG} - \vec{F}_{ENG} \cdot \vec{HNG} \quad (5.20)

or

TORQUE = \vec{F}_{ENG} \cdot [(1 + \kappa_{EQ}) \vec{CG} - \vec{HNG}] \quad (5.21)

But solving Equation 5.17

\vec{F}_{ENG} = \vec{F}_{TOT}/(1 + \kappa_{EQ}) \quad (5.22)

which can be substituted into Equation 5.21 to express the total offset torque in terms of the total force vector

TORQUE = \vec{F}_{TOT} \cdot [\vec{CG} - (1/(1 + \kappa_{EQ}))\vec{HNG}] \quad (5.23)

Here it may be noted that if the engine hinge point offset were zero the resulting torque would be

\[ \text{TORQUE} \mid_{\text{No hinge point}} = \vec{F}_{TOT} \cdot \vec{CG} \quad (5.24) \]
Therefore, the terms in the brackets may be interpreted as an equivalent cg offset, $\overline{CG_{EQ}}$, resulting in

$$\text{TORQUE} = \overline{F}_{TOT} \times \overline{CG_{EQ}} \quad (5.25)$$

where

$$\overline{CG_{EQ}} = \overline{CG} - \overline{HNG}/(1 + K_{EQ}) \quad (5.26)$$

The equivalent cg position, as described by equation 5.26, was used in the roll rate control algorithm (described in Chapter 6). The results of that simulation were compared with a simulation done under the same conditions with the exception that the actual cg and hinge point offset positions were used in the control algorithm. The result of the comparison was that for roll rate control purposes the use of $\overline{CG_{EQ}}$ was nearly identical to using the actual cg and hinge point offsets. To more clearly show the validity of using $\overline{CG_{EQ}}$ as a substitute for the actual cg and hinge point offsets, simulation runs were made in which the total offset torque was computed from the separate effects of aerodynamic and engine normal forces in combination with actual cg and hinge point offsets. In these same simulation runs, the total offset torque was also computed from Equation 5.25, based on the total normal force and the equivalent cg offset computed from Equation 5.26. These two total offset torques turned out to be virtually indistinguishable, as shown by the plots in Figure 5-5.

When a hinge point offset exists, the cg estimation procedure described in Section 5.3 yields an estimate of the quantity $\overline{CG_{EQ}}$ rather than $\overline{CG}$. The $\overline{CG_{EQ}}$ vector is actually what is needed for roll rate control as will be shown in the procedure described in Chapter 6. In simulation runs with full measurement noise and deterministic errors in roll angle IMU measurements, the estimation of $\overline{CG_{EQ}}$ when both hinge and cg offsets existed was as accurate as the estimate of $CG$ with no hinge point offset (see plot Figure 5-6).
Figure 5-5. Sum of CG and Hinge Torque Compared with Offset Torque Computed from Total Force and Equivalent CG Offset.
Figure 5-6. Equivalent CG Estimation.
CHAPTER 6

MODIFICATION OF YAW STEERING TO INCLUDE ROLL RATE CONTROL

The method chosen to control the roll rate is to provide yaw angle-of-attack commands such that the resulting normal forces produce roll torque components to oppose existing roll rates. By providing the roll control inputs only in the yaw plane (i.e., perpendicular to the pitch plane) the effect on the pitch-plane-steering control and trajectory shaping is minimized. The only coupling between pitch and yaw control is through the total-angle-of-attack limiter. Since the angle-of-attack limiter is normally used only briefly at the transition from the kick maneuver to the steering phase, and during peak wind disturbances, the overall effect of this coupling is negligible. The roll rate control method developed in this thesis does not operate during the kick maneuver. In the kick maneuver, the vehicle attitude, not angle of attack, is controlled to place the vehicle on the correct flight path to transition to steering. Therefore, there is no provision in the kick maneuver for employing a yaw-angle-of-attack command to control the roll rate. Reasonably large roll rates can occur during the kick maneuver due to large angles of attack generated by the rapid attitude maneuver. However, a worst-case analysis showed that the maximum roll rate in the kick maneuver was 35 deg/sec. Since this maximum was less than the assumed design limit of 50 deg/sec it was decided that it was not necessary to modify the kick maneuver implementation to allow roll rate control.
6.1 Determination of Magnitude of Roll Rate Control Command

