ANALYTICAL MODELS IN RAIL TRANSPORTATION:
AN ANNOTATED BIBLIOGRAPHY

by

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Introduction

Railroads in the United States have been facing fierce competition in the area of freight transportation during the past two decades. Their steadily falling market share in transport and major reorganizations provide a renewed stimulus to effect rationalized planning systems as well as more efficient rail operations. The use of analytical planning methods in conjunction with powerful information systems holds substantial promise in this direction.

This bibliography aims to collect the literature on analytical modelling of rail systems. Its scope covers the applications of Operations Research methods to rail. This has been a somewhat neglected area of research due to both the intimidating complexity of rail operations and the difficulty of establishing an interface between rail management and modellers with little specific knowledge of rail operations. The literature on rail modelling is rather scattered and mostly internal in nature. As a result few practitioners of Operations Research are sufficiently acquainted with issues of concern to rail operating management. This report attempts to provide an overview of the work already performed in this area.

Our direction in compiling this bibliography has been to cite most of the accessible references on analytical modelling. We have not chosen to be encyclopedic and have not cited internal documents (the reader can consult the AAR Applications Digest [AA1] in this latter regard). We acknowledge our bias in favor of optimization models as opposed to simulation studies. However,
due to the large role of simulation in rail modelling, we have included a number of references on simulation aiming to be representative rather than comprehensive. In all cases we have opted for references which reveal the logical or mathematical structure of a given problem in contrast to purely descriptive accounts. We have rarely cited papers dating before 1960. Finally our primary interest was freight and not passenger transport.

The classical reference on rail freight transportation is Beckmann et al. [BE1] whose description of rail operations from the optimization point of view has motivated our compilation effort. A few other references providing what we deem useful institutional knowledge to a modeller are also cited. We have included a number of survey articles on the applicability of mathematical methods to railways.

As a guide to this bibliography, we provide the following directory of references organized according to subject. The purpose of this directory is to highlight general areas of concentration in the bibliography. Obviously the categories are neither exhaustive nor mutually exclusive. The modelling of rail operations - routing, scheduling, hauling, classification yard activities, grouping, blocking, and makeup as well as their interactions - form our main focus. We have not included references on the interfaces of rail systems with other transportation modes. Similarly the economics of rail transportation and the general area of econometric modelling and forecasting lie beyond the scope of this report, as are the areas of car utilization and rail/truck competition and modal choice. For these omissions we refer the reader to the papers [RA2], [CT1], [KU1].

Notwithstanding, we hope that in the absence of any other compilation of a similar scope, our bibliography will be of use to both the rail planner and operations research communities.
Subject Directory to References

I. General and Institutional Background
   a) Descriptions of the rail system and its major issues:
      [BE1], [RA1], [RA2], [SE1], [TR1], [TS1].
   b) Costing in Railways:
      [KN1], [PO1], [ST1].
   c) Reports of modelling systems in particular railroads:
      [AA1], [GO1], [GR1],[K04], [RE2], [SA4], [TR3],[UR1],
      [WI5], [YA1].

II. Resource Acquisition and Facilities Installment
    [GU1], [KO1], [LE1], [MA1], [MA5], [PI1],[SO1].

III. Network and Yard Models
   a) Network Models based on Optimization:
      [FU1], [PE7-10], [SC1], [TH1], [TH2].
   b) Simulation Network Models:
      [AL2-5], [BE2], [GU1], [K03],[RS1], [SR1],
      [TS1], [WI4].
   c) Simulation models for yards:
      [CR2], [EB1], [NA1], [RS1], [SH1-2], [S01].
   d) Queueing models for yards:
      [BR1-2], [CR2], [FO4], [HE3-4], [KL1-2], [MI1],
      [PE1-2], [PE5].

IV. Car and Engine Movement
   a) Engine Scheduling - Allocation of motive power:
      [BA1], [FL1], [GL1], [HO2], [MC1], [SU1], [TO1].
b) Empty wagon distribution:
   [GO2-3], [HE5-6], [LE2], [MI2], [TR4], [WH1], [WY1].

c) Freight car movement:
   [GO2], [KO2], [MA3].

