THE DEVELOPMENT AND PRESENT STATUS
OF THE THEORY OF THE HEAT BALANCE IN THE ATMOSPHERE

by

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by Chaim Leib Feferis

ABSTRACT.

The purpose of the investigation is to formulate, insofar as it is at present possible, a theory of the heat balance in the atmosphere. This is done by presenting a critical discussion of the principal contributions in this field, and emphasizing their bearing on the general picture of the energy transformations in the atmosphere as suggested by latest observations.

In the introduction we summarize the existing observational material on the transformations of solar radiation in the atmosphere. This is followed by a discussion of the physics of the stratosphere. It is proven that Humphreys' theory of the temperature of the stratosphere is, contrary to his claims, strictly valid only under the assumption that the stratosphere is grey. It is emphasized that the two cases considered by Humphreys are intrinsically different, since in the first case there exists a state of thermodynamic equilibrium, while in the second case there exists only a state of radiative equilibrium.

In order to ascertain the extent to which the temperature of the stratosphere can be explained by the assumption of a state of radiative equilibrium, calculations were made of the radiative power of horizontal air-layers of various temperatures containing various quantities of water vapor. When \( \log I \) was plotted against \( \log T \), the curves obtained were nearly parallel straight lines so that, for the adopted range of temperatures and water vapor contents, the relation

\[
i = C T^n
\]

(1)

holds. From the slopes of the curves the value of \( n \) is found to be about 3.5.
The result is explained qualitatively. Next we discuss, in the light of C.G. Simpson's work, the probable spectral distribution of the long wave radiation incident at the bottom of the stratosphere. It appears that the bulk of radiation feeding the stratosphere originates in the tropopause. It follows, therefore, that the temperature of the stratosphere computed by Humphreys is overestimated because of his assumption that $n$ in (1) is 4 and also because of the high value for the effective temperature of the earth's radiation which he adopted. Similarly it is pointed out that Gold's crucial assumptions that turbulence carries heat only upwards is contrary to fact.

This is followed by a discussion of the cause of the convective state in the troposphere, the heat transport by the general circulation, the radiation currents in the atmosphere, and the theory of the latitudinal distribution of the temperature of the stratosphere.
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OF THE THEORY OF THE HEAT BALANCE IN THE ATMOSPHERE

by

C. L. PEKERIS

Cambridge, Massachusetts

1932
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PREFACE

In this paper an attempt is made to discuss and coordinate some important theoretical investigations aiming towards an explanation of the temperature distribution in the atmosphere. It is assumed that the elementary atmospheric processes of reflection, scattering, absorption, emission, evaporation, condensation, etc. have been quantitatively determined; the problem is then to find the temperature distribution that results from their combined action. It follows from this limitation of the undertaking that the experimental work done in determining the heat balance of the atmosphere will not be treated. To those who wish to become acquainted with this phase of the problem, a study of the four volumes of the Annals of the Astrophysical Observatory in Washington is indispensable. A concise summary of the measurements of solar radiation and of its depletion in the atmosphere appeared recently in a series of papers by H. H. Kinball [1, 2, 3]. Accounts of the present state of various special types of experimental investigations can also be found in recent communications by Ångström, Linke, Dobson and others.

It can hardly be doubted that our theoretical understanding of the heat balance in the atmosphere has been unable to keep pace with the rapid and systematic accumulation of experimental data. This study was undertaken at the suggestion of Prof. C. G. Rossby in the belief that a critical survey of the existing theories for the heat balance in the atmosphere is needed. The preliminary work was done during the summer of 1930 in the U. S. Weather Bureau in Washington, under the guidance of Dr. H. H. Kimball. Later the work was resumed in Cambridge under the direction of Prof. Rossby. While in Washington, the

*Reference to literature is made by numbers enclosed by brackets. The complete reference will be found at the end of the paper.*
author had the benefit of frequent discussions with Dr. W. J. Humphreys of the Weather Bureau and with Drs. F. E. Fowle and L. B. Aldrich of the Astrophysical Observatory of the Smithsonian Institution. The courtesy of Dr. C. F. Marvin, Chief of the Weather Bureau, and of Professor C. F. Talman, Librarian, in offering study facilities and free use of the Weather Bureau library should also be acknowledged.

Throughout the preparation of the manuscript Prof. Rossby was ever helpful with suggestions and criticisms for which I am very grateful. I am deeply indebted to Dr. Kimball and to Mr. I. F. Hand, who gave much of their valuable time to me during my stay in Washington.

Meteorological Laboratory,
Massachusetts Institute of Technology
Cambridge, March 1931
I. INTRODUCTION

The thermal state, or heat balance, of the atmosphere ultimately results from the various processes which the solar energy undergoes from the time of its incidence at the top of the atmosphere to the time of its return to space in the form of reflected short-wave solar radiation and long wave terrestrial radiation. Among these processes, some are of a purely mechanical or thermodynamical nature, such as the transport of heat by small-scale vertical turbulence or by the large-scale horizontal turbulence of the general circulation, as well as the transport of heat through evaporation and condensation of water vapor.

The principal task of the theory of the atmospheric heat balance is to explain the observed mean vertical and latitudinal temperature distribution and its variation with season. As happens often in geophysics these phenomena cannot easily be studied by proceeding from a cause or causes to an effect, but have to be considered as links in a closed chain of processes each one of which controls the others and is in turn controlled by them. Thus, the distribution of temperature, moisture, dust and clouds, the intensity of vertical and horizontal turbulence and even the geographical extent of snow-covered regions are all mutually dependent. However, radiation processes ought to receive primary attention in this cycle since they alone are responsible for the heat exchange between the earth and space and since the supply of radiation energy at the top of the atmosphere is fixed, or nearly so, and varies in a definite fashion with season and locality. Furthermore, any changes in the state of a radiating body manifest themselves instantaneously in corresponding modifications in the intensity and quality of its radiation. Thus, a diminution in the radiation of the surface layers of the atmosphere due to a sudden cooling or to advection of colder air will propagate itself with the velocity of light throughout the atmosphere, so that, for
example, almost instantaneously the stratosphere will sense
the change by a diminished radiation current from below. The
mechanical and thermodynamical modes of heat transportation,
on the other hand, are performed through the medium of moving
air or water vapor masses, whose rates of propagation, de-
termined by the thermodynamic stability or instability of the
atmosphere, are incomparably slower than that of radiation.

Not only is radiation the most efficient heat conveying
agent of the atmosphere, but it also lends itself most readily
to a qualitative and quantitative analysis with the aid of
well-known and perfectly definite radiation laws. A vast
amount of experimental data on various radiative aspects of
the atmosphere have been accumulated in the last 50 years,
mainly through the efforts of the Astrophysical Observatory
in Washington and associates in several parts of the world.
Through their painstaking work extended over several decades
a host of intricate phenomena have been disentangled and
brought down to a definite combination of elementary processes.

All this points to the conclusion that the natural course
to follow in the investigation of the heat balance of the
atmosphere is to center attention on the radiative processes
and treat the other relevant factors as complementary. This,
of course, does not exclude the possibility that during the
investigation we may discover the fallibility of the method;
all we say now is that it appears to be the path of least re-
sistance.

We can get a bird's eye view of the nature of our problem
if we recall briefly the modifications which the solar radia-
tion undergoes within the atmosphere of the earth. In doing
this we have to trace the effects of all the three elementary
processes by which radiation is depleted: absorption, scatter-
ing and reflection. The principal absorbing agent in the
atmosphere is water vapor with numerous and deep bands in the
infra-red end of the solar spectrum. The extreme ultra-violet
end of the spectrum (<0.18μ) is absorbed by oxygen above and
at fifty kilometers elevation; simultaneously and at the same
elevation the region from 0.23μ to 0.31μ suffers absorption
by ozone. Carbon dioxide has a few narrow absorption bands and there is also a slight amount of absorption by dust and the remaining permanent gases of the atmosphere. The scattering agents are dust and the atmospheric gases, of which water vapor appears to be, for equal numbers of molecules, most effective [4]. Reflection takes place either at the clouds, which are, of course, distributed in varying amounts over a wide range of elevations, or at the surface of the earth.

Using Aldrich's [5] value for the albedo of the earth as well as Ångström's and Kimball's [2] data concerning the transmission of solar radiation through clouds we can state that of the total energy incident at the top of the atmosphere

43% is reflected back to space,
12% is absorbed selectively by the water vapor in the atmosphere,
5% is absorbed in the atmosphere by the permanent gases, dust and clouds,*
40% is absorbed at the surface of the earth.

Part of the heat absorbed by the surface of the earth is spent through evaporation, the remainder necessarily being lost by radiation or through conduction and convection in the lower atmosphere. The atmosphere, which like the earth does not change its mean temperature, can lose heat energy chiefly by the selective long wave radiation of water vapor, carbon dioxide and ozone. Within the atmosphere turbulence carries

---

*Kimball [2] has estimated that the total energy received at the surface of the earth during completely overcast days is 22% of the possible sunshine (Ångström's value is 25%). According to Abbot [6] the average direct insolation and the diffuse sky radiation received on a unit horizontal area at sea level on clear days amount to 50% and 19% respectively, of the insolation in the absence of an atmosphere. With 52% cloudiness, the surface receives then,

1) in clear areas 0.48 * 0.69 = 33%
2) in cloudy areas 0.52 * 0.23 * 0.69 = 7.9%

Making a small deduction for the reflection at the surface we obtain, roughly, for the mean intensity of the insolation at the surface 40% of the insolation at the top of the atmosphere. This leaves the remaining 5% for absorption by clouds, dust and the permanent gases in the atmosphere.
heat from regions of high to regions of low entropy (equivalent-potential temperature). The general circulation carries heat from lower to higher latitudes.

With all these processes going on simultaneously, a definite mean temperature distribution is established at the surface of the earth and throughout the atmosphere; this temperature distribution must necessarily be such that for every layer the heat income from all available sources equals the total loss of heat. Were this not true, then the mean temperature would not be constant. With the given temperature distribution and the given distribution of the radiating elements the long wave radiation output of the earth is determined and this must equal the average solar radiation absorbed by the atmosphere and the surface of the earth, since the only possible way of squaring accounts with space is through radiation.

Now, the temperature distribution in the atmosphere is, as all meteorological elements, irregularly variable. Certain invariant characteristics are, however, manifested by the mean values. Generally the temperature decreases with elevation and at a uniform rate of 6 degrees per kilometer in the lower ten kilometers; thus, with a mean surface temperature of, roughly, 280°A the mean temperature at 10 km. is about 220°A. Above that level the temperature remains almost constant with elevation. The transitory level, called the tropopause, is higher the lower the latitude and has a maximum elevation at the equator; it also varies at any particular locality with the weather and season, being lower in cyclonic regions and higher in anticyclonic regions. These characteristics are represented in Fig. 1 and Table I, taken from a recent paper by G. C. Simpson [15].

It is, then, the task of the meteorologist to "explain" this mean temperature distribution and in particular such features as:

1) The uniform rate of decrease of temperature in the troposphere
2) The isothermal state in the stratosphere
3) The height of the stratosphere
4) The actual temperatures prevailing there and throughout the troposphere
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Fig. 1 Temperature distribution in the free atmosphere, after Simpson.
5) The temperature in the ozone layer, above and at 50 km,
6) The outgoing terrestrial radiation and the various radiation currents traversing the atmosphere
7) The latitudinal and seasonal variation of the above quantities.

Of special interest is the more difficult problem of finding the temperature distribution for a state of radiative equilibrium, in which every layer neither gains nor loses heat by radiation. Such a temperature distribution, if mechanically and thermodynamically stable, would probably be the first to be established, and is therefore of primary importance for an understanding of the temperature adjustability of the atmosphere.

In the following an attempt will be made to trace the development of the theory of the heat balance in the atmosphere with the purpose of ascertaining the extent to which the various problems enumerated above have been satisfactorily solved. It is also intended to examine the validity of the various methods of attack developed in this field to date and to coordinate them for the sake of the future investigator.

The history of this problem is not one of continuous progress; this, perhaps, is to be expected from its intrinsic complications. Precipitated by the then fresh discovery of the isothermal zone, there appeared between 1906 and 1909 a series of three papers by K. Schwarzschild [7], W. J. Humphreys [8] and E. Gold[9] dealing with the temperature of the stratosphere. They were followed in 1913 by a comprehensive paper by R. Emden [10] on the same subject, on the temperature distribution at radiation equilibrium and on the radiation currents in the atmosphere. Emden made a severe criticism, not altogether justified, of the papers by Humphreys and Gold, throwing doubt on the validity of their methods and the results obtained therefrom. In 1919 H. Hergesell [11] published a paper on the temperature distribution at radiation equilibrium and on the radiation currents in the atmosphere over Lindenberg, Batavia and California, proving Emden's temperature distribution at radiation equilibrium to be incorrect because of the enormous supersaturation it would give. Hergesell calculated anew
the nature of radiation equilibrium by introducing an empirical relation between atmospheric water vapor pressure and temperature, and obtained an isothermal state. This result has lately been disproved [12].

In all these papers the atmosphere was assumed to be grey, or semigrey, in other words, to have one or two constant coefficients of absorption for the entire spectrum. Only three years later E. A. Milne [13] published a paper in which he, while still adhering to the assumption of semigrey absorption, discussed the problem of the temperature distribution at radiation equilibrium in greater detail by taking into consideration a number of factors disregarded by previous writers. Milne learned about Emden's paper only after the major part of his work had been completed; hence, some of his results had already been given by Emden. Milne's criticism of Gold's work is similar to Emden's. The last paper properly to be considered as belonging to the "grey" period is one published by R. Mügge in 1926 [14]. There, an attempt was made to calculate the horizontal heat transport from lower to higher latitudes from the latitudinal temperature distribution in the stratosphere. The resulting distribution of terrestrial radiation with its maximum at the pole, which in view of the small amounts of water vapor there seemed to be untenable, furnished the incentive to a series of papers by G. C. Simpson, the first of which appeared in 1927 [15]. In it the radiative powers of the atmosphere with the given mean temperature and humidity distribution were calculated under the assumption of grey absorption and by using the coefficients of absorption adopted by Emden. The theory of grey absorption was there brought to a crisis by exhibiting its disagreement with observations. A new era was initiated in 1928 by the appearance of Simpson's second paper [16]. There, for the first time, methods were developed in which the selective absorption of the radiating elements in the atmosphere were taken into consideration. Finally a group of three papers by Abbot [17], Simpson [18] and M ügge [19] should be included, all dealing with the development of the new method and with the computation of the outgoing terrestrial radiation.
II. HUMPHREYS' THEORY OF THE TEMPERATURE OF THE STRATOSPHERE.

