CLEARANCE PRICING OPTIMIZATION AT ZARA

By

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Submitted to the MIT Sloan School of Management and the Department of Electrical Engineering and Computer Science in Partial Fulfillment of the Requirements for the Degrees of

Master of Business Administration AND
Master of Science in Electrical Engineering and Computer Science

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Abstract

In almost thirty-four years after opening the doors to its first store, the Inditex Group has grown to be one of the largest fashion distributors in the world. Today the group operates more than four thousand retail stores in seventy-three different countries and under eight different brand concepts.[1] Inditex's Zara brand division is renown for its high degree of vertical integration that allows it to maintain a tight control over the different stages of its supply chain and endows it with the flexibility to quickly react to current fashion trends.

With a yearly average of 173 new store openings, Zara's accelerated growth rate has forced it to seek innovation and continuous improvement in its operations in order to maintain the competitive advantages that characterize it. One of its biggest challenges deals with the management of its clearance sales where the remaining inventory at the end of its sales campaign must be sold at a discounted price. These clearance sales are fast-paced and pricing decisions must be made for more than 11,000 different fashion designs that Zara introduces each year, and considering the different market conditions that exist in the more than 70 countries where Zara operates.

The proposed project consists in the development of a pricing mathematical model based on a sales forecasting model that estimates consumer's reactions to price discounts and a linear optimization model that makes profit-maximizing optimal price assignments. The current thesis details the design, implementation, and live test of the proposed model based pricing methodology that resulted in an approximate increase of six percent to Zara's clearance sales profits.

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[1] Inditex. (2009). *FY2008 RESULTS, 1 February 2008 to 31 January 2009* .

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1 Introduction

1.1 Motivation for Thesis

The current thesis marks the conclusion of a collaboration internship between the MIT Leaders for Manufacturing (LFM) program and the fashion retailer and distributor chain Zara and its parent company Inditex, one of the largest fashion distributors in the world.[2] This thesis and internship also constitutes the second LFM internship that has been developed on-site at Zara's headquarters in the city of Arteixo in Spain, and is based on the research work of Professors Felipe Caro from the UCLA Anderson School of Management and Jérémie Gallien from the MIT Sloan School of Management[3]. The collaboration effort for the current project began early in 2007 when Professors Gallien and Caro began discussions with Zara's management team and proposed a project that would develop a new, data-driven methodology to make the pricing decisions during Zara's clearance sales in which the remaining inventory at the end of the regular sales season is liquidated or sold at a discounted price.

As illustrated in Figure 1, Zara's design collections and Sales Seasons are divided into a Summer Campaign that is gradually introduced towards the beginning of the calendar year, and a Winter Campaign that is introduced towards the middle of the calendar year:

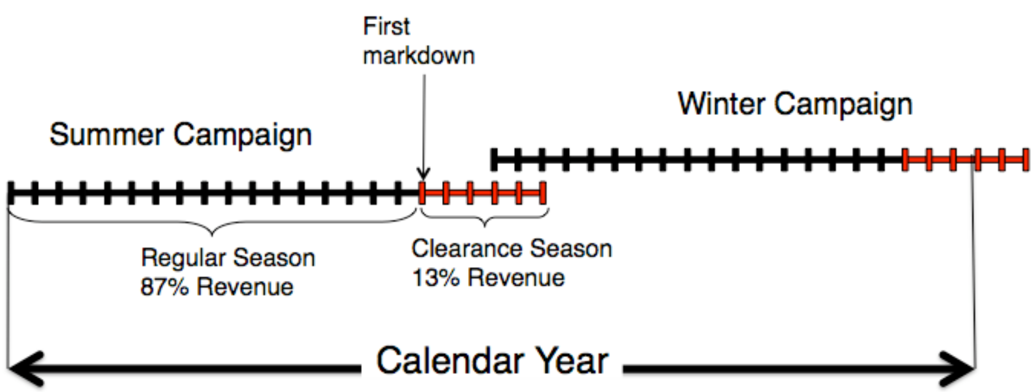

Figure 1: Zara's Sale Campaigns and Seasons

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[2] *INDITEX Group - Our group*, http://www.inditex.com/en/who_we_are/our_group (accessed April 21, 2009).

[3] Juan Correa, LFM Graduate of 2007, developed the first internship and thesis: Correa, J. R. (2007, June). Optimization of a Fast-Response Distribution Network.

Each Regular Season (marked in black on Figure 1) extends for roughly five months after which point all of the remaining inventory from the present Season is sold at a discounted price in time for the beginning of the next Season. Clearance Seasons, indicated in red in Figure 1, extend for approximately two months and temporarily coexist with the introduction of the new sales Season as the transition from Summer to Winter Campaigns is made, and vice versa. During the Clearance Season, garment prices are revised on a weekly basis and gradually increasing price discounts are issued in response to the observed sales and remaining levels of inventory. As illustrated by the profit distribution captions in Figure 1, a considerable amount of Zara's revenues are realized during the Clearance Season. However, when examining the revenue distributions, the importance of the Clearance Seasons is undermined by the fact that garments are sold at a discounted price during these Seasons. In order to better visualize the importance of the Clearance Season and its impact on Zara's bottom line profits, Figure 2 illustrates the distributions of inventory sold during the Regular and Clearance Seasons.

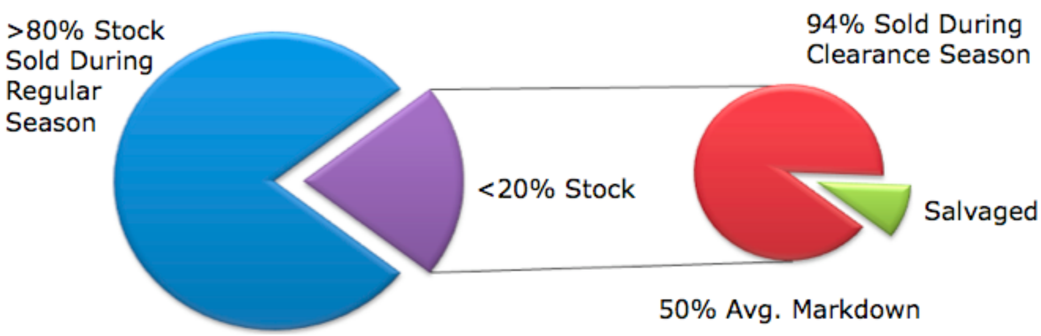

Figure 2: Inventory Sales Distribution

Of the entire inventory that is available on either the Summer or Winter Campaign, roughly over eighty percent is sold during the Regular Season sales while the remaining twenty percent, lower than the industry average, is offered at a discounted price during the Clearance Season.[4] Out of this less than twenty percent, ninety four percent of the inventory is sold during the Clearance Season, and at an average price markdown of fifty percent, while the remaining six percent is sold for a salvage value to third-party distributors who operate outside of Zara's target market.

[4] Data supplied by Zara.

Considering the fact that Zara posted revenues of 6,264 Million Euros in 2007[5], makes the prospect of even marginal improvements in the pricing methodology currently employed by Zara a promising venture. As a result, the design of a data-driven mathematical model, capable of making profit-maximizing pricing decisions during Zara's Clearance Season, became the motivation for this thesis.

1.2 Project Development

As mentioned earlier, work on the project on which the current thesis is based began as early as the summer of 2007 when Professors Caro and Gallien conducted a Zara store visit in order to observe and become familiarized with the operational aspects of Zara's management of the Clearance Season sales. Shortly thereafter, ideas on process improvements that would improve the profitability of the Clearance Season were discussed with Zara's management and a two-pronged approach was proposed:

1. Development of a new methodology to physically allocate the remaining inventory at the end of the Regular Season amongst Zara's stores in an optimal way that maximizes Clearance Season profits.
2. Given an initial physical distribution of the inventory, to design and implement a methodology for determining the price discounts that should be applied during the Clearance Season in a way such that an optimal balance is reached between sold inventory and its sale price, thus maximizing profits made.

In other words, the first step deals with "having the right inventory in the right place", while the second one addresses the issue of how to price it so that the perceived profits are maximized. In order to divide the problem into more manageable sections, it was decided that the current project would address the second point while the latter would be saved for a future LFM Internship. The timeline followed for the project is depicted in Figure 3:

[5] (2007). *Grupo Inditex Anual Report.* www.inditex.com.

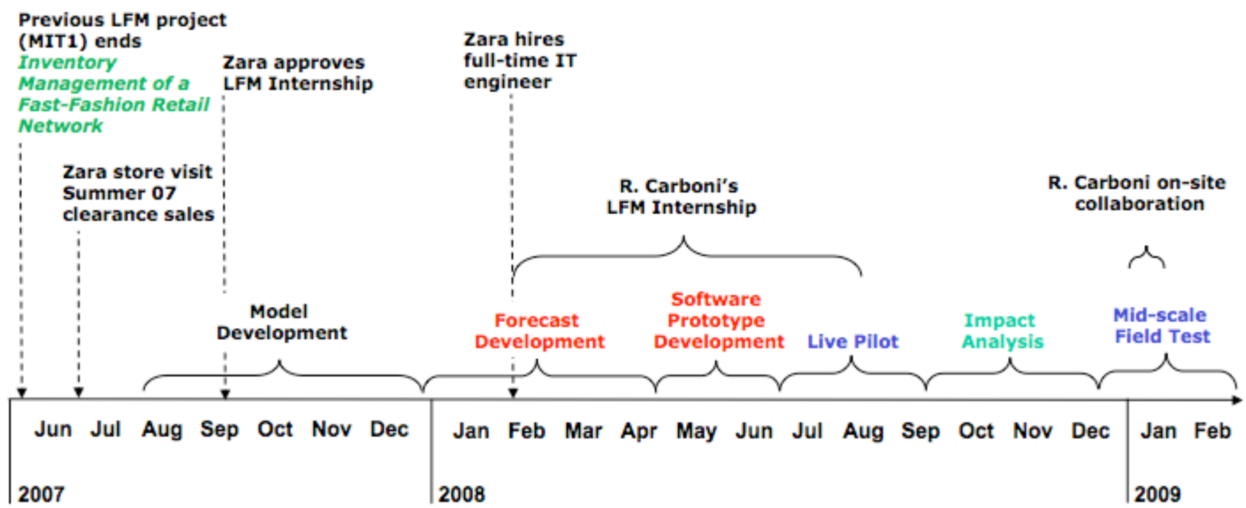

Figure 3: Project Timeline

Caro and Gallien began work on the mathematical models that support the developed pricing methodology during the summer of 2007[6]. Shortly thereafter, Zara authorized what would become its second LFM Internship, internally referred to as MIT2, which began in February of 2008 and took place on-site at Zara's headquarters in La Coruña, Spain. A full-time IT engineer was hired to handle the data-intense requirements of the project.

Based on the forecasting model proposed by Caro and Gallien, the first part of the internship was focused on implementing the model in Zara's IT infrastructure, experimenting with different choices for the model's parameters and variables, and working together with Zara's management in order to capture and incorporate part of their knowledge of the business into the model. Approximately four months into the Internship an Optimization Model, which takes the Forecasting Model's sales forecasts as one of its inputs and makes optimal Clearance Season pricing assignments was implemented. The third and last part of the on-site Internship consisted in the design and execution of a live pilot test in which the developed model was used to make the pricing decisions for four sample garment groups during the Summer Clearance Season for two mid-sized European countries.

[6] Caro, F., & Gallien, J. (2007, November 18). In preparation for: Clearance Pricing for a Fast-Fashion Retail Network. *Clearance Pricing Optimization at Zara* .

During the months following the conclusion of the Clearance Season, the results obtained from the pilot test were analyzed, and based on the measured positive impact on profits, a larger-scale field test was planned for the 2008 Winter Clearance Season. For this second test, a finalized version of the developed model that incorporated improvements made throughout the summer pilot, was tested in the same two mid-sized European countries, but this time across the entire inventory collection. As shown in the Project Timeline, this field test included a two-week site visit during the first weeks of the Clearance Season in order to make final adjustments to the models and collaborate on the execution of the test.

The project was finally concluded during the present year, after the results of the field test were analyzed and conclusive positive impact results over profits were measured. Based on the project's success, the developed model will now be extended to cover the entirety of Zara's Clearance Season pricing methodology.

1.3 Chapter Outline

This thesis is divided into seven chapters.

Chapter 2 provides company background and an overview of how the clearance season and pricing process are managed at Zara.

Chapter 3 describes the challenges that Zara faces when making clearance pricing decisions, how it currently makes its pricing decisions, and the new model based solution that will be presented in this thesis.

Chapter 4 describes the design and implementation process of a forecasting model that is capable of estimating Zara's clearance sales under variable discounted prices.

Chapter 5 provides the formulation of the linear optimization model that works together with the forecasting model and makes optimal clearance pricing decisions that maximize the realized profits.

Chapter 6 presents the experimental design and execution of two live tests that were conducted in order to evaluate the performance of the developed model based pricing methodology, and details the economical impact results and observations measured during the experiments.

Chapter 7 finally summarizes the project's impact to Zara, as well as the current status and next steps in its expansion and proliferation.

2 Background

2.1 Company Background

The Zara brand and stores are part of the Spanish textile industry conglomerate Inditex. With a 2008 posted net income of 1,262M Euros and a 2007/2004 Compound Annual Growth Rate of 25%, Inditex has placed itself as one of the worlds top fashion distributors[7]. The company is diversified across eight different store concepts, targeted at different market segments: Zara, Pull and Bear, Massimo Dutti, Berksha, Stradivarius, Oysho, Zara Home, and Uterqüe.

The history of Inditex began with the opening of the first Zara store in La Coruña, Spain in 1975 and in the last thirty-four has grown its operations into seventy-three countries where it operates 4,264 different stores and employs more than 89,000 persons.[8]

With 2008 net sales of 6,824 M EUR, the Zara brand represents two thirds of Inditex's business.[9] Zara's model or approach to the retail industry is renown by its high degree of vertical integration that allows it to maintain a tight control over the different stages of its supply chain, including textiles sourcing, design, manufacture, distribution, and retail. This, together with centralized distribution centers located in Spain from which inventory is constantly delivered to all of its stores, enables Zara to quickly respond to the predominant fashion trends in the market and deliver new fashion designs on a weekly basis. For instance, in any given year, more than eleven thousand different fashion designs are introduced into Zara's stores.

The great flexibility that Zara's large scale of vertical integration enables has been one of the keys for its success, and has led to an accelerated growth of its business. With an average of three new stores opened on each week of 2008 and increases in net sales and gross margins of ten percent over the previous year, Zara's growth has also presented significant challenges.[10] With increased capacity requirements and cost constraints, Zara has had to gradually move towards sourcing from locations that are geographically distant from its Spanish distribution centers, which means that it has had to

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[7] (2007). *Grupo Inditex Annual Report.*

[8] Inditex. (2009). *FY2008 RESULTS, 1 February 2008 to 31 January 2009* .

[9] Ibid.

[10] Calculated by the author based on Inditex. (2009). *FY2008 RESULTS, 1 February 2008 to 31 January 2009* .

further improve its supply chain, production, distribution, and retail operations in order to maintain its trademark and competitive advantage as a fast-fashion company.

As part of this improvement process, Zara first started collaborating with the LFM Program at MIT in 2006 and have since worked in projects focused around optimizing purchasing decisions, store distribution and replenishment orders, and management and pricing of clearance inventory, which is the subject presented in the current thesis. Zara is internally divided into three sections: Woman, Men, and Kids. Because the Woman Section represents sixty percent of Zara's business and revenues, it is usually chosen as the launching pad for the development and evaluation of improvement plans, which is the case for the project described in the current thesis as well.

2.2 Clearance Period Management Process

2.2.1 Introduction to Basic Terminology

2.2.1.1 Stock Keeping Units

Special terminology words relative to the current project and Zara's business, some of which have already been introduced, are capitalized throughout the thesis and when applicable, are defined in the Glossary as well. Some basic terminology words will be defined in the current Section while others will be defined later on as the subjects to which they relate to are introduced.

Zara's stock keeping units are determined by a Reference, Color, and Size coding that is visible in any of its garments' price tags:

Figure 4: Sample Price Tag

The Reference designation is itself composed by a four-digit number that identifies the garment's Model and a three-digit number that identifies its Quality, where the Model relates to the garments design pattern and its Quality to the textile material it is made of. Similarly, a three-digit number determines the Reference's Color. A single digit designates Size, but a more customer intuitive symbol, such as "M", "L" or "XL", is used on the printed price tags. Referring back to the example on Figure 4, it shows a designation of "5927/024/716 S", which uniquely identifies a garment as a Model 5927 in Quality 024 (or equivalently Reference 5927/024) in the Color 716 and Size small. Additionally, Country, Store, Campaign and Season designations accompany the Reference, Size, and Color designations for a highly detailed tracking of Zara's inventory and sales.

2.2.1.2 Sales Campaigns and Seasons

As mentioned in Chapter 1, Zara divides its design collections into two Campaigns: the Summer Campaign that is gradually introduced starting in January, and the Winter Campaign that is introduced starting in July[11]. Also, as it was previously explained, each of these Campaigns is in turn divided into a Regular Season and a Clearance Season.

2.2.1.3 Prices and Pricing

Zara's nominal prices, i.e., the possible prices at which garments might be sold, are compiled in the Commercial Pricing List, and are chosen based on their commercial appeal or the positive psychological impact that the numbering choice of the prices displayed on the store has on the customers. This practice is commonly followed across the retail industry and is the reason why you don't find prices like $21.27 or $63.21 on stores, but rather ones like $24.90 or $59.99. In Zara, there are two different Commercial Price List sets, one for the Regular Season and one for the Clearance Season. For example, in Euro currency countries, items priced at values below one hundred are usually ended in "*90*" decimal values (e.g., 89.90, 14.90, etc.) for the Regular Season or in "*95*" values for the Clearance Season (e.g., 19.95, 79.95, etc.) and in no-decimal numbers ending in "*9*" for items priced above one hundred (e.g., 149, 169, etc.). The number of different prices on the Commercial Price List is decided based on achieving a balance between pricing flexibility and store logistics costs and customer appeal to having a limited number of prices displayed on the

[11] Exact dates for the beginning and end of Campaigns vary by country.

stores. The Commercial Price List is based on Spain's Euro pricing and the price lists for other currencies and countries is partially determined through a Price Equivalence Table that for every Spanish price, lists an equivalent price in each of the other currencies. This currency equivalent is influenced by currency exchanged rates, but mainly responds to including prices that appeal to the customer markets in each country, e.g., having Clearance Season pricings such as 39.99 or 19.99 USD in the United States.

2.2.1.4 Organizational Groups

When References are introduced throughout the Regular Season, the Product and Purchasing department determines their prices based on a target price markup over the product costs. Product and Purchasing is itself divided into six different groups within Zara's Woman Division (e.g. Woman, Basic, etc.) and each of these groups is responsible for making the design and purchasing decisions for each respective subset of Zara's References.

Whereas initial pricing decisions during the Regular Season are made by the Product and Purchasing group, Clearance Season pricing decisions (or the price discounts applied over the original Regular Season prices) are made by the Pricing Team and Country Managers. The Pricing Team is conformed by a group of Zara's senior management who are knowledgeable on the product, distribution, financial, and brand image aspects of the Clearance Season. Country Managers are analysts assigned to specific countries or regions where Zara operates and who, amongst some of their responsibilities, oversee store sales performance, new product introductions, and handle general communications with store managers and clerks.

2.2.1.5 Clearance Groups and Clusters

During the Clearance Season, all References are grouped into one out of twenty two Clearance Groups, according to general garment types and Product and Purchasing ownership, e.g., Basic Jackets, TRF Dresses, T-shirts and Polos, Mesh, etc. Additionally, References within a Clearance Group are grouped into Price Clusters, based on their Regular Season prices, so that for example, the Price Cluster *59.90* within the TRF Skirts Clearance Group includes all References within this Group that were priced at 59.90 EUR during the Regular Season. Clearance Season sales and inventories are monitored on a daily basis, and new pricing decisions are made roughly on a weekly basis, based on the perceived need to refresh the current prices. In this way, the Clearance Season is itself divided into Clearance Periods, where the end of one Period and the start of the next one are determined by every time that new pricing decisions are made. Clearance Periods may or may not correspond to a calendar week and are therefore defined only by the points in time where the pricing decisions are made.

2.2.2 Clearance Store Management

Although more than five thousand different References are introduced in Zara's stores during each sales Campaign, they are introduced progressively throughout the Campaign so that customers visiting a store on a weekly basis will discover newly introduced garments with each visit. In order to avoid a cluttered presentation of the garments and allow customers to easily browse through the store's collection and quickly find garments in their particular size, References are presented in their minimum assortment on the store's exhibition floor. In general, a Reference is exhibited in each of its Colors and with one garment for each of its Sizes, or perhaps two of them for the most popular Sizes[12]. In order to execute this store layout concept, store clerks constantly monitor garment placement on the shopping floor, and review sales within the last hours in order to replenish any Colors or Sizes of a Reference that have become depleted. When inventory for a particular Reference is not enough to guarantee an adequate assortment for its display, or it exceeds a certain display life-cycle or its sales fall below a minimum threshold, it is pulled off the shelves and either stored in the store's backroom or sent back to Zara's central warehouses for later redistribution to stores during the Campaign end Clearance Season.[13]

At the store level, the Clearance Season is opened when the initial Clearance Season price assignment list, which details the new discounted prices at which all of the References within the store will be sold, is sent from Zara's headquarters some days in advance. As explained in Section 2.2.1, all References are separated into Clearance Groups and grouped by Price Clusters within them. Initially, price lists are communicated for each Clearance Group, and new prices are assigned to each Price Cluster so that for example, store clerks in charge of manually relabeling price tags

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[12] For a more detailed review of Zara's store display policies, read Correa, J. R. (2007, June). Optimization of a Fast-Response Distribution Network.

[13] This redistribution of the inventory corresponds to the first problem described in Section 1.2, which will be addressed by the MIT4 LFM Internship at Zara, scheduled to begin in June of 2009.

receive instructions along the lines of "all inventory within the Woman Blazer Clearance Group that was originally priced between 119 EUR and 99 EUR will now be priced at 69.95 EUR". In this way, different Price Clusters may be consolidated under a single price or Price Category at the opening of the Clearance Season. From that point on, Clearance Season sales performance are tracked by, and future price communications are made for each of these initial Price Categories, so that following our previous example, future pricing instructions to the store could be: "within the Woman Blazer Clearance Group, the initial Price Cluster grouping of 119-99 EUR will now be priced at 49.95 EUR".[14]

Once the initial price lists are received at the stores, all of the References must be manually relabeled to reflect their new discounted prices. With the exception of some countries where special regulations regarding discounted sales apply, References are relabeled by applying a price sticker over the original price label, leaving the original Regular Season price visible so that customers may compare the current discounted price to the original one:

Figure 5: Clearance Season Price Tag

Preparations at the store begin by relabeling inventory kept in the store's backroom, which is composed of the remnants of the Regular Season or of the inventory that is redistributed from Zara's central warehouses prior to the start of the Clearance Season. The night before the opening

[14] A small distinction between the current project's treatment of data and the way it is handled at the store level is that the developed models use the individual Price Clusters to designate References within a Clearance Group, as opposed to the initial Price Cluster *groupings*.

day of the Clearance Season, store clerks work hard in relabeling the entire inventory on the store's exhibition floor and the whole store exhibition layout is rearranged. As the inventory from the backroom is brought into the store, the number of different References that coexist on the exhibition floor far surpasses that which is normally available during the Regular Season. For this reason, References are no longer presented in their minimum assortment displays:

Figure 6: Regular Versus Clearance Season Store Layouts

As shown in Figure 6, References are grouped by Clearance Group and Price Category when displayed on the store's exhibition floor. Given the increased variety of References and assortment quantities within them, garments are presented under a more condensed space and visually grouped under large visible signs that identify the current pricing of each group.

Once the Clearance Season begins, further price discounts are communicated from Zara's headquarters roughly on a weekly basis. The list of new prices is received on the day before those prices become effective so that the store clerks have sufficient time for the relabeling process. Price tags are generally relabeled after closing hours and new price stickers are placed over the previous price stickers, so that only the original Regular Season price and the newly assigned one remain visible.

At the beginning of the Clearance Season, the store exhibition floor is almost entirely occupied by Clearance Season inventory. When the Clearance Season progresses, inventory levels are gradually depleted and as prices are progressively lowered, more Price Clusters are merged into a single Price Category and so, can be physically displayed together in the exhibition floor. As store space is freed up, new, non-discounted inventory from the following Campaign's Regular Season is gradually introduced. In this way, as it is evident from Figure 1, the Clearance Season from one Campaign briefly coexists with the Regular Season of the next Campaign. Inventory from the new Campaign is generally first placed towards the back of the store and the most visible exhibition space near the store's entrance is saved for the Clearance Season inventory. The area allotted to the new Campaign is then gradually increased until the store layout is once again shifted and the Clearance Season inventory is moved to the back of the store towards the end of the Clearance Season, thus completing the transition to the new Campaign's Regular Season.

2.3 Legacy Clearance Pricing Decision Flow

The current Section describes the basic *process flow* that is followed when determining the Clearance Season pricing assignments. The criteria utilized to make the pricing decisions themselves is described in Chapter 3.

2.3.1 Master Price List Process Flow

The process followed to determine the initial pricing lists for each Country and Clearance Group starts more than a month in advance from the actual beginning of the Clearance Season. Whereas the actual process followed by Zara encompasses more than fifteen operational steps involving 5 different department divisions, the main and most significant steps are summarized in Figure 7:

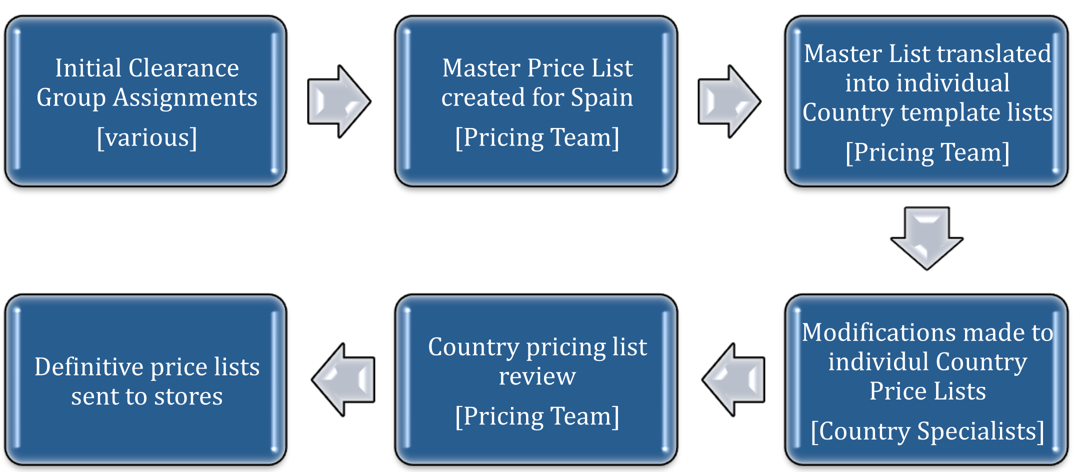

Figure 7: Master Price List Flow Summary

The process begins when, based on the Clearance Group assignments from the previous Campaign, every Reference from the current Campaign is assigned into one of the Clearance group. This association is based both on the Reference's garment type as well as on its respective Purchasing department and product family. This initial process involves considerable manual labor, as Clearance Group assignments for the current Campaign must be manually entered into the system.

