NUMERICAL PREDICTION OF HURRICANE MOVEMENT

by

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ABSTRACT

The non-divergent barotropic model with the pressure weighted
vertical average winds was used for the prediction of hurricane
movement. Two choice of averaging winds were made; one for the 10
levels, and the other for the multi-levels (23). The extra work re-
quired for the multi-level averaging procedures did not give any
better results of hurricane forecast.

The hurricane is considered as one of the disturbances in the
large scale flow. To reduce the truncation error and to conserve
the quadratic quantities of physical importance, the Arakawa (1966)
high order Jacobian scheme was applied. Perhaps due to the inconsis-
tency of the boundary conditions, the Arakawa Jacobian scheme could
not inhibit the increase of kinetic energy in the numerical integra-
tion of the non-divergent barotropic model. Operation of the mean
Jacobian so that the vorticity changes in the entire grid becomes
zero all the times significantly controled the kinetic energy increase.

In an attempt to improve the forecast of hurricane movement,
with the same sets of data, a series of Trials were made using vari-
ous boundary conditions and numerical schemes. Shuman's smoothing
and filtering procedure was applied rigorously to eliminate the
small wave lengths and to retain the longer wave lengths.

In an attempt to reduce the time consumed in the initial analy-
sis, a method of objective stream function analysis was applied, but
yielded no better forecast than did the subjective method, due to
the fact that no correction was applied to the initial guess in the
region where observations were not available.

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Title: Associate Professor of Meteorology
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The numerical computations were performed at the M.I.T. Computation Center. I am very grateful to Mr. Burpee whose help in the essential part of the computer programming was indispensable.

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I. INTRODUCTION

The hurricane or typhoon is capable of causing more widespread destruction in a short time than any other force known to us except perhaps an earthquake or nuclear bomb. In order to minimize such disasters, of paramount importance is the accurate prediction of hurricane movement.

Considerable research and some progress has been made in the prediction of weather phenomena, including hurricane motion by numerical methods during the past decade. In formulating the numerical prediction model of hurricane movement, there are several basic problems to be considered;

The first is whether or not we treat the atmosphere as barotropic or baroclinic. It is realized that perhaps one is oversimplifying the physics by treating the atmosphere as purely barotropic, thus neglecting its more complicated baroclinic nature. To be completely rigorous, one should also include diabatic and orographic effects. One feels justified however in treating the tropical atmosphere, as distinct from extratropical regions, as a predominantly barotropic fluid both theoretically, by rigorous scaling (Charney 1963), and from numerical experiments.

The second is whether we can treat the atmosphere geostrophically or non-geostrophically. The geostrophic model has been used successfully in the middle and high latitude regions but not in the tropics where hurricanes develop most frequently.
It is generally known that hurricanes are directed by the steering current of the general field. This promotes the question as to which level, or levels, has the most significant influence upon hurricane motion. Also, how do we treat the vortex pattern of the hurricane since it is of quite a different scale from the remaining general field?

So thirdly, we have to consider how we should choose the steering level or levels, a single level, say 500 mb as has been used operationally for a number of reasons, or several combinations of a few levels, for example, the 4 level Birchfield (1961) model, or inclusion of more levels as Sanders' (1961) and King's (1966) pressure-weighted 10-level average or the averaged multi-level which uses all the available information.

Fourthly, whether we eliminate the vortex pattern from the general field and then modify the remaining field as done by Kasahara (1957), Sasaki (1955) etc. or whether we treat the hurricane as one of the disturbances in the atmosphere without separation of the vortex pattern as done by King (1966), Birchfield (1961) etc.

The process adopted by the latter group is more natural and desirable in the sense of treating the actual hurricane field, but they have to suffer excessive truncation error unless some modification is applied. On the other hand, the process of the former group is too artificial, although the remaining field can be represented more or less by the steering flow without much destruction due to truncation error.
Bearing these considerations in mind, the numerical prediction model adopted is the simplified barotropic model using pressure-weighted vertical average winds. Two experiments are made. The number of levels used in forming the vertical average is, in one case, 28 and, in the other, 10. Also the hurricane is treated as an integral part of the large scale flow, but to reduce the truncation error and to conserve the quadratic quantities of physical importance such as kinetic energy and mean square vorticity, the Arakawa (1966) high order Jacobian scheme is applied.

In the influential region of hurricane motion, we are suffering from a scarcity of data. Therefore considerable time-consuming subjectivity is inevitable in the initial analysis to which the forecast is most sensitive. Since hurricanes are very destructive, quick and accurate forecasting is most essential, a kind of objective scheme of initial analysis was therefore attempted.
II. THE CONCEPT OF THE STEERING PRINCIPLE

A. Steering current

The steering principle of the hurricane motion became known to us from an early date. Strictly, steering refers to the forces that govern a relatively small disturbance in a broad current. Various efforts have been made to relate the movement of a hurricane to the basic current in which it is embedded. The width of the steering current as well as the depth through which the mean wind flow must be integrated may vary with the intensity of the hurricane and the region through which it is passing.

Jordan (1952) showed from her composite analysis surrounding the average hurricane center that the steering current obtained from the pressure-weighted mean of wind fields at all available levels from 4,000 to 30,000 ft in the band 2-4 degree of latitude from the centers closely approximates the mean forward velocity of the centers. However, there is no reason to assume a priori that the hurricane moves with the speed of a steering current, since the internal forces of the hurricane itself may well make some small contribution to the motion. Miller (1958) has extended the work of Jordan (1952) to 52,000 ft and concluded that moderate to intense storms move at the same velocity as the mean flow in the 500 to 200 mb layer extending 2-6 degree of latitude from the center. From the above findings the inclusion of more high level winds may suggest better forecasting.
B. The proper choice of the steering level

In recent research of Birchfield, Sanders and King, it was found that the 500-mb wind field alone is not a good representation of the steering level of the hurricane due to the complex vertical structure of the winds at low latitudes, although it is viewed as a good approximation to an equivalent barotropic atmosphere at middle and high latitudes. Inclusion of more levels where the hurricane circulation exists might be a better representation of the steering level. These influential levels were incorporated into a single steering level using the pressure weighted vertical average suggested by Sanders (1961);

\[ \left( \frac{V}{P} \right)_m \equiv \frac{1}{4} \left( \frac{P_r - P_b}{P_r} \right) \int_{P_b}^{P_r} \frac{V dP}{P} \]  \hspace{1cm} (1)

Birchfield (1961) chose 4 levels; 1,000 mb, 700 mb, 500 mb and 200 mb, and computed a weighting factor by the appropriate means of cubic polynominal (see table 1). Then he incorporated his averaged winds into the conventional equivalent barotropic model by assuming \( V^* = A^* V_m \). Sanders (1961) chose all the mandatory level data from 1,000 mb to 100 mb, inclusion and he demonstrated, by consideration of the angular momentum and time averaged flow, superiority of forecasting the hurricane movement using his 10 level average data over forecasts using 500 mb data only. King (1966) obtained better numerical results from 10 levels than Birchfield's 4 level and single 500 mb level gave.
Encouraged by the previous research, a study was made with an averaging process using all the available information; mandatory level plus the rawinsonde data between 1,000 mb and 100 mb. Weighting factor is computed by the same trapzoid rule using the U.S. Tropical Standard Atmosphere, and it is tabulated in table 1. However, the averaged winds of multi-level and 10-level differ, in general, by less than 1 m/s in speeds and a few degrees in direction. Seemingly this much difference is fully within the limitation of the data. The vector differences between multi (23) and 10-level of time averaged winds of Oct. 4th through 6th in the region of hurricane Flora (1963) gives little evidence of better forecasting.
The actual numerical forecasting using multi-level winds was
done for Flora cases (1963) and will be discussed later. The small
improvement obtained using the more accurate forecasting methods
does not justify the extra work involved. Therefore the optimum
levels for the vertical pressure-weighted averages was limited to
the Sander's 10 levels.

