SOME THERMAL CONSEQUENCES
OF A THEORY OF CONTINENT GROWTH

by
JAMES P. DOWNS

B.S., GEORGIA INSTITUTE OF TECHNOLOGY
(1956)

SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
DEGREE OF DOCTOR OF PHILOSOPHY

at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
June, 1960

Signature of Author

Department of Geology and Geophysics
May 11, 1960

Certified by ................................................ Thèsis Supervisor

Accepted by ...................................................... Chairman, Departmental Committee
on Graduate Students
# Table of Contents

<table>
<thead>
<tr>
<th>Title Page</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table of Contents</td>
<td>2</td>
</tr>
<tr>
<td>Table of Tables</td>
<td>3</td>
</tr>
<tr>
<td>Table of Figures</td>
<td>4</td>
</tr>
<tr>
<td>Abstract</td>
<td>5</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>6</td>
</tr>
<tr>
<td>Chapter I - Introduction</td>
<td>7</td>
</tr>
<tr>
<td>General</td>
<td>7</td>
</tr>
<tr>
<td>Chapter II - Formulation of Model</td>
<td>12</td>
</tr>
<tr>
<td>Physical Model</td>
<td>12</td>
</tr>
<tr>
<td>Mathematical Model</td>
<td>15</td>
</tr>
<tr>
<td>Pressure and Temperature Dependence on the Viscosity</td>
<td>22</td>
</tr>
<tr>
<td>Numerical Approximation</td>
<td>24</td>
</tr>
<tr>
<td>Parameters Used in Thesis</td>
<td>26</td>
</tr>
<tr>
<td>Chapter III - General Features of Model and Early Differentiation of Earth</td>
<td>29</td>
</tr>
<tr>
<td>Runs to Study Early Differentiation</td>
<td>29</td>
</tr>
<tr>
<td>Possible Early History of Earth</td>
<td>36</td>
</tr>
<tr>
<td>Initial Distribution of Acid Fraction for Near Surface Differentiation</td>
<td>40</td>
</tr>
<tr>
<td>Time Scale of Initial Separation</td>
<td>41</td>
</tr>
<tr>
<td>Comparison of Model to Other Models</td>
<td>43</td>
</tr>
<tr>
<td>Chapter IV - Near Surface Differentiation</td>
<td>45</td>
</tr>
<tr>
<td>General</td>
<td>45</td>
</tr>
<tr>
<td>Computer Runs</td>
<td>47</td>
</tr>
<tr>
<td>Discussion of Results</td>
<td>49</td>
</tr>
</tbody>
</table>
Chapter V - Conclusions and Suggestions for Further Work ........................................... 51

Conclusions .............................................. 51
Suggestions for Further Work ............................ 54
Appendix A - Viscosity ................................. 56
Pressure and Temperature Dependence .................... 56
The Effect of Volatiles on the Viscosity .................... 67
Value of Viscosity Parameters Used in Calculations .......... 70
Appendix B - Energy Sources ............................ 83
Possible Sources .......................................... 83
Appendix C - Numerical Approximation ..................... 92
General .................................................... 92
Stability and Convergence Conditions ...................... 93
Accuracy ................................................ 96
Difference Equations ....................................... 96
Biographical Note .......................................... 100
Bibliography ............................................. 101

TABLE OF TABLES

3.1 Description of Runs Used in Study of General Features of Model ................................. 31
3.2 Runs to Study Effect of Changes in V0 .................. 32
3.3 Runs to Illustrate Low Density Layer Feature ......... 35
3.4 Runs to Determine Distribution of N for Near Surface Differentiation Study ................. 42
4.1 Runs to Study Near Surface Differentiation ........... 48
5.1 Depth to Top of Low Density Layer for Several Cases ................................................. 52
### A.1 Viscosity Parameters for Some Silicate Glasses

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>62</td>
</tr>
</tbody>
</table>

### A.2 Least Squares Fit of Viscosity vs 1/T by Three Curves

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>73</td>
</tr>
</tbody>
</table>

### A.3 Least Squares Fit of Viscosity vs 1/T of Earth-Like Materials

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>74</td>
</tr>
</tbody>
</table>

### A.4 Comparison of Fit of Viscosity vs 1/T for Two Natural Lavaas by Three Forms of the Viscosity as a Function of Temperature

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
</tr>
</tbody>
</table>

### A.5 Difference Between Activation Energy for Wet and Dry Melts

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>69</td>
</tr>
</tbody>
</table>

### A.6 Data from Stewart (Unpublished) Used in Study of Effect of Water on Viscosity in Silicate Melts

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>69</td>
</tr>
</tbody>
</table>

### C.1 Comparison of Two Runs to Indicate Accuracy of Numerical Solution

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>97</td>
</tr>
</tbody>
</table>

### TABLE OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Temperature vs Depth Used as Initial Temperature Distribution</td>
<td>21</td>
</tr>
<tr>
<td>3.1</td>
<td>Melting Point vs Depth for Iron and Diopside</td>
<td>38</td>
</tr>
<tr>
<td>A.1</td>
<td>Energy of Activation for Viscous Flow in Binary Liquid Silicates</td>
<td>76</td>
</tr>
<tr>
<td>A.2</td>
<td>Relation Between the Preexponential Constant and Composition for Binary Liquid Silicates</td>
<td>77</td>
</tr>
<tr>
<td>A.3</td>
<td>Viscosity vs 1/T, 1/T-273, 1/T-473 for a Silicate Glass</td>
<td>78</td>
</tr>
<tr>
<td>A.4</td>
<td>Viscosity vs 1/T of Silicate Glass Fitted by Three Function Forms of the Viscosity</td>
<td>79</td>
</tr>
<tr>
<td>A.5</td>
<td>Viscosity vs 1/T for System Diopside-Albite-Anorthite</td>
<td>80</td>
</tr>
<tr>
<td>A.6</td>
<td>Viscosity vs 1/T for System Orthoclase-Albite</td>
<td>81</td>
</tr>
<tr>
<td>A.7</td>
<td>Viscosity vs 1/T for Two Natural Lavaas</td>
<td>82</td>
</tr>
</tbody>
</table>
Title: Some Thermal Consequences of a Theory of Continent Growth

Author: James P. Downs

Submitted to the Department of Geology and Geophysics on May 11, 1960 in partial fulfillment of the requirements for the degree of Doctor of Philosophy at Massachusetts Institute of Technology.

A mathematical model is formulated which is thought to contain the essential features of the process which would be active in an earth with continuously growing continents. The model is based on partial melting leading to the formation of a "glassy" or molten "acid fraction" with a viscosity given by \( \eta = \eta_0 \exp \left( \frac{E(1 + \alpha \lambda)}{k(T - C)} \right) \). The form of the viscosity and the value of the parameters used are based on experimental information reported in the literature for rocks and artificial melts.

Once the acid fraction melts, it is suggested that flow will result due to the action of the buoyancy force resulting from the density difference between the solid and glass if the melted zones are connected over a sufficient depth. The flow regime is approximated by a steady state Poiseuille flow through tubes.

The equations of the mathematical model were solved numerically on a computer for several values of the parameters involved in the formulation of the model.

The calculations indicate that the hypothesis of slowly growing continents requires an unmelted and comparatively unbroken near surface region in the early history of the earth. Unless these conditions are met, differentiation will be completed early in the earth's history. If these conditions are met, a "low density layer" seems to be a consequence of this hypothesis. The depth to the top of this layer is between 30 and 90 kms.

In the model, the lowest energy density necessary for later differentiation to occur in a volume 10 km (down to 100 kms) is \( 7.7 \times 10^{-14} \frac{\text{cal}}{\text{cm}^3 \cdot \text{sec}} \). This is equivalent to dissipation of the amount of energy carried in seismic waves for the whole earth in this volume. If this energy is provided, it seems that differentiation in this volume would occur with a time scale between one to fifty million years after the onset of the energy release.

Thesis Supervisor: H. Hughes
Assistant Professor of Geophysics
ACKNOWLEDGMENTS

The author wishes to express his sincerest thanks to Professor Harry Hughes who suggested the problem and offered valuable suggestions in the formulation and in the writing of the thesis.

The many conversations with the staff and students of the Department of Geology and Geophysics at M.I.T., in particular with Professors S. M. Simpson, T. R. Madden, and W. F. Brace were extremely helpful at critical moments. Professor Madden, among other things, read the final draft. Professor Simpson helped in the earlier stages of the programming. Professor Brace made valuable suggestions of source material.

The assistance of the staff at the M.I.T. Computational Center was invaluable in getting the program to work. The calculations were made on the IBM 704 at the M.I.T. Computational Center at no cost to the author.

The author is indebted to M.I.T. for awarding him a tuition scholarship for his first two years of study at M.I.T. (1956 and 1957). The National Science Foundation awarded the author a fellowship for his last two years in the Graduate School at M.I.T. (1958 and 1959).

Credit for the spelling in this thesis being conventional is due to the author's wife, Cecele H. Downs, who typed, proofread and helped organize the thesis.
CHAPTER I
INTRODUCTION

General

Theories have been advanced suggesting that the earth was formed by a process of cold accretion during which the temperature of the earth has never been much, if any, in excess of the temperature that is thought to prevail in the earth today. These theories have gained considerable acceptance in spite of the many unsolved problems associated with them. One of these unsolved problems is how the continents could have segregated out of an earth that was never molten, at least not near the surface. Wilson (1954) has suggested that the continents are now growing and have grown throughout geological time by a process of adding acidic material at the continental margins. The continents are thought to be growing at the expense of the ocean basins.

Wilson states that "Each continent can be divided into structures of three types - old stable shields, mountain ranges, and continental shelves - and it is becoming apparent that there is a mountain building process which converts shelves into mountains while mountains, by a process of erosion, are in time converted into provinces of shields. There seems to be a process by which the continents expand and encroach upon the ocean basins.

"There is a suggestion that the ocean floors represent those parts of the original crust of the earth that has been least altered and that they owe their general level to
that cause. They will be considered first and then shelves, mountains, and shields which seem to represent successive stages in the growth of the continents over former ocean floors."

The above quotation contains the main sequence of events that seem to be associated with the hypothesis of the continuing growth of the continents. First sediments are laid down off the shore of the current continents. These sediments are altered by the addition of acidic material from within the mantle and by heat and pressure into gneisses and elevated into mountain ranges. These mountain ranges are then reduced by erosion to shield areas and provide the sediments for the next stage of mountain building. The increase in the mass of the continents is due to the addition of the acidic material from within the mantle. The addition of the acidic material is not continuously occurring in any particular region, such as the shelves off the coast of any continent, at all times, but rather, it occurs in a particular region at a particular time. A consequence of this type of a process would be that the mantle under the oceans should be less differentiated than under the continents.

The geological arguments concerning the above process will not be discussed here and the reader is referred to the above paper by Wilson and the references listed there.

Current seismological information cannot resolve the question of whether the material under the oceans is
less differentiated than the material under the continents for it is a question of being able to determine chemical composition at depth (i.e., being able to distinguish between say eclogite and forsterite). The use of the "cosmic abundances" of the elements in conjunction with the seismological information can limit speculations but not solve the problem (Birch 1952).

Recent measurements have revealed that the heat flow in oceanic regions is not significantly different from what is observed in continental regions. Due to the lack of a granitic layer under the oceans, the heat sources needed to give the observed heat flow are thought to be distributed to depth in the mantle. In other words, the radioactive material under the oceans seems to be less differentiated than under the continents where, except for potassium, most of the radioactive materials are thought to be concentrated in the acidic material of the crust. This lack of segregation of the radioactive materials under the ocean may be taken as an indication that the mantle under the ocean is less differentiated than the material under the continents.

In the more or less qualitative geological arguments such as the above, it is desirable to abstract the physical processes that are thought to be important and include them in a mathematical model from which numerical values of the quantities involved can be obtained and evaluated as to their plausibility. Some arguments can be shown to be
implausible for the values of the quantities required to make the model work are in disagreement with the measured values by many orders of magnitude. Other arguments are found to be consistent with the laws of physics and the measured values of the quantities involved.

