Modeling of GaN/AlN Heterostructure-Based Nano Pressure Sensors

S. Patil\textsuperscript{a}, N. Sinha\textsuperscript{b} and R.V.N. Melnik\textsuperscript{a}

\textsuperscript{a}M$^2$NeT Lab, Wilfrid Laurier University, Waterloo, ON N2L 3C5, Canada
\textsuperscript{b}Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

**ABSTRACT**

We quantify the influence of thermopiezoelectric effects in nano-sized Al$_x$Ga$_{1-x}$N/GaN heterostructures for pressure sensor applications based on the barrier height modulation principle. We use a fully coupled thermoelectromechanical formulation, consisting of balance equations for heat transfer, electrostatics and mechanical field. To estimate the vertical transport current in the heterostructures, we have developed a multi-physics model incorporating thermionic emission, thermionic field emission, and tunneling as the current transport mechanisms. A wide range of thermal (0-300 K) and pressure (0-10 GPa) loadings has been considered. The results for the thermopiezoelectric modulation of the barrier height in these heterostructures have been obtained and optimized. The calculated current shows a linear decrease with increasing pressure. The linearity in pressure response suggests that Al$_x$Ga$_{1-x}$N/GaN heterostructure-based devices are promising candidates for pressure sensor applications under severe environmental conditions.

**Keywords:** Pressure sensor, Al$_x$Ga$_{1-x}$N/GaN heterostructures, thermoelectromechanical effects, coupled multiphysics models

**1. INTRODUCTION**

There has been a recent increase in demand for ultrasensitive, fast, portable and robust pressure sensors that can operate in harsh environments. Materials such as GaN and AlN exhibit favorable thermal, mechanical, and chemical stabilities and radiation hardness with minimal problems arising from the unwanted optical or thermal generation of charge carriers as a result of their large band gap and atomic bondings.\textsuperscript{1} Therefore they are suitable materials for constructing pressure sensors for applications in extreme environments. One of the unique advantages of GaN-based devices is that GaN/AlN heterostructures develop sheet charges at the heterointerfaces due to spontaneous polarizations and piezoelectric polarization.\textsuperscript{2–6} The stress-induced modulation of the barrier height in AlGaN/GaN structures has been recently investigated, demonstrating potential use of these structures in pressure or stress sensing.\textsuperscript{7–10} Also, portability of a sensor device, one of the prior concerns in sensor applications, can be achieved with recent progress in nano-fabrication techniques. Recent reports on micro pressure sensors based on Al$_x$Ga$_{1-x}$N/GaN heterostructures grown on 6H:SiC substrate\textsuperscript{7} show that the proposed heterostructure is a promising candidate. However, the device performance was limited due to the substrate material (6H:SiC), which can be improved further by using substrate from the same family of materials such as GaN.\textsuperscript{7} In order to meet technological requirements of such sensors, a systematic study of thermoelectromechanical loading effects in nano-sized heterostructures is required.

In this work, the pressure response of an Al$_x$Ga$_{1-x}$N/GaN is investigated using a generalized fully coupled thermopiezoelectric model. We use a fully coupled thermopiezoelectric formulation, consisting of balance equations for heat transfer, electrostatics and mechanical field. Accordingly, the changes in current due to thermionic emission, thermionic field emission and tunneling have been calculated. The results for the thermopiezoelectric modulation of the barrier height in these heterostructures have been obtained. The energy band...
gap shifts for the wide range of thermoelectromechanical loadings have been analyzed. We also study the effect of composition and dimensions of heterostructures on device performance. In particular, we have investigated the devices fabricated on a GaN substrate with the objective of determining the most promising AlGaN/GaN heterostructures for pressure sensing.

2. MODEL FORMULATION

In what follows, we formulate a mathematical model in order to study thermoelectromechanical effects in QDs. A general three-dimensional (3D) axisymmetric model is developed with coupled multi-physics governing equations. The problem is governed by a coupled system of equilibrium equations of elasticity, electrostatics and heat transfer.

