Search for $B^0$ meson decays to $\pi^0 K^0_S K^0_S$, $\eta K^0_S K^0_S$, and $\eta'$ $K^0_S K^0_S$

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We describe searches for $B^0$ meson decays to the charmless final states $\pi^0 K^0_S K^0_S$, $\eta K^0_S K^0_S$, and $\eta' K^0_S K^0_S$. The data sample corresponds to $467 \times 10^6$ $B\bar{B}$ pairs produced in $e^+ e^-$ annihilation and collected with the BABAR detector at the SLAC National Accelerator Laboratory. We find no significant signals and to determine the 90% confidence level upper limits on the branching fractions, in units of $10^{-7}$, $\mathcal{B}(B^0 \to \pi^0 K^0_S K^0_S) < 9, 10^{-7}$, $\mathcal{B}(B^0 \to \eta K^0_S K^0_S) < 10,$ and $\mathcal{B}(B^0 \to \eta' K^0_S K^0_S) < 20$. 

Charged particles from the $e^+ e^-$ interactions are detected, and their momenta measured, by a combination of five layers of double-sided silicon microstrip detectors and a 40-layer drift chamber. Both systems operate in the 1.5 T magnetic field of a superconducting solenoid. Photons and electrons are identified with a CsI(Tl) crystal electromagnetic calorimeter. Charged particle identification is provided by the average energy loss ($dE/dx$) in the tracking devices and by an internally reflecting, ring-imaging Cherenkov detector covering the central region (DIRC). A $K/\pi$ separation of better than 4 standard deviations ($\sigma$) is achieved for momenta below 3 GeV/c. Detector details may be found elsewhere [12].

The $B$ daughter candidates are reconstructed through their dominant decays: $\eta \to \gamma \gamma$, $\eta \to \pi^+ \pi^- \pi^0$ ($\eta_{3\pi}$) where $\pi^0 \to \gamma \gamma$, $\eta \to \pi^+ \pi^- \pi^0$ ($\eta_{3\pi}$) where $\eta \to \gamma \gamma$, and $\eta \to \rho^0 \gamma$ ($\eta_{\rho \gamma}$) where $\rho^0 \to \pi^+ \pi^-$. We require the laboratory energy of the photons to be greater than 30 MeV for $\pi^0$ in $\eta_{3\pi}$, 50 MeV for $\eta_{\rho \gamma}$ in $\eta_{\rho \pi \pi}$, and 100 MeV for $\eta_{3\pi}$, and for $\pi^0$ and $\eta_{\rho \gamma}$ produced directly from the $B$ decay. We impose the following requirements on the invariant mass (in MeV/c^2) of the candidate final states: $120 < m(\gamma \gamma) < 150$ for $\pi^0$, $510 < m(\gamma \gamma) < 585$ for $\eta_{\gamma \gamma}$ produced directly from the $B$ decay, $490 < m(\gamma \gamma) < 600$ for $\eta_{\gamma \gamma}$ in $\eta_{\rho \pi \pi}$, $538 < m(\pi^+ \pi^- \pi^0) < 558$ for $\eta_{\rho \pi \pi}$, $945 < m(\pi^+ \pi^- \pi^0) < 970$ for $\eta_{\rho \pi \pi}$, $930 < m(\pi^+ \pi^- \gamma) < 980$ for $\eta_{\rho \gamma}$, and $470 < m(\pi^+ \pi^- \rho^0) < 980$ for $\rho^0$. Tracks from $\eta$ and $\eta'\gamma$ candidate decays are rejected if the particle identification signatures from the DIRC and $dE/dx$ are consistent with those of protons, kaons, or electrons. Candidate $K^0_S$ decays are formed from pairs of oppositely charged tracks with $486 < m(\pi^+ \pi^-) < 510$ MeV/c^2, a decay vertex $\chi^2$ probability larger than 0.001, and a reconstructed decay length greater than 3 times its uncertainty.

We reconstruct the $B$ meson candidate by combining two $K^0_S$ candidates and a $\pi^0$, $\eta$, or $\eta'$ candidate. From the kinematics of the $Y(4S)$ decays we determine the energy-substituted mass $m_{ES} = \sqrt{s} - p_B^2$ and the energy difference $\Delta E = E_B - \frac{1}{2} \sqrt{s}$, where $(E_B, p_B)$ is the $B$ meson 4-
momentum vector, and all values are expressed in the Y(4S) rest frame. The resolution is 3.0 MeV/c² for $m_{ES}$ and in the range (12–32) MeV for $\Delta E$, depending on the decay mode. We require $5.25 < m_{ES} < 5.29$ GeV/c² and $|\Delta E| < 0.2$ GeV.

