EXPERIMENTAL STUDIES OF TURBULENCE
IN LIQUID-SOLID FLOWS
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WITHDRAWN
A series of laboratory experiments were performed to ascertain the extent and manner of turbulence modification induced by low concentration suspended loads of near neutral buoyancy. Hot-wire measurements of the fluid velocity field of free surface flows were obtained in a specially designed flume recirculating a dielectric liquid and 0.5 mm diameter spherical plastic particles. In the fixed Reynolds number flow (16,800) the data obtained at six different concentration levels, ranging from 0 to 3.5% by volume, indicate that the presence of particles produces substantial turbulence changes. Even at this low level the mean velocity profile shows an increasing gradient near the bed and sharp deviation from a logarithmic profile. The rms level of each of the velocity components \( u' \), \( v' \) and \( w' \) increases, indicating a general rise in turbulence intensity. The Reynolds stress \( \rho u'v' \) increases, and its maximum value shifts away from the bed. The overall scale of turbulence appears to remain unchanged.

The data indicate that offhand neglect of suspended particle presence is an oversimplification. There is a similarity between these data and those obtained under adverse pressure gradients. Some effort is made to clarify the altered turbulence production mechanism, and some future experimental work is proposed.
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I. INTRODUCTION

"That which is far off and exceedingly deep, who can find it out...?"

Ecclesiastes

In studies of fluid flow mechanics one need not go far afield to encounter complex unsolved problems. For over fifty years scientists and engineers have attempted to place the dynamics of sediment transport and other liquid-solid flows on a firm theoretical basis. Despite these efforts this problem area is still best described as an art rather than a science. The roots of the difficulty are readily apparent. To quote Batchelor (1966),

If I were asked to name the areas in fluid mechanics which are most frustrating from an all-round point of view, that is taking account of the conceptual, experimental, and analytical difficulties I should put high on the list first the general problem of turbulence and secondly the Lagrangian aspects of fluid flow. Both of these areas are involved in the motion of small particles in turbulent flows...

However, Batchelor was quick to point out that this level of complexity is for some investigators an attractive feature; those who thrive on simultaneous frustration and elation are well served.

Providing a challenging research area, studies of transport mechanics in various free streams and jets find application in a variety of scientific and engineering problems. In each the investigators attempt to correlate the mode and amount of material transported with measured parameters of the flow field. Studies by oceanographers and marine geologists on regional sediment transport, by
hydraulic engineers asked to predict beachfront erosion of the filling of navigable channels, by ecologists and biologists concerned with the dispersion of air pollutants and aerosols, by industrial engineers designing transport systems for coal, paper pulp, etc., and by chemical engineers of the many diffusion and catalytic processes involved in commercial production—all face basically the same problem. The highly nonlinear and time-dependent nature of the governing equations together with an imperfect understanding of the physics of the interactions involved precludes unique analytical solutions. Nevertheless these studies have produced empirical solutions for specific cases and a fairly detailed qualitative picture of the transport process. It has become clear that the individual features of this process are closely interdependent. To determine the macroscale properties it is not enough to detail and sum the microscale components. Selective interactions often take place. To examine some of these features in more detail I wish to confine this discussion to the transport of natural sediments and to noncohesive materials in particular.

Observations indicate that a bed of loose, noncohesive grains will tend to react in a well defined and repeatable manner when subject to shearing flows. Initially at rest, the particles show no tendency to move in very low velocity flows, whether laminar or turbulent. With an increase in mean velocity, slight movements are observed at separated locations on the bed. This "threshold of motion" is ill-
defined, and has been little studied. It is unclear whether the motion is the result of a general flow instability or simply a localized feature, possibly a simple lift or drag force on the individual particle.

Further increase in the velocity produces general agitation and motion of the surface particles. This level is marked by the onset of fully turbulent shear flow, the scattering of particles by the apparent impingement of turbulent jets or eddies, and the start of deformation of the initially plane bed into sediment ripples. Material transport, either by rolling or saltating (i.e. small hops), remains restricted to the bed surface. This "bed load" is the predominant transport mode for most natural regimes.

Necessarily, development of a rippled bed surface will tend to modify the mean velocity field. In turn, the resultant accelerations and decelerations will effect the evolution of the bed. An accurate description of the growth rate of these bed features requires a careful evaluation of the local velocity field and the resultant boundary shear stress. However, even with the required measurements in hand, it remains to apply the data within the framework of a coherent physical model. This problem has been addressed by numerous investigators (Benjamin, 1959; Kennedy, 1963). Their results are not entirely satisfying. While sufficiently accurate for some specific cases, their efforts do little to quantify bed-form evolution and subsequent velocity field modification. Although Smith (1969) has recently presented some more promising results, this area remains largely a fundamental unknown in the transport process.
Continuing to increase the mean velocity (but with the flow remaining subcritical) increases bed deformation, induces longer wavelength, large amplitude, features or dunes, and results in the transport of noticeable amounts of sediment above the bed as a suspended load. This suspended material tends to further complicate the transport mechanics. The already complex boundary conditions are aggravated by a concentration gradient, flow viscosity increases, momentum transfer will be subject to variations due to particle-particle collisions, and the structure of fluid turbulence will be modified as a function of suspended sediment concentration. In short, the problem initially posed as a single-phase flow with variable boundary conditions must now be posed in terms of two-phase or liquid-solid dynamics. Although this aspect is often neglected (being justified in terms of the low suspended concentration) it is becoming increasingly apparent that certain properties of the velocity field are sharply modified by extremely low suspended loads. Offhand neglect may be an oversimplification.

Further increase in the mean velocity will reveal few additional complicating features. Bed forms will continue to change, becoming plane at some high shear velocity. The suspended load increases and, in the presence of a free-surface, supercritical flows induce surface waves and the resultant bed forms termed antidunes. The latter features represent an additional influence on the velocity field. For the most part however, these problems represent an extension of the lower velocity conditions. Seemingly if a
A comprehensive model for subcritical flow was available, it could be made to serve for supercritical conditions.

This brief review of the qualitative features of the transport process points clearly to three prominent problem areas. First, what determines the threshold of motion and what is the nature of the forces involved? Second, what are the dynamics governing the onset and evolution of bed forms? Third, how does the presence of a suspended load influence the structure of fluid turbulence in shear flows? These areas form integral parts of the total transport process, and are to some extent mutually dependent. This interdependence is especially strong for the latter two areas, since any variations in the velocity field are necessarily reflected in the boundary shear stress and subsequently the bed profile. Accurate interpretations require careful evaluation of the character and extent of turbulence modification produced by a suspended load. Such a study in turn must recognize the existence of two mutually interacting fields of random motion: the particle and fluid turbulence fields. Only in the limit, as particle diameter approaches zero, do the measured properties of each become identical. The following investigation concentrates on modifications produced in the fluid turbulence field in hopes of clarifying one of the complex interactions involved in the transport process.
In reviewing the literature available on past efforts to assess flow modifications produced by suspended loads, one is confronted by a vast array of methods, results, and conclusions. Each investigator seems intent on proposing and supporting a new theory rather than clarifying or extending previous research. As a result it is difficult to arrive at a consensus of opinion on any one feature of this complex problem, a problem constrained first by the lack of a unified theory governing the structure and mechanics of turbulent shear flow and secondly by disagreement over the mechanism responsible for the suspension of particles.

The difficulties remain not for the lack of effort; as early as 1906, Einstein (1906) hypothesized that particles suspended by a free stream would modify the energy balance. To estimate quantitatively the increased energy dissipation, he introduced the concept of an "effective viscosity" as a function of suspended particle concentration. The simple partition of energy suggested by these laminar flow studies was soon questioned, however, when turbulent flow applications were attempted.

Gilbert (1914) published the results of an experiment designed to measure changes in stream flow resistance for various concentrations of suspended sediment. Contrary to the expected results, the data revealed increases in stream mean velocity with fixed bed conditions and increasing suspended load. This study, which suggests that a model pic-
turing dissipation of flow energy only through bed-form resistance and work to transport sediment is over-simplified, pointed out the need for a more detailed understanding of the dynamics of clear-liquid turbulence and its relationship to particle suspension.

An empirical relationship governing the vertical distribution of sediment suspended by turbulent flows was first published by O'Brien (1933) and later expanded by Rouse (1937). The work applied the theoretical concepts of turbulence developed by Taylor (1921) and Prandtl (1925) for clear fluids to sediment laden flows. Briefly, it assumed that fluid and particle motions were identical and that therefore the diffusion of sediment is analogous to the diffusion of momentum. Under steady flow conditions, the upwards diffusion of sediment across a given level must be equal to the amount falling under gravitational forces. With given conditions of shear stress and mean velocity it is straightforward to derive a governing differential equation.

Although based on several questionable assumptions, the close correlation between the predicted distribution of suspended sediment and field and flume data suggests that the proposed process was not very far from the truth. In particular, attention was focused on the vertical component of the turbulent velocity field as the agent responsible for particle suspension. Further attempts to quantify the suspension process led directly to detailed studies of flow modification.

Until the experimental work of Vanoni (1953) the presence of a suspended load, of normal field concentration, was as-
sumed to have little effect on the turbulent flow properties. Kalinske and Hsia (1945), for example, in their work with silt-size particles, noted that suspensions in excess of 11% by weight are required before there is modification of the fluid velocity field. Generally, considerations have been based only on the assumption that the overall flow remains Newtonian. The presence of sediment was related simply to changes in viscosity rather than to any changes in flow dynamics. The mean velocity profile was invariably described by using the logarithmic velocity defect law:

\[
\frac{U_1 - U_2}{u_*} = 2.3 \frac{\log_{10} \frac{Y_2}{y}}{k} \log_{10} \frac{Y_2}{y}
\]

with \(U_1\) and \(U_2\) velocities at height \(y_1\) and \(y_2\) respectively

\(u_* = \) the shear velocity \(= \sqrt{\gamma \tau / \rho}\)

\(\tau = \) the bed shear stress

\(\rho = \) the fluid density

In the analysis of a series of free-surface channel flows, Vanoni noted a consistent decrease in the von Karman constant \(k\) with increasing suspended load and a general departure of the velocity from a logarithmic profile in the wall region. He concluded that the decrease in \(k\) indicates a reduction in turbulence energy level (or intensity), an effect particularly noticeable in the high concentration area near the wall.

Subsequent studies by Einstein and Chien (1955) and Vanoni and Brooks (1957) have supported these original results.

Despite these carefully reasoned studies, the exact relationship between the von Karman constant and the turbulence energy level remains open to question. While certainly in-
indicating some modification in the turbulence field it is not clear that decreases in $k$ represent necessarily a decrease in energy level. Some investigators have even tended to discount completely the importance of turbulence as a primary suspension mechanism.

Bagnold (1954), combining some incisive physical reasoning with clever experiments, asserted that particle collisions provide the vertical stresses responsible for suspension. Citing the principle of least effect, he maintained that a collection of grains under shear will tend to rearrange themselves so as to minimize internal shear stress. According to Bagnold, the resultant dilation thus produces an excess pressure at the bed while the fluid turbulence is damped by the high shear resistance.

Although Bagnold was able to provide some experimental support for the "dilation pressure" and preliminary observational evidence for damping of turbulence, the importance of grain-to-grain collisions is still in doubt.

Recent experiments by Southard (1967) have been designed to study the exact nature of the excess normal stress noted by Bagnold. Lagrangian measurements of neutrally buoyant particle trajectories, in a rotating annulus, promise to provide a sensitive measure of momentum flux thus allowing assessment of the extent of grain collisions and the source of the observed wall pressure.

Efforts to evaluate the nature of turbulent entrainment and the extent of flow field modification produced by the presence of suspended particles are hampered by difficulties in
making direct measurements and the near impossibility of any substantial theoretical analysis.

Elata and Ippen (1961), using a Prandtl Tube and a capacitance gage pressure transducer, made turbulence intensity measurements in channel flows transporting neutrally buoyant plastic particles. Contrary to the expected result, their data showed turbulence intensity increasing with suspended load while the von Karman constant decreased and the mean velocity deviated sharply from the zero concentration profile. A model was suggested showing particle-flow interaction taking place primarily in the vicinity of the boundary layer. Using the structure of turbulent shear flow proposed by Townsend (1956), they suggested that the presence of particles upsets the flow equilibrium through increased production of small scale eddies and the resultant increase in eddy viscosity. Such imbalance would modify the momentum transfer throughout the flow and produce the indicated changes in the mean velocity profile. Interaction would be a maximum in the area where the turbulence length scales (eddy sized) were approximately equal to the particle diameter. This suggests that the density contrast $\frac{\rho_{\text{sed}}}{\rho_{\text{liquid}}}$ is of secondary importance and that particle size, shape, and volume concentration were the more relevant parameters.