The class of mathematical models which can be treated by numerical techniques using high speed computers include rather involved nonlinear systems. This permits consideration of many geological arguments which have a mathematical formulation that is beyond the reach of analysis alone. By solving the mathematical model for several values of the parameters involved, it is possible to determine the range of values if any for which the model gives a prediction that is realistic. These values can be compared with existing experimental values or they can be taken as suggestive of experiments that may be performed to check the argument.

Rubey (1951) and Wilson (1954) have argued that the volume and composition of the volatiles and lava that is escaping from the interior of the earth today is sufficient, if taken as typical of what has existed over geological time, to explain the composition and volume of the atmosphere, oceans and continents. All the previous examinations of the hypothesis of the slow growth of the continents do not show any serious conflict with the observational data available. Consequently, a further examination of the ideas contained in the hypothesis is desirable. In particular, the ideas
involved in the addition of acidic material from within the mantle to the near surface regions are susceptible to a mathematical model, and it is with this part of the above theory of continent formation that this thesis is concerned.

It is suggested that any process which is advanced to explain the upward migration of material from within the mantle must be based on a process involving the partial melting of material within a local region. The energetics of such a mechanism is discussed, and the processes that are found to be important are included in a mathematical model which is treated as described above.
phenomena (i.e., convection to the million years). They
show in silicates and contain that it is an extreme
mentual study of the intermediate layer.

In a system which is stuck to some degree,
the two mechanisms upon which a process can be based are the
differentiation of zones in a small or rapid, and hydrothermal flow
of which the cold fraction can be added near the surface.

By which the cold fraction must be some process
under the ocean. In particular, there seems to be differentiation to separate
that the cold fraction is concentrated in the crust in
The principal observation which must be explained is
show process interpreted

the base phase is not.

The base temperature for which the cold phase is motion and
temperature the material is a eutectic mixture, there will be a
denser base phase of density \( \rho_b \), occupying most of the volume.

a two phase mixture containing at least two phases (and a
any volatiles and radiogenic elements) of density \( \rho_a \) and a
mixture constituent of an eutectic phase

It will be assumed that the material can be represented by
with a trace of oxygen constituent similar to that of basalt.

The material in the mantle under the surface or

desertification of material

Invasive Model

FORMATION OF MOLTEN

CHAPTER II
conclude that "Diffusion rates would become appreciable only at temperatures well within the magmatic range, and large scale transformations (e.g., granitization) at temperatures around 400°C would require a span of time considerably in excess of reasonable geologic estimates. This conflicts with what some petrologists would consider irrefutable evidence from field observations. The only alternative seems that either granitization effectively occurs only at temperatures within the magmatic range (>700°C) or that the phenomenon is metasomatic."

Consequently, it seems as if any process which attempts to explain the formation of the crust must be based on a mechanism involving hydrodynamic flow in a region where at least partial melting has occurred. In the following, the possibility of diffusion in the liquid is neglected.

Physical Process

In regions where a process of partial melting has occurred, the acidic phase would be molten and occupy the pore spaces amongst the basic phase. Due to the density difference between the two phases and their presence in the earth's gravitational field, a buoyancy force exists which tends to drive the lighter acid phase towards the surface. This migration is possible if the pore spaces are sufficiently well connected.

The solid material in such a region would be in masses of various sizes and shapes, and would sink to the
bottom of the melted region. If the basic material had
infinite strength, these masses would settle until they
touched and supported themselves. This would force part
of the acid material towards the surface and retain part
in the enclosed pore space. However, the solid portion
has a finite strength, and if the stress differences
resulting from the material attempting to support itself
exceeds this strength, the material will fail either
elastically or plastically forcing the remaining fluid
part towards the surface if the pore spaces are still
connected. If the strength of material in the mantle is
of order $10^8$ dynes/cm$^2$ and the density difference is
$1/2$ cm/cm$^3$, the acidic phase would be squeezed toward the
top of any column with connected pores extending over 2 km.

Factors that govern the rate at which the fluid flows
towards the surface are the cross sectional area of the
pore spaces, the viscosity of the acid phase, and the
length of time it takes the basic phase to adjust to the
stresses built up in settling. It seems that the basic
solid should adjust as rapidly as the acidic material can
flow out of the voids.

A simple model is formulated which shows the relation-
ships between the important parameters and allows limits to
be set upon the values of the parameters in the earth for
reasonable time scales for the migration of the acidic
material. The justification for the simplifying assumptions
is found in the uncertainty in the values of the parameters appropriate to the earth.

**Mathematical Model**

**Velocity Equation**

The simplest one dimensional model which retains the physics of the problem is to assume that the pore spaces can be approximated by uniform tubes of length $L$ and radius 'a' orientated in a vertical direction. The coordinate system is selected so that the $Z$ axis is vertical with increasing positive $Z$ into the interior. Assuming that the viscosity $\eta$ of the material can be represented by some average value over the length $L$, the pressure gradient

$$\frac{\Delta p}{\Delta Z} = \Delta \rho g$$

where $\Delta \rho$ is the density difference between the phases and $g$ is the acceleration of gravity, the average steady state Poiseuille velocity of the acidic phase in a tube is

$$U = -\frac{1}{6\eta} \Delta \rho g a^2$$

2.1

The velocity of the acidic material is on the average always toward the surface. $U$ is negative in the coordinate system selected.

**Conservation of Mass Equation**

Let $N$ be the number of pores per unit area and $S$ be the fraction of the unit volume occupied by the acidic phase, then $S = Nwa^2$. $N$ is a function of depth in order to allow for varying concentration of the acidic phase. By
considering volume elements with linear dimensions large compared to 'a', the density of such a two phase model is

\[ \rho = \rho_b - \Delta \rho \pi a^2 N \]  \hspace{1cm} (2.2)

The change in the density of the individual phases with depth are neglected for they are smaller than the change in the gross density, \( \rho \), due to changes in the chemical composition.

The conservation of mass in the system can be written straightaway by realizing that the assumption that the density of the phases is independent of depth requires in the model that the volume of the basic phase which moves down be equal to the volume of the acidic phase moved up. With this condition, it is only necessary to consider the conservation of the acidic phase.

The net influx of mass of the acidic phase into a unit volume is

\[ -\pi a^2 \rho_a \frac{\partial (NU)}{\partial z} \]  \hspace{1cm} (2.3)

which must equal the time rate of change of the mass of the acidic phase per unit volume

\[ \pi a^2 \rho_a \frac{\partial N}{\partial t} \]  \hspace{1cm} (2.4)
Equating equations 2.3 and 2.4 gives the conservation of mass equation as

\[ \frac{\partial N}{\partial t} = \frac{\partial (NU)}{\partial z} \]  

where \( U \) is given by equation 2.1

The boundary conditions are that there is no mass transport across the surfaces \( Z = 0 \) and \( Z = H \) where \( H \) is the greatest depth to which the acidic material was distributed before the segregation process started. \( H \) is assumed to be the maximum depth at which melting occurs.

**Conservation of Energy Equation**

In Appendix B, it is argued that it is only necessary to consider the radioactive heat production and the strain energy released locally as heat in the energy balance. The conversion of potential energy into heat energy is negligible in the energy balance, but is included for this was not realized when the program was written. In the one dimensional model, the conservation of energy is expressed by

\[ \frac{3}{\partial t} \text{(heat/unit volume)} = \text{rate heat added by conduction} + \text{by convection} + \text{by conversion of potential energy} + \text{by heat generation per unit volume.} \]  

The rate heat is added by conduction is well known as

\[ R \frac{\partial^2 T}{\partial z^2} \]  

where \( R \) is the thermal conductivity.
The convective contribution is obtained in the usual manner by considering the inflow less the outflow of heat from a small volume and remembering that both the basic and solid phases carry heat. This gives

\[ \frac{\partial}{\partial t} \left( \rho T \right) = \frac{\partial}{\partial x} \left( \rho \frac{\partial T}{\partial x} \right) - \frac{\partial}{\partial y} \left( \rho \frac{\partial T}{\partial y} \right) - \frac{\partial}{\partial z} \left( \rho \frac{\partial T}{\partial z} \right) \]

where the specific heat, \( c \), of the two phases have been assumed equal.

Effectively, all of the decrease in potential energy due to the separation of the phases is converted into heat energy by viscous dissipation. The time derivative of the potential energy is

\[ \frac{\partial}{\partial \tau} \left( \frac{1}{2} \rho \left( \rho - \Delta \rho \right) \Delta \tau \right) \]

Using equation 2.5 and the assumptions that \( \rho_b, \Delta \rho \), and \( \varepsilon \) are constants, gives

\[ \frac{\partial}{\partial \tau} \left( \frac{1}{2} \rho \left( \rho - \Delta \rho \right) \Delta \tau \right) = \frac{1}{2} \rho \varepsilon \Delta \tau \]

Performing the integration, applying the boundary condition \( U(0) = 0 \) and dividing by a factor \( b \) to convert to gravitational potential energy into heat energy is

\[ \frac{1}{2} \rho \varepsilon \Delta \tau \]
The heat generation per unit volume consists of two parts. One is the contribution from radioactive decay and the other is the release of strain energy as heat. The strain energy released locally as heat is called \( \dot{Q}_E \) and is treated as a constant. The radioactivity contribution is obtained by assuming that the radioactive sources have a distribution proportional to the acidic material. If \( A \) is the rate of radioactive heat production/ce for a granite, the source term appropriate to the model under the above assumption is

\[
Q_R = A \pi a^2 N
\]

The total heat contained in a volume \( dV \) containing a two phase mixture is

\[
c T (p_b - \Delta \rho \pi a^2 N)
\]

where \( Q_0 \) is a constant. The time rate of change of this quantity is

\[
c T \frac{\partial}{\partial t} \left( p_b - \Delta \rho \pi a^2 N \right) + c \left( \rho_b - \Delta \rho N \overline{\pi a^2} \right) \frac{\partial T}{\partial t}
\]
The conservation of mass equation (equation 2.5) gives

\[-\pi a^2 c \Delta \rho T \frac{dN}{d\zeta} + \Delta \rho \frac{dT}{dt}\]  \hspace{1cm} 2.11

where as usual \( \rho = \rho_b - \Delta \rho \pi a^2 N \). Substituting equations 2.7, 2.8, 2.9, 2.10, 2.11 and \( Q_E \) in equation 2.6 gives the conservation of energy equation as

\[\rho \frac{dT}{dt} = \frac{d^2 T}{d\zeta^2} - \Delta \rho \frac{N' \pi a^2}{b} \left( \frac{1}{b} - c \frac{dL}{d\zeta} \right) + \frac{A_{\pi} a^2 N}{N} + Q_E\]  \hspace{1cm} 2.12

The boundary conditions are

\[T(0) = T_0\]
\[\frac{dT}{d\zeta} \bigg|_{\zeta = H} = 0\]

where the first condition reflects a constant surface temperature, and the second neglects any heat flow at the bottom of the partially melted region.

The selection of the initial temperature distributions used in the various models is rather arbitrary due to the uncertainty in the temperature that exists under the oceans. In the runs to study the general features of the model, the method used to select the initial temperature distribution was to assume a steady state temperature under the oceans (i.e., a parabola down to \( H \), the bottom of the region)
Figure 2.1. Plot of steady state temperature vs. depth for the two cases reported by MacDonald (1959) used as initial temperature distributions in the calculations.
containing heat sources, a constant below this depth and no heat flow between these regions). The specification of the surface temperature, $T_0$, and the temperature at the depth $H$, $T_H$, in addition to $rac{dT}{dz} = 0$ gives the initial temperature $T(z,0)$ as

$$T(z,0) = -\frac{H}{H^2} z^2 + 2\frac{H(T_H - T_0)}{H} z + T_0$$

2.13

Using the surface temperature, and the value of the temperature at the depth $H$ specifies the initial temperature. The distribution of heat sources $Q_1$ which gives this distribution is

$$Q(z) = 2R\frac{(T_H - T_0)}{H^2}$$

0 < z < H

2.14

For the cases to study near surface differentiation, the initial temperature was taken from a plot (Figure 2.1) of either model 4 or 6 in MacDonald's (1959) paper. The particular model used to give the initial temperature for a given case is given with a description of the case.