Backgrounds arise primarily from continuum $e^+e^- \rightarrow q\bar{q}$ events ($q = u, d, s, c$). We reduce these with a requirement on the angle $\theta_T$ between the thrust axis of the $B$ candidate in the Y(4S) rest frame and that of the rest of the charged tracks and neutral calorimeter clusters in the event [13]. The distribution is sharply peaked near $|\cos\theta_T| = 1$ for $q\bar{q}$ jet pairs and is nearly uniform for $B$ meson decays. The requirement is $|\cos\theta_T| < 0.9$. For the $\rho^0$ decays we also use $|\cos\theta_{\rho}|$ where the helicity angle $\theta_{\rho}$ is defined as the angle between the momenta of a daughter pion and the $\eta'$, measured in the $\rho^0$ meson rest frame. For $\eta_{\gamma\gamma}$ decays we use $|\cos\theta_{\eta}|$, where the decay angle $\theta_{\eta}$ is defined as the angle between the momenta of the most energetic daughter photon and the $B^0$ meson, measured in the $\eta$ meson rest frame. We require $|\cos\theta_{\rho(\eta)}| < 0.9$. Events are retained only if they contain at least one charged track in the decay products of the other $B$ meson ($B_{\text{tag}}$) from the Y(4S) decay. This requirement improves the precision of the determination of $B_{\text{tag}}$ thrust axis. The $B^0 \rightarrow \pi^0 K_S^0 K_S^0$ decay has background from $B^0 \rightarrow D^0 K_S^0$, with $D^0 \rightarrow \pi^0 K_S^0$, which has the same final state as the signal mode. In order to suppress this background, we define $m(\pi^0 K_S^0)$ as the closer of the two invariant-mass combinations to the nominal $D^0$ mass [14]. By requiring $m(\pi^0 K_S^0) < 1.815$–1.899 GeV/c², we veto 80% of this background.

We obtain the signal event yields from unbinned extended maximum likelihood (ML) fits. The observables used in the fit are $\Delta E$, $m_{ES}$, and a Fisher discriminant $\mathcal{F}$. The Fisher discriminant $\mathcal{F}$ [15] is a linear combination of four event shape variables and $|T|$, the absolute value of the continuous output of a flavor tagging algorithm [16]. The event shape variables used for $\mathcal{F}$ are the angles [16], with respect to the beam axis, of the $B$ momentum and the $B$ thrust axis in the Y(4S) frame, and the zeroth and second angular moments, $L_{0,2}$, of the energy flow about the $B$ thrust axis [17]. The moments are defined by $L_j = \sum_i p_i x_{i} \times |\cos\theta_i|^j$, where $\theta_i$ is the angle, with respect to the $B$ thrust axis, of track or neutral cluster $i$, and $p_i$ is its momentum. The sum excludes the $B$ candidate daughters. We use a neural network based technique [16] to determine the flavor at decay of the $B_{\text{tag}}$.

The coefficients of $\mathcal{F}$ are chosen to maximize the separation between the signal and the continuum background. They are determined from studies of Monte Carlo (MC) [18] simulated signal data and off-peak data. Signal MC events are distributed uniformly across the Dalitz plot. Correlations among the ML input observables are below 10%. The average number of candidates found per selected event is between 1.13 and 1.22, depending on the final state. We choose the candidate with the highest $B$ vertex $\chi^2$ probability, determined from a vertex fit that includes both charged and neutral particles [19]. From simulated events we find that this algorithm selects the correct candidate in (92–98)% of the events containing multiple candidates, depending on the final state, and introduces negligible bias.

We use a MC simulation to estimate backgrounds from other $B$ decays, including final states with and without charm. These contributions are negligible for the $\eta_{\gamma\pi\pi}$ mode. In all the other modes we introduce a non-peaking $BB$ component in the fit. In the $\pi^0 K_S^0 K_S^0$ analysis we also introduce a $BB$ background component that peaks in $m_{ES}$ and $\Delta E$, to take into account the main contribution to background from $B^0 \rightarrow K_S^0 K_S^0 K_S^0$ decay mode. We consider three components in the likelihood fit: signal, continuum, and $BB$ background. We have studied the possibility of misreconstruction of our $B$ candidates. We divide signal events into two subcomponents: correctly reconstructed (COR) signal and self cross feed (SCF) signal, where at least one $B$ candidate daughter has been exchanged with a particle from the rest of the event. The signal component is split according to this classification. The fractions of SCF events are fixed in the fit to the values found in MC simulated events, which are in the range (10–21)%, depending on the final state. For the $\pi^0 K_S^0 K_S^0$ decay mode, which has the lowest SCF fraction (6.6%), we use one signal component, comprising COR and SCF events. For each event $i$ and component $j$, we define the probability density function (PDF)

$$P_j = P_j(m_{ES})P_j(\Delta E)P_j(\mathcal{F})$$

and the likelihood function

$$L = e^{-(\sum n_j)} \prod_{i=1}^N \left( \sum_j n_j P_j^i \right),$$

where $N$ is the number of reconstructed events and $n_j$ is the number of events in component $j$ which is returned by the fit. We determine the PDF parameters from MC simulation of the signal and $BB$ backgrounds, while we use $m_{ES}$ and $\Delta E$ sideband data ($5.25 < m_{ES} < 5.27$ GeV/c², 0.1 < $|\Delta E| < 0.2$ GeV) to model the PDFs of continuum background.