The object of the theory of the heat balance in the stratosphere is to explain the existence of an isothermal region in the atmosphere as well as to calculate the observed mean temperature there. The problem was treated almost simultaneously by W. J. Humphreys [8] and E. Gold [9], although already in 1906 K. Schwarzschild [7] investigated the analogous situation in the atmosphere of the sun. The latter arrived at the conclusion that in the case of a grey atmosphere in radiation equilibrium (a notion, which, apparently, was then introduced by him for the first time into astrophysics--) the ratio of the temperature of the outer layer $T_2$ to the effective temperature of the radiation of the star $T$ is

$$\frac{T_2}{T} = \frac{1}{\gamma^2}$$

The significant feature of the thermal state of the stratosphere is the absence of any considerable amount of convection there, as demonstrated by the nearly isothermal lapse rate. Granted, then, that convection and with it any mechanical or thermodynamical heat transport is impossible, the prevailing temperature must be such that no layer gains or loses heat by radiation. Such a state we shall call from now on a state of "radiation equilibrium". The determining factors in the temperature distribution are then the absorbing elements like water vapor, carbon dioxide and ozone. The latter is concentrated in layers above those which we are considering (10-20 km). The other elements are present in very minute quantities so that the stratosphere, which constitutes one third of the total mass of the atmosphere, represents a much smaller fraction of its optical mass. This optical mass, the smallness of which results in a very slight difference between the stratospheric boundary temperatures, is spread out over a larger vertical distance than the troposphere, thereby making the temperature lapse rate vanishingly small. In general, then, whether the lapse rates be positive or negative, their absolute value must be small.
To find the temperature of the stratosphere we must recognize the fact that it is heated essentially from below by the long wave radiation of the surface of the earth and the troposphere. The absorption of solar radiation, Humphreys argues, is presumably not great, as is shown by the small seasonal variation in the temperature of the stratosphere. Also, because of the small quantities of absorbing matter involved the effect of the radiation of the stratosphere on the temperatures at lower levels is negligible. Now, if the stratosphere were contained between two radiating surfaces it would receive twice as much energy as when it is irradiated from one side only. Furthermore, in the former case it would assume the temperature of the radiating surfaces between which it is enclosed and this independently of its absorbing properties.

Were the atmosphere a grey body, having a uniform coefficient of absorption k, we could say immediately that the energy absorbed in the first case would be

\[ A_1 = k s T^4 + k s T^4 = 2k s T^4, \]  

(1)

where \( T \) is the effective temperature of the radiating surfaces and \( s T^4 \) the radiation incident on either side.* Similarly, in the second case (irradiation from one side only) the energy absorbed would be

\[ A_2 = k a T^4 \]  

(2)

As the total emission of a grey body is proportional to the fourth power of its temperature and as in each of the above two cases emission must equal absorption, the temperature \( T_s \) of the stratosphere in the second case can be determined from

\[
\frac{\text{Emission}_1}{\text{Emission}_2} = \frac{T_2^4}{T_2^4} = \frac{A_1}{A_2} = \frac{2k s T^4}{k s T^4} = 2, \]

\[ T_2 = \frac{T}{\sqrt{2}}. \]

---

* \( s \) is the Stefan-Boltzmann constant, \( 8.22 \cdot 10^{-11} \text{RT cal} \text{cm}^{-2} \text{min}^{-1} \text{°C}^{-4} \).
With a value of 1.94 cal/min/cm² for the solar constant and with Aldrich's generally accepted value of 0.43 for the albedo the energy E absorbed and emitted per cm² of the earth's surface is

\[ E = \frac{1.94 \pi R^2 \cdot 0.57}{4 \pi R^2} = 0.276 \text{ gr. cal./min/cm}^2, \quad (5) \]

(R is the radius of the earth) from which the effective temperature of the earth's radiation is obtained:

\[ T = \left( \frac{0.276}{8.22 \cdot 10^{-11}} \right)^{\frac{1}{4}} = 241^\circ \text{A} \]

Inserting this value in (4), we obtain for the temperature of the stratosphere \( T_2 = 0.841 \cdot 241 = 203^\circ \text{A} \). According to Humphreys [20], Aldrich's value of the albedo is overestimated, because the clouds which go to make up the adopted value of 0.52 cloudiness are not all thick enough to reflect 73% of the incident radiation, as Aldrich assumes. He prefers, therefore, to use the smaller value of 0.32 for the albedo, obtaining thereby from (5) a value of 252°A for the effective temperature of the earth's radiation, and from (4) a value of 212°A for the temperature of the stratosphere.

Humphreys claims relation (4) to hold also under the existing conditions of selective absorption, "if the spectral distribution of the radiation of the upper atmosphere is continuous, or nearly so (no matter how irregular), and not confined chiefly to lines with zero radiation between them" [Physics of the Air, 1929]. This contention can hardly be upheld. We must realize that the two states of the stratosphere considered by Humphreys are intrinsically more different than what would appear from mere energy considerations. It is not enough to say that when irradiated from one side by a body of given temperature the stratosphere receives half as much energy as when it is irradiated by bodies of the same temperature from both sides. For in the second case the stratosphere is completely enclosed within walls of constant temperature, resulting in black body radiation filling up the cavity, irrespective of the absorptive properties of the stratosphere and the
reflectivity of the surfaces of the cavity. On top of the state of radiation equilibrium we have there a state of thermal equilibrium. In the second case, which corresponds to actual conditions, the situation is different; the system is no longer closed, thermal equilibrium does not exist and the radiation coming up from the surface of the earth and the underlying troposphere is spectrally by no means identical with that of a black body at the effective temperature of the earth's radiation.

With the given distribution of absorbing elements throughout the troposphere every layer has its characteristic coefficient of absorption for every wavelength; similarly the part of the atmosphere superincumbent on any given layer has a definite transmission coefficient for every wavelength. The energy that leaves the troposphere is, then, composed of the fractions of radiation from each layer which are transmitted by the corresponding superincumbent atmospheres beginning with the surface of the earth, which radiates like a black body, and continuing up to the tropopause. Now, the stratosphere has certain coefficients of absorption for such diffuse radiation, say $k_X$. If the spectral distribution of terrestrial radiation leaving the troposphere be denoted by $K=\lambda$, then the energy absorbed by the stratosphere will be

$$\int_0^\infty K=\lambda k=\lambda d=\lambda$$

This must be lost through radiation towards space and towards the earth. Since the radiating matter involved is small, there will be no considerable temperature drop through the stratosphere and the radiation to both sides will be equal. Humphrey's principal idea can be expressed here by the fact that the energy radiated to either side is half the energy absorbed from terrestrial radiation. The emission of the stratosphere is given by

$$\int_0^\infty I(=\lambda,T) k=\lambda d=\lambda \quad (6)$$

where $I(=\lambda,T)$ is the intensity of black body radiation. A mechanical integration of (6) has been carried out and the result is given in table II. The spectral distribution of the
radiation of four layers containing .0001, .003, .046 and .455 cm. precipitable water vapor respectively, is computed from the coefficients of emission given by Abbot [17]. They were obtained from Fowle's [21] measurements of the absorption of infra-red radiation by water vapor under atmospheric conditions (containing normal amounts of carbon dioxide). In fig. 2 the logarithms of the total intensities (I) have been plotted against the logarithms of the temperatures. The result is a series of approximately straight and parallel lines. Therefore, within the temperature range considered

\[
\log I = n \log T + \text{constant}
\]

or

\[
I = \text{constant}. \, T^n
\]

where \( n \) has the approximate value 3.5. Thus, certainly, Stefan's fourth power law does not apply. The reason for this is that the rapid rate of increase of black body radiation with temperature is due largely to the steep rise of the black body radiation-curve in the neighborhood of the wavelength of maximum intensity. The stratosphere, whose radiation depends

<table>
<thead>
<tr>
<th>Calories per cm.² per min.</th>
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<tbody>
<tr>
<td>Temperature</td>
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<tr>
<td>Precipitable water vapor</td>
</tr>
<tr>
<td>0.0001 cm.</td>
</tr>
<tr>
<td>0.003 cm.</td>
</tr>
<tr>
<td>0.046 cm.</td>
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<tr>
<td>0.455 cm.</td>
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TABLE II. Showing the Radiation at Several Temperatures of Horizontal Atmospheric Layers Containing Various Quantities of Precipitable Water Vapor. The Units are gr-cal./min./cm.²

mainly on water vapor, is, however, almost transparent in that very region for all temperatures involved and, consequently,
Fig. 2 Radiation of horizontal atmospheric layers containing various quantities of precipitable water vapor.
its radiation increases less rapidly with temperature. Another feature brought out by the figure is that the dependence of radiation on temperature (given by the slope $n$ of the curves) is almost independent of the quantity of radiating matter.

Now, the radiation incident on the stratosphere, while equivalent in energy to radiation of a black body at 241°A has a quite different spectral distribution. If we decide with Abbot [17] that the moisture content of the stratosphere is minute, the radiation towards space computed by Simpson [16] will have essentially the same spectral distribution as that reaching the bottom of the stratosphere. In the transparent region of $8 \frac{1}{2}$-$11\mu$, the spectrum is that of a black body at a temperature equal to a mean of the temperature at the mean cloud level and the temperature of the surface of the earth. This mean is, of course, higher than 241°A. In the opaque region ($5 \frac{1}{2}$-$7\mu$ and $>14\mu$) and chiefly in the region above $14\mu$, where almost half of the terrestrial radiation is concentrated, the spectral energy distribution corresponds to that of a black body at the temperature of the tropopause. Everywhere else it corresponds to black body radiation at some mean temperature between the above two, which may be in the vicinity of 241°A. Roughly, the energies included in the three regions stand in the ratio 1:2:1. Of these the 25% originating in the surface layers lie in the transparent region and are, therefore, not absorbed by the stratosphere. It follows that the main bulk of the radiation feeding the stratosphere is contributed by the tropopause, so that the effective temperature of terrestrial radiation is only slightly above the temperature of the tropopause and certainly less than 241°A.

These considerations seem to bring out two deviations from conditions under which the Humphreys' relation

$$T_2 = \frac{T}{\sqrt{2}}$$

with $n = 4$ and $T = 241°A$ holds strictly. First, the value of $n$ is less than 4 (its value is about 3.5) and second, $T$ is considerably less than 241°A. Each deviation and especially the second one effects a smaller temperature of the stratosphere.
Since the average temperature of the stratosphere is about 220°A and since, in the light of the above, the value obtainable from Humphreys' theory is certainly less than 203°A or 214°A, as computed by Humphreys from the assumption of a smaller value (0.32) for the albedo, it would appear that the theory is yet far from being satisfactory. Of the two principal physical conditions underlying it and which may be held responsible for the disagreement between the derived and observed temperatures, the assumption of a state of radiation equilibrium is, in the light of the previous discussion, the least to be doubted. There remains, then, the second condition, that of irradiation from one side only, which might be modified by including the absorption of solar radiation from above. The effect of the radiation from the ozone layer at 50 km. seems too small to explain the discrepancy since it is concentrated mainly in the region for which water vapor is transparent. If after such an improvement we should still find, as we most likely will, a disagreement, we will be forced to take into account such quantitatively unexplored effects as the variation of the coefficients of absorption with pressure, density and temperature.
III. GOLD'S THEORY OF THE STRATOSPHERE.

It was mentioned before that Gold [9] treated the problem of the thermal state of the atmosphere almost simultaneously with Humphreys. His purpose, however, was not so much to compute the temperature of the stratosphere as to explain the division of the atmosphere into a convective troposphere and an isothermal stratosphere. His elaborate and exact method of computing the radiation currents crossing a given horizontal plane in the atmosphere and the radiation absorbed and emitted by a layer of finite thickness did not find much application in later meteorological papers but has gained much prominence in astrophysical work.

Gold began with a careful study of the available data on the absorptive power of the atmosphere. The long wave radiation currents from the surface of the earth and from the atmosphere are absorbed mainly by water vapor, since the effect of carbon dioxide is masked through the concurring absorption by the former. Two sets of data on water vapor absorption were available at that time, one by Paschen and Rubens and Aschkinass [22], who worked with steam, and one by Langley [23], who measured the absorption in columns of air under atmospheric temperatures and pressures. From these it appeared that if the absorption be taken as continuous throughout the spectrum about 95% of the earth's radiation would be absorbed by the normal quantity of water vapor present.* If the absorption were assumed to be spurious in the region 8 - 12.5μ the corresponding value would be 75%. When Gold later computed the radiation of the atmosphere to the earth and compared it with the observed values he found that the assumption of 75% transmission of the earth's radiation is the more correct one. He carried out his calculations for both sets of data and found the principal results to hold in both cases.

To determine the absorption of solar radiation he used the data in Vol. II of the Annals of the Astrophysical Observatory and concluded that it amounts to between 12% to 15%

*Gold assumes a normal atmosphere water vapor content of 2 cm of precipitable water.
of the solar energy incident at the top of the atmosphere.

Concerning the vertical distribution of water vapor the assumption was made that the water vapor density is proportional to

\[ \frac{\alpha}{q - p}, \]

where \( p \) is the pressure and \( \alpha \) and \( q \) are constants. Two values of \( q \) were introduced,

\[ q = \frac{9}{8} p_0 \quad \text{and} \quad q = \frac{5}{4} p_0 \quad (p_0 = \text{surface pressure}), \]

the first giving a more rapid diminution of water vapor with elevation than the observed and the second a slower. The value of \( \alpha \) was determined from the vertical transmission of long wave radiation, which, with \( \frac{\alpha}{q - p} \) as coefficient of absorption, has the value

\[ e^{\int_0^p \frac{\alpha dp}{q - p}} + \ln\left(\frac{q - p_0}{q}\right) \]

\[ = e^\alpha = \left(\frac{q - p_0}{q}\right)^\alpha = \left(\frac{1}{q^\alpha}\right) \]

The calculations were carried out for four cases corresponding to the four assumptions regarding the transmission of long wave radiation and the distribution of water vapor. The principal results held in all four cases. While the author made the doubtful statement that "owing to the large portion of the spectrum through which the constituents of the atmosphere radiate, their radiations may be taken to be proportional to the fourth power of the absolute temperature", he improved on it by assuming that

\[ p \sim T^4 \quad \text{and} \quad I \sim p, \]

where \( p \) is the pressure and \( I \) the intensity of the radiation emitted by a narrow layer. Actually, we have in convective equilibrium

\[ p \sim T^{3.41} \]

and, as shown in the discussion of Humphreys' theory,
making the radiation very nearly proportional to the pressure.

Having determined the radiative properties of the atmosphere Gold proceeded to calculate by exact methods the difference between absorption and emission in an atmosphere in adiabatic equilibrium throughout. It was found that in a layer limited by the top of the atmosphere and any plane below, absorption invariably exceeds emission even if the absorption of solar radiation is neglected. In this calculation the surface of the earth was assumed to radiate as a black body and to have the temperature of the atmospheric surface layer. The above result does not imply that the excess of absorption over emission increases with the thickness of the layer considered; in fact, the opposite is true; the excess diminishes, for the lower strata radiate more than they absorb. In the upper atmosphere, on the other hand, the continuous excess of absorption will raise the temperature, whereby emission will increase until radiation equilibrium is established. In the lower layers, the loss of heat by radiation must be balanced by heat carried upward from the surface of the earth by convection and by condensation of water vapor. According to Gold, convection can carry heat upwards only. Hence, in these layers a temperature distribution must be maintained, such, that transport of heat by convection is possible. As this transport is associated with mass movements the required state is one of maximum mobility compatible with stability, namely the adiabatic state. In the upper layers convection is impossible, for, according to Gold, additional heat would thereby be supplied from below, which together with the original excess would raise the temperature and establish, at least in the boundary region, stable lapse rates and thus check the convection. Hence, there is a tendency towards the development of stable lapse rates in the upper atmosphere and of adiabatic lapse rates in the lower atmosphere.

Just what particular temperature distribution will be established in the two regions, Gold does not investigate.
In accordance with observations he assumes the first to be isothermal and the second adiabatic, with the temperature of the isothermal region equal to the temperature of the upper boundary of the convective region. His problem is, then, to find such a level that an isothermal atmosphere above it will, as a whole, absorb as much energy as it radiates. We can perhaps see this method best by considering the temperature adjustment of the atmosphere from the impossible all-convective state. A narrow isothermal top layer will develop and penetrate downward, assuming the continuously increasing temperature of the upper boundary of the convective region. As its thickness increases its absorptive power grows; at the same time the radiative power grows due to the increasing temperature. Furthermore, the radiation available for absorption, being mainly atmospheric and terrestrial long wave radiation from below, will increase, due to the closer approach to the warmer black surface of the earth. The two factors, increased available incoming radiation and increased absorptivity, are counteracted by the increasing radiative power due to the higher temperature and the increasing emissivity. The two latter factors are the more effective; thus an equilibrium level is finally reached.

