ANALYSIS OF MESOSCALE FRONTOGENESIS
AND DEFORMATION FIELDS

by

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ABSTRACT

Three cold fronts which pass through the N.S.S.L. Beta Network are examined. The mesoscale deformation fields are computed and are used to evaluate frontogenesis for each case. In each case, deformation is found to be more important than convergence in frontogenesis. On this scale, cyclonic vorticity is found to be weakly correlated with the fronts.

Two models are proposed to explain why no apparent change in frontal intensity occurs, although huge rates of frontogenesis are calculated. One is rejected on the basis of the results, the other requires a balance between mesoscale frontogenetical processes and turbulent frontolitical processes.

In the final section, the relation between the wind-shift and temperature break is studied. Large rates of frontogenesis occur when these two events occur simultaneously. When the wind-shift outruns the temperature break, sizeable rates of frontogenesis are still found, and the mechanism responsible is investigated.

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Introduction

Fronts- The presence of frontal surfaces in the atmosphere has been recognized for many years. The achievement of understanding of this phenomenon, however, has been disappointingly slow. During the early 1900's the air-mass concept was introduced in Norway, synthesizing the earlier endeavors of such men as Fitzroy, Shaw, and Lempfert. The name "front" was most likely adopted because the battle between polar and tropical air-masses was analogous to the lines of implegimentation of the great armies facing each other in Europe at this time. A frontal zone arises from the juxtaposition of two air-masses of different origin brought to close proximity so as to form a zone of rapid temperature transition.

Classically, a boundary surface on which there is a discontinuity of a given element such as temperature is generally called a frontal surface. When the variable shows a discontinuity such as figure 1(a), it is said to have a discontinuity of zero order. When the variable is continuous figure 1(b), but its derivative is discontinuous across the surface, it has a discontinuity of the first order and so on. The intersection of this three dimensional surface with the ground is referred to as a front. Real discontinuities do not exist in nature, so the front so defined is somewhat of an idealization. However, due to the scale of synoptic charts, frontal zones may appear as
ZERO ORDER DISCONTINUITY

FIRST ORDER DISCONTINUITY

FIG. 1
COUGAR, OKLA.  MARCH 24, 1964

EL RENO, OKLA.  MARCH 23, 1965

FIG. 2
discontinuities in the temperature field, which can be shown to be convenient mathematically.

For hydrodynamic reasons, this idealized front must satisfy a dynamic boundary condition. This condition requires that pressure be continuous across an internal boundary. If a zero order discontinuity in pressure existed, we would be dealing with a finite change in pressure through an infinitely small distance and therefore with an infinite pressure gradient force. Since this is impossible, a front may have a theoretical zero order discontinuity with respect to temperature, but must have of first order with respect to pressure. If geostrophic flow is assumed, the dynamic boundary condition can be shown to require a wind-shift across the frontal surface.

Much confusion now exists in the field of Meteorology concerning fronts, their dynamics, and representation on daily weather maps. There are even those who deny the existence of fronts altogether, and it is to these skeptics that we address figure 2. The Weather Bureau has not helped this confusion. It has been suggested that fronts be omitted from their facsimile products so that local forecasters might analyze them independently. In the absence of precipitation, the two most important characteristics of a frontal passage are the temperature break and the wind-shift. The dynamic boundary condition requires a wind-shift to accompany the temperature break. Frequently these two events do not occur simultaneously, and therefore
some confusion exists regarding which event represents the true front. In this study a front will be understood as representing the previously mentioned "zone of rapid temperature transition." This choice is not made arbitrarily. It is made because we feel that the temperature break is the more significant of the two events. The timing between the wind-shift and the temperature break and its relation to frontogenesis and frontolysis will be examined in a later section.

Now that we have defined a front, we will examine the important process of frontogenesis.

**Frontogenesis**— Bergeron (1928) introduced the term "frontogenesis" as the tendency to create new fronts or intensify existing ones, and "frontolysis" as the tendency to destroy such zones. The frontogenetical function was first introduced by Petterssen (1936)

\[
F = \frac{d}{dt} \left| \nabla T \right|, \quad \nabla T = \frac{\partial T}{\partial x} \mathbf{i} + \frac{\partial T}{\partial y} \mathbf{j} + \frac{\partial T}{\partial z} \mathbf{k}
\]  

(1)

where the operator \( \frac{d}{dt} \) is a process following a specific air parcel, and \( T \) represents temperature a scalar quantity. A value of \( F > 0 \) gives frontogenesis, \( F < 0 \) frontolysis.

\[
F = \frac{d}{dt} \left| \nabla T \right| = \frac{1}{2} \frac{d}{dt} \left( \nabla T \cdot \nabla T \right) = N_T \cdot \frac{d}{dt} (\nabla T)
\]  