The primary purpose of the roll rate control is to limit the roll rate so that it does not exceed the maximum allowable value. Beyond that, an attempt is made to reduce the roll rate as much as possible, without unduly affecting the trajectory of the vehicle or complicating the control, so as to reduce the effects of roll rate on the IMU and vehicle staging. With this in mind, the alternative yaw steering system was modified to allow the normal yaw angle-of-attack command, based on cross-track velocity nulling, to be replaced at appropriate times by the yaw-angle-of-attack command that controls the roll rate. The numerical value of the alternative yaw command and the criteria for its use to reduce excessive roll rates were selected to achieve roll rate control objectives while having a minimum impact on yaw velocity steering.

To limit the use of the alternative roll-rate-control yaw angle-of-attack command it was decided to employ this command only if the roll rate exceeded a specified deadband. The control input itself, \( \alpha_{YRC} \), was initially tried as a constant value as in a bang-bang type control. The idea was to provide the maximum roll rate control input during the periods when roll rate control could be done so that during the periods when the yaw steering was not accepting roll rate control inputs, the roll rate would have been already reduced as much as possible. This was found to cause the roll rate to overshoot the deadband. To minimize the overshoot of the roll rate deadband, the magnitude of \( \alpha_{YRC} \) was reduced to zero as the roll rate approached the deadband value. Trial and error resulted in the selection of a deadband value for the roll rate of 1 deg/sec with \( \alpha_{YRC} \) determined from

\[
\alpha_{YRC} = \frac{\left| \dot{\phi} \right| - 1}{10} \quad \text{For} \quad \left| \dot{\phi} \right| > 1 \text{ deg/sec} \quad (6.1)
\]

and

\[
\alpha_{YRC} = 0 \quad \text{For} \quad \left| \dot{\phi} \right| \leq 1 \text{ deg/sec}
\]
6.2 Selection of Polarity of Roll Rate Control Command

The ability to estimate the equivalent cg torque and the raceway torque separately makes it possible to determine the total roll torque on the vehicle that would result from either polarity of \( a_y^{RC} \). This is in fact what is done to select a polarity of \( a_y^{RC} \) that will have the most effect in reducing the roll rate. The general plan of attack to do this is:

1. Determine the magnitude of \( a_y^{RC} \) based on \(|\dot{\phi}|\).
2. Compute the steady-state-roll torque values which would result from positive and negative polarities of \( a_y^{RC} \).
3. Select the polarity of \( a_y^{RC} \) that causes the resultant total roll torque to have the largest impact in reducing the roll rate. (This corresponds to the polarity of \( a_y^{RC} \) that produces the largest component of total roll torque that is in opposition to the roll rate.)

The detailed procedure for implementing these three steps is as follows:

1. Compute the magnitude of \( a_y^{RC} \) from Equation 6.1.

\[
a_TOTC = \sqrt{\alpha_p^2 + a_y^{RC^2}}
\]  

(6.2)

2. Using the computed value of \( a_y^{RC} \) and the current estimated pitch plane angle of attack, \( \alpha_p \), compute the total angle of attack command, \( a_{TOTC} \)

3. Compare the value of \( a_{TOTC} \) with the maximum allowable value of \( a_{TOTMAX} \) (which was based on the 4000 lb limit on normal
aerodynamic force). If \( \alpha_{TOTC} > \alpha_{TOTMAX} \), then limit the magnitude of \( \alpha_{YRC} \) according to the relationship \[
|\alpha_{YRC}| = \frac{\alpha_{YRC}}{\alpha_{TOTMAX} / \alpha_{TOTC}}
\] (6.3)

4. Assume \( \alpha_{YRC} \) is positive

5. Compute the normal aerodynamic and engine yaw forces in the unrolled body axis system resulting from the commanded \( \alpha_{YRC} \):
   
   The "commanded" yaw aerodynamic force is
   \[
   F_{AEROY_{RC}} = \alpha_{YRC} \quad \text{AREA} \quad C_{Na}
   \] (6.4)