We parameterize $P(m_{ES})$ as a Chrystal Ball function [20] for the COR and SCF signal subcomponents, an ARGUS function [21] for continuum and non-peaking $BB$ background components, and by an ARGUS function plus an asymmetric Gaussian distribution for peaking $BB$ background. The $P(\Delta E)$ distribution is described by an asymmetric Gaussian distribution plus an exponential tail (AGT) [22] for the COR signal subcomponent, an asymmetric Gaussian distribution plus a linear Chebyshev polynomial or an AGT for the SCF, and Chebyshev polynomials for continuum and $BB$ background components. The distribution of $\mathcal{F}$ is described with an asymmetric Gaussian distribution plus a Gaussian distribution.
TABLE I. Fitted signal yield in events and fit bias in events (ev), detection efficiency ε (%), daughter branching-fraction product \( \prod B_i \), significance \( S \), and measured branching fraction \( B \) with statistical error for each decay mode. For the combined measurements (in bold) we give \( S \) (with systematic uncertainties included) and the branching fraction with statistical and systematic uncertainties with the 90\% CL upper limit in parentheses.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Yield (ev)</th>
<th>Fit bias (ev)</th>
<th>ε (%)</th>
<th>( \prod B_i ) (%)</th>
<th>( S(\sigma) )</th>
<th>( B(10^{-7}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi^0 K_S^0 K_L^0 )</td>
<td>( 11.7^{+10.2}_{-14.5} )</td>
<td>( +1.0 \pm 0.7 )</td>
<td>17.5</td>
<td>47.9</td>
<td>0.7</td>
<td>( 2.7^{+4.3}_{-3.3} ) ± 0.6 ( &lt; 9)</td>
</tr>
<tr>
<td>( \eta_{\gamma \gamma} K_S^0 K_s^0 )</td>
<td>( 3.2_{-0.9}^{+0.0} )</td>
<td>( +1.1 \pm 0.7 )</td>
<td>17.5</td>
<td>18.8</td>
<td>0.3</td>
<td>( 1.4^{+5.9}_{-4.7} )</td>
</tr>
<tr>
<td>( \eta_3 \pi^0 K_S^0 K_L^0 )</td>
<td>( 2.2_{-2.3}^{+2.5} )</td>
<td>( +0.2 \pm 0.6 )</td>
<td>12.0</td>
<td>10.9</td>
<td>0.5</td>
<td>( 3.3^{+3.0}_{-2.7} )</td>
</tr>
<tr>
<td>( \eta K_S^0 K_L^0 )</td>
<td>( 0.5 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( 2.1^{+1.4}_{-1.3} ) ± 1.2 ( &lt; 10)</td>
</tr>
<tr>
<td>( \eta_{\eta \pi} K_S^0 K_s^0 )</td>
<td>( 2.4_{-2.3}^{+4.7} )</td>
<td>( +0.1 \pm 0.4 )</td>
<td>12.6</td>
<td>8.4</td>
<td>0.6</td>
<td>( 4.6^{+9.5}_{-6.9} )</td>
</tr>
<tr>
<td>( \eta_{\psi} K_S^0 K_L^0 )</td>
<td>( 13.4_{-13.1}^{+16.4} )</td>
<td>( +4.7 \pm 1.1 )</td>
<td>15.9</td>
<td>14.1</td>
<td>0.6</td>
<td>( 8.3^{+15.4}_{-13.5} )</td>
</tr>
<tr>
<td>( \eta K_S^0 K_s^0 )</td>
<td>( 0.8 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( 5.7^{+8.0}_{-6.5} ) ± 3.4 ( &lt; 20)</td>
</tr>
</tbody>
</table>

for the COR signal subcomponent, an AGT function for SCF signal events, an asymmetric Gaussian distribution plus a linear Chebyshev polynomial for continuum, and an asymmetric Gaussian distribution for \( B \bar{B} \) background subcomponents.