(2)
where $\mathbf{N}$ is unit vector in direction of $\nabla T$. Now expanding

$$\frac{d}{dt} \nabla T = \frac{\partial (\nabla T)}{\partial t} + u \frac{\partial (\nabla T)}{\partial x} + \nabla \frac{\partial (\nabla T)}{\partial y} + \omega \frac{\partial (\nabla T)}{\partial z}$$

$$= \nabla \left( \frac{\partial T}{\partial t} \right) + \nabla \left( \frac{\nabla \partial T}{\partial x} \right) + \nabla \left( \frac{\nabla \partial T}{\partial y} \right) + \nabla \left( \omega \frac{\partial T}{\partial z} \right)$$

$$- \frac{\partial T}{\partial x} \nabla u - \frac{\partial T}{\partial y} \nabla v - \frac{\partial T}{\partial z} \nabla w$$

$$= \nabla \left( \frac{dT}{dt} \right) - \left( \frac{\partial T}{\partial x} \nabla u + \frac{\partial T}{\partial y} \nabla v + \frac{\partial T}{\partial z} \nabla w \right) \tag{3}$$

it follows from equation (2) and (3)

$$F = \mathbf{N} \cdot \nabla \left( \frac{dT}{dt} \right) - \mathbf{N} \cdot \left( \frac{\partial T}{\partial x} \nabla u + \frac{\partial T}{\partial y} \nabla v + \frac{\partial T}{\partial z} \nabla w \right) \tag{4}$$

$$= \mathbf{N} \cdot \nabla \left( \frac{dT}{dt} \right) - |\nabla T| \frac{\partial V_n}{\partial n} \tag{5}$$

The first term on the right represents frontogenesis through diabatic processes. In the second term $|\nabla T|$ is the magnitude of the temperature gradient, $V$ is the velocity, and $n$ is in the direction normal to the isotherms and towards the cold air. Thus if diabatic processes are absent and the wind has a component normal to the isotherms which decreases in its downstream direction figure 3, frontogenesis is to be expected; if the reverse occurs we have frontolysis.

Bergeron (1928) investigated the behavior of isotherms in a linear field of temperature, superimposed on a linear field of
FRONTOGENESIS

FIG. 3
motion. He assumed that the velocity in the vicinity of a point may be represented by its Taylor Series approximation

\[ \mathbf{u} = u_0 + \left( \frac{\partial u}{\partial x} \right)_o x + \left( \frac{\partial u}{\partial y} \right)_o y + \frac{1}{2} \left( \frac{\partial^2 u}{\partial x^2} \right)_o x^2 + \frac{1}{2} \left( \frac{\partial^2 u}{\partial y^2} \right)_o y^2 + \frac{1}{2} \left( \frac{\partial^2 u}{\partial x \partial y} \right)_o xy + \cdots \]

\[ \mathbf{v} = v_0 + \left( \frac{\partial v}{\partial x} \right)_o x + \left( \frac{\partial v}{\partial y} \right)_o y + \frac{1}{2} \left( \frac{\partial^2 v}{\partial x^2} \right)_o x^2 + \frac{1}{2} \left( \frac{\partial^2 v}{\partial y^2} \right)_o y^2 + \frac{1}{2} \left( \frac{\partial^2 v}{\partial x \partial y} \right)_o xy + \cdots \]

In a sufficiently small area around the point in question, the higher order terms in \( x \) and \( y \) may be neglected. We may write this linear field

\[ \mathbf{u} \approx u_0 - \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)_o y + \frac{1}{2} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)_o x + \frac{1}{2} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)_o y \]

\[ \mathbf{v} \approx v_0 + \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)_o x + \frac{1}{2} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)_o y - \frac{1}{2} \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right)_o y + \frac{1}{2} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)_o x \]

(6)

where the velocity components are a combination of translation, rotation, divergence, stretching deformation and shearing deformation terms respectively. With the proper rotation of principal axes, it may be shown that the last terms are zero so that the velocity field can be written

\[ \mathbf{u} = (a + b) x - cy \]

\[ \mathbf{v} = cx + (b - a) y \]

(7)

where \( u_0 \) and \( v_0 \) vanish with a parallel transition, and where

\[ a = \frac{1}{2} \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) \] deformation, \( b = \frac{1}{2} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \) divergence, and \( c = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \) vorticity. If we choose the
angle $\beta$ between the $x'$ axis and the isotherms figure 4, and consider only the horizontal wind field then from equation (4)

$$F = D - \frac{\partial T}{\partial x'} \cdot \left( \frac{\partial T}{\partial x'} \cdot \nabla u + \frac{\partial T}{\partial y'} \cdot \nabla v \right)$$

where

$$D = \frac{\partial T}{\partial x'} \cdot \nabla \left( \frac{\partial T}{\partial t} \right)$$

and where the arbitrary coordinates $x'$, $y'$ are respectively along and perpendicular to the axis of dilatation. Using fact that

$$\frac{\partial T}{\partial x'} = \left| \nabla T \right| \cos \beta \quad \frac{\partial T}{\partial y'} = -\left| \nabla T \right| \sin \beta$$