   Under the assumption of steady-state torque conditions, the yaw engine force required to balance the commanded yaw aerodynamic force is
   \[
   F_{ENG_{Y_{RC}}} = \alpha_{YRC} \quad \text{AREA} \quad C_{Na} \quad L_{CP/CG} \quad X
   \] (6.5)

6. Form the vector representing the total engine force in the unrolled vehicle y-z axis system resulting from the commanded yaw engine force and the pitch engine force (based on measured engine deflection)
   \[
   F_{ENG_{RC}} = (0, F_{ENG_{Y_{RC}}}, F_{ENGZ})
   \] (6.6)

*Only \( \alpha_{YRC} \) needs to be limited here because \( |\alpha_p| \) does not exceed \( \alpha_{TOTMAX} \), since \( \alpha_p \) has already been subject to a limiter by Equation 2.35. This method of limiting \( |\alpha_{YRC}| \), when necessary, gave satisfactory results, but a more correct relationship to replace Equation 6.3 would be
   \[
   |\alpha_{YRC}| = \sqrt{\frac{2}{\alpha_{TOTC} - \alpha_p^2}}
   \]
7. Form the vector representing the total aerodynamic force in the unrolled vehicle y-z axis system resulting from the commanded yaw aerodynamic force and the estimated pitch aerodynamic force.

\[
\begin{align*}
\vec{F}_{AERO_{RC}} &= (0, F_{AERO_{RC}}, F_{AEROZ}) \\
\end{align*}
\]  

(6.7)

8. Transform the engine and aerodynamic forces into the (rolled) body coordinate frame using the measured direction cosine matrix \([C]_{MUB\rightarrow B}\) that transforms from unrolled to rolled coordinates

\[
\begin{align*}
\vec{F}_{ENG_{RC_B}} &= [C]_{MUB\rightarrow B} \vec{F}_{ENG_{RC}} \\
\vec{F}_{AERO_{RC_B}} &= [C]_{MUB\rightarrow B} \vec{F}_{AERO_{RC}} \\
\end{align*}
\]  

(6.8)  

(6.9)

9. From the y- and z-components of these two vectors calculate the components of the total normal force vector in the body-axis system

\[
\begin{align*}
F_{TOTY_{RC_B}} &= F_{AEROY_{RC_B}} + F_{ENGY_{RC_B}} \\
F_{TOTZ_{RC_B}} &= F_{AEROZ_{RC_B}} + F_{ENGZ_{RC_B}} \\
\end{align*}
\]  

(6.10)  

(6.11)

10. Calculate \(PHIXIA_C\) from the y and z components of \(F_{AERO_{RC_B}}\) using the procedure described in Section 2.1.2.2 and Equation 2.9
11. Calculate the commanded raceway torque coefficient \( C_{\ell C} \) by interpolation from the functionalized raceway torque coefficient table using PHIXIAC, Mach number, and \( \alpha_{TOTC} \)

12. Calculate the commanded raceway torque \( \text{RACEWAYTORQ}_C \) from \( C_{\ell C} \) using Equation 2.10

13. Calculate the commanded torque resulting from the cg and engine hinge point offsets. To do this the components of estimated cg equivalent position are used with the total force components computed in Step 9 as shown in the following equation

\[
\text{CGEQTORQ}_C = F_{TOTY_{RC}} - F_{TOTY_{EST}}
\]  
\[
\text{CGEQTORQ}_C = F_{TOTZ_{RC}} - F_{TOTZ_{EST}}
\]  
\[
\text{CGEQTORQ}_C = F_{TOTY_{RC}} - F_{TOTY_{EST}} \quad \text{(6.12)}
\]

14. Combine the commanded raceway torque and cg equivalent torque to find the total commanded roll torque due to a positive \( \alpha_{YRC} \)

\[
\text{TORQP}_C = \text{RACEWAYTORQ}_C + \text{CGEQTORQ}_C
\]  
\[
\text{TORQP}_C = \text{RACEWAYTORQ}_C + \text{CGEQTORQ}_C \quad \text{(6.13)}
\]