V. Planning Train Schedules and Timetables

a) Passenger train scheduling - Suburban transit:
   [CA1], [EI1], [IR1], [MO1], [NE1], [SA1], [SA2-3],
   [VU1], [YO1].

b) Freight train scheduling (longhaul and by-pass trains):
   [AC1], [CH1], [SU3], [TR2], [WI1].

c) Train formation and makeup:
   [BO2], [DU1], [KO4], [MA3].

d) Timetables on single track lines:
   [AM1], [CE1], [FR1], [MO1], [SZ1], [YO1].

VI. Other Topics

a) Reliability studies:
   [BE3], [CT1], [FO1], [KE1], [LA1-2], [MA4], [RE1],
   [SU1], [SU2].

b) Freight car stocking and allocation:
   [AD1], [AV1], [AL6], [JO1].

c) Analysis of track capacity:
   [HE1], [MI1], [PE3], [PE9], Chapters 2-4.

d) Simulation of over-the-line movement:
   [DR1], [WI3], [WI5].
[A11] (ASSOCIATION OF AMERICAN RAILROADS); Committee on Analytical Techniques: Applications Digest, Data Systems Division, prepared by the Subcommittee for Information Exchange (June 1969 revised Jan 1974).

contains one-page abstracts of the analytical models used by American and Canadian railroads and their addresses for contact purposes. Is revised periodically by AAR.


formulates a model to determine the number and composition of long haul trains on a rail network. The first phase schedules trains wherever there is enough traffic to warrant this and a second phase allocates trains for the remaining traffic. The resulting 0-1 integer program is solved by Branch-and-Bound and applied to an example involving the entire Swiss Rail network.

[AD1] ADL (Arthur D. Little); 'Economic Impact of Freight Car Shortages.' Arthur D. Little, Cambridge, Mass. Final Report prepared for Department of Transportation (May 1971)

estimates the economic impact of freight car shortages on the lumber, plywood, and grain industries in 1968. Develops a costing methodology for freight shortages and delays.


considers the problem of yard design in the light of the effects of overall traffic pattern. Transit time statistics of various shipping (routing) options, ranging from direct to point-to-point forwarding, are compared for best results.


describes an n-day simulation of a network with N yards. The network topology, train schedules, freight demand and yard policies are inputs as well as cost information (hauling, switching, etc.). The model outputs progress notices for the traffic plus cost and utilization statistics. Applies the model to a network with 20 yards and 85 regularly scheduled trains over a 10-day period.


[AL6] ALLMAN, W.P.; 'An Optimization Approach to Freight Car Allocation under Time-Mileage Per Diem Rental Rates.' Management Science, 18, 10, 567-574 (June 1974) uses linear programming to choose the optimal mix of cars for shipment so as to maximize the rental fees receivable-payable. The result is an assignment model for the allocation of cars to orders. Discusses an aggregation technique aggregating cars into order types to decrease the model size.

[AL1] AMIT, I. and D. Goldfarb; 'The Time-Table Problem for Railways.' pp. 379-387 in Developments in Operations Research, vol. 2, Gordon and Breach, New York (1971) (edited by B. Avi-Itzhak) formulates a model for a single track line of n sections (n+1 stations) and a given number of trains. Determining a timetable involves setting arrival/departure times of trains (moving in both directions) at stations. The objective is to minimize the weighted sum of passage times subject to track availability and delay (max. waiting time) constraints. Proposes a heuristic to solve the resulting large nonlinear program.

[AV1] AVI-ITZHAK, B., B.A. Benn, and B.A. Powell; 'Car Pool Systems in Railroad Transportation: Mathematical Models.' Management Science, 13, 9, 694-711 (May 1967) finds the number of cars required in a car-pool on the basis of the distribution of the number of busy units for two basic models: 1) large number of consignees place small orders randomly, and 2) shipper has only one consignee with bulk orders. Above two models are then combined to account for the case of a few consignees with bulk orders and a remaining class of consignees with small orders.
provides a method for the allocation of transport units (engines) to departing trains so that the minimum fleet size is obtained. The method essentially involves a pairing of arrivals and departures of engines at given yards.


provides what has become a classical account of rail operations and freight transportation organized as follows: Time element in transportation -- Freight Operations and classification policy -- Classification yards and the division of sorting work among them -- Scheduling trains to minimize accumulation delay -- Short-haul routing of empty boxcars. The division of sorting work and minimization of accumulation delay are posed as optimization problems and solutions are provided.