and equations 7 and 4

$$F = D - \left( \cos \beta \hat{i} + \sin \beta \hat{j} \right) \cdot \left| \nabla T \right| \left\{ \cos \beta \left( \frac{a + b}{c} \right) \hat{i} \right. \\
\left. - \hat{j} \right\} + \sin \beta \left[ c \hat{i} + (b-a) \hat{j} \right]^2$$

$$= D + \nabla T \left[ (a+b) \cos^2 \beta - (b-a) \sin^2 \beta \right]$$

$$= D + \left| \nabla T \right| \left( a \cos 2 \beta - b \right) \quad (8)$$

The argument leading to the derivation of this equation appears to be based on the assumption of a linear velocity field. It is common practice among Meteorologists to begin a derivation of equation 8 with this assumption. However, equation (6) is simply an identity utilizing the derivatives of $u$ and $v$ with respect to $x$ and $y$ and reduces to the statements
The fact that the velocity fields are actually non-linear does not invalidate the use of equation (6) except when the distances x and y are very large. We will assume linear velocity fields in this paper because finite difference methods are used and require the velocity to remain linear over a grid interval. Equation (6) will simply be used as a definition of divergence, vorticity and deformation. Therefore, although we will assume a linear velocity field, it is not a necessary condition for the validity of equation (8).

If we assume for the moment that the temperature is a conservative property, then $D = 0$ and the sign of $F$ in equation (8) is determined by the angle $\beta$ for any given field of motion. Only deformation and divergence can contribute to frontogenesis $F > 0$ and frontolysis $F < 0$. Furthermore, in
equation (8) convergence \( b < 0 \) promotes fronotgenesis; and divergence \( b > 0 \), frontolysis. The angle for which \( F = 0 \) in equation (8) is

\[
\beta' = \frac{1}{2} \cos^{-1} \frac{b}{a}.
\]

If the divergence \( b = 0 \), then \( \beta' = \pm 45^\circ \). When there is convergence \( b < 0 \), \( \beta' > \pm 45^\circ \); when there is divergence \( b > 0 \), \( \beta' < \pm 45^\circ \). In figure 4, we note that \( \beta' < \pm 45^\circ \) and so divergence has narrowed the frontogenetical sector between \( \pm \beta' \). At point A the angle \( \beta' \), between the tangent to the isotherm and the axis of dilatation, is less than angle \( \beta' \), at point B it is greater. Thus we have fronotogenesis at point A, frontolysis at point B, and the dashed line separates the two areas for this particular isotherm orientation.

**Purpose**—The purpose of this paper is to investigate the meso-scale processes involved in frontogenesis. To gain insight into these processes first it was necessary to determine what characteristics of fronts are germane to our investigation. Secondly, we use deformation fields in our analysis of frontogenesis in the hope that they will help us understand these highly complicated processes. No similar work on this scale has been done, although Williams (1962) investigated microscale deformation and frontogenesis patterns near squall lines. If
we compare equations (5) and (8),

\[ F = D - \frac{\sqrt{v}}{\frac{\partial v}{\partial n}} \]  \[ F = D + 1 \sqrt{v'} \left( a \cos 2\beta - b \right) \]

it appears that we have substituted a more complex expression of the velocity field for a simpler one. Historically the justification for this was that the Norwegian School originally pictured air-masses ascending and descending relative to each other at the frontal surface. This picture of frontogenesis implies convergence associated with these motions. Later, Bergeron (1928) showed that frontogenesis could be caused by deformation in the absence of convergence. In this paper, equation (8) is chosen to determine how important mesoscale deformation is to frontogenesis.

In the final section, the relation between the wind-shift and the temperature break accompanying frontal passage is investigated. We hope that it will shed some light on earlier investigations of this phenomenon by Plotkin (1965) and Sanders (1966).

Data—The opportunity to study the mesoscale structure of fronts was afforded by the establishment of the Beta Network by the National Severe Storms Laboratory. The network was created to study severe weather phenomena, i.e., thunderstorms and tornadoes, which often occur in the Southern Plains during the Spring.
In 1964 it consisted of 41 stations spaced at ten to fifteen mile intervals, located roughly between the Texas border and Oklahoma City. During 1965 eleven more stations were added to the southwest and north of the network. All stations record data continuously during the months March through June in graphic form consisting of station pressure, temperature, relative humidity, wind direction, wind speed, and rainfall. Of these, only wind speed, wind direction, and temperature were used. Wind speed and direction were measured from wind towers 20 feet above ground, wind direction being recorded at one-minute intervals and to 16 points of the compass. Wind speed is recorded continuously and can be read accurately to the nearest knot. Temperature was recorded continuously on a thermograph figure 2, accurate to nearest 0.5°F. Pressure was not analyzed because the barographs do not give station pressures accurately during the periods of interest. We will examine three cold front passages through the Beta network during the 1964 and 1965 seasons. These cases are March 24, 1964; March 23, 1965; and April 24, 1965. They vary in intensity, but have the common property of being essentially dry with little if any frontal precipitation. These results are at variance with those of Eliassen(1959) who professes that no fronts exist without associated precipitation.
Computations—The time when each of these fronts most nearly bisected the network was chosen for particular attention, and the winds and temperatures were plotted from the continuously recorded data. We made the rather crude assumption that each front had a characteristic orientation at this time. This orientation was chosen by drawing a straight line which most closely approximated the leading edge of the cold air. We decomposed the wind fields into $u$ and $v$ components relative to this orientation. Using finite difference methods, the quantities $\frac{\delta v}{\delta x}$, $\frac{\delta v}{\delta y}$, $\frac{\delta u}{\delta x}$, and $\frac{\delta u}{\delta y}$ were evaluated midway between grid points whose interval was nine nautical miles. The various combinations of these quantities yield those components of the velocity field discussed earlier.