Pressure and Temperature Dependence of the Viscosity

A discussion of the pressure and temperature dependence of the viscosity of silicate melts given in Appendix A shows that the viscosity of the acid fraction which must be a polycomponent silicate can be represented by two forms
\[ \eta = \eta_0 \exp \frac{E}{kT} (1 + \lambda z) \]  \hspace{1cm} 2.15

\[ \eta = \eta_0' \exp \frac{E'}{k(T-c)} (1 + \lambda z) \]  \hspace{1cm} 2.16

These two formulas describe the viscosity with the accuracy necessary in this thesis. \( E, E' \) are activation energies for the processes, \( \eta_0 \) and \( \eta_0' \) are the viscosity at infinite temperature, \( k \) is Boltzmann's constant, and \( \lambda (=1.5 \text{ pg/k} \) where \( K \) is the bulk modulus) gives the pressure dependence on the process. The hydrostatic approximation to the pressure has been made. It was not possible to neglect volume change in the formulation of the viscosity (included in \( \lambda \)) as in the energy balance equation.

The form 2.15 is shown to be applicable only when temperatures near the solidus point are not considered, for in this region, this form predicts values of the viscosity much lower than are observed. The form 2.16 is shown to be valid over a wide range of temperature including temperatures near the solidus point. Both physical arguments and experimental evidence requires the equation 2.16 to be used for complex melts consisting of many components which melt over a wide range of temperature when temperatures near the solidus point are considered.


**Numerical Approximation**

The equations 2.1, 2.5, and 2.12 with the appropriate form of the viscosity, initial and boundary conditions constitute the mathematical model. This set of nonlinear equations cannot be solved in closed form. Consequently, they are approximated by a set of finite difference equations which were solved on the ILM 704 at the M. I. T. Computational Center. The difference equations are:

For the velocity equation

\[
U^m_k = U^m_0 \exp \left\{ -\frac{E(1+\lambda k)}{kT^m_k} \right\}
\]

or

\[
U^m_k = U'_0 \exp \left\{ -\frac{E'(1+\lambda k)}{k(T^m_k - c)} \right\}
\]

where

\[
U^m_0 = \frac{\Delta \rho g a^2}{6\eta_0} \quad \text{and} \quad U'_0 = \frac{\Delta \rho g a^2}{6\eta'_0}
\]

for the conservation of mass equation

\[
N^{m+1}_k = N^m_k - \frac{\tau}{h} \left( N^{m+1}_k U^{m+1}_k - N^m_k U^m_k \right)
\]

and for the conservation of energy equation

\[
T^{m+1}_k = T^m_k + \frac{\tau}{\rho c^m_k} \left\{ \frac{R}{h^2} \left[ \frac{T^{m+1}_k - 2T^m_k + T^{m-1}_k}{h^2} \right] + U^m_k \frac{\Delta \rho a^2}{\rho} N^m_k \left[ \frac{\tau}{h} \left( T^{m+1}_k - T^m_k \right) - \frac{g}{h} \right] + A \pi a^2 N^m_k + Q_k \right\}
\]
where \( m \) and \( \lambda \) refer to the time and space location of a point in the difference net respectively.

The boundary conditions for the energy equation are

\[
T_0^m = T_0 \quad \text{at upper surface}
\]
\[
T_L^m = T_L^m + 1 \quad \text{at lower surface}
\]

The initial temperature is

\[
\tilde{T}_l^0 = -\frac{T_r - T_0}{\mu^2} \left( \frac{h}{l} \right)^2 + 2\frac{2(T_r - T_0)}{l} h \lambda + T_0
\]

or taken from Figure 2.1.

The boundary conditions for the conservation of mass equation are

\[
\nu_0^m = 0 \quad \text{at upper surface}
\]
\[
\nu_L^m + 1 = 0 \quad \text{at lower surface}
\]

and the \( N_\lambda^0 \) must be specified.

The requirements which the parameters must satisfy for stability are

\[
\frac{\tilde{c}_l^m \rho}{\rho_{l}^m c_l^2} < l_2
\]

2.20

and

\[
\left( \tilde{c}_l^m \right)_{\text{max}} < \frac{h}{(U_\lambda^m)_{\text{max}}}
\]

2.21
The length of the acidic phase extends to a depth of 

\[ h = \frac{H}{L} \]

where \( H \) is the depth to which the acidic phase extends and \( L \) is the number of space net points desired. The time interval \( t \) is selected during the running so that equations 2.20 and 2.21 are satisfied.

The computational scheme used was a simple forward marching type. In all of the cases run, 100 time increments were used.

**Parameters Used in Thesis**

The parameters involved in the formulation of the problem are:

- \( k \) = Boltzmann constant
- \( K \) = bulk modulus
- \( g \) = acceleration of gravity
- \( a \) = radius of tubes
- \( N \) = number of tubes per unit area
- \( \Delta \rho \) = density difference between the phases
- \( \rho_b \) = density of basic phase
- \( \lambda \) = pressure dependence of the viscosity
- \( \zeta, \eta' \) = preexponential constant
- \( E', E \) = activation energy for viscous flow
- \( R \) = thermal conductivity
- \( c \) = specific heat
- \( S \) = distribution of acidic phase

\( k, K, g, \varepsilon, R, \) and \( c \) need no special discussion. The values used whenever they occur in the calculations are:
\[ k = 1.38 \times 10^{-16} \text{ ergs} \]
\[ K = 10^{12} \text{ dynes/cm}^2 \]
\[ g = 10^3 \text{ cm/sec}^2 \]
\[ R = 0.003 \text{ cal/cm/sec/°C.} \]
\[ c = 0.25 \text{ cal/gm/°C.} \]

For detailed studies, the activation energy and the preexponential constant should be empirically determined for a melt of a given chemical composition. For the order of magnitude studies made in this thesis where the chemical composition of the melt is uncertain, \( E \) and \( \gamma' \) are estimated from MacKenzie (1957) which gives the activation energy and the preexponential constant appropriate to equation 2.15 as a function of chemical composition for dry melts. These values are then adjusted for the estimated effect of the presence of volatiles. This is done in Appendix A.

The values of the activation energy used in the calculations are \( E = 20, 35 \) and \( 50 \) kcal/mol, the value of the preexponential constant is \( \gamma = 10^{-4} \) poises. The values \( E = 35 \) kcal and \( \gamma' = 10^{-4} \) poises are thought to be appropriate to a "wet" plutonic granite, and the other two values are intended to allow for errors in the estimation of \( E \).

Less information is available to aid in the selection of \( E', \gamma' \) and \( C \). Viscosity as a function of temperature for some natural rocks (Birch et al. 1942) and glasses (Shartsis and Spinner 1951) was plotted and curves
were fitted in order to determine typical values of $E'$, $\eta_0$ and $C$. The values of $T_o$ determined by Dienes (1953) was considered in determining $C$. The values used in the thesis are $E' = 10, 20 \text{ kcal/mol, } \eta_0' = 10^{-2} \text{ poises}$ and $C = 700^\circ \text{ K}$.

The form of $\lambda$ is based on Frenkel (1946) and the value ($\lambda = 5 \times 10^{-9} \text{ cm}^{-1}$) is based on Hughes' (1955) measurement on the pressure dependence of the activation energy for cations in the ionic semi-conduction mechanism in peridot.

**Value for $\Delta \rho$**

An estimation of $\Delta \rho$ is obtained by using jadeite ($\rho_a = 3.3 \text{ g/cm}^3$) as representative of the acidic phase. Hypersthene ($\rho_b = 3.4 \text{ g/cm}^3$) and andradite ($\rho_b = 3.5 \text{ g/cm}^3$) are used as typical of the basic phase. For these two examples of the basic phase, $\Delta \rho = .1$ and $.2 \text{ g/cm}^3$. The presence of volatiles with a density of $2 \text{ g/cm}^3$ would lower the density of the acidic phase to say $3 \text{ g/cm}^3$. This gives $\Delta \rho = .4$ and $.5 \text{ g/cm}^3$. The values $\Delta \rho = .5 \text{ g/cm}^3$ and $\rho_b = 3.5 \text{ g/cm}^3$ are used in the calculations.

$a, N$ and $S$

To determine the initial values of $a$, $N$ and $S$ for the study of the near surface differentiation, it is necessary to obtain a better understanding of the general features of the model. This is the subject of Chapter III.
CHAPTER III
GENERAL FEATURES OF MODEL
AND
EARLY DIFFERENTIATION OF EARTH

**Runs to Study Early Differentiation**

**Description of Runs**

Before an estimation of the distribution and value of $a$, $N$ and $S$ for use in the study of near surface differentiation can be made, it is necessary to study the solution of several cases run on the computer using arbitrary values of the parameters in order to make clear certain general features of the model and the nature of some approximations. These cases give some insight into what the concept of the slow growth of the continents implies about the early history of the earth and will allow the initial distribution of the acid fraction to be determined.

For the runs, the form of the viscosity used is $\eta = \eta_0 \exp \frac{E}{kT}$. The form was used because the values of $\eta_0$ and $E$ are better known than $\eta'_0$ and $E'$, and only general features of the model are wanted.

$S$ was taken as a constant ($S = 1$) down to $Z = H = 300$ km. This is to approximate what would happen in an earth that had an initial uniform composition. The value 1 is arbitrary. It was selected by assuming the continents to be 30 km thick at the end of differentiation.
If the 30 km of material was initially distributed throughout 300 km, \( S \) must be .1. Due to the uncertainty in the gross chemical composition of the initial earth, no more meaningful selection of \( S \) can be made. The computations show that the value of \( S \) used in the model is only important in the convective transport of heat and the conversion of potential energy terms. The effect in these terms do not change the general features of the model.

The heat production per unit volume was selected by equation 2.14 to keep the temperature constant in time. This is done to make the effect of the temperature distribution clear. It neglects the change in the distribution of radioactive sources due to changes in the distribution of the acidic phase. This is treated later. The cases are described in Table 3.1.