We allow the continuum background PDF parameters to float in the fit. Large control samples of \( B^{-} \rightarrow D^{0} (K_S^{0})^{\pi^{+}} \pi^{-} \pi^{0}) \pi^{-} \) decays are used to verify the simulated \( \Delta E \) and \( m_{ES} \) resolution. Any bias in the fit, which mainly arises from neglecting the correlations among the discriminating variables used in the likelihood function definition, is determined from a large set of simulated experiments. For each experiment, the \( q \bar{q} \) background and non-peaking \( B \bar{B} \) background are drawn from the PDFs, and we embed the expected number of peaking \( B \bar{B} \) background and signal events taken randomly from fully simulated MC samples.

In Table I we show, for each decay mode, the fitted signal yields and their fit biases in numbers of events, the detection efficiencies, the product of daughter branching fractions, the significance \( S \), and the measured branching fractions. The detection efficiency is determined as the ratio of selected events in simulation to the number generated. The significance is given in units of \( \sigma \). We determine the corrected signal yields from the fitted signal yields and their fit biases, estimated using simulations. We use these values, detection efficiencies, daughter branching fractions, and number of produced \( B \) mesons, assuming equal production rates of charged and neutral \( B \) meson pairs, to compute the branching fractions. The statistical error on the signal yield is the change in the central value when the quantity \(-2 \ln \mathcal{L}\) increases by one unit from its minimum value. The significance is the square root of the difference between the value of \(-2 \ln \mathcal{L}\) (with systematic uncertainties included) for zero corrected signal yield and the value at its minimum. We combine results from different subdecay modes by adding the values of \(-2 \ln \mathcal{L}\). In order to account properly for systematic uncertainties when combining results from different subdecays, we convolve the \( \mathcal{L} \) of each subdecay mode with a Gaussian distribution with mean equal to zero and width equal to the uncorrelated systematic uncertainty of that decay mode. For the combined measurements we report the branching fractions, the statistical significances and the 90\% confidence level (CL) upper limits. The 90\% CL upper limit is taken to be the branching fraction below which lies 90\% of the total likelihood integral in the positive branching-fraction region.

Figure 1 shows projections of \( \pi^0 K_S^0 K_s^0 \), \( \eta K_S^0 K_L^0 \), and \( \eta' K_S^0 K_L^0 \) candidates onto \( m_{ES} \) and \( \Delta E \) for the subset of candidates for which the signal likelihood (computed without the variable plotted) exceeds a mode-dependent threshold.

The main sources of systematic error include uncertainties in the detection efficiencies, the PDF parameters, and
the maximum likelihood fit bias. We assign systematic uncertainties (13–20%) on the detection efficiencies due to nonuniformity of the efficiencies over the Dalitz plot. This contribution is taken to be the ratio between the standard deviation of the efficiency distribution over the Dalitz plot to its mean value. For the signal, the uncertainties in the PDF parameters are estimated by comparing MC and data control samples. Varying the signal PDF parameters in the PDF parameters are estimated by comparing MC to its mean value. For the signal, the uncertainty from the fit bias is taken as the sum in quadrature of 1.8% and 3.0% is assigned to the single photon and $\pi^0/\gamma\gamma$ meson reconstruction efficiencies, respectively. There is a systematic error of 0.9% for the reconstruction efficiency of each $K_S^0$. The uncertainty on the total number of $B\bar{B}$ pairs in the data sample is 1.1%. Uncertainties on the $B$ daughter branching-fraction products (3.5–4.9%) are taken from Ref. [14].

In conclusion we have searched for the $B^0$ decay modes to $\pi^0K_S^0K_S^0$, $\eta K_S^0K_S^0$, and $\eta' K_S^0K_S^0$ with a sample of 467 $\times 10^6$ $B\bar{B}$ pairs. We find no significant signals and set 90% CL upper limits for the branching fractions: $B(B^0 \to \pi^0K_S^0K_S^0) < 9 \times 10^{-7}$, $B(B^0 \to \eta K_S^0K_S^0) < 10 \times 10^{-7}$, and $B(B^0 \to \eta' K_S^0K_S^0) < 20 \times 10^{-7}$.

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[21] The threshold function is defined as $\alpha = 1 - x^2 \exp(-\xi(1-x^2))$, with $\alpha = 2m_{B\bar{B}}/\sqrt{s}$ and $\xi$ is a parameter that is determined by the fit: H. Albrecht et al. (ARGUS Collaboration), Phys. Lett. B 241, 278 (1990).
[22] We use the function $f(x) = N \exp(-\alpha_1(x-\xi_{L,R})/(\alpha_2(x-\xi_{L,R})(x-\xi_{L,R})))$, where $\mu$ is the peak position of the distribution, $\alpha_1,\alpha_2,\xi_{L,R}$ are the left and right tail parameters, and $N$ is a normalization factor.