2.1 Explicit form of governing equations for wurtzite nanostructures in cylindrical coordinates

Governing equations for wurtzite structures are axisymmetric, hence all thermal-, electric- and mechanical-field solutions are axisymmetric as well. Therefore, the original 3D problem can be reduced in this case to a simpler 2D problem.\(^4\) The electromechanical balance equations in the cylindrical coordinates for axisymmetric case take the following form:\(^11\)

\[
\begin{align*}
\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} &= 0, \\
\frac{\partial \sigma_{rz}}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{1}{r} \sigma_{rz} &= 0, \\
\frac{\partial D_r}{\partial r} + \frac{\partial D_z}{\partial z} + \frac{1}{r} D_r &= 0.
\end{align*}
\]

Here \(\sigma_{ij}\) are stress tensor components, \(D_i\) are electric displacement vector components, \(h_i\) are the components of heat flux vector. Coupling of equations (1-3) is implemented through constitutive equations. These equations are invariant with respect to rotations around the \(z\) axis (in spite of the lack of axisymmetry of the underlying wurtzite lattice, refer to detailed discussions in\(^12\)), hence solutions can be separated into a \((r, z)\) part and a \(\phi\) part, subject to adequate boundary conditions. The constitutive relations in equations then take the following form for wurtzite nanostructures:

\[
\begin{align*}
\sigma_{rr} &= C_{11} \varepsilon_{rr} + C_{12} \varepsilon_{\theta\theta} + C_{13} \varepsilon_{zz} - e_{31} E_z - \beta_{11} \Theta, \\
\sigma_{rz} &= C_{44} \varepsilon_{rz} - e_{15} E_r, \\
\sigma_{zz} &= C_{13} \varepsilon_{rr} + C_{13} \varepsilon_{\theta\theta} + C_{33} \varepsilon_{zz} - e_{33} E_z - \beta_{33} \Theta, \\
D_r &= e_{15} \varepsilon_{rz} + \varepsilon_{11} E_r, \\
D_z &= e_{31} \varepsilon_{rr} + e_{31} \varepsilon_{\theta\theta} + e_{33} \varepsilon_{zz} + \varepsilon_{33} E_z + p_3 \Theta + P^{sp},
\end{align*}
\]

where \(c_{ij}, e_{ij}, \varepsilon_{ij}\) and \(\kappa_{ij}\) are the elastic moduli, piezoelectric constants, dielectric constants and coefficients of heat conduction, respectively, while \(p_i\) and \(\beta_{ij}\) are pyroelectric and stress-temperature material constants, respectively. \(P^{sp}\) is the spontaneous polarization. At thermal equilibrium, the temperature change becomes spatially independent, effectively leading to the determination of solution of the equilibrium equations for mechanical and electric field only.\(^13\) The well-posedness of the corresponding mathematical models in this and in more general case was shown in\(^11,14\) (see also references therein) while the analysis of the special types of boundary conditions was carried out in.\(^15\)

To take into account the lattice mismatch, the strain tensor components take the following form:

\[
\begin{align*}
\varepsilon_{rr} &= \frac{\partial u_r}{\partial r} - \varepsilon^*_{a} \\
\varepsilon_{zz} &= \frac{\partial u_z}{\partial z} - \varepsilon^*_{c} \\
\varepsilon_{\theta\theta} &= \frac{u_r}{r} - \varepsilon^*_{a}
\end{align*}
\]
\[ \varepsilon_{rz} = \frac{1}{2} \left( \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right), \]  
(5)

with \( \varepsilon^*_a = \frac{a_m - a_{QD}}{a_m} \) and \( \varepsilon^*_c = \frac{c_m - c_{QD}}{c_m} \) inside the QD and zero otherwise. Quantities, \( a_m, c_m \) and \( a_{QD}, c_{QD} \) are the lattice constants of the matrix and the QD, respectively, while quantities, \( \varepsilon^*_a \) and \( \varepsilon^*_c \) are the local intrinsic strains (lattice mismatch) along \( a \) and \( c \) directions, respectively. The directions \( a \) and \( c \) correspond to the shorter and longer dimensions of the unit cell of the wurtzite crystal, respectively.