Effect of Changes in \( U_0 \)

Cases 1, 2, and 3 examine the effect of changes in \( U_0 \) on the results. The only difference between the three cases are the radius of the tubes and the initial value of the number of tubes per unit area which must be changed to accommodate the change in the radius. The extreme values of the radius used differ by a factor of \( 10^6 \). The relative number of tubes per unit area, the temperature, and the elapsed model time after a hundred time steps are given in Table 3.2. The ratio \( \frac{\text{final}}{\text{initial}} \) is the
### TABLE 3.1

**DESCRIPTION OF RUNS USED IN STUDY OF GENERAL FEATURES OF MODEL**

<table>
<thead>
<tr>
<th>Run</th>
<th>$T_o \circ K$</th>
<th>$T_H \circ K$</th>
<th>$a (\text{cm})$</th>
<th>$N(Z, O)$ Tubes/cm²</th>
<th>$\eta_o$ Poises</th>
<th>$E$ kcal/mol</th>
<th>$G_{1,1} \text{cal/cm}^3 \cdot \text{sec}$</th>
<th>$H (\text{km})$</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>300</td>
<td>2400</td>
<td>$10^{-6}$</td>
<td>$3.18 \times 10^{10}$</td>
<td>$10^{-4}$</td>
<td>35</td>
<td>$1.67 \times 10^{-14}$</td>
<td>300</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>300</td>
<td>2400</td>
<td>$10^{-4}$</td>
<td>$3.18 \times 10^{6}$</td>
<td>$10^{-4}$</td>
<td>35</td>
<td>$1.67 \times 10^{-14}$</td>
<td>300</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
<td>2400</td>
<td>$10^{0}$</td>
<td>$3.18 \times 10^{-2}$</td>
<td>$10^{-4}$</td>
<td>35</td>
<td>$1.67 \times 10^{-14}$</td>
<td>300</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>300</td>
<td>2400</td>
<td>$10^{-4}$</td>
<td>$3.18 \times 10^{6}$</td>
<td>$10^{-4}$</td>
<td>20</td>
<td>$1.67 \times 10^{-14}$</td>
<td>300</td>
<td>10</td>
</tr>
<tr>
<td>6</td>
<td>300</td>
<td>2400</td>
<td>$10^{-4}$</td>
<td>$3.18 \times 10^{6}$</td>
<td>$10^{-4}$</td>
<td>50</td>
<td>$1.67 \times 10^{-14}$</td>
<td>300</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>300</td>
<td>2800</td>
<td>$10^{-4}$</td>
<td>$3.18 \times 10^{6}$</td>
<td>$10^{-4}$</td>
<td>20</td>
<td>$1.67 \times 10^{-14}$</td>
<td>300</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>300</td>
<td>2800</td>
<td>$10^{-4}$</td>
<td>$3.18 \times 10^{6}$</td>
<td>$10^{-4}$</td>
<td>35</td>
<td>$1.67 \times 10^{-14}$</td>
<td>300</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>300</td>
<td>2800</td>
<td>$10^{-4}$</td>
<td>$3.18 \times 10^{6}$</td>
<td>$10^{-4}$</td>
<td>50</td>
<td>$1.67 \times 10^{-14}$</td>
<td>300</td>
<td>10</td>
</tr>
</tbody>
</table>

**NOTE:** For these cases $T_o = 100$ million years
TABLE 3.2

RUNS TO STUDY EFFECT OF CHANGES IN \( U_0 \). DEPTH, \( Z \), IN KM, TEMPERATURE, \( T \), IN \( ^\circ \)K

AND \( n = N(Z, t)/N(0, 0) \). ELAPSED MODEL TIME \( t_m \).

ACTIVATION ENERGY, \( \Delta E = 35 \) KCAL/MOL FOR ALL THREE CASES.

<table>
<thead>
<tr>
<th>Initial Data</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a = 10^{-6} ) cm ( t_m = 10^8 ) yr ( N(0, 0) = 3.18 \times 10^{10} ) tubes/cm(^2)</td>
<td>( a = 10^{-4} ) cm ( t_m = 5.18 \times 10^6 ) yr ( N(0, 0) = 3.18 \times 10^5 ) tubes/cm(^2)</td>
<td>( a = 1 ) cm ( t_m = 3.18 \times 10^{-2} ) yr ( N(0, 0) = 0.0318 ) tubes/cm(^2)</td>
<td></td>
</tr>
<tr>
<td>( Z )</td>
<td>( T )</td>
<td>( n )</td>
<td>( T )</td>
</tr>
<tr>
<td>0</td>
<td>300</td>
<td>1.000</td>
<td>300</td>
</tr>
<tr>
<td>30</td>
<td>700</td>
<td>1.000</td>
<td>699</td>
</tr>
<tr>
<td>60</td>
<td>1056</td>
<td>1.000</td>
<td>1056</td>
</tr>
<tr>
<td>90</td>
<td>1370</td>
<td>1.000</td>
<td>1371</td>
</tr>
<tr>
<td>120</td>
<td>1614</td>
<td>1.000</td>
<td>1613</td>
</tr>
<tr>
<td>150</td>
<td>1873</td>
<td>1.000</td>
<td>1873</td>
</tr>
<tr>
<td>180</td>
<td>2060</td>
<td>1.000</td>
<td>2060</td>
</tr>
<tr>
<td>210</td>
<td>2210</td>
<td>1.000</td>
<td>2206</td>
</tr>
<tr>
<td>240</td>
<td>2315</td>
<td>1.000</td>
<td>2309</td>
</tr>
<tr>
<td>270</td>
<td>2380</td>
<td>1.000</td>
<td>2371</td>
</tr>
<tr>
<td>300</td>
<td>2400</td>
<td>1.000</td>
<td>2392</td>
</tr>
</tbody>
</table>
relative number of tubes. There is no difference between case 2 and 3 in the relative number of tubes or temperature at the end of the same number of iterations. However, there is a significant difference in the elapsed model time.

There is a straightforward explanation of the difference in model time. In all cases, except where the velocity is so slow that the time interval is given by equation 2.20 as in case 1, the change in the time scale is inversely proportional to the change in the velocity. This is expected from equation 2.21. That the numerical solutions keep this relationship is another indication that the program is free of any serious errors. A change in \( U_0 \) causes a change in the time scale for the process.

**Effect of Initial Temperature and Activation Energy**

Cases 2, 4, 6, 7, 8, and 9 are used to show the effect of changes in the initial temperature distribution and the activation energy on the final distribution of the acid fraction. The final distribution is said to occur when there has been no significant change in \( N \) for a 100 million years period. To cover a sufficient amount of time, it was necessary to make successive computer runs for the same case. The reason is that there are only a hundred time steps programmed for each run. The length of a step can be anywhere from a fraction of a second to a million years depending on the velocity with a corresponding spread in the elapsed model time.
The initial conditions for a later run are the final conditions from the previous run. In order to lengthen the time interval in the later runs, the extent of the region, $H$, is redefined at the start of each run by dropping all points where the fraction of the acidic material is $0.01$ of its initial value. This removes the points where the temperature is highest thus lowering $(U_k^m)_{\text{max}}$ giving a longer time interval in succeeding runs. The same boundary conditions are applied to the new region.

$U_0$ was taken to be the same in all six models to eliminate the effects of changes in this quantity. Table 3.3 gives the temperature, number of tubes per unit area, and the elapsed model time for each run.

**Low Density Layer**

A feature of the model revealed by the six cases described in Table 3.3 is that there is a marked increase in the concentration of the acid fraction in a layer at depth $Z_1$ forming a low density layer. The depth $Z_1$ is seen to depend on the temperature distribution, and the value of the activation energy used. Since the time scale depends on the selection of $U_0$, this depth also depends on the value of $U_0$ used. In fact, there are selections of $U_0$ which allow the differentiation to be almost instantaneous all the way to the surface, and others which make the separation negligible at all depths. If the $1/(T - C)$ form of the viscosity had been used in the study, a selection of $U_0$
TABLE 3.3

RUNS TO ILLUSTRATE LOW DENSITY LAYER FEATURE AND TIME SCALE OF MODELS. IN THE TABLE, DEPTH, Z, IN KM AND TEMPERATURE IN °K. MODEL TIME IN PARENTHESES AFTER RUN NUMBER.

1 m'y. = 10^6 yr.

<table>
<thead>
<tr>
<th>INITIAL DATA</th>
<th>CASE 2</th>
<th>CASE 4</th>
<th>CASE 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>E=35 kcal/mol, a=10^4 cm</td>
<td>E=20 kcal/mol, a=10^4 cm</td>
<td>E=50 kcal/mol, a=10^4 cm</td>
<td></td>
</tr>
<tr>
<td>N=3.67x10^6 tubes/cm^2</td>
<td>N=3.67x10^6 tubes/cm^2</td>
<td>N=3.67x10^6 tubes/cm^2</td>
<td></td>
</tr>
</tbody>
</table>

**conditions at end of run**

<table>
<thead>
<tr>
<th>Z</th>
<th>T</th>
<th>N(0)</th>
<th>T</th>
<th>N(0)</th>
<th>T</th>
<th>N(0)</th>
<th>T</th>
<th>N(0)</th>
<th>T</th>
<th>N(0)</th>
<th>T</th>
<th>N(0)</th>
<th>T</th>
<th>N(0)</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>300</td>
<td>1.00</td>
<td>300</td>
<td>1.00</td>
<td>300</td>
<td>1.00</td>
<td>300</td>
<td>1.00</td>
<td>300</td>
<td>1.00</td>
<td>300</td>
<td>1.00</td>
<td>300</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>699</td>
<td>1.04</td>
<td>699</td>
<td>2.91</td>
<td>691</td>
<td>3.09</td>
<td>561</td>
<td>4.49</td>
<td>700</td>
<td>1.94</td>
<td>700</td>
<td>1.94</td>
<td>700</td>
<td>1.94</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>1056</td>
<td>4.04</td>
<td>1056</td>
<td>7.09</td>
<td>1021</td>
<td>6.91</td>
<td>667</td>
<td>5.45</td>
<td>1056</td>
<td>1.94</td>
<td>1056</td>
<td>1.94</td>
<td>1056</td>
<td>1.94</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>1370</td>
<td>4.87</td>
<td>1370</td>
<td>7.09</td>
<td>1021</td>
<td>6.91</td>
<td>667</td>
<td>5.45</td>
<td>1056</td>
<td>1.94</td>
<td>1056</td>
<td>1.94</td>
<td>1056</td>
<td>1.94</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>1644</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>1875</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td></td>
</tr>
<tr>
<td>180</td>
<td>2065</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td></td>
</tr>
<tr>
<td>210</td>
<td>2210</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>2315</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td></td>
</tr>
<tr>
<td>270</td>
<td>2380</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>2400</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td>1643</td>
<td>.58</td>
<td></td>
</tr>
</tbody>
</table>

**CASE 7**

E=20 kcal/mol, a=10^4 cm
N=3.67x10^6 tubes/cm^2

**CASE 8**

E=35 kcal/mol, a=10^4 cm
N=3.67x10^6 tubes/cm^2

**CASE 9**

E=50 kcal/mol, a=10^4 cm
N=3.67x10^6 tubes/cm^2

**conditions at end of run**

<table>
<thead>
<tr>
<th>Z</th>
<th>T</th>
<th>N(t)</th>
<th>T</th>
<th>N(t)</th>
<th>T</th>
<th>N(t)</th>
<th>T</th>
<th>N(t)</th>
<th>T</th>
<th>N(t)</th>
<th>T</th>
<th>N(t)</th>
<th>T</th>
<th>N(t)</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>300</td>
<td>1.04</td>
<td>300</td>
<td>1.04</td>
<td>300</td>
<td>1.04</td>
<td>300</td>
<td>1.04</td>
<td>300</td>
<td>1.04</td>
<td>300</td>
<td>1.04</td>
<td>300</td>
<td>1.04</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>775</td>
<td>1.12</td>
<td>775</td>
<td>2.68</td>
<td>772</td>
<td>2.68</td>
<td>772</td>
<td>2.68</td>
<td>772</td>
<td>2.68</td>
<td>772</td>
<td>2.68</td>
<td>772</td>
<td>2.68</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>1201</td>
<td>5.66</td>
<td>1200</td>
<td>2.84</td>
<td>1044</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>1675</td>
<td>7.15</td>
<td>1675</td>
<td>2.84</td>
<td>1044</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>2000</td>
<td>7.15</td>
<td>2000</td>
<td>2.84</td>
<td>1044</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>2175</td>
<td>7.15</td>
<td>2175</td>
<td>2.84</td>
<td>1044</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td></td>
</tr>
<tr>
<td>180</td>
<td>2400</td>
<td>7.15</td>
<td>2400</td>
<td>2.84</td>
<td>1044</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td></td>
</tr>
<tr>
<td>210</td>
<td>2575</td>
<td>7.15</td>
<td>2575</td>
<td>2.84</td>
<td>1044</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>2700</td>
<td>7.15</td>
<td>2700</td>
<td>2.84</td>
<td>1044</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td></td>
</tr>
<tr>
<td>270</td>
<td>2800</td>
<td>7.15</td>
<td>2800</td>
<td>2.84</td>
<td>1044</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td>737</td>
<td>2.29</td>
<td></td>
</tr>
</tbody>
</table>

*NOTE* Initial temp. used for these runs. \( Q_1 \) was not increased when \( H \) was shorted.
could be made that would allow the differentiation to be instantaneous to the depth where the temperature equaled \( C \). This is the shallowest depth in this model to which differentiation can occur.