Once the Clearance Group associations have been defined, the Pricing Team begins meeting in order to define a Master Price List that corresponds to the pricing assignments for each Price Cluster and Clearance Group and that will dictate the prices at the start of the Clearance Season. This Master Price List is based on Spanish stores and sales so, when elaborating it, the Pricing Team looks at the sales history and performance of each Clearance Group in Zara's Spain stores during the current Campaigns Regular Season, and at the behavior of the previous year's Clearance Season.

Once completed, the Master Price List is in essence the list of prices that will become effective at the beginning of the Clearance Season in Spain. Only as an initial template for the remaining countries, the Master Price List is then translated into each of the different country currencies through the Price Equivalence Table.

The next step in the process is for the Country Managers to make modifications on the template price lists for their countries. These modifications incorporate each Country Manager's unique knowledge of their country's market conditions, and through these modifications they ensure that the price lists remain competitive within those respective markets. For example, there might be a country where a particular style of dresses is particularly popular and so doesn't require such a substantive discount for it to sell well, or a country where the warm summer season is particularly short and beachwear must be more fiercely discounted.

After this, each Country Manager meets with the Pricing Team in order to discuss the suggested modifications on the template price list. This final process of approval with the Pricing Team is a good way to ensure congruency or homogeneity between the pricing in different countries and allows greater control on higher-level strategic and brand management decisions. One drawback of this approach however, is the fact that the Pricing Team is generally conformed by three to four members who have to meet with all of the Country Managers and review pricing lists for more than sixty different countries, and this causes a bottleneck. Time constraints are not as significant for the initial price lists as they can be prepared in advance to the beginning of the Clearance Season, but they in the case of the weekly decisions that must be made once the Clearance Season has begun, the time to review these lists is limited.

2.3.2 Clearance Pricing Decisions

Once the Clearance Season has begun, Country Managers track the sales performance of the stores in their countries of responsibility in order to get a sense for the customers' reactions to the current prices. In response to these, Country Managers issue new price lists that must then be revised during a meeting with the Pricing Team:

Figure 8: Clearance Period Pricing

Once the Clearance Season has begun, prices are reviewed on a weekly basis. Furthermore, new prices are usually assigned and sent to the stores on the day before its highest selling days within the week so that the respective store clerks can re-label the store's inventory in time for the weekly peak of store visitors to find new prices when they walk into the store the next day. For this reason, most of the pricing list reviews take place towards the second half of the week, which heightens the bottleneck effect and places added strain on the Pricing Team, which has to meet with Country Managers supervising the more than seventy countries where Zara operates. To counter this, different members of the Pricing Team or two-person teams meet with different Country Managers so that the review process can be done in parallel. Once the lists are approved, each Country Manager communicates them to the stores, and the cycle illustrated in Figure 8 is then repeated roughly on a weekly basis until the end of the Clearance Season.

2.3.3 Business Practices in Pricing

2.3.3.1 Hard Rules

On top of the pricing criteria that will be explained in the next chapter, there exist a series of pricing business practices that Zara's Pricing Team and Country Managers follow. As a business guideline, all items that were priced at a same price during the Regular Season must remain grouped under a same price all throughout the Clearance Season; this allows for grouping of different garments and so that clerks don't have to identify individual References but rather larger groups during the price tag relabeling process. Furthermore, a Reference that during the Regular Season is priced higher than another one must always have a price that is equal or greater than the price of that other Reference. In other words, the price order or References is preserved, mainly in response to brand and store protection purposes, e.g., higher quality design garments should logically be priced higher than less elaborate ones. Also, needless to say, once a price is lowered, it cannot be raised again, as this would generate distrust in Zara's customers. During the Clearance Season, as increasing discounts are issued, Price Clusters within a Clearance Group will tend to consolidate under a single price, and once they do, both Price Clusters must remain joined under a same price. This guideline responds to the way in which subsequent prices are assigned, i.e., as a method to avoid having to identify individual References when price tags are relabeled and relabeling all the References that are displayed under a single price sign instead.

2.3.3.2 Soft Rules

The guidelines discussed so far can be treated as rules, in the sense that their enforcement is not open to subjectivity, i.e., they are either followed or they are not. In contrast, there are other pricing business practices that have "softer" boundaries and are more like levers that the Pricing Team and Country Managers can adjust to determine the pace and management of the Clearance Season. For example, the minimum price discount that should be applied over a Reference's Regular Season price is usually determined to be a discount of at least fifteen percent. Minimum price discounts determine the ambience and price sensation within stores, e.g., a customer might be disappointed to walk into a store's Clearance Season only to find that items are barely discounted from their original prices and decide to visit a different brand store where discounts are more attractive.

Along these same lines, a Clearance Group's Exit Price denotes the highest price offered within the Group and has an important psychological impact over the customers, e.g., a higher Exit Price gives customers the impression that prices at the store in general are higher. For this reason, unless there is a critical mass in inventory that will be priced at the Exit Price, it might be preferable to liquidate that inventory at a lower price and avoid misperceptions on the brand's image

Finally, another way in which the pace of the Clearance Season can be managed is by placing restrictions on the maximum number of different Price Categories that are created for a Clearance Group. Limiting the number of different prices offered forces the consolidation of different Price Clusters, creates a more pleasant store environment, and simplifies the relabeling operations at the store level.

3 Pricing Problem

With approximately 11,000 new References introduced every year, distributed roughly in half between the Summer and Winter Campaigns, making pricing decisions for all of them on brief intervals of time is not a trivial task.

3.1 Pricing Challenges

Granted that creating a Master Price List for all Countries at the beginning of the Clearance Sales provides some level of homogeneity among the different Countries, it is only the best-known solution yet to a data intensive, time constrained problem. As we made it clear to Zara's Pricing Team all throughout the development of the current project, their business is extremely experience based, and given enough resources and time, their pricing suggestions would be harder to improve with a mathematical model. In other words, if the Pricing Team had infinite resources and time to review all of the References in detail, analyzing their sales during the Regular Season, identifying the specific garments they refer to, their remaining inventory levels, and then making pricing decisions for them, and on top of this, conducting a separate analysis for each individual Country, pricing decisions would more closely approach optimality. However, in reality, the time constraints are very significant and a Master Price List based on Spanish sales histories is chosen as the most representative starting point from which initial pricing lists are rooted for all of the Countries and decisions are made on aggregate, summarized data.

Once the Clearance Season starts, pricing decisions need to be made on a weekly basis by the Country Experts and they have limited access to the main Pricing Team that must review all of the price suggestion lists created by all Country Experts. At the same time, Country Experts are faced with the challenge of analyzing the Clearance Period sales, getting a sense for the customers' reactions to pricing, and weighting the remaining inventory against the number of days left in the Clearance Season. On top of this, Country Experts must try to adhere to the business practices just described in section 2.3.3 and assessing the economical impact of their decisions. Furthermore, having different persons making pricing decisions introduces different personal criteria into the pricing process and makes it difficult to establish uniform and homogenous guidelines at a corporate level.

In summary, Zara's pricing challenge during the Clearance Season can be described as making fast paced decisions that would ideally require the detailed analysis of massive amounts of information, and which influence a large percentage of its sales and inventory. For this reasons, any improvement in the current pricing process has the potential to significantly improve Zara's bottom line profits.

3.2 Manual Solution

As it was discussed in the previous chapter, people with store and inventory knowledge who have built their pricing expertise through years of experience are in charge of Zara's pricing. Due to the inherent time and resource limitations, Zara's Pricing Team and Country Managers must rely on aggregate, summarized reports to get a broad picture of how well different Clearance Groups sold during the Regular Season, how much inventory was left unsold and must therefore be sold during the Clearance Season, and in general, get a feel for how to best price this inventory during the allotted Clearance Season time.

In an effort to capture as much as possible of all the information that is available, the Pricing Team and Country Managers rely on various data reports. These reports allow them to follow the progression of the Clearance Sales and to compare the performance of the current Campaign amongst different countries, Clearance Groups, and Clearance Seasons from previous years. One of the most commonly used reports is the Clearance Group Sales Report, an excerpt of which is included in Figure 9:

		Daily and Cumulative Sales				Current Stock	Days Worth Inventory	Success
	Precio Saldo	Venta día 17/01/2009	Venta día 18/01/2009	Venta día 19/01/2009	Venta Acumul 19/01/2009	Stock Tienda 19/01/2009	Stock/ VentaDia 19/01/2009	% Éxito
FROM 49.90 TO 29.90	19,95	24	5	12	534	1.218	102	31
OF 24.90	14,95	21	8	8	466	1.006	126	32
OF19.90	9,95	16	14	6	420	384	64	54
FROM 14,90 TO 12,90	6,95	21	22	12	519	322	27	64
	Totales...	**82**	**49**	**38**	**1.939**	**2.930**	**77**	**41**
TOTAL SECCION		5.887	3.978	2.491	107.419	108.094	43	**52**

Figure 9: Sample Clearance Group Sales Report

In this automated report, sales results are shown for each Country and Clearance Group and a summary of average values at the Clearance Group and Country level is shown at the end of each section. The excerpt shown in Figure 9 corresponds to data for a single Clearance Group. In the report, each row corresponds to one of the initial Price Categories created at the beginning of the Clearance Period, e.g., the first row corresponds to the initial Price Category composed of the Price Clusters from 49.90 to 29.90. The data shown on the columns to the right correspond to the current Clearance Price for each Price Category (i.e., the price at which it is currently being sold), so for example, it can be seen that there are currently four distinct Price Categories within this group, one for each of the different prices on the first column of the report. The following columns show the sales figures for the last three days, the cumulative sales to date, the remaining inventory, and the number of days of sales this inventory represents based on the previous days' sales. The right-most column that is highlighted in grey corresponds to the success rate, a measure of the percentage of inventory that has been sold during the Clearance Season. From this report, it is possible to observe the sales trend for the last days and measure the remaining inventory against the number of days left in the Clearance Season. Also, by looking at the overall average figures, it is possible to determine what Clearance Groups and Price Categories within them have days worth inventory levels above the total average level and assess whether further markdowns are needed to reactivate its sales.

Other things that the Pricing Team can look at are the remaining inventory levels in order to consolidate smaller sized price categories into a single price label so that the corresponding References may be physically consolidated in the store.

Although this Clearance Group sales report is a very useful tool that summarizes massive amounts of information in a report that can be browsed literally in seconds, it suffers from several disadvantages. For example, browsing the days worth of inventory for a Price Category and comparing it to the Country average might prompt the analyst to introduce a price cut, but it doesn't provide an indication of how big a markdown is necessary, or if marking down is even the most profitable solution. For example, there may be cases where, despite having considerable remaining inventory for a certain Price Category, it is more profitable to maintain a price and end up with more unsold inventory because the possible increase in sales that would be obtained with a markdown would not compensate for the less profit made by selling at a lower price. Similarly, it is difficult to evaluate or weight the profit repercussions of selecting one price over another, and even more challenging to make future profit projections for the remaining Clearance Periods.

3.2.1 Pricing Decision Case Study

As an illustrative example of the thought process that an analyst might follow when making pricing decisions for a Clearance Group, we can picture the case of a Country Manager who is working on the Price Suggestion list for her country of responsibility. She has just printed a copy of the Clearance Group Sales Report detailed in Figure 9, and needs to come up with her Price Suggestion list within the next thirty minutes, at which time she will be meeting with two members of the Pricing Team who will review and validate her suggestions. Furthermore, we can pretend that the illustrated report was issued on day number twenty-five of the Clearance Season and that the average values for the entire country being considered are the following:

	Precio Saldo	Venta día 17/01/2009	Venta día 18/01/2009	Venta día 19/01/2009	Venta Acumul 19/01/2009	Stock Tienda 19/01/2009	Stock/ VentaDia 19/01/2009	% Éxito
TOTAL SECCION		5.887	3.978	2.491	107.419	108.094	43	**52**

Figure 10: Sample Clearance Group Sales Report, Country Summary

Starting from the data shown in Figure 9, the Country Manager would most probably focus her attention on Price Categories of 19.95 and 14.95, as most of the inventory (1218 and 1006 units respectively) is concentrated on these categories. When looking at the sales figures from the first three columns, the analyst would first properly frame the observed sales trend based on the fact that the first column corresponds to the date 17/01/09, Saturday, when sales are generally amongst the largest within the week. Given this context, and based on her business experience, the Country Manager could then assess the general trend in sales for each of the Price Categories and make projections into the future if the current prices were to be kept unchanged. Returning to the inventory levels, she could then examine the number of days worth of inventory based on the previous day's sales and would realize that the first two Price Categories have 102 and 126 days worth of inventory, values that are considerably higher than the Clearance Group's average value of 77. Furthermore, looking at the Country average values from Figure 10, she would corroborate that the average days worth of inventory for all of the Clearance Groups is only 43 days. Similarly, looking at the right-most column in the report, she would observe that the success rates for the first two Price Categories are only 31 and 32 percent, whereas the Clearance Group and Country averages are considerably higher at 41 and 52 each.

Based on this analysis, the Country Manager would most likely decide that the prices for the Price Clusters currently priced at 19.95 and 14.95 should be further lowered, while the remaining Clusters could stay at their current prices. Now, however, the Country Manager faces the non-trivial task of deciding how aggressive a discount is necessary to get the sales of these Price Clusters "back on track". Will lowering their prices to 12.95 and 9.95 be enough? And if so, will the resulting increase in unit sales be large enough to make up for the lower revenues from selling at a lower price? And if not, should she even discount these Price Clusters at all?

This is an example of the uncertainties that Country Managers and members of the Pricing Team must deal with when making important pricing decisions that directly impact Zara's bottom line revenues. Furthermore, the Clearance Group from Figure 9 and Figure 10 was specifically chosen for its unusually high days worth of inventory and low success rate in order to illustrate a case where further price discounts would normally be issued, but this is not representative of the majority of Zara's Clearance Groups, where the decision to lower prices or maintaining them is not this trivial.

The Country Manager in this sample case would ultimately rely on her experience of the business to make her final decision. But given different the different backgrounds and experience levels of different Country Managers, there really is no saying over whether one pricing suggestion is optimal over another one, or whether two different Country Managers would reach a consensus over which price list will result in bigger earned profits. In order to contrast the legacy pricing methodology with the methodology presented in the current thesis, this case will be revisited in Section 6.6.4, once the methodology has been thoroughly presented and explained.

3.3 Proposed Solution

The solution proposed in the current project is a Linear Optimization Model that makes optimal pricing decisions in order to maximize profits during the Clearance Season. For every Country and Clearance Group, the Optimization Model works with a forecast for sales corresponding to each possible price from the Clearance Price List and selects and assigns the optimal prices that will guarantee the maximum profits possible. The model assigns a price to each individual Price Cluster so for example, a possible model output for a Clearance Group is to assign a price of 19.95 EUR to the 24.90 EUR Price Cluster, i.e., all of the References that were priced at 24.90 EUR will be marked down to 19.95 EUR. In this way, the model is run periodically throughout the Clearance Season every time prices need to be revised, and each time the model analyzes the most recent data available and issues a new list of assigned prices in which some of the Price Clusters may be further marked down while other's current prices may be maintained. And because different price markdowns are possible at different time points during the Clearance Season, when making price assignment decisions, the Optimization Model will also make internal projections for sales during the remaining weeks, testing different price progression scenarios and choosing the optimal one for the current Clearance Period. Furthermore, the model when assigning the optimal prices also considers the fact that any unsold inventory at the end of the Clearance Period can be sold for a salvage value. As shown in Figure 11, the proposed project will include both the just described Optimization Model as well as a Forecasting Model that computes the actual sales forecasts that serve as an input for the Optimization Model.

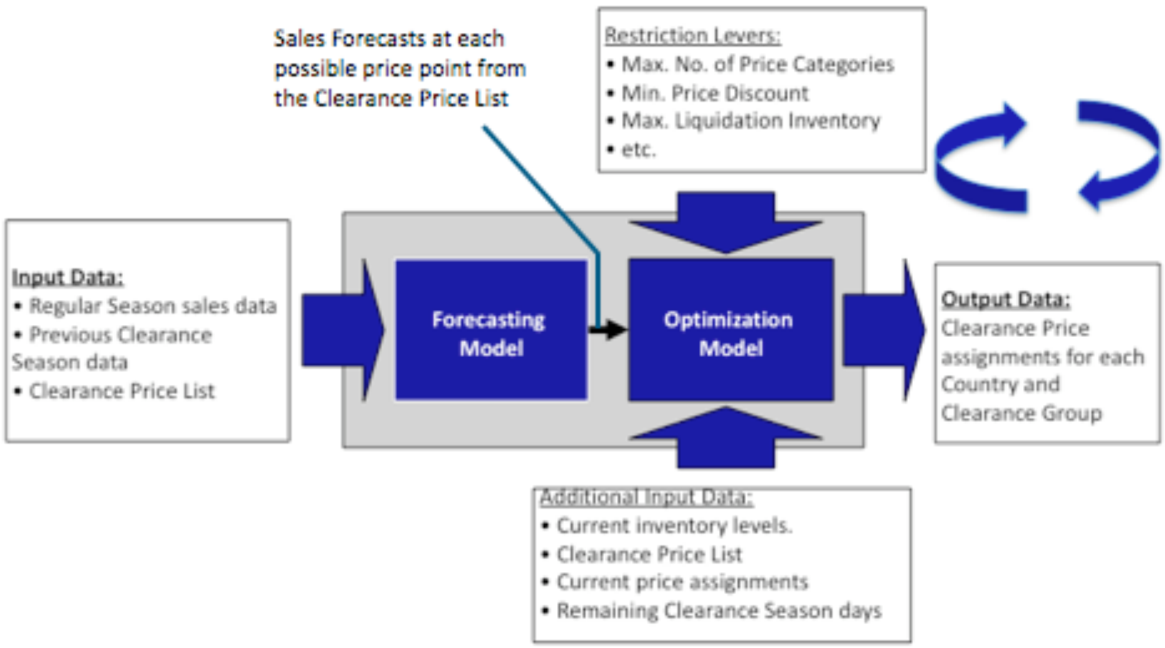

Figure 11: Proposed Solution

Fitting a Linear Regression multiplicative functional form that relates the conditions of several relevant Reference variables such as purchase size, time from introduction to stores, and inventory position to the observed sales during the current Regular Season creates the basis for the Forecasting Model. Furthermore, estimating and measuring the consumer's price elasticity of demand, the Forecasting Model is capable of producing sales forecasts for References when they are priced at any of the prices from the Clearance Price List.

In addition to the sales forecasts from the Forecasting Model, the Optimization Model takes inputs such as the current inventory levels, the time remaining in the Clearance Season, and the prices currently assigned to the different Price Clusters in order to optimize its price selection. Furthermore, the user of the model has the option to adjust or select a series of Restriction Levers in order to influence the output price assignment list. For example, the user might want to restrict the maximum number of different Price Categories created in order to consolidate several Price Clusters into a single Price Category so that, at a store level, more References are priced equally and can therefore be displayed together thus saving space. Similarly, the Optimization Model captures

Zara's pricing business practices such as having a decreasing price progression or observing minimum price discounts from one Clearance Period to the other as constraints.

As shown in Figure 11, when the models' data has been populated through automated databases, the Price Team or Country Managers can run the model to obtain price list assignments for any specific Clearance Group and Country. Furthermore, following an iterative process, they can continue to adjust the Restriction Levers if necessary and evaluate the repercussions that placing restrictions on the output price list has on the expected profits.

The most salient advantage of the proposed solution is the fact that it optimizes the overall profits made during the Clearance Season as opposed to the liquidation of remaining inventory. Working with an automated Forecasting Model, the Optimization Model is capable of making pricing decisions that take into consideration the sales history and characteristics of every single Reference within a Clearance Cluster and that are specifically tailored for each Country and Clearance Group, therefore providing a detailed level of analysis that would otherwise not be possible. By considering and iterating through different possible price progression scenarios, the Forecasting Model can look beyond the current Clearance Period that it is run in, and makes pricing decisions that are globally optimal for the complete Clearance Season.

Finally, following the proposed solution provides the added benefits of having a single, homogeneous decision process, through which all of Zara's pricing decisions are made. Additional positive collateral effects result in relieving the bottleneck effect that arises from the Pricing Team having to meet with all of the Country Managers, since the proposed solution provides a near-to-final pricing list and only minor revisions over it are required. The pricing communication logistics will also be enhanced as the submission of selected prices to the warehouses, and ultimately to the store-point cash registers, may be automated because the solution is closely embedded within Zara's IT systems.

4 Forecasting Model

4.1 Introduction

The Forecasting Model needs to provide a forecast for every possible price on the Clearance Price List for each Price Cluster within each individual Clearance Group and Country. In other words, its purpose is to estimate the expected unit sales when References that were priced at the same price during the Regular Season are priced at any of the possible discount prices.

In order to create the model, an adequate metric for observed sales volumes is designed, qualitative and quantitative variables that influence these sales are determined, and a functional form for the model is chosen. The model is then fitted through a minimum-squared difference linear regression. A methodology for measuring price elasticity of demand is implemented and incorporated in the model and the desired sales forecasts are then possible.

4.2 Data Mining

The first step in developing the forecasting model was to determine the available data, and most importantly, the data that would normally be available when the model is first run at the beginning of the Clearance Sales and subsequently, each time a pricing list needs to be generated. Most of this information was known from the previous experience of the MIT1 project or from Caro and Gallien's initial visits to Zara.[15] In addition, the scope of the forecasting model and grouping of data was based both on the Clearance Groups used in the legacy pricing methodology (2.3) and on achieving the right balance between detail and data reliability.

4.2.1 Time Periods

Zara logs point-of-sale records on a daily basis and results from a day's sales are generally available and consolidated at their main headquarters during the morning of the next day. This allows us the flexibility to have a very fine level of detail that we can also summarize on a higher level when convenient.

[15] Correa, J. R. (2007, June). Optimization of a Fast-Response Distribution Network.

Such is the case of the Regular Season sales data that is available when the model is first run before the first week of Clearance Sales. Because of the seasonality in sales that occurs from one day of the week to the other, and between weeks within the year, the most sensible grouping of the data is to group it by calendar weeks. This means that for the Regular Season, all the available data (i.e., sales, inventories, shipments, etc.) is summarized by week when it is extracted from the repository database and made available to the model. In this way, the volume of the data that needs to be handled is reduced significantly without sacrificing the value of its information.

Similarly, for the Clearance Season, the most sensible grouping of data registers corresponds to grouping by Clearance Periods, which are defined as the periods between the times when successive pricing decisions are made. The idea behind this reasoning is to summarize the data under time units at which the References did not suffer any price changes. In order to avoid overlapping periods between different Price Clusters in a Clearance Group, the Clearance Periods are terminated or split whenever *any* of the Clusters within any of the Groups suffers a price change. In other words, a Clearance Period ends and another one begins every time that the pricing team at Zara meets to define prices and sends its pricing decisions out to the stores; this corresponds to the moment when the currently discussed model is run to generate a pricing suggestion list.

In addition to the current Regular Season data, information from previous years' Regular and Clearance Seasons is also available and provides useful insight on the behavior of Clearance Sales and its relation to the Regular Season sales when the current Clearance Sales have yet to begin.

Once the Clearance Sales start, new data is gathered on a daily basis and we no longer have to rely solely on previous years' data to evaluate the effect of selling References at a discount, as we now have relevant data from the actual References being sold.

4.2.2 Data Treatment

Zara introduces approximately 11,000 new References (Model/Quality) every year and maintains individual daily point-of-sale records at a Store, Color, and Size level of detail. Given this, one may be tempted to design a model that forecasts sales for each Store, Reference, Size, and Color, however, business intuition and data reliability dictate otherwise.

From a revenue perspective, it makes no difference whether a Reference was sold in one Store or another, or in one particular Color or Size as they all are priced equally, i.e. the profits generated in all the Stores in that Country are unaffected, so this increased level of detail in forecasting does not add value. If we go to the other extreme, we might want to generate a sales forecast for all References within a Clearance Group that are sold at the same price, however, different References exhibit very different sale behaviors that we can only accurately capture when we forecast each Reference separately.

One additional point to support this approach is that we found out that inventory and sales data in Zara is most accurate when observed at an aggregate Reference level. This is due to error in the manual entry of data at stores and to Zara's consolidation of invoice differences that is done at a Reference level.

For these reasons, sales forecasts are made at a Reference level. However, as previously discussed in Chapter 2.3, during Clearance Sales, References are grouped by Clearance Group (e.g. Blazers Woman) and Price Cluster (e.g. References with a Regular Season price of 129 EUR), and a single price is assigned to each grouping, so that the pricing decision is made at this aggregated level. Consequently, the data treatment can be more accurately described as: sales are first *forecasted* at a Reference level and are then aggregated by Price Cluster and fed into the Optimization Model that makes the *pricing decision* at this aggregated level.

4.2.3 Instances and Model Scope

Because each Season, Section, Country and Clearance Group exhibits different sales behavior, a separate instance of the model is fitted for each of these combinations (e.g. summer, women's sales in Germany for the Blazers Basic group). In other words, a different model is run and a price list generated for every Clearance Group in each of the Sections in each Country so if, for example, the model was to be implemented for all of the more than seventy countries where Zara operates, and for each of its more than twenty Clearance Groups, then more than 1,400 instances of the model would be invoked every time that pricing decisions were made. This configuration also allows for the flexibility of including different model variables for different groups and countries, should it be required.

4.3 Measuring Sales

Zara's business is characterized by well-defined seasonal trends and sales of a particular Reference will vary depending on the Store at which it is sold, and the Colors and Sizes that it is offered in. Therefore, before selecting the variables that are explanatory of sales and fitting a linear regression model, it is first necessary to design an unbiased metric for sales that will remove the nonlinear effects of seasonality and product assortment so that the true relationship between the explanatory variables and sales can be determined.