<table>
<thead>
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<th>levels</th>
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<th>10 levels</th>
<th>multi-levels</th>
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<td>.0761</td>
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Table 1: Weighting factor for vertical average of the winds
for 4, 10 and multi-level.
III. THE PREPARATORY CONSIDERATIONS FOR THE DYNAMICAL PREDICTION MODEL

A. The scale of motion in the tropic and extra-tropics

By taking the curl of the horizontal equation of motion for an inviscid atmosphere, we obtain the vorticity equation in the following form:

\[
\frac{\partial \omega}{\partial t} + \nabla \times \mathbf{V} = \frac{\partial \mathbf{V}}{\partial t} + \omega \nabla \times \mathbf{V} + \omega \frac{\partial \mathbf{V}}{\partial z} + \mathbf{a} \times \mathbf{V} + \frac{\gamma \nabla^2 \mathbf{V}}{\rho^2} - \frac{\nabla \times \mathbf{V} \cdot \mathbf{R}}{\rho^2} = 0
\]  

(2)

where \( \omega \) is the relative vorticity and \( f \) is the coriolis parameter.

The horizontal velocity \( \mathbf{V} \) can be expressed as the sum of a non-divergent part \( \mathbf{V}_n \) and an irrotational part \( \mathbf{V}_I \). Thus

\[
\mathbf{V} = \mathbf{V}_n + \mathbf{V}_I = \mathbf{a} \times \mathbf{V}_n + \nabla \mathbf{X}
\]  

(3)

On the basis of scale analysis (Charney 1963, Phillips 1963) each term of equation (2) can be expressed in orders of magnitude as follows:

\[
O(1), \ O(1), \ O(\frac{2 \pi L}{U} \cos \phi), \ O(\frac{W}{B} \mathbf{R}_o), \ O(\frac{V}{B} \mathbf{R}_o), \ O(\frac{H}{B} \mathbf{R}_o), \ O(\frac{W}{B} \mathbf{R}_o), \ \text{o}(FR\mathbf{R}_o^2 \mathbf{F})
\]  

(4)

by introducing non-dimensional parameters,

\[
\mathbf{R}_o = \frac{U}{fL}, \ \mathbf{F} = \frac{U^2}{gH}, \ \mathbf{R}_o = \frac{\kappa}{f}
\]  

(5)

where \( U \) and \( W \) are the characteristic horizontal and vertical wind speed, \( L \) and \( D \) are the characteristic horizontal distance and vertical height of large synoptic scale of motion and \( a, \Omega, \phi \) are the earth's radius, angular speed and latitude respectively and \( \kappa \) is the stability factor \( (\kappa = \frac{\partial \theta}{\partial z}, \theta \text{ is potential temperature}) \).
The main assumption behind the scale analysis in the extratropical region (Charney and Stern 1962) are that; (1) the motion is quasi-hydrostatic \( D/L \ll 1 \) and slow (the characteristic time scale is less than the advective time scale), (2) the motion is quasi-geostrophic \( \mathcal{R}_o \ll 1 \), (3) the Froud number \( F \) is small and (4) the beta-plane approximation \( L/a \sim \mathcal{R}_o \) is satisfied.

The above assumption, except for the quasi-geostrophic assumption, may be applied to the tropical region. In the case of the extra-tropical region, the fifth, sixth and seventh terms in equation (2) and (4), \( O(LU^{-1}WD^{-1}) \) is small as deduced by Charney (1962) incorporating with the first law of thermodynamics \( \frac{1}{\mathcal{R}_o} \frac{\partial \psi}{\partial x} \approx \frac{F}{\mathcal{R}_o} \times 10^{-4} \) (by setting \( F \approx 10^{-3}, \mathcal{R}_o \approx 10^{-1}, \mathcal{X} \approx 10^{-1} \)). But the fourth term may be appreciable because of the additional factor \( \mathcal{R}_o^{-1} \) and the last term is \( \mathcal{F} \mathcal{R}_o^{-2} \approx 10^{-1} \). In the case of \( \mathcal{R}_o \gg 1 \) (tropical region), the term including \( LU^{-1}WD^{-1} \) are even small, since \( WD^{-1} \) is small, and the fourth term (the most important difference between extra-tropical region and tropic region) becomes smaller and the last term is \( \mathcal{F} (\approx 10^{-3}) \). In both cases, the third term is of order of unity from the beta-plane approximation.

Incorporating these in the equation of continuity of a homogeneous incompressible atmosphere of thickness \( D \) and with the aid of similar scale analysis, the quasi-geostrophic potential vorticity equation can be obtained in the following form (Kasahara and Platzman 1963):

\[ -9- \]
\[ \frac{\partial \mathbf{v}}{\partial t} = - \nabla \cdot (f + \mathbf{v}) \]
\[ \mathbf{v} = \frac{\mathbf{A}}{\theta} \times \nabla \psi \]
\[ \mathbf{r} = (\nabla^2 - \lambda^2) \psi \]
\[ \lambda \equiv \frac{f}{g D_0} \]

(6)

where \( f_0 \) and \( D_0 \) denote the mean value of \( f \) and \( D \) and the term involving \( \lambda \) expresses vorticity changes induced by horizontal divergence associated with deformation of the upper boundary (free surface).

Rossby (1939) discussed the behavior of the atmospheric longwave and Bolin (1956) and Cressman (1958) claimed improvement by inclusion of tropopause as a divergent term in the Barotropic model. To proceed in this way in the numerical forecasting, we need more accurate climatological data and reasonable justification for using in the large global area.

The previous scale argument is not valid in the case of condensation due to the smallness of the stability factor. Considering the climatological fact that the most frequent occurrence and progression of hurricane are neither in the inter-tropical convergence zone, which is known as the primary source of condensation, nor in the extra-tropical region, where the divergence effect might be important, the quasi-horizontal and quasi-non-divergent (\( \lambda = 0 \)) Barotropic model is well adopted for the forecasting of hurricane movement. Accordingly the equations (2), (3), (6) are reduced to (7), (8), (9):

\[ \left( \frac{\partial}{\partial t} + \nabla \cdot \mathbf{v} \right) (\mathbf{S} + f) = 0 \]  
(7)

\[ \mathbf{v} = \mathbf{v}_\psi = \frac{\mathbf{A}}{\theta} \times \nabla \psi \]  
(8)

\[ \mathbf{S} = \frac{\mathbf{A}}{\theta} \cdot \nabla \times \mathbf{v}_\psi = \nabla^2 \psi \]  
(9)
where \( \nabla \) and \( \nabla^2 \) are the well known two-dimensional del and laplacian operators.

The hurricane vortex and the steering flow

Any hurricane has a relatively large vertical and horizontal velocity for its relatively small horizontal extent. So strictly speaking, the previous scale analysis is not applicable in the small region of hurricane circulation. To extend the scale analysis reasoning to the hurricane problem, the total flow can be written as the sum of vortex flow \( (\mathbf{v}^*) \) and steering flow \( (\overline{\mathbf{v}}) \). Then the non-divergent barotropic vorticity equation can be decomposed in the following:

\[
\begin{align*}
\frac{\partial \mathbf{v}^*}{\partial t} + \mathbf{v}^* \cdot \nabla \mathbf{v}^* + \frac{\partial \overline{\mathbf{v}}}{\partial t} + \overline{\mathbf{v}} \cdot \nabla \overline{\mathbf{v}} &= - \mathbf{v}^* \cdot \nabla \mathbf{v}^* - \overline{\mathbf{v}} \cdot \nabla \overline{\mathbf{v}} - \nabla \cdot \overline{\mathbf{v}} - \nabla \cdot \mathbf{v}^* \\
\mathbf{v}^* &= \mathbf{x} \times \nabla \mathbf{v}^* \\
\overline{\mathbf{v}} &= \mathbf{x} \times \nabla \overline{\mathbf{v}} \\
\mathbf{v}^* &= \nabla^2 \mathbf{v}^* \\
\overline{\mathbf{v}} &= \nabla^2 \overline{\mathbf{v}}
\end{align*}
\]  

(10)

(11)

(12)

where star denote the vortex flow and bar denote the remaining general flow. By the elementary scale analysis of Kasahara and Platzman (1963), the terms of equation (10) have relative orders of magnitude as follows:

\[
O(a^2b^2) + O(1) + O(ab^2 - R_v) + O(1) = O(a^3b^3) + O(ab^3) + O(a) + O(1)
\]

(13)

\[
a = U_v / U_s > 1, \quad b = L_s / L_v > 1, \quad R_v = R_s \cdot a \cdot b
\]

where \( U_v \) and \( U_s \), \( L_v \) and \( L_s \) denote scale velocities and lengths for vortex and steering flow, and \( R_v \), \( R_s \) are the corresponding Rossby numbers of vortex and steering flow. Assuming that \( a \) and \( b \) are about the same order of magnitude, the first three advection terms on the right hand side of equation (13) are four to one orders of magnitude.
greater than the last one. So the main problem for the steering method is how to partition each term of equation (10) in the vortex and steering flow. There is no unique way of partitioning as pointed out by Kasahara and Platzman (1963).