The uncertainty in the value of \( C \) appropriate to the acid fraction coupled with the uncertainty in the temperature under the oceans rule out an accurate estimation of the depth at which the motion ceases. If \( C \) is in the range 400–800°C, and MacDonald's estimation of the temperatures under the oceans, plotted as figure 2.1 is assumed as being representative, the minimum depth to the top of the low density layer would be between 30–90 km.

These calculations show that if partial melting occurs in a region, the mobile material must move toward the top of the melted region in a comparatively short time. The shape of the masses in which this fraction would collect should have a larger horizontal dimension than vertical. As argued in Chapter II, the vertical dimension of these masses can be no longer than the maximum height that the basic material can support.

Possible Early History of Earth

Now that the formation of the low velocity layer in the model has been described and some understanding gained as to the parameters which govern its depth, it is possible to further develop the physical description of the model. The next few paragraphs are a description
of a process which might have occurred in the earlier history of the earth. The discussion is given in order to fix ideas for a later section and to suggest possible extensions of this work.

No one is more aware of the incompleteness of the following discussion than this author. For a complete discussion, considerably more experimental data on the flow properties of rocks particularly near their melting points is necessary. It is also necessary to extend the work of this thesis to cover a three phase system including the terms involving pressure which were omitted in the model formulated for the upper part of the earth.

Assume that the earth was initially of uniform chemical composition including radioactive elements and that the material consisted of three fractions: iron, basic, and acid. Furthermore, assume that the temperature at the time of formation was everywhere below the melting point of the material.

Due to the burial of the radioactive material, there would be a general warming up of the earth. This process would continue until one of the fractions melted. Studies (e.g. MacDonald 1959, p. 1979) of the melting point gradient of basic materials and iron have been made, and the melting point of iron is $200^\circ-2000^\circ$ C. less than that of diopside for depths greater than 300 km (see figure 3.1). No studies
Figure 3.1 Adapted from MacDonald (1959) to give the extrapolated melting relations for iron and diopside within the earth. Iron I curve is Strong's (1959) estimate. Iron II curve is extrapolated by using the initial slope obtained by Strong and Gruneisen's constant as appropriate at high pressures.
of the melting point of acidic silicates with depth have been made but a reasonable assumption is that the melting point of the acidic fraction at depths lies between that of the iron and basic phases.

Due to its lower melting point, the iron fraction would be the first fraction to melt in the warming earth. Once the iron fraction melted, differentiation would begin with the dense iron phase settling towards the center of the earth. In settling, gravitational potential energy would be released and be converted into heat energy through viscous dissipation. The temperature rise accompanying this release of heat would help in the melting of the acidic fraction.

Once melted, the acid fraction containing a large fraction of the radioactive elements and volatiles would be forced towards the surface. The upward migration of the acid fraction would continue until an unmelted zone (i.e., low temperature zone) was invaded. The upward motion of the acid fraction of this material would stop in this zone forming a low density layer at the bottom of the unmelted region. The acid fraction would cool and solidify unless the unmelted region was fractured so that the acid fraction could move adiabatically upwards through the channels opened by fracturing or else the near surface material was melted by some process.

Geophysical evidence indicates that there are no large scale (i.e., of oceanic dimensions) fluid chambers
in the upper part of the mantle. The assumption that differentiation was not completed in the early stages of the earth's history implies that the near surface region was to a very large extent unbroken and unmelted and that the acid fraction has solidified.

The existence of a low velocity layer is implicit in theories of slow accretion of the continents. It provides a physical feature that may be found by seismic or gravity methods. This investigation would determine if the ideas of incomplete differentiation are plausible. The technique to be used in such a study is straightforward and need not be elaborated on here.

**Initial Distribution of Acid Fraction for Near Surface Differentiation**

To determine the initial distribution of the acid fraction for use in the following study of near surface differentiation, it was necessary to run several cases on the computer using the form of the viscosity given in equation 2.16. The initial temperature distribution was taken from model 6 in figure 2.1. The requirement of no significant change in N in 100 million years was used as a criterion of when a stable configuration was reached. The initial value of N makes use of the observation that differentiation in the model below 100 km is so swift that the distribution after the initial run can be approximated by a function of the form
\[ S_0 \quad 0 \leq z \leq 60 \text{ km} \]
\[ N = S_1 \quad 60 \text{ km} \leq z \leq 90 \text{ km} \]
\[ 0 \quad z > 90 \text{ km} \]

The strain energy contribution was taken as zero. The runs are described and the results presented in Table 3.4. The concentrations for the last run in the cases listed in Table 3.4 are used as initial distributions for the study of near surface differentiation in Chapter IV.

Larger values of the radius were used but the time scale for the separation was so short that it was not meaningful. The time scale for \( a = 1 \text{ cm} \) can be used as a lower limit for larger values of the radius.

**Time Scale of Initial Separation**

The time scale of such a process could be estimated from the equations given if the value of the radius of the tubes appropriate to the melted region could be estimated. However, this is not possible from available information. The time scales given in Table 3.3 for \( a = 10^{-4} \text{ cm} \) give an upper limit to the possible time scale. The change in the time scale caused by assuming a larger radius can be estimated as argued previously. The previous runs indicate that if the effective radius of the pore spaces in the melted regions are centimeters or larger, the initial differentiation would be very swift, requiring 100,000 years or less once the partial melting of the acidic fraction occurred.
TABLE 3.4
RUNS TO DETERMINE DISTRIBUTION OF N FOR NEAR SURFACE DIFFERENTIATION STUDY. DEPTH, Z, IN KM, TEMPERATURE, T, IN °K. n = N(Z,t)/N(0,0), Q_e = 0 FOR ALL CASES. A = 5.3 x 10^{13} cal/cm^2·sec. ELAPSED MODEL TIME GIVEN IN PARENTHESES AFTER RUN NUMBER. 1my = 10^6 yr

<table>
<thead>
<tr>
<th>INITIAL DATA</th>
<th>CASE 10</th>
<th>CASE 12</th>
<th>CASE 14</th>
</tr>
</thead>
<tbody>
<tr>
<td>E' = 10 kcal/mol, a = 10^{-4} cm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N(0,0) = 3.18 x 10^6 tubes/cm^2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>conditions at end of run</td>
<td>1 (16 my)</td>
<td>2 (100 my)</td>
<td>1 (100 my)</td>
</tr>
<tr>
<td>Z</td>
<td>T</td>
<td>n</td>
<td>T</td>
</tr>
<tr>
<td>----</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>0</td>
<td>300</td>
<td>1.00</td>
<td>300</td>
</tr>
<tr>
<td>15</td>
<td>523</td>
<td>1.00</td>
<td>517</td>
</tr>
<tr>
<td>30</td>
<td>693</td>
<td>1.00</td>
<td>742</td>
</tr>
<tr>
<td>45</td>
<td>980</td>
<td>1.39</td>
<td>1000</td>
</tr>
<tr>
<td>60</td>
<td>1238</td>
<td>5.00</td>
<td>1237</td>
</tr>
<tr>
<td>75</td>
<td>1493</td>
<td>5.00</td>
<td>1237</td>
</tr>
<tr>
<td>90</td>
<td>1693</td>
<td>5.00</td>
<td>1237</td>
</tr>
<tr>
<td>CASE 17</td>
<td>CASE 25</td>
<td>CASE 28</td>
<td></td>
</tr>
<tr>
<td>E' = 20 kcal/mol, a = 1 cm</td>
<td>E' = 10 kcal/mol, a = 10^{-2} cm</td>
<td>E' = 20 kcal/mol, a = 10^{-2} cm</td>
<td></td>
</tr>
<tr>
<td>N(0,0) = .0318 tubes/cm^2</td>
<td>N(0,0) = 3.18 x 10^2 tubes/cm^2</td>
<td>N(0,0) = 3.18 x 10^2 tubes/cm^2</td>
<td></td>
</tr>
<tr>
<td>conditions at end of run</td>
<td>1 (20 yr)</td>
<td>2 (.17 my)</td>
<td>3 (100 my)</td>
</tr>
<tr>
<td>Z</td>
<td>T</td>
<td>n</td>
<td>T</td>
</tr>
<tr>
<td>----</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>0</td>
<td>300</td>
<td>1.00</td>
<td>300</td>
</tr>
<tr>
<td>15</td>
<td>523</td>
<td>1.00</td>
<td>523</td>
</tr>
<tr>
<td>30</td>
<td>693</td>
<td>1.00</td>
<td>694</td>
</tr>
<tr>
<td>45</td>
<td>980</td>
<td>1.26</td>
<td>980</td>
</tr>
<tr>
<td>60</td>
<td>1238</td>
<td>5.00</td>
<td>1513</td>
</tr>
<tr>
<td>7.5</td>
<td>1493</td>
<td>5.00</td>
<td>1238</td>
</tr>
<tr>
<td>9.0</td>
<td>1693</td>
<td>5.00</td>
<td>1238</td>
</tr>
</tbody>
</table>
At the end of this stage of differentiation, the earth would have the iron core, a basic mantle and a mixed near surface region containing excess acid material collected in masses buried at some depth. Most of the radioactivity would be in the low density layer so that the heat production in the deep interior from this source would be negligible.

Comparison of Model to Other Models

This model is based on the current ideas of partial melting as contrasted to the earlier earth models which were based on total melting of the earth.

The essential differences between models based on partial melting and those involving total melting are that partial melting permits the retention of a solid outer shell at all times. It is only necessary to supply energy to melt a small fraction of the earth in order that differentiation occur. The completely fluid earth requires differentiation to be more or less completed early in its history. This model requires the existence of an intermediate undifferentiated near surface region.

The feature of a solid outer shell allows the consideration of the formation of the earth by a process in which the temperature of the outer shell is the equilibrium temperature resulting from a balance between the absorption and radiation of the sun's energy at the surface. Requiring only a small fraction of the earth to melt, reduces the
energy needed to cause differentiation in an earth formed by a cold process. Both of these features are helpful in explaining differentiation in an earth formed by the current cold accretion hypotheses.

In the models where the earth is assumed to be fluid at some stage in its history in order that differentiation can occur, it is still necessary to appeal to "fluid filaments" in the final stages of the separation process. The following quotation from Jeffreys (1959) illustrates this. "... the fluid filaments left at any stage carried up the latent heat and the new heat by convection, and that as solidification proceeded the residual fluid was forced up by deformation of the crystals, so that solidification and the raising of the radioactive material to the upper layers were part of the same process." This mechanism suggested by Jeffreys is the mechanism used in this thesis to explain the differentiation of a partially melted earth. Consequently, the mathematics in this thesis can be used to describe the last stages in the differentiation of an initially molten earth.
CHAPTER IV
NEAR SURFACE DIFFERENTIATION

General

From the discussion in Chapter III, it seems that the hypothesis of an incomplete differentiation of the mantle requires that the acid fraction be more or less concentrated in masses at depths somewhere between 30 and 90 km. The question is how can this material be brought nearer to the surface so that it can be assimilated into the continents. It was suggested in Chapter I that any process which attempts to bring this material up from these intermediate depths must be based on a process of partial melting, and thus, another question is how can this melting occur. Melting is used to mean that the acid fraction has been converted from the crystalline state into the glassy state. Once glassy, equation 2.16 describes the viscosity of the acid fraction.

The partial melting of material for this near surface differentiation is visualized as the result of the concentration of strain energy released as heat in a rather localized region. Some mechanism similar to that described by Crowan (1960) and discussed in Appendix B of this thesis or to Treitel's (1958) suggestion that shock waves might be very important in the region of failure may provide a mechanism for this concentration of the release of strain energy.
It is not certain that material in the mantle behaves as Orowan suggests, but after studying his arguments, it seems possible that some similar mechanism is active in the earth.