The isotherm field was constructed from the plotted values of temperature in whole degrees Fahrenheit. The values were determined by using finite differences over the same grid interval. We neglect the differences in station elevation and gradients of pressure, but the maximum possible error is $1^\circ F$ per nine nautical miles. Since the temperatures are read only to the nearest degree, and since the actual gradients are as large as $19^\circ F$ per nine nautical miles, this approximation is considered sufficiently good for the cases studied.

Using an average frontal orientation the angle $\beta$ was computed in the following manner following Saucier 1953. If the $u$ and $v$ components of velocity and derivatives in the initial
(x, y) coordinate system are related to the velocity components \( u_1 \) and \( v_1 \) and derivatives in the \((x_1 y_1)\) coordinate system, rotated counter-clockwise through the arbitrary angle \( \theta \).

\[
M = M_1 \cos \theta - V_1 \sin \theta; \quad \frac{\partial}{\partial x} = (\cos \theta) \frac{\partial}{\partial x_1} - (\sin \theta) \frac{\partial}{\partial y_1},
\]

\[
V = U_1 \sin \theta + V_1 \cos \theta; \quad \frac{\partial}{\partial y} = (\sin \theta) \frac{\partial}{\partial x_1} + (\cos \theta) \frac{\partial}{\partial y_1}.
\]

Then

\[
\frac{\partial u}{\partial x} = [(\frac{\partial u}{\partial x_1}) \cos \theta - (\frac{\partial u}{\partial y_1}) \sin \theta] \cos \theta - [(\frac{\partial u}{\partial y_1}) \cos \theta - (\frac{\partial v}{\partial y_1}) \sin \theta] \sin \theta
\]

\[
\frac{\partial v}{\partial y} = [(\frac{\partial u}{\partial x_1}) \sin \theta + (\frac{\partial u}{\partial y_1}) \cos \theta] \sin \theta + [(\frac{\partial v}{\partial x_1}) \sin \theta + (\frac{\partial v}{\partial y_1}) \cos \theta] \cos \theta
\]

\[
\frac{\partial v}{\partial x} = [(\frac{\partial u}{\partial x_1}) \sin \theta + (\frac{\partial u}{\partial y_1}) \cos \theta] \sin \theta - [(\frac{\partial v}{\partial x_1}) \sin \theta + (\frac{\partial v}{\partial y_1}) \cos \theta] \cos \theta
\]

\[
\frac{\partial u}{\partial y} = [(\frac{\partial u}{\partial x_1}) \cos \theta - (\frac{\partial u}{\partial y_1}) \sin \theta] \sin \theta + [(\frac{\partial v}{\partial x_1}) \cos \theta - (\frac{\partial v}{\partial y_1}) \sin \theta] \cos \theta
\]

a) \( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = [(\frac{\partial u}{\partial x_1}) - (\frac{\partial v}{\partial y_1})] \cos 2\theta - [(\frac{\partial u}{\partial y_1}) + (\frac{\partial v}{\partial y_1})] \sin 2\theta \)

b) \( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = [(\frac{\partial u}{\partial x_1}) + (\frac{\partial u}{\partial y_1})] \cos 2\theta + [(\frac{\partial v}{\partial x_1}) - (\frac{\partial v}{\partial y_1})] \sin 2\theta \)
The two types of deformation are dependent upon the orientation of the coordinates, and with the proper orientation one of these deformations may be eliminated. Choosing to eliminate the shearing deformation

c) \[ \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = \left[ \left( \frac{\partial u}{\partial x} \right) - \left( \frac{\partial v}{\partial y} \right) \right] \cos 2\theta \]

d) \[ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = \left[ \left( \frac{\partial u}{\partial x} \right) - \left( \frac{\partial v}{\partial y} \right) \right] \sin 2\theta \]

dividing \[ \tan 2\theta = \frac{\left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)}{\left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right)} \] (9)

thus the angle of required rotation is obtained from the ratio of the deformations in the initial coordinate system.