BELLMAN, J.A.; 'Railroad Network Model.' pp. 148-154 of [PR2].

describes the Frisco network simulation model developed for San Francisco Railway. Traffic data, train schedules, yard grouping policies and train take-lists, as well as cost information, are inputs to the model. The model simulates the movements of 51 trains on a 25 node network.


studies the reliability of line haul movements, i.e., excluding delays at intermediate yard stops. Four classes of delays are specified. Analysis of data suggests that running and departure time variances as well as intermediate yard stop variance have greater effects on reliability than operating and mechanical failure delays. On this basis recommendations for reducing unreliability are made.


suggests a method for sending the maximal flow of trains through a capacitated network of railway links. The method involves a "flooding technique" which was a precursor of the Max-Flow algorithm.

provides a theoretical description of a practical method for train formation used in French railways since 1932. The problem involves finding a procedure for moving cuts from classification onto assembly tracks.


applies a queueing model to the reception sidings of a yard. Trains arrive according to a Poisson distribution and the service time is assumed constant. The occupancy distribution of the sidings is computed and used as a basis for determining the required number.


is a follow-up to [KL2] computing the capacity of a siding when the cars undergo an additional fixed delay in addition to the fixed collection cycle time. The capacity of a set of such sidings is discussed as well as optimal departure times for the trains (whose frequencies are determined by the collection cycle).


describes a computer simulation model of a rail transportation system capable of dealing with train schedules, vehicle size, speed and turnaround time and right-of-way or headway restrictions. The model provides statistics on car utilization, load factors, train sizes, and passenger waiting times.

[CE1] ČERNY, J. and R. Vašíček; 'The GOP-1 Method and its Use in the Time-Table Preparation.' Rail International, 9, 2, 97-103 (Feb. 1977)

describes a probabilistic method of allocating time reserve (leeway) for a train passing through a series of stations with random interstation running times. Contrasts this method to the current practice of adding reserves times proportional to the running times for a Czechoslovak express train.

adopts a crew formulation (rather than a car formulation) for the movement of traffic in a terminal switching railroad. An integer linear program determines the minimum cost allocation of crew to a given set of feasible routes. The output is the number of crew-engine packages assigned to each leg of the routes.


studies the problem of setting prices and providing service on an itinerary of a transport network. Assumes two firms are in competition for clients and the demand varies according to a known distribution whose parameters depend on price. Considers choices for the objective function (maximize expected profit, the probability of profit being greater than a given value, and the difference in the two firms' revenues) in a chance-constrained formulation of the problem.


analyzes yard operations to derive performance curves of man-days of labor required per day as a function of the traffic level at the yard for different crew-engine resource levels. The minimum cost crew-engine allocation can then be found for different traffic levels. Was an actual study carried out for the Western Maryland Railway Company.

CRANE, R.R.; 'Some Recent Developments in Transportation Research.' Naval Research Logistics Quarterly, 4, 3, 173-179 (Sep. 1957)

contains an account of the early work done in analytical modelling of rail systems.


uses a Monte Carlo simulation of a classification yard modelled as two queueing systems in series corresponding to inspection and classification activities. Also investigates the effect of feedback mechanisms (such as the addition of inspection crew once the queue size becomes large) in the delay distribution.
[CT1] CTS-MIT; 'Railroad Reliability and Freight Car Utilization: An Introduction.' The Industry Task Force on Reliability Studies and the Center for Transportation Studies, MIT. Report No. CTS 75-8 (July 1975) describes a research program on car utilization presented to AAR. The previous research on reliability performed at MIT is brought to bear on the utilization issue. Proposes a methodology for making experiments and provides the example [SU1]. The appendix contains a discussion of rail freight service models.

[DR1] DRUCKER, R.W., B.L. Jewell, and R.P. Borden, 'A Mini-Network Computer Simulation Model for Railroad Planning.' Rail International, 4, 11/12, 1193-1198 (Nov.-Dec. 1973) outlines a computer simulation model to duplicate over-the-road operations on single and multiple tracks. The model is applied to evaluate the effects of changing the maximum continuous duty time from 14 to 12 hours for the Chessie system.

[DU1] DUVALYAN, S.V.; 'Working Out Optimal Freight Train Make-up Plan on Electronic Computer and Distribution of Classification Work on Large Railway Networks.' Rail International, 4, 10, 1069-1071 (Oct. 1973) considers the problem of choosing train make-up plans and scheduling yard-pass trains to optimize with respect to two conflicting criteria of reducing classification work and accumulation delay. Outlines a heuristic solution method based on 'largest savings.'