4.3.1 Seasonality

There are two types of seasonality inherent in Zara's sales trends: intra-week and inter-week seasonality. Intra-week seasonality refers to the seasonality observed from one day of the week to the other, e.g. sales on a Friday tend to be greater than sales on a Monday. We need to normalize the observed sales for this seasonality because otherwise, in fitting a regression model, we would look at instances in which one same Reference, under equal conditions, exhibits different sales which can be explained by none other factor than the day of the week the sales took place. In order to determine this intra-week seasonality, we look at the aggregate sales levels throughout the Regular Season and assign a weight to each day of the week and normalize it so that the sum of all weights is equal to seven[16]. Intra-week seasonality weights for a sample Country and Clearance Group for three consecutive Summer Campaigns are shown in Figure 12, and it can be seen how the intra-week seasonality trends are very consistent from one year to the other:

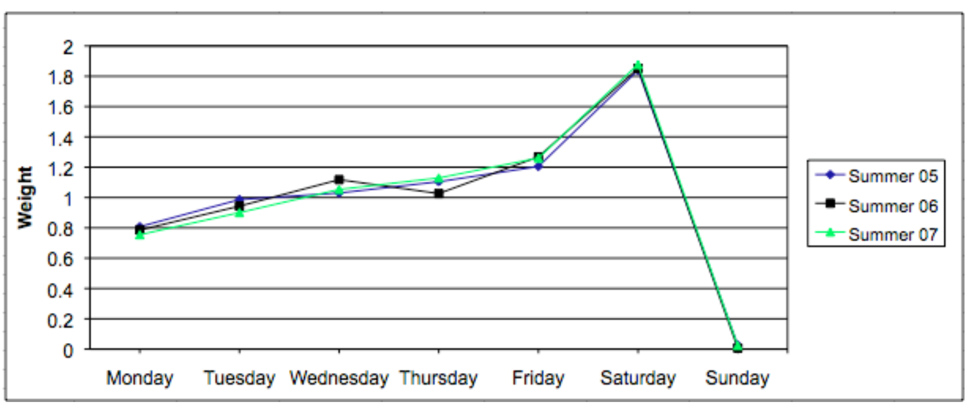

Figure 12: Sample Intra-Week Seasonality Weights for the Summer Campaign

[16] Regular Season data, as opposed to Clearance Season data, is used in order to observe the true seasonality of sales in the absence of price discounts.

Similarly, inter-week seasonality measures the different sales patterns from one week of the Regular Season to the other, and accounts for regular sales trends within the year and other special events such as national holidays, which can affect the observed sales. Figure 13 shows the weighting of the inter-week seasonality for a sample Country and Clearance Group:

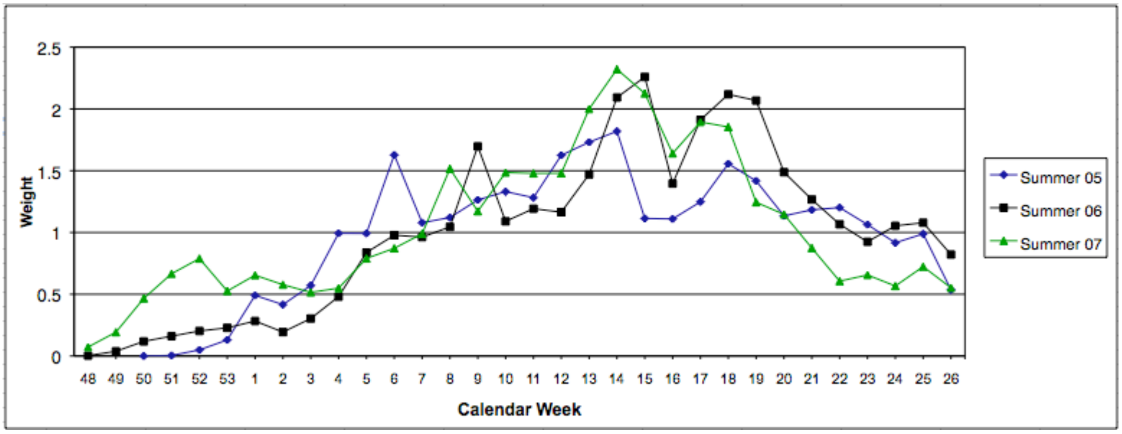

Figure 13: Sample Inter-Week Seasonality Weights for the Summer Campaign

In this example, sales for the Summer Campaign begin to ramp up during the last weeks of the previous year and come to a halt at around week number twenty-six, when the Clearance Season begins.[17] Inter-week seasonality weights are calculated as the fraction over the total Regular Season sales that each week's sales represent, and normalized so that the sum of the weights is equal to the number of weeks considered.

Different Countries and different Clearance Groups exhibit different seasonality trends, e.g. Thursdays are important sales days in The Netherlands but not quite as important in Spain, so this exercise is repeated for each Country and Clearance Group. Finally, as discussed in section 4.2.2, because the Regular Season data is grouped by calendar week, only the inter-week seasonality weighting needs to be applied to this data. On the other hand, because Clearance Season data is grouped by Clearance Periods, the individual intra-week weightings are applied to each specific day-of-the-week within each Clearance Period.

[17] A slight decline in sales is evident towards the end of the Regular Season, as customers anticipate the advent of the Clearance Season, so data is truncated in order to correct for this effect.

4.3.2 Location, Color and Size

In the same way that the day of the week and week of the year play an important role on the sales observed for a particular Reference, so do the Color and Size of that Reference and the Store that it was sold in. For example, if we observe the sales of the exact same Reference (for now, we can assume that it is only offered in a single Color and Size), in the same day of the year, but at two different Stores, then it is natural to expect the Reference to sell better at that Store that has the most traffic or higher overall sales. In the exact same way, if we observe sales of a Reference in a particular Store and Color but different Sizes, or sales in a particular Store and Size but different Colors, we find that there are some Colors that are more popular and sought out than others, as well as more popular Sizes that fit the majority of the customer population better.

For these reasons, if we want to build a regression model and identify the variables that explain sales for a given Reference, we first must account for, or normalize for the Color, Size, and Store at which that Reference was sold.

4.3.3 Demand Metric: Time Scaling

In order to measure sales in a way such that the effects of seasonality and Reference Color, Size, and Store, Caro and Gallien proposed the weighted demand metric illustrated in Equation 4-1 where the index *w* refers to a time period, i.e. a calendar week for Regular Season or a Clearance Period during the Clearance Season, and *r* represents a Reference[18].

$$\lambda_r^w = \frac{Sales_r^w}{Time_r^w}$$

Equation 4-1

This demand metric provides a normalized measure of the sales for a Reference observed during a particular period where the effects of seasonality and location, color, and sizing are taken into consideration and adjusted for. The numerator for Equation 4-1 is obtained by adding the unit sales for the Reference *r* under consideration in each of its offered Colors *c*, Sizes *s*, and Stores *j*, for the time period *w* under consideration:

[18] Caro, F., & Gallien, J. (2007, December 10). In preparation for: Clearance Pricing for a Fast-Fashion Retail Network.

$$Sales_r^w = \sum_{c,s,j} Sales_{rcsj}^w$$

Equation 4-2

The normalization of the demand measure is accomplished through time scaling on the denominator of Equation 4-1:

$$Time_r^w = \sum_{c,s,j} \alpha_{rcsj} * \tau_{rcsj}^w$$

Equation 4-3

The time scaling is achieved by: 1) An *alpha* factor that assigns a weight based on the sales strength of a particular Color, Size, and Store combination, and 2) By a *Tau* factor that adjusts for the number of days an item was under display within the considered time period and a weight based on the seasonality of the time period being considered.

Alpha factors are calculated for each Color, Size, and Store combination by measuring the fraction of the total sales for the Reference *r* under consideration that each combination represents:

$$\alpha_{rcsj} = \frac{Sales_{rcsj}}{\sum_{c,s,j} Sales_{rcsj}}$$

Equation 4-4

In this way, as illustrated by Equation 4-4, each Color, Size, and Store variation within each Reference is assigned a weight with a possible range between zero and one such that the sum of all the *alpha* factors for each reference is equal to one. These factors are calculated for each Reference based on the entire Regular Season sales, which provide an unbiased weighting of the sales of each Reference combination.

Tau factors are calculated for a particular period *w* by first estimating the number of days that the reference was displayed at the store floor, and then applying a *delta* seasonality factor:

$$\tau_{rcsj}^{w} = \sum \delta_d \left(DaysOnDisplay_j^w - DaysNotOnDisplay_{rcsj}^w \right)$$

Equation 4-5

The number of days that a Reference was displayed at the store by calculating the number of days that the store opened for business in that period, *DaysOnDisplay*, and subtracting the number of days *DaysNotOnDisplay* for which store data is available (i.e. the store opened) but there was no inventory available for that Reference in any particular Color, Size combination at that store. In using this formulation, we are making the assumption that if there is inventory in the store, it is present on the store floor where customers can see it and purchase it. In reality, this is not entirely true, as when inventories for a particular Reference are "scarce", the inventory is no longer displayed and is kept in the backroom or store warehouse awaiting the Clearance Sales where everything is brought out for liquidation. In determining an appropriate threshold that could determine when inventory was "scarce", we experimented with different alternative threshold levels below which we could assume that a Reference was no longer being displayed at the store but came to the conclusion that these thresholds would vary greatly from one case to the other and thus decided to make the stated assumption and rather capture the effect of non-displayed References as part of a Broken Assortment variable in the regression model, which will be discussed briefly.

Delta factors are applied to the number of days that a Reference was on display during a time period in order to weight and adjust them for seasonality effects. The *delta* factors are simply the seasonality weights presented in section 4.3.1. In this way, when the *lambda* reference demand level presented in Equation 4-1 is calculated for sales observed in the Regular Season, inter-week *delta* factors are applied, and when demand levels for the Clearance Season are estimated, intra-week *delta* factors are applied.

In essence, what the time scaling method presented in Equation 4-1 does is assign different values to each day of store display depending on whether the Reference is offered in one Color, Size and Store or another, or whether that day or week is a seasonally high or low on sales. In other words, by scaling the observed sales by the *Time* factor, different merits are assigned to each observed unit sale of a Reference, e.g., twenty units of a Reference sold in its most popular Color and Size, in a high traffic store and on an important shopping holiday might be scaled to the same *Lambda* demand level as 5 units of the same Reference but in a less popular Color and Size, sold at a smaller store and on a regular weekday.

By using the proposed *Lambda* demand level metric, we remove the biases caused on observed unit sales by seasonality effects and Color-Size-Store combination effects and are able to focus on the qualitative Reference variables that explain the observed demand levels and that can be used to forecast them.

4.4 Explanatory Variables

The process for designating appropriate Explanatory Variables was centered around gathering Zara's legacy business experience and working together with the Pricing Team in order to determine which factors are determinant in sales and to design metrics around them that could then be translated to variables in a linear regression model.

4.4.1 Legacy Business Learning

After meeting with the Pricing Team at Zara and experienced Country Managers, two main factors were identified as the most relevant in determining the sales of a particular Reference: its assortment and fashion component. A reference's assortment refers to the variety of Colors and Sizes in which it is offered at a store. Having the adequate display of Colors and Sizes in a store is an important part of Zara's business model and having this assortment is crucial for the product to sell well in a store.[19] A Reference's fashion component refers to its trendiness or how much it differs from basic garments, e.g., a trendy dress with an artsy flower pattern vs. a plain green sweater. References with a higher fashion component are characterized by more volatile sales trends and shorter peak seasons while References with a lower fashion component have stable sales levels and continue to sell from one year to the other. Also important to note, is a Reference's age or position within its life cycle, as it follows a predictable pattern as it progresses from a novelty article when it first reaches the store and gradually looses this edge with time.

[19] For more information on in-store assortment and its role in Zara's business model please refer to Correa, J. R. (2007, June). Optimization of a Fast-Response Distribution Network.

4.4.2 Formulation of Variable Metrics

When consulting with Zara's managers about the possible factors that influence sales, most of the identified factors were experience-based as opposed to quantifiable and tangible data, i.e., they are factors which are readily observed by a store manager walking around his store but not straightforward to grasp from point-of-sale and inventory databases. For instance, attributes such as a Reference's intended seasonal weather use, e.g. swimming trunks for use in summer, are important in determining its sales, especially when it is soon to go out of season.[20] In this respect, the available data is limiting because even though there exists ample data, including descriptions and photographs for most garments, there is no straightforward way of analyzing this data and incorporating it into a model. Designing and testing heuristics to capture the identified factors is therefore a crucial step in the variable formulation.

In considering a Reference's assortment, heuristic methods for designating a minimum threshold for the number of garments available in each Color and Size at each Store can be calculated to determine the point where a Reference can be considered "well assorted". Similarly, indicators of whether a garment is available in the most highly demanded colors, e.g., black or white, can be used to explain its sales.

The fashion component of a Reference can be determined from other factors that are highly correlated to it, like for example, the initial global purchases for it; purchases for riskier, high fashion component garments are naturally smaller than less risk, lower fashion component ones.

Further additional possible candidate variables were identified after working together with Zara's Pricing Team. The number of days past since a Reference is first introduced in a store is a good predictor for its demand level, as it starts at a high level and gradually decline with time.[21] Also indicative of demand level, a Reference's success ratio measures the total cumulative sales made over the total inventory delivered to a store and provides a way to identify its popularity among customers.

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[20] Although Zara separates their campaigns into Summer and Winter Campaigns, this nomenclature refers to budget allowances and not necessarily for the garment's intended weather use.

[21] Once seasonality and other biasing effects are removed.

When considering the impact of price discounts over demand levels during the Clearance Season, there are several ways in which the discount may be measured. One of them is to take the quotient of the current price over the original Regular Season price and quantify the discount relative to the original price. Another way is to measure the absolute difference between the original price and the discounted one. Given that Zara's customers can read both the current discount price and the original price off a garments price tag, yet another factor that can be considered is whether the price discount represents a change in the tens' position of the price number, e.g. a price change from 24.90 to 19.90 tends to seem more appealing than a change from 19.90 to 14.90, even though they both represent a markdown of five. Finally, the current price itself may be used itself as a categorical or dummy variable, since there are certain prices that are of a psychological appeal to customers and therefore explain higher demand levels.

4.5 Regression Analysis

Once we compiled a list of explanatory variables, a model functional form and different combinations of the variables were fitted through regression analysis and the ones that worked best were chosen as the standard variables for the model. In this way, the demand level metric introduced in Equation 4-1 is explained by these chosen variables.

4.5.1 Model Functional Form

An exponential functional form was chosen for the Forecasting Model, based on its documented positive results[22]:

$$F\left(x_1,\ldots,x_n\right)=e^{\beta_1 * x_1} * \ldots * e^{\beta_n * x_n}$$

Equation 4-6

[22] Smith, S. A., McIntyre, S. H., & Achabal, D. D. (1994). A Two-Stage Sales Forecasting Procedure Using Discounted Least Squares. *Journal of Marketing Research , 31* (1), 44-56.

In this functional form, the *lambda* demand level is a function of the explanatory variables x_i,[23] and β_i parameters are fitted through minimum squared error linear regression analysis. This functional form is fitted with variables and demand level data for each reference and a different set of *beta* parameters is calculated for each Clearance Group and each Country.

4.5.2 Selection Criteria

The main criteria for selecting what variables to use from the pool of candidate variables that are tested with the model was the correlation factor or R^2. Additionally, as a rule of thumb, the t-statistic considered acceptable for incorporating a variable to the model is taken to be 2.00. Finally, simplicity in the model formulation is highly desired, so only variables that significantly increase the model's correlation are incorporated into it, and the number of variables utilized is kept to a minimum. Along these same lines, variables for which the fitted *beta* parameters have a small weight that only contributes marginally to the forecast value, were not selected.

It is important to note that ideally, the variable selection criteria that would make the most sense would be one where variables are selected according to the Forecasting Model's final sales prediction errors. However, as the reader will soon realize, the forecasting procedure involves several stages and the massive database calculations required make such an iterative process impractical.

4.5.3 Selected Variables

After several iterations in which different candidate variables were tested, five of them were selected for the final version of the forecasting model.

4.5.3.1 Purchase

The variable *Purchase* refers to the overall purchase or manufactured number of units of a Reference for selling in Zara's worldwide stores. As mentioned earlier, the fashion component of a Reference is a good indicator of its demand level. In turn, the global purchase size is correlated to the fashion component and therefore, provides a means for measuring it. The global purchase, as opposed to the purchase for each particular country, provides a better measure of the fashion component, as it is not biased by a country manager's expectations for sales in his country.

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[23] Natural logarithm conversions are applied to variables, depending on the range of their possible values for scaling purposes

References with low fashion content are purchased in larger quantities and so there is a positive correlation between the purchase and the observed demand level.

4.5.3.2 Age

The *Age* of a Reference is defined as the number of days past since it was first shipped out to the stores for selling. The peak of a Reference's sales occurs shortly after it is first delivered out to the stores, and as the weeks go by, its sales gradually decrease, and so, age is negatively correlated to the demand level.

4.5.3.3 Demand Level for the Previous Period

The demand level is auto correlated and abrupt changes in the demand level of a Reference from one week to the other are not expected[24], so the regression objective *lambda* itself is included as a variable in the model with a one period lag, i.e., the forecast for the demand level of the present week is a function of the demand level measured on the previous week, and is represented by the variable λ^{w-1}.

4.5.3.4 Broken Assortment

Including the stock at the beginning of the period as a variable made good business sense as a factor that would explain the observed demand level. As an alternative to the initial stock, the Inventory Position, calculated as initial stock plus stock delivered throughout the week provided a yet better correlation.

The Inventory Position parameter, though significant, brings up the question of whether a large inventory causes large sales, or if they are merely correlated factors. On the other hand, a low inventory position can signal a broken assortment and explain diminished demand.[25] In order to study both of these two effects separately, the Inventory Position variable was split into its two components:

[24] This is only true during the Regular Season. Previsions for the Clearance Season will be discussed shortly.

[25] Smith, S. A., & Achabal, D. D. (1998). Clearance Pricing and Inventory Policies for Retail Chains. *Management Science , 44* (3), 285-300.

$$LN\left[\max\left(1,\frac{InvPos}{f*C*S}\right)\right]$$

Equation 4-7

$$LN\left[\min\left(1,\frac{InvPos}{f*C*S}\right)\right]$$

Equation 4-8

Here, a threshold is defined by: C, the number of Colors in which a Reference is offered; S, the number of sizes in which it is offered; and f, a scalar which is adjusted individually for each Country and Clearance Group. In this way, Inventory Position levels that are above this threshold are captured by the variable shown in Equation 4-7 and those below it by Equation 4-8.

Conceptually, the scalar *f* is a function of the number of stores in the Country, and represents the number of References that should be available for each Color and Size in which it is offered. For example, a scalar value of forty for a country with twenty stores means that the Reference is considered to be well assorted as long as there are at least two units of Inventory Position for each Color and Size in which it is offered.

It is important to note that this formulation only looks at the aggregate Inventory Position level and not at a Color and Size level, e.g., an Inventory Position of 400 for a Reference offered in one Color and two Sizes could as well be evenly distributed amongst both offered Colors or concentrated in one. Although more elaborate inventory weighting methods were tested, the simplest approach of utilizing an aggregate measure yielded the best results. Inventory distribution however, is considered when forecasted demand levels are disaggregated and converted into sales figures, as it will be discussed shortly.

Finally, even though both components of the Inventory Position resulted significant in the regression analysis, only the Broken Assortment component was included in the model. This decision was driven by both the causality issue just discussed for high levels of inventory, and the desire to avoid complications in the Optimization Model that works with the forecasting model.[26]

4.5.3.5 Price Discount

The *beta-5* parameter that is calculated for the variable that represents the price discount corresponds to the price elasticity of demand. In turn, the variable that represents the price discount is calculated as the natural logarithm of the ratio of the discounted price to the Regular Season price. Both constant and non-constant elasticity models were tested and it was this constant elasticity version that yielded the best results.

4.5.4 Forecasting Functional Form

The completed version of the Forecasting Model is presented in Equation 4-9, where P_r^w represents a Reference's current price for week *w*, and P_r^T represents its Regular Season price:

$$\lambda_r^w = F(Purchase_r, Age_r^w, \lambda_r^{w-1}, InvPos_r^w, P_r^w) =$$

$$e^{(\beta_{0r})} * e^{(\beta_1 LN(Purchase_r))} * e^{(\beta_2 Age_r^w)} * e^{(\beta_3 LN(\lambda_r^{w-1}))} * e^{\left(\beta_4 LN\left(MIN\left(1, \frac{InvPos_r^w}{f * C_r * S_r}\right)\right)\right)} * e^{\left(\beta_5 LN\left(\frac{P_r^w}{P_r^T}\right)\right)}$$

Equation 4-9

A single set of six *beta* parameters is initially fitted through minimum squared-error linear regression for each Clearance Group and Country.[27] The input data used for the regression is composed of all the Reference-week combinations within the Clearance Group during the Regular Season, i.e., there is a regression data point for each week in the Regular Season and each Reference sold during that week.

A *beta-0* intercept is initially calculated and then, as illustrated in Equation 4-9, an individual *beta-0* intercept is calculated for every Reference. This is done by first fitting a generic set of six *beta*

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[26] As it will be discussed in Chapter 5, the Optimization Model needs to project the actual forecast into different rolling-horizon alternatives by replicating the forecasting model's treatment of Inventory Position.

[27] The actual linear regression is done by taking the natural logarithm of both sides of Equation 4-9, so that the function is transformed to a sum of *beta* parameters multiplied by their corresponding Reference variables.

parameters for all References, and then individually modifying the generic *beta-0* parameter while holding all other *beta* parameters constant) for each Reference so that the squared difference for that Reference is minimized. In other words, the data for all References is pooled and a common set of parameters *beta-1* through *beta-5* is calculated, and then, individual *beta-0* intercept parameters are fitted for each individual Reference.

Only Regular Season data is used to fit the *beta* parameters in this linear regression because it is the only data available at the time when the Forecasting Model is run before the start of the Clearance Period. Furthermore, because ordinary References are not discounted during the Regular Season, the ratio of the current price to the Regular Season price is always equal to 1 and the price elasticity of demand parameter *beta-5* cannot be calculated. For this reason, although the functional form presented in Equation 4-9 is followed, a different approach is used to calculate the price elasticity parameter *beta-5*.

4.6 Measuring Price Elasticity of Demand

4.6.1 Residual Method for Parameter Estimation

The price elasticity of demand parameter *beta-5* is estimated by a residual method. Conceptually, the fitted model with parameters *beta-0* through *beta-4* could be used to predict Regular Season demand. If this same model were to be used to predict Clearance Season demand, it would fall short of the real actual demand, as it does not take into account the fact that the References are sold at a discount and they therefore experience an increase in demand. Following this logic, we can calculate this increase in demand as a residual, or difference between the actual demand and the demand $\hat{\lambda}_r^w$ forecasted by the model fitted with parameters *beta-0* through *beta-4*:

$$\phi_r^w = \lambda_r^w - \hat{\lambda}_r^w = \lambda_r^w - e^{(\beta_{0r})} * e^{(\beta_1 LN(Purchase_r))} * e^{\left(\beta_2 Age_r^w\right)} * e^{\left(\beta_3 LN\left(\lambda_r^{w-1}\right)\right)} * e^{\left(\beta_4 LN\left(MIN\left(1, \frac{InvPos_r^w}{f * C_r * S_r}\right)\right)\right)}$$

Equation 4-10

A series of all residuals ϕ_r^w for all References *r* in a Clearance Period *w* can be calculated and then the price elasticity parameter *beta-5* for that period can be calculated by fitting a linear regression in which the residuals are explained by the price discount variable:

$$\phi_r^w = e^{\left(\beta_5 LN\left(\frac{P_r^w}{P_r^T}\right)\right)}$$

Equation 4-11

Furthermore, if there are any other *beta* parameters besides *beta-5* whose weight might need to be modified for the Clearance Season, it could be left out from the residual calculation shown in Equation 4-10 and its parameter re-calculated by including the corresponding variable on the right hand side of Equation 4-11. In fact, this is the case for the parameter *beta-4*; while the explanatory weight of parameters *beta-0* through *beta-3* remains relatively stable throughout the Regular and Clearance Seasons, the effect of the Broken Assortment variable, and the price elasticity of demand do change as the Clearance Season progresses. For this reason, the regression method is used to calculate both the *beta-4* and *beta-5* parameters in the Forecasting Model. Additionally, we need to modify the way in which the variable for the Demand Level for the Previous Period is calculated, as it no longer makes sense to use this variable now that there may be abrupt changes in the demand level as different price discounts are applied from one Clearance Period to the other. To remedy this problem, although the *beta-3* parameter is still calculated with the demand level for the previous period and using Regular Season data, when forecasts are made or parameters estimated through the residual method, the "previous period" is taken to mean the whole Regular Season. In other words, in these cases, the variable that accompanies the *beta-3* parameter is calculated as the overall *lambda* or demand level for all the weeks contemplated within the Regular Season. After incorporating these modifications, the equation used for calculating the residuals is the following:

$$\phi_r^w = \lambda_r^w - \hat{\lambda}_r^w = \lambda_r^w - e^{(\beta_{0r})} * e^{(\beta_1 LN(Purchase_r))} * e^{\left(\beta_2 Age_r^w\right)} * e^{\left(\beta_3 LN\left(\lambda_r^{RS}\right)\right)}$$

Equation 4-12

And the *beta-4* and *beta-5* parameters are calculated by running an instance of the following linear regression for all References in a Clearance Group and in a specific Clearance Period:

$$\phi_r^w = e^{\left(\beta_4 LN\left(MIN\left(1, \frac{InvPos_r^w}{f * C_r * S_r}\right)\right)\right)} * e^{\left(\beta_5 LN\left(\frac{P_r^w}{P_r^T}\right)\right)}$$

Equation 4-13

4.6.2 Key Learnings and Adjustments

Originally, more complex ideas were tested out for estimating the price elasticity of demand. For example, formulations in which the elasticity is calculated as a function of each Reference's success level were tested. However, this approach proved to be too sensitive to provide a generalized solution for making a demand forecast with an elasticity that would work well with all of the References. In other words, depending on a Reference's success levels, its final estimated elasticity could go beyond the limits of what would be considered a normal elasticity, i.e., a negative elasticity that reflects the true nature of the business where price discounts result in increased demand levels. For this reason, the more general approach of fitting a single elasticity factor for all of the References in a Clearance Group was followed.

Even when this general approach is followed, the calculated elasticities must be revised to make sure they fall within certain standards. For example, it is possible to measure positive elasticities that suggest that increases in demand level could be obtained by raising the prices, and although this might suggest that customers are behaving irrationally, the truth is that there factors that the model is not considering, like the placement of inventory within the store; moving inventory from a high-visibility location within the store such as an exhibition table or roundup by the store's entrance to a shelve towards the back of it can itself cause the Reference's demand to drop. Because of this, we incorporated limits and checks on the calculated elasticities, based on the experience of Zara's management and customer behavior. Elasticities that deviate from a minimum expected elasticity, which varies from one Clearance Period to the other, or elasticities with a low regression fit are substituted by either historical measurements or elasticities from similar groups within the same country.

The general trends that are observed for the measured parameters *beta-4* and *beta-5* is an increasing trend in the Broken Assortment parameter and a decreasing one for the price elasticity of demand. This shows that when the Clearance Period begins, the customers' sensitivity or response to price discounts is greatest and it then decreases until reaching an almost inelastic behavior towards the end of the Clearance Period. Conversely, the effect of the Broken Assortment parameter increases as the Clearance Period advances and the most sought-out-for inventory is depleted and the assortment broken.