We shall now give the main results of Gold's work. He first computes the lowest possible temperature in the atmosphere for a given surface temperature. This will be the temperature assumed at radiation equilibrium by the uppermost skin-layer of an atmosphere in convective equilibrium throughout, for such a layer receives the minimum possible radiation from below. (The absorption of the solar radiation is neglected.) Gold finds for a ground temperature of 300°C the following values, corresponding to the four cases treated: 198°C, 173°C, 193°C and 154°C. Of these the first and third values correspond to the more correct assumption of 75% transmission.* It is

*In this hypothetical case of an all-convective atmosphere there must be, between the tropopause and the infinitely thin stratosphere a sudden temperature increase equal to the above values, for, with convective equilibrium, the temperature of the tropopause must be zero (T-p0.288, p=0). That the above temperatures do not differ more from the observed temperature at the top of the stratosphere is due to the fact that the radiation from the stratosphere is small for a wide range of temperatures.
only this phase of Gold's work which is analogous to Humphrey's, while otherwise the two investigations do not overlap.

Gold then computes the absorption and radiation of an isothermal stratosphere extending down to $1/2 \, p_0$ and finds that there is an excess of emission over radiation in all four cases, too large to be balanced by the absorption of solar radiation. This means that the bottom layers, where this excess has its maximum value, will cool and cause an extension of the convective state to higher elevations.

His next step is to consider the heat balance of a stratosphere extending only down to $1/4 \, p_0$. He finds that were the stratosphere in convective equilibrium too, its radiation would be less than the absorption. (This is in agreement with the discussion on page 18). If, however, the stratosphere is isothermal, then emission does not exceed absorption (apart from the absorption of solar radiation). The actual values of the absorption, excluding solar radiation, are

$$0.10\pi I, 0.275\pi I, 0.13\pi I, 0.32\pi I,$$

while the radiation is

$$0.07\pi I, 0.28\pi I, 0.09\pi I, 0.37\pi I.$$

Here $\pi I$ is the black body radiation of the surface of the earth. The slight excess of radiation over absorption in the second and fourth cases, which correspond to the assumption of 95% transmission, is, according to Gold, balanced by the absorption of solar radiation. The emission of the convective layer extending from $p_0$ to $\frac{3}{4}p_0$ is found to exceed the absorption and this excess must be balanced by the slight absorption of solar radiation and by non-radiational heat supplies. Moreover, it is found that in the region between $\frac{1}{2}p_0$ and $\frac{3}{4}p_0$ the excess of radiation over absorption is very small and Gold concludes that convection must be of slight intensity there.

We shall now review two comments on this paper by R. Emden [10] and E. A. Milne [13]. Gold arrived at the conclusion that in the case of an isothermal atmosphere heated by black body radiation from the surface of the earth and by solar radiation from above"the temperature of the isothermal state
must be such that a full radiator at that temperature would radiate with an intensity equal to the average vertical component of the intensity of solar radiation."

Emden proved that a selectively absorbing isothermal atmosphere cannot be in radiation equilibrium unless it is irradiated by black body radiation of its own temperature from both sides. From this he concluded that our atmosphere cannot be isothermal. The proof is based on Schwarzschild's theory of radiation transfer (see page 50); he also pointed out how Gold was led to the incorrect conclusion.

Another difficulty in Gold's theory was treated by Milne from another point of view. He proved that while the isothermal layer extending to \( p = \frac{1}{2} p_o \) is as a whole in radiation equilibrium its upper portions emit more and its lower portions emit less than they absorb. This would disturb the isothermal state. By applying Schwarzschild's equations and by assuming grey radiation in the infra-red region involved, he could show that the excess of emission over absorption at the top just equals the excess of absorption over emission at the bottom. One can arrive at his results by elementary considerations. Let the intensity of the diffuse radiation incident at the bottom of the isothermal layer be \( I_o \) and let the fraction \( a \) be absorbed while passing through it. As the layer is isothermal its radiation, equal to both sides, will be \( aI \), where \( I \) is black body radiation at the given temperature. Since the layer as a whole is in radiation equilibrium we must have

\[
a I_o = 2aI; \quad I_o = 2I,
\]

the familiar Humphreys relation. Consider now an infinitesimal layer at the upper boundary having a mass \( dm \) and a coefficient of absorption \( k \) for diffuse radiation. Its total emission is then \( 2I k \ dm \). The radiation current traversing it \( (R) \) is composed of the residual radiation \( I_o (1-a) \) and of the radiation of the isothermal layer, \( aI \):

\[
R = I_o (1-a) + aI = I(2-a)
\]

The net gain, absorption minus emission, is therefore
(2I - aI - 2I) kdm = -aIkdm.

A similar infinitesimal layer at the bottom will be traversed by $I_0$ and the downward radiation from the isothermal layer,

$$R' = I_0 + aI = 2I + aI$$

Thus we have for the net gain

$$(2I + aI - 2I)kdm = aIkdm,$$

which is just the negative of the value found for the top layer. Moreover, Milne argues, as this net gain is a continuous function it will be positive in the uppermost layers of the convective region. Since, according to Milne, this region receives heat by convection from below its temperature will rise rapidly and thus build up a temperature gradient in the lower part of the stratosphere. Hence, unless $a$ is vanishingly small (meaning minute quantities of absorbing matter in the isothermal layer) the isothermal state is impossible.

Finally, it should be mentioned that Gold's crucial assumption that convection carries heat upward only is untenable in the light of the modern theory and observations on atmospheric turbulence. The principal result of that theory is that for the prevailing less-than-adiabatic lapse rates heat is always carried downward by turbulence. This flow is partially, but apparently not completely, compensated by the transport of water vapor (latent heat) to higher levels. Even in low latitudes where a state of conditional instability and a net transport of heat upward seem to be characteristic of the lower troposphere the situation appears to be reversed from an elevation of about 5 km and up [24].

Thus, the validity of Gold's results may be questioned. His theory, however, leaves us with a mathematical apparatus which eventually will have to be used in the final solution of the problem of the heat balance in the atmosphere. His careful analysis of the data then available on atmospheric absorption is exemplary. The importance of his elaborate and severe attack on the problem, should, therefore, not be underestimated because of the only apparent victory.
IV. EMDEN'S THEORY OF RADIATION EQUILIBRIUM AND ATMOSPHERIC RADIATION.

Considerable progress towards the understanding of the heat balance in the atmosphere was made in 1913 by Emden in his celebrated paper on the Theory of Radiation Equilibrium [10]. We saw that under the existing conditions of selective absorption, and on account of the qualitative difference between solar and terrestrial radiation an isothermal atmosphere would not be in radiation equilibrium. However, were the atmosphere grey over the whole spectrum, that is equally sensitive to short wave and to long wave radiation, it would assume uniformly the effective temperature of the solar radiation. As this isothermal state would have high mechanical stability and a minimum of convection it is obvious that, starting from any initial condition, the atmosphere would tend towards the isothermal state and eventually assume it.* Since, however, the observed temperature distribution is not uniform, we conclude that the assumption of grey absorption represents too crude an approximation and must be abandoned. After this unfortunate experience with grey absorption one naturally becomes curious as to the temperature distribution at radiation equilibrium under the existing conditions of selective absorption. For, should this temperature distribution be thermodynamically stable, it would be the first to be established. To carry out an exact solution of the problem was unfeasible at Emden's time because of lack of data on the coefficients of absorption of the atmosphere. Also, even if such empirical data had been available, the mathematical difficulties appeared to be unsurmountable. Emden therefore, used the following scheme: he considered the solar short wave and the terrestrial long wave radiation currents separately and assigned to each a uniform characteristic coefficient of absorption ($k_1$, $k_2$). The values of these coefficients he determined from Abbot's and Fowle's early (1908) estimates [25], that the atmosphere transmits 0.9 of the solar radiation not reflected back to space and 0.1 of terrestrial radiation. As the absorption of both kinds of radiation is due primarily to water vapor Emden further simplified the problem by considering this the only absorbing element.

*This argument does not, as Emden claims, invalidate Gold's results, for Gold nowhere assumes the absorptivity for short wave to be the same as for long waves, but considers the atmospheric absorption of solar radiation as a secondary heat source.
With these assumptions, the fraction of vertical radiation absorbed by an elementary layer of thickness $dz$ would be

$$kdm_w = k\rho_w dz,$$

(1)

where $\rho_w$ is the water vapor density and $k$ is a factor of proportionality (coefficient of absorption) to be determined later. Emden however, uses instead of (1) the expression

$$k'fdm = k' f\rho \, dz,$$

(2)

in which $\rho_\alpha$ is the total density of the air, $f$ the water vapor pressure*, and $k'$ is another constant of proportionality, a step which at first sight does not seem to be justified. However, since, very closely,

$$f = \frac{8}{5} \frac{\rho_w}{\rho_\alpha} p$$

(p = total pressure)

(2) can be written

$$k'f \, dm = \frac{8}{5} k' \rho \rho_w \, dz,$$

(3)

which differs from (1) by the proportionality of the absorption on the local total pressure. The factor $8/5$ in (3) is immaterial, for the constants are in each case determined from the known value of the total transmissions,

$$-\int_0^\alpha k\rho \, dz \quad -\int_0^\infty \frac{8}{5} k' \rho \rho_w \, dz$$

or

We know that actually the coefficient of absorption varies with total pressure and whatever evidence there is for this effect in water vapor absorption seems to support the implicit assumption involved in (2). In fact, in correlating certain empirical data on the radiation of the atmosphere obtained at different elevations, A. Ångström [20] found that such an assumption was necessary.

*Although Emden talks explicitly about water vapor density, he uses Hann's empirical formula [38] for the distribution of water vapor pressure and the latter's proportionality to the cube of the superincumbent airmass.
The next feature is to choose a suitable mathematical apparatus to handle this idealized situation. With the above approximations it would be superfluous to use Gold's exact equations of radiation transfer with their serious analytical difficulties. These complications arise from the fact that an elementary horizontal layer within the atmosphere receives radiation of an intensity that varies with direction, since the length of the radiating columns increases with zenith distance. Moreover, this anisotropy of the incident radiation is itself a function of the elevation of the elementary layer considered; also, the path of the incident radiation through this layer is proportional to the secant of the zenith angle. Were the incident radiation everywhere uniform, we could avoid the latter difficulty by assuming all the radiation to propagate along the vertical direction and by choosing a mean effective value for the path through the layer. It can easily be shown that if an infinitesimal horizontal layer absorbs a fraction \( k \) of vertical radiation, it absorbs a fraction \( 2k \) of uniformly diffuse radiation. The substitution of parallel for diffuse radiation involves, then, only a doubling of the coefficient of absorption. It is this scheme that Emden adopts and it is the one which was first developed by K. Schwarzschild in 1906 [7]. That the approximation of uniform radiation is tolerable we can see from the data on the absorption coefficients of water vapor that became available in 1917 and 1918. As mentioned before, these data show two predominating regions of total absorption and of total transparency with a narrow intermediate region. Now, for wave lengths of strong absorption, even the vertical beam of the radiation incident on the elementary layer will originate within the neighboring skin-layer (it will have the intensity of black body radiation at its temperature) and this applies a fortiori to all other directions. The only deviation from the state of uniform irradiation will be introduced by the radiation in the semi-transparent region.

Special attention must be given to the solar radiation which Emden assumes to be diffuse for the globe as a whole. Let us consider the mean position of the earth to be one in
which the equatorial plane and the ecliptic coincide (zero declination). Then the solar irradiation of the earth's surface would be uniformly diffused if the total area illuminated per unit time by radiation of a given obliquity were independent of this obliquity. Actually the area illuminated at an angle enclosed by $\alpha$ and $\alpha + d\alpha$ is proportional to $\cos \alpha \cdot d\alpha$, so that, at the top of the atmosphere, a greater portion is irradiated by vertical ($\alpha = 0$) than by oblique radiation. However, this variation in obliquity is of importance only when one considers the heat balance as a function of latitude. For the globe as a whole Emden rightly uses Abbot's estimated values for the absorption of solar radiation in which the difference in length of path was duly taken account of.

We have then, according to Emden, an atmosphere stratified in horizontal layers (the earth's curvature can be neglected), traversed by diffuse short and long wave radiation currents. The down-going current consists of the solar radiation $B_1$ and the long wave radiation $B_2$ from the superincumbent atmospheric layers; the upgoing current $A_2$ consists of long wave radiation only, originating at the surface of the earth and in the underlying atmosphere. Each of these currents, while passing an elementary horizontal layer of mass $dm$ is weakened by the absorption and strengthened by the self-radiation of that layer. The fraction absorbed is $k_1 dm$ or $k_2 dm$, where $k$ is the coefficient of absorption. This coefficient is, in Emden's scheme, proportional to the local water-vapor-pressure which according to a formula by Hann, is roughly proportional to the cube of $m$, where $m$ is the mass of a vertical air column of unit crosssection extending down from the top of the atmosphere to the level considered. Remembering that $m$ is counted positive downward we have the following differential equations of radiation transfer:

\[
\begin{align*}
\frac{dB_1}{dm} &= -k_1 B_1 \\
\frac{dB_2}{dm} &= -k_2 B_2 + k_2 E \\
\frac{dA_2}{dm} &= +k_2 A_2 - k_2 E
\end{align*}
\]
E represents black body radiation at the local temperature. The solution of (4) is
\[
\int_0^m k_1 \text{d}m = \sigma e \cdot \int_0^m k_2 \text{d}m,
\]
in which \( \sigma \) is the mean intensity of the not reflected part of the solar radiation at the top of the atmosphere. Since \( E = sT^4 \), we can solve (5) and (6) if we know the temperature as a function of elevation or of \( m \), obtaining,
\[
\int_0^m k_2 \text{d}m = \int_0^m k_1 \text{d}m = \int_0^m k_3 \text{d}m = \int_0^m k_4 \text{d}m
\]
(8)
\[
A_2 = \sigma e - e \cdot \int_0^m E \cdot e \cdot k_1 \text{d}m
\]
(9)
In (8,9) we have imposed the boundary condition that the outgoing long wave radiation at the top of the atmosphere equals \( \sigma \) and that the incoming long wave radiation there is zero.

For \( m = 0 \), \( A_2 = \sigma \) and \( B_2 = 0 \).

However, Emden's main problem is just the reverse: by imposing the condition of radiation equilibrium he wants to calculate the vertical temperature distribution in the atmosphere. The condition of radiation equilibrium implies that every elementary layer radiates as much as it absorbs. This can be expressed by the equation:
\[
2k_2 E = k_2 B_2 + k_1 B_1 + k_2 A_2
\]
(10)
which together with (4, 5, 6) forms a system of four equations for the determination of the four unknowns \( E, A_2, B_2 \) and \( B_1 \).