[EB1] EBERHARDT, J.S.; 'The Effects of Shorter, more Frequent Trains on Railroad Classification Yards: A Computer Simulation.' Bulletin of the International Railway Congress Association, Cybernetics and Electronics on the Railways, 3, 9-10, 411-439 (Sep-Oct. 1966) investigates the effect of shorter trains on yards. For a specific yard with fixed total traffic, the train lengths (and frequencies) are varied to explore the changes in delay time using a yard simulation model in GPSS II. Drawn from the author's M.S. Thesis at Northwestern University.

[EL1] EISELE, D.O.; 'Application of Zone Theory to a Suburban Rail Transit Network.' Traffic Quarterly, 22, 1, 49-67 (Jan., 1968) describes the concepts of zone-stop and skip-stop schedules used in [SA2] and [M01].
finds the optimal distribution and mix of engine types to provide motive power for a given schedule of trains at minimum capital and operating costs. Applies Benders Decomposition to the flow problem on a space-time network representing 216 train movements with good convergence results. Also experimented with a larger network of 430 train movements.

investigates the effect of various operating policies on reliability by using two basic simulation models: a network model simulating day-to-day car movement through a portion of the rail network; and a model simulating the journey of a single car through a series of yards. The policies studied are: a) holding outbound trains for more traffic, b) cancelling outbound trains, c) altering connection times, d) running shorter, more frequent, trains.

presents and analyzes data on train performance and yard operations. The former includes train arrivals at yards, correlation of arrival time with train length and weight, arrivals by day of week, train line-haul times, and train departures from yard. Yard data includes time spent in the receiving yard, total yard times, and results on missed connections at yards.

provides a survey of analytical network models in two groups: i) simulation models (Allman [AL2], Frisco [E1], Canadian National, AAR [AAL] etc.) and ii) optimization models (Thomet [TH1], Queen's University Group [F1], [PE10]).

applies a simple queueing model with Poisson arrival of trains to calculate load coefficients on the set of reception sidings. Analytically derived formulae for such load factors are then used to determine the number of sidings necessary.


studies two-way traffic on a single line for periodic transport systems. The traffic capacity, cycle time of the trains, and number of trains needed for traffic transport are discussed.

[FU1] FULLERTON, H.V.; 'A Railcar Network Model of the Canadian Pacific Railway System.' Canadian Institute of Guided Ground Queen's University at Kingston, Ontario (Jan. 1972)

describes the Railcar Network Model which determines car movements (routing) to minimize total delay over a rail network. Over-the-road delays and yard delays are expressed as convex functions of link (or yard) flow. A traffic assignment algorithm then finds the optimal flow pattern. See also [PE7, 8, and 9].


contains two parts. In the first Gleaves describes the nature of the assignment and routing problem for motive power units with the complications due to consist and inspection specifications. In the second part Bartlett and Charnes extend the approach in [BA1] to deal with this more complicated setting. The motive units are modelled as flows on what amounts to a space-time graph.


presents the research program of UIC (International Railways Union) and its consultant Metra International. Provides a list of problems and areas of application for analytical and simulation models.
focuses on car movement scheduling for a rail system with preset blocking and classification strategies and train schedules. Provides a space-time network representation as in [WH1] and discusses the extension to the multifleet case which involves a multicommodity flow problem.

discusses two methods of empty wagon distribution: intermittent distribution where demands are accumulated over some time period and continuous allocation where wagons are assigned at the moment the demand is reported. For the first a "compensation method" based on supplying wagons from neighboring points is used as a heuristic. In the second case the assignment takes probability considerations into account.

discusses the series of models used sequentially by Canadian National Railways (CN) to perform forecasting, car fleet planning, traffic management (by a network simulation model), and motive power planning (using a linear programming model).

[GU1] GULBRANDSEN, O.; 'Optimal Planning of Marshalling Yards by Operational Research.' pp. 226-233 of [PR1]
uses a simulation model to plan the profile, the technical equipment of reception and sorting yards, and the number of tracks in the reception yard of a marshalling yard. For each choice of the above three factors the simulation model calculates the total costs (fixed, service, and waiting time costs). The combination with least total costs is chosen.

solves the problem of sequencing various categories of trains over a critical line section so as to minimize average total occupation time. The solution technique is a variant of the assignment method.


uses a two-stage model to derive the mean waiting time on the approach to the yard where cars are placed on arrival sidings and processed through the hump as soon as it becomes available. The derivation is based on an approximation method.