4.7 Demand Forecasting at Different Prices

With the residual method for measuring the Broken Assortment and elasticity parameters during the Clearance Period, the forecasting model presented in Equation 4-9 is complete, and can be used to predict the demand level for every reference. However, one of the biggest challenges in the forecasting process is forecasting for the first week or period of the Clearance Season. Because Zara doesn't discount its References during the Regular Season, there is no information that can be used to predict how the customers will react to price discounts when the clearance sales begin. Once the first Clearance Period is over, a wealth of information, that can be used to improve the subsequent forecasts, becomes available. This includes the actual price elasticity and Broken Assortment parameters that were measured on this first period, as well as the accuracy and prediction errors for the forecasts made.

4.7.1 Forecasting at the Beginning of the Clearance Period

When the Forecasting Model is first run before the beginning of the Clearance Season, only data for the Regular Season is available, so the parameters *beta-0* through *beta-3* can be calculated. The Broken Assortment and elasticity parameters *beta-4* and *beta-5* are determined from historical data by taking the parameters that were measured for the same Clearance Group and Country on the two previous years, i.e., the linear regression from Equation 4-13 is run with input data from the first Clearance Period of the two previous years.

4.7.2 Exponential Smoothing

Once the first Clearance Period is over, we can go back and measure the actual Broken Assortment and elasticity parameters for each Clearance Group and Country. In order to incorporate this knowledge into the forecasts that are made for subsequent periods, a weighting or exponential smoothing method is used, in which the *beta-4* and *beta-5* parameters used for the forecast of a Clearance Period are based on the weighted sum of the forecast parameter for the previous period, and the actual measured parameter for that same period:

$$\hat{\beta}_4^w = w * \overline{\beta}_4^{w-1} + (1-w) * \hat{\beta}_4^{w-1}$$

Equation 4-14

$$\hat{\beta}_5^w = w * \overline{\beta}_5^{w-1} + (1-w) * \hat{\beta}_5^{w-1}$$

Equation 4-15

The weighting parameter *w* is a number between zero and one that determines the distribution between the measured value $\overline{\beta}$ and the forecasted value $\hat{\beta}$. The value of this parameter is set for every Country and Clearance Group based on historical data of the previous two years so that the squared difference between the forecasted parameters and the real measured ones is minimized. This value is typically around 0.8, meaning that the majority of the weight is placed on the value for the parameter that was measured during the previous Clearance Period.

4.8 Final Sales Forecasting

4.8.1 Procedure for Disaggregating Demand

The demand level metric introduced in section 4.3.3 provides an unbiased indicator of sales for which a linear regression model can be fitted. Once a model is fitted for every Clearance Group and Country, forecasts are made for each Reference by inputting the variables for each of them. The only unknown variable is the current price at which the Reference will be sold during the upcoming Clearance Period. For this reason, a demand level forecast λ_{rk} must be made for every Reference *r* and possible price *k* on the Clearance Price List.

Once demand level predictions are made however, we need to convert these back into real unit sales. In order to convert demand level predictions into sales figures, we need to: determine the length of the Clearance Period for which the forecast will be made, and consider the existing inventory for each Reference and its distribution in Color, Sizing, and Stores. This is achieved by the following formulation:

$$\hat{\theta}_{rcsjk}^w = \alpha_{rcsj}^D * \lambda_{rk}^w * \sum_{d \in w} \delta_d$$

Equation 4-16

$$sales0_{rk} = \sum_{c,s,j} \mathrm{E}\left\langle Sales_{rcsj}^{w} \middle| \hat{\theta}_{rcsjk}^{w}, I_{rcsj}^{w} \right\rangle$$

Equation 4-17

In Equation 4-16, demand levels λ_{rk} are multiplied by the duration of the period, or the sum of the weight of the intra-week seasonality factors for the days *d* within the considered Clearance Period *w*. In practice however, because price lists are generated and sent out to the stores on a weekly basis, this summation is simplified and replaced by a constant of seven days. In order to consider the distribution of the inventory in Colors, Sizes, and Stores, the aggregate Reference-level demand level λ_{rk} must be disaggregated into its component Color, Size, and Store distribution. A way of doing this could be to use the *alpha* parameters introduced in Equation 4-4, however, real life business practices require us to modify this value. The original *alpha* values are calculated based on Regular Season data, but because during the Clearance Sales unsold inventory is redistributed from the central warehouses, it is possible that specific Color, Size variations that had never been sold in a Store now become available during the Clearance Period. Therefore, if the original *alpha* parameters were to be used, part of the disaggregated demand λ_{rk} would be lost. For this reason, a new series of *disaggregating-alpha* parameters are calculated as follows:

$$\alpha_{rcsj}^{D} = \frac{\sum_{s,j} Sales_{rcsj}}{\sum_{c,s,j} Sales_{rcsj}} * \frac{\sum_{s} Sales_{rcsj}}{\sum_{s,j} Sales_{rcsj}} * \frac{\sum_{r,c,s} Sales_{rcsj}}{\sum_{r,c,s,j} Sales_{rcsj}}$$

Equation 4-18

The only difference in this calculation is the fact that a generalized weight for the Store sales distribution is used. The furthermost right summation in Equation 4-18 corresponds to this weight that is assigned to each store based on the total sales for all references sold in that store over the total sales for the Country in question.

Once Reference demand level forecasts are scaled by time and disaggregated into its components as illustrated in Equation 4-16, the disaggregated *theta* sales are input into a Gamma Distribution in which the *theta* values are distributed among the inventory levels I^{w}_{rcsj} that exist at the time when the forecast is being made. This process simulates the stock-out implications of having the expected sales for a specific Color-Size-Store combination lost when the corresponding inventory level is depleted, i.e., $I^{w}_{rcsj}=0$.

In this way, as shown by Equation 4-17, the *theta* sales estimates are distributed amongst the existing inventory levels and once again aggregated on a Reference-Price level $sales0_{rk}$ (where *'0'* represents "current period"). These Reference-Price sales estimates constitute the final output of the Forecasting Model and are the input data for the Optimization Model.

4.8.2 Prediction Error Adjustments

After the first Clearance Period is over, we can also look back at the predictions made and compare them with the sales observed in reality. We can learn from forecasting errors and adjust for them in future forecasts because some of them are caused by factors that the Forecasting Model fails to consider but which nonetheless have an impact on sales.

Whereas underestimation or overestimation errors represent equal deviations from the real value, when applied to Zara's business, i.e., using the sales forecasts to decide Clearance Season prices, adds a different connotation to each of these forecasting errors. Looking ahead to the implementation of the Optimization Model that sets prices in order to maximize profits, and to the nature of decreasing sales levels with the pass of time, the progression of the Clearance Period sales can be characterized by a decreasing upper envelope on the prices. For this reason, Zara's management decided that overestimation errors are less desirable than underestimation errors, and so adjustments are made on the forecasts for a Clearance Period based on the overestimation errors committed for the same Price Cluster during the previous Clearance Period. For example, if an overestimation error of fifteen percent is committed for a Price Cluster in a given Clearance Period, the next forecast for this Price Cluster will be adjusted down by this same percentage.

4.9 Forecasting Process Summary

The different steps involved in the forecasting procedure as well as the different treatment of the References and their Colors, Sizes, and Stores is at times a source of confusion for persons not familiar with the Forecasting Model and warrants further description.

4.9.1 Data Detail Treatment

In tracing the flow of data through the Forecasting Model, it is important to remember what the desired output for it is: estimations for the expected sales for each Price Cluster, i.e., all the References from a same Clearance Group that were priced the same during the Regular Season, when priced at any of the possible discount prices from the Clearance Price List. Working backwards from this desired output, one might suggest building a model that makes forecasts at a Price Cluster level, however, different References clearly behave differently and thus a forecast for each individual Reference is first made and then aggregated on a Price Cluster level.

Another frequently asked question is why data is broken down to a Reference-Color-Size-Store level when the required forecasts are on a Reference level. There are two reasons why data is analyzed at this level of detail. The first one is to remove the seasonality and Color, Size, and Store effects discussed in section 4.3.2, and once the *lambda* demand level metrics are calculated, data is treated on a Reference level. The second point were this Color-Size-Store detail is used is when the *theta* demand level forecasts are distributed among the existing inventory levels through the *alpha* parameters, as explained in section 4.8.1.

One may further argue that since sales-point data is available on a Reference-Color-Size-Store level of detail, that forecasts should be made at that level and then aggregated at a Price Cluster level. However, several considerations make this impractical. First, as it was previously explained, Zara's inventory levels are most accurate when observed at a Reference level. Also, the massive amounts of data that would be required to maintain separate records for each Color, Size, and Store make this an undesirable option.

4.9.2 Process Summary

The Forecasting Model is first fitted or created some days before[28] the start of the Clearance Season, when most of the Regular Season data is known. Using the *theta* demand level metric introduced in section 4.3, the functional form illustrated in Equation 4-9 is fitted and the *beta* parameters *beta-0* through *beta-4*, that define the model, are calculated for each Clearance Group and Country. Having no current season data to estimate the price elasticity demand for the first Clearance Period, an

[28] A time allowance is necessary for store logistics consideration such as manual relabeling of the inventory.

estimate is obtained by looking at the historical data from the two previous years. This *beta-5* parameter that corresponds to the elasticity is calculated through the residual method presented in section 4.6.1, together with the *beta-4* Broken Assortment parameter, which is recalculated since it varies when moving from the Regular Season to the Clearance Season. In this way, the demand level for each Reference when sold at any of the possible Clearance Prices is forecasted. This demand level forecast is translated to actual unit sales by expanding it to cover a period of seven days and distributing the forecast among the existing inventory at the stores. Finally, the sales forecasts are aggregated on a Price Cluster level and this data is passed as input data to the Optimization Model. As the Clearance Season progresses, new and valuable data becomes available and is used to update the Broken Assortment and elasticity *beta* parameters through an exponential smoothing, and sales forecasts for the next Clearance Period are adjusted for overestimation forecasting errors that were observed.

5 Clearance Profit Optimization

5.1 Introduction

The objective of the Optimization Model is to assign optimal prices to each of the Price Clusters within a Clearance Group and Country in a way such that the perceived revenues and profits are maximized. When the Optimization Model is run at the beginning of each Clearance Period, it will output the price list assignments that should be implemented within that Clearance Period if the *entire* Clearance Season profits are to be maximized. A distinction is made here between maximizing profits during the current Clearance Period, and the entire Clearance Season. In order to make these optimal price assignments the Optimization Model must take into account the estimated sales for each possible price during the current Clearance Period, given by the Forecasting Model, and make projections for the expected sales during the remaining Clearance Periods considering that different price assignments will be possible and that any remaining inventory at the end of the Clearance Season may be sold for a salvage value.

The Optimization Model is formulated as a Mixed Integer Program model coded in AMPL programming language and solved using the ILOG made CPLEX linear programming Solver.[29] Current Clearance Period sales estimations made by the Forecasting Model as taken as input data and future Clearance Period sales are estimated based on this input data and future inventory level projections that are modeled internally. Zara's pricing business practices described in Section 2.3.3 are coded into model constraints, and additional constraints are provided in order to allow the Model's user to influence the output of the pricing decisions by incorporating business knowledge that is not otherwise considered by the model.

5.2 Model Formulation

5.2.1 Indices and Index Sets

Before going through the Optimization Model's formulation, the relevant indices and sets are presented.

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[29] *ILOG CPLEX: High-performance software for mathematical programming and optimization.* (n.d.). Retrieved April 2009, from ILOG: http://www.ilog.com/products/cplex/

- $k \in$ CLEARANCE PRICE LIST = {1,…,K}: The prices available from the Clearance Price List are indexed by *k*, and its corresponding prices are assumed by the parameter p_k, that is increasing by convention, i.e. $p_1 \leq p_2 \leq ... \leq p_K$. Here, p1 is the minimum allowable selling price or the Salvage Price.
- $n \in$ PRICE CLUSTERS = {1,…,N}: References are grouped by their initial Price Cluster groupings, i.e., by their original Regular Season price. The Optimization Model assigns and outputs a price to each individual Price Cluster. When two Price Clusters are assigned the same price, they form a Price Category.
- $c \in$ PRICE CATEGORIES : The list of current Price Categories, i.e., the different prices that are currently assigned to the Price Clusters within the Clearance Group.
- $r \in n$: References are thus grouped in one *n* out of N Price Clusters.
- $w \in W = \{1, \ldots, W_c\}$: Weeks or Clearance Periods are indexed by *w*, where *w* = 1 refers to the current Clearance Period for which the Optimization Model is being run and W_c is the number of weeks into the future through which the Clearance Season will extend, i.e., the number of weeks remaining before the Clearance Season is over. W_c is specified as an input parameter to the model.

5.2.2 Main Input Data

In addition to the just described Clearance Price List and the list of Price Clusters for the appropriate Country and Clearance Group under which the Optimization Model is instanced, the following data is input to the model:

- $Inv0_n$: Inventory of Price Cluster *n* currently remaining, i.e., the aggregated inventory of all the references within the Price Cluster, as shown in Equation 5-1:

$$Inv0_n = \sum_{r \in n} Inv0_r$$

Equation 5-1

- cp_n: Current price for Price Cluster *n*

- $sales0_{nk}$: Expected sales for Price Cluster *n* when it is priced at price *k*. This sales forecast is simply the output from the Forecasting Model and as discussed previously, the individual forecasts for each reference *r*, as calculated with Equation 4-17, are aggregated by Price Clusters *n* as illustrated by Equation 5-2:

$$sales0_{nk} = \sum_{r \in n} sales0_{rk}$$

Equation 5-2

- $precioT_n$: The current price for each Price Cluster is supplied as an input parameter to the model.

- psaldero : Salvage Price (from *"precio saldero"* in Spanish), or price at which inventory remaining at the end of the Clearance Period may be sold

5.2.3 Decision Variables

- $x_{nk}^{w} \in \{0,1\}$: Binary decision variable that assumes the value *1* to indicate whether cluster *n* is assigned a clearance price less than or equal to p_k during Clearance Period *w*. In other words, if during the Clearance Period *w'*, the Price Cluster *n'* is priced at $p_{k'}$, then all the variables $x_{n'k}^{w'}$ such that $k > k'$ will be set to *1*. As it will be discussed shortly, this implementation enables model constraints that respond to Zara's particular business practices during the Clearance Season.

- $y_{nk}^{w} \in \{0,1\}$; $y_{n1}^{w} = x_{n1}^{w}$; $y_{nk}^{w} = x_{nk}^{w} - x_{nk-1}^{w} \forall k > 1$: This binary decision variable is defined as a function of the previously defined variable *x*, and becomes active, i.e., assumes the value of *1*, if the price p_k, is assigned to Price Cluster *n* during Clearance Period *w*.

- L_{nk}^{w}: A decision variable is used to model the forecasted sales for Price Cluster *n* in the remaining Clearance Periods $w \in \{2,...,W_c\}$ that remain in the Clearance Season after the current Period for which the Optimization Model is being run. As it will be shown briefly, the Optimization Model considers the remaining Clearance Periods in its objective function such that the pricing decisions made for the current Period maximize the *overall* profits for all of the Clearance Season. For this reason projections of expected future sales are internally modeled by the decision variable L_{nk}^{w}.

- INV_n^w : Similarly, this decision variable is used to internally model the inventory levels of each Price Cluster *n* at the beginning of each Clearance Period. This variable is initialized to reflect the actual levels of inventory $Inv0_n$ at the beginning of the current Clearance Period, i.e., $INV_n^1 = Inv0_n$. Because the initial inventory levels for each of the remaining Clearance Periods $w \in \{2,...,W_c\}$ are contingent on the prices selected by the Optimization Model, this variable is used to model these levels of inventory as the different pricing scenarios are considered. Additionally, $INV_n^{W_c+1}$ is defined to represent the remaining inventory at the end of the last decision period, i.e., the inventory that will be sold for a salvage price.

- $Z_k^w \in \{0,1\}$: Binary decision variable indicating whether clearance price p_k is currently assigned to any of the Price Clusters ($Z_k^w = 1$) or not ($Z_k^w = 0$) during any given Clearance Period *w*. This variable provides a means to count and control the number of different prices from the Clearance Price List that are assigned within the Clearance Group being considered.

5.2.4 Future Sales Estimation

As it was briefly hinted at, the Optimization Model not only considers the expected sales for the current Clearance Period when making the optimal pricing assignments, but also takes into account the fact that there is a number W_c of remaining Clearance Periods on which Price Clusters can assume different prices as the Clearance Season advances, and furthermore, that any remaining inventory at the end of the Clearance Sales can be sold at a salvage price *psaldero* for further profit. In this way, the model is not "short-sighted" in the sense that it makes pricing assignments that optimize the *overall* profits during the entire Clearance Season. However, difficulties arise in making forecasts for future sales because, as we discussed in Chapter 4, sales are highly dependent on the Inventory Position variable, which is an uncertain variable, as it can assume different values depending on the pricing assignments that are made for Clearance Periods in the future. When the Optimization Model is run, only the current Inventory Positions at the beginning of the current Clearance Period are known; Inventory Positions at the beginning of subsequent Clearance Periods will depend on the prices that are assigned for the Clearance Periods preceding them. Ideally, to solve this problem, as the Optimization Model iterates through the different possible price assignments for the current Clearance Periods, it would calculate the initial Inventory Positions at the beginning of the next Clearance Period by subtracting the expected sales for the pricing assignments selected for that current iteration from the initial Inventory Positions of the current Clearance Period, and *then* follow the same process described in Chapter 4 to forecast sales for the upcoming Clearance Period. This process would then have to be repeated for each subsequent Clearance Period and for every iteration and permutation of pricing selections on the preceding Clearance Periods. This second step however, is not feasible. The procedure described in Chapter 4 for the Forecasting Model (i.e. forecast a demand level for each reference and then disaggregate it and convert it into a unit sales figure) involves lengthy calculations that under reasonable computing power take close to twenty minutes to calculate[30], and so expecting the Optimization Model to internally run these calculations for every iteration of the pricing assignment selections and for multiple Clearance Period and different combinations of pricing assignments within them, is far from possible.

For this reason, the approach that we follow is to first internally model future inventory levels with the decision variable *INV*, but *then* estimate future sales based on a *simplified* version of the procedure followed by the Forecasting Model. In this simplified procedure, future sales estimates are based on the sales forecasts for the current Clearance Period and the process followed by the Forecasting Model is emulated by replicating the factors that influence clearance sales when moving from one Clearance Period to the next one.

When examining the forecasting procedure that the Forecasting Model follows, as explained in Chapter 4, a single set of *beta* parameters is calculated at the beginning of the Clearance Season, based on data from the Regular Season (and historic data for the price elasticity and Broken Assortment effects). As the Clearance Season progresses, only the parameters *beta-4* and *beta-5*, for the Broken Assortment and price elasticity are recalculated based on the smoothing procedure detailed in Section 4.7.2. Therefore, in defining an abbreviated procedure for estimating future Clearance Period sales, two things are necessary: 1) Modeling the decreasing levels of inventory that

[30] Based on preliminary tests run on mid-sized Countries. And although database calculations can be further optimized, execution times of even one minute would render the discussed solution unfeasible.

are reflected in the Inventory Position variable that is in turn weighted by the *beta-4* parameter, and 2) To model the changes in sales and consumer behavior as the Clearance Season progresses (which would otherwise be captured by the *beta-4* and *beta-5* parameters that are recalculated at each new Clearance Period by the smoothing procedure from Section 4.7.2, once data from the previous Clearance Periods becomes available).

There is one last hurdle that must be overcome in taking the process followed by the Forecasting Model and emulating it from within the Optimization Model. As the reader might recall, in the original forecasting procedure followed by the Forecasting Model, the Inventory Position variable is input into Equation 4-9 in order to calculate a forecast for the *lambda* demand level estimate that is then transformed into a unit sales forecast through Equation 4-16 and Equation 4-17. Within the "realm" of the Optimization Model however, sales figures are in unit sales and not in demand level units, so a means of incorporating the effect of the Inventory Position variable into a unit sales forecast is necessary. To this end, the following additional input data is provided to the Optimization Model:

- U_c: This parameter represents a means of capturing the Broken Assortment parameter *beta-4* from the Forecasting Model and converting it into a sensitivity parameter that may be utilized to adjust final unit sales figures. In other words, the *beta-4* parameter that quantifies the impact of the Broken Assortment on *demand levels* is translated into the parameter U_c that quantifies the impact of the Broken Assortment effect on *unit sales*. The parameter U_c is calculated directly from the *beta-4* parameter based on the close form solution to the minimization of the distance between the functional forms for the Forecasting Model and the Optimization Model[31]:

$$U_c = \frac{3 * \beta_4^{\,2} + 9 * \beta_4}{2 * \beta_4^{\,2} + 6 * \beta_4 + 4}$$

Equation 5-3

[31] Caro, F., & Gallien, J. (2008, June 10). In preparation for: Clearance Pricing for a Fast-Fashion Retail Network. *Clearance Pricing Optimization at Zara* .

- $f0_{nk}$: This input parameter corresponds to the unit sales forecast for Price Cluster *n* when priced at price p_k, but adjusted to remove the Broken Assortment Effect. This parameter is based off the original Reference-level sales forecast $sales0_{rk}$ and adjusted by the current Inventory Position of the reference as shown in Equation 5-4:

$$f0_{rk} = \frac{sales0_{rk}}{1 - U_c + U_c \dfrac{Inv0_r}{F_c * CS_r}}$$

Equation 5-4

Here, F_c corresponds to the Broken Assortment threshold that was described in Section 4.5.3.4 and similarly, CS_r corresponds to the arithmetic product of the number of Colors and the number of Sizes in which Reference *r* is available. The parameter $f0_{rk}$ is then aggregated on a Price Cluster level as shown in Equation 5-5:

$$f0_{nk} = \sum_{r \in n} f0_{rk}$$

Equation 5-5

- $fav0_{nk}$: The weighted average of the broken assortment-adjusted sales forecasts, based on each reference's inventory weight within the whole Price Cluster:

$$fav_{nk} = \sum_{r \in n} \frac{Inv0_r}{Inv0_n} f0_{rk}$$

Equation 5-6

This input data, together with the previously introduced decision variables INV_n^w and L_{nk}^w are used to define a forecasting functional form for the Optimization Model in order to model future sales through the use of the two following model constraints[32]:

$$L_{nk}^w \le \left[sales0_{nk} * y_{nk}^w \right] * \kappa^{w-1}, \forall w > 1, n, k$$

Constraint 1: price_demand[33]

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[32] Smith, S. A., & Achabal, D. D. (1998). Clearance Pricing and Inventory Policies for Retail Chains. *Management Science , 44* (3), 285-300.

$$L_{nk}^{w} \leq \left[f0_{nk} * \left(1 - U_c\right) + U_c * fav0_{nk} * \frac{INV_n^w}{F_c} \right] * \kappa^{w-1}, \forall w > 1, n, k$$

Constraint 2: brkassrtmnt_demand

Constraint 1 serves two purposes, the first being to force the sales forecast to zero for those prices p_k not currently assigned to a Price Cluster *n* ($y_{nk}^{w} = 0$), and the second, to fix an upper limit to the sales forecast L_{nk}^{w} that is equal to the sales forecast $sales0_{nk}$.

Furthermore, the *kappa* parameter introduced in Constraint 1 serves the purpose of modeling the second factor that was previously identified as necessary in order to emulate the forecasting procedure followed by the Forecasting Model: to model the decrease in sales that occurs from one Clearance Period to the other. To determine the *kappa* parameter, data for References that were priced at the same price during two or more consecutive Clearance Periods during previous years' Clearance Seasons is selected and the $f0_{rk}$ parameter introduced in Equation 5-4 is calculated for each Reference, with the distinction that a different parameter is calculated for each of the Clearance Periods and the sub index *k* is dropped as it looses significance given the choice of data made:

$$f0_r^w = \frac{sales0_r^w}{1 - U_c + U_c \dfrac{Inv0_r^w}{F_c * CS_r}}$$

Equation 5-7

Then, as described by Caro and Gallien, the set ν is chosen to represent triplets *(r, w, w')* such that *w* represents a Clearance Period where Reference's *r* price has just been changed, and *w'* represents a Clearance Period that is posterior in time to *w* and for which the price p_k originally set in period *w* is maintained.[34] The set ν is then populated with data from the two previous years' Clearance Seasons for the Country and Clearance Group in question, and the *kappa* parameter is fitted to minimize the following squared difference:

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[33] Optimization Model constraints are labeled with the original AMPL code designations so that the reader may identify them in the code that is provided in the Appendix.

[34] Caro, F., & Gallien, J. (2008, July 9). In preparation for: Clearance Pricing for a Fast-Fashion Retail Network. *Clearance Pricing at Zara , July 9, 2008*.

$$\sum_{(r,w,w')\in v}\left(\kappa^{w'-w}-\frac{f0_r^{w'}}{f0_r^{w}}\right)^2$$

Equation 5-8

In other words, the *kappa* parameter is fitted to explain the exponential fall in sales that is observed when moving from one Clearance Period to the other, e.g., a *kappa* parameter of 0.78 means that when a Reference's current price is held constant, for each new Clearance Period, its sales will fall exponentially by a factor of 0.78. Constant price data is chosen to fit the *kappa* parameter so that only the effect of time on observed sales, without the influence caused by price discounts, is captured. Similarly, the parameter $f0_{rk}^{w}$, as opposed to the unadjusted $sales0_{rk}^{w}$, is used so that the effect of the inventory assortment is filtered out.

Coming back to Constraint 2, this constraint is only binding for the case were the Inventory Position INV_n^w does not surpass the Broken Assortment threshold, i.e., there is a Broken Assortment effect; in all other cases Constraint 1 is the binding constraint on variable L_{nk}^{w}. In other words, Constraint 2 further limits the expected future sales if the foreseen inventory levels in the future present a Broken Assortment.[35]

5.2.5 Objective Function

As previewed in the previous section, the objective function for the Optimization Model is composed by three elements:

max CURRENT_PROFIT + DISC_FUT_PROFIT + PROFIT_SALDERO

Equation 5-9

Where each of these elements represents the profit generated in the current Clearance Period for which the Optimization Model is invoked, the profit generated over the remaining Clearance Periods, and the profits made when the unsold inventory at the end of the Clearance Season is sold at the salvage price.

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[35] Formal derivations for Constraint 1, and Constraint 2 can be found in Caro, F., & Gallien, J. (2008, July 9). In preparation for: Clearance Pricing for a Fast-Fashion Retail Network. *Clearance Pricing at Zara* .