Neither deformation gives individually the total deformation unless the other is zero. To compute fields of deformation, we must combine the two types to give a resultant deformation. This is achieved in equations c) and d) where rotation of the coordinate axes through such an angle that in the new system one type of deformation vanishes. Thus if the shearing deformation is eliminated

\[ D_{E_2} = \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) \sec 2\theta \] (10)
If this arbitrary angle $\Theta$ is assumed to be equal to the angle $\beta$, we can compute the axis of dilatation for each station using equation 9. The absolute rate of deformation may be found using the appropriate shearing deformation and angle $\beta$, equation 10. Finally with the temperature gradient, divergence, deformation, and angle $\beta$, the kinematic term of frontogenetical function may be evaluated by equation (8) if we neglect the diabatic term D. This term and its contribution to the total rate of frontogenesis will be discussed in following sections.

In the final section, the structure of each of the three fronts is examined at all stations for the period surrounding the initial temperature break. The time of the break was estimated by use of thermograph traces, and was generally evidenced by an unmistakable break figure 2. The time of wind-shift was determined from the continuous wind records, and in all cases this shift was at least 22.5 degrees. The difference between these two times is calculated, and the wind field is examined in the immediate vicinity of the temperature break.
Results

Case I - March 24, 1964

The surface chart for 1200 GMT figure 5a shows a cold front just northwest of the network with a closed low in the vicinity of Lake Superior. The front does not appear to be intense at this time, nor does it undergo any dramatic change while in the network. But upon leaving, a wave developed on the front which eventually became a major storm affecting the Northeast. The front was in the network between 1018 and 2023 CST and the hourly positions are indicated in figure 5b. It maintains an average speed of nine knots and has an average orientation of 250-070 degrees. It appears to move faster for the first four hours, but slows down after 1400 CST.

The nine-minute temperature drops at each station with usable data are shown above the station figure 5b with maximum drops of 14°F recorded at three stations, and an average drop for the network of 8.9°F. A small wave west of Oklahoma City increases amplitude with time, and appears to have developed into the major storm which affected the Northeastern States two days later. Although no significant precipitation accompanied the front, low ceilings associated with the developing wave were reported on the hourlies at stations around the network. This prevented direct insolation before frontal passage. A few stations in the middle of the network reported scattered clouds prior to frontal passage. The destructive
March 24, 1964

figure 5 (a) Surface Chart.

(b) Hourly positions of front and nine-minute temperature drops.
effect of insolation might explain some of the relatively small nine-minute temperature drops between 1300 CST and 1500 CST.

The wind field at 1400 CST figure 6a indicates the surface wind observations in knots. The divergence and vorticity fields at this time figures 6b and 6c are drawn for each 15 units of $10^{-4}$ sec$^{-1}$. Large values of convergence are found along the front with a maximum, in the vicinity of the small wave. The vorticity field has a maximum west of the maximum convergence zone, and has large values surrounding the wave with small values outside the frontal area.

The thermal field at 1400 CST figure 6d has a maximum temperature gradient in the eastern half of the front. A thermal ribbon exists within a grid interval behind the front and as we progress to the West, we note the destructive influence of the Wichita Mountains, north of Lawton. In the cold air to the north, there is a temperature differential greater than 30°F, while in the warm air to the south there is only 5°F differential.

The axes of dilatation, indicated by the double-ended arrows, are generally parallel to the front. At a distance greater than 15 miles from the front they become more normal.

The absolute magnitude of the resultant deformation figure 6e computed from equation 10 is drawn for each 15 units of $10^{-4}$ sec$^{-1}$. It also has a maximum in the vicinity of the
March 24, 1964

figure 6  (a) Surface wind observations in knots.
(b) Divergence in units $x 10^{-4}$ sec$^{-1}$.
(c) Vorticity in units $x 10^{-4}$ sec$^{-1}$.
(d) Isotherms in °F and axes of dilatation.
(e) Absolute magnitude of the resultant deformation in units $x 10^{-4}$ sec$^{-1}$ computed from equation (10),
(f) Frontogenetical function in units °F/NM/HR, computed from non-diabatic terms in equation (8).
of the wave. Although it resembles the divergence pattern, the deformation is larger on the average and encompasses a broader area as we might expect.

The frontogenetical function figure 6f is given by equation (7) and naturally has its maximum where the favorable fields of convergence, deformation and temperature are coincident. Large values of frontogenesis measured in °F/nm/HR are found close to the leading edge of the front and extend well into the thermal ribbon behind the front. Frontolysis is noted well outside the frontal zone and should be expected to the South where there is negligible temperature gradient. To the north, there is a moderate temperature gradient but the axes of dilatation are mostly normal to the isotherms producing net frontolysis. It should be understood that this is an instantaneous picture of frontogenesis, and that although large values of frontogenesis will remain in the vicinity of the front, the maximum areas may migrate along the front.

Case II - March 23, 1965

During the 1965 season the Beta network added eleven new stations on the southwestern and northern boundaries of the network. At 0000 GMT figure 7a a rather diffuse stationary front is located north of Oklahoma. In the period of a few hours, it intensifies and comes through the network figure 7b with an average speed of 24 knots, producing nine-minute
March 23, 1965

figure 7 (a) Surface Chart.