[HE4] HEIN, O.; 'The Optimum Number of Sidings in a Marshalling Yard.' Rail International, 5, 6, 414-422 (June 1972)

finds the optimum number of arrival sidings to minimize total delay considered to be the sum of two delays terms: waiting time on the approach to the sidings (which decreases as m increases) and waiting time on the sidings on the approach to the hump (which increases with m). The analytic expressions for these delays are taken from [HE3].


see below.


The above two papers describe the distribution model used by Swiss Federal Railways since March 1976. The basic solution technique is the Out-of-Kilter algorithm. However, a number of complications such as an unmanageable network size, car substitutability, priorities, and insufficient supply are dealt with.


contains the papers [AL2], [URL], and [WI2].

provides a general account of the Bashpeak suite of programs used by British Rail to determine locomotive schedules.
Mathematical programming is used in conjunction with manual post-processing to obtain operational schedules to which the crew schedule is subsequently added.

predicts the passenger load pattern on the Tokyo commuter rail system. Assumes passengers' travel pattern follows a minimum cost network flow. Also evaluates the effect of adding new services on the flow pattern.

describes the freight car distribution problem with special attention to fluctuations of buffer inventories and its causes, suggests an improved pattern of information flow and procedures for inventory control taking distribution into account.


uses a queueing theory approach to derive occupation statistics for the locomotive change sidings assuming Poisson train arrivals and exponential service times at the sidings.


considers the process of wagon collection for outgoing trains under stochastic arrival of cars. Derives analytic expression for waiting times, train loads and frequencies, etc. under two different collection policies: 1) fixed collection periods, variable trainloads with specified maximum load; 2) variable collection periods with fixed trainload.


contains an econometric study of the production function in rail operations used to derive a long-run cost function on the interfirm level. Also discusses cost functions on the intrafirm level for yards and marginal costs for car movements. Provides a list of performance measures that can be used in freight studies.


describes a graphical procedure for the selection of steps taken to improve the capacity of a single line railway given predictions of future demand. Uses the concept of transitions between states in the upgrading process.


formulates the problem of distributing loaded and empty cars over an n-day planning horizon among N sectors (zones) as a linear program. The objective is to minimize empty car-mileage and total shortage costs. A decomposition approach by time period (daily) is also discussed.
KOOMANOFF, F.A. and J.A. Bontadelli; 'Computer Simulation of Railroad Freight Transportation Operations.' Journal of Industrial Engineering, 18, 1, 3-8 (Jan. 1967)

describes the work of Battelle Memorial Institute in developing simulation models for classification yards, mainline movement, servicing diesel units, distribution and assignment of motive power.


considers the question of train formation (makeup) as a tradeoff between shunting time at classification yards for trains of inhomogeneous makeup (which require sorting) and the accumulation delay required for the formation of direct trains. Suggests a number of heuristic methods to solve the problem.


investigates the modal choice decision of intercity freight shippers. Relates inventory stockout costs and unreliability to modal choice. Criticizes previous analytical studies in this area and uses linear logit models to determine the market share of each mode.


contains an analysis of road train delays due to mechanical failures and derailments classified by failure type and possible causal factors -- train tonnage, track profile, and train length. Data analysis shows the last factor is more important. Distribution of car delays as a function of train length is given.


summarizes the research described in earlier reports [LA1], [MA4], [BE3], [RE1], [F01], and [F02]. Presents research conclusions and policy recommendations.

describes a procedure for the control of car speeds by retarders at a hump yard to ensure safe coupling thus minimizing overspeed coupling damage.


solves a network improvement problem where the coefficient of shipping costs on each arc is lowered by improvements at a known cost. The decision variables are the cost coefficient and the flow of traffic for each arc. Shows how such a nonconvex, nonconcave problem can be transformed to a concave transshipment problem.


describes the development and implementation of a 2-day empty car distribution program using the linear programming transshipment model. The method is applied to a fleet of 1500 cars resulting in higher utilization rates and improved customer service.


contains a variety of routing and scheduling models for general transportation systems formulated as 0-1 integer linear programs. Suggests the solution technique of Land and Doig for such programs.