15. Assume \( \alpha_{YRC} \) is negative

16. Repeat Steps 5-14 to obtain the commanded torque \( \text{TORQN}_C \) for a negative \( \alpha_{YRC} \)

17. Select the polarity of \( \alpha_{YRC} \) that produces a component of total roll torque that is in opposition to the roll rate. This is done by comparing \( \text{TORQP}_C \) and \( \text{TORQN}_C \) based on the roll rate direction as follows:

\[
\alpha_{Y_{RC}} = -|\alpha_{Y_{RC}}| \quad \text{if} \quad \text{TORQN}_C < \text{TORQP}_C \quad \text{(6.14)}
\]
or
\[
\alpha_{Y_{RC}} = \left| \alpha_{Y_{RC}} \right| \quad \text{if } \text{TORQN}_C \geq \text{TORQP}_C \quad (6.15)
\]

If \( \dot{\phi} < -1 \text{ deg/sec} \),
\[
\alpha_{Y_{RC}} = \left| \alpha_{Y_{RC}} \right| \quad \text{if } \text{TORQN}_C < \text{TORQP}_C \quad (6.16)
\]
or
\[
\alpha_{Y_{RC}} = -\left| \alpha_{Y_{RC}} \right| \quad \text{if } \text{TORQN}_C \geq \text{TORQP}_C \quad (6.17)
\]

18. At one second intervals, determine whether the normal yaw command mode using \( \alpha_{Y_{VC}} \) or the alternative command mode using the roll rate control command \( \alpha_{Y_{RC}} \) is to be used. This is done according to the procedure discussed in the following section and shown in Figure 6-2. The selected command mode will be used for a period of one second although the steering commands are updated every 0.1 secs. This is done to prevent rapid cycling between yaw steering modes and to allow sufficient response time to the commands in each mode.

6.3 Control Logic for Selecting Yaw Steering Command

The final step in the modification of the yaw steering to include roll-rate-control commands is to provide the logic to decide at one second intervals whether to use the alternative yaw angle-of-attack command for roll control or the normal yaw angle-of-attack command for cross-track velocity nulling. These two yaw commands may or may not be of different polarity. In some cases the normal velocity-nulling command may increase the roll rate, and in other cases it may decrease it. The following analysis of the basic physical characteristics will give some
insight into the options which should be considered in developing the yaw command selection logic.

First, when both a roll rate and cross-track velocity error exist, the polarity of $\alpha_{yRC}$ and $\alpha_{yVC}$ will be the same every 180 degrees of roll. Thus one of these control inputs will result in some measure of desired control of the other. Secondly, when pitch plane forces are high enough that the resulting $\alpha_p$ approaches $\alpha_{TOTMAX}$, the magnitude of the allowable $\alpha_{yRC}$ becomes limited to a small value. Consequently, only a small amount of roll rate control is achieved. The existence of this situation can be determined by comparing $TORQP_C$ and $TORQN_C$. If they are of the same sign, then $\alpha_{yRC}$ is relatively ineffective in controlling the roll rate and therefore velocity control is permitted to take over. An exception to this situation is made if $|\dot{\phi}| > 20$ deg/sec. In this case, roll rate control is given priority, regardless of its effectiveness, in order to keep $|\dot{\phi}| < 50$ deg/sec. Thirdly, when the vehicle rotates to an orientation where the raceway torque and the equivalent cg torque are nearly equal and opposite or both small for positive and negative values of $\alpha_{yRC}$, then roll rate control will be ineffective regardless of pitch-plane forces. For example, when the vehicle is close to the orientation shown in Figure 6-1, with the equivalent cg offset as illustrated, forces parallel to the unrolled y axis do not generate an appreciable roll torque component. Therefore, attempts to use $\alpha_{yRC}$ to control the roll rate during these times would only unnecessarily increase the cross-track velocity error and not significantly affect the roll rate. Conversely, using $\alpha_{yVC}$ to control the velocity during these periods would have a negligible effect on the roll rate. Once again these orientations can be determined to exist by comparing $TORQP_C$ and $TORQN_C$. It was determined experimentally that when

$$|TORQP_C - TORQN_C| < 40 \text{ lb-ft} \quad (6.18)$$
unrolled y body axis forces have little effect on the roll rate and therefore the velocity steering command is used in this case. An exception is again made if $|\dot{\phi}| > 20$ deg/sec. In the absence of any of the above conditions, the choice of whether to use $\alpha_{YRC}$ or $\alpha_{YVC}$ is made by using a priority system based on the magnitudes of the cross-track velocity, $V_y$, and $\dot{\phi}$. The logic diagram summarizing the above discussion is shown in Figure 6-2.
Figure 6-2. Control Logic for Selecting Yaw Steering Command.
CHAPTER 7