If the acid material under the oceans is concentrated in masses at some depth, it should be closer to its melting point and contain a larger percentage of volatile material than basic material at the same depths. It seems plausible that the masses of acid material would be the weakest regions in the mantle. The release of strain energy should occur in these masses when sufficient stress differences have been set up. This leads to their remelting.

Using 300 cal/cm$^3$ as a typical value of the latent heat, it would require $4 \times 10^5$ or $4 \times 10^6$ years to melt the acid fraction in $10^7$ km$^3$ ($100 \times 1000 \times 100$) for $Q_E = 7.7 \times 10^{-13}$ or $7.7 \times 10^{-14}$ cal/cm$^3$·sec respectively. The acid fraction occupies a third or less of this volume. This sets a lower limit on the time required for differentiation after the onset of intense seismic activity in a region. This time must be added to the model times obtained in the runs reported in this chapter, for no provision for latent heat was included in the model. The model assumes that the material is in a glassy state at the temperatures existing and that flow will occur for $T > 0$. 
If the shear zones where some melting has occurred extend from these masses up toward the surface, the buoyance force would cause the upward motion to restart. The shear zones are not visualized as voids, but rather as regions that have become viscous. These fluid channels permit upward motion to occur. It is a process of this type that the equations in Chapter 1 are intended to approximate. The radius of the tubes in the mathematical model may be associated with an effective radius of the channels in the shear zones.

Computer Runs

Description

The cases run to determine the time scale of differentiation after remelting as a function of the viscosity parameters, radius of the channels, and the rate strain energy is fed into the system are extensions of the cases reported in Table 3.4. The description of the cases and the results are reported in Table 4.1. The case in Table 3.4 that the initial conditions were taken from is indicated by the number in parentheses beside the case heading.

Summary of Runs

Cases 18, 26, and 29 indicate that \( q_E = 7.7 \times 10^{-14} \) cal/cm\(^3\)-sec is the lowest value of this term for which near surface differentiation would occur in this model. This is the value of \( q_E \) which is given by \( n = 1 \) in equation B.1 and is equivalent to dissipating an amount of energy in
<table>
<thead>
<tr>
<th>TABLE 4.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>RESULTS OF RUNS TO STUDY SURFACE DIFFERENTIATION. ELAPSED MODEL TIME GIVEN IN PARENTHESES AFTER RUN NUMBER. E(my) = 10 YEARS. INITIAL CONDITIONS TAKEN FROM CASE IN TABLE 3.4 WITH THE NUMBER GIVEN IN PARENTHESES BY CASE HEADING. T IN °K</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CASE 15</th>
<th>CASE 16</th>
<th>CASE 18</th>
<th>CASE 19</th>
</tr>
</thead>
<tbody>
<tr>
<td>INITIAL DATA</td>
<td>INITIAL DATA</td>
<td>INITIAL DATA</td>
<td>INITIAL DATA</td>
</tr>
<tr>
<td>E=10 kcal/mol a=1 cm</td>
<td>E=10 kcal/mol a=1 cm</td>
<td>E=20 kcal/mol a=1 cm</td>
<td>E=20 kcal/mol a=1 cm</td>
</tr>
<tr>
<td>Q=7.7 x 10^3 cal/cm^2/sec</td>
<td>Q=7.7 x 10^3 cal/cm^2/sec</td>
<td>Q=7.7 x 10^3 cal/cm^2/sec</td>
<td>Q=7.7 x 10^3 cal/cm^2/sec</td>
</tr>
<tr>
<td>conditions at end of run</td>
<td>conditions at end of run</td>
<td>conditions at end of run</td>
<td>conditions at end of run</td>
</tr>
<tr>
<td>Z</td>
<td>n</td>
<td>T</td>
<td>n</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>0</td>
<td>1.00</td>
<td>300</td>
<td>1.00</td>
</tr>
<tr>
<td>15</td>
<td>1.00</td>
<td>523</td>
<td>1.00</td>
</tr>
<tr>
<td>30</td>
<td>28.5</td>
<td>693</td>
<td>28.5</td>
</tr>
<tr>
<td>45</td>
<td>977</td>
<td>1.00</td>
<td>947</td>
</tr>
<tr>
<td>CASE 26</td>
<td>CASE 27</td>
<td>CASE 29</td>
<td>CASE 30</td>
</tr>
<tr>
<td>E=10 kcal/mol a=10^2 cm</td>
<td>E=10 kcal/mol a=10^2 cm</td>
<td>E=20 kcal/mol a=10^2 cm</td>
<td>E=20 kcal/mol a=10^2 cm</td>
</tr>
<tr>
<td>Q=7.7 x 10^3 cal/cm^2/sec</td>
<td>Q=7.7 x 10^3 cal/cm^2/sec</td>
<td>Q=7.7 x 10^3 cal/cm^2/sec</td>
<td>Q=7.7 x 10^3 cal/cm^2/sec</td>
</tr>
<tr>
<td>conditions at end of run</td>
<td>conditions at end of run</td>
<td>conditions at end of run</td>
<td>conditions at end of run</td>
</tr>
<tr>
<td>Z</td>
<td>n</td>
<td>T</td>
<td>n</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>0</td>
<td>1.00</td>
<td>300</td>
<td>1.00</td>
</tr>
<tr>
<td>15</td>
<td>1.00</td>
<td>523</td>
<td>1.00</td>
</tr>
<tr>
<td>30</td>
<td>28.5</td>
<td>693</td>
<td>28.5</td>
</tr>
<tr>
<td>45</td>
<td>977</td>
<td>1.00</td>
<td>947</td>
</tr>
<tr>
<td>CASE 26</td>
<td>CASE 27</td>
<td>CASE 29</td>
<td>CASE 30</td>
</tr>
<tr>
<td>E=10 kcal/mol a=10^4 cm</td>
<td>E=20 kcal/mol a=10^4 cm</td>
<td>E=20 kcal/mol a=10^4 cm</td>
<td>E=20 kcal/mol a=10^4 cm</td>
</tr>
<tr>
<td>Q=7.7 x 10^3 cal/cm^2/sec</td>
<td>Q=7.7 x 10^3 cal/cm^2/sec</td>
<td>Q=7.7 x 10^3 cal/cm^2/sec</td>
<td>Q=7.7 x 10^3 cal/cm^2/sec</td>
</tr>
<tr>
<td>conditions at end of run</td>
<td>conditions at end of run</td>
<td>conditions at end of run</td>
<td>conditions at end of run</td>
</tr>
<tr>
<td>Z</td>
<td>n</td>
<td>T</td>
<td>n</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>0</td>
<td>1.00</td>
<td>300</td>
<td>1.00</td>
</tr>
<tr>
<td>15</td>
<td>1.00</td>
<td>523</td>
<td>1.00</td>
</tr>
<tr>
<td>30</td>
<td>28.5</td>
<td>693</td>
<td>28.5</td>
</tr>
<tr>
<td>45</td>
<td>977</td>
<td>1.00</td>
<td>947</td>
</tr>
</tbody>
</table>

* THE WRONG INITIAL CONDITIONS WERE USED IN THESE CASES. 11.6 SHOULD BE SUBTRACTED FROM THE LARGEST VALUE.

\[ n = \frac{N(Z,t)}{N(0,0)} \]

where \( N(0,0) = 0.318 \times a^2 \)
the region of failure equal to the annual amount of energy carried in seismic waves.

The elapsed model times indicate that the time required for the material in the low density layer to reach the near surface region (i.e., near 15 km) is not appreciably different from the time required to melt the acid fraction. With \( Q_E = 7.7 \times 10^{-13} \text{ cal/cm}^3\text{sec} \), .4 million years is the estimated time required to melt the acid fraction. The time required to move the material to the surface is 12 million years in case 11 and one million years for case 16. This leads to the conclusion that if the energy sources are available, differentiation would require 1-50 million years after the onset of intense seismic activity in a region.

The elapsed model time depends rather strongly on the value of the radius and the viscosity parameters. Very little can be said about the effective radius of the tubes. \( a = 10^{-4} \) cm is probably the smallest the tubes can be, and if the tubes are larger than one centimeter, the time of separation is negligible compared to the time of melting. The viscosity parameters were determined independently and are thought to span the possible values. Consequently, there is reason to believe that the time scales are representative.

**Discussion of Results**

The most important result of the calculations is that the minimum rate of strain energy fed into a region
of failure required for near surface differentiation must be at least as large as the annual rate of energy release indicated by the energy in seismic waves for the entire earth. Unless one of the suggested mechanism or some other process can provide such concentrations of energy, an intermittent near surface differentiation does not seem possible, and the major differentiation of granitic material in the earth must have occurred at some time in its past history. The existence of molten lavas indicates that the melting of material is possible. This implies that either the energy sources are available or the estimation of the required energy is too high.

If the energy source is available and the low density layer was formed, the existence of such differentiation seems possible. The time required for the differentiation of a local region is not unreasonable and the process is straightforward.
Conclusions

This study indicates that there is a physical process based on partial melting of silicate rocks that could be responsible for the segregation of an initially homogeneous earth formed by "cold accretion". Differentiation would not be complete in the early history of the earth if the surface region were unmelted and relatively unbroken.

A feature of this process is the formation of a low density layer containing an excess of acid material at depths of 30-90 km. Table 5.1 gives the depth to the top of the layer for the cases run on the computer. The existence of such a layer is shown to be a consequence of the hypothesis that the formation of continents is a continuing process and any model which explains the slow growth of continents must contain a similar feature.

The temperature distribution required for differentiation is not in excess of the temperature thought to exist in the earth today. Computations indicate that the time scale of this early differentiation is 100,000 years or less after the acid fraction melts.

It is suggested that the periodic addition of acid material to the near surface region is a result of the remelting of acid material in local regions of the low density layer. The remelting is attributed to the
TABLE 5.1

DEPTH TO TOP OF LOW DENSITY LAYER. $Z_L = \text{DEPTH TO TOP OF LOW DENSITY LAYER IN KM.}$

$\eta = 10^{-2}$ FOIRES FOR ALL CASES. DATA IS ABSTRACTED FROM TABLE 3.4.

<table>
<thead>
<tr>
<th>Case Number and Description</th>
<th>$Z_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 10: $a = 10^{-4}$ cm</td>
<td>45</td>
</tr>
<tr>
<td>$E' = 10$ kcal/mol</td>
<td></td>
</tr>
<tr>
<td>Case 12: $a = 10^{-4}$ cm</td>
<td>60</td>
</tr>
<tr>
<td>$E' = 20$ kcal/mol</td>
<td></td>
</tr>
<tr>
<td>Case 14: $a = 1$ cm</td>
<td>30</td>
</tr>
<tr>
<td>$E' = 10$ kcal/mol</td>
<td></td>
</tr>
<tr>
<td>Case 17: $a = 1$ cm</td>
<td>45</td>
</tr>
<tr>
<td>$E' = 20$ kcal/mol</td>
<td></td>
</tr>
<tr>
<td>Case 25: $a = 10^{-2}$ cm</td>
<td>45</td>
</tr>
<tr>
<td>$E' = 10$ kcal/mol</td>
<td></td>
</tr>
<tr>
<td>Case 28: $a = 10^{-2}$ cm</td>
<td>60</td>
</tr>
<tr>
<td>$E' = 20$ kcal/mol</td>
<td></td>
</tr>
</tbody>
</table>
local release of elastic energy as heat due to failure by some mechanism similar to that suggested by Crowan (1960) or Treitel (1958). Once the acid material is remelted, it would move closer to the surface through partially melted shear zones above the melted regions of the low density layer due to the action of the buoyancy force.

The acid fraction in this stage of differentiation does not reach the surface (i.e., volcanic phenomena), but rather stops at depths somewhat above 30 kms. No attempt has been made to discuss the difficult problem of the assimilation of this material into the continents.