Concurrently Kada and Hanratty (1960), studying the diffusion of potassium chloride solutions in fully turbulent pipe flows, also found particle size and the slip velocity (i.e. the relative velocity between particle and fluid) to be the factors governing flow modifications.
The hydraulic characteristics of liquid-solid flows suggested by these studies were examined in more detail by Daily and Chu (1961) and later by Daily and Roberts (1966). Isolating first concentration effects and then particle size effects, they concluded that fine particles primarily produce changes in the boundary layer region by suppressing the laminar sublayer. This results in an increase of centerline velocity (pipe flows), producing sharper mean velocity profiles. Coarse particle suspensions tend to display blunter profiles indicative of improved momentum transport due to grain collisions.

Theoretical verification of these experimental results has been limited. While individual particle motions in a turbulent stream have been described with some success, notably by the work of Tchen (1947), Saffman (1956), and Batchelor (1965), a general, quantitative description of flow modifications produced by a suspended load is lacking. By deriving energy and acceleration balance equations, Hino (1963) was able to predict the effect of various suspended particle concentrations on the von Karman constant and the turbulence intensity. For neutrally buoyant particles the results were in close agreement with the experimental data. For higher density contrast, the method indicated extremely little turbulence damping over a wide range of concentrations and a rapid increase in eddy dissipation. These latter results still require experimental verification.

Although this work represents a significant step towards quantifying basic liquid-solid flow characteristics, it also
points to the continuing need for additional experimental data on the structure of turbulent shear flows and the extent of changes produced by suspended loads.
III. EXPERIMENT

The following experimental investigation has two primary aims:

1) development of a laboratory experimental system sufficiently reliable to insure accurate, repeatable measurements of the structure of fluid turbulence in liquid-solid flows using standard hot-wire anemometers;

2) employment of the system to observe the effects produced by an increasing concentration of neutrally buoyant particles in fully turbulent, free-surface channel flows.
The Experimental Apparatus

THE TURBULENCE FLUME

Figure 1 shows in outline and Fig. 2 the overall view of the turbulence flume used for this study. While built along the lines of a standard tilting-recirculating flume, this facility incorporates features designed specifically for turbulence studies using hot-wire anemometers.

The four meter tunnel is supported by a section of aluminum channel braced along each side by smaller aluminum channels and at each end by adjustable support braces. This combination insures a rigid, planar surface.

The support beam rests on four resilient shock mounts which in turn are fastened to two aluminum tables. One table is supported by a scissors jack (Fig. 3) and the other by twin swivel links (Fig. 4). The entire structure is supported by stainless steel legs bolted to a heavy wooden base. The jacking arrangement allows the flume to be tilted and then locked in place. This, together with the mass of the tables and the rigidity of the legs, provides a very secure support system.

The rectangular channel and divergent end sections were fabricated using standard window glass (thickness 0.635 cm). Pieces were hand selected from stock to be flaw-free, and then cut to shape and edge finished. All seams were filled using Dow Corning building sealant. The channel rests on a Neoprene pad glued to the support beam. Lateral constraint
Figure 1. Turbulence Flume Outline Drawing

Legend

1. Drain and Filtration Tap
2. Aluminum Support Beam
3. Scissors Jack
4. Standing Liquid Manometer
5. Venturi Flowmeter
6. 3.8 cm. Return Line
7. Resilient Shock Mounts
8. Dayton 1/2 H. P. VariDrive
9. Labawco Centrifugal Pump
10. Safety Catch Box
11. Ball Type Dump Valve
12. Modified 15 cm. Glass Elbows
13. Adjustable Support Brace
14. Glass Channel Test Section
15. Convergent Entrance and Diffuser Section
16. Aluminum Side Supports
FIG. 2. TURBULENCE FLUME OVERALL VIEW.
FIG. 3. SCISSORS JACK
FIG. 4. TWIN SWIVEL LINKS
and vertical support is provided by four pairs of adjustable aluminum angles. In combination with three internal spreaders, these serve to fix and hold the inside channel dimensions at 20 cm x 10 cm.

At each end of the channel modified glass elbows effect the transition from rectangular to circular cross section. These mate with 4:1 reducers which lead into the 3.8 cm return line. The elbows, reducers, and return line are all Pyrex acid-waste piping. This composite glass construction makes for easy cleaning and provides a system capable of circulating a wide variety of liquids.

The return piping loop is broken near the downstream end of the tunnel to provide access for the circulating pump. This pump and its associated drive unit is bolted directly to the wooden base. Flexible Tygon tubing provides coupling into the return line as well as providing a degree of vibration isolation and the freedom required to tilt the flume.

The centrifugal pump used in this study was designed with large impeller clearances to permit the pumping of slurries and abrasive materials. In combination with a mechanical variable-speed drive, it has performed well over a moderately wide range of particle concentrations and flow rates. There is virtually no evidence of impeller wear or of damage to the particles.

Mass flow rates are measured by a Venturi flowmeter placed in the return line. Pressure differences across the nozzle are sensed by a standing liquid manometer which is open to the atmosphere. By applying the calibration data
FIG. 5.
FILTRATION SYSTEM
obtained under controlled conditions (see Appendix A) these values are easily converted to flow rates.

A prime requisite for successful hot-wire velocity measurements is that the flow liquid be free of room dust and lint. These materials tend to coat the wire and offset the calibrations. To minimize this possibility, the rectangular channel was fitted with weather stripping and a set of dust-tight lids. In addition a tap in the return line was fitted with a small control valve to permit removal of the liquid through an in-line filter. The filtered liquid could be returned to the tunnel by siphoning or be stored in glass carboys for future use (Fig. 5). Particles were retained in the tunnel by a fine screen over the suction port.

To reduce the possibility of spreading liquid spills due to rupture of the channel or return line, a safety catch box was constructed. A large (2.54 cm) ball valve discharges into this box for situations requiring rapid flume drainage.

THE INSTRUMENT CARRIAGE

Figure 6 shows the instrument carriage assembly in place on the flume. The unit was designed to replace one of the covers so that the tunnel remains dust-tight.

The carriage and vertical feed ride on a cross-stream slide supported by a pair of longitudinal slides bedded into the Plexiglas base. The milled access allows a longstream travel of 46 cm and a cross-stream traverse of 20 cm. The
FIG. 6. INSTRUMENT CARRIAGE ASSEMBLY
base is supported by three pairs of adjustable legs resting on the aluminum channel. Once leveled, the assembly is locked in place with C-clamps.

The requirement that the tunnel remain sealed necessitated a rather unusual longitudinal drive. A worm and wheel gear pair drives two hard rubber rollers, one at each end of the assembly. These rollers alternately take up or let out a thin Teflon sheet which is attached to the instrument port on the cross-stream slide. In this way the carriage may be moved in the longstream direction without uncovering the access. The carriage position may be read to the nearest 0.1 cm by means of a steel rule fixed to the assembly base.

Cross-stream position is also determined using a steel rule. Mounted on the instrument port, it allows position measurements to the nearest 0.1 cm.

The instrument housing on the vertical feed has been designed to accommodate a variety of probes. These extend through a Plexiglas port on the cross-stream slide and have full vertical and transverse freedom without disturbing the dust-tight integrity. Vertical position is measured to the nearest 0.001 cm with a vernier scale mounted on the carriage.

THE DATA ACQUISITION SYSTEM

Figure 7 shows schematically the system used to measure, record, and analyze the structure of fluid turbulence in various liquid-solid flows. In detail, this system consisted of the following equipment:
STAGE 1

CONSTANT TEMP.
HOT WIRE
ANEMOMETER
FLOW CORPORATION
MODEL 900-A

DIGITAL
VOLT
METER
NLS X-3

HOT
WIRE
PROBE

VARIABLE
FILTER
KROHNHITE
MODEL 3202

ANALOG-
DIGITAL
CONVERTER
DEC MODEL
ADO8-A

DIGITAL
COMPUTER
DEC
PDP-8S

PUNCHED
PAPER
TAPE

Oscilloscope
Tektronix
Model 502

STAGE 2

PAPER TAPE
TO CARD
PUNCH
IBM
MODEL 047

DATA ACQUISITION SYSTEM

DIGITAL
COMPUTER
IBM OS/360

PROCESSED
DATA
CALCOMP
PLOTS

FIG. 7.
The Velocity Probe

Hot-wire probes mounted on the instrument carriage constitute the actual velocity sensors. The possibility of high mechanical stresses due to particle-wire collisions required the use of unusually heavy elements. Kovar wire (0.00381 cm diameter) was selected to provide the required durability while maintaining moderate sensitivity and frequency response.

Two probes were constructed: a two wire, horizontal X-array (Probe no. 1) and single wire inclined at 45 degrees to the vertical (Probe no. 2). In each, stainless steel tubing (30 cm long x 0.341 cm o.d.) was terminated by an insulating cap. Pairs of long needles bedded into the cap served as the wire supports. Needle spacing on Probe no. 1 established a wire aspect ratio (length/diameter) of 120 while spacing on Probe no. 2 resulted in a ratio of 93.4. No insulating coating was required on the wire or their supports since all experiments were made in a dielectric medium.

The Hot-Wire Anemometer

A two-channel, constant-temperature anemometer (Flow Corp.) was used to excite the hot-wire probes and to sense the variations in rate of flow past the wire. The operating principles of this kind of apparatus are well known (Corrsin, 1963) and will be described only briefly.

A heated wire placed in a free stream of fluid will be cooled as a function of the rate of heat transfer at its surface. This heat transfer depends on

1) flow velocity

2) temperature difference between wire and fluid (i.e.
the overheat ratio)

3) physical properties of the fluid
4) dimensions and physical properties of the wire

(Hinze, 1959).

Hot-wire anemometry usually attempts to correlate fluctuations in the cooling rate with fluctuations in the velocity field. However, even with the remaining parameters known and constant an exact functional relationship is difficult to derive.

King (1914), assuming potential flow and continuum dynamics, developed a theoretical expression for the heat transfer from a wire. Using a rotating arm, he verified the results experimentally and arrived at an expression which can be written as

\[
\frac{I^2 R_w}{R_w - R_g} = \frac{e \frac{k_g l}{\rho}}{1 + \sqrt{\frac{2 \pi \rho f^2 c_p U d}{k_g}}} 
\]  

where:
- \( I \) - current through the wire.
- \( R_w \) - total electrical resistance of the wire after heating.
- \( R_g \) - electrical resistance of the wire at the fluid temperature.
- \( e \) - a conversion constant (See Hinze, 1959).
- \( k_g \) - heat conductivity of the fluid.
- \( l \) - wire length.
- \( R_0 \) - wire resistance at some reference temperature.
- \( \theta \) - temperature coefficient of electrical resistivity of the wire.
- \( \rho \) - fluid density.
- \( c_p \) - specific heat of the fluid at constant pressure.
- \( d \) - wire diameter
- \( U \) - total velocity
Eq. 1 may be written in its more standard form

\[ \frac{I^2R_w}{R_w-R_f} = A + B\sqrt{U} \] (2)

where the constants A and B are simply dependent on the physical properties of the wire and liquid.

The rather idealized assumption of flow conditions and temperature distribution and the questionable experimental methods used by King have prompted numerous additional investigations. Despite extensive effort the essential form of the equation derived by King remains unchanged.

Kramers (1946), for example, studied in detail the nature of heat transfer from a wire. With the exception of modified expressions for A and B his equation has the same form as Eq. 2.

More recently, Collis and Williams (1959) experimentally investigated the mechanism of heat transfer. Their work extended from low velocities, where free convection effects may predominate, up to the range where vortex streets form in the wake of the wire. The experiments lead to the following expression:

\[ N \left( \frac{T_m}{T_\infty} \right)^{0.17} = A + BR^n \] (3)

where

- \( N \) – Nusselt Number (a dimensionless heat transfer coefficient)
- \( T_m \) – mean temperature between stream and wire
- \( T_\infty \) – surface temperature of the wire
- \( A \& B \) – constants dependent on the wire and fluid properties
- \( n \) – coefficient dependent on the flow velocity
- \( R \) – wire Reynolds number.
The equation indicates the complicated functional dependence of the heat transferred on the fluid and wire properties as well as on the velocity. Noteworthy is the use of a variable exponent $n$ rather than the fixed exponent (0.5) employed by King.