PERFORMANCE EVALUATION

This chapter presents the results of simulations used to evaluate: (1) the trajectory-family pitch-plane steering method in the absence of roll-rate control and (2) the modified yaw steering approach for roll-rate control in combination with the pitch-plane steering concept. The simulations performed to evaluate the pitch-plane steering without roll-rate control used worst-case conditions that produced the maximum uncontrolled roll rates. In this connection, it should be pointed out that results from zero-roll-rate simulation runs, which are not presented in this thesis, showed that there were only negligible differences in the pitch-plane trajectory as compared to the uncontrolled-roll-rate case presented in the next section, 7.1. Moreover, the modified yaw steering for roll-rate control was designed so that this modification would not significantly alter the pitch-plane trajectory, as shown by simulation runs in section 7.3.

7.1 Trajectory-Family Pitch-Plane Steering Without Roll-Rate Control

Uncontrolled-roll-rate simulation runs were made for no wind, head wind, tail wind, and crosswind conditions for various launch angles, using the trajectory-family pitch-plane steering method developed in this thesis. The effects of off-nominal thrust and thrust estimation errors were also examined. The results show that the steering method developed in this thesis provides good performance in terms of controlling terminal dynamic pressure.
The following table (Table 7-1) shows the performance of the steering scheme, under nominal thrust conditions, in terms of the dynamic pressure achieved at burnout. It should be noted that as a result of the compensation described in Section 2.3.1.7, variations in launch angle have almost no effect on the terminal dynamic pressure. Plots of runs 1 through 4 are shown in Figures 7-1 through 7-4. (Note that roll rate related information is included in the plots of runs 2 through 4. These plots will be referred to again in Section 7.2 for comparison with the roll rate control results.)

Table 7-1. Trajectory Steering Results, Nominal Thrust.

<table>
<thead>
<tr>
<th>Run #</th>
<th>Launch Angle (deg)</th>
<th>Wind</th>
<th>Error Sources</th>
<th>End of Stage Dynamic Pressure lb/ft²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>119</td>
<td>No</td>
<td>1</td>
<td>1200.6</td>
</tr>
<tr>
<td>2</td>
<td>119</td>
<td>Head</td>
<td>1</td>
<td>1240.7</td>
</tr>
<tr>
<td>3</td>
<td>119</td>
<td>Tail</td>
<td>1</td>
<td>1160.5</td>
</tr>
<tr>
<td>4</td>
<td>119</td>
<td>Cross</td>
<td>1</td>
<td>1201.3</td>
</tr>
<tr>
<td>5</td>
<td>119</td>
<td>No</td>
<td>2</td>
<td>1202.7</td>
</tr>
<tr>
<td>6</td>
<td>119</td>
<td>Head</td>
<td>2</td>
<td>1239.7</td>
</tr>
<tr>
<td>7</td>
<td>119</td>
<td>Tail</td>
<td>2</td>
<td>1162.9</td>
</tr>
<tr>
<td>8</td>
<td>119</td>
<td>Cross</td>
<td>2</td>
<td>1202.3</td>
</tr>
<tr>
<td>9</td>
<td>61</td>
<td>Head</td>
<td>1</td>
<td>1242.1</td>
</tr>
<tr>
<td>10</td>
<td>61</td>
<td>Tail</td>
<td>1</td>
<td>1161.3</td>
</tr>
<tr>
<td>11</td>
<td>90</td>
<td>Head</td>
<td>1</td>
<td>1242.2</td>
</tr>
<tr>
<td>12</td>
<td>90</td>
<td>Tail</td>
<td>1</td>
<td>1162.5</td>
</tr>
</tbody>
</table>

Error Sources: 1. IMU measurement noise and deterministic errors, with no errors in the angle-of-attack estimator coefficients.