Almost all treatment of the vortex in the steering flow are too artificial, assuming there to be no change in shape or intensity of the vortex field during the forecast. For example, Kasahara (1957) made the partition as follows:

\[
\frac{\partial \mathbf{f}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{f} = - \nabla \cdot \mathbf{V} \mathbf{f}
\]

assuming the vortex has no influence upon the evolution of the steering flow. He found in his prediction (based on the above steering flow model) bias to the right of the actual hurricane path. Then later Kasahara and Platzman (1963) included the interaction terms (last two terms of equation (14)) in the prediction for the steering flow and claimed that the major part of the rightward bias could be reduced.

On the other hand, Birchfield (1961) and King (1966) predicted the hurricane movement on the basis of the hurricane circulation being an integral part of the large-scale flow, and the hurricane is treated as one of the disturbances in the atmosphere. The interaction and coupling between the vortex and the steering flow are implicit in the approach of the integral group. This approach is more natural in the sense that each individual hurricane can not
exist alone. However, in this method excessive truncation error occurs if the coarse grid is used as it stands. Birchfield (1961) used fine grids of 150 km and King (1966) did 165 km. Reducing the grid distance further does not do any good since the interpolation error in the initial analysis is becoming large, apart from the consideration of economical problems. Therefore, the non-divergent barotropic model, equation (7) - (9) was used in this research, but numerical schemes are investigated in an attempt to reduce truncation error.

IV. NUMERICAL INTEGRATION OF NON-DIVERGENT BAROTROPIC VORTICITY EQUATION

A. Map factor

To proceed with the numerical integration of the barotropic vorticity equation, the spherical earth was mapped onto a plane conformally and a cartesian coordinate system (X,Y) in the plane was introduced. The map chosen in our research was the Lambert conformal projection with standard parallels at 30 and 60 degree of latitude and its map scale is 1:13,000,000. Since the region of our interest would be well covered in this map as shown in figure (2). The distortion would not be large except at the southern boundary as shown in figure (2). The scale variation on a Lambert Conformal Projection is a function of latitude only (Saucier 1953):

\[ m = \frac{\sin \phi}{\sin \phi / (\tan \phi / 2)} \]  

(16)
where \( \phi \) is the variable latitude, \( \phi_0 \) is the standard latitude and \( n \) is a constant which is determined from two simultaneous equations (16) for 30 and 60 degree of latitude. The convenient property of this projection is that certain differential invariants (the most frequent usages are for horizontal Laplacian and Jacobian operators) retain this mathematical form after transformation to the map coordinates.

### B. Finite Difference

The barotropic vorticity equation (7) can be rewritten as

\[
\nabla^2 \frac{\partial \eta}{\partial \xi} = J(\eta, \psi) \quad (17)
\]

\[
\eta = \xi + \xi, \quad \nabla^2 \psi = \xi
\quad (18, a, b)
\]

where \( \eta \) is the absolute vorticity and \( J \) is the Jacobian operator with respect to the rectangular coordinates \( X \) and \( Y \) in this plane.

The next step is to introduce the centered finite-difference approximation, for example:

\[
S = \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y} \approx \frac{m}{d^2} (V_{i+1,j} - V_{i-1,j} - U_{i,j+1} + U_{i,j-1}) \quad (19)
\]

\[
\nabla^2 \psi = \frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} \approx \frac{m^2}{d^4} (\psi_{i+1,j} + \psi_{i-1,j} + \psi_{i,j+1} + \psi_{i,j-1} - 4 \psi_{i,j}) \quad (20)
\]

\[
J(\eta, \psi) = \frac{\partial \eta}{\partial X} \frac{\partial \psi}{\partial Y} - \frac{\partial \eta}{\partial Y} \frac{\partial \psi}{\partial X} \approx \frac{m^2}{d^4} \left[ (\eta_{i+1,j} - \eta_{i-1,j})(\psi_{i,j+1} - \psi_{i,j-1}) - (\eta_{i,j+1} - \eta_{i,j-1})(\psi_{i+1,j} - \psi_{i-1,j}) \right] \quad (21)
\]

where \( i \) and \( j \) are the finite grid indices in \( X \) and \( Y \), \( U \) and \( V \) are eastward and northward wind components with respect to grid north, and \( d \) is the grid distance.
The grid distance chosen in our research is 165 km which happened to correspond exactly to half an inch in the map and to a grid of 4 column spaces and 3 line spaces in the computer outputs. The grid points are altogether 1715 points (49 x 35). The relative vorticity at each interior points \( 1 < i < 49 \) \( 1 < j < 35 \) is computed by equation (19) on the basis of initial wind information at each of the 1715 points. Equations (17) and (18 b) are Poisson equation for \( \frac{\partial \psi}{\partial t} \) and \( \psi \) which it is possible to solve numerically by specifying the boundary conditions and forcing function at the interior points.

C. Relaxation technique

The next problem is to compute \( \psi_{i,j} \) at the interior points from the Poisson equation:

\[
\nabla^2 \psi = \tilde{\psi}_{i,j}
\]

with the specified boundary values (for example, equation (29)). In fact, \( \psi \) can be any variable \( (\phi) \). The most well known method to solve equation (22) is the iterative relaxation techniques in which an approximate solution is determined by successive corrections of an initial value. There are several relaxation methods; For instance, Richardson's simultaneous method:

\[
\phi_{i,j}^{n+1} = \phi_{i,j}^n + \lambda \mathcal{R}_{i,j}^n
\]

\[
\mathcal{R}_{i,j}^n = (\tilde{\phi}_{i,j}^n + \tilde{\phi}_{i+1,j}^n + \tilde{\phi}_{i-1,j}^n + \tilde{\phi}_{i,j+1}^n + \tilde{\phi}_{i,j-1}^n - 4 \phi_{i,j}^n) - \overline{\phi}_{i,j}
\]

where \( \lambda \) is the relaxation coefficients and \( \mathcal{R}_{i,j} \) is the remainder. This method is mathematically simple but convergence is slow.
A more efficient method of solving equation (22) is using Liebmann relaxation. In this method the values of $\Phi_{ij}^{n+1}$ are computed from $\Phi_{ij}^n$ and $\Phi_{ij}^n$ for a definite sequence of points. The values so corrected are used in all subsequent operations in the iteration steps. The sequence, in general, proceeds forward the right across the first row of points, then jumps to the first point on the second row and so on. Therefore, in this method, the remainder can be written as

$$R_{ij}^{n+1} = \Phi_{ij}^n + \Phi_{ij}^n + \Phi_{ij}^{n+1} + \Phi_{ij}^{n+1} - 4\Phi_{ij}^n - \Phi_{ij}^n \quad (25).$$

In these two methods, the relaxation coefficient $\alpha = \frac{1}{2}$ is used. Faster convergence was obtained using over-relaxation ($\alpha > \frac{1}{2}$).

Frankel (1950) determined the optimum value of the over-relaxation coefficient analytically. The iteration for the error $\epsilon_{ij} = \Phi_{ij}^n - \Phi_{ij}^n$ is obtained by substituting in equation (23) and (25) in the following brief form:

$$\epsilon_{ij}^{n+1} = K(\alpha) \epsilon_{ij}^n \quad (26)$$

where $K(\alpha)$ is a linear operator depending on the parameter $\alpha$.

He examined the spectrum of eigen values of $K(\alpha)$ and showed that the optimum value of $\alpha$ is that which minimize the maximum of $|K_{ij}|$.

The optimum value so determined is

$$\alpha_{opt} = \frac{1}{2} \left(1 + \sin \theta \right) \quad (27)$$

$$\cos \theta \equiv \frac{1}{2} \left( \cos \frac{n}{m} + \cos \frac{m}{n} \right)$$

where $n$ and $m$ are number of grid points per row and column respectively.
Using his optimum equation (27) in our grid system, $\alpha$ is determined to be approximately .463. However, the most suitable value for $\alpha$ can be determined empirically as Frankel himself stated in his note (1950). We used $\alpha = .46$ for solving equation (22).