describes a deterministic simulation model for the optimal location of a new yard. The set of feasible sites, the traffic flow, and the changes in classification policies caused by the new location are inputs. Total costs of accommodating traffic are computed for each site (using analytic formulae) and the least expensive site is selected.

uses an aggregate production smoothing approach for a yard with quadratic cost functions for crew, overtime, and backlog (unswitched cars) costs. The minimization of the total cost function results in linear decision rules whereby the number of cars switched and the crew needed on a particular day are derived from forecasts of cars arriving on successive days. Compares the results with actual yard performance.


formulates the problem of assigning blocks of traffic to a given feasible set of operating trains. In a first formulation blocks are assigned as indivisible units leading to a 0-1 decision problem. In a second version this restriction is relaxed and a mixed integer program results. The problem of inhomogeneous empty wagon fleet distribution is also treated as a special case.


contains the following topics: the role of unreliability in the inventory costs of a logistics system. Measures and quantifiable definitions of reliability. Analysis of Origin-Destination trip time data using trip distance, number of intermediate yards, and unreliability as regression variables. Conclusions on the role of reliability in mean trip times.


describes a very general model of acquiring transport capacity for a multimodal transportation system with traffic classes (distinguished according to maximum allowed travel time) over a horizon of several years. In the most general case this results in a convex program with convex constraints.

investigates the effect of sequencing trains of different speed on the total occupation time of a section of line. Formulates the problem as a linear integer program.


uses the Out-of-Kilter algorithm on a space-time network of given train schedules to distribute locomotives and cabooses and to develop cyclic operation plans for different classes of motive power.


gives a tutorial account of queueing theory and its application to a classification yard. Emphasizes the effect of increasing facilities on reducing waiting times.


gives an exposition of how linear programming and the transportation method may be used to minimize empty wagon-hours while balancing supply and demand for empty cars.


approaches the suburban rail problem via a two-stage planning process. First explores combinations of zones, express and local service, number and size of trains resulting in the selection of a train schedule plan. Then, the second stage proceeds to construct the timetable using dynamic programming.

describes the simulation model used at New York Central System (NYC) for a large classification yard. The model is in GPSS and moves a transaction (a train or a cut) through the system's operations.

[NE1] NEMHAUSER, G.L.; 'Scheduling Local and Express Trains.' Transportation Science, 3, 2, 164-175 (May 1969).

considers a rail route where two kinds of services are available to passengers: A nonstop express service on the route and a local service which has intermediate stops. Seeks a jointly optimal schedule of local and express services to maximize revenues. Solves the problem by means of an efficient Dynamic Programming algorithm.


presents a survey of areas of application of Operations Research to rail and a list of applications in major national railways by country. One of a series of such reports appearing periodically in the journal cited above.


uses a queueing model for train assembly with Poisson arrivals of cars (at a given rate) which are then processed in batches of up to λ cars per train. The service rate, i.e. the rate of train departures, is also specified.


models the waiting time for pickup at a yard as a bulk service queue with random batch size (since the size of the block set on a train stopping at an intermediate yard is a random variable). Assumes Poisson arrival of railcars and an Erlang distribution of order k for train interarrivals. Studies the effects of varying the train arrival pattern from completely random (k=1) to completely regular (k=∞).

determines the mean running time for train's operating at different speeds in both directions over a single track. Evaluates delays due to meets and overtakes for a given priority, scheme and uniformly distributed random train departure times.


describes an algorithm for traffic assignment on a network with convex delay functions. This algorithm is used as part of the Railcar Network Model [PE9].


uses queueing models to analyze yard operations of classification and train assembly as well as connection delay for various types of railyards. Service rates for such operations are obtained from [PE1]. The output of the model is the distribution of put-through times for traffic which is then compared to actual histograms.


examines yard operations of classification and train assembly based on a computation of yard switching workload. The ratio of number of switches required per cut entering the yard is derived for a yard with a given number of tracks and multiple classification engines and used to compute classification and assembly times and rates.


a description of the model detailed in [FU1] and [PE9].


summarize the contents of Fullerton's paper [Ful].


contains an early description of the Railcar Network Model at its inception. See [PE9] for the final version of the model.