Although representing a significant improvement in reliability, the form of Eq. 3 does not readily lend itself to applied hot-wire anemometry. A series of investigations described by Richardson and McQuivey (1968) incorporated the work of Collis and Williams and showed that for continuum, incompressible flows the relationship between heat loss and mean velocity could be written

$$\frac{I^2R_\omega}{R_\omega (R_\omega - R_g)} = A + B U^n$$

which is noticeably similar to Eq. 2 but incorporates the variable exponent $n$.

In practice Eq. 4 is only slightly less difficult to apply than Eq. 3. While the wire current $I$ and the resistance values are easily measured, the constants $A$ and $B$ are best obtained by individual calibration rather than by calculation. This is complicated by the fact that the exponent $n$ shows a marked dependence on flow conditions. Collis and Williams found that it could range between 0.3 - 0.45. In an attempt to eliminate this difficulty, Eq. 4 was used in the present experiment simply as a guideline for a computer curve fitting routine.
Under calibration or experimental conditions, the electrical output of the hot-wire anemometer provides the information necessary to solve the left-hand side of Eq. 4 directly. The wire "cold" resistance $R_g$ can be read on the front panel after balancing an internal bridge. The "hot" resistance $R_w$ is fixed for liquid flows at 1.1 $R_g$. This insures an overheat ratio $k$ of 1.1.

In the constant temperature mode of operation the electrical resistance of the heated wire is held constant. An electronic feedback network in the bridge circuit corrects any fluctuations in the wire temperature. The feedback current $I$ is passed through a fixed precision resistor. The resultant drop provides the voltage available at the anemometer output.

The response of this system, in combination with a wire compensation circuit, insures a frequency response many times higher than suggested by the inherent time constant of the uncompensated sensor.

The right-hand side of Eq. 4 contains the unknown velocity $U$ and constants $A$, $B$, and $n$. The constants were determined using a calibration procedure described in Appendix B. The values together with the resistances and the overheat ratio, were then used to determine the relationship between output voltage and velocity.

The Monitors and Filter

As shown in Fig. 7 the anemometer output was monitored using a digital voltmeter and a dual-beam oscilloscope.
The voltmeter displayed the DC level while the oscilloscope was used to set the feedback gain, and to monitor the level of voltage fluctuations and the filter output.

The variable filter (Krohn-Hite) was used in a low-pass mode to eliminate that portion of the output signal produced by system vibration and electronic noise (~60 hz). In addition the moderately sharp cut-off provided an upper frequency limit. This was used in selecting a data sampling rate (see Data Reduction Considerations).

The Digital Recording System

The voltage output from the variable filter was processed in a single channel analog-to-digital converter (Digital Equipment Corporation Model AD08-A). Using the technique of successive approximation, this unit produced a ten-bit binary work representing the analogue of the input signal. System accuracy was 0.1% of full scale, ±1/2 of the least significant bit (quantizing error).

The buffer register of the A-to-D converter was sampled using a small digital computer (Digital Equipment Corporation PDP-8S). Since the size of core memory available (4096 12-bit words) precluded statistical computations, this unit served simply as a data storage system. The program used (see Appendix C) allowed selection of a variable scan rate and the acquisition of 2556 samples. The record length was thus dependent on the scan rate used.

Output from the PDP-8S was via teletype and a punched paper tape unit (ASR-33). Tape format was in ASCII (8) code punched at a rate of ten characters per second.
Data Conversion and Processing

All data processing was performed on an IBM 360/65 system (IBM Manual A22-6821-7). Since direct input of paper tape data was not possible on this machine, all tape was converted to punched cards. The cards, each with 12 samples, served as raw data input to the processing programs. The programs are tabulated in Appendix C. All data reduction considerations were based on a series of preliminary experimental runs (see Data Reduction Considerations).

THE FLOW LIQUID

All experimental runs were conducted in silicone oil with a kinematic viscosity of 1.0 centistroke (for physical properties see Appendix D). This stable dielectric liquid was selected because it

1) permits the use of uncoated hot-wire sensors. This eliminates the possibility of coating damage due to particle-wire collisions and increases the frequency response of the sensor.
2) is available in a viscosity which simplifies Reynolds number similarity to water flows.
3) eliminates the need for "wetting" the particles.
4) reduces attraction between particles due to surface charge.
5) eliminates the possibility of algal growth which often develops in water flows.
6) is safe to handle and will not react with the plastic particles. These features in combination with its on-
the-shelf availability tend to offset its relatively high cost.

THE EXPERIMENTAL PARTICLES

Expandable polystyrene particles (Sinclair-Koppers, Dylite F-40-C) were selected for use in the experiment. This material contains 5-8% by weight of a volatile, saturated paraffinic hydrocarbon. When heated above 77°C the hydrocarbon acts as a "blowing agent" causing the particles to expand. Careful metering of the heat applied allows control of the expansion. This technique was used to produce particles with a specified density.

For the neutrally buoyant case, particles with a specific gravity of 1.05 were first expanded using steam. Care was taken to insure uniform heat distribution so that the particles remained spherical. Next an effort was made to determine the relationship between particle size and density. Floating samples of differing nominal diameter in a container of flow liquid indicated that only particles from the 30 mesh (U.S. Sieve Designation) screen possessed the desired density range; smaller particles were too heavy, larger particles were too light.

After sieving each expanded lot, the selected particles were handculled using a double flotation process. This tedious routine finally yielded spherical particles with a specific gravity range of 0.817-0.84 at 25°C and a nominal diameter of 595 micron.
THE CONCENTRATION SAMPLER

Figure 8 shows the apparatus used to sample the concentration (by weight) of suspended particles. A section of stainless steel tubing (0.318 cm o.d. x 0.0279 cm wall) was shaped to form the probe. Mounted on the instrument carriage, it is connected to a 50 ml centrifuge flask by a length of surgical tubing and a glass petcock.

Samples are drawn by siphoning. The adjustable stand allows the pressure head to be set so that the flow rate in the probe is equal to the local fluid velocity in the channel.

Two balances were used in weighing the samples: an Ohaus (Model 310) beam balance, with an accuracy of 0.01 g, and a Sartorius digital analytical balance, with an accuracy of 0.001 g.
FIG. 8. CONCENTRATION SAMPLER
The Experimental Procedure

FLUME FLOW CHARACTERISTICS

Prior to the experimental runs, a careful study of the flow characteristics of the turbulence flume was performed.

The instrument assembly was clamped in place spanning Stations 150-200 (an arbitrary location near the end of the test section). After calibration, hot-wire Probe no. 1 was removed from the test port and mounted on the instrument carriage. The channel was then filled to a depth of 3 cm with clear silicone oil and carefully leveled. The hot-wire probe served as a point gage.

The requirement that all experiments be conducted under fully turbulent subcritical conditions with no loose bed was most simply met by maximizing the pump discharge just short of the onset of impeller cavitation. This resulted in the following flow characteristics:

- Discharge \( Q = 1100 \text{ cm}^3/\text{sec} \)
- Running Depth \( D = 2.84 \text{ cm} \)
- Width/Depth Ratio 7:1
- Hydraulic Radius \( R_h = 2.22 \text{ cm} \)
- Mean Velocity \( U = Q/A = 19 \text{ cm/sec} \)
- Reynolds Number \( R = \frac{U4R_h}{\nu} = 17,800 \)
- Froude Number \( F = \frac{U}{\sqrt{gD}} = 0.36 \)
- Liquid Temperature \( T = 25^\circ C \)

No effort was made during this initial study to use an algorithm for the conversion of hot-wire voltage to velocity.
For the qualitative information required, the mean velocity based on discharge was sufficiently accurate.

A preliminary traverse over the cross-section at Station 198.5 revealed noticeable unsteadiness in the mean flow, most likely the result of return line and entrance condition. This unsteadiness required the installation of a honeycomb baffle in the diffuser section. This baffle was constructed from plastic drinking straws (20 cm long x 0.318 cm i.d.) filling the channel entrance and held in place by a Plexiglas wedge.

The cross-stream profiles shown in Fig. 9 indicate that in combination with the 4:1 expansion and the convergent section the baffle effectively eliminates flow irregularities produced in the return line. The slight asymmetry at Station 185 (y/D=0.562) was caused by an uncorrected thermal shift in the hot-wire calibration.

Figure 9 also shows that over a large portion of the channel cross-section the flow is essentially two-dimensional, i.e. varying only with depth and cross-stream position. This characteristic was considered essential for the proposed experiment and was the reason for the choice of the high width-to-depth ratio (7:1).

The indicated long-stream development of the vertical velocity profiles proved to be the result of a slight non-uniformity in running depth. This was eliminated by modifying the channel slope. The resulting profiles at Stations 185 and 198.5 were found to be identical (Fig. 10).
FIG. 16

MEAN VELOCITY COMPARISON

WIRE NO. 1 \perp TO MEAN FLOW
RUNNING DEPTH \( D = 2.85 \text{ cm} \)

key: • STA. 199.5
    + STA. 105

RELATIVE DEPTH \( y/d \)

0.75

0.50

0.25

0

VELOCITY \( \text{cm/sec} \)

0

15

30
On the basis of these preliminary studies, Station 198.5 was selected as the test section. Here operation near the channel centerline provided a fully turbulent regime with a velocity profile that is uniform and two-dimensional in the mean. In addition, this station was sufficiently far upstream to assure freedom from possible exit effects.

VOLTAGE-TO-VELOCITY CONVERSION

Before establishing the data-reduction criteria it was necessary to develop a routine for converting the anemometer output voltage to velocity.

The method proceeds from the assumption that the heat transfer and therefore the output voltage is directly related to the flow velocity. The remaining variables, including the physical properties of the wire and fluid, are known and constant.

The voltage-velocity dependence for each wire of Probe no. 1 was obtained using the procedure described in Appendix B. The data indicate that the calibration curves can be closely approximated by a series of straight lines. Over each linear segment the transfer coefficient \( n \) will be a constant. If the mean velocity of the experimental run is known, the proper value or values of \( n \) can be selected from the tabulation in Appendix B. Combining this with the derived values of \( A' \) and \( B' \) allows computation of the cooling velocity \( U \) given the output voltage \( E \) using the formula

\[
E^2 = A' + B'U^n
\]  
(5)
This algorithm and the linear ("piece-wise") approximation was employed in the main computer program (Appendix C) for all voltage-velocity conversions.

It remains to determine the relationship between the cooling velocity $U$ and the individual components of the velocity field.

If, as shown in Fig. 11, the velocity field is described in terms of a mean velocity $\bar{U}$ and fluctuating components $u'$, $v'$, and $w'$, then a wire placed perpendicular to the mean flow will be subject to a total cooling velocity $U$ with a magnitude given by

$$U^2 = (\bar{U} + u')^2 + (v')^2 + (w')^2$$  \hspace{1cm} (6)

Expanding Eq. 6 and rewriting yields

$$\frac{U^2}{\bar{U}^2} = 1 + 2 \frac{u'}{\bar{U}} + \left( \frac{u'}{\bar{U}} \right)^2 + \left( \frac{v'}{\bar{U}} \right)^2 + \left( \frac{w'}{\bar{U}} \right)^2$$  \hspace{1cm} (7)

Measurements in the turbulence flume at a Reynolds number of 17,800 indicated that $\bar{U}$ was some 20-40 times larger than the fluctuating components. For most studies, therefore, the quadratic terms in Eq. 7 can be neglected. The output voltage can be considered to be simply a function of the mean velocity $\bar{U}$ and its first order fluctuations $u'$.

In practice, when the turbulence level $u'/\bar{U}$ is low, the heat transfer is assumed to be governed by the mean flow field alone. The small turbulent fluctuations are said to have little or no effect on the heat transfer. Studies by Sandborn (1962) have indicated this to be true over most
FIG. 11. COORDINATE SYSTEM
FREE SURFACE
CHANNEL FLOWS
ranges of interest.

If the wire is perpendicular to the mean flow, then this assumption allows the mean velocity to be computed directly using Eq. 5 and the DC level of the anemometer output. Also, since the velocity fluctuations are assumed to have no effect on the heat transfer, the same expression (i.e. Eq. 5) can be used to compute $u'$ by making use of the instantaneous voltage output. This technique was employed in all experimental runs to determine $\bar{U}$ and $u'$.