Now we know how to solve the Poisson equation numerically if suitable boundary conditions are specified. The boundary condition used for the computation of $\psi$ are

$$\frac{\partial \psi}{\partial n} = V_s, \quad \frac{\partial x}{\partial s} = 0$$ \hspace{1cm} (28)

in the sense of inward differencing which minimize the kinetic energy of the velocity potential (King 1966). The finite difference form at boundaries for equation (18 b) are

$$\psi_{k,1} + 1 = \psi_{k,2} + \frac{d}{d} \left( U_{k,1} + U_{k,2} \right)$$
$$\psi_{k,3} + 1 = \psi_{k,3} + \frac{d}{d} \left( U_{k,3} + U_{k,35} \right)$$
$$\psi_{i,j} + 1 = \psi_{i,j} + \frac{d}{d} \left( V_{i,j} + V_{i,2} \right)$$
$$\psi_{k,i} + 1 = \psi_{k,i} + \frac{d}{d} \left( V_{k,i} + V_{k,i} \right)$$ \hspace{1cm} (29)

So the stream function can be determined without much difficulty from the equations (19), (22) and the boundary specification (29). However, for the numerical integration of the barotropic vorticity equation (17), there are further problems to be discussed.

D. Jacobian computation

In the barotropic vorticity equation (17) the integrals of the vorticity ($\nabla \times \nabla \times \psi$), the vorticity squared ($\nabla \times \nabla \times \psi$)$^2$ and the kinetic energy ($\nabla \psi^2$) are conserved in the region if we assume the tangential
gradient of \( \eta \) or \( \psi \) along the boundary line vanishes (Arakawa 1966), that is, their integrals over a fixed region in space can only be changed by transport through the boundaries of the region. The finite difference form of the Jacobian (equation 21) is compatible with this:

\[
\iint J^{++}(\eta, \psi) \, dx \, dy = \iint J(\eta, \psi) = 0
\]  
(30)

\[
J^{++}(\eta, \psi) \equiv \frac{m^2}{fd^2} \left[ (\eta_{i+j} - \eta_{i-j})(\psi_{j+i} - \psi_{j-i}) - (\eta_{i+j} - \eta_{i-j})(\psi_{j+i} - \psi_{j-i}) \right]. 
\]  
(31)

However, given that there is no flux through the boundary, it is unlikely that vorticity squared or the kinetic energy is conserved in the finite difference representation of equation (31), as pointed out by Lilly (1965) and Arakawa (1963):

\[
\iint \psi J^{++}(\eta, \psi) \, dx \, dy \neq \iint \psi J(\eta, \psi) \, dx \, dy = 0
\]  
(32)

\[
\iint \xi J^{++}(\eta, \psi) \, dx \, dy \neq \iint \xi J(\eta, \psi) \, dx \, dy = 0
\]  
(33)

\[
\frac{\partial}{\partial t} \iint (\psi \psi)^{1/2} \, dx \, dy \neq 0
\]  
(34)

\[
\frac{\partial}{\partial t} \iint \xi^2 \, dx \, dy \neq 0
\]  
(35)

Arakawa (1966) has developed a class of second-order finite-difference schemes which conserve the mean vorticity and both of the mean quadratic quantities, kinetic energy and squared vorticity, from the finite difference analogue of the Jacobian for a square grid:
\[ \mathcal{J}_1(\eta, \psi) = \frac{1}{3} \left[ \mathcal{J}^{++}(\eta, \psi) + \mathcal{J}^{+-}(\eta, \psi) + \mathcal{J}^{-+}(\eta, \psi) \right] \] (36)

where
\[ \mathcal{J}^{\pm \pm}(\eta, \psi) \equiv m^2 \left[ \eta \left( \psi_{x^2} - \psi_{y^2} \right) - \eta \left( \psi_{x} \psi_{y} - \psi_{y} \psi_{x} \right) \right] \] (37)

\[ \mathcal{J}^{\pm \mp}(\eta, \psi) \equiv m^2 \left[ \eta \left( \psi_{x} \psi_{y} - \psi_{y} \psi_{x} \right) - \eta \left( \psi_{x^2} + \psi_{y^2} \right) \right] \] (38)

\[ \mathcal{J}^{\mp \mp}(\eta, \psi) \equiv m^2 \left[ \eta \left( \psi_{x} \psi_{y} - \psi_{y} \psi_{x} \right) - \eta \left( \psi_{x^2} + \psi_{y^2} \right) \right] \] (39)

In this scheme the conservation is satisfied with the same boundary condition as equations (32) and (33):
\[ \int \int \mathcal{J}_1(\eta, \psi) \, dxdy = \int \int \mathcal{J}_2(\eta, \psi) \, dxdy = 0 \] (40)

Lilly (1965) also discussed the conservation property of the quadratic quantities by spectral analysis and rederived Arakawa's difference schemes (1963) systematically.

Arakawa (1966) further extended the Jacobian finite-difference scheme using additional grid points to improve the accuracy, without losing the computational stability, that is:
\[ \mathcal{J}(\eta, \psi) = 2 \mathcal{J}(\eta, \psi) - \mathcal{J}_2(\eta, \psi) \] (41)

where \( \mathcal{J}_1 \) is given by equation (36) and
\[ \mathcal{J}_2(\eta, \psi) \equiv \frac{1}{3} \left[ \mathcal{J}^{xx}(\eta, \psi) + \mathcal{J}^{xx}(\eta, \psi) + \mathcal{J}^{xx}(\eta, \psi) \right] \] (42)

\[ \mathcal{J}^{xx}(\eta, \psi) \equiv \frac{m^2}{\delta x^2} \left[ \eta \left( \psi_{x} \psi_{y} - \psi_{y} \psi_{x} \right) - \eta \left( \psi_{x^2} + \psi_{y^2} \right) \right] \] (43)

\[ \mathcal{J}^{xx}(\eta, \psi) \equiv \frac{m^2}{\delta y^2} \left[ \eta \left( \psi_{x} \psi_{y} - \psi_{y} \psi_{x} \right) - \eta \left( \psi_{x^2} + \psi_{y^2} \right) \right] \] (44)

\[ \mathcal{J}^{xx}(\eta, \psi) \equiv \frac{m^2}{\delta z^2} \left[ \eta \left( \psi_{x} \psi_{y} - \psi_{y} \psi_{x} \right) - \eta \left( \psi_{x^2} + \psi_{y^2} \right) \right] \]
To examine the truncation error, we have to consider the time truncation error and the space truncation error at the same time, since nothing is gained, as pointed out by Charney and Phillips (1953), if the truncation error from one independent variable is reduced, while the truncation error from the other variable remains larger. However, since the spatial truncation error of the Jacobian is known as the largest of all (Shuman and Vanderman 1965), we consider only this. Expanding $\eta$ and $\psi$ into Taylor series around the point $(i,j)$, the orders of magnitude of the Jacobians (equations 31, 36 and 40) are

$$J^+ (\eta, \psi) = J (\eta, \psi) + O(\alpha^2)$$  \hspace{1cm} \text{(45)}

$$J_1 (\eta, \psi) = J (\eta, \psi) + O(\alpha^2)$$  \hspace{1cm} \text{(46)}

$$J (\eta, \psi) = J (\eta, \psi) + O(\alpha^2)$$  \hspace{1cm} \text{(47)}.$$

We tested the three different Jacobian schemes (equations 31, 36 and 40) for our trial series and the result will be discussed later.

The aliasing error (Phillips, 1958) introduces spurious interactions which may cause unconditional computational instability in the ordinary Jacobian scheme. However, in the quadratic conserving scheme of Arakawa (equation 36), these spurious interactions are balanced in triads and do not cause computational instability. Arakawa's quadratic-conserving schemes of Jacobian computation are primarily concerned with the long-term integration in a closed system. Such conservation is not necessarily applicable to the short-range operational forecasts in our bounded region and
consequently, his no-flux boundary is not a good representation for the physical boundary in our model.

In order to reduce the spatial truncation error, we may apply high order centered differences (for example, 5 point or 7 point grids for each dimension) but, in addition, eliminating the aliasing error and preserving computational stability we selected the 13-point high-order Arakawa's Jacobian scheme (equation 40) for the interior grid points and the 9-point Jacobian (equation 36) at the computational boundary grid points.