discusses a branch-and-bound procedure to solve a network design problem involving the selection of train routes on a given network. Uses 3 criteria: a) shortest routes should be used; b) number of routes should be minimized; and c) the number of passengers with direct routes (no connections) should be maximized.


gives a detailed analysis of cost accounting procedures for various parts of the rail system including freight, hauling, way train, and fuel consumption costs.


contains a general discussion of the use of mathematical methods in different aspects of rail management.


reports the result of a study of the potential of Operations Research applications to freight management. Describes models for various parts of the rail system and their links using a mix of analytical and simulation methods. Also tests the model in a 42 yard example.


extends the Ford-Fulkerson algorithm to deal with a network problem with bundle constraints arising from scheduling on a two-way traffic section with priority. Also discusses engine scheduling to improve turn around time.


gives a description account of the problems listed in the title of the paper and their interconnections as operations to be optimized.


contains the paper [SU3] and [SZ1].


contains a number of papers and discussions on the application of simulation methods for railroads. In particular, includes reports of the work done by the Railroad Systems Research Group (RSRG) at Battelle Memorial Institute.


considers passengers transport on a line of stations leading to a Central Business District (CBD). Supposing that the train stops at every station, gives a linear programming formulation of train loading to maximize total
number of passengers transported. Extends this to K trains and proves the existence of integral solutions. The problem is then solved by Dynamic Programming. Next considers scheduling trains on the line to minimize total carriage mileage extending certain results of [SA2]. Also contains a heuristic for solving a network design problem for rail.


develops timetables for a railway line without branches from optimal stop schedules (which train stops at what station). Uses two criteria for optimality: total number of passenger stops and total carriage miles, both to be minimized. Such optimal schedules are found by dynamic programming.


develops a model for minimizing the number of railcars needed for peak operations of a suburban railway system of rays emanating from a central business district. The model decides where to cut railcars back to the center along their route by solving a 0-1 integer program. A case study with 4 lines was performed for the City of Adelaide (South Australia).

describes the information system and car movement control system developed by Missouri Pacific to determine a route and movement schedule for each car.


is a comparison volume to [PE9] providing instruction for using the model described therein.

discusses the particular features of transporting transcontainers on rail with attention to the blocking and scheduling issues.

describes the work of Battelle Memorial Institute in model-building for rail systems. Discusses, in particular, a rail terminal simulation model processing the traffic of incoming trains into appropriate outbound trains. See also the following paper:


provides equations on the average time required by various operations on a car (sorting, uncoupling, etc.) at a yard. Uses the measure of converted car hours for total yard workload. Discusses various shipping options (one-yard or two-yard pass trains, etc.) with respect to this measure.


describes the mathematical modelling of yard operations and the use of Monte-Carlo methods to derive a relation for car detention at a yard as a function of the volume of traffic flowing into that yard. Also formulates the longer term problem of facilities improvement for a yard over a given planning horizon.


familiarizes the user with the SIMTRAN network simulation model developed at Southern Railway.


discusses the traditional way of railway cost accounting and a new method of evaluating costs for various rail operations.

deals with the problem of planning train movements so as to balance the costs of deadheading and costs attached to fleet size. Formulates this as a minimum cost circulation problem subject to certain inspection and operational constraints. The Out-of-Kilter method is then applied to the resulting space-time network.


provides a summary account of [LA2].


summarizes the results of a case study of the detection of unreality and the recommendation, implementation, and evaluation of procedures for improving reliability. Results show greater reliability can be obtained without increasing costs.


formulates the problem of scheduling by-pass (direct) trains on a rail network as an integer program. Minimizes the total transit time of cars accounting for the tradeoff between speedier transport of bypass trains and increased accumulation delay for cars making up such trains. Solves a problem with 36 yards and 20,000 cars by a heuristic add-drop method.

SZPIGEL, B.; 'Optimal Train Scheduling On a Single Track Railway.' pp. 343-352 of [RO1].

considers the problem of arranging for meets and passes on a single track line as analogous to scheduling jobs on a set of machines (trains on track sections). Formulates an integer program minimizing total travel time to be solved by branch-and-bound. Largest problem solved involves 5 track sections and 10 trains.

presents an optimization model for freight routing which considers both railroad operating costs (motive units, classification, etc.) and shippers cost. Starting with a large set of possible trains, one for each origin-destination pair of nonzero demand, a heuristic algorithm cancels trains by a greatest savings criterion to arrive at a satisfactory set of trains accommodating the demand. Provides documentation and coding of the optimization procedure.


gives a summary of the model detailed in [TH1].