(b) Hourly positions of front and nine-minute temperature drops.
temperature drops of 20°F. The average drop for the 49 stations with usable data over the nine-minute period, is 13.0°F, and of the 49 stations, 43 demonstrates an unmistakable break similar to figure 1. Two waves are present and the one to the northeast appears to die as it leaves the network. The other, in the lee of the Wichita Mountains, also appears to lose amplitude with time. The front appears to be moving slightly faster as it enters the network than when it leaves. It seems to undergo some modification while within the network, at least with respect to the nine-minute temperature drops. There is an unusual lack of cloudiness in this case, and the typical reporting station experiences at most only broken conditions. The early hour in which this front came through the network would indicate that the source of smaller nine-minute drops in the temperature to the south, is other than insolation.

The wind field at 0400 CST figure 8a shows strong northerly flow behind the front with weak southerly flow in advance. The divergence pattern figure 8b has two large convergent areas with the one to the southwest the more intense. Some small areas of divergence are located behind the front which are evident from the wind analysis figure 8a. The vorticity pattern figure 8c has a large maximum behind the front to the northeast and a secondary maximum to the southwest. An unusual value occurs in the vicinity of the small wave east of the Wichita Mountains. This maximum of anticyclonic vorticity occurs along the front.
March 23, 1965

figure 8, notation same as in figure 6.

(a) Surface wind observations.
(b) Divergence.
(c) Vorticity.
(d) Isotherms and axes of dilatation.
(e) Absolute magnitude of the resultant deformation.
(f) Frontogenetical function.
and it can be accounted for in the following manner. The orographic influence of the Wichita Mountains has a modifying effect on the normal component of the wind, and thus as we proceed eastward along the front in the positive $x$ direction, the normal component of wind increases and we get a large negative $\frac{\partial v}{\partial x}$ ($v$ being measured positive in the $y$ direction.) This large value of $-\frac{\partial v}{\partial x}$ dominates the term $\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$, and therefore anticyclonic vorticity is found along the front, contrary to geostrophic expectations.

The temperature field figure 8d shows a very strong temperature gradient which extends 10 to 15 miles behind the front. Again the effect of the Wichita Mountains is seen to the west of Lawton. The temperature field in the warm air exhibits little gradient while the contrast of temperatures in the cold air is large. The axes of dilatation are mostly parallel to the isotherms in the region of maximum temperature contrast and remain parallel well into the warm air in advance of the front. At distances greater than 25 miles, the axes become more normal to the isotherms.

The deformation field figure 8e appears to be concentrated slightly behind the front with large values of deformation encompassing a major portion of the larger temperature gradients. Again we note that the deformation field is broader and has larger absolute values than the divergence field. The Wichita Mountains are included in the southwestern maximum, which should
be expected as the wind must contract as it approaches the mountains, producing deformation.

The frontogenetical function is concentrated in two areas figure 8f. The effect of the divergence and lack of appreciable temperature gradient is evident east of Lawton. The huge rates of frontogenesis to the northeast and to the southwest correspond well with the nine-minute temperature drops near and downstream from these maxima figure 7b.

Case III - April 24, 1965

At 1200 GMT figure 9a, an unorganized stationary front is located north of the network. It intensifies and twelve hours later passes Oklahoma City as a weak cold front. Initially the front is dry and therefore similar to the two previous cases. However during the afternoon, thunderstorms developed in the southern and southwestern portions of the network, and this complicates the analysis of thermograph traces at a few stations. The hourlies indicate scattered to broken sky conditions exist before and after frontal passage at those stations north of the 2000 CST position. Maximum diurnal heating occurs in this area, with temperatures reaching the mid eighties before passage. South of the 2000 CST position figure 9b, thunderstorms reduce the temperature at many stations and after sunset, sky conditions were favorable for radiational cooling. The front was in the network from 1649 CST to 0026 CST. It has an average speed of
April 24, 1965

figure 2  (a) Surface Chart.

(b) Hourly positions of front and nine-minute temperature drops.
12 KTS, and an average frontal orientation of 230-050 degrees. The maximum nine-minute temperature drops are 13°F with a network average of 6.6°F for the 50 stations with readable data. Rain associated with thunderstorms \((R)\) is reported at almost all stations in the bottom two rows and naturally modifies the temperature drop which would have otherwise occurred. One small wave is present to the west and increases amplitude near the Wichita Mountains, but loses its amplitude soon after.

The wind field at 2000 CST figure 10a, indicates broad convergence in the frontal zone. The divergence and vorticity fields figures 10b and 10c, are generally symmetric with respect to the front. Three large convergent areas lie along the front, with a general broadening of the convergent region to the southwest. A large divergent area to the south results from weak southeasterly flow figure 10a at these stations with maximum values of southeasterly flow closer to the front. The vorticity pattern exhibits two maxima, one on either side of the small wave. A large area of positive vorticity lies along the front, and two small anticyclonic areas lie to the north.

The thermal gradient figure 10d is well organized in the southwest and somewhat less well organized to the northeast. The weak gradients to the northeast seem to be well correlated with the small nine-minute temperature drops figure 9b. The temperature field to the south is flat except for a moderate
April 24, 1965

figure 10, notation same as in figure 6.