The variable CURRENT_PROFIT represents the estimated profit for sales made during the current Clearance Period. This corresponds to the sales forecasts made by the Forecasting Model, for the prices p_k currently selected as optimal:

$$CURRENT\_PROFIT = \sum_{n,k} sales0_{nk} * p_k * y_{nk}^1$$

Equation 5-10

The variable DISC_FUT_PROFIT represents the profits expected during the remaining Clearance Periods. When solving for an optimal solution, the Optimization Model will in essence assign all of the prices for all Price Clusters and for every Clearance Period, but in practice only the pricing assignments for the current period are taken, whereas the pricing for the remaining Clearance Periods will be obtained in the same way when the Optimization is once again run at the beginning of each Clearance Period.

$$DISC\_FUT\_PROFIT = \sum_{n,k,w\in\{2..Wc\}} L_{nk}^w * p_k * tighten\_slack^{(w-1)}$$

Equation 5-11

The maximizing nature of the objective function ensures that the decision variables L_{nk}^w assume the largest values permitted by Constraint 1 and Constraint 2, therefore correctly modeling their respective estimated future sales. However, some situations arise in which the model will be indifferent between selling a Price Cluster in one Clearance Period or the other, e.g., if after the current Clearance Period there is only one unit of inventory left for a particular Price Cluster that is currently priced at 14.95 EUR, then, economically speaking the Optimization Model is indifferent between selling that unit in the next Clearance Period or any other period after that as it will always perceive a profit of 14.95 EUR, but reality dictates that given limited inventory and demand for it, it will be sold sooner than later. In order to ensure that the estimated future sales are modeled correctly, the discount factor parameter *tighten_slack* is introduced. The impact to the objective function created by this parameter is insignificant as its value is in the order of $1\text{-}1\text{x}10^{-4}$, but it is still sufficient to ensure the correct modeling of the future sales estimates.

Although the inventory remaining after the end of the Clearance Season may be sold at a salvage value to third party wholesalers, there are limits to the amount that inventory that these wholesalers can absorb. For this reason, the variable PROFIT_SALDERO, that represents the estimated profit made when the inventory remaining at the end of the Clearance Season is sold at the salvage price *psaldero*, is formulated in a way in which these constraints may be considered.

PROFIT_SALDERO =

$$disc\_saldero * psaldero * Lsaldero + \left(1 - disc\_saldero\right) * psaldero * \sum INV_n^{W_c+1}$$

Equation 5-12

Here, *Lsaldero* is a decision variable that represents the amount of inventory that is sold at the full salvage price *psaldero*, while any remaining inventory over the quantity *Lsaldero* is sold at the salvage price *psaldero* but at a discount given by the parameter *disc_saldero*.[36] Because the terms in Equation 5-12 correspond to profits, the parameter *disc_saldero* may be seen as either a discount over the salvage price *psaldero* for the salvaged inventory over the quantity given by *Lsaldero*, or as the fraction of the inventory over the quantity given by *Lsaldero* that may be solved for a salvage value, i.e., it may be seen as a discount on the salvage price or as a cap on the inventory that can be sold at all. The decision variable *Lsaldero* is in turn defined by the following constraints:

$$Lsaldero \leq \sum_n INV_n^{W_c+1}$$

Constraint 3: remain_inv

$$Lsaldero \leq CAP_c$$

Constraint 4: cap_liq

The first constraint simply states that the inventory that is sold at the full salvage price may not exceed the remaining inventory at the end of the Clearance Season. The second constraint allows the user of the Optimization Model to define a cap CAP_c on the maximum amount of inventory that

[36] Equation 5-12 is equal to the simplification of: Lsaldero*psaldero + (1-disc_saldero)*($\sum INV_n^{Wc+1}$-Lsaldero) *psaldero

can be sold at the full salvage value. In this way, by controlling the input parameters CAP_c and *disc_saldero*, the user may define the maximum level of inventory that can be absorbed by third party wholesalers at the full salvage value, and define a discount over the salvage price at which any inventory remaining after that may be sold, e.g., a value of *0.5* for *disc_saldero* and a value of 1.500 for CAP_c would mean that 1.500 may be sold at the full salvage price and any quantity over that will be sold at a fifty-percent discount over the salvage price.

Finally, mainly for the sake of continuity, the variable that is actually included in the objective function is DISC_PROFIT_SALDERO:

$$DISC\_PROFIT\_SALDERO = PROFIT\_SALDERO * tighten\_slack^{(W_c)}$$

5.2.6 Model Constraints

In addition to the already introduced model constraints, further constraints are introduced in the current Section:

$$INV_n^1 = inv0_n \forall n$$

Constraint 5: initial_inventory

This constraint simply initializes the inventory decision variable to reflect the current levels of inventory that are provided to the model as an input paramter.

$$INV_n^2 = inv0_n - \sum_k sales0_{nk} * y_{nk}^1 \forall n$$

Constraint 6: inventory_endof_actual

Here, the initial inventory for the Clearance Period after the current one is updated to discount the inventory that is sold during the current period.

$$INV_n^{w+1} = INV_n^w - \sum_k L_{nk}^w \forall w > 1, n$$

Constraint 7: estimated_inventory

Similarly, this constraint acts recursively and models the initial inventory levels for the remaining Clearance Periods by discounting the sales from the previous periods.

$$x_{nk}^{w} \leq x_{nk+1}^{w} \forall w,n,k < K, \; x_{nK}^{w} \leq 1$$

Constraint 8: price_k_or_higher

Constraint 8 is a structural constraint and enables the telescoping effect that was described when the decision variable *x* was introduced in Section 5.2.3.

$$x_{nk}^{w} \leq x_{n+1k}^{w} \forall w,n < N,k, \; x_{Nk}^{w} \leq 1$$

Constraint 9: ordered_clusters

Constraint 9 ensures that a Reference *r* that was priced higher than another reference *r'* during the Regular Season, is never priced lower than that same Reference *r'* during the Clearance Season, in accordance to Zara's pricing policies.

$$x_{nk}^{w} = xc_{ck}^{w} \forall w,n,k : p_k = precioT_c$$

Constraint 10: category_groups

All throughout the Clearance Season, Price Clusters are treated individually, even though they will naturally merge into Price Categories when they are priced at the same price, and will remain merged under a single price for the remainder of the Clearance Sales, in accordance to Zara's pricing policies. For this reason, a mirror decision variable xc_{ck}^{w}, identical in nature to the original variable xc_{nk}^{w}, but with the distinction that Price Category instead of Price Cluster indexes it, is defined. Then, through Constraint 10, all of the Price Clusters currently priced at the same price $precioT_n$ will be merged into a single pricing decision variable xc_{ck}^{w}, so that the Optimization Model assigns the same price to all Price Clusters currently joined under a Price Category.[37] Naturally, this constraint is not enforced for the first Clearance Period in the Clearance Season.

[37] The constraint untangling and simplification is left to the Solver's pre-processing algorithms. Furthermore, this constraint only guarantees that Price Categories remain merged for the pricing assignments of the current Clearance Period, so it is possible that the internally estimated pricing assignments for future periods do not follow Zara's price merging policy. This is however partially offset by the non-increasing trend in the number of Price Categories enforced by Constraint 13.

$$x_{nk}^{w} \le x_{nk}^{w+1} \forall w < W_c, n, k$$

Constraint 11: decreasing_prices

Constraint 11 enforces a nonincreasing trend for the prices of all Price Clusters from one Clearance Period to the other, i.e., once a price is set, it can either be maintained or further lowered during future Clearance Periods, but never raised.

$$y_{nk}^{w} \le z_{k}^{w} \forall w, n, k$$

Constraint 12: restrict_price

As noted when the binary variable z_k^w was introduced, through Constraint 12, the number of different prices selected on a given Clearance Period *w'* can be calculated by the sum $\sum_k z_k^{w'}$.

$$\sum_k z_k^{w+1} \le \sum_k z_k^{w} \forall w < Wc$$

Constraint 13: dec_num_prices

Given that the variable z_k^w is an indicator of the number of Price Categories or number of different prices assigned for a Clearance Period, Constraint 13 thus forces the number of Price Categories to follow a non-increasing trend.

5.2.7 Configuration Levers

In addition to the constraints detailed in the previous Section, additional constraints allow the user of the optimization to "shape" the outcome of the pricing assignments by adding additional constraints in response to business needs that he or she foresees but that are not automatically considered by the Optimization Model. We call these constraints "configuration levers", and users of the model will fix these input levers based on their knowledge of the business, so that their expertise may be incorporated into the Optimization Model. Two of these levers: *disc_saldero* and CAP_c were already introduced in Section 5.2.5; the remaining levers are presented here.

$$x^1_{nk} = 1 \forall n, \max(k) \ni p_k < precioT_n * (1 - \min discount)$$

Constraint 14: initial_discount

This constraint is only enforced for the first Clearance Period in the Clearance Season and enables the user to specify the minimum discount *mindiscount* at which all Price Clusters must be marked down with respect to their Regular Season price.

$$y^1_{nk} = 0 \forall n, k \ni p_k > precioT_n * \left(1 - \min discount\right) \wedge p_k \neq precioT_n$$

Constraint 15: next_discount

Starting on the second Clearance Period of the Clearance Season, Constraint 15 replaces Constraint 14, and it states that Price Clusters can either be maintained at their current price, or further discounted if the new price represents a minimum discount of *mindiscount* over the current price. A minimum discount considers the added costs of relabeling material at a store, i.e., it might not be worth it to physically re-label a Price Category at a store if the change in price is not significant.

$$y^1_{nk} = 0 \forall n, k \ni precioT_n < umbraldisc2 \wedge p_k > precioT_n * \left(1 - \min discount2\right) \wedge p_k < precioT_n$$

Constraint 16: next_discount2

Constraint 16 is similar to Constraint 15, but it only targets Price Clusters currently priced below a threshold price *umbraldisc2* given as an input parameter, and forces the minimum discount required to the input parameter *mindiscount2*. Imposing a separate and bigger minimum discount for low priced items serves the same purpose of considering store logistics costs and it is necessary because percentage discounts on lower priced items logically correspond to lower absolute currency discounts that may not justify relabeling the inventory at the store.[38] For example, a discount of 15% may justify relabeling garments priced at 119 EUR (a discount of 17.85 EUR) while that same discount could not justify relabeling garments priced at 9.95 EUR (a discount of 1.49 EUR).

[38] Another reason behind these constraints is the fact that Zara's customers are used to visit the stores on a weekly basis and will frown upon "petty" discounts form one Clearance Period to the other.

$$\sum_k z_k^1 \leq C_c$$

Constraint 17: max_num_prices_act

The user may limit the maximum number of different Price Categories, or different number of prices, that she desires for the Optimization Model's pricing assignment output. Having smaller numbers of Price Categories simplifies store operations as more inventories may be displayed together under the same price sign, and the relabeling process is also simplified when new prices are assigned, e.g., store clerks will receive instructions such as "all t-shirts currently priced at 19.95 EUR need to be relabeled at 14.95 EUR".

$$\sum inv0_n * y_{nk}^1 \geq Q_c * z_k^1 \forall k$$

Constraint 18: min_inventory

When too little inventory of a particular Price Category remains, it is common practice to merge that Price Category with another one in order to facilitate store logistics, given that store clerks must keep each Price Category physically separated in the store, and allotting shelf space for a small group of garments may result impractical.

5.3 Implementation

5.3.1 Coding

The Optimization Model mathematical model was implemented in AMPL modeling language.[39] Following the model layout recommended by the authors[40], the model was programmed or embedded in three sections:

1. MIT2_MODEL.mod: This section contains the variable and parameter declarations together with the optimization model definition, including the objective function and constraint declarations and definitions.

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[39] *Modeling Language for Mathematical Programming.* (n.d.). Retrieved August 2008, from http://ampl.com/

[40] Fourer, R., Gay, D. M., & Kernighan, B. W. (2002). *AMPL: A Modeling Language for Mathematical Programming.* Duxbury Press.

2. MIT2_DATA.dat: This module contains the SQL queries necessary to load all of the external input parameters, including the sales forecasts and related variables.
3. MIT2_RUN.run: The model and data files are invoked from this run file. Given a finalized version of the two previous files, the user needs only to interact or work with this file in order to invoke the Optimization Model. In this file, the templates for the Country, Clearance Group and Clearance Period are selected and the configuration levers are adjusted. The runfile, after loading the model and data components, invokes the Solver and then presents the results in a series of reports.

The actual AMPL code for these three files is presented in the Appendix.

5.3.2 Solver

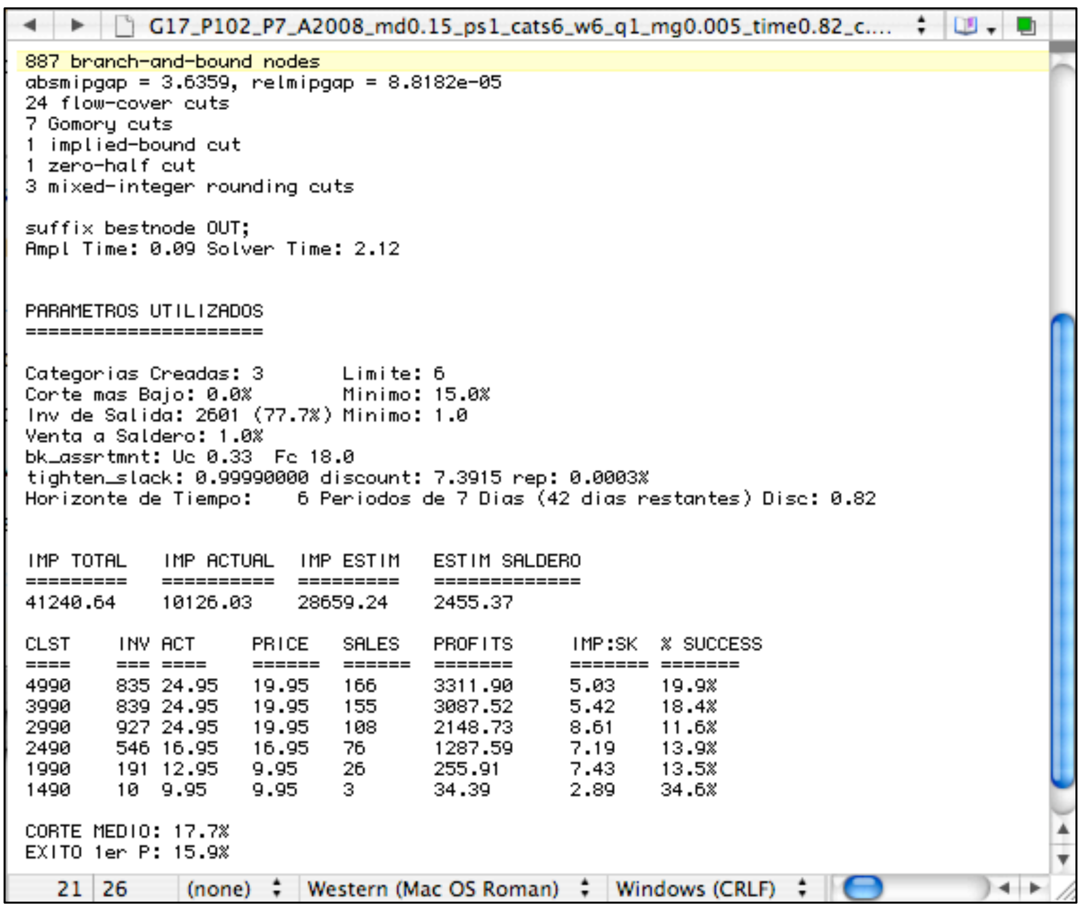
ILOG CPLEX was used as the optimization Solver for the Optimization Model[41]. CPLEX, originally named so for its use of the Simplex method for solving linear optimization problems and its C computer language programming interface, is internally invoked by the AMPL run file described in the previous Section, and the results of the optimization, which include the optimal values for all of the decision variables as well as the objective function value achieved are then returned to the main AMPL program so that the results can be parsed and presented as desired.

5.3.3 Optimization Results Presentation

As discussed previously, the results or solution to the optimization problem is presented in a series of reports that are implemented in the AMPL code for the MIT2_RUN.run file described earlier. The first and more direct report is a summary that is presented on-screen right after the optimization problem is solved, an example of which is shown in Figure 14. In this report, the settings for the manually set input parameters and configuration levers are summarized, and then the objective function values achieved are presented. The total expected profits for the remainder of the Clearance Period are shown to the left, and then this value is detailed into its three components: expected profits for the current Clearance Period, expected profits during the remaining Clearance Periods, and expected salvage profits. For instance, on the example presented in Figure 14, the total profits correspond to 41.240 EUR, of which 10.126 EUR are expected for the current Clearance

[41] http://www.ilog.com/products/cplex/

Period, 28.659 EUR more during the remaining Clearance Periods, and 2.455 EUR that correspond to the salvage profits. Afterwards, a table summarizing the results for each Price Cluster is presented. In this table, pertinent data such as the current inventory levels for each Price Cluster or their current prices is presented together with the optimal price assignments and the associated expected sales and profits for each Price Cluster. The column labeled "ACT" corresponds to the actual prices and the one labeled as "PRICE" corresponds to the new suggested prices. In our example, these results could be interpreted as: Price Clusters 4990 through 2990 should be priced at 19.95 EUR, cluster 2490 should remain at the current price of 16.95 EUR, and the cluster 1990 should be merged with cluster 1490 at the price of 9.95 EUR. Similarly, store clerks might receive instructions on the lines of: References currently priced at 24.95 EUR need to be relabeled at 19.95 EUR, and those currently priced at 12.95 EUR need to be relabeled at 9.95 EUR.



G17_P102_P7_A2008_md0.15_ps1_cats6_w6_q1_mg0.005_time0.82_c....

```
887 branch-and-bound nodes
absmipgap = 3.6359, relmipgap = 8.8182e-05
24 flow-cover cuts
7 Gomory cuts
1 implied-bound cut
1 zero-half cut
3 mixed-integer rounding cuts

suffix bestnode OUT;
Ampl Time: 0.09 Solver Time: 2.12


PARAMETROS UTILIZADOS
=====================

Categorias Creadas: 3       Limite: 6
Corte mas Bajo: 0.0%        Minimo: 15.0%
Inv de Salida: 2601 (77.7%) Minimo: 1.0
Venta a Saldero: 1.0%
bk_assrtmnt: Uc 0.33  Fc 18.0
tighten_slack: 0.99990000 discount: 7.3915 rep: 0.0003%
Horizonte de Tiempo:    6 Periodos de 7 Dias (42 dias restantes) Disc: 0.82


IMP TOTAL    IMP ACTUAL   IMP ESTIM    ESTIM SALDERO
=========    ==========   =========    =============
41240.64     10126.03     28659.24     2455.37

CLST     INV ACT      PRICE   SALES   PROFITS      IMP:SK  % SUCCESS
====     === ====     ======  ======  =======      ======= =======
4990     835 24.95    19.95   166     3311.90      5.03    19.9%
3990     839 24.95    19.95   155     3087.52      5.42    18.4%
2990     927 24.95    19.95   108     2148.73      8.61    11.6%
2490     546 16.95    16.95   76      1287.59      7.19    13.9%
1990     191 12.95    9.95    26      255.91       7.43    13.5%
1490     10  9.95     9.95    3       34.39        2.89    34.6%

CORTE MEDIO: 17.7%
EXITO 1er P: 15.9%
```

21 | 26 | (none) | Western (Mac OS Roman) | Windows (CRLF)

Figure 14: Optimization Model Results Screenshot

Finally, the ratios of profits to inventory and percentage of inventory expected to sell during the current Clearance Period are presented, and global inventory-weighted averages for the average discount and percentage of sold inventory are displayed.

Additional reports created by the Optimization Model's run file include detailed spreadsheet files that show the models internal calculations and expected future sales and profits during each Clearance Period, as well as a general log file that reports execution related data and general results. These reports provide valuable insight into the Optimization Model's reasoning and are useful for debugging purposes as well.

6 Live Testing

6.1 Introduction

Throughout the development of the Forecasting and Optimization Models, tests were conducted to ensure forecasting reliability and adequate or logical responses to changing the optimization parameters and levers, but as we told Zara's management from the beginning, the true test would be to measure profit increases that result when the developed methodology and models are used to make Clearance Season pricing decisions. For this reason, a live Pilot Test was planned and executed during the 2008 Summer Campaign Clearance Sales. This initial, small-scale test was used to test the developed models in real-time and to refine them throughout the execution of the pilot, while still being able to measure a statistically significant positive impact on the profits made. Based on the positive results measured for the initial pilot, a second, larger-scale Field Test was planned for the 2008 Winter Campaign Clearance Season. This mid-scale Field Test was planned as an experiment that would revalidate the measured impact for the initial Pilot Test, and as a final checkpoint before applying the newly developed Clearance Season pricing methodology to all of Zara's business.

6.2 Experimental Design

6.2.1 Summer Campaign Pilot Test

The Summer Campaign Pilot Test was designed to test the impact of utilizing the newly developed model-based pricing methodology over an experimental subgroup of Clearance Groups within a mid-sized country. For this experimental subgroup, or Test Group, four out of twenty one Clearance Groups were chosen as a representative sample:

Figure 15: Pilot Test Groups

In this way, the Optimization Model was used to make the pricing decisions for the four Clearance Groups shown in Figure 15, while the remaining groups were priced through the regular legacy pricing method described in Section 2.3. From these remaining groups, four Clearance Groups with similar characteristics to those in the Test Group were chosen as the Control or Legacy Group, which is used as representative sample of the legacy pricing method in order to compare it with the new proposed pricing methodology. The Pilot Test was conducted in Belgium, which by the year 2008 had twenty-stores and is therefore considered a mid-sized country by Zara's standards.[42] This selection for the country fulfilled the requirements of minimizing the business risks of experimenting with pricing while still providing significant data for impact analysis.

Some Clearance Groups are naturally more profitable than others so this natural difference must be first accounted for before measuring the profit results for the Test and Control Groups and assessing the impact of the pricing model. For instance, if the four Clearance Groups chosen for the Test Group were generally more profitable than the groups chosen for the Legacy or Control Group, the experiment would be biased towards making it seem that using the new pricing methodology results in increased profits. As a means of normalizing these intrinsic differences, France was chosen as the Point of Comparison country, where pricing for all of the Clearance Groups is done according to the regular legacy pricing method. In this way, by observing the

[42] All analyses are done based on a "same-stores" metric, i.e., considering stores that have been open for at least one year, so actual the actual number of stores used varies for some of the countries.

Clearance Season in France, the natural differences that exist between the Clearance Groups in Belgium's Test and Control Groups, may be quantified and corrected for when measuring the impact of the new pricing methodology. The experiment design for the Summer Campaign Pilot Test is summarized in Figure 16:

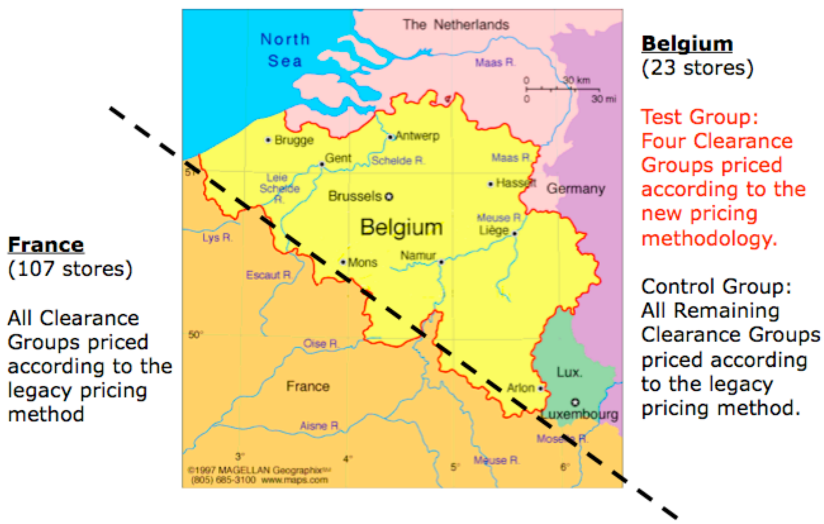


Figure 16: Summer Campaign Pilot Test Design[43]

6.2.2 Winter Campaign Field Test

After successfully conducting the Summer Campaign Pilot Test, a more ambitious larger-scale Field Test was planned for the Winter Campaign Clearance Season. In this second experimental design, Ireland and Belgium were chosen for conducting the experiment while the rest of Western Europe was used as the Point of Comparison.[44] This time, as part of the Test Group, all of the Clearance Groups were priced according to the new pricing methodology: half of them in Belgium, and the other half in Ireland; the other halves of the Clearance Groups in each country were priced through the regular legacy pricing method. This experimental design allows for more conclusive observations because if the pricing methodology under test truly has a positive effect on Clearance Season profits, then one half of the Clearance Groups in Belgium will out-perform the other half,

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[43] Image (n.d.). Retrieved from http://www.maps.com

[44] Germany was excluded from the Point of Comparison because its clearance pricing policies differ from the rest of Western Europe, due to legislative regulations.

while the opposite thing will happen in Ireland, i.e., the "loosing half" from Belgium will become the "winning half" in Ireland, thus proving the pricing model's effectiveness regardless of the Clearance Groups it is tested on. In any case, the Point of Comparison once again provides a mean for accounting for the profitability differences that exist between one set of Clearance Groups and the other under regular conditions, i.e., in the absence of an experiment. The Field Test design for the Winter Campaign Clearance Season is summarized in Figure 17:

Figure 17: Winter Campaign Field Test Design[45]

6.3 Experiment Execution

6.3.1 Process Flow

The regular clearance pricing flow from Section 2.3 was followed during the execution of the experiment, with Country Managers creating price suggestion lists for their countries of responsibility and then presenting them to members of the Pricing Team over a meeting. This time however, the price assignments made by the Optimization model were printed in a report much like the one shown in Figure 14 and presented during this meeting. The Pricing Team would then review the price assignments made by the Optimization Model for those Clearance Groups belonging to a Test Group, and the price suggestions made by the Country Manager for those

[45] Image (n.d.). Retrieved from http://vlib.iue.it/images/europe-map.jpg

Clearance Groups in the Control Groups. Pricing lists for the Point of Comparison countries were elaborated and reviewed with no intervention whatsoever from the ongoing experiment.

6.3.2 Model Acceptance

The experiment execution, where a mathematical model was used to make pricing decisions that have an impact over Zara's bottom line profits, marked a paradigm shift for both the Pricing Team and the Country Managers. While conscious of the fact that the pricing model's suggestions could differ from the ones that the Pricing Team would make, we made a conscious effort of ensuring that the reasoning behind the model was transparent and made business sense, so that the model was not seen as a "black box", as many third-party software packages or applications often do.

A consensus was reached with the Pricing Team in which the pricing suggestions made by the Optimization Model would be honored and implemented whenever the reasoning behind the model's decisions was clear. However, we agreed to evaluate the business risk of making a real-life pricing decision based on the stakes involved and the degree to which the model's suggestions differed from those made by the Country Managers. In the rare cases where there was a perceived high risk of adopting one of the model's pricing decisions, we decided to implement the pricing suggestions made by the Country Manager instead.