7.7 x 10^{-14} \text{cal/cm}^3 \cdot \text{sec} is the minimum rate energy can be fed into the region in order that remelting and the near surface differentiation can occur. This is equivalent to dissipating the same amount of energy carried by seismic waves in the region of failure. Unless such a source of energy is provided, it is doubtful that near surface differentiation can occur. However, the molten lava in volcanoes indicates that perhaps such sources do exist or that the estimation of the energy required is too high.

If the sources required do exist, calculations indicate that the time scale for the upward movement to be completed after the onset of intense seismic activity is not impossibly long. For example, if the effective
radius of the shear zones are between $1-10^{-4}$ cm, the
time scales are between $1-50$ million years for the values
of the viscosity used. This includes an estimation of
the time required to remelt the material. These results
are reported in Table 4.1

Suggestions for Further Work

The low density layer seems to be a required
feature of earth models which attempt to explain conti-
nent formation by a continuing process. Therefore, a
thorough investigation should be made to determine if such
a layer exists.

It is well known that seismic techniques have not
been able to determine whether such a layer does or does
not exist. Gutenberg (1959) and others show that there
is evidence of the existence of a low velocity layer.
However, the evidence is not conclusive as to the existence
of a low density layer, for the low velocity layer may be
interpreted as due to a decrease in the elastic constants
resulting from the temperature increase.

An approach using the orbital path of satellites
may yield an answer to this problem. For if the mantle
underneath the oceans is undifferentiated compared to
continental regions, there should be an asymmetry in the
gravity field of the earth that is not due to the presence
of the continents. It would take the form of a mass
deficiency in the oceanic regions. Since the satellites are very sensitive to large scale deviations of small magnitude, this asymmetry would measurably affect their orbits.
APPENDIX A

VISCOSITY

Pressure and Temperature Dependence

The flow of silicate melts has been described very successfully in the glass industry by empirical relationships based on the assumption that the flow is Newtonian and that temperature dependence of the viscosity has one of the two forms

\[
\log \eta = \log \eta' + E \frac{1}{k} (T - C) \quad \text{(A.1)}
\]

and

\[
\log \eta = \log \eta' + E \frac{1}{kT} \quad \text{(A.2)}
\]

where \( \eta' \) and \( \eta' \) are the viscosity at infinite temperature and \( E \) and \( E' \) are activation energies. The temperature is measured in °K. The Kelvin scale of temperature will be used for all temperatures unless other scales are specifically indicated.

The considerations of several workers in the field will be reviewed and extended in order to determine the temperature range over which each of the above two expressions are applicable and the values of the constants which are appropriate to earth materials.
Frenkel (1946) has examined the viscosity of liquids as being the result of the movement of discrete ions and obtains

\[ \eta = \frac{kT \tilde{\tau}}{\sigma^{3}} \exp \left( \frac{\gamma P}{\alpha kT} \right) \exp \left( \frac{E}{kT} \right) \]  

where \( \tilde{\tau} \), interatomic distance; \( \gamma \), a relaxation time; \( k \), the Boltzmann constant; \( \gamma \), the activation energy for the flow; \( \alpha \), the coefficient of thermal expansion; \( k \), the bulk modulus; \( P \), the pressure; and \( \gamma \), a parameter giving the pressure dependence. It is noted that the equation given by Frenkel has the temperature dependence of equation A.1 with the additional feature of pressure dependence.

The following interpretation of the constant \( \sigma \) due to Professor Hughes, permits an estimation of the magnitude of the pressure dependence. \( \gamma \) is defined in Frenkel by

\[ E = E_o - \sigma kT \]

writing

\[ E = E_o + \frac{\partial E}{\partial T} T \]

and using \( \frac{1}{V} \frac{\partial V}{\partial T} = \alpha \frac{\partial E}{\partial T} \frac{\partial E}{\partial V} \frac{\partial V}{\partial T} \) which assumes that \( E \) is only a function of the interatomic distance gives

\[ E = E_o + E \frac{\partial E}{\partial V} \frac{\partial V}{\partial T} \alpha T \]
Giving

\[
\gamma = \frac{\alpha E}{k} \frac{\partial E/\epsilon}{\partial V/\nu}
\]  

where \( V \) is the specific volume.

The experimental value given by Hughes (1955) for the activation energy for cations in the ionic semiconduction mechanism for peridot

\[
\frac{\partial E/\epsilon}{\partial V/\nu} = 1.5 \pm 0.5
\]

is used in order to evaluate the magnitude of \( \gamma \). Taking \( \alpha = 10^{-5}/\degree K \), \( k = 1.38 \times 10^{-16} \text{ ergs}/\degree K \), and \( E = n \times 10^{-12} \text{ ergs} \) gives

\[
\gamma = 1.74 \eta
\]

Dane and Birorh (1938) examined the effect of pressure on the viscosity of boric anhydride glass for pressures up to 2,000 bars at two temperatures. The best fit to the data is given by

\[
\eta = \eta_0 \exp \alpha p
\]

with \( \alpha = 4.6 \times 10^{-4} \text{ cm}^2/\text{kg} \) at 516\degree C., \( 15 \times 10^{-4} \text{ cm}^2/\text{kg} \) at 359\degree C. There is insufficient data to draw any firm
conclusions from this except that the pressure dependence does exist.

Mackenzie (1957) studied the effect of chemical composition on viscosity by examining polycomponent systems of silicates. He finds that equation A.1 is universally obeyed for binary alkali and alkaline-earth liquid silicates, and that the activation energy $E$ is independent of the cationic species at all compositions up to 55% metal oxide in alkaline-earth systems and to 35% metal oxide in alkali systems. Figures A.1 and A.2 reproduced from Mackenzie's paper give the behavior of the activation energy and the preexponential constant, respectively, on the composition, and cationic species.

Mackenzie (1957) demonstrates that the behavior of polycomponent liquid silicates can be deduced by an analysis of the binary systems which are included. If the cations are of the same group, the activation energy is dependent on the mole fraction of the metallic oxides. If the cations are a mixture of different groups and the melt is regarded as an "ideal" mixture of the individual binary melts, the activation energy is given by

$$ E = n_a (E_a)_x + n_b (E_b)_x $$

where $n_a$ and $n_b$ are the mole fractions of the two metal oxides and $(E_a)_x$ and $(E_b)_x$ are the activation energies for
for the two groups at the total metal oxide concentration x.

Equation A.6 can be used to estimate the activation energy and the preexponential constant for silicate melts that have not been measured. This is applied to a plutonic granite to illustrate the use of this equation. Barth (1952) gives the composition of a plutonic granite as being 70% SiO₂ and 30% metallic oxides. The potassium and sodium oxides are 5.5%. Treating the other 24.5% of the metal oxides as alkaline-earth oxides and using figures A.1, A.2, and equation A.6, gives the activation energy for a melt of this composition as $E = 45 \text{ kcal/mol}$. The preexponential constant $\eta_0$ is approximately $10^{-4}$ poises for a 30% metal oxide melt.

Brookins and Lowe (1954) studied the activation energy of the system CaO-SiO₂ as a function of temperature and percent metal oxide in the melt. The interpretation of the results was given in terms of the flow of a discrete silicate anions. If the melt contained 52% or more metal oxide, the flow unit was a silicate anion chain. At 66% metal oxide, the chain was one unit long. For the range 12–52%, the flow unit was of different structure than a chain. For 12% or less metal oxide, the flow unit was thought to be the silica molecule. The behavior of $\log \eta$ vs. $1/T$ was found to have some curvature. It is suggested in the above paper that the change in activation energy at the higher temperatures can be explained by assuming that the bonds of the flow unit are weakened by the addition of thermal energy and that the
shearing stress is more easily relaxed by a breakdown of the complex ion into simpler (and smaller) species than by their flow. The smaller anions have a lower activation energy. This curvature in the plot of $\log \eta$ vs. $1/T$ may be explained by the introduction of an activation energy for short range order as follows.

Dienes (1953) considers the effect of short range order on the viscosity by using an expression given by Frenkel for the temperature dependence of the short range order and obtains

$$\eta = \eta_0 \exp \left( \frac{E}{kT} \right) \left( 1 + \exp \left( \frac{U}{k(T-T_0)} \right) \right)$$  \hspace{1cm} (A.7)

where $E$ is the activation energy for viscous flow in disordered liquid, $U$ is the structure activation energy of Frenkel and $\eta_0$ is the viscosity at infinite temperature. If $U = 0$ or $T$ is large, equation A.7 reduces to equation A.1 and if $U \gg E$ and $eU/k (T-T_0) \gg 1$, it reduces to equation A.2

In equation A.7, Dienes finds that $T_0$ is a constant and is approximately 500° K. for some silicate glasses. Table A.1 which is part of Table 1 from the above paper by Dienes is given to show some values of the above quantities, and the closeness of fit is obtained by the use of equation A.7. The range of viscosity covered by the data in Table A.1 is based on $10^2 - 10^{14}$ poises and the temperatures are between 540° C.-1400° C.
Parameters for some silicate glasses fitted by Curve A.7. This is part of Table 1 in Dienes (1953).

<table>
<thead>
<tr>
<th>Silicate Glasses</th>
<th>Log$_{10}$ $\eta$ in poises</th>
<th>$T_0$ in °K</th>
<th>$E$ in kcal/mol</th>
<th>$U$ in kcal/mol</th>
<th>Avg. deviation in Log$_{10}$ $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td># 2</td>
<td>-2.135</td>
<td>560</td>
<td>18.28</td>
<td>11.77</td>
<td>.073</td>
</tr>
<tr>
<td># 10</td>
<td>-1.446</td>
<td>560</td>
<td>4.57</td>
<td>17.11</td>
<td>.029</td>
</tr>
<tr>
<td># 330</td>
<td>-2.249</td>
<td>562</td>
<td>18.28</td>
<td>10.47</td>
<td>.002</td>
</tr>
</tbody>
</table>

NOTE: Viscosity of glass samples are reported by Robinson and Peterson (1944).

Dienes describes a computational scheme for calculating the parameters in equation A.7. However, the available data for earth materials which consists of three or four measurements per sample does not justify the application of A.7. In fact, even $C$ in equation A.2 cannot be determined with any reliability by a computational scheme. A typical value of $C$ for equation A.2 is determined later which is used in the calculations.

A better understanding of the effect of the parameter $C$ and its importance in the model can be obtained from a study of the measurements of Shartsis and Spinner (1951) on optical silicate glasses. The results included are for sample #8198 which is typical of all the samples studied.

Three curves are plotted in Figure A.3 to show the effect of the parameter $C$. Log viscosity in poises is plotted vs. $1/T$, $1/T-273$, and $1/T-473$. The curves were fitted by eye.
As can be seen, there is a selection of \( C \) which removes the curvature.

The difference in the predicted viscosity between a curve including \( C \) and one omitting it is shown in Figure A.4. In this figure, log viscosity was plotted vs. \( 1/T \). A least squares fit of this data by four curves

\[
\begin{align*}
\eta &= \eta_o + P \frac{1}{T} \\
\eta &= \eta_o' + g \left( \frac{1}{T} \right)^2 \\
\eta &= a + b \frac{1}{T} + c \left( \frac{1}{T} \right)^2 \\
\eta &= \eta_o'' + \frac{r}{t - 4.73}
\end{align*}
\]

was made where \( \eta = \log \eta, \eta_o, \eta_o', \eta_o'' \) and \( a = \log \eta_o \) and \( t = T x 10^{-3} \).

In fitting curve A.10, the term \( b/t \) was found to be so small that this form could not be distinguished from A.9. Therefore, the form A.10 is omitted from the discussion.

The curves corresponding to equations A.8, A.9, and A.11 are plotted in Figure A.4. Table A.2 gives the values of the parameters for the three curves and the differences between the measured values and the values given by these curves.