Emden first derives an important characteristic of a system in radiation equilibrium, which is found also in cases of selective absorption and anisotropic radiation (it is an integral of the exact equations of radiation transfer). By adding (4) and (5) and subtracting (6) from the sum, we obtain
\[
\frac{d}{dm} [ \text{Bi} + \text{B}_2 - \text{A}_2 ] = -k_2 [ \text{A}_2 + \text{B}_2 ] - k_1 \text{B}_1 + 2k_2E,
\]

which vanishes on account of (10). The integral

\[
\text{Bi} + \text{B}_2 - \text{A}_2 = \text{constant}
\]

(11)

means that the difference of the downgoing and upgoing currents is constant throughout the atmosphere. Much in the same way as in the case of heat transfer by molecular conduction, it is possible to have a state of radiation equilibrium with steady temperatures but at the same time with a steady net transport of heat. This phenomenon was first discussed by Schwarzschild and applied to the sun. However, since in Emden's case the two radiation currents are equal at the top of the earth's atmosphere they must be equal throughout. Now, of the total downward radiation the solar beam Bi suffers absorption at a rate which depends on the distribution of the water vapor, but not on the temperature distribution. As we descend through the atmosphere the thickness of the layer overhead increases at the expense of the underlying one, favoring B2 over that part of A2 which is due to atmospheric radiation, while the part of A which is due to the transmitted radiation from the surface of the earth increases. The condition of radiation equilibrium demands such a temperature distribution that the net long wave radiation current depending on it, \( A_2 - B_2 \), shall equal Bi, or

\[
A_2 - B_2 = B_1 = \sigma e \int_0^m \kappa dm
\]

(12)

We notice that the temperature of the surface of the earth, which determines the value of A2 at the ground and hence also for m = 0 is not arbitrary but must have such a value that the upper boundary condition (9') is satisfied. In other words, a definite long wave radiation current must emerge at the top of the atmosphere of which one part equals 1/10 of the radiation of the surface of the earth and the remainder originates within the atmosphere. The former part is, then, to be adjusted in accordance with the temperature distribution and the resulting upward radiating power of the atmosphere.
The temperature distribution can be obtained from $E_2 = sT^4$ by (10) if we know $A_2+B_2$ as a function of $m$. This can be obtained by adding (5) and (6) and substituting for $A_2-B_2$ the equation (12).

$$\frac{d}{dm}(A_2+B_2) = k_2(A_2-B_2) = k_2 \sigma e^{-\frac{m}{k_2}} = k_2 B_1,$$

which, with the proper boundary values, integrates into

$$A_2 + B_2 = \sigma + \int_0^m k_2 B_1 dm. \quad (13)$$

The integral equation for the temperature distribution at radiation equilibrium becomes

$$2E_2 = \sigma + \int_0^m k_2 B_1 dm + \int_0^m k_1 dm \left( B_1 = \sigma e^{-\frac{m}{k_2}} \right) \quad (14)$$

where $k_1$ and $k_2$ are functions of $m$.

In Emden's case

$$k_1 = b_1 m^3, \quad e^{\frac{m}{k_2}} = 0.9; \quad k_2 = b_2 m^3, \quad e^{\frac{-m}{k_2}} = 0.1; \quad \frac{k_1}{k_2} q = \text{constant} \quad (15)$$

Thus,

$$\int_0^m k_2 B_1 dm = \int_0^1 k_1 dm$$

and

$$2E = \sigma \left[ \int_0^1 \left( e^{-\frac{m}{k_2}} \right) + \int_0^m k_1 dm \right]$$

$$= \sigma \left[ \frac{q+1}{q} \left( 1 - (1-q) e^{-\frac{m}{k_2}} \right) \right] = \sigma \frac{q+1}{q} \left[ 1 - (1-q) e^{-\frac{b_1 m^4}{k_2}} \right]$$

(16)
Applications: The boundary temperatures are easily obtained by inserting in (16) \( m = 0 \) and \( m = 1 \), respectively. This gives

\[
T_0 = 288.8^\circ + 15.8^\circ C, \quad T_\infty = T_1 = 213.7^\circ = -59.3^\circ C.
\]

Of these, the upper boundary temperature, given by

\[
T_1 = 254 \sqrt{\frac{1 + \frac{k_1}{K_2}}{2}},
\]

is doubtful because water vapor was assumed to be the only absorbing element, while actually, due to its scarcity at high elevations, it may become subordinate to carbon dioxide and the inert gases. At any rate, the value of \( q \) which was obtained from the absorption of the whole atmosphere is certain to be underestimated for high elevations and this deviation, Emden argues, most likely increases with elevation. If, then, \( q \) increases from the bottom of the stratosphere upward, (17) would give an increasing temperature in the same direction, as seen in the accompanying table computed by Emden.

**TABLE III.**

<table>
<thead>
<tr>
<th>( q )</th>
<th>( T_1 - 273 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-19°C</td>
</tr>
<tr>
<td>1/2</td>
<td>-35</td>
</tr>
<tr>
<td>1/5</td>
<td>-49</td>
</tr>
<tr>
<td>1/10</td>
<td>-54</td>
</tr>
<tr>
<td>1/23</td>
<td>-57</td>
</tr>
<tr>
<td>0</td>
<td>-59</td>
</tr>
</tbody>
</table>

The temperature of the surface of the earth, \( T_e \), is obtained from

\[
sT_e^4 = A_\infty(\text{ground})
\]

The latter quantity can be obtained by adding (12) and (13),

\[
2A_\infty = \sigma \left[ 1 + \frac{1}{q} - \left( \frac{1}{q} - 1 \right) e^{-\frac{b_1 m_4}{2}} \right], \quad A_\infty(\text{ground}) = 2.1^\circ.
\]
Substituted in (18) we have,

\[ T_e = 254 \sqrt{2.1} = 309^\circ A = +36^\circ C, \]  

(19)

giving a temperature drop of 20°C between the surface of the earth and the atmospheric surface layer.

The most important feature of the results so far obtained will be disclosed as soon as we investigate the thermodynamical stability of the atmosphere at radiation equilibrium. To simplify calculations Emden develops the exponential function in (16) in series and retains only the first power in \( m^4 \). If \( T_i \) denotes the inversion temperature given by (17), the approximate formula becomes

\[ T^4 - T_i^4 = \left[ 1 + (b_2-b_1) \frac{m^4}{4} \right] \]  

(20)

Let \( z \) represent the vertical coordinate, counted positive downwards.

Then

\[ \frac{dp}{\rho} = g \rho dz, \quad p = R T \rho, \quad p = mg, \]

\[ \frac{dp}{\rho} = \frac{dm}{\rho} = \frac{gdz}{RT}. \]

Combined with (20), differentiated logarithmically, it gives

\[ \frac{T^4 dT}{T^4 - T_i^4} = \frac{dm}{m} = \frac{gdz}{RT} \quad ; \quad \frac{dT}{dz} = \frac{g}{R} \left[ 1 - \left( \frac{T_i}{T} \right)^4 \right] \]  

(21)

For stability we must have

\[ \frac{dT}{dz} < \frac{g}{5.5 R}, \text{ or } T < T_i \sqrt{\frac{5}{7}} = 234.8 = -38.2^\circ C \]  

(22)

Thus, in the atmosphere below the level of the temperature 
\(-38.2^\circ C\) we would have superadiabatic lapse rates. To complete the picture Emden integrates (21) and obtains \( T \) as an explicit function of elevation. The result is shown in table IV.
In order to keep the atmosphere above 3130m in radiation equilibrium the temperatures below must increase towards the ground at a higher rate than is compatible with thermodynamical stability. Were not the physical structure of this whole theory so schematic we should agree with Emden that therein lies the cause for the convection in the troposphere. The breakdown of instability below 3130m. would bring about a near adiabatic equilibrium, while the transport of water vapor upwards would carry away heat from the surface of the earth and help wipe out the temperature discontinuity at the ground*. This process will bring about a cooling of the ground and a heating throughout the convective region, which, as can easily be seen, must extend above the level of 3130m. Emden does not calculate the limit of the convective region because he would have to consider the effect of the general circulation in the atmosphere. Now, Emden argues, the temperatures of the atmosphere above the convective region will still be given by(16) or, on account of the small vapor content there, by(17). For, the downgoing current crossing the boundary will not be effected by the convection below and, as it must equal the upgoing current, the radiational environment of the upper atmosphere will remain unchanged. While this argument is essentially correct it may appear to be too formal. If, however, we

*As long as superadiabatic lapse rates prevail turbulence will also carry heat upward.
stress the fact that the radiating matter in the non-convective region is small enough so as to contribute very little to the earth's radiation to space, we see immediately that (17) is simply Humphreys' relation, corrected for the slight absorption of solar radiation. In this case the long wave radiation entering the stratosphere will practically equal the current emerging to space, the latter being fixed by the insolation and the albedo, independently of the temperature distribution within the atmosphere. In short, as stated by Emden, the change in temperatures within the convective region must be such that the resulting upward radiation equals its original value.

This process of transition from a state of radiative to convective equilibrium Emden claims to be realized in nature in the local thunder-storms. While the usual notion is that the high lapse rates established before the development of such local disturbances are due to heating from below, he emphasises the equally possible cooling from above, as shown by the low upper air temperatures in the case of radiation equilibrium. Emden claims that one can thereby explain the development of such disturbances over the tropical ocean, where the surface temperature is practically constant, as well as the maximum occurrence of ocean storms in the second half of the night, when the cooling from above has lasted for a long time.

As to the variation of the stratosphere temperature, Emden recognizes the relevant factors which should be included in his general scheme. He admits that the assumption of a uniform distribution of solar radiation over the globe is incorrect. Furthermore, one cannot account completely for the latitudinal variation by assigning to σ values proportional to the insolation for the ever-existing latitudinal heat-transport by the general circulation tends to diminish the latitudinal variation in heat supply. As to the seasonal variation of stratosphere temperature he points out the important fact that the increased water vapor content of the stratosphere during the summer with the resulting low value of q counteracts the reverse effect that would follow from the increased radiation currents.
The last part of his paper Emden devotes to the problem of computing the radiation currents and radiation balance for a given temperature distribution. If the emission $E$ is a known function of $m$, then, semi-grey radiation assumed, the currents $A_2$ and $B_2$ can be obtained by solving (5) and (6). The initial values are determined from the fact that $B_2$ is zero for $m=0$ and that $A_2$ equals the radiation of the surface of the earth (determined by its temperature) for $m=1$. The excess of radiation over absorption is, then, for every level

$$dQ = 2k_aE - k_aB_a - k_aA_a - k_1B_1$$

We shall not give an account of these calculations by Emden, since they were later repeated by Hergesell under a more satisfactory assumption with respect to the distribution of water vapor with elevation. Certain of Emden's results are, however, worth noticing. He first considers the cooling at night of an isothermal atmosphere and finds, as we did in accounting for Milne's criticism, that every layer extending from the top of the atmosphere to any level within looses heat at a rate equal to its radiation to space, which is merely the absorption coefficient of the layer multiplied by black body radiation at its temperature. He also derives the result that in such an atmosphere there is an internal layer of maximum cooling. The significance of this fact, which he found to hold in any polytropic atmosphere, was later more fully recognized by Hergesell.

Next of importance are Emden's considerations of the heat shielding effect of the atmosphere. First we want to remember that the atmosphere not only furnishes the ground with a current of long wave radiation but it also adversely affects the incoming solar radiation by reflection at clouds and by upward molecular scattering. When we talk of the shielding effect of the atmosphere we mean its modifying influence on the temperature of the surface of the earth. To understand the general problem which we are treating, it is of great importance to acquire a clear conception of the physical causes for the existence of this shielding effect.
Seeing the atmosphere absorb only a fraction of the solar radiation, one may naturally wonder how this atmosphere, radiating on both sides, can reemit towards the earth's surface more than it thus absorbs. While Emden, more than anybody else, searched for the elementary physical processes, which would account for this strange behaviour, it would seem that he did not quite succeed in eliminating completely the mystery surrounding it. The following comments may throw some additional light on the subject. To begin with we must rid ourselves of the notion that transformations of large quantities of radiant energy are always associated with high temperatures. There certainly is no one-to-one correspondence between these two quantities in steady or equilibrium states and especially not in radiation equilibrium. A definite quantity of absorbable short wave radiation incident on an opaque surface being given, the temperature assumed by that surface at radiation equilibrium will depend on its radiating power for long waves. The lower this radiating power the higher will be the equilibrium temperature and vice versa. If a surface is to radiate a given quantity of energy and by some means its radiating power is diminished it will necessarily have to assume a higher temperature. Now, primarily because of atmospheric transparency to solar radiation and because of the resulting heating from below and the cooling from above, the temperature decreases with elevation. Starting from the surface layers, every layer absorbs a fraction of the upgoing radiation and contributes to it a smaller quantity due to its lower temperature, with the result that this upgoing current is continuously diminished with elevation. The radiation emerging from the system earth plus atmosphere is, thus, less than the original radiation of the surface of the earth, but at the same time more than the radiation of the uppermost layers, because part of the outgoing radiation was contributed by warmer layers from below. As far as the total outgoing energy is concerned, it corresponds to the radiation of some layer within the atmosphere (having the effective temperature of the outgoing radiation). Hence, under the particular terrestrial conditions of heating from below the
effective temperature of the earth's radiation must be less than the ground temperature. If the system as a whole is to radiate the same quantity of energy towards space as in the absence of an atmosphere, the temperature of the ground must be raised. That under any conditions the mean radiation of a planet must equal the radiation absorbed from space is obvious from the fact that the continuous and monotone temperature change in the unsteady state would regulate the radiating power of the planet until its output equals the radiation received. Obviously, too, if the atmosphere had a constitution such that the absorption of solar radiation mainly took place at its top, the temperature would in the extreme case increase with elevation and the effect of the atmosphere would be to lower the temperature of the surface of the earth.

So far we have only considered the effect of the atmosphere on that part of solar radiation which remains after reflection. Emden makes the following interesting investigation of the combined effect of the atmosphere as a reflector, scatterer, absorber and radiator. He computes, for several latitudes, the temperature which the surface of the earth would assume if it were to reradiate the mean annual insolation, assuming a solar constant of 2.0 g. cal./min/cm$^2$. These values he compares with the observed mean temperatures and attributes the deviation entirely to the intervening atmosphere, which, outside of its effects enumerated above, also acts as an heat conveying agent from lower to higher latitudes. In the same way he computes separately the temperatures corresponding to the mean insolation of the summer-half-year and the winter-half-year and compares them with the observed temperatures for July and January, respectively [table V.]
As seen in the accompanying table the heating effect of the atmosphere in all its capacities is negative at the equator and positive everywhere else. Most interesting is the intense atmospheric shielding during the winter. Emden suggests that the heat necessary to maintain the relatively high temperatures at high latitudes during winter is supplied from lower latitudes through the general circulation, which is especially intense then.
V. HERGESELL'S INVESTIGATION OF RADIATION EQUILIBRIUM AND THE HEAT BALANCE OVER LINDENBERG.

One sees that with Emden's paper a host of new ideas were introduced. For the first time a non-empirical quantitative study of the temperature distribution in the atmosphere had been undertaken with methods which were not too complicated and yet seemed accurate enough for the purpose. The assumption of semi-grey absorption was simple and entirely new. Though the numerical values obtained agreed fairly well with observations, the lasting significance of Emden's work lies entirely in its methodical features. Thus, the whole idea of radiation equilibrium, associated with a temperature distribution which, on account of its possible thermodynamical instability, may be the cause of the convection in the atmosphere, will be helpful to the investigator studying the heat balance in the atmosphere. Moreover, the quantitative treatment of the shielding effect of the atmosphere was original and instructive and precipitated a number of similar investigations.