Using the Jacobian computation (equations 36 and 40) and the boundary condition $\frac{\partial \psi}{\partial t} = 0$, we solved the Poisson equation (17) again by the extrapolated Liebman relaxation technique as described previously:

\[
\frac{\partial \psi}{\partial t} \psi_{ij} = \frac{\partial}{\partial x} \left( \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \right) + \alpha \left[ \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \right] - \frac{1}{4} \frac{\partial}{\partial x} \left( \mathcal{J}_i \eta \psi \right)
\]

\[
\frac{\partial \psi}{\partial t} \psi_{ij} = \frac{\partial}{\partial x} \left( \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \right) + \alpha \left[ \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \right] - \frac{1}{4} \frac{\partial}{\partial x} \left( \mathcal{J}_i \eta \psi \right) \eta \left( \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} \right)
\]

where $\nu$ is the iteration index of the relaxation passage and $\mathcal{J}$ and $\mathcal{J}_i$ are defined in equations (40 and 36). Here we chose the over-relaxation coefficient $\alpha = 0.30$ and the tolerance of convergence was restricted to \( |\frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y}| \leq 1 \times 10^{-6} \) at every grid point as already determined by King (1966). From the computed value of \( \frac{\partial \psi}{\partial x} \psi \) at every time step ($\tau$), we can obtain the new stream function at all interior points and using a forward finite difference.
for the initial step (equation 50) and thereafter centered time differences (equation 51):

\[
\psi_{ci,j} = \psi_{ci,j,0} + \frac{\delta \psi}{\delta t} \Delta t \quad (\tau = 0) \tag{50}
\]

\[
\psi_{ci,j}^{c+1} = \psi_{ci,j}^{c-1} + \frac{\delta \psi}{\delta t} \Delta t \quad (\tau > 0) \tag{51}
\]

The new relative vorticity \( \omega \) is obtained by:

\[
\Delta \omega_{ci,j} = \nabla^2 \omega_{ci,j} = \frac{m^2}{\Delta d^4} \left( \psi_{ci,j+1} + \psi_{ci,j-1} + \psi_{ci,j+1} + \psi_{ci,j-1} - 4 \psi_{ci,j} \right) \tag{52}
\]

at all interior points \( 2 \leq i \leq 48, \quad 2 \leq j \leq 38 \).

Since the boundary conditions are purely arbitrary and there is no way of knowing the exact value (in the region of our concern) for the inflow boundary, we prescribed \( \gamma = 0 \) and for the outflow boundary we adopted the following:

\[
j = 1; \quad \psi > 0 \Rightarrow \psi_{ci,j+1} = 0, \quad \psi \leq 0 \Rightarrow \psi_{ci,j-1} = 0 \Rightarrow \psi_{ci,j} = 2 \psi_{ci,j+1} - \psi_{ci,j-1} \tag{53-1}
\]

\[
j = 3; \quad \psi > 0 \Rightarrow \psi_{ci,j+1} = 0, \quad \psi \leq 0 \Rightarrow \psi_{ci,j-1} = 0 \Rightarrow \psi_{ci,j} = 2 \psi_{ci,j+1} - \psi_{ci,j-1} \tag{53-2}
\]

\[
i = 1; \quad \psi > 0 \Rightarrow \psi_{ci,j+1} = 0, \quad \psi \leq 0 \Rightarrow \psi_{ci,j-1} = 0 \Rightarrow \psi_{ci,j} = 2 \psi_{ci,j+1} - \psi_{ci,j-1} \tag{53-3}
\]

\[
i = 4; \quad \psi > 0 \Rightarrow \psi_{ci,j+1} = 0, \quad \psi \leq 0 \Rightarrow \psi_{ci,j-1} = 0 \Rightarrow \psi_{ci,j} = 2 \psi_{ci,j+1} - \psi_{ci,j-1} \tag{53-4}
\]

\[
\psi_{ci,1} = (\psi_{ci,2} + \psi_{ci,2})/2, \quad \psi_{ci,35} = (\psi_{ci,34} + \psi_{ci,35})/2 \tag{52-5}
\]

\[
\psi_{ci,35} = (\psi_{ci,34} + \psi_{ci,35})/2, \quad \psi_{ci,35} = (\psi_{ci,34} + \psi_{ci,35})/2
\]
For the stream function we extrapolated from the interior points, instead of fixing it for all time step, as follows:

\[ \psi_{i,j} - c = 2 \psi_{i,j} - \psi_{i,j} \]

\[ \psi_{i+1,j} = 2 \psi_{i,j+1} - \psi_{i+1,j} \]

\[ \psi_{i,j+1} = 2 \psi_{i+1,j} - \psi_{i,j} \]

\[ \psi_{i,j-1} = 2 \psi_{i+1,j} - \psi_{i,j} \]

\[ \psi_{i-1,j} = 2 \psi_{i+1,j} - \psi_{i,j} \]

We assumed the normal gradient of the stream function adjacent to the boundary to be the same as at the boundary. The new stream function and relative vorticity at the boundaries are used to compute the relative vorticity and Jacobian in the next time step.

In the solution of equation (50 and 51) the time increment \( (\Delta t) \) should be restricted by the computational stability criterion:

\[ \frac{d}{\Delta t} \geq \sqrt{m} \left| V \right|_{max} \]  

where \( \left| V \right|_{max} \) is the maximum particle speed in the forecast region and \( m \) is the map factor as previously defined. For our chosen grid distance \( d=165 \text{ km} \), maximum map distortion \( m=1.2 \) and particle speed \( V_{max}=50 \text{ m/s} \), the maximum allowable time interval is about 33 minutes. Since we chose \( \psi \) as the motion of history as in the equations (50 and 51), we have to determine a new \( \psi \) field for every time step. Accordingly, the stability criterion equation (55) should be satisfied for the ordinary two-dimensional advection equation. Therefore, we chose a forecasting time interval \( \Delta t \) of half an hour.
E. Smoothing and Filtering

Truncation errors arising in the course of numerical com-putation by finite differences become large in the region of small-scale disturbances and may amplify the high-frequency component in the final product beyond physical reality. Therefore, in the numerical prediction of the large-scale disturbances, it is desirable to filter out the small high-frequency components. Shuman (1957) developed a smoothing and filtering technique in the numerical method of weather prediction. First he defined the one-dimen-
sional smoothing element:

\[ \tilde{E}_{ij} = \Phi_{ij} + \frac{1}{2} k (\Phi_{i+1,j} - 2 \Phi_{i,j} + \Phi_{i-1,j}) \]  

(56)

where \( k \) is the smoothing element index.

Then he extended the above smoothing element into a two-dimensional operator by smoothing each dimension independently:

\[ \tilde{E}_{ij} = \tilde{E}_{ij} + \frac{1}{2} k (-k) (\tilde{E}_{i+1,j} + \tilde{E}_{i-1,j} + \tilde{E}_{i,j+1} + \tilde{E}_{i,j-1} - 4 \tilde{E}_{ij}) \]  

(57)

He introduced an alternative 5-point operator of two-dimensional smoothing:

\[ \tilde{E}_{ij} = \tilde{E}_{ij} + \frac{1}{2} k (\tilde{E}_{i+1,j} + \tilde{E}_{i,j+1} + \tilde{E}_{i,j-1} + \tilde{E}_{i-1,j}) \]  

(58)

However, the 9 point operator is used exclusively since the 5-point operator reduces only one-half of the one dimensional operator (56). Both the 5 and 9 point operators were used in the initial objective analysis. If one chooses a value of the filtering element so as to eliminate the high frequency components, the
low frequency ones are not represented very accurately. If one desires to represent the low frequency as well as possible, the high frequency components can not be eliminated, and so it is necessary to apply smoothing and unsmoothing procedures.

For our numerical integration we used the two-dimensional 9-point operator (equation 57) at the interior points and the 3-point one-dimensional operator (equation 56) at the boundary grid points. We chose indices of smoothing $k_1 = +0.5$ and unsmoothing $k_2 = -0.6$ in the first and second passes respectively. This choice of smoothing and unsmoothing operator almost completely filters out the short wave length components of less than 2 grid distances and at the same time the large scale components retains their original feature very closely (King 1966).