2. IMU measurement noise and deterministic errors, IMU boresight error, 0.25 degree measurement error in nozzle deflection, 12% error in the coefficients of angle-of-attack and rate estimators
Off-nominal thrust effects are given in Table 7-2 for ±20% thrust variations. These results assume that the exact value of thrust is available for steering computations.

Table 7-2. Thrust Variation Effect on Burnout Dynamic Pressure, No Wind

<table>
<thead>
<tr>
<th>$K_{BURN}$</th>
<th>$\Delta Q (\text{lb/ft}^2)$ From Nominal Thrust</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>6.6</td>
</tr>
<tr>
<td>0.8</td>
<td>-10.9</td>
</tr>
</tbody>
</table>

The effect of thrust estimation errors was also evaluated. As stated in section 2.2.2, it should be possible to estimate the thrust within 2%; however, the investigation included both ±2% and ±5% thrust estimation errors. The evaluation assumed that the estimated thrust was equal to the nominal thrust, where the actual thrust was set at values ±2% and ±5% from the nominal thrust. The results of simulation runs using these assumed errors are shown in Table 7-3.

Table 7-3. Effect of Thrust Estimation Error on Burnout Dynamic Pressure, No Wind

<table>
<thead>
<tr>
<th>Deviation of Actual Thrust from Nominal</th>
<th>$\Delta Q (\text{lb/ft}^2)$ at Burnout From Nominal Thrust Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2%</td>
<td>-5.9</td>
</tr>
<tr>
<td>+2%</td>
<td>+6.7</td>
</tr>
<tr>
<td>-5%</td>
<td>-18.4</td>
</tr>
<tr>
<td>+5%</td>
<td>+22.5</td>
</tr>
</tbody>
</table>

Assuming that the members of the family of trajectories are linearly related as demonstrated in Section 2.3.1 and assuming that $\gamma_{\text{BIASC}}$ has been added to eliminate launch angle effects on burnout dynamic pressure, the values of $\Delta Q$ shown in Tables 7-2 and 7-3 can be merely added to Table 7-1 to find the associated effects on burnout dynamic pressure for various wind and error combinations.
Figure 7-1. Trajectory Steering Run #1.
Figure 7-1. Trajectory Steering Run #1 (cont.).
Figure 7-2. Trajectory Steering Run #2.
Figure 7-2. Trajectory Steering Run #2 (cont.).
Figure 7-2. Trajectory Steering Run #2 (cont.).
Figure 7-3. Trajectory Steering Run #3.
Figure 7-3. Trajectory Steering Run #3 (cont.).
Figure 7-3. Trajectory Steering Run #3 (cont.).
Figure 7-4. Trajectory Steering Run #4.
Figure 7-4. Trajectory Steering Run #4 (cont.).
Figure 7-4. Trajectory Steering Run #4 (cont.).
Figure 7-4. Trajectory Steering Run #4 (cont.).
7.2 Roll Rate Control with Trajectory-Family Pitch-Plane Steering

The evaluation of the performance of the modified yaw steering for roll-rate control, in combination with the trajectory-family pitch-plane steering, was carried out using headwind, tailwind, and crosswind conditions. The simulation runs that were made for this evaluation study used the orientations that gave the maximum roll rate conditions determined in section 3.2. The worst case conditions were also chosen for the random center of gravity and engine hinge point offsets.