As a summary and means of visualizing the acceptance and implementation of the Optimization Model's pricing suggestions, Table 1 shows the pricing suggestions that were made for the test groups during the Winter Campaign Field Test, how they differed from the Country Manager's own pricing suggestions, and the extent to which they were implemented:

Table 1: Winter Campaign Field Test Acceptance

Country – Group		Week 100	101	102	103	104	105	106	107	108
Belgium	1	◆	◆		◆	◆	◆	◆		
	2	◆	◆	◆		◆		◆	◆	
	3	◆		◆			◆	◆		
	4		◆			◆		◆	◆	
	5	◆	◆	◆			◆	◆	◆	
	6	◆	◆	◆	◆					
	7	◆	◆	◆	◆		◆	◆	◆	
	8	◆	◆	◆	◆	◆	◆		◆	
	9	◆	◆	◆			◆	◆	◆	
	10	◆	◆	◆			◆		◆	
	11	◆	◆		◆	◆	◆	◆		
	12	◆	◆		◆	◆	◆		◆	
Ireland	13	◆	◆	◆	◆			◆		◆
	14	◆		◆	◆		◆	◆	◆	◆
	15	◆		◆	◆	◆	◆	◆	◆	◆
	16	◆	◆	◆	◆		◆	◆	◆	◆
	17	◆		◆	◆	◆	◆	◆	◆	◆
	18	◆		◆	◆	◆	◆	◆		◆
	19	◆		◆	◆	◆	◆	◆	◆	◆
	20	◆	◆	◆	◆		◆			

◆ = model and commercial suggested different markdowns
= model's suggestion was followed
= model's suggestion was followed constraining salvaged stock
= model's suggestion was followed restricted to two prices max
= commercial's suggestion was followed

Table 1 lists all of the Clearance Groups that were priced according to the model based pricing methodology in Belgium and Ireland, and illustrates the pricing outcomes for each of the Clearance Periods in the Clearance Season.[46]

As shown in the legend, a black rhombus marks the cells where the Optimization Model's price list differed from the one that was prepared by the Country Manager. The fields shaded in medium green and which cover the majority of the table, correspond to the instances where the Optimization Model's pricing suggestion list was honored and implemented at the stores. And on the other hand, fields shaded in yellow correspond to those rare cases where we perceived a business risk associated with following the Model's pricing suggestion and therefore decided to implement the Country Manager's list instead. For example, Clearance Group 20, was represented by a considerably large volume of inventory and the Optimization Model's price suggestions differed greatly from the Country Manager's, so in the interest to avoid taking unnecessary business risks, it was decided to honor the Country Manager's suggestions for three of the Clearance Periods. The fields shaded in yellow for Clearance Period 108 in Ireland however, correspond to a decision that was made to

[46] For database implementation purposes, Clearance Periods are sequentially numbered starting at 100.

liquidate the remaining inventory by drastically discounting its prices. This is one of the special scenarios that were mentioned in which business considerations such as freeing up exhibition space at the stores takes precedence at the expense of having to further discount the remaining Clearances Season inventory. Along these same lines, fields shaded in dark and light green correspond to scenarios where the output of the Optimization Model was controlled by adjusting the configuration levers described in Section 5.2.7, as a means of accelerating the pace of the Clearance Season.

As evidenced in Table 1, the great majority of the Optimization Model's price suggestions were honored and implemented. And although the fields highlighted in different tones of green correspond to cases where additional constraints were placed on the Optimization Model, the implemented lists still correspond to a profit-maximizing optimal pricing assignments. As mentioned earlier, these additional constraints or configuration levers allow the Country Managers and the Pricing Team to use their unique knowledge of the business and make previsions for special situations that the Optimization Model was not designed to handle. Additional examples of cases when these configuration levers are used will be discussed towards the end of the current Chapter.

6.4 Experimental Results

6.4.1 Evaluation Metric

The metric used to evaluate the results of the Pilot Test and Field Test is based off one of the profitability metrics that Zara uses to evaluate its Clearance Season profits. This metric is calculated as the ratio of revenues made during the Clearance Season to the inventory's original, Regular Season price valuation at the beginning of the Clearance Season:

$$Y = \%\ \text{Income} = \frac{\text{Clearance Season Revenue} + \text{Salvage Revenue}}{\text{Initial Stock Valued at Regular Season Prices}}$$

Equation 6-1

In other words, the *Y* metric represents the percentage of the initial inventory's value that was realized during the Clearance Season, and conversely, the value *(1-Y)* represents the percentage of the inventory's value that was lost on account of selling it at a discounted price. Based on this metric, a profit impact analysis was performed in accordance to the different experimental designs that were chosen for the Test Pilot and the Field Test.[47]

6.4.2 Summer Campaign Test Pilot Results

As noted in Section 6.2.1, the experimental design consisted in a Test Group and a Legacy or Control Group that were tested in Belgium's Summer Clearance Season. Additionally, the results for both groups were observed in France's Clearance Season, which constituted the Point of Reference country.

Table 2: Summer Campaign Y-Metric Comparison for 2008

2008	Y - Pilot Test Groups	Y - Legacy Groups	Avg Difference
Belgium	44.0%	42.0%	1.9
France	42.4%	42.2%	0.1
B - F	**1.6**	**-0.2**	**1.8**

The first row in Table 2 shows the Y-metric results for Belgium's Summer Clearance Season, where the first column corresponds to the Y-metric for the Test Groups and the second one corresponds to the four Clearance Groups that were chosen as the Control Group and priced according to the legacy pricing method. The last column shows the average difference between the first two columns based on all of the stores in the country, i.e., the difference between the Test and Control Group Y-Metrics is calculated for each individual store in the country and an average value is calculated:

$$Avg\_Difference = \frac{\sum_{Stores}\left(Y_{Test\_Group} - Y_{Legacy\_Group}\right)}{\#\,Stores}$$

Equation 6-2

The results from the first row in Table 2 can therefore be interpreted as saying that the Y-metric or the percentage income that was realized for the Pilot Test Groups in Belgium, was greater by two percentage points to the percentage income realized for the Control Group that was priced according to the legacy pricing method. As a means of evaluating whether the average difference measured for Belgium is a consequence of having used the proposed pricing methodology to price

[47] Caro, F., & Gallien, J. (n.d.). Clearance Pricing Optimization at Zara.

the Clearance Groups in the Test Group, or if it is just a consequence of the group selections themselves, data for the Point of Comparison country, France, is shown on the second row of Table 2. Here, the average difference that was measured between the Y-metric for the Test and Legacy groups, only amounts to one fifth of a percentage point. Therefore, the results of the first two rows can be stated by saying that under regular conditions, i.e., in the Point of Comparison country, the percentage income for the groups in the Test Groups was measured to be higher than that of the Control Group by 0.2 percentage points, but when the model based pricing methodology was used in Belgium, the same difference was higher by 1.8 percentage points. These results are summarized in the last row of Table 2.

As a final test to ensure that the observed results are not influenced or biased by the choice of Belgium and France as the Pilot Test and Point of Comparison countries, data for the two previous years is shown in Table 3 and Table 4, when the Optimization Model did not exist yet, and everything was priced according to the legacy pricing method.

Table 3: Summer Campaign Y-Metric Comparison for 2007

2007	Y - Pilot Test Groups	Y - Legacy Groups	Avg Difference
Belgium	45.5%	50.2%	-4.7
France	41.7%	45.8%	-4.1
B - F	**3.8**	**4.4**	**-0.6**

Table 4: Summer Campaign Y-Metric Comparison for 2006

2006	Y - Pilot Test Groups	Y - Legacy Groups	Avg Difference
Belgium	41.8%	46.3%	-4.5
France	41.5%	46.3%	-4.9
B - F	**0.3**	**0.0**	**0.4**

As evident in the third line of these tables, the percentage income difference between the Clearance Groups in the Test Group and those in the Legacy Groups was only marginally greater than zero for 2006, and negative for 2007, further reinforcing the positive difference of 1.8 that was obtained when the pricing model was used for the Summer Test Pilot of 2008.

Further analysis was conducted at the individual level of each of the Clearance Groups that form the Test and Control Groups for the experiment:

Table 5: Summer Campaign, Clearance Group level Y-Metric Performance

Y-Metric		2006 B	2006 F	2006 B - F	2007 B	2007 F	2007 B - F	2008 B	2008 F	2008 B - F	2008 Avg(B - F)
Test Group	Woman Blazer	35.3%	27.8%	7.5	38.8%	33.5%	5.3	39.1%	35.4%	3.7	2.2
	Basic Blazer	43.6%	37.4%	6.2	50.2%	45.2%	5	48.7%	46.4%	2.3	
	TRF Trouser	47.6%	45.7%	1.9	45.5%	40.2%	5.3	40.1%	40.0%	0.1	
	T-Shirt	42.8%	43.1%	-0.3	45.9%	43.2%	2.7	47.6%	44.9%	2.7	
Legacy/ Control Group	Woman Trouser	36.5%	31.6%	4.9	51.3%	44.9%	6.4	36.6%	38.1%	-1.5	-2.6
	Basic Trouser	56.3%	49.3%	7	48.9%	48.8%	0.1	40.6%	46.6%	-6	
	TRF Blazer	46.8%	34.5%	12.3	34.1%	45.5%	-11.4	42.1%	43.9%	-1.8	
	TRF Skirts	52.0%	43.4%	8.6	41.3%	31.7%	9.6	38.4%	39.4%	-1	

As seen in Table 2, the Y-Metric for the Test Group in Belgium for 2008, highlighted in the table to designate when the Test Pilot was run, resulted in an average difference of 2.2 percentage points over the Y-Metric for the Point of Comparison country France, while the performance for the Legacy Group was 2.6 percentage points *below* those observed in France. Furthermore, looking at the $B - F$ data over the previous years, it can be seen that the percentage income for Belgium over France has followed a downward trend, but for 2008, the Clearance Groups in the Test Group were still superior to those in the Legacy Group.

Finally, it can be checked that Belgium and France had comparable average inventory levels, as shown in Table 6, where the average inventories for each year and Clearance Group are shown, with the percentage increase over the previous year's inventory shown in parenthesis:

Table 6: Summer Campaign Average Inventory Levels

Average Stock		2006 B	2006 F	2007 B	2007 F	2008 B	2008 F
Test Group	Woman Blazer	252 (-)	202 (-)	113 (-55%)	123 (-39%)	259 (129%)	245 (100%)
	Basic Blazer	220 (-)	226 (-)	210 (-5%)	233 (3%)	230 (10%)	229 (-2%)
	TRF Trouser	1479 (-)	1750 (-)	1453 (-2%)	1584 (-9%)	1458 (0%)	1533 (-3%)
	T-Shirt	6770 (-)	5812 (-)	5726 (-15%)	4897 (-16%)	5062 (-12%)	4866 (-1%)
Legacy/ Control Group	Woman Trouser	550 (-)	534 (-)	371 (-33%)	373 (-30%)	801 (116%)	782 (109%)
	Basic Trouser	1155 (-)	1234 (-)	663 (-43%)	710 (-42%)	688 (4%)	723 (2%)
	TRF Blazer	54 (-)	67 (-)	23 (-58%)	24 (-64%)	85 (270%)	76 (215%)
	TRF Skirts	975 (-)	1324 (-)	264 (-73%)	328 (-75%)	224 (-15%)	238 (-27%)

6.4.3 Winter Campaign Field Test Results

The evaluation of the Winter Campaign Field Test was conducted in a similar way, with the difference that testing the pricing model on half of the Clearance Groups in Belgium and the complementary half of the Clearance Groups in Ireland allows for calculating the differences between both Clearance Group halves within each country, comparing them, and adjusting them by the Y-Metrics observed for the same groups in the rest of Western Europe. Consequently, the following Y-Metric differences were calculated for each of the stores in each of the countries:

$$Difference = Y_{Clearance\_Groups:1-12} - Y_{Clearance\_Groups:13-20}$$

Equation 6-3

As mentioned earlier, this allows for a "double check" where the difference between the Y-Metrics should tend to larger/positive values for Belgium, where Clearance Groups one through twelve are priced through the model based methodology, and to smaller/negative values for Ireland where Clearance Groups thirteen through twenty are the ones priced according to the model based methodology, i.e., if the model based pricing methodology is indeed better than the legacy method.

Following a similar procedure to the one used for the Pilot Test evaluation, the Y-Metric the store-level average values of both halves of the Clearance Groups were calculated according to Equation 6-2 with the difference that the summation on the numerator is done over the Y-Metric difference calculation presented in Equation 6-3. The results for the average Y-Metrics and differences are tabulated in Table 7:

Table 7: Winter Campaign Y-Metric Comparison for 2008

2008	Y - Groups 1-12	Y - Groups 13-20	Avg Difference
Belgium	43.1%	42.6%	0.5
Ireland	41.9%	46.7%	-4.8
Rest of Western Europe (RWE)	40.5%	42.7%	-2.2
Belgium - RWE	**2.6**	**-0.1**	**2.7*** **
Ireland -RWE	**1.4**	**4.1**	**-2.6***

For clarity, the Clearance Groups that were priced under the model based pricing methodology are highlighted in red; all other Groups were priced under the legacy pricing method. Looking at the average difference values in the last column, the Clearance Groups that were priced according to the model based method resulted in a percent income that was higher than the one for the groups priced under the legacy method by 0.5 percentage points. Similarly, the same was true for the Clearance Groups that were priced by the model based method in Ireland, whose percent income was on average 4.8 percentage points higher than the Groups priced under the legacy method.[48] However, from the average difference for the rest of Western Europe, under uniform conditions, Clearance Groups thirteen through twenty generally have a higher percent income than Groups one through twelve, by 2.2 percentage points. Considering this Point of Reference value, the adjusted percent income or Y-Metric improvements obtained by the model based pricing methodology are shown on the bottom right corner of Table 7.

Once again, Y-Metric values were calculated for the two previous years in order to ensure that the choice of countries for the experiment did not bias the results:

Table 8: Winter Campaign Y-Metric Comparison for 2007

2007	Y - Groups 1-12	Y - Groups 13-20	Avg Difference
Belgium	44.0%	44.2%	-0.2
Ireland	42.9%	43.7%	-0.8
Rest of Western Europe (RWE)	42.2%	40.6%	1.5
Belgium - RWE	**1.8**	**3.5**	**-1.7***
Ireland -RWE	**0.8**	**3.0**	**-2.3**

Table 9: Winter Campaign Y-Metric Comparison for 2006

2006	Y - Groups 1-12	Y - Groups 13-20	Avg Difference
Belgium	45.4%	48.6%	-3.2
Ireland	40.9%	44.6%	-3.7
Rest of Western Europe (RWE)	41.2%	43.7%	-2.4
Belgium - RWE	**4.1**	**4.9**	**-0.8**
Ireland -RWE	**-0.3**	**0.9**	**-1.2**

As an indication of the statistical significance of the results shown in the bottom right corners of each of the previous tables, the following legend was used:

---

[48] As already mentioned, the average difference value for Ireland is negative because of the way the difference is calculated in Equation 6-3. In other words, a negative value for Ireland means that the percent income was higher for the Clearance Groups priced under the model based methodology.

Table 10: P-value Statistical Significance Legend

*:	$p \leq 0.05$
**:	$p \leq 0.01$
***:	$p \leq 0.0005$

Here, *p* refers to the statistical significance p-values that were obtained for the calculation of the average difference values, according to the values observed for the each of the stores analyzed. The percent income increases measured in Belgium and Ireland that are shown in Table 7, are therefore attributable to the use of the model based pricing methodology with a statistical significance of 0.9995 and 0.95. Here, however, looking back at the Field Test pricing acceptance summary shown in Table 1, it is questionable whether the results obtained for Clearance Group 20 should in fact be considered in evaluating the model based pricing methodology, given that the Optimization Model's pricing suggestion was bypassed on several occasions in favor of the pricing proposal made by the Country Manager. Therefore, we repeat the calculations from Table 7, but this time the data for Clearance Group 20 in Ireland is omitted:

Table 11: Winter Campaign Y-Metric Comparison for 2008 (w/o Ireland Group 20)

2008	Y - Groups 1-12	Y - Groups 13-19	Y - Groups 13-20	Avg Difference	
Belgium	43.1%		42.6%		0.5
Ireland	41.9%	48.8%		-6.9	
Rest of Western Europe (RWE)	40.5%	44.3%	42.7%	-3.8	-2.2
Belgium - RWE	**2.6**		**-0.1**		**2.7*** **
Ireland -RWE	**1.4**	**4.5**		**-3.1** **	

In this table, the calculations for Belgium's average difference or percent income increase are left unchanged and shown on the last column. For Ireland however, an additional Y-Metric column for Clearance Groups thirteen to nineteen, i.e., excluding Group 20, was calculated and the corresponding average difference is shown on the next to last column. As evidenced by the results in Table 11, the suspicions of Clearance Group 20 introducing statistical noise into the calculations were well founded, as the increase in percent income that was measured in Ireland is now 3.1 percentage points, and is attributable to the model based pricing methodology with a significance of 0.99.

6.5 Economical Impact to Zara

Based on the statistically significant results for the measured increases in the profitability of the Clearance Season when the model based pricing model is used to make the pricing decisions, the economical impact that the developed project will have on Zara's entire business can be quantified. In this way, by projecting the obtained results to the entirety of Zara's Clearance Season sales, an approximation can be obtained for the increased earnings that will result when Zara uses the developed pricing model to make all of its Clearance Season pricing decisions.

In order to calculate the economical impact of the developed model, we start by looking at the aggregated data for Zara's entire Clearance Season during the most recent years' Summer Campaign. Parting from the initial valuation of the inventory at the beginning of the Clearance Season, i.e., when it is priced at its Regular Season undiscounted price, and then taking all of the Clearance Season revenues, including proceeds from the salvage of unsold inventory at the end of the Clearance Season, the real Y-Metric that was observed during each year is calculated:

Table 12: Economical Impact Calculation, Summer Campaign[49]

Assumptions		
New Methodology Y-Metric Increase		1.8
EUR/USD Conversion Rate		1.3
Winter Campaign Clearance Season	**2007**	**2008**
Initial Inventory, valued at Regular Season Price (M EUR)	743.6	997.1
Clearance Season Revenues, including Salvage (M EUR)	334.6	448.7
Y-Metric with the Legacy Methodology	45.0%	45.0%
Y-Metric with New Methodology	46.8%	46.8%
Revenue Increase (M EUR)	13.4	17.9
Revenue Increase (%)	4.0%	4.0%
Revenue Increase (M USD)	**17.4**	**23.3**

As shown in Table 12, these values corresponded to 45.0% for the years 2007 and 2008. Now, the projected Y-Metric that would have been obtained, had the developed pricing model been used to make the Clearance Season pricing decisions, can be calculated by adding the average increase in the percent income that was experimentally measured to be equal to 1.8 percent point, as it was previously reported in Table 2. In this way, the projected Y-Metrics for the new pricing methodology are projected, and based on them, a revenue increase can be calculated by applying the

[49] Calculations made by the author.

new Y-Metric to the initial valuation of the inventory and subtracting the Clearance Season Revenues from the second row of Table 12.

As a result, revenue increases of 13.4 and 17.9 million Euros were calculated for the years 2007 and 2008, which correspond to an increase of four percent in Zara's Clearance Season revenues and profits. The corresponding figure in United States dollars is shown at the bottom of Table 12.

A similar economical impact analysis can be done for Zara's Winter Campaign Clearance Season by following the described procedure, but taking data from Zara's last three Winter Campaigns. The value of the increase in the Y-Metric that is attributable to the pricing model is calculated as the average of the two values that were measured for Belgium and Ireland, and reported in Table 11. The resulting economical impact analysis in shown in Table 13:

Table 13: Economical Impact Calculation, Winter Campaign[50]

Assumptions			
New Methodology Y-Metric Increase			2.9
EUR/USD Conversion Rate			1.3
Winter Campaign Clearance Season	**2006**	**2007**	**2008**
Initial Inventory, valued at Regular Season Price (M EUR)	864.9	1026.9	1238.5
Clearance Season Revenues, including Salvage (M EUR)	392.8	455.1	571.5
Y-Metric with the Legacy Methodology	45.4%	44.3%	46.1%
Y-Metric with New Methodology	48.3%	47.2%	49.0%
Revenue Increase (M EUR)	24.9	29.6	35.4
Revenue Increase (%)	6.4%	6.5%	6.2%
Revenue Increase (M USD)	**32.4**	**38.5**	**46.0**

In this case, revenue increases between 24.9 and 35.4 million Euros were calculated for the years between 2006 and 2008. It is important to note that the calculated increases in revenue are the result of a change in the pricing decision process itself and thus require no additional capital investments. Also, because there are two Clearance Seasons within a year, Zara's revenue increases can be calculated by adding the results from Table 12 and Table 13. For example, using the developed pricing model to make all of Zara's Clearance Season pricing decisions for 2008 would have resulted in a yearly revenue increase of 53.1 M EUR, or 69.3 M USD. Furthermore, given that, Zara has already incurred on all of the costs of goods sold for its Clearance Season inventory, these revenue increases fall directly to the company's bottom line profits.

[50] Calculations made by the author.

6.6 Key Learnings and Observations

6.6.1 Forecasting Accuracy

The forecasting accuracy of the Forecasting Model, and specifically, the minimum accuracy that should be targeted, was a source of uncertainty at the beginning of the project. And while we realized that the question of "how good is good enough?" is one that does not have a straight forward answer, we had a clear understanding of the fact that the ultimate test for the developed project's success would be a live test where conclusive results could be measured. Nevertheless, the accuracy of the Forecasting Model was followed closely all throughout the development of the project and error metrics are automatically calculated for all of the forecasts made. And as detailed in Chapter 4, extensive trials were conducted in order to improve the Forecasted Model in any way deemed possible.

For the present thesis, focus is placed on the live experiment results as a means of evaluating the overall project's success, but as a high-level illustrative summary of the Forecasting Model's accuracy during the Winter Campaign Field Test, Table 14 illustrates the revenue forecasting error for Belgium and Ireland for every Clearance Period:

Table 14: Field Test Revenue Forecasting Errors

Clearance Period	100	101	102	103	104	105	106	107	108	Overall
Belgium	-22%	0%	-15%	-19%	-8%	34%	-12%	11%	31%	-9.2%
Ireland	-22%	-3%	-7%	-13%	-34%	-10%	-11%	-3%	1%	-13.8%

The error metrics shown in Table 14 are calculated based on the expected revenues estimated by the Forecasting Model, i.e., the Model's forecast at the assigned prices, multiplied by the prices themselves. So for example, the value of -11% for Ireland's Clearance Period 106 means that the revenue forecasts made by the Forecasting Model for that Clearance Period, underestimated the real revenues by eleven percent.

As evident in the table's last column, the overall forecasting errors for the whole Clearance Season correspond to sub-estimation errors that do not surpass a fourteen percent error. The fact that the forecasting errors correspond primarily to sub-estimations can be partially explained by the fact that forecast adjustments are only made when the forecasting errors for the previous Clearance Period corresponded to an overestimation. As it was explained in Section 4.8.2 a sub-estimation error means that the Forecasting Model's sales estimates were surpassed in real life, and therefore are preferred to overestimation errors on the account of minimizing risk.

6.6.2 Price Trajectories and Clearance Season Pace

One of the most challenging aspects of making "manual" Clearance Season price assignments is determining how exactly the public and customers will react to a discounted price, and whether the perceived increase in sales will compensate for the foregone revenue resulting from a discounted price. As an illustrative example of the Model's reasoning when achieving this task, and how it compares to the pricing decisions made through the legacy pricing methodology, Figure 18 summarizes the average prices that prevailed throughout the execution of the Winter Clearance Field Test:[51]

Figure 18: Field Test Price Trajectory

As seen in Figure 18, the price trajectory for Clearance Groups 1-12 is more conservative in Ireland, where this Clearance Groups were priced according to the model based pricing methodology, and especially towards the beginning of the Clearance Season. Conversely, the price trajectory for Clearance Groups 13-20 is higher in Belgium, where these Groups were priced according to the pricing model's output.

Similar results had been previously observed during the Summer Campaign Pilot Test:

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[51] Average prices are determined by dividing the overall revenues obtained during each Clearance Period by the number of units sold.

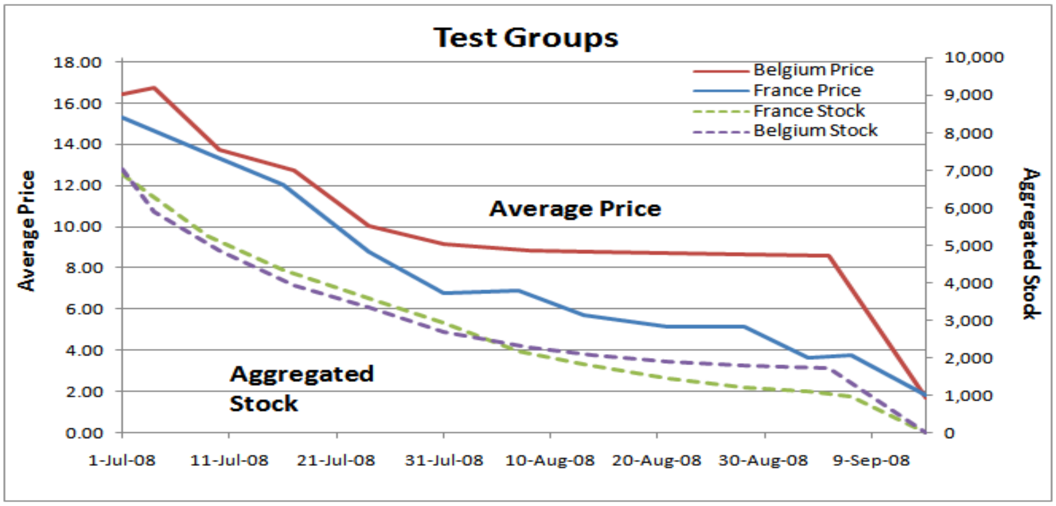

Figure 19: Pilot Test Price and Stock Trajectory

In Figure 19, the average prices for the Pilot Test groups are contrasted with the average prices of the corresponding Clearance Groups in the Point of Comparison country France. Here, the conservative nature of the average pricing in Belgium is more notorious. Figure 19 also plots the per-store inventory levels for both countries during the Clearance Season as detailed in the right hand side axis. As it can be seen, both countries started off with comparable levels of inventory and despite the more conservative pricing that was implemented in Belgium, both inventory levels remain at comparable levels up until the last weeks of the Clearance Season when France's inventory falls slightly below Belgium's.

Figure 18 and Figure 19 both capture the great value added that the developed model based pricing methodology brings to Zara. While the Pricing Team acknowledges that there may be times when they suffer from being "trigger happy" with price discounts, meaning that they issue more aggressive price discounts than what would be necessary to maximize profits, the task of determining what prices really need to be discounted, and by how much, is not a trivial task. By directly capturing the customers' price elasticity of the demand, the Forecasting Model provides the necessary information to evaluate the tradeoff between increased sales and the lower revenue per unit sold when prices are discounted, and the Optimization Model makes the optimal pricing decisions that will maximized realized revenues.