The vertical averaging process was done by the I.B.M. 7094 computer, and the numerical integration of the barotropic model and the objective scheme of the initial analysis were programmed for the I.B.M. 360-at the Computing Center of M.I.T.
V. DATA PREPARATION AND INITIAL ANALYSIS

An attempt was made to improve the initial averaged wind data by including all rawinsonde and mandatory level wind between 1,000 and 100 mb. To eliminate irregular terrain effects the 1,000 mb winds were represented by those at 500 m above the surface (see table 1) or, if missing, the nearest available above that. If a wind report were missing at some level the weighting factors at the adjacent levels above and below were increased by a fraction of the weighting factor at the missing level, according to the pressure interval between the adjacent and missing levels. And if wind were not reported beyond certain level, it was assumed that all the missing wind-data was the same as that at the last level present. All the missing level were counted to use as an aid in the initial analysis.

Hurricane Flora (Oct. 4, 5, 6, 1963) was chosen for multilevel averages and compared with the 10 levels which were already pre-analyzed by King (1966). There are slight differences in the averaging wind data—such as taking the wind speed to the first decimal place rather than truncating to an integer as done by King (1966) and using a slightly more accurate determination of the wind direction when nearly due east or west.

A. Subjective Analysis

After concluding that 10-levels was the optimum number of levels for averaging in the study of Flora (the appropriate figures and tables of the results were discussed in the next section),

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7 case studies of Hurricane Donna (1960) were made from Sept. 5th 00Z to 11th 00Z. Additional wind data at constant pressure levels over the Caribbean area were obtained from the Data Processing Division of Air Weather Service, Climatic Center of U.S. Weather Bureau, Asheville, N.C.

The analyzed constant-pressure charts (1,000 mb, 700 mb, 500 mb, and 200 mb) provided by the National Hurricane Research Laboratory were used as a guide in the analysis over the Atlantic ocean where it is almost impossible to get the 10-level averaged winds. For the Mexican area where only several pibal data are available, we used the subjective interpolation from the time cross section at several stations. As there were almost no observation in the SW and SE corner of our map, we used the climatological 500 mb winds for this period as an aid.

Still we have to suffer the lack of data right near the hurricane. Therefore, Prof. Sanders prepared its composite wind analysis from the aircraft reconnaissance data within a 300 n.m. radius from the center of the hurricane. For each composite analysis, 48 hour periods were chosen, assuming only moderate temporal and spatial changes near the hurricane. A vertical cross section of relative wind speeds of Hurricane Hilda (Oct.1,1964) prepared by N.H.R.L. was used as an aid to isotach analysis in the radial distance from the hurricane eye.

Using all these available information together with vertically averaged 10-level winds, a subjective analysis was made with isogon
Fig. 2: The 10-level averaged winds and heights (Sept. 9th, 00Z, 1960), speeds in m/s and heights in m.
Fig. 3; The isogon (solid lines) and isotach (dashed lines) analysis for the initial data at each grid of fig. 2 (Sept. 9th, 00Z, 1960, 10 levels)
at every 30° and isotachs at every 5 m/s and intermediate speeds as shown in fig. 3. From this subjective wind analysis, again the subjective reading of wind direction and speed at each grid point were carried out for using the machine computation as the initial data. Up to this point, the individual errors were inevitable due to too much subjectiveness. Forecasting results with the same set of data were demonstrated by 5 different teams in the laboratory of 19.45T and dispersion becomes significant after 36 hours as shown in fig. 4. Therefore, as a more desirable method an objective initial analysis was attempted.
B. **Objective Analysis**

Objective analysis can be described as a systematic method transforming a mass of irregularly spaced observational data into at a pre-arranged network of points with minimum manual subjectiveness. With the advent of the computer, objective analysis of meteorological fields have been obtained through the use of statistical, polynomial fitting and successive approximation approaches. The method of fitting some kind of polynomial to the data by least squares was suggested by Panofsky (1949), later modified by Gilchrist and Cressman (1954), Bushby and Huckle (1957) and Corby (1961). This least-square method did not prove overly successful in areas of sparse data.

Therefore, we are mainly concerned with the method of successive approximations, or the correction method, proposed by Berthorssen and Doors (1955) and modified by Cressman (1959). This method consists essentially of using the reported data to make successive corrections to an initial guess which should approximate the final field as accurately as possible. The first guess we have chosen in our scheme is the previous day 12 hour forecast stream field.

The analysis is then performed in a series of "scans" i.e. successive passes through the grid points in the entire region. Since we use only the wind as our information, the correction can be obtained by Taylor series expansion of the initial stream function at the grid points around the observation points:

\[
C_v = W_f \left[ \frac{1}{h} \left( v_{\Delta x} - u \Delta y \right) - \frac{v}{\Delta y} \right]
\]  

(59)
where \( C_v \) is the correction to be added to the initial guess, \( \psi_o \) and \( \psi_g \) are the stream functions of the first guess at the observation point and the grid point, respectively. \( W_f \) is a weighting factor given by Cressman (1959):

\[
W_f = \frac{N^2 - q^2}{N^2 + q^2}
\]

(60)

where \( q \) is the distance between the grid point and the observation, and \( N \) is the scan radius at which \( W_f \) goes to zero. The curve of weighting function \( W_f \) vs. Distance \( q \) is shown in figure (5).

Fig. 5: Weighting function \( W_f \) vs. Distance \( q \). Dashed line refers to \( W_f \) on 4th scan (=1).

It is seen that the weighting factor is larger closer to the grid points. In the last scan (fourth), \( W_f = 1.0 \) is used for a better horizontal resolution of the smaller scale components as found by Cressman (1959) in the 500-mb contour field.

In the equation (59), \( \psi_o \) is obtained by linear interpolation of \( \psi_g \) between the surrounding four grid points since only the \( \psi_g \) field is given as an initial guess. The scan radius \( N \) for successive scans is given in table (2) as suggested by Cressman (1959).
scan No.       \( N \) (grid lengths)
1             4.75
2             3.60
3             2.20
4             1.80

Table 2: scan radius for each scan.

In the areas of relatively dense and redundant data, the above scheme may be reasonable. But near the hurricane eye, almost no vertically averaged winds can be obtained, therefore it is desirable to use some arbitrary hurricane with the easily obtainable hurricane parameter. Vanderman (1962) defined the wind speeds in the hurricane vortex from the concept of gradient wind as follows:

\[
\begin{align*}
V_r &= \frac{\partial \psi}{\partial r} = C_1 \frac{r}{R^*}, & 0 \leq r \leq R^* \\
V_2 &= \frac{\partial \psi}{\partial r} = C_2 \frac{r^{-5/8}}{R^*}, & R^* \leq r \leq R
\end{align*}
\]

where \( r \) is the distance from the vortex center, \( R^* \) is the observed eye radius and \( R \) is the outside mean radius of the hurricane circulation.

Integrating equations (61-1 and 61-2) with respect to radial distance from the center, assigning the maximum wind speed \( V_{\text{max}} \) at the edge of the vortex eye, and requiring the vortex stream function to be zero at the outside radius \( R \), the stream functions are obtained in following form:

\[
\begin{align*}
\psi_r &= \frac{V_{\text{max}}}{2R^*} \left[ r^2 - R^{*2} \right] + \frac{C_1}{2} \left( R^* \right)^2, & 0 \leq r \leq R^* \\
\psi_2 &= \frac{C_2}{3} \frac{V_{\text{max}}}{R^*} \left[ r^{5/8} - R^{3/8} \right], & R^* \leq r \leq R
\end{align*}
\]
where $V_{\text{max}}$ is the maximum velocity, $\psi_1$ and $\psi_2$ are vortex stream functions inside and outside the eye respectively, continuity is enforced by putting $\psi_1 = \psi_2$ for $r = R^*$. 

Assuming that the exact location of the hurricane center, the maximum wind speed, the radii of the eye and the edge of the vortex are easily obtainable by some other means for example by reconnaissance flight, the above vortex stream function is computed to add to the general stream field such that the initial vortex stream is eliminated from the 12 hours forecast stream field-the initial guess.

The 5 point smoothing operator equation (58) was applied ($k = 0.5$) once between scans 2 and 3 and once between scans 3 and 4 to reduce the discontinuity, and 9-point smoothing operator equation (57) was applied at every scan to eliminate the high frequency components of wave length of 2 grid distances and to retain the longer waves.
VI. RESULTS AND DISCUSSION

A. 10-level and Multi-level

Having found some systematic differences of wind speed between the multi-level and 10-level, although of small magnitude, less than $\pm 1$ m/s except for two extreme stations where it was $\pm 1.5$ m/s as shown in figure (6), a forecast was made with the multi-level speed while retaining the 10-level wind direction using King's original Computer program with some change of smoothing process. The positions of maximum absolute vorticity in the hurricane region were tracked as the forecast position of hurricane center because the positions of minimum stream function values were difficult to locate, quite often it vanishes after 36 hour forecast.