TOUNG, J.; 'Automatic Allocation of Locomotives on the SNCF.' pp. 179-182 of [PR3].

contains a method developed by the French railways (SNCF) to assign tractive units (locomotives) to trains. The objectives are to minimize locomotive fleet size and the number of light runs. The procedure is based on the Hungarian method for solving assignment problems.


aims to provide a primer on railroad yards by describing their basic operations and functions. Also contains a discussion of present operating practices and the problems yards face. Contains an extensive bibliography.


considers the train formation problem as balancing accumulation and classification delays. Given routes for traffic flows the problem is to determine the flow creating direct train services and an optimal classification policy for the remaining flows. A heuristic search algorithm proceeds sequentially to determine the direct trains and the intermediate classification yards.

describes a planning system which uses a mix of mathematical programming, simulation, heuristic, and manual techniques to plan the gamut of rail operations. The system was developed for the Polish State Railways.


formulates the problem of empty car rolling stock distribution over a planning horizon of n days. The result is a linear program of transshipment type whose cost structure can be modified to penalize overdue deliveries and take priorities into account.

[TS1] TSC (Transportation Systems Center); 'Data Specifications for Models used in MIR Analysis.' (internal document).

describes the basic structure, data specifications, and comparability of the following rail models: FRA Network Model and Path Analyzer; SRI Blocking and Train Statistics Programs; SRI Yard Activity Program; PMM (Peat Marwick & Mitchell) Train Dispatching simulator and Parametric Analysis Program; TSC Cost Model.


contains a brief discussion of three examples of Operations Research applied to railways: (1) A simulation model of a commuter transport system. (2) A regression model estimating the number of accidents per station for prevention studies. (3) A procurement/distribution model for locomotive fuel.


uses the Out-of-Kilter method to find a schedule of arrivals and departures which minimizes the vehicle-kilometers accumulated by empty trips.


considers the problem of determining the numbers and locations of stations on a line emanating from a central
terminus. The input data includes passenger distribution along the line and the objective is to minimize total passenger travel time. The optimal locations are obtained by dynamic programming and satisfy a set of simultaneous difference equations.


models the empty car distribution (balancing) problem as a transshipment problem on a space-time network. For a homogeneous fleet of empty cars, the Out-of-Kilter algorithm is suitably specialized to solve the problem efficiently.


provides the model to decide upon the mix of regular and special trains to meet demand over a given planning horizon. A search over the regular trains fleetsize is performed to minimize a simple objective function. Also discusses the dispatching of such trains in time.


gives an account of the institutional problems in the way of applying Operations Research to rail systems. Discusses modeling approaches in the areas of classification yards, line haul operations, and equipment allocation and distribution.

[WI3] WILSON, P.B. and C.J. Hudson; 'Simulation of Over-the-Road Operations.' pp. 142-149 of [PR1].

describes two simulation models used by Canadian National Railways (CNR): 1) The Train Performance Calculation (TPC) which simulates train movement taking account of its length, horsepower, speed, etc. but neglecting interference by other trains. 2) The Single Track Capacity Analyzer (STCA) which simulates a dispatcher in arranging for meets between interfering trains.
WILSON, P.B. and C.J. Hudson; 'Development Validation and Application of the CN Network Model.' pp. 61-65 of [PR3].

describes the structure of the CN Network Model which incorporates line haul and yard operations (with limited yard resources). A test case of 5 yards was modeled as a network of 41 nodes and 38 links with 100 trains per day. Model and actual results were in good agreement.


contains an update of [WI3] describing an extension and replacement of STCA called SIMTRAC for dispatching trains over single track lines for a period of up to 10 days. Also describes a siding-to-siding simulation of loaded car moves used for analyzing the service time variability (i.e. reliability).


presents a detailed discussion of the empty car distribution problem with emphasis on implementation issues as experienced by the Polish State Railways. Represents the problem as a space-time network.


reviews the history and applications of Operations Research in Rail systems with a list of related publications. Also discusses the problem of locating a railway workshop.


develops timetables by maximizing an objective function based on operating costs, revenues, and travelers benefits (computed from a model specifying traveler's willingness to pay). A dynamic programming algorithm develops timetables for each vehicle in the system.