6.6.3 Salvage Inventory Levels

Another observation that can be drawn from Figure 19 is how the model based pricing methodology can often lead to increased levels of inventory at the end of the Clearance Season which must then be sold to third-party wholesalers at a salvage price. This however, does not mean that having extra remaining levels of inventory is less profitable than selling them before the end of the Clearance Season and salvaging a smaller quantity in the end. On the contrary, as it was pointed in Section 5.2.5, the Optimization Model makes price assignment decisions based not only on the future expected sales and revenues made during the Clearance Season, but on the fact that any remaining inventory will be sold for a salvage value. And because a non-negative salvage price is obtained for each unit of inventory salvaged, there may be cases where issuing a more aggressive price discount to reduce the units of inventory left unsold, may result in sub-optimal realized revenues. So for example, adding to the observations made in the previous section: in order to justify a price discount, it must result in an increase in sales that will compensate the revenues lost on account of the more discounted price, and *furthermore*, the increase in sales will at the same time result in less units salvaged at the end of the Clearance Season, and consequently less salvage revenue realized. In this way, if we take a Price Cluster that is currently priced at price P_1, the expected change in revenue R when further discounting the Cluster to price P_2, is given by the following relationship:

$$\Delta R = \Delta S * \left(P_2 - P_{SL}\right) + S_1 * \Delta P$$

Equation 6-4

Where ΔS represents the increase in sales that results from changing the price from P_1 with sales S_1, to a lower price P_2 with sales S_2, and P_{SL} represents the salvage price. The first term on the right hand side of Equation 6-4 is always positive, as any price during the Clearance Season is higher than the final salvage price, and the change in sales expected from lowering a price is positive. Furthermore, dividing both sides of the equation by ΔP and having Γ represent the price premium $(P_2 - P_{SL})$ between the target price and the salvage price leads to the following relationship:

$$\frac{\Delta R}{\Delta P} = \frac{\Delta S}{\Delta P} * \Gamma + S_1$$

Equation 6-5

Where the fraction or rate of change on the first term on the right hand side of Equation 6-5 can be seen as a back-of-the-envelope calculation for the price-elasticity of demand, and the equation shows the relationship between the price elasticity of demand and revenue increases. This relationship can therefore be interpreted as saying that the expected change in revenue with respect to the change in price can be expected to be positive if and only if this approximation to the price-elasticity of demand, scaled by the price premium between the target and salvage prices surpasses the unit level of sales at the current price P_1. Consequently, for a fixed price elasticity of demand and sales level S_1, revenues will be proportional to the price premium Γ, or the price distance between the target price and the final salvage price, and also, for a fixed price premium Γ, the increase in revenues is proportional to the price elasticity of demand.

An additional relationship that can be extracted from Equation 6-5, is that as Clearance Season prices approach the final salvage value, and the price elasticity of demand decreases, further price discounts become increasingly difficult to justify with the promise of increased revenues. And although the case presented here only considers a single stage horizon and does not take into account multiple Clearance Periods, it can be used to explain the phenomenon that occurs towards the end of Zara's Clearance Season, when prices approach the terminal salvage value and the customer's price elasticity of demand gradually wanes as the novelty of the Clearance Season fades away.

In reality however, there are cases, when different business situations such as the need to free up space in the store's exhibition space take precedence over maximizing profits, especially towards the end of the Clearance Season when the majority of the inventory has been sold. An example of this situation was given for Ireland's last Clearance Period during the Field Test and illustrated in Table 1. For this reason, as it was explained in Section 5.2.6, a configuration lever was included in the Optimization Model that allows the user to impose a discount on the nominal salvage price. This lever introduces an incentive into the Model to reduce the amount of salvaged inventory. Coming back to Equation 6-5, the described lever is equivalent to increasing the price premium Γ, and in fact, if the salvage price is adjusted to zero, the equation simplifies to an approximation of the basic price elasticity of demand relation:

$$\frac{\Delta R}{\Delta P} = \frac{\Delta S}{\Delta P} * P_2 + S_1$$

Equation 6-6

6.6.4 Pricing Decision Case Study Revisited

In order to examine the key learnings and observations that have been so far mentioned, but this time under the practical context in which they arise in Zara's business, it is convenient to revisit and continue the case study that was introduced in Section 3.2.1, towards the beginning of this thesis. As you might recall, a Country Manager was working on her price suggestion list before her meeting with the Pricing Team, and came across a couple of Price Categories within a Clearance Group for which she considered a further discount necessary. Not surprisingly, this case was adapted and created with real life data from the Field Test, when the developed model based pricing methodology was being tested during Zara's Winter Clearance Season.

On her way to the meeting, the Country Manager kept thinking about the Clearance Group in question and what had happened during the previous week's meeting. She had followed the regular (legacy) procedure and determined that the Price Category in question needed to be further discount as their days-worth of inventory levels were clearly above the country's average. However, when the MIT2 model's pricing suggestion was reviewed, she was surprised to find that the model suggested that the Price Cluster's current prices be maintained. With some convincing from the Pricing Team, she had reluctantly accepted the model's suggestion, mostly for the sake of allowing the ongoing experiment to continue. Nevertheless, throughout the last week, sales for that Category continued to deteriorate and the days-worth of inventory for them surpassed the Country's average values by a long shot: this week the prices had to be discounted by even more than what she had suggested on the previous week.

During the meeting, when the time came to discuss the pricing for the Clearance Group in question, the Country Manager couldn't believe her eyes when she saw the pricing proposal made by the MIT model: maintain the prices at their current levels. The pricing suggestions of both the Manager and the model for the last two weeks are summarized in Table 15.

Table 15: Case Study Revisited, Price Suggestions

Clearance Period:	P102	P103			P104		
Price Category	Price	Dw-Inv/Avg	MIT2 Suggestion	Specialist Suggestion	Dw-Inv/Avg	MIT2 Suggestion	Specialist Suggestion
49.9-29.90	19.95	1.8	19.95	14.95	2.4	19.95	12.95
24.90	14.95	2.6	14.95	9.95	2.9	14.95	9.95
19.90	9.95	0.8	9.95	5.95	1.5	9.95	9.95
14.90 -12.90	6.95	0.6	6.95	5.95	0.6	6.95	6.95

Additionally, the ratio of days-worth inventory over the country average was calculated for each Price Category. As shown in Table 15, the MIT2 model had suggested maintaining the prices of Price Clusters 49.90-29.90 and 24.90 at their current Price Categories of 19.95 and 14.95 for two weeks in a row now, despite them having days-worth of inventory over twice as much as country average value.

Sensing desperation in the Country Manager's expression, the project team proceeded to explain the reasoning behind the model's predictions to the Country Manager and Price Team, but pending a clear and reasonable explanation, the model's price suggestions would most certainly not be followed. After having conducted a Pilot Test over the previous Summer's Clearance Season, the team had developed a sense for what Zara's concerns about the project might be and had since prepared backup reports that explained the reasoning behind the model's suggestions and which they now brought with them to the meetings. The team pulled out a series of reports that they had prepared for Price Clusters 49.90 to 29.90, and began by reviewing the exponential smoothing procedure through which the Forecasting Model estimated the price elasticity of demand for each Clearance Group.

Figure 20: Case Study Revisited, Price Elasticities

Pointing at the graph in Figure 20, they highlighted how the predicted elasticities had closely matched the real elasticities until that moment. Furthermore, pointing at the predicted elasticity for Clearance Period 104, they explained how elasticity values around -1 correspond to a weak, almost neutral reaction to price discounts. In other words, this means that further discounting the Price Clusters will not result in a sufficient increase in unit sales that would make up for the lost revenues on account of issuing the price discount. This effect was then highlighted in Figure 21, which corresponds to the Forecasting Model's revenue forecasts. This plot shows the revenue forecasts for each of the Price Clusters when they are priced at each of the Clearance Price List prices from the x-axis. Figure 21, thus illustrates the compromise between increased sales and the foregone revenues from further discounting the price:

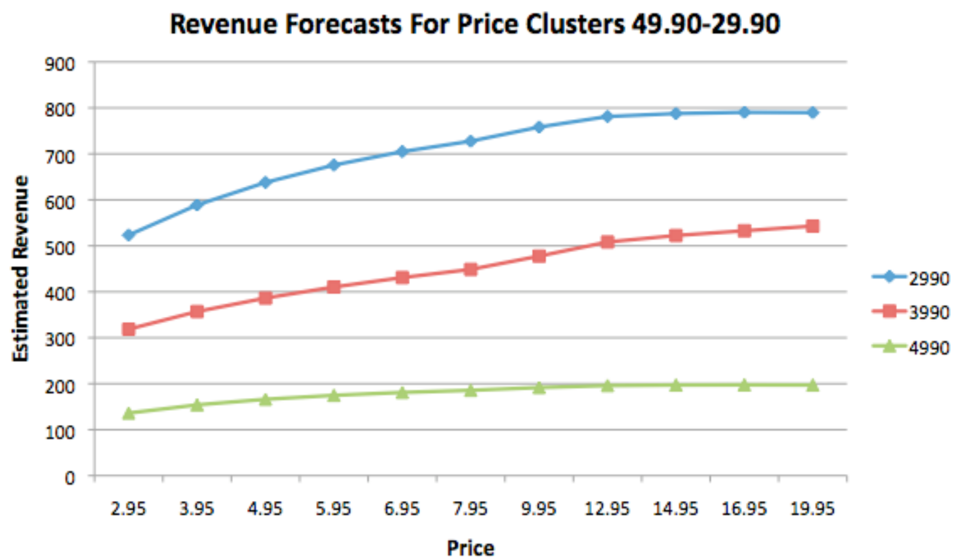

Figure 21: Case Study Revisited, Revenue Forecasts

The fact that all three revenue plots in Figure 21 are downward sloping when starting from the current price of 19.95 means that the expected increase in sales from lowering this price, will not be big enough to make up for the lower revenues associated with a lower price. Consequently, based on this input data, the Optimization Model makes the decision to maintain the current prices.

Table 16: Case Study Revisited, Profit Simulation

CLUSTER	Price	INV_1	P1	S1	Rev1	P2	S2	Rev2	P3	S3	Rev3	P4	S4	Rev4	INV_SL	P_{SL}	Rev_{SL}
4990	19.95	105	19.95	10	197.10	19.95	8	160.79	19.95	7	131.39	19.95	5	107.43	75	2.24	168.20
3990	19.95	355	19.95	27	542.89	19.95	22	437.03	19.95	18	356.64	19.95	15	291.30	273	2.24	612.42
2990	19.95	518	19.95	40	789.84	19.95	32	647.67	19.95	26	528.33	19.95	22	431.32	398	2.24	891.17
2490	14.95	477	14.95	52	774.42	14.95	42	628.12	14.95	34	513.30	14.95	28	419.73	321	2.24	718.53
1990	9.95	390	9.95	75	745.26	9.95	61	605.20	9.95	50	492.92	9.95	40	401.97	164	2.24	368.12
1490	6.95	178	6.95	19	133.70	6.95	16	109.32	6.95	13	89.40	6.95	11	73.14	120	2.24	268.01
1290	6.95	230	6.95	32	225.69	6.95	27	184.25	6.95	22	150.28	6.95	18	122.69	132	2.24	295.10

Going through Table 16, a report output from the Optimization Model, the team then explained how the Optimization Model makes price, sales, and revenue predictions for the three remaining future Clearance Periods, and makes pricing assignment decisions for the current Clearance Period that maximize the *overall* Clearance Season revenues and profits, including the salvage revenues after the end of Clearance Season.

Quickly pulling out her calculator and summing the salvage inventories on the third column to the left from Table 16 for Price Clusters 49.90 to 29.90, the Country Manager pointed out the fact that according to the Optimization Model's previsions, seventy percent of the current inventory would have to be salvaged at the end of the Clearance Season, and that could not be profitable. After acknowledging how the model's logic might seem counterintuitive at first, the team went back to the plot on Figure 21 and explained that if for example the Price Category's price were to be lowered from 19.95 to 12.95, the price difference of 7 would be lost on every sale made, and furthermore, each additional sale made as a result of having lowered the price, would imply one less unit that will be sold for the salvage value of 2.24 described in Table 16.

Much more at ease after understanding the reasoning behind the model's price suggestions, the Country Manager had one last question: "And how do we know the model's forecasts are correct?". Having foreseen this question coming up, the team pulled out one last graph in which the aggregated revenue forecasts for Price Clusters 49.90 to 29.90 are plotted side by side with the real revenues:

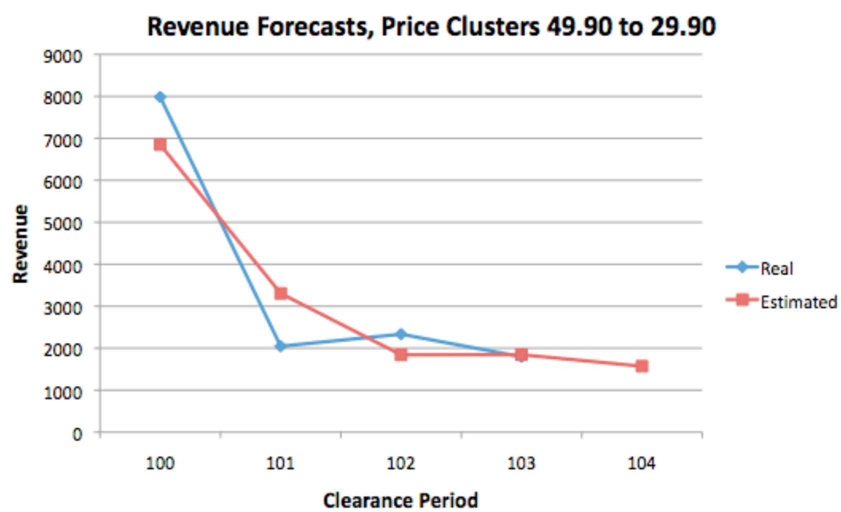

Figure 22: Case Study Revisited, Aggregated Revenue Forecasts

Having seen how closely the model's estimates had followed the actual Clearance Season revenues obtained up to that point, the Country Manager nodded in approval as a member of the Pricing Team humorously assented: "Agreed, we do what the 'machine' says then. Send this price list out to the stores."

7 Conclusion

7.1 Impact to Zara

After almost two years since the current project was first set into motion, a new mathematical model based methodology for making Clearance Season pricing decisions has been designed, implemented, and successfully tested for Zara's Woman Section. Ever since the conception of the project, the expectations for its potential have been high. And after having obtained conclusive evidence that supports the fact that the implemented methodology results in bottom-line clearance profit increases of up to six percent that could amount to more than thirty-five million Euros, we can say that our expectations have been met.

Here, it is important to highlight the fact that the resulting savings not only go straight to Zara's bottom-line profits, but are also the result of a near zero sum capital investment. Aside from the company's sponsorship of the LFM Program and the man-hours invested in the development and support of the project, the resulting savings come as the result of a *change* in the decision criteria for pricing, while the original process flow remains in place unmodified, and did not require any major restructuring efforts. And if anything, this flow has been made more efficient, as the automation of several of the steps described in the legacy pricing flow like Clearance Group and Price Cluster assignments and possibly the automation of new price assignment communications to the stores will result as collateral benefits of the project's implementation.

Another important factor in the project's success that should be highlighted is the fact that the developed model was specifically built to fit Zara's business characteristics. Zara's unique approach to the textile, distribution, and retail business has been key to placing it at the top of its industry, and the company's management recognizes these unique characteristics of its business and has consequently been known to turn down offers from enterprise software vendors who sell "customized" industry solutions. The developed pricing model, on the other hand, was built while catering to the specific needs and characteristics of Zara's business and by incorporating the experience-based knowledge and expertise of its managers and analysts. Furthermore, the involvement of the company's IT engineers, managers, and Country Managers during the development of the project gave Zara a transparent tool which they can continue to use, improve, and adapt as deemed necessary by the changing conditions of its market.

Finally, in addition to the positive economic impact that the project attained, it achieved a paradigm shift on the company's mentality and created an opportunity to experiment with new and innovative approaches to solving old problems. In fact, part of the learning that resulted from the developed model's live tests transcended the scope of the experiment. As Zara's Pricing Team acknowledged, profit increases in the Clearance Season of other countries that were not part of the experiments, resulted on account of their internalization of the proposed model's line of reasoning.

7.2 Future Work

At the time of publishing the present thesis, Zara has made the decision to implement the designed pricing model on the entirety of its operations. This includes all Clearance Groups in all three of its Woman, Men, and Children Sections, and its more than 1,300 non-franchised stores across the world. In preparation for the upcoming Summer Campaign Clearance Season, Zara's IT department is currently implementing the required database calculations and designing a graphical user interface that will allow Country Managers to generate pricing suggestion lists for their individual countries.

Zara's IT engineers, having been involved closely with the project are well familiarized with the Forecasting and Optimization Models and are more than qualified to continue to develop further improvements on them, and future expansion projects such as the proliferation of the model based pricing methodology to manage the Clearance Season for other Inditex brand stores are possible.

Finally, MIT4, the fourth project on the line of LFM and Zara collaboration projects, is scheduled to begin at the beginning of June of the present year. This new project is deeply related with the one currently being concluded under this thesis, as it relates to developing a methodology for the optimal distribution of remaining inventories from Zara's central warehouses to its stores at the beginning of the Clearance Season. Once again, expectations are high for this project and once completed, the combination of having an optimal distribution and pricing of Clearance Season inventory is expected to further positively impact Zara's bottom-line profits. More importantly, the synergy of both projects working together will empower Zara to tightly control the management of its fast-paced Clearance Season while still being able to capture the experience-driven knowledge of its analysts and managers.

GLOSSARY

Broken Assortment (Effect): Refers to the observed decrease in demand for a garment when it is no longer offered in all of its initially available Colors and Sizes. This effect is heightened by the fact that it is the most popular Colors and Sizes that are sold first with the other, lower demand ones remain.

Clearance Group: A grouping of garments that is made for Clearance Season pricing purposes. Garments are grouped according to garment type and purchasing department criteria and subdivided into Price Clusters within each Clearance Group according to their initial Regular Season price.

Clearance Group Sales Report: A sales report frequently used to determine Clearance Season pricing lists before and during the Clearance Season. This report details the sales and inventory status for all countries, Clearance Groups, and Price Categories and allows pricing analysts to determine which Price Categories' prices need to be further discounted.

Clearance Period: A Clearance Season is divided into Clearance Periods that are delimited by the points in time where prices are revised, i.e., every time that prices are discussed and decisions made, marks the beginning of a new Clearance Period. Clearance Period are roughly equal to seven calendar days, but may vary depending on when the actual pricing decisions are made.

Clearance Price List: The list of possible prices that may be chosen to assign to a Clearance Season item. The Clearance Price List is chosen to include commercially attractive prices that vary depending on the currency or country in question.

Clearance Season: The Clearance Season corresponds to the end of a Campaign in which the remaining inventory that could not be sold during the Regular Season of that Campaign is sold at a discount. Clearance Seasons extend for approximately two months, depending on the country in which they take place.

Commercial Price List: The list of possible prices that can be assigned to a Regular Season item. As its name implies, the Commercial Price List includes prices that are commercially attractive to Zara's customers.

Control Country: During a test experiment, a Control Country refers to a country where pricing decisions are made as they were normally made in the absence of the current project, i.e., following the Legacy Pricing Method.

Control Group: During a test experiment, a Control Group is a Clearance Group that is priced following the Legacy Pricing Method, in order to serve as a point of comparison for evaluation of a Test Group.

Country Managers: Country Managers are analysts in charge of overseeing the performance of the stores within their country of responsibility. Country Managers oversee the new product introduction process and handle communications with the store clerks and managers in general. During the Clearance Season, Country Managers are in charge of making pricing suggestion lists for their country which they then review with the Pricing Team and communicate to the stores after that.

Exit Price: The Exit Price refers to the highest priced garment within a Clearance Group. The Exit Price has a psychological effect on the customers whose perceptions of the stores general pricing are anchored towards the most expensive articles. So for example, even a few high priced items will give customers the feeling that prices at the store in general are high.

Legacy Pricing Methodology: The general pricing process that Zara had traditionally followed before the introduction and testing of the methodology presented by the current thesis.

Master Price List: The Master Price List is the initial pricing list for the beginning of the Clearance Season. In the Legacy Pricing Methodology, the Master Price List is designed and thought for Spain, but is also used as a starting point when making the pricing decisions for the rest of the countries.

Price Category: At any given point during the Clearance Season, any articles within a same Clearance Group that are currently priced equally, form part of the Price Category indicated by that price. In other words, Clearance Groups are divided into Price Clusters, and all the Price Clusters within a Clearance Group that are priced at the same price during the Clearance Season, are joined under a Price Category.

Price Cluster: Clearance Groups are subdivided into Price Clusters, based on the original (non-discounted) Regular Season prices, i.e., a Price Cluster is formed by all the garments in a Clearance Group that were priced at the same price during the Regular Season, before the Clearance Season begins.

Price Equivalency Table: This table establishes a relationship between all the different prices for different countries in their respective currencies. It can be used as a rough guideline to convert the Clearance Season Master List to be used in other countries, although the main criteria for pricing is to assign prices that maintain Zara's competitive edge in each market or country.

Price Suggestion List: A Price Suggestion List shows tentative price assignments for a Clearance Group and country in particular, and once approved bye the Pricing Team, it is communicated to the stores and becomes the current pricing definition. Price Suggestion Lists are created either by Country Managers or by the models being developed in the current thesis.

Pricing Team: The Pricing Team is formed by a group of Zara's management who are knowledgeable in the areas of finance, product management, and distribution, and who oversee all of Zara's pricing decisions.

Product and Purchasing: This department within Zara is divided by different product families and is responsible for making purchasing and ordering decisions as well as setting the prices for newly introduced products.

Regular Season: The Regular Season covers the initial part and majority of a Campaign and consists of the period of roughly five months where garments for the Campaign are gradually introduced and sold at their full, undiscounted price.

Restriction Levers: In the context of the Optimization Model, Restriction Levers represent those settings in the model that allow the user to incorporate his knowledge of the business into the model's selection criteria.

Salvage Price: The Salvage Price for a Clearance Group corresponds to the price at which any remaining inventory by the end of the Clearances Season will be sold. This remaining inventory is sold to third-party wholesalers who operate in countries or regions outside of Zara's target market.

Summer/Winter Campaign: Zara divides its design collections into two Campaigns, the Summer and the Winter Campaign, which span for approximately seven months and temporarily coexist when one Campaign ramps up while the other winds down.

Test Group: During a test experiment, the Test Group corresponds to the group that is receiving treatment, i.e., the group that is priced according to the new methodology under test.