The vertical and cross-stream fluctuations $v'$ and $w'$, and the Reynolds stresses $u'v'$ and $u'w'$, could be determined by making use of the angular sensitivity of the hot-wire.

Implicit throughout the above discussion is the assumption that only velocity components normal to the wire axis exert a cooling effect. Geßner (1964) has shown that this characteristic can be used to determine the Reynolds stress and the mean square fluctuating velocities in a manner that is independent of wire calibration. For wires oriented at 45 deg. to the mean flow he derived the following expressions:

$$\overline{v'^2} = \overline{u'^2} \frac{M_{A-B}^2}{M_{A+B}} \text{ } \text{xy plane} \tag{8}$$

$$\overline{w'^2} = \overline{u'^2} \frac{M_{A-B}^2}{M_{A+B}} \text{ } \text{xz plane} \tag{9}$$

$$\overline{u'v'} = \overline{u'^2} \frac{(M_A^2 - M_B^2)(M_A^2 + M_B^2)}{2M_{A+B}^2 (M_A^2 + M_B^2)} \text{ } \text{xy plane} \tag{10}$$

$$\overline{u'w'} = \overline{u'^2} \frac{(M_A^2 - M_B^2)(M_A^2 + M_B^2)}{2M_{A+B}^2 (M_A^2 + M_B^2)} \text{ } \text{xz plane} \tag{11}$$
where $M_A$ and $M_B$ are equal to the rms voltage output with the wire oriented at $+45^\circ$ (subscript A) and $-45^\circ$ (subscript B) to the mean flow and $M_{A+B}$ and $M_{A-B}$ equal the rms of the sum and difference of the A and B outputs respectively.

These equations were designed for use with matched wires and were shown to work as well for single wire rotation as for a fixed X-array. The latter feature was of use in this experiment since limitations imposed by the single channel data-acquisition system precluded use of an X-array. As a result, all data were provided by a single wire placed alternately at $\pm 45^\circ$ to the mean flow. Probe no. 1 was used in the xz plane, and Probe no. 2 in the xy plane.

The "Gessner" reduction program (Appendix C) combines the data obtained using the perpendicular wire with that obtained using inclined wires and provides as output the Reynolds stress $u'v'$, $u'w'$, and the mean square fluctuations $v^2$ and $w^2$. No effort was made to obtain the actual values of $v'$ and $w'$, since the rms values are of primary interest.

DATA REDUCTION CONSIDERATIONS

Any data acquisition and reduction system is inherently a source of signal distortion. Electronic noise and lack of response in combination with algorithm and sampling errors often produce a result that is a poor analogue of the natural process under investigation.

The system characteristics of the hot-wire anemometer were measured under actual operating conditions. Heated and
immersed in silicone oil, the wire was subjected to a square wave signal produced by an internal calibrator. The response of the feedback loop to this 10 mv pulse was measured on the oscilloscope and is shown in Fig. 12. As provided in the operating instructions, the upper frequency limit \( F_u \) of the system can be computed by determining the time interval \( \Delta t \) (in \( \mu \)sec) required for the signal to reach the 3 mv level from its peak value and substituting this into the formula

\[
F_u = \frac{275}{\Delta t} \quad \text{kHz.} \quad (12)
\]

Fig. 12 shows \( \Delta t \) to be approximately 1.0 msec and the upper frequency limit therefore to be 275 hz. The results were identical for all wires.

This calibration procedure was conducted under zero-velocity conditions. At higher flow rates the response will be slightly increased, thus insuring a sufficiently high frequency level for the proposed turbulence experiments.

The electronic noise of the anemometer system, which appears as "hash" on the oscilloscope trace (Fig. 12), proved to be negligible. Its 5 mv level is below the resolution of the A-to-D converter. Also, preliminary runs indicated an expected signal-to-noise ratio of 10 for the fluctuating voltages and 400 for the DC signal level. Such values assure accurate signal discrimination.

With the exception of the inclined-wire data, all voltages were converted to velocity before analysis. It is
FIG. 12.

ANEMOMETER FREQUENCY RESPONSE

Scale

Voltage = 10 mv/cm

Time Base = 0.5 msec/cm
recognized that the voltage-to-velocity algorithm discussed in the previous section is likely to introduce a finite error because of the linear approximation and the use of first-order heat transfer theory. Estimations based on previous investigations (Sandborn, 1962) and on the calibration data indicate that this error can be kept smaller than 1-2% by selecting small linearizing intervals and flows with low turbulence levels. If not amplified by sampling or other analysis errors, this is a modest value.

Prior to any processing, all voltage data were converted from their continuous analog form into a series of discrete digital values. This quantizing process, because it necessarily involves some approximation, may introduce errors in both the amplitude and the time or frequency domains.

The degree of amplitude error is governed by the accuracy of the A-to-D converter. For the system used this was quoted at 0.1% of Full Scale ±1/2 of the least significant bit. The resultant error can usually be ignored.

Accurate reproduction of the spectral character of the analog signal by its digital representation is determined by the sampling rate. When a continuous signal \( F(t) \) is sampled at a rate of \( 1/T \) samples/sec, frequency components in the resultant digital record greater than \( \pi/T \) radians/sec cannot be distinguished from those in the 0-\( \pi/T \) range. This folding or "aliasing" of the frequency spectrum can be el-
minated by selecting a sampling interval $\Delta t$ such that $\Delta t = \frac{1}{2F_n}$ with the "Nyquist" or cutoff frequency $F_n$ chosen such that there is no significant spectral energy above it. The selection of this value for the proposed experiment was based on a series of preliminary runs.

Examination of the anemometer output during these runs revealed a noticeable spectral peak around 60 hz. On the assumption that this was a product of pump vibration and electrical pickup, the peak was eliminated by low-pass filtering. Then as a check, a qualitative spectral study was made using a selected cut-off frequency of 40 hz and sampling rate of 80/sec.

The results, shown in Fig. 13, indicate that the bulk of turbulent energy lies below 15 hz and that it approaches zero as the 40 hz limit is approached. This decay tends to support the assumption as to the source of the 60 hz peak.

The 40 hz cutoff and the resultant 80/sec sampling rate was used in all subsequent experimental runs.

With available core memory in the PDP-8S computer set at 2556 words, choice of sampling rate automatically fixed the single-pass record length. Using a rate of 80/sec, this was approximately 32 sec, providing an attainable frequency resolution $\Delta f$ of 0.0391 hz. To a great extent these values determine the accuracy of the statistical computations. Particular attention was given to the reliability of the energy spectrum and its dependence on record length.
FIG. 13. ENERGY SPECTRUM

PRELIMINARY RUNS II-12
CLEAR LIQUID
1.0 cs SILICONE OIL
\( \gamma_D = 0.175 \)
\( R = 17,800 \)
For 2N real data points $X_0, X_1, \ldots, X_{2N-1}$, the scientific subroutine RHARM (IBM Manual H20-0205-3) uses the Fast Fourier Transform (Cooley and Tukey, 1965) to compute the real coefficients $a_0, a_1, \ldots, a_{N-1}$ and $b_1, \ldots, b_{N-1}$ in

$$X_j = \frac{1}{N} a_0 + \sum_{k=1}^{N-1} \left( a_k \cos \frac{2\pi j k}{N} + b_k \sin \frac{2\pi j k}{N} \right) + \frac{1}{N} a_N (e^{j})^j$$

where $j=0,1,\ldots,2N-1$

This method is used in the main reduction program (Appendix c).

A plot of $(a_k^2 + b_k^2)/\Delta f$ for each frequency component represents the raw "periodogram", an indication of the spectral distribution of energy density. However, since each of these points has only two degrees of freedom (Bingham et al., 1967), this plot is a rather poor spectral estimate. Reduction of the large error bars, with a fixed record length, requires either additional runs and the formation of an ensemble, or sacrifice in frequency resolution by averaging over a number of adjacent points in a single record. Whichever method is selected, the increase in degrees of freedom is a simple multiple of two, e.g. the average of five points has ten degrees of freedom.

The procedure used in the turbulence experiments allows a combination of the above options based on the requirements of each individual run.

At each vertical location where spectral information is required, two 32-sec records are obtained. Representing a compromise necessitated by the slow speed of the PDP-8S
output punch, these data can be averaged over the ensemble or the individual record in a manner designed to provide optimum reliability with suitable frequency resolution. For example, the spectra of several preliminary runs represented ten point averages resulting in a resolution of 0.391 hz and giving each point 40 degrees of freedom.

CONCENTRATION MEASUREMENTS

The average weight concentration of suspended particles was used to set the initial conditions for each liquid-solid run. Particles were added to the known volume of oil in the flume in carefully weighed amounts. Then, at the completion of each run, after computation of the mean velocity profile, measurements were made to obtain the actual concentration over the vertical.

The concentration sampler was used to obtain three 50 ml samples at each of six vertical locations. Use of the velocity data allowed the internal flow rate of the probe to be set equal to the local fluid velocity at each point.

Next, total weight (liquid + particles) was determined for each sample. The particles were then separated, dried, and weighed. The resultant weight concentration could be converted to volume concentration by using the relationship

\[
\% \text{ By Volume} = 0.80 \times \% \text{ By Weight}
\]

This relationship is governed by the packing density, and was determined by observing the volume increase in a graduated vial caused by the addition of a known weight of experimental particles.
SUMMARY OF THE EXPERIMENTAL PROCEDURE

Each experimental run proceeded in the following fashion:

1) The turbulence flume was filled to the selected operating depth with a filtered and weighed amount of silicone oil. Depth and surface checks were performed using hot-wire Probe no. 1 as a point gage.

2) A weighed sample of neutrally buoyant particles were introduced and the average weight concentration calculated.

3) In still liquid, the null resistance $R_n$ of each hot-wire was measured and the ambient temperature recorded. During the experimental run, pump and frictional heating caused a slow increase in liquid temperature. This was carefully checked and $R_n$ adjusted to maintain an overheat ratio of 1.1.

4) With the wires heated, the feedback and compensation networks were tuned.

5) The circulating system was started and the pump speed adjusted to provide the required discharge. This discharge was constant for all experimental runs.

6) The flow depth was measured and channel slope adjusted to insure uniform flow.

7) The instrument carriage was fixed in place at Station 198.5 and the probe positioned on the channel centerline. (All primary experimental data were obtained at this location.) The recorded raw voltages represent the following measuring sequence:

   a. With hot-wire Probe no.1 perpendicular to the mean flow, two 32-sec records were taken at each of six
locations along a vertical.

b. Next, Probe no. 1 was placed first at +45 and then -45 degrees to the mean flow. In each position the resultant output signal was sampled for 32-sec, with the measurements being repeated at each of four vertical locations.

c. Then Probe no. 2 was mounted on the instrument carriage and used to take ±45° measurements at the same vertical locations as in step b.

8) The experimental data was processed as follows:

a. All punched paper tape output was transferred to punched cards.

b. The voltage data from step a of the measuring sequence (above) were converted to instantaneous velocities \( u_h \) using a voltage-to-velocity algorithm. After this step, the main computer program was used to calculate:

The mean velocity

\[
\overline{U} = \frac{1}{N} \sum_{k=1}^{N} u'_{hk} \tag{14}
\]

with \( N \) = the number of samples

The root-mean-square velocity

\[
\sqrt{\overline{u'^2}} = \left[ \frac{1}{N} \sum_{k=1}^{N} (u'_{hk} - \overline{U})^2 \right]^{1/2} \tag{15}
\]

The auto-correlation function

\[
R_{jj} = \frac{1}{N-j+1} \sum_{k=1}^{N-j} (u'_{hk} - \overline{U})(u'_{hk+j} - \overline{U}) \tag{16}
\]

with \( j=1,2,3,... \) representing the number of time lags.
The Fourier coefficients $a_k$ and $b_k$ in

$$\chi_j = \frac{1}{2} a_0 + \sum_{k=1}^{N-1} \left( a_k \frac{\cos \pi j k}{N} + b_k \frac{\sin \pi j k}{N} \right) + \frac{1}{2} a_\infty$$

with $j=0,1,2,3,...,2N-1$

c. The data obtained in steps b and c of the measuring sequence were not converted to velocities; instead the respective rms levels were used in the Gessner Reduction Program (Appendix C) to yield the Reynolds stresses $u'v'$ and $u'w'$ and the cross-stream velocity components $v'$ and $w'$.

9) After computation of the mean velocity gradient, the actual concentration of suspended particles was determined using the concentration sampler.