### 2.2 Barrier height calculations

In order to highlight further our point regarding the influence of thermoelectromechanical effect, we consider a two band model that consists of conduction band \((C_1)\) and valance band \((HH_1)\). It is instructive in this context to think of electrons and holes as being particles moving in effective potentials, \( V_{Ceff} \) and \( V_{Heff} \), respectively. We express the relations for the effective conduction and valence band edges (for wurtzite crystals):

\[ E_{C1} = V_{Cedge} + a^\parallel_e \varepsilon_{rr} + a^\perp_z \varepsilon_{zz} - V, \]  
(6)

\[ E_{HH} = V_{Hedge} + (d_2 + d_4 - d_5) \varepsilon_{rr} + (d_1 + d_3) \varepsilon_{zz} - V, \]  
(7)

where \( V_{Cedge} \) and \( V_{Hedge} \) are the unstrained conduction and valence band-edges, respectively, \( a^\parallel_e \) and \( a^\perp_z \) are the conduction band deformation potentials along \( a \) and \( c \) directions, while \( d_i \) (i=1,2,3,5) are valance band deformation potentials.\(^{16}\)

Hence, the effective band gap \((C_1 - HH_1)\) is,

\[ E_{\text{eff}} = E_{C1} - E_{HH} \]  
(8)

\[ = E_g(\Theta) + \left( a^\parallel_e - (d_2 + d_4 - d_5) \right) \varepsilon_{rr} + \left( a^\perp_z - (d_1 + d_3) \right) \varepsilon_{zz}, \]

where

\[ E_g(\Theta) = E_g(0) - \frac{\alpha_\Theta \Theta^2}{\beta_\Theta + \Theta}, \]  
(9)

along with \( \alpha_\Theta \) and \( \beta_\Theta \) as Varshni coefficients.\(^{18}\)

In this relatively simple case the above equation indicates that the effective energy band gap is independent of electric potential and is dependent on temperature and strain. The electric potential \((V)\) tilts the energy bands, however, the effective band gap remains unchanged.\(^{2}\)

### 2.3 Current calculation

As mentioned previously, we calculate transport current under pressure by incorporating thermionic emission and tunneling as the current transport mechanisms. The detailed expressions for the current densities in corresponding cases are expressed as follows. The charge density \( \rho \) near the junction is given by\(^{19}\)

\[ \rho(x) = \begin{cases} -qN_a & -x_p < x < 0 \\ +qN_D & 0 < x < x_N, \end{cases} \]  
(10)

where \( N_a \) is the net acceptor concentration in the \( p \) side and \( N_D \) is the net donor concentration on the \( N \) side. Using Gauss’s law, one can write

\[ \frac{d}{dx} E(x) = \begin{cases} -\frac{qN_a}{\varepsilon_p} & -x_p < x < 0 \\ +\frac{qN_D}{\varepsilon_N} & 0 < x < x_N, \end{cases} \]  
(11)

where \( E \) is the electric field, and \( \varepsilon_p \) and \( \varepsilon_N \) are the permittivity in the \( p \) and \( N \) regions, respectively. According to Gauss’s law, the slope of the \( E(x) \) profile is given by the charge density divided by the permittivity, Therefore,

\[ E(x) = \begin{cases} -\frac{qN_a(x+x_p)}{\varepsilon_p} & -x_p < x < 0 \\ +\frac{qN_D(x-x_N)}{\varepsilon_N} & 0 < x < x_N, \end{cases} \]  
(12)
in the depletion region and zero outside. The relationship between the electric field and the electrostatic potential distribution \( \phi(x) \) across the junction is given by

\[
E_x = -\frac{d}{dx}\phi(x).
\]

If the reference potential is chosen as zero for \( x < -x_p \), we have

\[
\phi(x) = \begin{cases} 
0 & x \leq -x_p \\
qN_a(x + x_p)^2 \frac{2\varepsilon_p}{2\varepsilon_p} & -x_p \leq x \leq 0 \\
qN_a x^2 + qN_D(2x_N - x^2) & 0 \leq x \leq x_N \\
V_0 & x_N \leq x,
\end{cases}
\]

where

\[
V_0 = V_{0p} + V_{0N};
\]

\[
V_{0p} = \phi(0) = \frac{qN_a x_p^2}{2\varepsilon_p},
\]

\[
V_{0N} = \frac{qN_D x_N^2}{2\varepsilon_N};
\]

where \( V_0 \) is the total potential drop across the junction, \( V_{0p} \) is the portion of the voltage drop on the p side and \( V_{0N} \) is the portion of the voltage drop on the N side. For nondegenerate semiconductors, one can write

\[
E_{CN} - F_N \approx -k_B T \ln \left( \frac{N}{N_{CN}} \right),
\]

\[
F_p - E_{vp} \approx -k_B T \ln \left( \frac{p}{N_{vp}} \right).
\]