The results show that the roll rate control can achieve significant reductions in the vehicle roll rate after the kick maneuver without causing appreciable changes in the pitch-plane trajectory. These results are shown in Table 7-4 and Figures 7-5 through 7-7. It should be noted that the maximum roll rates with roll control in Table 7-4 occur just after the kick maneuver and are quickly reduced as shown in the plots, whereas the uncontrolled roll rate stays at a high level. Plots of runs 1 through 3 are shown in Section 7.1, Figures 7-2 through 7-4 respectively. Plots of runs 4 through 6 are shown in Figures 7-5 through 7-7.
## Table 7-4. Roll Rate Control Results

<table>
<thead>
<tr>
<th>Run #</th>
<th>Roll Control</th>
<th>Wind Sources</th>
<th>Error Sources</th>
<th>Initial Roll (deg)</th>
<th>Max Roll (deg/sec)</th>
<th>End of Stage Roll (deg/sec)</th>
<th>Q (lb/ft²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>No Head</td>
<td>1</td>
<td>-10</td>
<td>10</td>
<td>40.35° @ 24.8s</td>
<td>26.25</td>
<td>1240.7</td>
</tr>
<tr>
<td>2</td>
<td>No Tail</td>
<td>1</td>
<td>30</td>
<td>-10</td>
<td>-45.96° @ 40.6s</td>
<td>-25.67</td>
<td>1160.5</td>
</tr>
<tr>
<td>3</td>
<td>No Cross</td>
<td>1</td>
<td>10</td>
<td>-10</td>
<td>-60.90° @ 37.4s</td>
<td>-46.20</td>
<td>1201.3</td>
</tr>
<tr>
<td>4</td>
<td>Yes Head</td>
<td>1</td>
<td>-10</td>
<td>10</td>
<td>37.60° @ 14.0s</td>
<td>-15.60</td>
<td>1247.2</td>
</tr>
<tr>
<td>5</td>
<td>Yes Tail</td>
<td>1</td>
<td>30</td>
<td>-10</td>
<td>-39.03° @ 14.4s</td>
<td>1.82</td>
<td>1169.1</td>
</tr>
<tr>
<td>6</td>
<td>Yes Cross</td>
<td>1</td>
<td>10</td>
<td>-10</td>
<td>-38.90° @ 13.6s</td>
<td>1.90</td>
<td>1209.4</td>
</tr>
<tr>
<td>7</td>
<td>Yes Head</td>
<td>2</td>
<td>-10</td>
<td>10</td>
<td>38.60° @ 14.6s</td>
<td>-18.98</td>
<td>1249.5</td>
</tr>
<tr>
<td>8</td>
<td>Yes Tail</td>
<td>2</td>
<td>30</td>
<td>-10</td>
<td>-40.13° @ 14.4s</td>
<td>-9.31</td>
<td>1167.3</td>
</tr>
<tr>
<td>9</td>
<td>Yes Cross</td>
<td>2</td>
<td>10</td>
<td>-10</td>
<td>-39.46° @ 13.8s</td>
<td>-9.80</td>
<td>1206.0</td>
</tr>
</tbody>
</table>

Simulation Conditions:  
- 119° launch angle, nominal thrust  
- Random center of gravity, CGZ = +.0167 ft  
- Random hinge point offset, HNGZ = -.0133 ft  
- Error Sources 1. IMU measurement noise and deterministic errors, angle-of-attack estimation with no errors in coefficients  
- Error Sources 2. IMU measurement noise and deterministic errors, IMU boresight error, 0.25 degree nozzle deflection measurement error, 12% error in coefficients of angle of attack and rate estimators
Figure 7-5. Roll Rate Control Run #4.
Figure 7-5. Roll Rate Control Run #4 (cont.).
Figure 7-5. Roll Rate Control Run #4 (cont.).
Figure 7-5. Roll Rate Control Run #4 (cont.).
Figure 7-5. Roll Rate Control Run #4 (cont.).
Figure 7-5. Roll Rate Control Run #4 (cont.).
Figure 7-5. Roll Rate Control Run #4 (cont.).
Figure 7-6. Roll Rate Control Run #5.
Figure 7-6. Roll Rate Control Run #5 (cont.).
Figure 7-6. Roll Rate Control Run #5 (cont.).
Figure 7-6. Roll Rate Control Run #5 (cont.).
Figure 7-6. Roll Rate Control Run #5 (cont.).
Figure 7-6. Roll Rate Control Run #5 (cont.).
Figure 7-6. Roll Rate Control Run #5 (cont.).
Figure 7-7. Roll Rate Control Run #6.
Figure 7-7. Roll Rate Control Run #6 (cont.).
Figure 7-7. Roll Rate Control Run #6 (cont.).
Figure 7-7. Roll Rate Control Run #6 (cont.).
Figure 7-7. Roll Rate Control Run #6 (cont.).
Figure 7-7. Roll Rate Control Run #6 (cont.).
Figure 7-7. Roll Rate Control Run #6 (cont.).
8.1 Conclusions