10) At the completion of the experimental runs, the hot-wire calibration was repeated in order to evaluate any shift in the wire constants $A'$ and $B'$. 
TABLE 1

<table>
<thead>
<tr>
<th>SUMMARIZED EXPERIMENTAL HYDRAULIC DATA</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RUNNING DEPTH</strong></td>
</tr>
<tr>
<td>$D = 1.748 \text{ cm}$</td>
</tr>
<tr>
<td><strong>CROSS-SECTIONAL AREA</strong></td>
</tr>
<tr>
<td>$A = 35.52 \text{ cm}^2$</td>
</tr>
<tr>
<td><strong>CHANNEL SLOPE</strong></td>
</tr>
<tr>
<td>$S = 0.00421 \text{ cm/cm}$</td>
</tr>
<tr>
<td><strong>DISCHARGE</strong></td>
</tr>
<tr>
<td>$Q = 1000 \text{ cm}^3/\text{sec}$</td>
</tr>
<tr>
<td><strong>LIQUID PROPERTIES</strong></td>
</tr>
<tr>
<td>$\rho = 0.918 \text{ gm/cc}$ @ $25^\circ \text{C}$</td>
</tr>
<tr>
<td>$\nu = 1.0 \times 10^{-2} \text{ cm}^2/\text{sec}$</td>
</tr>
<tr>
<td><strong>HYDRAULIC RADIUS</strong></td>
</tr>
<tr>
<td>$Rh = 1.491 \text{ cm}$</td>
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<tr>
<td><strong>DISCHARGE VELOCITY</strong></td>
</tr>
<tr>
<td>$U_d = \frac{Q}{A} = 28.15 \text{ cm/sec}$</td>
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<tr>
<td><strong>CHANNEL REYNOLDS NUMBER</strong></td>
</tr>
<tr>
<td>$Re = \frac{U_d 4Rh}{\nu} = 16,800$</td>
</tr>
<tr>
<td><strong>SHEAR VELOCITY</strong></td>
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<tr>
<td>$u_* = \sqrt{gRhS} = 2.480 \text{ cm/sec}$</td>
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<td><strong>FRÖUDE NUMBER</strong></td>
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<tr>
<td>$Fr = \frac{U_d}{(gD)^{1/2}} = 0.680$</td>
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<td>RUN DESIGNATION</td>
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<tr>
<td>Clear liquid</td>
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<td>RUN DESIGNATION</td>
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IV. RESULTS AND CONCLUSIONS

Concentration Measurements

Hot-wire measurements were obtained at six different mean values of suspended load. The initial run (NC 0) was performed in clear silicone oil and serves as the reference for each subsequent run. Concentration gradients for the liquid-solid runs are shown in Fig. 14. Run designations NC 1 to NC 5 are in order of increasing mean concentration. Each linear plot represents a best fit (by eye) to the raw data. No effort was made to average or to produce a least-squares fit.

The noticeable departure of each of the concentration gradients from a constant value is caused by the slight variations (~2%) in particle density. The deviation from neutral buoyancy produces a particle assemblage with a finite fall velocity and results in maximum concentration values at the bottom boundary. To estimate the average fall velocity the transit time for each of 75 particles falling in a large-diameter, graduated column of silicone oil was measured. The assemblage average was 0.371 cm/sec @ 24°C.

In addition to the obvious mass variations in the vertical, the suspended load also produces an increase in flow viscosity. The relative magnitude can be estimated using

\[
\left( \frac{\nu}{\nu_0} \right)^{1/2} = 1 + \frac{2.5 C_0}{2 (1 - 1.35 C_0)}
\]

(18)

with

- \( C_0 \) = volume concentration
- \( \nu \) = kinematic viscosity @ \( C_0 \)
- \( \nu_0 \) = kinematic viscosity @ \( C_0 = 0 \)
an expression originally derived by Eilers (1941) for laminar flows but holding with sufficient accuracy for many turbulent, liquid-solid flows (see Fig. 15, Elata and Ippen, 1961).

Solving Eq. 18 over the concentration ranges indicated in Fig. 14 (0-3.5%) shows the maximum change in viscosity to be roughly 8%, with a 1% variation over an individual run. Efforts to support these calculations with direct measurements were unsuccessful. The slight viscosity changes were below the viscometer resolution and resulted in widely scattered, inconsequential data. There is no doubt, however, that the flow dynamics remain completely Newtonian in behavior at the low suspended concentrations employed in this experiment.

Mean Velocity Comparisons

A comparative plot of the mean velocity profiles over the vertical is shown in Fig. 15. The sampling locations were confined to the central region of the flow to insure hot-wire measurements free of surface or bottom boundary effects. In addition, efforts to extend the measurements close to the bed during the liquid-solid runs were hampered by the tendency of the particles to lodge beneath the wire. As a result, close proximity measurements were possible only in the clear-liquid case. The remaining profiles extended to within one or two particle diameters of the bed.

It is apparent that the presence of a suspended load produces rather marked effects on the mean velocity structure. Over the measured region, increasing concentrations
result in a general flow deceleration. One may assume, if the volume discharge remains constant, that the external region of the flow must concurrently experience an acceleration. These results are qualitatively similar to previous experimental data for small particles (Daily and Roberts, 1966). Of note, however, is the fact that the suspended concentrations used in this experiment are substantially lower than those of previous investigators. Daily and Roberts, for example, report findings obtained at 15% concentrations, while Elata and Ippen (1961) worked with volume concentrations up to nearly 30%. That rather large variations are observed at low concentrations in this experiment is believed to be the result of the increased sensitivity afforded by the use of a hot-wire anemometer. This instrument hopefully would represent some improvement over the standard Pitot tube.

An exact analytical description of the mean velocity profiles, valid over the entire range of suspended particle concentrations, is difficult to derive. As shown in Fig. 16, the clear-liquid profile is for the most part adequately described by a logarithmic law of the form

\[
\frac{U_m - U}{u^*} = -2.3 \log \frac{y}{k y_m}
\]  

\[ (19) \]

with

- \( U_m \) = mean velocity @ \( y_m \)
- \( U \) = mean velocity @ \( y \)
- \( u^* \) = Shear velocity
- \( k \) = von Karman constant

Some slight departure is noted as the boundary layer region is approached. Laufer (1951) found this transition point
Fig. 16 VELOCITY DEFECT COMPARISON

- CLEAR LIQUID
- NC 1
- NC 2
- NC 3
- NC 4
- NC 5

$\frac{U_{m} - U}{u_{*}}$
near \( y = 30 \nu / u^* \) which in this experiment with \( u^* = 2.480 \) cm/sec gives \( y = 0.1209 \) cm and a \( y/D = 0.0692 \). This is in fair agreement with the data of Figs. 15 and 16, although additional points near the boundary are required for an accurate check. One should note that this boundary layer thickness is of the order of two particle diameters—a point which may be of some importance in later discussions of turbulence modification.

With increasing suspended load the mean velocity deviates rather sharply from the clear-liquid profile. Over the linear regions in Fig. 16 the variation can still be described by a logarithmic function with decreasing values of the von Karman constant (ranging from 0.385 to 0.240). At higher concentrations, however, this single-parameter fit must give way to more complicated multiple-parameter functions. Elata and Ippen (1961), by extending the reasoning used by Prandtl (1925) in his "mixing length" theories, introduced an additional function \( f_2 \) dependent on the concentration gradient. Substitution into Eq. 19 yields

\[
\frac{U_m - U}{u^*} = -\frac{2.3}{k} \log \left[ \frac{y}{Y_m} + f_2(C_0; \frac{y}{Y_m}) \right]
\]

(20)

with \( C_0 = \) volume concentration at \( y/Y_m \)

Although, as noted in their report, this equation serves to present experimental data in some plausible form, the required graphical determinations introduce wide scatter. As a result the general applicability of Eq. 20 is unclear. Increased confidence seems to require some check of the vali-
dity of the assumptions used in its derivation. In turn such information can be provided only by a far more detailed understanding of the structure of turbulence in liquid-solid flows than is presently available. Some partial insight into this problem can be provided by the mean velocity profiles themselves.

The trend displayed in Fig. 15 seems indicative of a general reduction in vertical momentum transport with increasing suspended load. The initially blunt turbulent profile is tending towards the sharper laminar profile. This would seem to support the hypothesis that turbulence damping is to be expected at high particle concentrations. Such reasoning, very much in vogue until recently, overlooks the fluctuating components of the turbulent velocity field. As detailed by Townsend (1956), the structure of turbulent shear flow is a function of the interaction of these components with the mean velocity. A reduction of fluid momentum close to the boundary is not necessarily correlated with a general reduction in turbulence level. Indeed, Schubauer and Klebanoff (1951), studying boundary layers in adverse pressure gradients, found increasing reduction in momentum to be correlated with increasing turbulence levels. It seems clear that any effort to assess turbulence modifications must study not only the mean velocity field but also its fluctuating components.
Turbulence Intensity

Figs. 17 and 18 show the vertical distribution of the longstream velocity fluctuations relative to the shear velocity $u_\star$. The clear-liquid profile displays a steady increase in rms level as the boundary is approached. The characteristic peak near the bed is indicative of the region of maximum turbulence production and the general dominance of viscosity in this region. Again a reduced sampling interval is required to delimit the maximum accurately.

The behavior of the rms levels with increasing suspended-particle concentrations is shown in Fig. 18. A general restructuring of the distributions is evident. Initially there is a slight reduction throughout the vertical and an indication of some turbulence damping. With increasing suspended load, however, the trend is definitely towards increasing turbulence intensity. Although difficult to discern, there is the suggestion of a much less rapid increase in the wall region ($y/D = 0.15$) than further out. With the exception of NC 5 each of the intervening rms levels is below that of NC 1 at the bed sampling point ($y/D = 0.080$).

In the light of these results and because so much importance is placed on the accurate measurement of turbulence intensity, two possible sources of error should be discussed.

Any hot-wire probe operating in a liquid-solid flow is subject to collisions between the particles and the wire. Prior to the experimental runs it was reasoned that if the wire survived, these collisions would produce high frequency components in the output signal (very similar to "strumming"
FIG. 18  RMS of VELOCITY FLUCTUATIONS

key
• NC 1
x NC 2
+ NC 3
* NC 4
Δ NC 5

\[ \sqrt{\frac{u'^2}{u_{ref}} } \]
the wire). These anomalies should be above our range of interest (0-40 Hz) and could therefore be filtered out.

At the start of the experimental runs it became evident that this was not entirely the case. Visual examination of the oscilloscope trace revealed that simple low-pass filtering still permitted vibration components to pass. Particle-wire collisions apparently result in a rather broadband introduction of energy. Admittedly this energy level is quite low, particularly at low suspended loads. If the concentration was carried beyond 5%, however, there is a possibility of significant error.

At the concentrations employed in this experiment the errors introduced by particle-wire collisions were deemed negligible. Their cumulative effect was very much smaller than the energy levels of the velocity field. If this were not the case, it would seem that in regions of high concentration, i.e. near the boundary, and therefore increased collisions, the rms levels would be substantially higher than further out. As has been indicated, this is not the case.

The second possible source of error involves the shear velocity \( u^* \). There is some indication (see Discussion) that increasing particle concentrations may result in decreased boundary shear stress \( \tau_0 \). Any such reduction would necessarily imply a reduction in the shear velocity (since \( u^* = \sqrt{\frac{\tau_0}{\rho}} \)) and would result in some errors in the data points shown in Fig. 17 and 18. The constant \( u^* \) value used in the normalizing of each of these points was dictated by the slope
of the free surface. Within the measurement accuracy attainable this remained constant over each of the liquid-solid runs.

This possible source of error was also disregarded since any reduction in shear velocity would only result in an increase in the normalized values. This in turn would tend to reinforce the previous findings that rms levels were increasing with increasing suspended particle concentrations.

A comparative plot of the rms levels for each of the fluctuating velocity components $u'$, $v'$, and $w'$ is shown in Fig. 19. Each point has been normalized using a selected mean velocity value $U_0$ (see Table 2).

The accuracy of the $v'$ and $w'$ levels is somewhat below that of the $u'$ measurements. As shown in Goldberg (1966) a slight angular misalignment can cause rather substantial errors in X-array measurements. Viewed qualitatively however, the comparison plot displays some rather interesting features.