With these good points, Emden's theory suffered from a serious defect which was pointed out in 1919 by Hergesell [11]. In a problem like Emden's, where a temperature distribution is to be determined from a given moisture distribution, some provision must be made to insure that the final temperatures shall not be so low as to give supersaturation for the already adopted moisture distribution. To the condition of radiation equilibrium must be added the condition of non-saturation. This limiting condition may be made more definite on account of the observed correlation between humidity and temperature in the free atmosphere. Hergesell found that Hann's formula for the distribution of water vapor pressure as a function of air mass disagrees widely with observations, a conclusion at which Fowle had previously arrived from spectroscopic observations [27]. As a substitute Hergesell [28] offered the formula

\[ f = f_0 e^{\beta \frac{t}{T}}, \quad \beta = 23.56, \]  

(1)
in which \( f \) is the local water vapor pressure, \( f_0 \) its value at the level of \( 0^\circ C \), \( T \) the absolute temperature and \( t \) the same temperature on the Centigrade scale. It is a modification of Thiesen's formula for the saturation water vapor pressure and gives decreasing relative humidity with decreasing temperature.

That the adoption of some such formula as (1) is necessary can be seen from the fact that Hergesell, in calculating the relative humidities corresponding to Emden's solution, found supersaturation values in all but the surface layer. The formula (1) implies that the moisture distribution adapts itself to the temperature distribution. Hergesell set out to recompute the temperature distribution at radiation equilibrium by accepting the empirical formula (1) for the moisture distribution instead of Hann's formula. This, of course, complicated matters considerably, for instead of a single relation between the fixed moisture distribution and the resulting temperature distribution a system of interlinking relations between the two quantities is obtained, out of which both are to be determined. More definitely, we are now looking for a temperature distribution and a moisture distribution at radiation equilibrium.

Obviously the new equations can be obtained by substituting for \( f \) its expression in terms of the temperature, equation (1). This was done by Hergesell, but in carrying out the detailed calculation he committed an error which invalidated his results. In the determination of the constant \( b \) from the equation

\[
-e^{-b_2 \int_0^1 f \, dm} = 0.1 ,
\]

which expresses the fact that the atmosphere transmits one tenth of terrestrial radiation, the integral

\[
\int_0^1 f \, dm = \int_0^{f_0} e^{\frac{T}{f}} \, dm
\]

is now a function of the sought temperature distribution and must therefore be considered as one of the results of the calculation. Hergesell adopted for it a value determined
from the observed mean temperature and moisture distribution over Lindenberg. Since the troposphere over Lindenberg is not in radiation equilibrium the integral (3) proved to be \(17.3\) times too large \([12]\). As this overestimate makes \(b_2\) correspondingly too small it may be interpreted, in a certain sense, as an assumption of a very high atmospheric transparency resulting in a negligible temperature difference between the two boundaries of the atmosphere. In determining the temperature distribution Hergesell assumed that the temperature at the elevation for which \(m = 0.19\) has the observed value of \(-54.6^\circ\text{C}\). As was to be expected, he obtained a very low temperature next to the ground \((-54.1^\circ\text{C})\). Had he introduced instead the observed surface air temperature as an empirically given constant he would have obtained a correspondingly much warmer isothermal atmosphere with the stratosphere only about half a degree colder than the surface layer. His conclusion that under the assumption of an automatic adjustment of the moisture distribution to the prevailing temperatures, the temperature distribution at radiation equilibrium would be isothermal appeared to invalidate Emden's results. Hergesell's result was largely accepted as final \([29]\), and one may perhaps attribute to this fact the neglect of meteorologists to investigate further the temperature distribution at radiation equilibrium*.

The remaining part of Hergesell's paper contains a number of interesting results. He computed the radiation currents and the radiation heat balance in the atmosphere over Lindenberg from the observed mean values of temperature and water vapor pressure. We have already emphasized the fact that in the Emden-Schwarzschild scheme the initial value of the ascending long wave current \((A_2)_{\text{ground}}\) is determined by the condition that the part of the radiation from the surface of the earth which is transmitted to space plus the total upward radiation of the atmosphere must equal the mean intensity of the insolation \((\sigma)\). Obviously, if the temperature distribution is such that the ascending atmospheric radiation is more than \(\sigma,A_2\)_{ground}

\*Milne, who did write an extremely valuable paper on this subject in 1922 \([13]\) was apparently unaware of Hergesell's work.
would have to be assumed to be negative, a conclusion which we would be reluctant indeed to accept. Hergesell was confronted with just this difficulty in the case of the atmosphere over Lindenberg. Having found that no plausible changes in the values of the albedo and the atmospheric transmission would give him a positive value for he concluded that the radiative power of the atmosphere had been overestimated. He then found that a small diminution of the exponent 4 in Stefan's law would make $A_2$ positive in the surface layers. While this idea is in agreement with our previous conclusion with regard to the radiative power of the stratosphere (see page 13), we must not be too hasty in accepting it. To begin with, the smallness of the correction in the exponent is only apparent, for not only do we want to make $A_2$ positive in the surface layers but, since in an atmosphere with an upward decreasing temperature the radiation current, even in the case of selective absorption, must decrease upwards, it also must have its maximum value there and, thus, be at least by $\sigma$ greater than 0, if $\sigma$ represents the net outgoing radiation. Secondly, we have no reason to force the outgoing radiation at Lindenberg to equal the annual mean insolation there. Emden repeatedly emphasized and strikingly demonstrated the effectiveness of the general circulation as a heat conveying agent. Thus, at the latitude of Lindenberg ($50^\circ$N) one should expect more energy radiated away to space than is received there. Furthermore, while in Emden's case of radiation equilibrium the temperatures were determined from certain conditions imposed on the radiation currents, we are now dealing with the reversed problem of determining the actual radiation currents from the observed temperatures. Thus, the boundary conditions ought to be determined from the observed temperatures and in no other way. If, after that, it should turn out that the atmospheric radiation to space differs from $\sigma$, that in itself would be an important result of the calculation.

In his calculations Hergesell actually assumed $A_2$ to equal black body radiation at the temperature of the surface air layer. He divided the atmosphere in a number of layers
each having small optical thickness and computed the radiation current by assigning to each a mean temperature. If the values of the optical masses

\[ x = \int_{0}^{m} k dm = \int_{0}^{m} b f dm \]

at the bottom and the top of a layer are \( x' \) and \( x'' \) respectively and if the value of the ascending current at the bottom is \( A_{2}' \), then its value at the top will be

\[ A_{2}'' = A_{2}' \ e^{-(x'-x'')} + E(1-e^{-(x'-x'')}) \]

In a similar way the downgoing currents \( B_{2} \) and \( B_{1} \) can be computed. For the mean insolation he assumes the value of \( A_{2} \) at the top of the atmosphere.

The points of interest brought out are the following:
There is a net loss of energy by long wave radiation (corresponding to conditions at night) extending up to 3 kilometers, while the gain of energy from there on upwards is small enough to be negligible. If the absorption of short wave radiation be included we still find a net loss of energy by radiation in a layer extending from the ground up to about the same elevation and having a maximum at the altitude of 1.42 kilometers. This implies that as far as radiation balance is concerned the temperature at 1.42 kilometers is abnormally high; consequently this level receives a maximum amount of energy from non-radiating sources. This fits in nicely with the distribution of latent energy released by condensation processes and the above level of maximum cooling is to be identified with the principal condensation level. These considerations of Hergesell throw some new light on the nature of the condensation level. It is usually regarded as the level to which adiabatically cooled air must be raised from the ground to become saturated. We now see that even in the absence of convection, or turbulence, a level of excessive cooling and, consequently, condensation, exists in the atmosphere, at about 1.4 km.
Similar calculations were carried out by Hergesell for Batavia, a typical tropical station. A net loss of heat by long wave radiation is found throughout the troposphere with a maximum at 3850 meters. The absorption of solar radiation is here quite large due to the high moisture content, so that in the surface layer it almost compensates the net loss of heat by long wave radiation. When the former is subtracted from the latter we get the heat supplied at each level by other than radiational sources and this quantity is found, again, to have a maximum at 3850 meters. The condensation level thus determined is more than twice as high in Batavia as in Lindenberg, while at the same time the rate of cooling is more intense in Batavia. It is also found that the return radiation of the atmosphere over Lindenberg amounts to 84% of the radiation of the surface of the earth (assumed to have the temperature of the surface air layer), while in Batavia it is 97%.

In the last part of his paper Hergesell uses Ångström's data on the radiation of the atmosphere at Lone Pine (1140 m) and Mt. Whitney (4420 m), both in California, to compute by the Emden-Schwarzschild equations the atmospheric transmission for long wave radiation as well as the magnitude of the long wave radiation coming down from the stratosphere. Ångstrom found from his numerous measurements of atmospheric radiation in several parts of the world that a dry atmosphere would radiate about 50% of that of a black body at the temperature of the place of observation. This radiation is supposed to originate from the ozone layer in the upper atmosphere. Having chosen a day when satisfactory measurements of atmospheric back radiation at both Lone Pine and Mt. Whitney were obtained and upper air soundings in that vicinity were available, Hergesell used the measurements at both stations to determine the above two unknowns. He found for the stratosphere radiation a value comparing favorably with that measured by Ångström, while for the transmission of the atmosphere in the vicinity of Mt. Whitney he found a value of 24% as compared with Abbot and Fowle's early estimates of 10%. (See page 24) With these quantities determined and from the available data on the
temperature and moisture distribution at the above California stations, he computed, as for Lindenberg and Batavia, the radiation currents and the other relevant elements. The boundary between the regions of loss and of gain of heat by long wave radiation was found at a lower elevation than before. This result is doubtful, since the back radiation from the stratosphere, originating largely from the ozone layer is concentrated in the region of 10\(\mu\) where water vapor is transparent.

The merit of Hergesell's work lies in his demonstration of the necessity to avoid supersaturation in a theoretically determined temperature distribution and in his analysis of the radiation elements over Lindenberg, Batavia and California. For the first time it was made plausible that there is a net loss of heat by radiation throughout the lower troposphere. The maximum loss was found to occur at a level which, roughly, coincides with the principal condensation level.
At this point it is perhaps proper to remind the reader that there is no particular reason to wax enthusiastic over any results derived or derivable from such approximate methods as those of Schwarzschild and Emden. From the outset we realized that we were confronted with a typical geophysical problem with all its complexities; the analytical treatment was expected to be difficult and to proceed very slowly. When we hear that certain features of this problem can be easily explained, our first reaction is one of scepticism, for, constitutionally, our problem lacks simplicity. Essentially, the weakness of these approximate theories lies in the assumption that certain highly variable functions, such as the coefficient of absorption, the insolation, the moisture distribution and the cloudiness, are constant or can be represented by average values. One then uses a few laws (most of which are either direct results of new formulations of the second law of thermodynamics) to investigate the relative effectiveness of these factors in regulating the thermal state of the atmosphere. The above simplifications make the analytical treatment very flexible enabling us to watch the simultaneous action of a great number of parameters. In this way one succeeds in establishing certain relations between the atmospheric elements (say, the dependence of the temperature on the humidity distribution, the insolation, etc.) which constitute the framework for a more detailed analysis in which the proper variable functions are substituted for the above parameters and the relations correspondingly modified. Considered from this viewpoint all the work discussed so far and much of the following is of a preliminary nature. There is, then, no reason for satisfaction or disappointment when the theory in its early stage is brought face to face with experience. The only demand that can be made on the successive developments of the theory is that the restrictions on the variable parameters shall be progressively removed and that at each stage all the possible consequences shall be brought to light.
The importance of Milne's work [13] which we are going to discuss now lies in his investigation of the variation with latitude of the temperature distribution at radiation equilibrium. The principal factor variable with latitude is the intensity and relative absorption at different levels of solar radiation. Emden was interested in the mean temperature distribution for the globe as a whole; thus it was sufficient for his purposes to use an average value for the solar intensity and to assume it to be uniformly diffuse. Milne had to take into account the latitudinal variation of obliquity of the solar beam. This he could do by a generalized interpretation of Emden's formula

\[ 2sT^4 = \sigma \left( \frac{q+1}{q} \right) \int_0^m k_1 \, dm \]

for the temperature distribution at radiation equilibrium. In (1) \( q \) represents the ratio of the coefficient of absorption for solar radiation \( (k_1) \) to that for terrestrial radiation \( (k_2) \).

As both of these radiation currents were assumed to be diffuse, \( q \) represents also the ratio of the optical masses, the differentials of which are given by

\[ k_1 \, dm \text{ and } k_2 \, dm \]  

The application to Milne's problem is made possible by substituting for \( q \) the ratio of the optical depths. The elements of optical depth (2) vary with the coefficients of absorption as well as with the effective value of \( dm \). It can be shown that the effective value of \( dm \) for vertical radiation is just half the corresponding value for diffuse radiation. Furthermore, if the parallel radiation is inclined at an angle \( \alpha \) with the normal to the surface the effective optical depth is

\[ \frac{k}{2} \sec \alpha \, dm \]

Hence the ratio of the optical depth for the oblique solar radiation to that for the diffuse long wave terrestrial radiation is
If one further remembers that the effective insolation* is now \( \alpha \cos \alpha \), the generalized Emden's formula becomes, according to Milne,

\[
sT^4 = \frac{g}{q} \frac{\cos \alpha + \left(\frac{q}{2} - \cos \alpha\right)e^{-\frac{\sec^m \alpha}{k \text{dm}}}}{\cos \alpha + \left(\frac{q}{2} - \cos \alpha\right)e^{-\frac{\sec^m \alpha}{k \text{dm}}}}
\]  

(4)

If, for the sake of simplicity, we assume the axis of the earth to be perpendicular to the ecliptic, the average diurnal insolation may be easily calculated and the pure latitudinal temperature variation obtained. For a given latitude \( \phi \) and hour-angle \( h \) the zenith distance (\( \alpha \)) is given by

\[
\cos \alpha = \cos \phi \cdot \cos h.
\]  

(5)

If this expression is substituted in (4) we obtain the temperature distribution as a function of the hour-angle and the latitude (instantaneous adjustment of the atmosphere to the current insolation assumed). A 24-hour integration over \( h \) then gives the desired latitudinal distribution of temperature.

One could also introduce the exact relation between zenith distance, latitude, solar declination and hour-angle and by averaging over the latter obtain a seasonal as well as a latitudinal variation of the temperature distribution at radiation equilibrium. It suffices to state that Milne for the first time provided us with a tool for the exact analysis of the latitudinal as well as of the seasonal variations of atmospheric temperatures and of the causes that maintain convection in the troposphere.

Let us now return to equation (1) which for a given latitude has the form (4). It gives the temperature distribution at radiation equilibrium in an atmosphere traversed by two

* \( \sigma \) is equal to \( S(1-A) \), where \( S \) is the solar constant and \( A \) the albedo.
types of radiation, one of which comes from the sun and traverses an optical mass $q$ times that traversed by the other type, the long wave radiation. It is seen that although the boundary conditions are determined so that the radiation to space equals the effective solar radiation, the temperature distribution is not isothermal. If $q$ is less than 1, as it generally is in our case, the temperature decreases upwards, for then the solar radiation penetrates freely towards the earth's surface leaving the upper layers to be warmed by ascending radiation only. When $q$ is equal to one we have isothermal conditions, for then the radiation from above is equally effective as the radiation from below. When $q$ exceeds 1, which in our case will happen when

$$\sec \alpha > \frac{2}{q}$$

the main heating comes from above and the temperature will increase with altitude. This result Milne offers as a partial explanation of the observed latitudinal temperature distribution in the stratosphere. For, although the actual stratosphere temperature given by (4),

$$sT_0^4 = \frac{5}{2} (\cos \alpha + \frac{q}{2})$$

(6)

decreases with increasing latitude, its ratio to the effective temperature of the earth's radiation ($sT_1^4 = \sigma \cos \alpha$) namely

$$\frac{T_0}{T_1} = \left[ \frac{1(1 + \frac{q}{2} \sec \alpha)}{2} \right]^{\frac{1}{4}}$$

(7)

increases very rapidly in the same direction.