(a) Surface wind observations.
(b) Divergence.
(c) Vorticity.
(d) Isotherms and axes of dilatation.
(e) Absolute magnitude of the resultant deformation.
(f) Frontogenetical function.
gradient ahead of the small wave. The dilatation axes are mostly parallel to the isotherms in the frontal zone with the notable exception of the northeastern sector. This is well correlated with the temperature drops. Outside a ten mile distance, the axes are poorly organized and tend to be normal to the isotherms.

The deformation field figure 10e is quite similar to the divergence pattern with two maxima along the front. The field is more intense and broader than the divergence field. Notice that the deformation field encompasses the area where large values of divergence were located in figure 10b.

The frontogenetical function figure 10f shows a huge rate of frontogenesis just behind the front where there is a favorable coincidence of convergence, deformation, and temperature gradient. This large area occurs in a region where the temperature gradient was relatively weak, but where the additive values of convergence and deformation produce huge rates of frontogenesis. With reference to the nine-minute temperature drops figure 9b, the front on the average appears to intensify the following hour, but then diabatic processes weaken it.
Conclusions

We have investigated the mesoscale kinematics of three cold fronts. These fronts were dissimilar first with respect to intensity if the nine-minute temperature breaks are to be used as criteria. Secondly the speeds differed considerably. One case required ten hours to traverse the network, while another required less than four hours. There is no obvious correlation among cases between frontal speed and temperature drop. The one case which moved the fastest did experience the largest temperature drops, but the case which shows the smallest drops has the second fastest movement.

The calculated convergence associated with each case is what should be expected. The maximum convergence in all cases occurs in the cold air, and extends into the warm air. The values of divergence and other kinematic quantities are two orders of magnitude greater than on synoptic scale. Although this results from the small grid interval chosen, it appears to be representative of the mechanisms in operation.

Vorticity has the poorest correlation with the fronts, and has no systematic pattern. On the average, cyclonic vorticity is found near the front with the exception of one area of anticyclonic vorticity in the March 23, 1965 case. Maximum values of cyclonic vorticity are at best weakly correlated with the small waves on the fronts. This is contrary to geostrophic
expectations and to the results of Petterssen and Austin (1942). The scale of analysis must be an important factor, as Williams (1962) found even weaker correlations in microscale vorticity patterns.

The temperature field in each case is what should be expected. The maximum gradients in the cold air occur within ten miles of each front. The dilatation axes are generally parallel to the isotherms in the frontal zones due to well organized wind fields. The orientation of these axes is less favorable once removed from the frontal zones.

The absolute magnitude of resultant deformation is the most systematic kinematic quantity investigated. Large values of deformation are characteristic of the frontal zones in each case. A comparison of the divergence and deformation fields indicates that deformation fields are both broader and more intense in all cases.

The frontogenetical function has a maximum in the cold air where the largest temperature gradients are located. In the March 24, 1964 case the fields of convergence, deformation, and temperature gradient are favorably coincident yielding large value of frontogenesis throughout the frontal zone. By comparison, it appears that this case is undergoing the greatest frontogenesis at the time chosen. The other two cases have isolated areas of large frontogenesis which are highly correlated with the deformation fields.
We calculate huge rates of frontogenesis in all cases, but find negligible change in frontal intensity as it moves through the network. There are two possible models which might account for this. First the front may propagate through the air acting on different parcels. This would occur if it were not a substantial surface. In this model, large values of frontogenesis would be generated in advance of the front and equally large values of frontolysis to the rear. We must reject this model on the basis of results, which do not exhibit large values of frontolysis behind the fronts.

The second possibility is that the front moves with the air as a substantial surface but there is a balance between mesoscale frontogenetical processes and turbulent frontolytical processes. These frictional dissipative processes would have to be of the same order to negate the huge rates of frontogenesis which we have computed. This model is the more appealing of the two in light of the results. Diabatic effects must naturally be important in any model of frontogenesis. It is impossible to analytically calculate this process, but the results of one case indicate that it may be of the same order of magnitude as the frontogenetical function.

We have found that deformation fields are extremely important in mesoscale frontogenesis. The general impression is that these fields are characterized by large hyperbolic streamline patterns which are evident on synoptic scale charts.
It is hoped that we have shown that this type of pattern is not a requirement in mesoscale analysis and that other types of flow can produce larger rates of deformation important to frontogenesis.
Investigation of Wind-Shift and Temperature Break

Plotkin (1965) showed that the wind-shift at a cold front is a dual phenomenon whereby a directional wind-shift precedes the temperature break, and a secondary surge in wind or vector shift accompanies or immediately precedes the break. The peak wind speed thus occurs at the beginning of the temperature break and we find diffluence in the zone of maximum temperature gradient producing frontolysis. The timing in this case, however, was crude because of inadequate time checks and instrumentation. Sanders (1966) investigated the March 23, 1965 case in the Beta Network and was able to measure the temperature break and wind-shift more precisely. His conclusions indicate that maximum convergence occurs ahead of the break and extends a small distance into the cold air, producing large rates of frontogenesis. Elsewhere in the cold air there is divergence and frontolysis on the average.