The behavior of each component is similar. With decreasing relative depth ($y/D$) there is a gradual increase in intensity, with a peak value near the bed. The lateral fluctuations $w'$ display a more gradual increase, while the $u'$ and $v'$ values are nearly identical. In liquid-solid runs NC 1 through NC 5 there is a steady increase in the rms level for each component above $y/D = 0.15$. Closer to the boundary and relative to the clear liquid run, this
FIG. 19  TURBULENCE INTENSITY COMPARISON

key
- CLEAR  • NC 3
• NC 1.
+ NC 2
• NC 5

\[
\frac{\sqrt{v^2}}{U_0} \times 10^2
\]

\[
\frac{\sqrt{v^2}}{U_0} \times 10^2
\]

\[
\frac{\sqrt{v^2}}{U_0} \times 10^2
\]

\[
\frac{\sqrt{v^2}}{U_0} \times 10^2
\]
consistent increase is noted only at the higher concentrations. These variations clearly indicate that the presence of a suspended load does not simply produce a redistribution of turbulence intensity (for example, increasing $u'$ levels with decreasing $v'$ or $w'$ intensity) but rather results in a general increase in turbulence intensity, implying some substantial alteration in the turbulence production mechanism.

Reynolds Stress

The changing levels of turbulence intensity discussed in the previous section are reflected in the plot of Reynolds stress $\overline{u'v'}$ shown in Fig. 20. For the clear liquid run there is a close correlation between the level of maximum turbulence intensity and peak Reynolds stress. The production appears to be induced by the interaction of these virtual stresses with the local mean velocity gradient, both displaying maximum values in this region.

With increasing suspended load, the point of maximum stress tends to move away from the bed. The peak values are noticeably altered, and there is a general reduction in gradient. The steady increase in Reynolds stress at $y/D = 0.572$, combined with the increasing velocity gradient, implies increased production at this level. Conversely the reduced near-bed values (runs NC 2 through NC 4) very likely indicate a reduction in turbulence production. As was indicated above, these facts are weakly evidenced in the plots of turbulence intensity.
FIG. 20 REYNOLDS STRESS COMPARISON

key
- CLEAR
- NC 1
- NC 2
- NC 3
- NC 4
- NC 5

\[
\frac{u'v'}{U_0^2} \times 10^5
\]
Although the data presented in Fig. 20 are instructive, additional points are required to allow accurate determination of the point of maximum Reynolds stress. Also the measurements, apparently the first in liquid-solid flows, require corroborative support. Despite these weaknesses it is quite clear that the presence of suspended particles in quite low concentrations markedly alters the turbulence structure.

Power Spectra and Scaling

The frequency distributions of the longstream velocity fluctuations $u'$ are shown in Fig. 21 (in viewing plots, note that all clear-liquid runs are grouped in a single plot and subsequent liquid-solid runs are grouped by relative depth). For each experimental run, 15 points of a two-member ensemble were averaged to provide a spectrum with 60 degrees of freedom and a frequency resolution of 0.5865 Hz. The Calcomp plots represent the normalized power spectrum $F(n)$, where

$$ F(n) = \frac{P(n)}{\bar{u}^2} = \frac{\omega/\Delta f}{\bar{u}^2} \left( \sum_{n=1}^{15} \left( \tilde{a}_n + \tilde{b}_n \right)^2 \right) $$

and therefore

$$ \int_0^\infty F(n) \, dn = 1 $$

with $n$ = frequency,

$P(n)$ = power spectrum

$\Delta f$ = frequency resolution of the raw periodogram

$\bar{u}^2$ = mean square velocity @ $y/D$ of interest

$\tilde{a}_n, \tilde{b}_n$ = ensemble average Fourier coefficients

Each plot clearly displays a dominant low-frequency content with the bulk of the energy lying below 20 Hz. Qualitatively,
FIG. 21. NORMALIZED POWER SPECTRA
CLEAR LIQUID

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\[ y_D = 0.430 \]

\[ y_D = 0.287 \]

\[ y_D = 0.143 \]

\[ y_D = 0.063 \]
$Y_d = 0.572$

NC 1

NC 2

NC 3

NC 4

NC 5
$Y_D = 0.430$

NC 1

NC 2

NC 3

NC 4

NC 5
$\chi_D = 0.287$

NC 1

NC 2

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$\psi_D = 0.080$

NC 1

NC 2

NC 3

NC 4

NC 5

FREQUENCY (Hz)

F (N) [SEC]

FREQUENCY (Hz)
the spectral character of each remains essentially un-
changes for all experimental runs. At the higher concen-
trations there is a suggestion of increased high-frequency
components, particularly at 0.572 and 0.430. This feature
is possibly indicative of increased production of small-
scale turbulence or improved distribution in the outer flow
regions, but a definitive finding must wait on longer records
or larger ensembles and the resultant increase in accuracy.

To study the spectral decay characteristics of the ex-
perimental runs, the data plotted in a linear fashion in
Fig. 21 were replotted on log-log paper. A sample of the
results is shown in Fig. 22. The overall characteristics
of the curves in this comparison plot appear to be nearly
identical. The steadily increasing negative slope varies
from -0.5 at the low frequency end to -6 at the high fre-
quency extreme. Quite similar in shape to those obtained
by Raichlen (1967) in water flows, each curve passes rather
rapidly through the -5/3 or Kolmogoroff region and displays
the dominance of viscous effects above 10 hz.

The variation in high-frequency energy content from
clear liquid to NC 5 conditions is apparent. This suggests
that some truncation error was produced by limiting the
high concentration liquid-solid runs to a 40 hz cutoff
frequency. Efforts to analyze this error by studying
changes in the rms level at higher filter settings revealed
no noticeable increase. The truncation error, if any,
must be quite small.

The single point hot-wire measurements precluded direct
turbulence scale determinations. If a curve fitting routine, after the manner of Laufer (1951) or Raichlen (1967), is used with the data of Fig. 21, the close similarity between these curves forces the conclusion that there was no change in turbulence macroscale with increasing suspended load. While this indeed appears to be the case, support for these results, as well as the additional (independent) determination of microscale variations, requires more data or, ideally, simultaneous multiple point measurements.

Autocorrelation Function

A study of the autocorrelation function for each experimental run revealed the same degree of similarity evidenced by the power spectra. The behavior of the example plotted in Fig. 23 is typical. Such a result further tends to support the hypothesis that only slight changes in turbulence scale resulted from the increasing suspended loads. Using the area delimited by the characteristic sharp decline near the origin and the first zero point (0.5 sec) as a measure of the macroscale, Fig. 23 shows that only at NC 5 is there a suggestion of a significant change in scale. Although admittedly this is a rough estimation, owing to the periodic variations present beyond 0.5 sec., it is believed to be valid, since this external flow region (y/D = 0.572) should be especially sensitive to any significant increase in small scale eddy production. Nearer the boundary such changes could be masked by the already dominant small-scale turbulence.
FIG. 23  AUTOCORRELATION CURVES

\( \gamma_D = 0.572 \)
Discussion

Perhaps the most significant result of this experimental investigation is that there are substantial alterations of the turbulent flow characteristics at low suspended particle concentrations. The departure of the mean velocity from a logarithmic profile, the general increase in turbulence intensity, and the variations in momentum transport all occur at suspended load levels not far above those encountered in many natural regimes. These findings clearly indicate that in most instances offhand neglect of suspended load effects will be an oversimplification. What is needed is a simple analytical description of the observed phenomena which could be systematically included in the derived equations of motion. This in turn requires a more detailed understanding of the physics of turbulence production in liquid-solid flows. Towards this goal, can the experimental results be of any assistance?

It seems apparent that the presence of particles induces an additional source of stress over and above the usual pressure, viscous, and Reynolds stresses. Moreover it was observed that the action of the experimental particles induced a change in flow characteristics quite similar in form to the observed longitudinal variations in adverse pressure gradient flow. In a gradual linear gradient, for example, a comparison of results over a section far upstream of any separation point shows the same trend in mean velocity (Schubauer and Tchen, 1961), a similar increase in rms levels (Schubauer and Klebanoff, 1951), a general increase
in Reynolds stress with peak levels moving away from the bed (Goldberg, 1966), and the concurrent small change in energy spectra (Moses, 1964). While it is not suggested that the presence of particles induces an adverse pressure gradient per se (clearly precluded by the uniform flow conditions), the turbulence modifications in each case are strikingly similar and limited use of the analogy might be instructive.

Within a model based on such an analogy it seems improbable that particle presence produces the observed turbulence changes simply by increased production of small-scale eddies. As suggested by Elata and Ippen (1961), this mechanism was supposed to be a function of scaling similarity between particle diameter and the viscous sublayer thickness. This is an intuitively pleasing construct, but, as noted above, not evidenced in the spectral data. Moreover, such matching is hard to envision in adverse pressure gradient flow where similar turbulence changes are noted.

Rather it is suggestive to reason that the suspended particles exert primary influence through their effect on the gradient of mean velocity. The flow retardation noted in Fig. 15 results in increased shear and increases $u'v'$ correlation. In turn the interaction of the increased Reynolds stress $\overline{u'v'}$ and the mean velocity gradient results in increased turbulence production as indicated by the higher rms levels. Such a mechanism could produce a higher energy turbulence field with little change in spectral
character. Since optimum effects would be produced in the region of maximum turbulence production, it is reasonable to assume that particle diameter would still remain a significant parameter in combination with flow Reynolds number, suspended concentration and the mean velocity gradient. The density contrast \( \frac{\rho_{\text{m}}}{\rho_{\text{w}} \omega} \) serves only to determine the concentration gradient.

This is admittedly a rather simplistic line of reasoning. Mean velocity gradient, Reynolds stress, and turbulence intensity are pictured as independent variables, when in fact they are mutually interdependent. Nevertheless the experimental results indicate that the proposition is not entirely specious. The increasing levels for each variable are clearly observed, and although it remains to be shown that the primary particle effect is through the mean velocity gradient, this feature is the easiest of the three to work with in terms of a descriptive or predictive model.

A clarifying study would proceed first to improve the quality of the spectral record. A tenet of the above proposition is that no significant increase in high frequency energy is noted at the higher particle concentrations.

Next a visual study of the boundary layer region (cf. Kline et al., 1967) in liquid-solid flows could provide an insight into the character of the near-wall velocity gradient and the nature of the stresses introduced by the particle presence. Both could be used to provide a check on the applicability of the adverse pressure gradient analogy and the functional dependence of the turbulence
production on particle size and concentration in various Reynolds number flows.

As in the case in the majority of turbulence investigations and particularly liquid-solid flow studies, the results of this experiment are difficult to generalize. However, the clarity of the data and the reliability of the apparatus are encouraging. A continuing systematic study of this problem appears to be quite possible. In combination with the information provided by the above investigations, this approach promises to permit the development of a sufficiently accurate model for most field applications and at the same time yield the necessary additional insights into the mechanics of turbulence in liquid-solid flows.
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APPENDIX A

Venturi Flowmeter Calibration

Fig. Al shows the principal features of the Venturi flowmeter used in the turbulence flume. Measurement of the volume rate of flow is governed by the equation

\[ Q_{\text{actual}} = \frac{C_d A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2g (P_1 - P_2)}{\gamma}} \]  

(1)

where:

- \( Q_{\text{actual}} \) = volumetric discharge
- \( C_d \) = discharge coefficient
- \( A_1 \) = inlet area (Fig. A1)
- \( A_2 \) = throat area (Fig. A1)
- \( g \) = acceleration due to gravity
- \( P_1 \) = pressure measured at the inlet (Fig. A1)
- \( P_2 \) = pressure measured at the throat (Fig. A1)
- \( \gamma \) = specific weight of the flow liquid.

Calibration of a flow meter consists of determining the discharge coefficient. Normally, if the meter is constructed using the guideline established by the American Society of Mechanical Engineers\(^2\), the coefficient can be read directly from tabulated values based on inside dimensions of the flowmeter and the discharge. In this case calibration was


FIG. A1
VENTURI FLOWMETER
required since the inside diameter (3.8 cm) was slightly below the smallest tabulated value (~5 cm). Individual calibration of a meter should also increase the attainable measurement accuracy.

Calibration Procedure

Measurements of the pressure drop across the flowmeter as a function of the discharge were made with the meter placed in a piping section exactly similar to the flume return line. Water from the laboratory supply flowed through the section and into a calibrated container. The discharge was measured using a stopwatch (gallons/minute) while an open manometer indicated the Venturi pressure drop. A precision steel rule was used for the head measurements (cm).