Another factor regulating the temperature of the stratosphere, which was for the first time discussed quantitatively by Milne, is the general circulation. Since it carries heat from lower to higher latitudes, the radiation to space must in the former be less and in the latter exceed the local incoming radiation. In fact, if the excess of the outgoing over the incoming radiation per unit time and area is $F$, the stratosphere
temperature is given by

$$sT_0^4 = \frac{1}{2} F + \frac{\sigma}{2} (\cos \alpha + \frac{q}{2})$$  (8)

While these ideas may seem plausible we must not trust them before it is shown that the radiating power of the system earth plus atmosphere varies correspondingly with latitude. We shall see later that from such considerations the modern theory, based on the selectiveness of atmospheric absorption, evolved. Suffice it to mention here that already in 1913 E. Gold (30) argued along similar lines. Gold found from his theoretical investigations of 1909 that the radiating power of the earth plus atmosphere decreases with increasing atmospheric moisture content; thus the long wave radiation available to heat the stratosphere increases with latitude. However, since he did not consider the effect of the general circulation, the above latitudinal distribution of outgoing radiation appeared to him as an improbable result. To get around this difficulty he suggested, in 1913, that there exists in the earth's radiation a region for which the atmosphere is perfectly transparent; thus part of the radiation emanating from the surface of the earth is effective in balancing the incoming radiation but not in heating the stratosphere. Such a region actually does exist and it was even pointed out by Gold in his paper of 1909.

To sum up, Milne points out two factors tending to give a stratosphere temperature increasing with latitude. First, the warming by solar radiation will not diminish with latitude as rapidly as the total insolation; second, the effect of the general circulation is to favor the outgoing radiation in higher latitudes over that in lower latitudes and this is just what we would expect from the outward radiating power of the atmosphere as effected by the latitudinal distribution of water vapor.

In general, it may be said that Milne completely exhausted the possibilities of the theory of radiation equilibrium in a semi-grey atmosphere. Some of the ideas introduced by him were later expressed, apparently independently, by other investigators and were, as we shall see, very stimulating towards further research in this subject.
VII. R. MÜGGE'S CALCULATION OF THE HORIZONTAL HEAT TRANSPORT IN
THE ATMOSPHERE FROM THE TEMPERATURE OF THE STRATOSPHERE.

When we stop to reflect on the work done in the field of
the heat balance in the atmosphere during the first quarter of
this century the most outstanding achievement appears to be
Emden's explanation of the existence of a convective tropo-
sphere from the unstable stratification that characterizes
radiation equilibrium. Artificial as his model atmosphere may
be, we are left with the impression that very likely a similar
unstable stratification will be obtained in the actual atmos-
phere. We have here an instance where the theory of the radia-
tion balance in the atmosphere transcends its natural boundari-
es and throws some light on the phenomena of vertical convec-
tion. Not only does the theory aim to explain the existence
of vertical convection but it suggests the possibility of
estimating the intensity of convection from the degree of in-
stability. Another such application of the theory to a
"weather" problem was made by R. Mügge [14] in 1926 in his com-
putation of the horizontal latitudinal heat transport from the
stratosphere temperatures. Mügge was apparently unaware of
Milne's work and had committed himself, along with Emden, to
semi-grey absorption. Therefore, the increase of the strato-
sphere temperature with latitude could mean to him only that
the outgoing terrestrial radiation varies in the same way un-
less one is willing to consider the very obscure realm of a
variation of \( q \) and of the albedo with latitude. It then fol-
 lows that the difference per vertical column of unit cross-
section between the incoming radiation and the outgoing radia-
tion, which latter maintains the local stratosphere temperature,
equals the net loss of energy due to the horizontal flow
through the walls of the column. This net loss \( (R_\varphi)^* \), which
goes to maintain the general circulation, will be positive at
the equator and negative at the pole, changing sign at some
intermediate latitude. Mügge's principal idea was to estimate

*Mügges \( R_\varphi \) is identical with Milne's \(-F\).
this quantity and others related to it from the observed stratosphere temperatures. Obviously, he had to modify Emden's formula to take account of the latitudinal variation of insolation, the very thing that was accomplished by Milne. In doing so, he took into account the variation of the intensity only and not that of the optical path. In the simplified case of zero declination and of local balance between incoming and outgoing radiation this neglect of the variation in the optical path makes the stratosphere temperature at the pole equal to zero. Actually, as pointed out by Milne, when the solar beam approaches grazing incidence, it is completely absorbed in the upper skin layer and maintains there a finite temperature. In the case of such an infinitely thin layer of mass dm on which light of intensity I is incident at an angle α the fraction absorbed per unit horizontal area is

\[ -\frac{dI}{I} = kdm \cdot \sec \alpha \cdot \cos \alpha = kdm \]

and, therefore, independent of α.

If in Milne's formula (8)

\[ sT_0^4 = \frac{\sigma}{2} (\cos \alpha + \frac{\pi}{2}) - \frac{1}{\sigma} R \varphi \]

we substitute the value for \( \alpha \) obtained from

\[ \cos \alpha = \cos \varphi \cdot \cos h \]

and integrate to obtain a 24-hour average, we obtain the stratosphere temperature as a function of latitude,

\[ sT_0^4 = \frac{\sigma}{2\pi} (\cos \varphi + \frac{\pi}{4} q) - \frac{1}{\sigma} R \varphi \]  \hspace{1cm} (1)

Instead of this Muggge uses

\[ sT_0^4 = \frac{0.363}{2} \cos \varphi (1 + q) - \frac{1}{\sigma} R \varphi \left( \frac{\sigma \cos \varphi (1+q) - \frac{1}{\sigma} R \varphi}{2\pi} \right) \]  \hspace{1cm} (2)

where \( 0.363 = \frac{2(1-0.43)}{\pi} \), 2 being the solar constant, 0.43 the albedo and \( 1/\pi \) the factor obtained by averaging. With the
same values for the solar constant and for the albedo, Milne's constant becomes 0.182. It happens, however, that in the particular application that Mügge makes, (1) and (2) give identical results.

Equation (2) cannot be solved for $R_\phi$ unless $T$ is known as a function of $\varphi$. As the stratosphere temperature is sufficiently well known only for middle latitudes, Mügge assumes it to be a linear function of the sine of the latitude,

$$T_\phi = p + \chi \sin \varphi$$

so that the entire distribution may be determined from the values at two latitudes. One such generally accepted value is

$$T = 220^\circ A = -53^\circ C \text{ for } \varphi = 52^\circ$$

$$p = 220 - 0.789 \ell.$$  

The other relation for the determination of $p$ and $\chi$ he obtains by the following reasoning. The energy contributed by a circular belt of width $rd\varphi$ at latitude $\varphi$ to the general circulation is

$$2\pi r \cos \varphi \cdot rd\varphi \cdot R_\phi$$

and the total energy contributed between the equator and this latitude is

$$\overline{R}_\phi = 2\pi r^2 \int_0^\varphi R_\phi \cos \varphi \, d\varphi$$  (4)

Since in the long run no local accumulation of energy is possible this quantity of energy will cross the latitude circle every minute. $\overline{R}_\phi$ will increase from the equator polewards throughout the region where $R_\phi$ is positive and will begin to decrease from the latitude where $R_\phi$ vanishes. If the globe as a whole is to be in radiation equilibrium with space the excess of insolation over outgoing radiation in the region of positive $R_\phi$ must equal the corresponding defect in the regions of negative values of $R_\phi$. This implies that

$$\overline{R}_\phi = \int_{-\frac{\varphi}{2}}^{\frac{\pi}{2}} 2\pi r^2 R_\phi \cos \varphi \, d\varphi = 0,$$  (5)
where \( R_\phi \) is a function of \( T \), which in turn depends linearly on \( p \) and \( l \). Thus (5) gives us the second relation between these quantities. By numerical calculation one obtains the values

\[
p = 190, \; l = 38.0.
\]

Figure 3, taken from Mügge's paper represents \( T_\phi \) as thus obtained from (3). The contrast between the temperatures at the equator and the pole is obviously too large and is due to the assumption that the averaging over the seasons can be accomplished by assuming the declination to be zero. Actually the ratio between the annual insolation at the pole and the equator is, according to Angot [3]

\[
\frac{144}{348} = 0.414
\]

and not 0, as would follow from the above assumption. The characteristics of the general circulation deducible from the results thus obtained are represented graphically in the accompanying figures taken from Mügge's paper. Figure 4 gives \( R_\phi \) as computed from (2). At latitude 35° the incoming radiation balances the outgoing. Its value at the pole is higher than the insolation at the equator. Figure 5 gives the energy crossing unit length of a latitudinal circle \( (S_\phi) \),

\[
S_\phi = \frac{\overline{R}_\phi}{2\pi \cos \phi}.
\]

Finally, if we assume the general circulation, insofar as it effects transportation of heat, to be limited to the troposphere and adopt for the height of the latter the formula

\[
h_\phi = (17 - 9\sin \phi) \text{ km},
\]

then \( S_\phi/h_\phi \) will give the intensity of the heat transport.

From figure 6 it may be seen that the maximum occurs in the region of maximum cyclonic activity.

The numerical results obtained by Mügge are quite uncertain and his work is incomplete in so far as he did not investigate the radiative power of the atmosphere. Nevertheless, the
attempt to calculate such a vital meteorological element as the intensity of the general circulation from the temperature distribution in the remote stratosphere is striking and the methods developed here are certain to prove useful in the future.
Fig. 3 Latitudinal variation of stratosphere temperature, after Mugge (1926).

Fig. 4 The quantity of heat $R_p$ which is removed from ($R_p > 0$) or added to ($R_p < 0$) a vertical atmospheric column of unit cross section during one minute, after Mugge (1926).

Fig. 5 The quantity of heat $S_p$ crossing per minute a rectangle of one cm width along the circle of latitude $\varphi$ and of the height of the troposphere, after Mugge (1926).

Fig. 6 The heat-current density $\nu$, which crosses per minute a vertical cm$^2$ in latitude $\varphi$. The dotted curve shows the assumed height of the troposphere, after Mugge (1926).
VIII. G. C. SIMPSON'S STUDIES IN TERRESTRIAL RADIATION.

As mentioned before Mügge's work was incomplete inasmuch as he did not investigate whether the atmosphere actually radiates in such a way as to maintain the computed stratosphere temperatures. Such an investigation should have served as a check not only on his calculations but also on the soundness of the entire calculating apparatus which had been gradually developed since the days of Emden's paper. In fact, G. C. Simpson [15] "was very much exercised to find some explanation of how a cold polar atmosphere with little water vapor can emit more energy than the hot equatorial atmosphere heavily charged with water vapor." This difficulty, whether real or only apparent, was rather beneficial to the development of our subject. The more Simpson looked into it the more it puzzled him until at last he was convinced that the then existing theory deserved but little respect. It was through these intellectual pangs that the new "selective" theory was born. During the short period of its existence it has proven quite healthy and it looks as if in the near future it will mature enough to digest all the relevant matter in the problem of the heat balance of the atmosphere.

The difficulties that arose from an examination of the radiative power of the assumed grey atmosphere were three in number. First, it appeared that the outgoing radiation was practically constant all over the globe so that, unless the variation of the stratosphere temperature with latitude should be fictitious, the problem of the stratosphere became again shrouded in mystery. Further, the mean outgoing radiation was for all possible values of the parameters involved much in excess over the incoming radiation as computed from the current value of the solar constant and from Aldrich's value for the albedo. Finally, it was found that the radiating mechanism of the atmosphere resulting from the assumption of grey radiation was so clumsy that it could not change its output in response to changes of insolation. Since considerable changes in the solar output are reported to occur the last drawback is quite serious.
In calculating the outgoing radiation, the same task as was undertaken by Hergesell, Simpson devised a simple and quite satisfactory method. In variance with his predecessors, he realized immediately that if you attribute the entire atmospheric radiation to the water vapor, the other elements do not exist at all for radiation problems and there is, therefore, no necessity to fix the water vapor content on the scale of the total air mass. One need not worry about the exponent \( n \) in the formula

\[
\rho_w = m^n
\]

which gives the water vapor density or pressure as a function of the superincumbent atmosphere \( m \), since, for our purposes, the steam atmosphere alone is relevant. Hence the superincumbent mass of water vapor ought to be selected as the proper running coordinate. He, then adopted Hergesell's empirical formula [28] for the relative humidity as a function of temperature,

\[
\log r = 1.8333 + 1.603 \frac{t}{T}
\]

(1)

\( r \) is the relative humidity, \( t \) the temperature on the centigrade scale and \( T \) the same temperature on the absolute scale. The atmosphere was divided into layers each having a temperature range of 6°C. Hence, since the average tropospheric lapse rate is 6°C per kilometer, each such layer is one kilometer deep. From the mean temperature of each layer and the relative humidity given by (1) the water vapor content in millimeters of precipitable water was determined. Since the lapse rate was assumed to be constant with latitude and since the water vapor content, on account of (1), depends on temperature only the complete state of the atmosphere is determined once the latitudinal temperature distribution of the ground is known.

With grey radiation and with a given coefficient of absorption the emission from each layer is easily computed. Similarly, the net transmission through a number of layers is the product of the individual transmissions. Thus, starting with the black radiation of the earth's surface and adding to
it the radiation from each successive layer, the fractions transmitted to space can be computed and their sum gives the total outgoing radiation for the particular latitude. The coefficient of absorption was determined from Abbot and Fowle's early estimates [25] that the atmosphere absorbs 9/10 of the earth's radiation, which, with a mean water vapor content for middle latitudes of 18 mm gives 12% absorption for one mm. It is proper to remark here, and this applies to Hergesell's work as well, that in his treatise on "Water-Vapor Transparency to Low-Temperature Radiation," published in 1917, Fowle [21] gives the following data:

<table>
<thead>
<tr>
<th>Precipitable water in mm</th>
<th>0.03</th>
<th>0.3</th>
<th>3.0</th>
<th>50.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical trans, of black body radiation of 287°A</td>
<td>51%</td>
<td>43%</td>
<td>34%</td>
<td>25%</td>
</tr>
</tbody>
</table>

Fowle estimates that one cm of precipitable water vapor will transmit 28% of the earth's diffuse radiation. From the above table it follows that the absorption of diffuse radiation by 1 mm of ppt. water is about 64%. This value could not be used in a scheme such as Simpson's since the above data show considerable deviation from Beer's law. However, whether or not Simpson's value for the coefficient was the most probable at the time, it accomplished its historical mission by showing up the weakness of the old theory and by inducing him to revise it.