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Appendix

Appendix A: Optimization Model, AMPL Model Implementation

```
#*********************************************************************************************
# MIT2 MODELO DE OPTIMIZACION                                                                 *
# FECHA: 16/1/2009                                                                            *
# VERSION: 23                                                                                 *
#                                                                             Rodolfo Carboni *
#*********************************************************************************************
# 16/01/2009: Se cambió la definiciÛn de disc_saldero y el importe saldero, ahora un maximo de
CAPc unidades se venden a psaldero y el resto a (1-discsaldero)*psaldero (RCB)
# Se añade el parámetro Ic y la restriccion max_fin_inv para limitar el inventario final con que
puede terminar el saldo
# 08/01/2009: Se aÒadiÛ segundo umbral de descuento minimo mindiscount2 (RCB)
# xx/12/2008: El divisor de fav0 en el constraint de L[w,n,k] pasa de (Fc * ct[n]) a Fc (FPP,
CSS)

### DECLARACION SETS Y PARAMETROS ###

param Wc > 1;                                   #Numero de Periodos de Decision
param Qc >= 1, default 1;                       #Inventario Minimo Requerido Por CategorÌa
param Cc >= 1, default 99;                      #N˙mero M·ximo de CategorÌas Distintas
param Tc {2..Wc} >= 1, default 1;               #Longitud de Periodos de Decision
param Sc >= 1, default 1;                       #Inventario Minimo Requerido Para el Precio de
Salida
param Ic >= 0;                                  #Inventario M·ximo al Terminar el Saldo
param mindiscount  >= 0, <= 1 default 0;        #Rebaja Minima Sobre el Precio Anterior
param mindiscount2 >= 0, <= 1 default 0;        #Rebaja Minima Sobre el Precio Anterior Para
Precios Bajo un Umbral Determinado
param umbraldisc2 >= 0, default 0;              #Umbral Bajo el Cual se Aplica mindiscount2
param tighten_slack >= 0, <= 1 default 0.9999;  #Discount Factor para ajustar L a su bound
param CAPc >= 0;                                #Capacidad en unidades de absorcion del Saldero
param disc_saldero >= 0, <= 1 default 1;        #Porcentaje de descuento sobre psaldero al que se
logra vender el inventario remanente sobre el nivel CAPc
param tiempo > 0, <= 1;                         #Factor para simular caida de ventas debida al
tiempo

### VALORES CARGADOS DE TABLAS ###

set PRECIOS ordered;                                    #Precios Comerciales Disponibles
param psaldero >= 0;                                    #Precio Saldero de LiquidaciÛn Final
set CLUSTERS ordered;                                   #Clusters de Precio Inicio Saldo
param precioT{CLUSTERS} >= 0;                           #Precio de Temporada
set CATEGORIES = setof {n in CLUSTERS}precioT[n];       #Categorias de Precio Periodo Anterior
param inv0 {CLUSTERS} >= 0;                             #Inventario Inicial del Periodo Actual
param ct {CLUSTERS} >= 0;                               #Suma de productos de Colores y Tallas de
cada MC en el Cluster
param sales0 {CLUSTERS, PRECIOS} >= 0;                  #Sales Estimate Para el Periodo Actual
param f0 {CLUSTERS, PRECIOS} >= 0;                      #Tasa de Demanda Para el Periodo Actual
param fav0 {CLUSTERS, PRECIOS} >= 0;                    #Tasa de Demanda Ponderada Por Inventario
param Fc >= 0;                                          #Threshold Broken Assortment
param Uc >=1, <= 1;

### CHECKS ###

#CHK 1: Revisa que el precio anterior m·s bajo sea m·s alto que el precio digno
check : precioT[last(CLUSTERS)] >= first(PRECIOS);
#CHK 2: Revisa que el descuento minimo no sobrepase el precio digno     ***PERIODO I
#check : precioT[last(CLUSTERS)]*(1-mindiscount) >= first(PRECIOS);

### VARIABLES DE DECISION ###
```

```
var X {1..Wc, CLUSTERS, PRECIOS} binary;  #Selecciona los precios >= al precio de decision

var XC {1..Wc, CATEGORIES, PRECIOS} binary; #Agrupa los Clusters seg˙n precios del periodo
anterior

var Y {w in 1..Wc, n in CLUSTERS, k in PRECIOS} =
        if k = first(PRECIOS) then
                (X[w,n,k])
        else
                (X[w,n,k] - X[w,n,prev(k)]);            #Selecciona el precio de decisiÛn

var L {2..Wc, CLUSTERS, PRECIOS} >= 0;                  #Modela la Tasa de Demanda

var Lsaldero >= 0;      #PorciÛn del inventario no saldado que es absorbida por el Saldero

var Z {1..Wc, PRECIOS} binary;  #Habilita o deshabilita los Precios Comerciales utilizados

var INV {1..(Wc+1), CLUSTERS} >= 0;                     #Modela el Inventario disponible

var CURRENT_PROFIT =
        sum{n in CLUSTERS, k in PRECIOS} sales0[n,k]*k*Y[1,n,k];        #Ingresos Periodo Actual

var EST_FUT_PROFIT =
        sum{w in 2..Wc, n in CLUSTERS, k in PRECIOS} L[w,n,k] * Tc[w] * k; #Ingresos Periodos
Siguientes

var DISC_FUT_PROFIT =
        sum{w in 2..Wc, n in CLUSTERS, k in PRECIOS} L[w,n,k] * Tc[w] * k * tighten_slack^(w-1);

#Ingresos Periodos Siguientes Descontados
var PROFIT_SALDERO =
        disc_saldero * psaldero * Lsaldero + (1-disc_saldero) * psaldero * sum{n in CLUSTERS}
INV[Wc+1,n];   #Ingresos Liquidacion Saldero

var DISC_PROFIT_SALDERO =
        (disc_saldero * psaldero * Lsaldero + (1-disc_saldero) * psaldero * sum{n in CLUSTERS}
INV[Wc+1,n]) * tighten_slack^(Wc);

### FUNCION OBJETIVO ###

maximize Total_Profit:

        CURRENT_PROFIT + DISC_FUT_PROFIT + DISC_PROFIT_SALDERO;

### RESTRICCIONES ###########################################################

### ESTRUCTURALES ###

# DEMANDA
#Considerar solo la demanda para el precio seleccionado
subject to price_demand {w in 2..Wc, n in CLUSTERS, k in PRECIOS}:
        L[w,n,k] <= sales0[n,k]*Y[w,n,k]*tiempo^(w-1);

#Considerar el decremento en la demanda causado por el broken assortment
subject to brkassrtmnt_demand {w in 2..Wc, n in CLUSTERS, k in PRECIOS}:
        L[w,n,k] <= (f0[n,k]*(1 - Uc) + Uc*fav0[n,k]*INV[w,n]/Fc) * tiempo^(w-1);

#El inventario absorvido por el saldero no puede ser mayor que el inventario remanente ni que la
capacidad CAPc del Saldero
subject to remain_inv:
        Lsaldero <= sum{n in CLUSTERS} INV[Wc+1,n];
subject to cap_liq:
        Lsaldero <= CAPc;

# INVENTARIO
#Sujeto al inventario inicial en el periodo actual
subject to initial_inventory {n in CLUSTERS}:
        INV[1,n] = inv0[n];
```

```
#Sujeto al inventario inicial para el proximo periodo
subject to inventory_endof_actual {n in CLUSTERS}:
        INV[2,n] = inv0[n] - sum{k in PRECIOS} sales0[n,k]*Y[1,n,k];

#Sujeto al inventario estimado en los siguientes periodos
subject to estimated_inventory {w in 2..Wc, n in CLUSTERS}:
        INV[w+1,n] = INV[w,n] - (sum{k in PRECIOS} L[w,n,k])*Tc[w];

# SELECTORES

#Estructural: selecciona los precios >= al precio seleccionado
subject to price_k_or_higher {w in 1..Wc, n in CLUSTERS, k in PRECIOS}:
        X[w,n,k] <=
                if k = last(PRECIOS) then
                        1
                else
                        X[w,n,next(k)];

#Los Clusters se deben mantener ordenados por precio decreciente
subject to ordered_clusters {w in 1..Wc, n in CLUSTERS, k in PRECIOS}:
        X[w,n,k] <=
                if n = last(CLUSTERS) then
                        1
                else
                        X[w,next(n),k];

#Los Clusters de inicio de saldo deben ser agrupados seg˙n los precios del ˙ltimo periodo
subject to category_groups {c in CATEGORIES, n in CLUSTERS, w in 1..Wc, k in PRECIOS: precioT[n]
= c}:
        X[w,n,k] = XC[w,c,k];

# PRECIOS

#Los precios son iguales o decrecientes de un periodo al otro
subject to decreasing_prices {w in 1..Wc, n in CLUSTERS, k in PRECIOS: ord(w) < Wc}:
        X[w,n,k] <= X[w+1,n,k];

#Se debe escoger un precio para el primer periodo
subject to choose {n in CLUSTERS}:
        sum{k in PRECIOS} Y[1,n,k] = 1;

#Sujeto a precios restringidos por Z[k], el # de CategorÌas (en el periodo actual)
subject to restrict_price_act {n in CLUSTERS, k in PRECIOS}:
        Y[1,n,k] <= Z[1,k];

#Sujeto a precios restringidos por Z[k], el # de CategorÌas (en los demas periodos)     ***
subject to restrict_price_all {w in 2..Wc, n in CLUSTERS, k in PRECIOS}:
        Y[w,n,k] <= Z[w,k];

#El n˙mero de Categorias es igual o decreciente (en los dem·s periodos)                 ***
subject to dec_num_prices {w in 1..Wc: ord(w) < Wc}:
        sum{k in PRECIOS} Z[w+1,k] <= sum{k in PRECIOS} Z[w,k];

### PALANCAS ###

# REBAJA INICIAL
#Todos los Clusters se deben rebajar al inicio del Saldo
subject to initial_discount {n in CLUSTERS}:
        X[1,n,max {k in PRECIOS: k <= precioT[n]*(1-mindiscount)} k] = 1;

#Los descuentos deben cumplir con un % fijo con base al periodo anterior
subject to next_discount {n in CLUSTERS, k in PRECIOS: k > precioT[n]*(1-mindiscount) and k <>
precioT[n]}:
        Y[1,n,k] = 0;

#Los descuentos para clusters con precio actual menor a umbraldisc2 deben ser de al menos un %
fijo
```

```
subject to next_discount2 {n in CLUSTERS, k in PRECIOS: precioT[n] < umbraldisc2 and k >
precioT[n]*(1-mindiscount2) and k < precioT[n]}:
        Y[1,n,k] = 0;

# NUMERO DE CATEGORIAS
#Sujeto a un n˙mero m·ximo de CategorÌas (en el periodo actual)
subject to max_num_prices_act:
        sum{k in PRECIOS} Z[1,k] <= Cc;

# INVENTARIO MINIMO POR CATEGORIA
#Sujeto a un inventario mÌnimo por CategorÌa (para el periodo actual)
subject to min_inventory {k in PRECIOS}:
        sum{n in CLUSTERS} inv0[n]*Y[1,n,k] >= Qc*Z[1,k];

# INVENTARIO MINIMO PARA PRECIO DE SALIDA
#Sujeto a un inventario mÌnimo para el precio maximo seleccionado
subject to inv_salida {k in PRECIOS: k <> first(PRECIOS)}:
        sum{n in CLUSTERS} inv0[n]*(1-X[1,n,prev(k)]) >= Sc*Z[1,k];

# LIMITA EL INVENTARIO FINAL CON QUE PUEDE TERMINAR EL SALDO
#Sujeto a un inventario final maximo
subject to max_fin_inv:
sum{n in CLUSTERS}INV[Wc+1,n] <= Ic;
```

Appendix B: Optimization Model, AMPL Input Data Loading

```
#*************************************************************************************************
# MIT2 CARGA DE DATOS                                                                            *
# FECHA: 16/1/2009                                                                               *
# VERSION: 23                                                                                    *
#                                                                               Rodolfo Carboni *
#*************************************************************************************************

### DEFINICION DE PARAMETROS ###

param Wc := 5;                          #Numero de Periodos de Decision
param tighten_slack := 0.9999;          #Discount Factor para que el L se ajuste a su upper bound

### DECLARACIONES DE TABLAS ###

table PreciosComSaldo IN "ODBC" "AS400" ("SQL=SELECT * FROM MIT2.MIT2OPTPRECIOSCOMERCIALES WHERE"
& $SelectGroup & "AND ESPRECIOSALDERO = 0 ORDER BY PRECIO"):
        PRECIOS <- [PRECIO];

table PrecioSaldero IN "ODBC" "AS400" ("SQL=SELECT * FROM MIT2.MIT2OPTPRECIOSCOMERCIALES WHERE" &
$SelectGroup & "AND ESPRECIOSALDERO = 1"):
        [], psaldero ~ PRECIO;

table InvPeriodoActual IN "ODBC" "AS400" ("SQL=SELECT CATEGORIA, MAX(0, STOCK) AS STOCK,
COLORES_POR_TALLAS, PRECIOANT FROM MIT2." & $CASO & "_MIT2OPTPRUEBA WHERE" & $SelectGroupPer & "
ORDER BY CATEGORIA DESC"):
        CLUSTERS <- [CATEGORIA], inv0 ~ STOCK, ct ~ COLORES_POR_TALLAS, precioT ~ PRECIOANT;

table SalesPeriodoActual IN "ODBC" "AS400" ("SQL=SELECT CATEGORIA, PRECIO, SALES_CF, F, MAX(0,
FNWA) AS FNWA FROM MIT2." & $CASO & "_MIT2OPTF WHERE" & $SelectGroupPer):
        [CATEGORIA, PRECIO], sales0 ~ SALES_CF, f0 ~ F, fav0 ~ FNWA;

table BrkAssrtmntU IN "ODBC" "AS400" ("SQL=SELECT MAX(U) AS U FROM MIT2." & $CASO &
"_MIT2OPTPRUEBA WHERE" & $SelectGroupPer):
        [], Uc ~ U;

table BrkAssrtmnt IN "ODBC" "AS400" ("SQL=SELECT NMIN FROM MIT2." & $CASO & "_MIT2BRKASSORTPARAM
WHERE" & $SelectGroup & "AND TEMPORADA = 'S'"):
        [], Fc ~ NMIN;

### LECTURA TABLAS ###

read table PreciosComSaldo;
read table PrecioSaldero;
read table InvPeriodoActual;
read table SalesPeriodoActual;
read table BrkAssrtmntU;
read table BrkAssrtmnt;
```

Appendix C: Optimization Model, AMPL Run File

```
#******************************************************************************************
# MIT2 PLANTILLA DE EJECUCION                                                             *
# FECHA: 16/1/2009                                                                        *
# VERSION: 23                                                                             *
#                                                                         Rodolfo Carboni *
#******************************************************************************************
# 16/01/2009: Se cambio la definiciÛn de disc_saldero y el importe saldero, ahora un maximo de
CAPc unidades se venden a psaldero y el resto a (1-discsaldero)*psaldero (RCB)
#       Tambien se limpia el codigo de pruebas de sensibilidad viejas, se restablece la opcion de
utilizar MGAP, cambia el nombre de archivos de salida, y la linea de reporte execlog
#       Se aÒade el par·metro Ic y la restriccion max_fin_inv para limitar el inventario final con
que puede terminar el saldo
# 08/01/2009: Se aÒadiÛ segundo umbral de descuento minimo mindiscount2 (RCB)
# 20/8/2008: cambio time_disc a tighten_slack, habilitar para el ultimo periodo (poner Wc=2,
tiempo = 0)
# 11/8/2008: prueba para monitorear sensibilidad a errores en la prediccion
# 17/7/2008: incorpora porcentajes de caida de venta en el reporte de profits
# 17/7/2008: restriccion para forzar a escoger un precio para el periodo actual
# 09/7/2008: restriccion de L por un descuento sobre el sales0
# 07/7/2008: restriccion para un descuento minimo en los periodos > 100
# 01/7/2008: ya no se utilizan gammas, Uc viene dado por B4 unicamente
# 30/6/2008: habilitado con Fn y Fav, Salessincorreccion
# 23/6/2008: habilitado para P > 100 (mindiscount se hace 0), agrupa por categorias XC
# 20/6/2008: ahora lee tablas por periodo

reset;

### OPCIONES PLANTILLA ################################

option VERSION 22;
option CASO 'B';
option CENTROCOMPRA D;
option SECCION 1;
option ANO 2008;
option CAMPANA 2;
option PAIS 2;
option GRUPO 14;
option PERIODO 103;
option COMMENT '';
#option MGAP 0.005;

######################################################

### OPCIONES ENV ###

option rootdir 'C:\MIT2I2007\DESARROLLO_RCB_temp\';
option SelectGroup    (" CENTROCOMPRA = '" & $CENTROCOMPRA & "' AND SECCION = " & $SECCION &" AND
ANO = " & $ANO & " AND CAMPANA = " & $CAMPANA & " AND PAIS = " & $PAIS & " AND GRUPO = " & $GRUPO
& " ");
option SelectGroupPer (" CENTROCOMPRA = '" & $CENTROCOMPRA & "' AND SECCION = " & $SECCION &" AND
ANO = " & $ANO & " AND CAMPANA = " & $CAMPANA & " AND PAIS = " & $PAIS & " AND GRUPO = " & $GRUPO
& " AND PERIODO = " & $PERIODO & " ");

### OPCIONES CONF AMPL ###

option prompt1 'MIT-2> ';
option show_stats 1;

### OPCIONES CPLEX ###

option cplex_options ("bestnode ");
#option cplex_options ("bestnode mipgap=" & $MGAP);
```

```
### CARGAR MODELO Y DATA ###

model ($rootdir & 'MIT2_MODEL.mod');
data  ($rootdir & 'MIT2_DATA.dat');

### SE ESTABLECEN A 0 LOS DATOS DE LOS PRECIOS QUE YA NO SE PUEDEN USAR
let {n in CLUSTERS, k in PRECIOS: max {c in CATEGORIES} c < k} sales0[n,k] := 0;
let {n in CLUSTERS, k in PRECIOS: max {c in CATEGORIES} c < k} f0[n,k] := 0;
let {n in CLUSTERS, k in PRECIOS: max {c in CATEGORIES} c < k} fav0[n,k] := 0;


### PALANCAS ##################################################################################

#let tiempo := 0.82; #Ajuste por la caida de ventas de periodo a periodo (0 si queda 1 periodo)
let mindiscount := 0.15;       #Descuento Minimo
let mindiscount2:= 0.35;       #Descuento Minimo para Clusters con precios bajo umbraldisc
let umbraldisc2 := 10;#umbral determinado para cada moneda (a partir de tabla de eq de precios)
let CAPc := 0;  #Hasta un m·ximo de CAPc de las unidades no saldadas se venden a psaldero, el
resto a (1-discsaldero)*psaldero
let disc_saldero := 0; #Descuento sobre psaldero al que se venden las unidades sobrantes por
encima de CAPc

drop max_fin_inv; #No limitar el inventario maximo con que puede quedar al final del saldo
#let Ic := 1600;  #Cantidad maxima de inventario al final del saldo

let Cc := 6;                            #Numero maximo de Categorias

drop inv_salida;                        #No Requerir Cantidad Minima de Inventario de Salida
#let Sc := 1500;                        #Cantidad Minima de Inventario de Salida

drop min_inventory;                     #No Requerir un Inventario Minimo Por Categoria
#let Qc := 3000;                        #Cantidad Minima Inventario Por Categoria
#drop min_inventory[first(PRECIOS)];    #No requerir Cantidad Minima de Inventario para precio
menor

##############################################################################################

### CONDICIONES PERIODOS ###

if num($PERIODO) = 100 then {drop category_groups};
if num($PERIODO) = 100 then {drop next_discount};
if num($PERIODO) = 100 then {drop next_discount2};
if num($PERIODO) > 100 then {drop initial_discount};

### LOGFILES ###

option outfile1 ($rootdir & $CASO & '_Propuestas_' & $ANO & '\P' & $PERIODO & '\G' & $GRUPO &
'_P' & $PERIODO & '_P' & $PAIS & '_A' & $ANO & '_csld' & CAPc & '_ps' & disc_saldero & '_cats' &
Cc & '_w' & Wc & '_mg' & $MGAP & '_time' & tiempo & '_c' & $COMMENT & '.xls');
option execlog  ($rootdir & $CASO & '_Propuestas_' & $ANO & '\P' & $PERIODO & '\execlog.tab');
option log_file ($rootdir & $CASO & '_Propuestas_' & $ANO & '\P' & $PERIODO & '\G' & $GRUPO &
'_P' & $PERIODO & '_P' & $PAIS & '_A' & $ANO & '_csld' & CAPc & '_ps' & disc_saldero & '_cats' &
Cc & '_w' & Wc & '_mg' & $MGAP & '_time' & tiempo & '_c' & $COMMENT & '.out');

### RESOLVER ###

print;
print ctime()," Version ",$VERSION;
print "Comments: ",$COMMENT;
print "ANO ",$ANO," PAIS ",$PAIS, " GRUPO ", $GRUPO, " PERIODO ", $PERIODO;
print "SOLVING...";
print;
solve;
printf "Ampl Time: %0.2f\tSolver Time: %0.2f\n\n",_ampl_time, _solve_time;

### LOG SOLVER EXECUTION ###
```

```
printf "%s %s %s %s %s %i %i %i %0.2f %0.2f %0.2f %i %s %0.2f %0.2f %0.2f %0.2f %0.4f %0.4f %0.5e
%s %i %0.2f %0.2f %0.2f %0.2f %0.4f %s %s\n",
$CASO,$ANO,$PAIS,$GRUPO,$PERIODO,card(CLUSTERS),card(PRECIOS),Wc,mindiscount,umbraldisc2,mindisco
unt2,
Cc,$MGAP,
CURRENT_PROFIT,EST_FUT_PROFIT,PROFIT_SALDERO,Lsaldero*psaldero,
Total_Profit,Total_Profit.bestnode-Total_Profit,(Total_Profit.bestnode-
Total_Profit)/Total_Profit,
solve_result,solve_result_num,_ampl_time,_solve_time,_solve_user_time,_solve_system_time,
tighten_slack,$COMMENT,ctime() >> ($execlog);

### PRESENTAR RESULTADOS ###

print;
printf "PARAMETROS UTILIZADOS\n";
printf "=====================\n\n";
printf "Categorias Creadas: %i\t\tLimite: %i\n",card{k in PRECIOS: sum{n in CLUSTERS}Y[1,n,k] >
0},Cc;
printf "Corte mas Bajo: %0.1f\%\t\tMinimo: %0.1f\%\n",min({n in CLUSTERS} (precioT[n] - sum{k in
PRECIOS}Y[1,n,k]*k)/precioT[n])*100,mindiscount*100;
printf "Desc Minimo2: %0.1f\%\t\tUmbral: <%0.2f\n",mindiscount2*100,umbraldisc2;
printf "Inv de Salida: %i (%0.1f\%)\tMinimo: %0.1f\n",
sum{n in CLUSTERS} inv0[n]*Y[1,n,sum{k in PRECIOS} Y[1,first(CLUSTERS),k]*k],
100*(sum{n in CLUSTERS} inv0[n]*Y[1,n,sum{k in PRECIOS} Y[1,first(CLUSTERS),k]*k])/sum{n in
CLUSTERS}inv0[n],
Sc;
printf "Capacidad Saldero: %i  Precio: %0.2f  Descuento Sobre Capacidad: %0.1f\%\n",CAPc,
psaldero, disc_saldero*100;
printf "bk_assrtmnt: Uc %0.2f  Fc %0.1f\n",Uc,Fc;

printf "tighten_slack: %.8f discount: %.4f rep: %.4f\%\n",tighten_slack,EST_FUT_PROFIT -
DISC_FUT_PROFIT,(EST_FUT_PROFIT - DISC_FUT_PROFIT)/max(1,EST_FUT_PROFIT);
printf "Horizonte de Tiempo:\t%i Periodos de %i Dias (%i dias restantes) Disc:
%.2f\n\n",Wc,7,Wc*7,tiempo;

printf "\nIMP TOTAL\tP ACTUAL\tP FUTUROS\tT SALDERO\tSCAP SLD\n";
printf "=========\t==========\t=========\t=========\t========\n";
printf "%.2f\t%.2f\t%.2f\t%.2f\t%.2f\n\n",CURRENT_PROFIT + EST_FUT_PROFIT + PROFIT_SALDERO,
                                        CURRENT_PROFIT,
                                        EST_FUT_PROFIT,
                                        PROFIT_SALDERO,
                                        PROFIT_SALDERO - psaldero * Lsaldero;

printf "CLST\tINV\tACT\tPRECIO\tVENTAS\tIMPORTE  \tIMP:SK\t\% EXITO\n";
printf "====\t===\t====\t======\t======\t=======  \t=======\t=======\n";
printf {n in CLUSTERS} "%i\t%i\t%0.2f\t%0.2f\t%0.0f\t%0.2f  \t%0.2f\t%0.1f\%\n",
n,
inv0[n],
precioT[n],
sum{k in PRECIOS} Y[1,n,k]*k,
sum{k in PRECIOS} sales0[n,k]*Y[1,n,k],
sum{k in PRECIOS} sales0[n,k]*Y[1,n,k]*k,
inv0[n] / max(1,sum{k in PRECIOS} sales0[n,k]*Y[1,n,k]),
100*(sum{k in PRECIOS} sales0[n,k]*Y[1,n,k])/max(1,inv0[n]);
print;

printf "CORTE MEDIO: %0.1f\%\n",
#100*(sum{n in CLUSTERS}inv0[n]*(precioT[n] - sum{k in PRECIOS}Y[1,n,k]*k)/precioT[n])/sum{n in
CLUSTERS}inv0[n];
100*(sum{n in CLUSTERS}inv0[n]*(precioT[n] - sum{k in PRECIOS}Y[1,n,k]*k))/(sum{n in
CLUSTERS}precioT[n]*inv0[n]);
printf "EXITO 1er P: %0.1f\%\n\n",
100*(sum{n in CLUSTERS,k in PRECIOS} sales0[n,k]*Y[1,n,k])/sum{n in CLUSTERS}inv0[n];


### Excel
```

```
#Mostrar progresion de precios
table price_prgrsn OUT "ODBC" ($outfile1):
        {n in CLUSTERS} -> [CLUSTER],
        precioT[n] ~ Precio_Regular,
        sum{k in PRECIOS} Y[1,n,k]*k ~ PRECIO_1,
        (precioT[n] - sum{k in PRECIOS} Y[1,n,k]*k)/precioT[n] ~ DESC_1,
        {w in 2..Wc}    <sum{k in PRECIOS} Y[w,n,k]*k ~ ('PRECIO_' & w)>,
        psaldero ~ PRECIO_S;

#Simular la trayectoria de profits
table profits_sim OUT "ODBC" ($outfile1):
        {n in CLUSTERS} -> [CLUSTER],
        precioT[n] ~ Precio_Regular,
        INV[1,n] ~ INV_1,
        sum{k in PRECIOS} Y[1,n,k]*k ~ PRECIO_1,
        sum{k in PRECIOS} sales0[n,k]*Y[1,n,k] ~ SALES_1,
        (sum{k in PRECIOS} sales0[n,k]*Y[1,n,k]-(sum{k in PRECIOS} L[2,n,k])*Tc[2])/max(1,sum{k
in PRECIOS} sales0[n,k]*Y[1,n,k]) ~ DWN_1,
        sum{k in PRECIOS} sales0[n,k]*Y[1,n,k]*k ~ PROF_1,
        {w in 2..Wc-1}  <sum{k in PRECIOS} Y[w,n,k]*k ~ ('PRECIO_' & w),
                        (sum{k in PRECIOS} L[w,n,k])*Tc[w] ~ ('SALES_' & w),
                        ((sum{k in PRECIOS} L[w,n,k])*Tc[w]-(sum{k in PRECIOS}
L[w+1,n,k])*Tc[w+1])/max(1,(sum{k in PRECIOS} L[w,n,k])*Tc[w]) ~ ('DWN_' & w),
                        (sum{k in PRECIOS} L[w,n,k]*k)*Tc[w] ~ ('PROF_' & w)>,
        sum{k in PRECIOS} Y[Wc,n,k]*k ~ ('PRECIO_' & Wc),
        (sum{k in PRECIOS} L[Wc,n,k])*Tc[Wc] ~ ('SALES_' & Wc),
        (sum{k in PRECIOS} L[Wc,n,k]*k)*Tc[Wc] ~ ('PROF_' & Wc),
        INV[Wc+1,n] ~ INV_REM,
        psaldero ~ PRECIO_S;

#Simular la trayectoria de stock
table stock_sim OUT "ODBC" ($outfile1):
        {n in CLUSTERS} -> [CLUSTER],
        precioT[n] ~ Precio_Regular,
        INV[1,n] ~ INV_1,
        sum{k in PRECIOS} Y[1,n,k]*k ~ PRECIO_1,
        sum{k in PRECIOS} sales0[n,k]*Y[1,n,k]/max(1,INV[1,n]) ~ EXITO_1,
        sum{k in PRECIOS} sales0[n,k]*Y[1,n,k]*k/max(1,INV[1,n]) ~ PRtoSTK_1,
        {w in 2..Wc}    <INV[w,n] ~ ('INV_' & w),
                        sum{k in PRECIOS} Y[w,n,k]*k ~ ('PRECIO_' & w),
                        (sum{k in PRECIOS} L[w,n,k])*Tc[w]/max(1,INV[w,n]) ~ ('EXITO_' & w),
                        (sum{k in PRECIOS} L[w,n,k]*k)*Tc[w]/max(1,INV[w,n]) ~ ('PRtoSTK_' & w),
                        INV[w+1,n]/max(1,(sum{k in PRECIOS} L[w,n,k])*Tc[w]) ~ ('ROT_' & w)>,
        INV[Wc+1,n] ~ INV_REM,
        psaldero ~ PRECIO_S;

#Mostrar el efecto del Broken Assortment en la Demanda
table brk_assrtmnt OUT "ODBC" ($outfile1):
        {n in CLUSTERS} -> [CLUSTER],
        INV[1,n] ~ INV_1,
        ct[n] ~ COL_TALL,
        sum{k in PRECIOS} sales0[n,k]*Y[1,n,k] ~ SALES_1,
        {w in 2..Wc}    <INV[w,n] ~ ('INV_' & w),
                        sum{k in PRECIOS} sales0[n,k]*Y[w,n,k]*tiempo^(w-1) ~ ('F0_' & w),
                        sum{k in PRECIOS} f0[n,k]*Y[w,n,k]*tiempo^(w-1)*(1-Uc) ~ ('ASRT_A_' & w),
                        sum{k in PRECIOS} fav0[n,k]*Y[w,n,k]*tiempo^(w-1)*Uc*INV[w,n]/(Fc *
ct[n]) ~ ('ASRT_B_' & w),
                        sum{k in PRECIOS} L[w,n,k] ~ ('LAMBDA_' & w)>,
        INV[Wc+1,n] ~ INV_REM;

#Precios Comerciales disponibles para el Saldo
table cs_prices OUT "ODBC" ($outfile1):
        {k in PRECIOS} -> [PRECIOS_SALDO];


### ESCRIBIR RESULTADOS

write table price_prgrsn;
```

```
write table profits_sim;
write table stock_sim;
write table brk_assrtmnt;
write table cs_prices;
```