Rewriting Equation I,

\[ C_d = \frac{Q_{\text{actual}}}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \]  

For the meter used, \( A_1 = 11.35 \text{ cm}^2 \)
\( A_2 = 2.85 \text{ cm}^2 \)

Using \( P = fgh = \gamma h \) and \( h_1 - h_2 = \Delta h \), Equation II becomes

\[ C_d = \frac{Q_{\text{actual}}}{\sqrt{1960 \cdot \Delta h}} \]  

(with \( \Delta h \) in cm and \( Q \) in cm\(^3\)/sec, also, 1 gallon = 3790 cm\(^3\)).

Sample Calculation

@ 7.4 gpm and \( \Delta h = 12.50 \text{ cm} \)

\[ C_d = \frac{7.4 \left(\frac{3790}{60}\right)}{\sqrt{2.95/1960 \cdot (12.5)}} = 1.01 \]
Calibration Results

Fig. A2 shows the results of a series of runs with discharges ranging from 1 to 12 gallons per minute. The series was performed with the pressure taps aligned along the side of the test section. This configuration assured the absence of air bubbles in the manometer line and minimized the possible accumulation of particles in later liquid-solid runs.

Early scatter in the data points was eliminated with the addition of a honeycomb baffle placed in the entrance to the test section. Consisting of plastic drinking straws (20 cm long x 0.318 cm i.d.), this baffle was also placed in the flume return line and used in all experimental runs.

After several test series it became apparent that the discharge coefficient was very close to unity except at low flow rates (<4gpm). In order to insure fully turbulent flows in the rectangular section, the discharge would have had to be several times this value. Therefore, for our purposes the discharge coefficient was taken equal to unity.

After determining the discharge coefficient it remained necessary to compute and tabulate pressure head versus discharge for flows of one-centistoke silicone oil. This simply required a correction for the density difference, since the viscosity effect was negligible\(^1\).

Since \( \rho_{\text{silicone oil}} = 0.818 \text{ gm/cc @ 25°C} \), Equation III can be rewritten

\[
Q_{\text{ave}} = \sqrt{1950(0.818)\Delta h} \times 2.95
\]

with \( C_d = 1.0 \)

This equation is plotted in Fig. A3 for various flow rates.
FIG. A2. VENTURI FLOWMETER CALIBRATION

WITH HONEYCOMB BAFFLE

$H_2O$ at 24°C

FIG. A3. VENTURI FLOWMETER

DISCHARGE VS. PRESSURE HEAD

1.0c SILICONE OIL at 25°C
APPENDIX B
Hot-Wire Anemometer Calibration

Beginning with

\[ \frac{I^2 R_w^2}{R_w (R_w - R_g)} = A + B U^n \]  

(V)

where \( I \) = wire feedback current
\( R_w \) = hot wire resistance
\( R_g \) = wire resistance @ fluid temperature
\( A \) & \( B \) = constants dependent on wire and fluid properties
\( U \) = cooling velocity
\( n \) = heat transfer exponent, dependent on flow conditions

one could write

\[ I^2 R_w^2 = R_w (R_w - R_g) (A \\neq B U^n) \]  

(VI)

\[ = R_g (k) (k-1) (A \\neq B U^n) \]  

(VIa)

with \( k = \) overheat ratio = \( R_w / R_g \)

which leads to

\[ E^2 = A' \neq B' U^n \]  

(VII)

with \( E \) equal to the mean level of anemometer output voltage
and \( A' \) and \( B' \) representing the values of \( A \) and \( B \) modified
by the constant resistance and overheat values.

Under the assumption of a known or constant \( n \) measurements of \( E \) versus flow velocity under controlled conditions
would permit determination of \( A' \) and \( B' \). The exponent \( n \)
however, is dependent on flow conditions. This is especially
true for X-array operation. The small vertical separation
of the wires causes noticeable free convection effects at
low flow rates.
The following scheme was used to overcome this difficulty. Measurements of output voltage \( \bar{E} \) as a function of mean velocity \( U \) were obtained by using the flume return line as a calibration tunnel. The filter tap (see Fig. 1) was fitted with a sealed gland to permit insertion of the hot-wire probe. The DC output from the anemometer at various flowrates was read using a digital voltmeter (Non-Linear Systems). Venturi pressure drop was used as a measure of the discharge, and centerline velocity in the pipe was assumed to be

\[
U = \frac{V}{A^*} \tag{VIII}
\]

where \( U \) = mean velocity

\( V \) = discharge

\( A^* \) = cross-sectional area corrected for displacement thickness

The location of the test port some 45 pipe diameters downstream from the Venturi insured flow characteristics that were steady and well developed. Cross-stream measurements confirmed this assumption.

Each wire of Probe no. 1 was calibrated using the above procedure. The data plotted in Fig. B1 and Fig. B2 were obtained with both wires heated and alternately placed at right angles to the mean flow at the pipe centerline.

To obtain the value of the exponent \( n \), Figures B1 and B2 were used to graphically determine \( \frac{d\bar{E}}{dU} \). The values were then plotted versus mean velocity of log-log paper (Fig. B3). The slope of these curves is simply related to \( n \), since differentiating Eq. VII to eliminate \( A' \) leads to

\[
\frac{d\bar{E}}{dU} = \frac{nB'}{2\bar{E}} U^{n-1} \tag{IX}
\]
Taking logs,
\[ \log \frac{dE}{dU} = \log nB' + n-1 \log U \]  \hspace{1cm} (X)

indicates that the slope is proportional to \(n-1\).

A sharp dependence of \(n\) on the velocity is apparent. Only with flow rates greater than 60 cm/sec does \(n\) maintain a constant value over a wide range of velocities. This feature was used in calculating \(A'\) and \(B'\).

Using the exponent value selected from the high-velocity portion of the curve, \(B'\) was calculated using Eq. IX. This value was assumed to be constant over the entire calibration range. While this may not be entirely correct, the exact variation of \(B'\) is of academic interest only, since at this point the procedure becomes entirely an effort to fit the available data to Equations IX and VII.

The value of the exponent \(n\) was computed for various velocity ranges by substituting \(B'\) into Eq. IX and making use of the graphical values of \(d\overline{E}/dU\) and the available calibration data.

\(A'\) was calculated by substituting \(B'\) and \(n\) into Eq. VII. Its value was found to be essentially constant over the calibration range.

The results of these computations are tabulated in Table B1.

The agreement between the measured calibration data and values computed using the derived constants is shown in Fig. B4. One would expect the fit to be excellent except in the very low velocity range, where buoyancy effects pro-
duce anomalous results causing the voltage-velocity curve to approach zero in a fashion poorly reflected by the extra-
polation.
FIG. B1. CALIBRATION DATA
WIRE NO. 1

E (D.C. VOLTS OUT)

X-ARRAY
KOVAR WIRE
1.0 CS SILICONE OIL
K = 1.1

VELOCITY CM/SEC

3.0

2.0

1.0

0

30

60

90

FIG. B2. CALIBRATION DATA
WIRE NO. 2

E (D.C. VOLTS OUT)

X ARRAY
KOVAR WIRE
1.0 CS SILICONE OIL
K = 1.1

VELOCITY CM/SEC

3.0

2.0

1.0

0

30

60

90
FIG. B3. CALIBRATIONS

CALIBRATION DATA

WIRE NO. 1

X-ARRAY
KOVAR WIRE
LO CS SILICONE OIL
E=9.1

key: • MEASURED
+ COMPUTED

VELOCITY
CM/SEC

CALIBRATION DATA

WIRE NO. 2

X-ARRAY
KOVAR WIRE
LO CS SILICONE OIL
E=1.1

key: • MEASURED
+ COMPUTED

FIG. B4. DATA COMPARISONS
TABLE B1
HOT-WIRE CALIBRATION CONSTANTS

<table>
<thead>
<tr>
<th>WIRE</th>
<th>VELOCITY cm/sec</th>
<th>n</th>
<th>A'</th>
<th>B'</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50 - 90</td>
<td>0.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>35 - 50</td>
<td>0.43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. 1</td>
<td>29 - 35</td>
<td>0.40</td>
<td>3.45</td>
<td>7.26</td>
</tr>
<tr>
<td></td>
<td>12 - 29</td>
<td>0.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.5 - 12</td>
<td>0.30</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>45 - 90</td>
<td>0.43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. 2</td>
<td>20 - 45</td>
<td>0.41</td>
<td>3.60</td>
<td>7.10</td>
</tr>
<tr>
<td></td>
<td>12 - 20</td>
<td>0.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.5 - 12</td>
<td>0.30</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX C

COMPUTER PROGRAMS
The Data Acquisition Program

Language: Pal III Symbolic Assembler (See Digital Equipment Corporation Manual 8-3-S)
Loading Sequence See DEC Manual DEC 08-NGCA-D
The PDP-8 Console Manual

*0232/ VARIABLE SCAN RATE A-D CONVERSION PROGRAM W. F. BOHLEN
TCF
KCC/CLEAR ALL FLAGS
BEGIN, CLA
   TAD MCNT
   DCA CNTR
   TAD TOLL
   DCA STORE
   TAD K
   DCA CLEAR ≠ 6
   TAD OUT
   DCA SPAS/ INITIALIZE
CLEAR, TAD LOOP
   DCA BAG/ RESET LOOP
   ADCV
   ADSF
   JMP .-1
   ADRB/ A-D CONVERT
   DCA I STORE/ STORE DATA
   ISZ STORE
   ISZ BAG
   JMP .-1/ VARIABLE LOOP
   ISZ CNTR
   JMP CLEAR
CLA
   TAD MCNT
   DCA CNTR
   TAD TOLL
   DCA STORE/ REINITIALIZE
   TAD L
   DCA DATA
DATA, TAD I STORE
   ISZ STORE
   JMS SSPRNT/ SIGNED DECIMAL PRINTOUT SUBROUTINE (8-23-U)
   JMS IS WHIZ/ TYPE A SPACE SUBROUTINE (8-19-U)
   ISZ SPAS
   JMP ≠ 5
CLA
   TAD OUT
   DCA SPAS
   JMS I KID/ CARRIAGE RETURN-LINE FEED SUBROUTINE (8-19-U)
   ISZ CNTR
   JMP DATA
MCNT,-5000
CNTR, 0
K,3710
TOLL,2000
STORE,0
OUT, -15
SPAS,0
L,1710
WHIZ, TYSP
KID, TYCR
LOOP, -50 / OR WHATEVER VALUE IS SET IN BY HAND
BAG, 0

PAUSE

The following routine was employed to change the length
of the variable loop and therefore the scanning rate:
Load Address 0315
Depress Exam 0417 should appear in the memory buffer
Key in the desired loop length, e.g.
Record Length 64 sec. key 7310 for sampling rate of 40/sec
  32 sec. - 7545 - - - - - - - - 80/sec
  12.8 sec. - 7710 - - - - - - - - -200/sec
Load The Starting Address 0232
Press Start....
MAIN REDUCTION PROGRAM

C THIS PROGRAM COMPUTES MEAN AND RMS VELOCITY
C THE FOURIER COEFFICIENTS AND THE AUTOCORRELATION
C COEFFICIENTS FOR TURBULENT FREE SURFACE
C CHANNEL FLOWS. IT IS DESIGNED TO REDUCE
C DATA OBTAINED USING HOT-WIRE ANEMOMETRY
C IN FLOWS OF ONE CENTISTOKE SILICONE
C OIL. BOTH WITH AND WITHOUT SUSPENDED
C PARTICLES.

DIMENSION F(26CC),A(2600),R(5CC),INV(300),S(300)

JN=5

C SET UP LCCC FOR DATA READING

5 JA=1

6 N=JA+11

READ (5,21) (E(I), I=J4,')

IF (E(N) .EQ. 0) GO TO 7

JA=JA+12

GO TO 6

7 WRITE (6,31) N

CC 10 I=1,N

E(I)=E(I)/412.

CC CONTINUE

C CHECK TEN POINTS OF THIS CONVERSION

WRITE(6,22) (E(I),I=1,10)

C CONVERT VOLTS TO VELOCITY

DC 14 I=1,N

E(I)=((E(I)**2+3.45)/3.26)**2.702

14 CONTINUE

WRITE (6,32)

140 WRITE (6,23) (E(I),I=1,10)

C SET UP MATRIX FOR COMPUTING AUTOCORRELATION FUNCTION

DC 19 I=1,N

A(I)=E(I)

19 CONTINUE

L=400

CALL ALTO (A,N,L,R)

CD 200 J=1,400

WRITE (6,39) J,R(J)

R(J)=R(J)**10.*3

200 CONTINUE

WRITE (7,30) (R(I),I=1,400)

C WRITE OUT MEAN AND RMS VELOCITY

R(1)=R(I)*10.