The amount of radiation from a particular layer that manages to escape depends on the original intensity, as determined by the temperature and the radiating matter of the layer, and on the transmission of the superincumbent layers. With respect to the first factor, the earth's surface, being black and having the highest temperature is most favored while, with respect to the second factor, it is the top layers. In fact, the surface radiation is, except in the polar regions, largely suppressed and it is only from layers well above the earth's surface that appreciable radiation escapes to space. This is shown in figure 7 (taken from Simpson).
In the lower half of the figure the distance from the ordinate-axis to the boundary of the shaded area represents, on the scale attached in the left hand corner, the amount of radiation from the particular elevation that emerges to space. The shaded area, thus, gives a measure of the total outgoing radiation. It will be noticed that everywhere the effective radiating layers are well up in the troposphere and that at the equator the ground and the atmospheric layer occupying the two kilometers next to it do not contribute anything and that, proceeding towards the pole, the shaded area approaches the ground. Important is the fact that only layers within the temperature range from 220°A to 286°A are capable of contributing radiation to space. Those having lower temperatures than 220°A are poor radiators because of lack of water vapor, those having higher temperatures than 286°A are very good radiators, but, lying under the blanket of the absorbing atmosphere they are completely masked. The above implies that if, without change in lapse rate, the temperatures should either rise or drop it still will be the layer of 220°-286°A that will be effective. The only difference will be that in the case of a rise in temperature it will be displaced to a higher elevation and vice versa in the case of a general drop in temperatures. Obviously, the total outgoing radiation will be effected only in the polar regions where the earth's surface intersects the critical interval. In the upper part of the figure the abscissa is the sine of the latitude and the ordinate represents the outgoing radiation obtained under the assumption that 1mm of water vapor absorbs 30% of the incident radiation*. Over three quarters of the globe the outgoing radiation is constant, while the decrease in the polar region is slight. The dotted curves represent the effect of a general increase or decrease of temperatures by 12°C. The area under the curve which represents the total radiation to space, is affected only slightly.

It is easy to predict now the effect of a general change in relative humidity while keeping the temperature distribution constant.

*The area of a latitudinal belt having a width rdφ is \( 2\pi r \cos \varphi \, rd\varphi \) or \( 2\pi r^2 d(\sin \varphi) \).
As absorption, which primarily determines the limits of the region of effective outgoing radiation, is independent of temperature, the effective region starts at the top of the atmosphere and extends down to a level above which there is the same water vapor quantity as there is under normal conditions with the region of 220° to 286°A. A rise in relative humidity will thus bring about an upward displacement of the effective region, lowering the temperatures within it and effecting a net decrease in the outgoing radiation. Similarly, if the lapse rate is decreased, each elementary layer becomes thicker and the quantity of water vapor represented by the mean temperature of the layer increases. Since the temperature at the top of the atmosphere is unaltered and is equal 220°A, this will result in a decrease of the outgoing radiation. Needless to say, an increase in the coefficient of absorption is equivalent to a rise in humidity.

The effect of clouds can be estimated by similar considerations. A thick cloud absorbs all the upgoing radiation and radiates itself as a black body at the temperature of its upper surface. Thus, a cloud region is equivalent to a ground surface of the same temperature with an atmosphere such as that above the cloud. Simpson computes the maximum effect of clouds on the outgoing radiation by considering the clouds to be concentrated at a level of 260°A, corresponding to an elevation of 4 kilometers in latitude 50°, and by adopting a value of 0.50 for the mean cloudiness.* He finds a maximum decrease of 7% in the outgoing radiation or, a probable decrease of 5%.

The distribution of the outgoing radiation as shown by figure 7 is certainly in contradiction with Mügge's results. Simpson tried several values for the coefficient of absorption and in every case the outgoing radiation was uniform over more than three quarters of the globe, while whatever variation there was in the polar regions was in the opposite sense to Mügge's. The discrepancy was explained when he discovered from a study of the latest available soundings that the temperatures well

*In this value only thick clouds are included.
within the stratosphere, where convectional and advectional
effects of the troposphere and substroposphere are completely
eliminated, group themselves surprisingly close to 220°A. In
other words, in those parts of the stratosphere where you
would expect a state of radiation equilibrium a latitudinal
variation of the temperature does not exist. It is by using
this value for the constant temperature of the stratosphere
and by employing Humphrey's relation to determine the required
upgoing current, that he arrives at 0.30 for the absorption
of one mm of water vapor.

With a value of 1.953 for the solar constant and 0.43
for the albedo the effective incoming radiation is

\[
\frac{\pi r^2 \cdot 1.953 \cdot 0.57}{4 \pi r^2} = 0.278 \text{ cal/min/cm}^2
\]

In table VI are given the outgoing radiations for various
values of the coefficient of absorption (for one mm). In every
case the mean value is much higher than 0.278. Similarly,
Simpson computed the effect of changing the relative humidity
by reasonable amounts and finds it to be small. Hence the
assumption of grey radiation gives the impossible excess of
outgoing over incoming radiation.

The third difficulty mentioned above is by now self-evident.
We saw before that the outgoing radiation depends entirely on
the temperature distribution in the region containing a definite
quantity of water vapor. If an increase in insolation
should not change the lapse rate the region of 220°A to 266°A
will be merely lifted. On account of the fixed relation be-
tween humidity and temperature the internal structure of the
layer would be unchanged. Only in the polar regions where the
effective region reaches the earth's surface will there be a
change in the outgoing radiation, and as is shown by figure 7,
the effect is quite small. Further, Simpson finds that an
increase in the lapse rate from 0.6 to the maximum value of
1.0 would increase the outgoing radiation by only 7%. This
increase would in part be offset by the increase in relative
humidity that would result from increased turbulence associ-
ted with the higher lapse rates. Hence, it would appear that
the earth responds very reluctantly to variations in the solar radiation.

The most illuminating feature of the paper is Simpson's conclusion as to where the remedy is to be looked for, and what changes in the radiational behaviour of the atmosphere as pictured above one ought to expect. He says: "There can be no doubt that the general conclusion that the outgoing radiation is almost independent of the surface temperature, and is practically the same in all latitudes, will continue to be true." Also: "This leaves only (for modification) the assumption that water vapor radiates as grey body."

One cannot help being impressed with the completeness of the treatment. All the meteorological phenomena which could testify in this case of the theory of grey radiation were investigated and the insufficiency of the theory was thereby well established.

A few months after the publication of the first paper a second paper by Simpson [16] appeared. It starts out with a discussion of the absorption coefficients of water vapor and carbon dioxide, the only two appreciably absorbing atmospheric elements. From Hettner's [32] measurements of the absorption by water vapor and from Rubens' and Aschkinass' [33] measurements of the absorption by carbon dioxide, Simpson prepared a

<table>
<thead>
<tr>
<th>Latitude</th>
<th>Surface Temperature °A.</th>
<th>Total outgoing radiation.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>A</td>
</tr>
<tr>
<td></td>
<td></td>
<td>z=.12</td>
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<tr>
<td>Pole</td>
<td>250</td>
<td>.320</td>
</tr>
<tr>
<td>70°</td>
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<td>.377</td>
</tr>
<tr>
<td>60°</td>
<td>268</td>
<td>.403</td>
</tr>
<tr>
<td>50°</td>
<td>280</td>
<td>.440</td>
</tr>
<tr>
<td>40°</td>
<td>286</td>
<td>.449</td>
</tr>
<tr>
<td>Equator</td>
<td>296</td>
<td>.452</td>
</tr>
<tr>
<td>Average:</td>
<td></td>
<td>.430</td>
</tr>
</tbody>
</table>

TABLE VI. Outgoing radiation, after Simpson.
chart giving the spectral absorption by 0.03 grams of H₂O and 0.06 grams of CO₂. These quantities were supposed to exist in the stratosphere, the values having been obtained on the assumption that at the base of the stratosphere the air is completely saturated with water vapor and that the partial pressure of carbon dioxide is throughout the troposphere 0.03% of the total pressure. It was further assumed that the gases are distributed in the stratosphere according to Dalton's law. Those who are not convinced with the validity of this method of calculating the quantities of absorbing matter in the stratosphere may associate the name "stratosphere" with that upper part of the atmosphere which contains .3 mm ppt. water vapor and Simpson's results will, in the main, still hold if the mean temperature of the layer does not differ appreciably from the temperature of the stratosphere. There may be some doubt about the propriety of using Hettner's data on the absorption of water vapor in the atmosphere. Fowle's [21] measurements on water vapor under atmospheric conditions give values everywhere less than Hettner's, the cause of the discrepancy being probably the difference in concentration and temperatures. Simpson, therefore, modified Hettner's values by adopting zero absorption coefficients in the region $6\frac{1}{2}-11\mu$ as was observed by Fowle. As this region contains the peak of the black body radiation curve at atmospheric temperatures, the correction thus introduced is probably sufficient. The combined absorption curve of H₂O and CO₂ for parallel and vertical radiation obtained in this fashion is shown in figure 8. Simpson postulates that for the actual diffuse radiation the absorption in the stratosphere of wavelengths greater than $14\mu$ is complete; in addition there is complete absorption in the region $5\frac{1}{2}-7\mu$. He then states the following radiation laws, which, as we shall see immediately, are sufficient for the computation of the outgoing radiation. First: "If a layer of gas at a uniform temperature T throughout, completely absorbs radiation of wavelength $\lambda$, then it will emit radiation of this wavelength exactly as if it were a black body at temperature T." Second, "If a layer of gas rests on a black surface of infinite extent at temperature T, and the temperature within the gas
decreases from the surface outwards so that at its outer surface the temperature of the gas is \( T_1 \), then the flux of radiation outwards of any wavelength cannot be greater than that of a black body at temperature \( T \) nor less than that of a black body at temperature \( T_1 \). Both of these laws are evidently, formulations of Kirchoff's law. In the second law we need only remember that in the case of an atmosphere with temperatures decreasing upward the radiation current decreases too, for each layer absorbs radiation coming from the underlying warm layers and emits the same kind of radiation at its own, lower, temperature. The outgoing radiation cannot be equal to the current leaving the bottom surface, for portions of the outgoing radiation originate in colder layers; similarly, it must be less than corresponding to the upper boundary, for part of it consists of the transmitted fractions of the radiation from the interior.

With these two laws, the approximate calculation of the outgoing radiation at a given latitude is rather simple and can be accomplished, with the appropriate degree of accuracy without the knowledge of the temperature distribution between the given surface temperature and the temperature of the stratosphere. In figure 9 are shown black body radiation curves for these temperatures at latitude 50°. The outgoing radiation in the opaque regions of \( 5\frac{1}{2}-7 \mu \) and \( 14\mu-\infty \) originates, according to the first principle, in the stratosphere and is represented in the figure by the vertically hatched areas GHM AND QKF. In the transparent region of \( 8\frac{1}{2}-11 \mu \) the radiation originates at the surface of the earth and is represented by the horizontally hatched area NODP. For the intermediate regions of \( 7-8\frac{1}{2} \mu \) and \( 11-14 \mu \) Simpson, guided by the second principle, ascertains that they will be represented by areas intermediate between those given by the two curves. As, furthermore, the distribution of the outgoing radiation is expected to be continuous, he joins the known points D, K, and H, C at the boundaries by smooth curves, obtaining for the spectral intensity of the outgoing radiation the curve GHCDKF. The numerical values for each of the component parts are given in table VII.
Wave-lengths $5\frac{1}{2}\mu$ to $7\mu$.

Outgoing radiation from black body at $218^\circ A$. = 0.003 cal./cm.²/min.

Wave-lengths $7\mu$ to $8\frac{1}{2}\mu$.

Mean of outgoing radiation from black bodies at $280^\circ A$ and $218^\circ A$ = $\frac{0.041 + 0.007}{2} = 0.024$

Wave-lengths $8\frac{1}{2}\mu$ to $11\mu$.

Outgoing radiation from black body at $280^\circ A$. = 0.079

Wave-lengths $11\mu$ to $14\mu$.

Mean of outgoing radiation from black bodies at $280^\circ A$ and $218^\circ A$ = $\frac{0.091 + 0.038}{2} = 0.059$

Wave-length greater than $14\mu$.

Outgoing radiation from black body at $218^\circ A$. = 0.128

Total outgoing radiation = 0.293 cal/cm²/min.

TABLE VII. Outgoing Radiation from Latitude 50°, after Simpson.

Results of similar calculations for several latitudes are shown in table VIII. It will be noticed that the greatest contributions come from the stratosphere's radiation in wavelengths greater than 14 and from the earth's radiation in the transparent region of $8\frac{1}{2}\mu$ to $11\mu$. Next in magnitude is the transparent region of $11\mu$ to $14\mu$.

All this applies to a cloudless atmosphere. The effect of clouds will be to substitute for the temperature of the surface of the earth the average temperature at the top of the clouds. Assuming, as was done before, this effective cloud temperature to be $260^\circ$ everywhere, one can compute the outgoing radiation with completely overcast skies. As in this case the only variable quantity entering the calculation will be the temperature of the stratosphere, one would expect an increase in the
outgoing radiation with latitude. This is illustrated in figure 10. Under the actual conditions of 50% cloudiness the outgoing radiation is given by the mean of the curves for clear and cloudy skies. The area under this curve is 0.271 as compared with the value of 0.278 obtained for the incoming radiation from the known value of the solar constant and Aldrich's value for the albedo. Hence the new method seems to lead to a satisfactory value for the outgoing radiation.

<table>
<thead>
<tr>
<th>R in cal./cm.²/min.</th>
</tr>
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<tbody>
<tr>
<td>Latitude</td>
</tr>
<tr>
<td>0°</td>
</tr>
<tr>
<td>Stratosphere temperature</td>
</tr>
<tr>
<td>Surface temperature</td>
</tr>
<tr>
<td>Wave-lengths</td>
</tr>
<tr>
<td>5.2 μ to 7 μ</td>
</tr>
<tr>
<td>7 μ to 8.5 μ</td>
</tr>
<tr>
<td>8.5 μ to 11 μ</td>
</tr>
<tr>
<td>11 μ to 14 μ</td>
</tr>
<tr>
<td>&gt; 14 μ</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

TABLE VIII. Outgoing Radiation from Clear Skies, after Simpson.

Simpson makes an interesting application of his method to the computation of "nocturnal" or "effective" radiation, by which is understood the difference between black body radiation at the temperature of the surrounding atmosphere at a given level and the atmospheric radiation received there from above. This quantity, which was extensively measured by Ångström, is usually positive and at the ground it gives a measure of the rate of loss of heat at night by the black surface of the earth. Since the quantity of 0.3 mm of water vapor is contained, under normal humidity conditions, in an atmospheric layer sufficiently thin to have almost the same temperature as that of the surface, the effective radiation in the opaque regions is nil.
The main loss is in the transparent region of $\frac{1}{2} - 1 \mu$ and equals black body radiation at the temperature of the surface of the earth in that interval from the ozone layer. The loss in the semi-transparent regions will vary with the humidity and the temperature distribution. As these areas are the only variable ones they determine the limits of the possible variations of effective radiation. With clear skies the transparent region will always give a definite loss, while in the semi-transparent regions the loss will be complete in an extremely dry atmosphere and will diminish with increasing humidity. In fact, from Dine's [34] measurements of nocturnal radiation at Benson, near Oxford, Simpson is able to demonstrate a remarkable correlation between humidity and the variable part of effective radiation.

The effect of clouds is the same as that of increased humidity, giving always a decrease in effective radiation. In cases of low clouds with a temperature above heat of the surface, the effective radiation may even become negative, for clouds radiate as black bodies.