In the introduction, it was mentioned that some Meteorologists use the wind-shift as the criterion for frontal location. We choose the temperature break as the criterion in this paper, because we find it less ambiguous as discussed above. We will now investigate the relation between these two events in determining rates of frontogenesis. Because of the duality in the wind-shift, we will refer to the initial shift in direction as the wind-shift and the secondary surge as the vector shift.
Results
March 24, 1964

The maximum nine-minute temperature drops figure 5b are found during the first few hours. As the front progresses, there appears to be a slight decrease in these drops and possibly a slight increase as it is leaving the network. The difference between the times of temperature break and wind-shift figure 11a are indicated to nearest minute. The time of temperature break was read from thermograph traces such as figure 2 and the wind-shift, which in all cases was at least 22.5 degrees in one minute, was read from the continuous wind records. The front has an average speed of nine knots and average orientation of 250-070 degrees. The average difference in time of the two events figure 11a is 1.54 minutes for 35 stations with usable records. There is an extremely good correlation between this difference and the magnitude of the nine-minute temperature drop figure 5b. The zero isoline encompasses the maximum drops. As the time between the two events decreases, there is a corresponding growth in the temperature drop.

The vector shift is generally coincident with the wind shift at the stations in the northern half of the network, producing large rates of frontogenesis. In the middle of the network, the wind-shift outruns the temperature break and the vector shift is weak producing small rates of frontogenesis. In the southern half of the network the vector shift and wind-shift are again coincident producing vigorous frontogenesis.
figure 11  (a) Difference in minutes between times of temperature break and wind-shift for March 24, 1964 case.
(b) March 23, 1965 case, same notation as in (a).
(c) April 24, 1965 case, same notation as in (a).
March 23 1965

The largest nine-minute temperature falls figure 7b occurred in this case, which was previously investigated by Sanders(1966). Maximum drops occur in the northern portion of the network and are reduced by one half before leaving. The front moves with an average velocity of 24 knots and has an average orientation of 240-060 degrees. The difference between shifts figure 11b indicates that the wind-shift occurs increasingly ahead of the temperature break as the front progresses, with a maximum value of twenty-seven minutes. The large areas of simultaneous shifts are absent, and the average difference for 51 stations is 4.86 minutes. The three minute isoline encompasses the majority of the large nine-minute temperature drops.

The vector shifts occur in the cold air at all stations and are more vigorous to the north of the network. The front is undergoing frontogenesis in the north and as it progresses the vector shift decreases in magnitude. Normally the largest values of convergence are associated with the directional wind-shift and smaller values with the vector shift. Because the vector shift is occurring in the cold air, it must be the mechanism responsible for producing frontogenesis. As the vector shift decreases, the turbulent scale processes dominate and we have net frontolysis in the southern half of the network.
April 24 1965

This case produced the smallest nine-minute temperature drops. The front has an average speed of twelve knots and average orientation of 230-050 degrees. The temperature drop field figure 9b is not as well organized as the two previous cases. Two possible reasons for this are the inequality of solar radiation due to cloud conditions, and the outbreak of thunderstorm activity in the southern half of the network. There are indications of small increases in the nine-minute temperature drops in the middle of the network. The difference between shifts figure 11c shows a poorly organized pattern with an average difference of 2.04 minutes for the 51 stations.

The vector shift is weak to the north, but increases in magnitude as the front progresses through the network. These increases in velocity last only one to two minutes after which the wind field becomes strongly difluent. In this case, it is possible to examine the wind records and predict the general magnitude of the nine-minute temperature drops. If the thermograph traces figure 2 suffer from too large a time constant, it is possible that near zero order temperature discontinuities do in fact exist in the atmosphere. Huge rates of frontogenesis then could result from a vector shift occurring in these regions.
Conclusions

We find a definite correlation between the rate of frontogenesis and the time difference between the temperature break and the wind-shift. In areas where the two are coincident, huge rates of frontogenesis occur because the area of largest convergence is acting on the zone of maximum temperature gradient. As the wind shift outruns the temperature break, advective processes and frontogenesis produced by the vector shift create the temperature drops. In near zero order temperature discontinuities exist, the vector shifts may be capable of producing huge rates of frontogenesis. When the vector shift becomes weak, the weak mesoscale frontogenetical processes are dominated by turbulent frontolytical process resulting in net frontolysis.

In Plotkin's case (1965), the peak wind speed occurs in the warm air, while it occurs in the cold air for all three cases studied here. A model which might reconcile this, is as follows. The peak wind speed originally is well within the cold air, but propagates somewhat faster than the temperature break. So long as it remains within the cold air we have frontogenesis. When it reaches the frontal boundary or enters the warm air we have frontolysis.
BIBLIOGRAPHY