R(1)=SQR(R(1))

WRITE (6,28) R(1)

VBAR=C.

CC 13 I=1,N

VBAR=VEAR+E(I)

13 CONTINUE

B=N

VEAR=VBAR/B

-------------------
WRITE (6,24) VBAR
C REMOVE MEAN AND SET UP MATRIX FOR FOURIER ANALYSIS
CC 16 I=1,N
E(I)=E(I)-VBAR
CONTINUE
DO 17 I=1,2048
A(I)=E(I)
CONTINUE
CALL RHARIV(A,K,INV,S,EFERR)
WRITE (6,25) A(1),A(2)
WRITE (6,26) (A(I),I=3,2048)
J=2
DO 18 I=3,2047,2
A(I-J)=A(I)*2*A(I+1)*2
A(I-J)=A(I-J)*10.*3
J=J+1
CONTINUE
C WRITE FOURIER COEFFICIENTS AND PUNCH C(N) SCALED
WRITE (6,27) (A(I),I=1,10)
WRITE (6,28) A
JN=JN-1
IF (JN.EQ.0) GO TO 6C
CONTINUE
GO TO 5
STOP
21 FORMAT (12F6.1)
22 FORMAT (1CF1C.5//)
23 FORMAT (' VELOCITY CHECK'//1CF1C.5//)
24 FORMAT (' TEMPORAL MEAN VELOCITY'//F1C.5//)
25 FORMAT (' A(C)='//F10.5,1CX,' E(C)='//F1C.5//)
26 FORMAT (4(10X,' A(N)',1CX,' P(N)'),//E(6X,F10.5))
27 FORMAT (' C(N) SCALED'//10F1C.5//)
28 FORMAT (' RMS VELOCITY'//F1C.5//)
30 FORMAT (8F10.8)
31 FORMAT (1H1,' L='//16//)
32 FORMAT (' WIRE NO.1'/)
34 FORMAT (16//)
35 FORMAT (2(14,10X,' CORRELATION COEFFICIENT='//F1C.8//))
This program computes Reynolds stress making use of an algorithm that is independent of the wire calibration data in use obtained using a single wire oriented at + and - 45 degrees to the mean flow.

Dimension E(2600), A(2600), B(2600), C(2600), D(2600)

K = 4

1. READ (5, 24) UGNE
   JN = 0

2. JA = 1

3. N = JA + 1

4. RFAC (5, 260) (E(I), I = JA, N)

5. IF (E(N) .EQ. C.) GC TC 7

6. JA = JA + 12

7. GC TC 6

8. DO 10 I = 1, N


10. CONTINUE

11. XSUM = 0.

12. DO 11 I = 1, N

13. XSUM = XSUM + E(I)

14. CONTINUE

15. XBAR = XSUM / F

16. DO 12 I = 1, N

17. E(I) = E(I) - XBAR

18. CONTINUE

19. ESTABLISH THE COMPUTATIONAL ARRAYS

20. JN = JN + 1

21. GC TC (5C, 6C), JN

22. DO 13 I = 1, N

23. A(I) = E(I)

24. CONTINUE

25. GC TC 5

26. DO 14 I = 1, N

27. B(I) = E(I)

28. CONTINUE

29. DO 15 I = 1, 2500

30. C(I) = A(I) + P(I)

31. D(I) = A(I) - B(I)

32. CONTINUE

33. ARMS = 0.

34. BRMS = 0.

35. CRMS = 0.

36. DRMS = 0.

37. DO 16 I = 1, 2500

38. ARMS = ARMS + (A(I)**2)
BRMS = BRMS * P(I) ** 2
CRMS = CRMS * C(I) ** 2
DRMS = CRMS * (C(I) ** 2)

16 CONTINUE
ARMS = SQRT(ARMS / 2500.0)
BRMS = SQRT(BRMS / 2500.0)
CRMS = SQRT(CRMS / 2500.0)
DRMS = SQRT(DRMS / 2500.0)
WRITE (6, 21) ARMS, BRMS, CRMS, DRMS

USE THE COMPUTED VALUES TO SOLVE FOR THE REYNOLDS STRESS
RNUM = ((ARMS ** 2) - (ARMS ** 21) * (CRMS ** 2) * (DRMS ** 2))
RDCM = 2 * ((CRMS ** 2) * ((ARMS ** 2) + (BRMS ** 2)))
RSTRES = UCNF * (RNUM / RDCM)
WRITE (6, 22) RSTRES
VSQ = UCNF * ((DRMS / CRMS) ** 2)
WRITE (6, 23) VSQ

K = K - 1
IF (K .EQ. 0) GOTO 39
GOTO 4

39 STCP
20 FORMAT (12F6.1)
21 FCWRAT(IH1, 'ARMS='*, F11.8//, 'BRMS='*, F11.8//, 'CRMS='*, F11.8//, 'DRMS='*, F11.8//)
22 FORMAT ('REYNOLDS STRESS='*, F11.8//)
23 FORMAT ('MEAN SQUARE WPRIME='*, F11.8//)
24 FORMAT (F10.8)
END
APPENDIX D

EQUIPMENT AND MATERIAL SPECIFICATIONS

1. Circulating Pump

Manufacturer: Labawco Pumps, Inc.,
Belle Mead, New Jersey
Type: Centrifugal

<table>
<thead>
<tr>
<th>CAPACITY Gallons/Minute</th>
<th>HEAD Meters</th>
<th>SPEED rpm</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>1.50</td>
<td>3450</td>
</tr>
<tr>
<td>34</td>
<td>2.45</td>
<td>&quot;</td>
</tr>
<tr>
<td>33</td>
<td>3.05</td>
<td>&quot;</td>
</tr>
<tr>
<td>30</td>
<td>4.60</td>
<td>&quot;</td>
</tr>
<tr>
<td>27</td>
<td>6.10</td>
<td>&quot;</td>
</tr>
<tr>
<td>18</td>
<td>9.15</td>
<td>&quot;</td>
</tr>
<tr>
<td>12</td>
<td>10.50</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

2. Drive Unit

Manufacturer: Dayton Electric Manufacturing Co.,
Chicago, Illinois
Type: Mechanical Vari-Drive
Motor: 1/2 Horsepower Capacitor Motor
115/230v 60cycle 1725 rpm
Speed Control: Adjustable Pitch Pulleys and Deep-cog Belt, speed adjustable
705-4230 rpm

3. Composition of Kovar

Manufacturer: The Driver Corporation

Nominal Composition

<table>
<thead>
<tr>
<th>Element</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ni</td>
<td>29%</td>
</tr>
<tr>
<td>Co</td>
<td>17%</td>
</tr>
<tr>
<td>Fe</td>
<td>54%</td>
</tr>
</tbody>
</table>

 Resistivity.................300 ohms/c.m.-ft.
 Temperature Coefficient
 of Resistance..............0.003 ohm/°C
 Melting Point.............1430°C
 Tensile Strength..........90-100 x 10³ psi.
 Magnetic and somewhat subject to corrosion
4. Flow Liquid

Manufacturer: Dow Corning Corporation
Midland, Michigan
Material: D. C. 200 Silicone Oil

Specifications

Viscosity @ 25°C - 1.0
Flash Point - 100°F
Pour Point - -123°F
Specific Gravity @ 77°F - 0.818
Viscosity Temperature Coefficient - 0.37
Coefficient of Expansion - 0.00134 cc/cc/°C
Refractive Index @ 77°F - 1.382
Surface Tension @ 77°F - 17.4 cyne/cm
Thermal Conductivity @ 77°F - 0.00024 gm-cal/sec/cm²/°C.cm thick
Boiling Point - 305°F @ 760mm
Specific Heat - 0.348 cal/gm/°C @ 40°C
Dielectric Constant - 2.18 @ 10² Hz

5. Experimental Particles

Manufacturer: Sinclair-Koppers Company
Pittsburgh, Pennsylvania
Material: Dylite Expandable Polystyrene
F-40-C

Specifications

Specific Gravity Unexpanded - 1.02-1.05
Expanded - 0.82-0.84

Size (Unexpanded)
Sieve Analysis (U.S. Standard)
on No. 30 - 60% max. (by weight)
on No. 40 - 70% max.
on No. 45 - 3% max.
on No. 50 - 0.5% max.
Volatiles (% weight) - 6.0-6.5

6. Analog-To-Digital Converter

Manufacturer: Digital Equipment Corporation
Maynard, Massachusetts
Type: AD08-A

Specifications

Analog Input:
Voltage - 0 to 10V (standard)
Impedance - 1,000 ohms (standard)
Resolution - 1 part in 1024 (10 mv)
Accuracy - 0.1% of Full Scale ± 1/2 Least Significant Bit
Temperature coefficient - 0.5 mv/°C
Operation temperature - 0 to 50°C
Conversion Rate - 100 khz maximum
Digital Output - A 10-Bit Binary Number

7. Variable Filter

Manufacturer: Krohn-Hite Corporation
Cambridge, Massachusetts
Type: Model 3202

Specifications
Range - 20hz-2Mhz
Passband Gain - Unity (Odb)
Attenuation Rate - 24 db/octave
Maximum Attenuation - Greater than 80db
Output Hum and Noise - Less than 100 µv

8. Digital Voltmeter

Manufacturer: Non-Linear Systems
Palo-Alto, California
Type: Series X-3 Multi-purpose Digital Voltmeter

Specifications (on the 10v range)
Resolution - 10 mv
Accuracy - 0.1% LSB ≠ 1 digit
Scan Rate - 3 samples/sec

9. Hot-Wire Anemometer

Manufacturer: Flow Corporation
Watertown, Massachusetts
Type: Model 900-A

Specifications

<table>
<thead>
<tr>
<th></th>
<th>D.C. Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage</td>
<td>0.1% or 0.1 mv</td>
</tr>
<tr>
<td>Hot Sensor Current</td>
<td>0.1% or 0.01 ma</td>
</tr>
<tr>
<td>Cold Sensor Resistance</td>
<td>0.1% or 0.01 ohms</td>
</tr>
<tr>
<td>Fluid Temperature</td>
<td>0.1% or 0.03°C</td>
</tr>
<tr>
<td>Fluid Velocity</td>
<td>1% or 0.1 fps</td>
</tr>
</tbody>
</table>
Voltage 0.05% or 0.05 mv
Hot Sensor Current 0.05% or 0.005 ma
Cold Sensor Resistance at 1 ma 0.05% or 0.0025 ohms
Fluid Temperature (50ohm sensor) 0.05% or 0.015°C
Fluid Velocity 0.5% or 0.01 fps

Output
in 1 kHz: 10

Voltage 0.002% or .02 mv
Hot Sensor Current 0.002% or 0.002 ma
Cold Sensor Resistance at 1 ma 0.0025 ohms
Fluid Temperature (50ohm sensor) 0.015°C
Fluid Velocity 0.01% or 0.001 fps

10. Viscometer

Manufacturer: Brookfield Engineering Laboratories Inc., Stoughton, Massachusetts
Type: LVT Series

Specifications
Accuracy - 1.0% of Full Scale
Sensitivity - 0.2%
Reproducability - 0.2%
BIOGRAPHICAL SKETCH

The author was born in Tarrytown, New York on June 21, 1938 and raised in nearby Dobbs Ferry, New York.

After receiving a Bachelor of Science degree from the University of Notre Dame in 1960, he entered the U.S. Navy and served for two years as a shipboard engineering officer.

In 1962 he joined the staff of the Woods Hole Oceanographic Institution as a research assistant in the Geophysics Group, and participated in several scientific cruises studying oceanic thermal structure and methods to measure it continuously from an underway ship.

In November of 1963 he joined the staff of an oceanographic consulting firm located in Washington, D.C. For the next year he was engaged in the design and construction of a mineral prospecting ship and participated in its first cruise.

At the completion of this project he returned to Woods Hole and joined the second leg of the International Indian Ocean Expedition aboard the R.W. ATLANTIS II. This cruise ultimately led to his enrollment in M.I.T. in September of 1965.

The author is a member of the Marine Technology Society, the American Geophysical Union, and the Society of Sigma Xi.

In October of 1967 he married the former Elisabeth Pope of Silver Spring, Maryland.