In table IX and figure 11 are shown data and curves for the transport of heat by the general circulation. The values of the outgoing radiation were multiplied by the factor $273/271$ to effect a mean outgoing radiation of 0.278. As for the rest, the method is similar to that of Mugge. Finally, Simpson removes the last difficulty of his first paper, namely the inadjustability of the earth's outgoing radiation to changes in insolation. He finds that the most effective, and therefore the most probable, means of adjustment would be the change in albedo resulting from a change in cloudiness. Should the incoming radiation increase, the contrast in temperature between equator and pole as well as between continents and oceans will grow and bring about an intensification of the general circulation accompanied by increased cloudiness and precipitation. As clouds reflect, according to Aldrich, as much as 78%, the major part of the extra insolation will be reflected, the remainder being just sufficient to maintain the increased circulation. From further considerations it becomes plausible that the temperatures on the three planets Venus, Earth, and Mars are about the same, the effective
<table>
<thead>
<tr>
<th>Latitude</th>
<th>Equator</th>
<th>10°</th>
<th>20°</th>
<th>30°</th>
<th>40°</th>
<th>50°</th>
<th>60°</th>
<th>70°</th>
<th>80°</th>
<th>Pole</th>
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</thead>
<tbody>
<tr>
<td><strong>Effective incoming solar radiation cal./cm.$^2$/min.</strong></td>
<td>.339</td>
<td>.334</td>
<td>.320</td>
<td>.279</td>
<td>.237</td>
<td>.232</td>
<td>.193</td>
<td>.160</td>
<td>.144</td>
<td>.140</td>
</tr>
<tr>
<td><strong>Outgoing terrestrial radiation cal./cm.$^2$/min.</strong></td>
<td>.271</td>
<td>.282</td>
<td>.284</td>
<td>.284</td>
<td>.282</td>
<td>.277</td>
<td>.272</td>
<td>.260</td>
<td>.252</td>
<td>.252</td>
</tr>
<tr>
<td><strong>Difference between incoming and outgoing radiation cal./cm.$^2$/min.</strong></td>
<td>+.068</td>
<td>+.052</td>
<td>+.036</td>
<td>+.013</td>
<td>-.015</td>
<td>-.045</td>
<td>-.079</td>
<td>-.100</td>
<td>-.108</td>
<td>-.112</td>
</tr>
<tr>
<td><strong>Total horizontal flow of heat across circles of latitude per min. cal./2 R$^2$</strong></td>
<td>.0000</td>
<td>.0106</td>
<td>.0183</td>
<td>.0221</td>
<td>.0219</td>
<td>.0183</td>
<td>.0122</td>
<td>.0066</td>
<td>.0020</td>
<td>.0000</td>
</tr>
<tr>
<td><strong>Horizontal flow of heat per cm. of circle of latitude per min. cal./10$^7$</strong></td>
<td>.00</td>
<td>0.69</td>
<td>1.34</td>
<td>1.63</td>
<td>1.83</td>
<td>1.82</td>
<td>1.56</td>
<td>1.23</td>
<td>0.73</td>
<td>.00</td>
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</tbody>
</table>

**TABLE IX.** The Horizontal Heat Transport by the General Circulation, after Simpson.
Fig. 7 Latitudinal distribution of outgoing radiation, after Simpson (1928).

Fig. 8 Absorption by the stratosphere and radiation curve for black body at 2200A, after Simpson (1928).

Fig. 9 Outgoing radiation from latitude 50°, after Simpson (1928).

Fig. 10 Latitudinal distribution of outgoing radiation, after Simpson (1928).

Fig. 11
Curve I Effective solar radiation.
Curve II Outgoing terrestrial radiation.
Curve III Total horizontal heat flow across circles of latitude.
Curve IV Horizontal heat flow per cm. of circle of latitude.

(After Simpson, 1928).
insolation being regulated by the albedo which is close to unity at Venus, one half at the earth and zero on Mars.

In a third paper Simpson invades a new field, which had been touched upon only slightly by Emden, namely, that of the variation of terrestrial radiation with season. Since it is hoped through these investigations to acquire a qualitative and possibly a quantitative knowledge of the nature of the general circulation and its variations, Simpson's new line of attack seems to strike at the very heart of the problem. Whether or not the theoretical methods developed by Simpson are refined enough to tackle the new detailed problem it is still too early to tell.
IX. C. G. ABBOTT ON THE RADIATION OF THE PLANET EARTH TO SPACE.

Abbot [17] points out that Simpson's value of 0.3 mm for the water vapor content of the atmosphere is overestimated. In the course of the Smithsonian work on the determination of the solar constant spectroscopic methods were developed for the determinations at the top of Mt. Whitney (4,420 m) and Mt. Montezuma (2,710 m) it appeared that often the total atmosphere above the two places would contain 0.3 mm only. This would mean that the stratosphere contains only a fraction of 0.3 mm.

Abbot proceeded to calculate the intensity of the outgoing radiation by using Fowle's [21] measurements of the absorption of long wave radiation by water vapor under atmospheric conditions. The absorption for different water vapor masses were read off for each narrow band of wavelengths from the experimentally determined curves; this procedure eliminated the necessity of considering the validity of Beer's law. The state of the atmosphere was assumed to be given by Simpson's representation discussed in his first paper, in which the troposphere has a constant lapse rate of 6°C per kilometer and the moisture content depends on the temperature according to Hergesell's formula. Since the latter gives decreasing relative humidity with elevation its adoption in this investigation automatically deprives the stratosphere of any appreciable moisture content. The program was to compute the outgoing radiation in the manner of Simpson (in his first paper) modified by the consideration of the selectiveness of absorption and radiation. In the determination of the coefficients of absorption for the different layers their moisture content was doubled to take into account the diffuse character of the radiation. This does not mean that the coefficients were doubled but that to a given quantity of water vapor a coefficient was assigned equal to that of a quantity of vapor twice as large. The spectrum was divided into 13 narrow bands; for each band and for each Simpson layer the radiation was computed from the mean temperature of the layer and from its coefficient of emission. Similarly, the transmission of the superincumbent
layers was determined from the experimental values for the total water vapor contained in them. Furthermore, a value of 16\% black body efficiency was allowed for the ozone band between 9\mu and 11\mu and 30\% absorption was assumed for the stratosphere in the opaque region of 13\mu to 50\mu. The effect of clouds was taken into consideration by assuming, as was done by Simpson, that a mean temperature of 280^\circ A prevails at the top of the clouds. This temperature is the same as the surface temperature at latitude 70^\circ; thus, for half of the time the outgoing radiation was assumed to be everywhere the same as in latitude 70^\circ with clear skies.

The results obtained are shown in table X. The advantage of this method over Simpson's is that the contribution from the ground can be traced separately; as is seen, this quantity is quite uniform with latitude. The radiation from the atmosphere itself decreases with latitude, while the total outgoing radiation for half-cloudy skies shows a larger variation with latitude than was found by Simpson. This result revives in an acute form the difficulty in the explanation of the latitudinal distribution of the temperature in the stratosphere.

Evidently, Abbot's laborious method is superior to Simpson's, although it still largely depends on the propriety of using Simpson's model atmosphere. There is also the question of the dependence of water vapor absorption on temperature and total pressure, effects which undoubtedly exist, but which are quantitatively still largely undetermined. We know that A. Ångstrom had to consider these effects in order to correlate measurements on atmospheric radiation at different pressures. The necessity of further experimental research on the absorption of water vapor is evident.
<table>
<thead>
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<th>Smithsonian Results</th>
<th>Simpsonian Results</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Clear Sky</td>
<td>Half Cloudy Sky</td>
</tr>
<tr>
<td></td>
<td>Atmosphere</td>
<td>Surface</td>
</tr>
<tr>
<td>0°</td>
<td>0.220</td>
<td>0.105</td>
</tr>
<tr>
<td>40°</td>
<td>0.192</td>
<td>0.107</td>
</tr>
<tr>
<td>50°</td>
<td>0.182</td>
<td>0.105</td>
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<tr>
<td>60°</td>
<td>0.132</td>
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<td>70°</td>
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</tr>
<tr>
<td>90°</td>
<td>0.129</td>
<td>0.096</td>
</tr>
</tbody>
</table>

**TABLE X. The Radiation of Earth and Atmosphere to Space, after Abbot.**

*Calories per cm.² per min.*
According to Aristotle three is a perfect number, for it has a beginning, a middle, and an end. When Simpson proved that the assumption of grey radiation led to absurd results and when he later brought to our attention the nature of the absorption spectrum of water vapor, we should have expected that somebody would answer the call for the consideration of the selectiveness of absorption by replacing Emden's single coefficient by three: one for the opaque region, one for the transparent region and a third one for the remaining part of the spectrum. This was done by R. MÜGGE in a paper of 1929 [19].

He expresses the idea that the variation of the temperature of the stratosphere with latitude, a question which was left open by Simpson, may be accounted for by the increasing intensities of the opaque radiation emitted by the warmer layers in the tropopause, which is the most effective agent in maintaining the temperature of the stratosphere. His argument for the increase with latitude of the opaque radiation is as follows: Simpson has proven that the total outgoing radiation varies only slightly with latitude and since the radiation in the transparent region, originating at the ground, decreases with latitude the remaining part must increase in the same direction. Mügge also shows that the radiation in the intermediate region, which depends on the moisture distribution and the temperatures in the atmosphere and at the ground, does not vary with latitude due to the compensation effect discussed by Simpson in his first paper. The clouds, which effect a decrease in this type of radiation, will increase the outgoing radiation at higher latitudes, for their amount is less there than in equatorial regions.

In this connection mention should be made of one feature of the effect of clouds on the outgoing radiation, pointed out by Mügge, which does not seem plausible. He gives curves, for several latitudes, showing the total outgoing radiation (from the clouds and the superincumbent atmosphere) as a function of the height of the clouds. At each latitude the radiation increases as the cloud level is raised from the ground to a
critical height in the lower atmosphere, from where it decreases. This critical level, which decreases with increasing latitude, has a maximum value at the equator of about 4 kilometers. Now consider in two situations the outgoing radiation from an atmosphere with an upward decreasing temperature and with a normal humidity distribution: one when the cloud is at an elevation \( h \) and another when the cloud is at a higher elevation \( H \). It is understood that in each case the cloud is thick enough to absorb all the radiation coming from below and that, therefore, it radiates as a black body at the temperature of its surroundings, i.e., that temperature at \( h \) (\( T_h \)) in the first case and the temperature at \( H \) (\( T_H \)) in the second case. In this latter case the outgoing radiation consists of the radiation of the atmosphere above \( H \) and the fraction of the radiation from the clouds which it transmits. In the first case the radiation from the part of the atmosphere above \( H \) is the same. However, instead of having black body radiation at the temperature \( T_H \) transmitted through it we have now the radiation from the atmospheric layer (\( h-H \)) and the fraction of the black body radiation at the temperature \( T_h \) transmitted by the layer (\( h-H \)). This combined radiation reaching the level \( H \) is more intense than black body radiation at the temperature \( T_H \) for parts of it originate in the lower, warmer, layers. Hence, we ought to expect that, as long as the temperature decreases upward, the effect of clouds will always be to decrease the outgoing radiation and this effect grows in a monotone fashion as the clouds are raised from the ground to higher levels.

It does not seem probable that Mügge's new approximate method will be able to survive alongside Abbot's more exact method. The detailed calculating apparatus needed in Simpson's scheme and without which it would soon have dissipated its initial impulse, was adequately, and in proper time, produced by Abbot. Abbot's method is free from any artificial assumptions and has already, in contrast to Mügge's, yielded definite results as to the spectral distribution and total intensity of the outgoing radiation.

As to the treatment of the problem of the temperature distribution in the stratosphere the natural course would be
to use Abbot's data for the outgoing radiation, supplemented by Fowle's data on the absorption of solar radiation by water vapor, and to compute the temperatures that could be maintained at radiation equilibrium. The absorption of diffuse solar radiation returning to space after reflection at the clouds should also be considered. If the result agrees with the observed temperatures the problem is solved; if not, we will at least know that the difficulty is real. One would then have to investigate in more detail the possible effect of the radiation from the ozone. Our physical picture of the situation is clear enough and no simplifying assumptions should be permitted any longer.

With these comments we may consider our survey completed although a number of papers dealing with various details of our problem have been left out so as to leave the framework of the general theory unobscured. Similarly, and in view of the excellent symposium [35] available on this subject, we have not treated the problem of the temperature distribution in and above the ozone layer to which notable contributions were recently made by E. H. Gowan [36].
XI. SUMMARY.

It must be admitted that our achievements in this field of endeavor are at present very meagre. Of the problems enumerated in the introduction none have yet been completely solved, although a few have been given a satisfactory qualitative treatment. Of these, the central one, that of the division of the atmosphere into a convective troposphere and an isothermal stratosphere, was most adequately treated by Emden. He showed that under certain assumptions as to the radiative and absorptive powers of the atmosphere, the state of radiative equilibrium would be characterized by unstable lapse rates in the lower atmosphere. The mixing that would occur during the establishment of a stable stratification would not only change the original temperature distribution but would also adversely effect the state of radiation equilibrium by introducing new types of heat supply, such as the latent heat of condensation and the heat transported by convection, and by changing the absorptive and radiative powers of the atmosphere through the altered moisture distribution. In short, the mixing would give us the convective troposphere as we observe it. The height to which convection would reach would depend on the extent of the original instability and would be greater the larger the latter. Above that height the state of radiation equilibrium would persist and give us the stratosphere. The force of this whole argument, is, however, weakened by the simplifying assumptions on which it was based since the latter were proven by Simpson to lead to absurd results when applied to the outward radiative power of the atmosphere. Since the nature of radiation equilibrium under the existing conditions of selective atmospheric absorption and radiation has not yet been investigated, this problem, then, still awaits its final solution.

Concerning temperature of the stratosphere we have the two theories by Humphreys and by Emden of which the latter differs from the former mainly by the consideration of the slight effect of absorption of solar radiation within the stratosphere. Under the assumption of grey or semi-grey
radiation the temperatures computed by the above two authors are close to the observed values. However, as soon as this assumption is dropped and the spectral distribution of the upgoing current available for absorption is investigated, it becomes evident that the temperature that could be maintained would be considerably less than the observed value. An exact calculation of this type has not been carried out yet although the spectral distribution of the outgoing radiation is available in a recent work by Abbot.

The latitudinal variation of the mean temperature of the stratosphere was discussed qualitatively by Milne. He points out one factor which may, in some measure, contributes to the relatively high stratosphere temperatures in high latitudes, namely the greater absorption of solar radiation in the upper atmosphere there, due to the increased obliquity of the solar beam. Moreover, the general circulation causes the distribution of outgoing radiation to be more uniform than that of the incoming radiation. Calculations by Abbot as well as by Simpson show a slightly decreasing outgoing radiation with latitude. The factors brought out by Milne would therefore seem to be insufficient to explain the observed latitudinal variation of temperature in the stratosphere.

The most serious handicap in our work is the lack of data on the absorption and emission by water vapor and carbon dioxide. It is the contention of present day physicists [37] that band spectra are not continuous but consist of discrete lines. The available experimental data on the absorption of water vapor, those of Fowle and Hettner, are conflicting; this spectrum has not yet been interpreted theoretically, so that the uncertainty about the experimental values is still unre- moved. The effect of the total pressure, the density and temperature on absorption is still a much debated problem. As long as the radiative behaviour of H₂O and CO₂ under atmospheric conditions is not definitely known, theoretical investigations of the temperature distribution in the atmosphere will, except in special cases, be inconclusive. When in the future such data become available the investigator will find in the papers here discussed a number of well developed methods of attack.
It is hoped that the present pamphlet has brought out the necessity of closer critical cooperation between various investigators, to prevent duplication and the use of antiquated methods in this branch of meteorological research. If, in addition, it should succeed to induce new experimental investigations of the water vapor and carbon dioxide, our efforts will be amply rewarded.
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<td>The radiation of the planet earth to space. Smithsonian Miscellaneous Collections, Vol. 82, No. 3, 1929.</td>
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