Conclusions and recommendations are presented below for the two areas of investigation covered in this thesis: (1) the development of a pitch-plane steering method based on the functionalization of the family of zero-angle-of-attack trajectories that terminate at the same dynamic pressure at Stage 1 burnout and (2) the development of a modified yaw steering method to limit the roll rate in the Stage 1 boost phase.

8.1.1 Pitch-Plane Steering

This investigation has shown that the dynamic pressure of a boost vehicle at staging can be controlled by steering the vehicle to the "nearest" of a family of zero-angle-of-attack trajectories based on the vehicle's current values of flight path angle, altitude, and magnitude of earth-relative velocity. Furthermore, these trajectories were demonstrated to be linearly related and therefore conveniently functionalized. A technique was also developed to eliminate the effects of pitch-plane launch angle variations on the dynamic pressure at staging. This technique was found to be effective not only for the steering method investigated here but also for Fader's Altitude with Flight Path Angle Feedback Steering method [2].

8.1.2 Modified Yaw Steering for Roll Rate Control

A simulation method was developed to determine the worst-case maximum uncontrolled roll rate for an assumed asymmetrical boost
vehicle. It was shown how this method could be applied to demonstrate that the worst-case roll rate would exceed the assumed design limit. It was then shown that it is feasible to employ a modified yaw steering method to change the lateral forces on the vehicle so as to reduce the roll rate that would otherwise result from these forces. This study considered the roll torques that would be produced by a raceway on one side of the vehicle as well as the additional torques that would result from lateral offsets in the vehicle center of gravity and engine hinge point. The study showed that even small unpredictable offsets could result in roll torques that are comparable to or in excess of the roll torque produced by a typical raceway. Therefore, it was concluded that the modified yaw steering must estimate and take into account the effects of these offsets. It was found that an "equivalent center of gravity offset" could be defined that would result in a roll torque equal to the net roll torque resulting from both center of gravity and hinge point offsets. An estimator design was developed that provides a satisfactory estimate (within 15%) of the equivalent center of gravity offset. This estimator was incorporated into the modified yaw steering concept. The modified yaw steering concept was developed to provide an alternative yaw angle-of-attack command based on estimates of the effects of this command on the roll torque components. Logic was developed for employing this alternative command in lieu of the normal velocity-nulling yaw steering command, as required, to limit the roll rate without significantly affecting the cross-track velocity error or the pitch-plane trajectory.

8.2 Recommendations

8.2.1 Pitch-Plane Steering

The success in using a functionalized family of trajectories for pitch-plane steering suggests the possibility of obtaining even better performance by using an in-flight predictive simulation to periodically update a gravity-turn relationship used to compute the commanded flight
path angle, \( \gamma_C \). This predictive simulation would determine the change in current \( \gamma_C \) that would produce a zero-angle-of-attack trajectory ending at the desired vehicle state at the predicted engine burnout time. In addition to providing the flexibility to accommodate, without functionalization, a wide variation in launch attitude, engine thrust level, and wind conditions, this predictive steering approach might be used, without modification, for a variety of desired burnout conditions. The differential equations that would have to be integrated in the predictive simulation are very simple in form and may provide satisfactory performance with as much as a two-second integration time increment. It is recommended that this predictive steering approach be investigated.

8.2.2 Modified Yaw Steering for Roll Rate Control

For vehicles with roll torque characteristics such as described in this thesis, the roll rate control method described appears to be a viable alternative to the use of roll control jets. Also, in control applications that require knowledge of the lateral cg offset, the estimation technique developed may be useful. Finally, for vehicles that require roll rate control, consideration should be given to the relative design of raceway compared to cg and engine hinge point offsets. If the vehicle's raceway could be designed so as to have an effective roll torque greater than that of the cg and hinge point offsets, a simpler roll rate control method based on raceway position alone could be used.
LIST OF REFERENCES