The forecasting results from the speed modified 10-level does not differ much from the complete 10-level forecast as shown in figure (7), moving parallel to each other with the former placed to the left of the latter. The maximum difference of predicted positions during the course of the 72 hour forecast is 20 n.m. Since King's 10-level averaged speed was printed out to the nearest 1 m/s and the author's multi-level wind speed was printed out to the nearest .1 m/s, it is difficult to detect any significant difference initially.

To compare the 10-level and multi-level more rigorously, three more cases of Flora (Oct. 4, 5, 6, 00Z, 1963) were carried out with complete analysis of wind speed and direction.
Fig. 6: The speed difference between 10 and multi-level.

Fig. 7: The forecast result of 10-level and speed modified multi-level (Oct 4, 00Z, 1963)
The forecast results (as shown in fig. 8, 9, 10) of both 10-level and multi-level show the systematic rightward bias for a forecast of more than 36 hours, but the multi-level tends to show
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**Average** E_d(m) 29.3

**Table 3:** Multi-level and 10 level, P_0; Observed displacement, P_f; Predicted displacement, E_d; Displaced vector error, R_f; The ratio of the predicted to observed displacement, R_e; The ratio of the magnitude of the vector error to the observed displacement, subscript m and 10 denote multi and 10 levels respectively, values within the parenthesis denote 12 hours displacement.

However, as shown in the table (3), the magnitude of the error vector $\mathbf{E}_d$ for the multi-level is, in general, greater than that for the 10-level. Both 10 level and multi-level move faster than the observed tracks for the case of Oct. 4, but the multi-level moves faster than the 10-level does. For the case of Oct. 5 and 6, both the 10-level and multi-level move slower than the observed tracks except for the 60 hours and 72 hours forecast of Oct. 5.

Considering the overall results, the multi-level does not give any better forecast than 10-level does. In fact, it is even worse as far as the acceleration of the hurricane movement is concerned. This particular hurricane Flora (1963) was rather weak and it seems to me that for slowly moving weak hurricane, the inclusion of more high level wind information as in the multi-level case does not necessarily give better steering. Also it has the disadvantages of not using the averaged height information as a guide in the initial analysis.
B. Trial Series

In an attempt to improve the hurricane forecast a series of trial experiments was made with the same initial analysis (Sept. 9th, 00Z, 1960 Donna);

**Trial (1);** refers to the version of King's (1966) original computer program (FAP, 7094) in which an error in the smoothing process is corrected. He used a constant grid distance in the region, and 2 rows and columns at each boundary were excluded from the forecast.

**Trial (2);** refers to the inclusion of a map distortion factor giving a grid distance varying with latitude. Hereafter, a map factor is included in all succeeding trial series and \( \psi \) values of every 12 hour (even time step) and of preceding time step (odd) are smoothed.

**Trial (3);** refers to Arakawa's Jacobian (equation 36) employing a condition of no flux at the boundary \( \frac{\partial \psi}{\partial z} = 0 \).

**Trial (4);** refers to the Arakawa 9-point Jacobian computation (equation 36) and here, after extending the relative vorticity values at the outer boundary linearly from the interior grids, the forecast field included one more row and column.

**Trial (5);** refers to the application of Arakawa's 13-point high order Jacobian (equation 40) at the interior points and the 9-point Jacobian at the computation boundaries. The boundary specification is the same as in trial (4).
Trial (6); refers to the subtraction of the average of Jacobian over the entire grids from Arakawa's high order Jacobian (equation 40) at each grid point for every time step before solving for \( \frac{\partial \psi}{\partial t} \), so that the average change of vorticity \( \frac{\partial \psi}{\partial t} = 0 \) becomes zero all the times. Here the boundary conditions on \( \psi \) and \( f \) were specified by equations (53 and 54). The absolute vorticity values \( \eta \) were smoothed every 6 hours besides smoothing \( \psi \) every 12 hours to reduce any instability developing at the boundary.

The series of forecasting results is shown in figure (11). Trial (2) gives decreasing values of the gradient of stream function at the lower latitudes which are far from the standard latitude, and the forecast results show little improvement from Trial (1) at 60 and 72 hour (see figure 11 and table 4). However, Trial (3), which was originally intended by Arakawa (1963) for long-term integration over a closed system gives the worst result due to an unrealistic boundary condition. Here we are concerned with short period forecasting over bounded region, and the specification of zero flux through the boundaries is inapplicable.

As far as the forecasting of this particular hurricane is concerned, Trial (4) gives the best results as shown in figure 11 and table 4; the discrepancy with observation being less than 150 n.m. even at 72 hours. But for 60 and 72 hours, the forecast movement shows deceleration which is quite opposed to the observed motion, and the average kinetic energy at 72 hours shows an increase of 66% over
Fig. 11: Forecast results of Trial series (Sep. 9th, 00Z, 1960)
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Table 4: Forecast results of trial series. ( ) subscripts denote the Trial series (Sept. 9th, 00Z, 1960), P₀: Observed displacement, Pᶠ: Predicted displacement, Eᵈ: Displaced vector error, Rᶠ: The ratio of the predicted to observed displacement, Rᵉ: The ratio of the magnitude of the vector error to the observed displacement.
Table 5: Average Kinetic Energy, units = ( ) x 10^4 erg/g, the values inside the parenthesis are the percentage increment from the initial 00 hr values.

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Table 6: Average absolute vorticity and vorticity squared, units of absolute vorticity squared = ( ) x 10^{-10} sec^{-2}, units of absolute vorticity = ( ) x 10^{-6} sec^{-1}.

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the initial values. The increase of K.E. might be due to the flux flowing through the boundary. The maximum vorticity values near the hurricane region was truncated to 43.3% as shown in table (7).

The high order Jacobian scheme (Trial 5) forecasts systematically faster hurricane movement than the observed tracks as shown by $R_f(5)$ in the table 4. The maximum vorticity values was truncated to only 35.1% which is minimum in the series. Kinetic energy increased less rapidly than the case of Trial 4 did, but still not to our satisfaction.
Finally the Trial (6) gives more realistic curvature although its forecast has a leftward bias as most other Trial do. The only real improvement is seen for the 72 hours forecast. There is a tendency for acceleration with forecasting time as is observed in the actual motion. The average kinetic energy had increased by only 10.7% at the 72 hour forecast. Therefore the average Jacobian controls the increment of the average kinetic energy fairly well. The intensity of the hurricane is also relatively well preserved as shown in table 7.

Most of the above schemes maintain the average vorticity and the averaged squared vorticity almost constant. This is not really so in the case of Trial (5), may be owing to the improper boundary condition involving extrapolation of relative vorticity values from the interior points, regardless of inflow and outflow boundary. Considering the forecast results and the physical importance of above meteorological and mathematical parameters, we decided to use Trial (6) as our numerical experiments of another hurricane cases.

C. Hurricane Donna (1960)

Using Trial scheme (6), 7 forecasts were made of Donna (1960) from Sept. 5th to 11th (002). Hurricane Donna originated near the Cape Verde Island, off the west coast of Africa, as did many of the notable hurricanes to hit Florida, and passed through the east coast of U.S. She was the major hurricane of the season and the most destructive ever to strike the east coast states. Donna's movement during
the first few days was towards the westnorthwest at about 17 knots, somewhat above the climatological mean speed. A well established high pressure area to the north prevented recurvature and the hurricane rapidly increased in size and intensity. On Sept. 9 Donna skirted the northeast coast of Cuba, then headed toward the Florida Keys. For the chosen period of the numerical experiment, the hurricane was already in the mature and intensifying stage (Butson 1960).

Each individual forecasting results is drawn in figure 12 through 18, and the summaries of every 12 hour forecast were drawn in the polar diagram as shown in figure 19 through 21, the observed displacement ($P_o$), the predicted displacement of maximum vorticity ($P_{fv}$) and of the minimum stream function ($P_{fs}$), the magnitude of error vector of the predicted displacement of maximum vorticity ($E_{dv}$) and the minimum stream function ($E_{ds}$), and the ratio of $P_{fv}$ and $E_{dv}$ to the $P_o$ are given in table 8.