Using the boundary condition \( N_a x_p = N_D x_N \), the expressions for \( V_0, V_{0p}, V_{0N} \) and the total width of the depletion region \( x_w \) from

\[
x_w = x_p + x_N,
\]

one can derive

\[
x_p = \frac{N_D}{N_a + N_D} x_w; \quad x_N = \frac{N_a}{N_a + N_D} x_w.
\]

Therefore, \( x_w \) and \( V_0 \) are related as

\[
x_w = \left[ \frac{2\varepsilon_p V_0}{qN_a N_D \left( N_D + \frac{2\varepsilon_p}{\varepsilon_N} N_a \right)} \right]^{1/2} (N_a + N_D).
\]

The band edge \( E_v(x) \) from the \( p \) side to the \( N \) side is expressed as

\[
E_v(x) = \begin{cases} 
-q\phi(x) & \text{on } p \text{ side} \\
-\Delta E_v(x) - q\phi(x) & \text{on } N \text{ side},
\end{cases}
\]

The conduction band edge \( E_c(x) \) is above \( E_v(x) \) by an amount \( E_{gp} \) on the \( p \) side and by an amount \( E_{GN} \) on the \( N \) side. \( E_c(x) \) is always parallel to \( E_v(x) \):

\[
E_c(x) = \begin{cases} 
E_v(x) + E_{gp} & x < 0 \\
E_v(x) + E_{GN} & x > 0,
\end{cases}
\]
Using the one-dimensional WKB approximation, the transmission probability for the barrier is expressed as

\[ T(E_x) = \begin{cases} \exp\left(-\frac{4\pi}{\hbar} \int_0^{X_E} [2m_n^* (E_c(x) - E_x)]^{1/2} dx\right), & E_{\text{min}} \leq E_x < E_{\text{CN}}, \\ 1, & E_{\text{CN}} \leq E_x, \end{cases} \]

where \( h \) is Planck constant, \( m_n^* \) is the electron effective mass, \( E_x \) is the energy component in the \( x \) direction and \( E_{\text{min}} = \max\{E_{cp}, E_c(W)\} \).

Assuming the Boltzmann energy distribution, the electron current density \( J_n \) across the heterointerface is expressed as

\[ J_n = J_{n1} - J_{n2} = -\frac{A^* T}{k} \int_{E_{\text{min}}}^{x} f_1(E_x)T(E_x)dE_x + \frac{A^* T}{k} \int_{E_{\text{min}}}^{x} f_2(E_x)T(E_x)dE_x, \]

where \( k \) is the Boltzmann constant, \( A^* \) is the effective Richardson constant for electrons and \( f_{1,2}(E_x) \) is the occupation probability in each region. We have

\[ J_{n1}(0^-) = -qv_{n1}(1 + \delta)n_1(0^-)\exp\left(-\frac{\Delta E_c}{kT}\right), \]

where

\[ n_1(0^-) = N_{c1}\exp\left(\frac{E f_{n1}(0^-) - E_c(0^-)}{kT}\right), \]

\[ v_{n1} = \frac{A^* T^2}{qN_{c1}}. \]

\[ \delta = \frac{\exp\left(\frac{E_{c}(0^+)}{kT}\right)}{kT} \int_{E_{\text{min}}}^{E_{c}(0^+)} \exp\left(-\frac{E_x}{kT}\right) \]

\[ \times \exp\left(-\frac{4\pi}{\hbar} \int_0^{X_E} [2m_n^* (E_c(x) - E_x)]^{1/2} dx\right) dE_x. \]

Similarly, the equation for the opposing flux of electron current can be expressed as

\[ J_{n2}(0^+) = -qv_{n2}(1 + \delta)n_2(0^+), \]

where

\[ n_2(0^+) = N_{c2}\exp\left(\frac{E f_{n2}(0^+) - E_c(0^+)}{kT}\right), \]

\[ v_{n2} = \frac{A^* T^2}{qN_{c2}}. \]

where \( v_{n1,2} \) is the mean electron thermal velocity, \( N_{c1,2} \) is the effective density of states in the conduction band in each region, and \( n_1(0^-) \) and \( n_2(0^+) \) are the electron densities on either side of the heterointerface.

The net electron current density crossing at the heterointerface is

\[ J_{n,i} = -qv_{n1}(1 + \delta)n_1(0^-)\exp\left(-\frac{\Delta E_c}{kT}\right) +qv_{n2}(1 + \delta)n_2(0^+) \]

The thermionic emission is taken into account through the parameter \( \delta \). When the tunneling mechanism is neglected (\( \delta = 0 \)), this equation becomes the thermionic boundary condition.
We study the dependence of thermoelectromechanical loadings on barrier height and current calculated for applied potential for 10 nm and 20 nm thick Al$_x$Ga$_{1-x}$N. The calculations are performed for a range of Al-concentration (0 to 1), temperature (0 K to 300 K), pressure (0 GPa to 10 GPa) and for 20 nm thick Al$_x$Ga$_{1-x}$N in 60 nm × 140 nm matrix of GaN. Figure 1 shows the band gap and conduction band edge variation under different

thermoelectromechanical loadings. The band gap energy of Al$_x$Ga$_{1-x}$N increases with increase in pressure and decreases with increase in temperature. $E_g$ varies from 3.45 eV for $x = 0$ to 6.2 eV for $x = 1.0$ at T=0 K and P=0 GPa, while smaller band gaps are observed at T=300K and P=0GPa, 3.35 eV for $x = 0$ and 6.1 eV for $x = 1.0$. For T=0K and 300K, the change $\Delta E_g$ is $\sim 0.1$eV at $x = 0.3$ which further decreases to $\sim 0.0$eV at $x = 1.0$. On the other hand, the conduction band edge ($E_c$) profiles for different $x$ and pressure at T=0K 3 and T=300K 3 show that the change in $E_c$ due to change in pressure, $\Delta E_c$ increases with increase in $x$. For T=0K and 300K, the change $\Delta E_c$ is $\sim 0.8$eV at $x = 0.3$ which further decreases to $\sim 1.1$eV at $x = 1.0$.

The Fermi level, $\eta$ for intrinsic semiconductor can be calculated from $E_g$ and the barrier height, $\phi_B$ is known from the profile of $E_c$ for corresponding conditions. The smaller values of $\Delta E_g$ with increase in Al-concentration, may result in less sensitivity to pressure with higher Al-concentrations in Al$_x$Ga$_{1-x}$N. On the other hand, higher values of $\Delta E_c$ with increase in Al-concentration will increase the sensitivity of the pressure sensor. However, the effect of relatively higher values of $\Delta E_c$ may overcome the effect of smaller values of $\Delta E_c$ with increase in Al-concentration and effectively may lead to increase in sensitivity of the pressure sensor at higher Al-concentrations.

Figure 2 shows the pressure dependence of the current at different Al-concentrations ($x = 0.3, 0.5, 0.7$ and 1.0).
The results indicate a decrease in the magnitude of the current that is nearly linear over the range of pressures from 0 to 10 GPa. This indicates that Al$_x$Ga$_{1-x}$N/GaN heterostructures are promising candidates for pressure sensing applications under severe environmental conditions.

4. CONCLUSIONS

In this paper, a systematic study on the effects of thermomechanical loading on Al$_x$Ga$_{1-x}$N/GaN heterostructures has been performed. In order to achieve this objective, a fully coupled multiphysics model incorporating balance equations for heat transfer, electrostatics and mechanical field has been developed. Using the barrier height modulation principle, we have demonstrated that Al$_x$Ga$_{1-x}$N/GaN heterostructures can be employed as pressure sensors under a wide range of thermal and pressure loadings. The linear trends in vertical transport current suggest the potential of these heterostructures in pressure sensor applications under severe environmental conditions.

REFERENCES