There were two extremely bad forecasts, namely the first two days (Sept 5 and 6), and two extremely good forecasts (Sept 7 and 11). The two bad forecasts may be due to poor initial analysis. There was severe lack of data in the regions surrounding the hurricane and almost no information over the North Atlantic ocean on these days, therefore the initial analysis is purely arbitrary. The forecast positions of minimum stream function were, in general, located to the left of the maximum vorticity position, due to the fact that the well established anticyclone is located toward the right of the hurricane movement throughout the forecasting periods.
Fig. 15; Forecast of Donna (Sep 8, 00Z, 1960)

Fig. 16; Forecast of Donna (Sep 9, 00Z, 1960)

Fig. 17; Forecast of Donna (Sep 10, 00Z, 1960)
Therefore the extreme rightward bias for the first two cases can be reduced to a certain extent, but basically the most significant error comes from the wrong choice of the arbitrary position of the anticyclone to the right of the hurricane movement. It seems to me that the large high should be located due north rather than split into two or one high to the northeast on Sept 5 and 6 respectively.

In general, the position of the maximum vorticity tracks gave better results, particularly in the early part of the forecast. Very good agreement with observation was obtained for the case of Sep 7, the displacement error of the 72 hour forecast being only 30 n.m. On the date of 11th, the forecast position after 48 hours was difficult to locate because of the immersion of the vortex into the big trough, but the acceleration given by $R_{fv}$ in the table (8) was very good.

The average 72 hour displacement errors of maximum vorticity
Fig. 19: Displacement errors of maximum vorticity and minimum stream functions from the observed displacement of Donna (1960) at 12 and 24 hours.
Fig. 20: Forecast displacement errors of maximum vorticity and minimum stream functions from the observed displacement of Donna (1960) at 36 and 48 hours.
Fig. 21: Forecast displacement errors of maximum vorticity and minimum stream functions from the observed displacement of Donna (1960) at 60 and 72 hours.
<table>
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Table 8: Forecast results of Donna series (Sep 5, 6, 7, 1960)
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Table S (cont.); Forecast results of Donna series (Sep 8, 9, 10, 1960)
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<td>140</td>
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<td>1172</td>
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<td>n.m.</td>
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<td>365</td>
<td>670</td>
<td>1055</td>
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<td>$P_{fs}$</td>
<td>180</td>
<td>460</td>
<td>735</td>
<td>1140</td>
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<td>50</td>
<td>95</td>
<td>70</td>
<td>170</td>
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<td>$E_{ds}$</td>
<td>25</td>
<td>80</td>
<td>165</td>
<td>90</td>
<td>100</td>
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<td>0.954</td>
<td>0.900</td>
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<td>$R_e$</td>
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<td>0.275</td>
<td>0.100</td>
<td>0.145</td>
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</table>

| $E_{dv}$ | 37.3  | 74.6  | 135.3 | 245.6 | 359.2 | 427.5 n.m. |
| $E_{ds}$ | 40.6  | 71.7  | 143.9 | 237.6 | 366.7 | 345.0*    |
| $E_{dv1}$ | 29.6  | 57.6  | 97.4  | 172.8 | 211.3 | 215.0    |
| $E_{ds1}$ | 34.4  | 66.0  | 118.0 | 172.0 | 232.5 | 191.3    |

Table 8 (cont.): Forecast results of Donna (Sep 11, 1960) and averages, $P_o$; the observed displacement, $P_{fv}$ and $P_{fs}$; the predicted displacement of maximum vorticity and minimum stream function, $E_{dv}$ & $E_{ds}$; the predicted vector error of max vorticity and min stream, $R_f$ & $R_e$; the ratio of $P_{fv}$ and $E_{dv}$ to $P_o$ respectively, wiggles denote average and suffix 1 denote average excluding first two, * denotes that data of Sep 5 was not included for averaging.

and of the minimum stream function positions were 427.5 n.m. and 345.0 n.m. or excluding the first two days only 215 n.m. and 191 n.m. respectively.

All forecasts except those of 7th and 11th moved faster than the actual hurricane. The first three cases were forecast to the right of the hurricane path and the remaining four cases to the left. Therefore, any systematic left or rightward bias was not found in this experiment as more clearly shown in figure 19 through 20.
D. Results of Objective Initial Analysis and Forecast

The technique of objective initial analysis described previously was tested for the case of Donna (Sep 9, 00Z, 1960). The 12 hour forecast of the previous day (Sep 8, 00Z) stream function values was used as the initial guess (see figure 22). In the region of hurricane Donna, the maximum wind speed, the radii of hurricane eye and vortex were chosen to be 40 m/s, 25 km, 350 km respectively.

The results of the objective initial stream function analysis on 3rd and 4th scan were shown in figure 23 and 24 respectively. The root-mean square difference (RMSD) between the objective and subjective stream function (figure 25) analysis was computed over the entire grid and over grids restricted to the continental areas for the successive scans.

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<th>3</th>
<th>4</th>
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<td>19.135</td>
<td>19.217</td>
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<td>RMSD(c)</td>
<td>18.616</td>
<td>18.673</td>
<td>18.586</td>
<td>18.775</td>
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</table>

Table 9: Root-mean square difference of objective analysis from the subjective initial analysis, subscripts (e) & (c) denote RMSD of the entire grid and of the continental region.

As shown in the table 9, it was found that the RMSD was least on the third scan. It is of special interest to note that on the fourth scan, where a fixed weighting factor was used, the RMSD gave the highest values over the continental area, contrary to the suggestion of Cressman from his contour analysis (1959). This may be due to the fact that the equal weighting of all observations, regard-
Fig. 22: The initial guess (12 hour forecast of Sep 8, 00Z). Note that the vortex stream was eliminated.

Fig. 23: Objective initial analysis of stream functions after 3rd scan (Sep 9, 00Z, 1960).
Fig. 24; Objective initial analysis of stream functions after 4th scan (Sep 9, 00Z, 1960).

Fig. 25; The computed 00 hour stream function from subjective isogons and isotachs analysis (Sep 9, 00Z, 1960)
less of distance between observation and grid point may yield good results in Cressman's case using both wind and height observations but not in the present case using wind data alone.

In both cases of the initial analysis of 3rd and 4th scan, the non-divergent feature of stream function appeared well over the region of dense observations, but there were no change in the initial guess over the Atlantic ocean and the Mexican area (compare the figures 23, 24 and 22). On the 4th scan, the small scale disturbances appeared at several points and the anticyclone magnitude were also increased a little.
The forecast results of the objective initial analysis of 3rd and 4th scan are shown in the figures 26 and 27. Neither forecast using objective initial analysis gave any better results than did those using the subjective initial analysis. The forecast displacement using the objective initial analysis did not curve or accelerate realistically as shown in figures 26 and 27. The main reason for this is due to the fact that the anticyclone of the initial guess over the area of no observations to the left of Donna still exists in the product of the objective analysis, so that the forecast resembles the previous day's forecast (see figures 15, 26 and 27).

Therefore, objective initial analysis can not guarantee any better forecasting unless it is operated in the region of heavily populated observations.

VII. CONCLUDING REMARKS

The non-divergent barotropic model is, in general, acceptable for the prediction of hurricane movement as shown in the numerical experiments. The inclusion of up to 10 levels gives a good representation of the steering level. For the moderate hurricane, there is no advantage in the inclusion of further levels, since the difference in results is within the limitations of errors of initial analysis.
If the hurricane is treated as an integral part of the large scale flow, the fine grid and the high order differencing scheme are necessary to reduce the excessive truncation error in the region of the hurricane. Arakawa's Jacobian scheme for the numerical integration in the non-divergent barotropic model controls the mean vorticity and the associated quadratic quantity, but does not moderate the increase of kinetic energy to our satisfaction, perhaps due to the inconsistency at the boundary. The operation of mean Jacobian, in addition to Arakawa's high order Jacobian scheme, may be useful to meet both the requirements of reduction of truncation error and of conservation of quadratic quantities in the bounded region.

The forecast results of the trial cases of Donna were fairly satisfactory close to the continent but the first two cases gave rather poor results due to the incorrect initial analysis over the Atlantic ocean.

The method of objective initial analysis is rather satisfactory in the region of dense observation but should be devised more carefully in the region of sparse observation, otherwise it is nothing but the initial guess field. The choice of a carefully analysed 24 hour forecast from the previous day may be better for the initial guess than the present choice of 12 hour previous day forecast.
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