From Figure A.4, it is seen that both the linear form and the \( 1/T-C \) form give a good fit to the data in the
interval of temperature where data was taken. The differences between using either of these two forms in the calculation in this interval of temperature away from the melting point are much less than the error associated with the uncertainty in the chemical composition of the material. However, the agreement between equation A.8 and equations A.9 and A.11 is rather poor in regions of low temperatures. The values of viscosity given by the form A.8 are much lower.

Birch et al (1942) lists the measurements of the viscosity as a function of temperature for several rocks and artificial materials with a composition similar to natural rocks. The data given for seven of these samples are plotted in Figures A.5, A.6, and A.7. Values of \( \gamma_0 \) and \( E \) determined by a least squares fit of the data are given in Table A.3. \( \gamma_0' \) and \( E' \) was determined for two natural lavas using \( C=700^\circ K \) (the selection of the value is explained later) and they are given in Table A.3. The curves for these two lavas are plotted in Figure A.7. Table A.4 gives the comparison between the fit obtained with \( 1/T \) and \( 1/T-C \) temperature dependence.

There is a definite relationship between the chemical composition and the value of the activation energy as expected with the more basic material having the lower activation energy. However, the values obtained for \( E \) and \( \gamma_0 \) using a \( 1/T \) dependence are higher than those
predicted by use of the curves in Mackenzie's paper for a
glassy melt with the same per cent of metal oxide. This
discrepancy arises from the curvature in plot of the vis-
cosity vs. $1/T$ for the natural rocks. The high values of
the viscosity at the low temperature end increase the slope
of the line and decrease the intercept at infinite temperature.

The physical interpretation of this is not certain.

There are two possible explanations that have some merit.
The first is even if it is assumed that the equation $A.8$ is
valid down to the solidus point of an individual component,
this relationship cannot be valid in a polycomponent system
due to the dependence of $E$ and $\gamma$ on the chemical composi-
tion of the melt. When the temperature of the polycomponent
melt is lowered, crystals form over a wide temperature range.
These crystals remove a greater portion of metal oxide than
silica. The remaining fluid would contain a smaller per-
cent of metal oxide. This would lead to a higher activation energy and a smaller $\gamma$ for the remaining fluid since
the activation energy is larger for more acid melts. The
temperature scale of the measurements reported in the litera-
ture would not permit a discrete change in the slope of
$\log \eta$ vs. $1/T$ to be detected if it existed. The apparent
change in slope would be continuous. If care is not taken
to insure that equilibrium conditions were reached, no
experiment would show a discrete change. Hence, even if
the $1/T$ dependence of the viscosity is correct for a simple
system, it is necessary to use a $1/T-C$ dependence in a complex system such as a granite to describe the viscosity as a function of temperature when temperatures in the range containing the melting point of the various components are included.

The second explanation is that there is an increase in the short range order as suggested by Dienes as the solidus point is approached even for a single component system, and that the curvature is due to this phenomena. There is too little known about the behavior of the viscosity as a material passes through its melting point to decide the question of which of these explanations is the most realistic.

One group of authorities look on the transition from a crystalline to a liquid state as being analogous to the liquid to gas transition. While others view the crystalline to liquid transition as being essentially a discontinuous transition. Borelius (1953) gives a review of the arguments of both groups, and presents his arguments for accepting a smooth transition from the crystalline to liquid state. The question in an oversimplified form is, does a critical point exist for the crystal to liquid transition. Experimental evidence on this point does not seem to be conclusive.

For the purpose of the calculations in this thesis, it does not matter which of these two viewpoints are "correct".
It is enough that the viscosity is described by either equation A.1 or A.2 for the temperature range of interest. In either case, the form A.1 is a poor predictor formula when temperatures near the solidus point must be considered.

Two forms of the viscosity are used in the thesis. They are

\[
\eta = \eta_0 \exp \left( \frac{E}{kT} (1 + \lambda z) \right)
\]

and

\[
\eta = \eta_0' \exp \left( \frac{E}{k(T - c)} (1 + \lambda z) \right)
\]

where the hydrostatic approximation, \( P = \rho g z \), for the pressure has been used and \( \lambda = 1.5 \, \rho g / k \).

The first relationship is used when the melt is at temperatures relatively far above its solidus point, and the second form is used at lower temperatures. In the calculations in the thesis, the first form is used in the study of the initial separation and the second form is used in the near surface differentiation.

**The Effect of Volatiles on the Viscosity**

Observers have remarked on the effect of water on the viscosity of silicate melts. It seems that "wet melts" are less viscous than "dry melts" of the same chemical composition except for the water. A rather extensive
literature search did not disclose any experimental work contrasting the viscosity of a "wet melt" and a "dry melt" of the same chemical composition from which any quantitative estimation of the effect of the water could be made. Therefore, it is necessary to use what data is available to estimate the effect of the water.

Minakami (1951) reports that the viscosity of lava from the 1951 eruption of the volcano Oo-sima as measured in the field was $4 \times 10^3$ poises at $1150^\circ$ C. Samples of the same gross chemical composition when remelted in the laboratory had a viscosity of $5 \times 10^4$ poises at the same temperature. The difference in the viscosity was attributed to the richer volatile content of the natural lava. If it is assumed that the effect of the volatiles is to change the activation energy for viscous flow, the relationship

$$\Delta E = \frac{|cT_1|}{\log e} \left( \log \frac{\eta_{\text{dry}}}{\eta_{\text{wet}}} \right)$$

where $\Delta E = E_{\text{dry}} - E_{\text{wet}}$ and $T_1$ is the temperature in $^\circ$K at which the measurements were made. Equation A.12 can be used to estimate the change in the activation energy due to the presence of the volatiles. Table A.5 gives $\Delta E$ for several values of $\eta_{\text{dry}}/\eta_{\text{wet}}$ at $T_1 = 1423$ $^\circ$K. $\Delta E = 7$ kcal/mol is indicated by the measurement on the Oo-sima lavas. The activation energy for such basaltic lavas is approximately 30-40 kcal/mol. A tentative estimation of the effect of
the presence of the volatiles is that it can cause a change of 20% in the activation energy $E$.

**TABLE A.5**

<table>
<thead>
<tr>
<th>$\gamma_{\text{dry}}/\gamma_{\text{wet}}$</th>
<th>$\Delta H \text{ kcal/mol}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3.9</td>
</tr>
<tr>
<td>12</td>
<td>7.0</td>
</tr>
<tr>
<td>100</td>
<td>13.0</td>
</tr>
</tbody>
</table>

David B. Stewart of the United States Geological Survey has furnished some unpublished information on the settling rate of crystals in hydrous silicate liquids from which the viscosity of the liquids can be estimated. The data and the calculated viscosity for four of the measurements are given in Table A.6. The viscosity was calculated using Stokes approximation for the slow fall of spheres in a fluid (i.e., $U = \frac{2 \Delta \rho g a^2}{9 \eta}$ where $a$ is the radius of the sphere and the other symbols have the same meaning as before).

**TABLE A.6**

<table>
<thead>
<tr>
<th>Water Pressure (Bars)</th>
<th>Particle Size (Microns)</th>
<th>Temp (°C)</th>
<th>Compo-</th>
<th>Dis-</th>
<th>Viscosity in Poises</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,000</td>
<td>5.5</td>
<td>1320</td>
<td>An80Si20</td>
<td>.6</td>
<td>.25</td>
</tr>
<tr>
<td>5,000</td>
<td>10.5</td>
<td>1190</td>
<td>&quot;</td>
<td>.3</td>
<td>.35</td>
</tr>
<tr>
<td>10,000</td>
<td>12.5</td>
<td>1040</td>
<td>An70Si30</td>
<td>.15</td>
<td>.43</td>
</tr>
<tr>
<td>10,000</td>
<td>12.5</td>
<td>900</td>
<td>&quot;</td>
<td>.10</td>
<td>.43</td>
</tr>
</tbody>
</table>

**NOTE:** An stands for anorthite and Si for silica.
The data is not sufficiently extensive to determine the activation energy and the preexponential constant for these substances. However, the extremely low values of the viscosity found for these wet melts indicates that the presence of water vapor is a very important parameter, and that the estimation of a 20% change in the activation due to this cause is probably not too high.

The density differences of .4 gm/cm³ between the hydrous silicate liquid and the crystals indicates that the value of .5 gm/cm³ used in the thesis as the density difference between the basic solid and the liquid acid fraction is not too high.

The data supplied by Stewart also indicates the importance of crystals settling out of even such a relatively simple system as given here. In particular, Stewart mentions that the settling of crystals is an unwanted phenomena for equilibrium is difficult to obtain when settling occurs.

Value of Viscosity Parameters used in Calculations

\( E \) and \( \eta \)

The chemical composition of a plutonic granite should be typical of the composition of any "acid fraction" which can exist except for the volatile content. Consequently, the value of the activation energy of a plutonic granite previously calculated from Figure A.1 reduced by 20% is used as typical of the value of the activation energy for an "acid fraction". This value is 35 kcal/mol. Figure A.1
indicates that the activation energy cannot radically differ from this value for any melt containing 20% or more metal oxide. Therefore, three activation energies are used in the calculations where equation A.1 is used as the form of the viscosity to include all possible acid fractions. They are $E = 20, 35, \text{ and } 50 \text{ kcal/mol}$. The range could be narrowed if the quantitative effect of the water vapor were better understood. The value of the preexponential constant, $\gamma_0$, is taken as $10^{-4}$ poises. This value is not in error by more than an order of magnitude for melts containing 10-50% of metal oxide.

$E', \gamma_0', \text{ and } C$

As has been pointed out, there is a scarcity of data on the temperature dependence of the viscosity in the region near the solidus point. A result of this scarcity of data is that the values of $E'$ and $\gamma_0'$ are not known as well as $E$ and $\gamma_0$, but as has been shown, the $1/T-C$ dependence must be used for temperatures in the melting range of a complex silicate. Therefore, it is necessary to estimate as well as possible the values $E'$, $\gamma_0'$ and $C$ for use in the thesis.

The values $E' = 10$ and 20 kcal/mol are used as being typical of the activation energy of an acid fraction. The values are typical of the activation energy of the natural lavas and the glasses of Shartsis and Spinner. $\gamma_0' = 10^{-2}$ poises was used for the preexponential constant.
$\Omega = 500^\circ K.$ is a typical value for the glasses reported in Shartsis and Spinner. For the natural lavas studied, $\Omega$ fell in the range $700^\circ K.-1000^\circ K.$ $\Omega = 500^\circ K.$ is the value reported by Dienes for his glasses. The value of $\Omega$ adopted for use in the calculations is $\Omega = 700^\circ K.$ The uncertainty in $\Omega$ is indicated by the preceding discussion.

In the calculations where $\Omega$ is used, the viscosity is made infinite for $T < \Omega.$ This ignores flow at very low temperatures. The main effect of changing $\Omega$ is to change the depth at which flow becomes negligible (i.e., the viscosity is made infinite). The uncertainty in the temperatures that exist under the oceans would not permit determination of a particular value for this depth even if $\Omega$ were very well known.
### TABLE A.2

**Least Squares Fit of Data for Sample #8198 (Shartsis and Spinner, 1951)**

For three curves. Viscosity in poises and temperature in °K.

\[
\begin{align*}
\xi_{A.8} &= -4.670 + 10.309x \\
\xi_{A.9} &= -.559 + .161x \\
\xi_{A.11} &= -1.409 + 3.710/t - .473
\end{align*}
\]

\[X = 1/t = 10^3/T\]

<table>
<thead>
<tr>
<th>(x_m)</th>
<th>Measured (\xi_m)</th>
<th>(\xi_{A.8})</th>
<th>(\xi_{A.9})</th>
<th>(\xi_{A.11})</th>
<th>(\xi_m - \xi_{A.8})</th>
<th>(\xi_m - \xi_{A.9})</th>
<th>(\xi_m - \xi_{A.11})</th>
</tr>
</thead>
<tbody>
<tr>
<td>.6398</td>
<td>1.961</td>
<td>1.859</td>
<td>1.968</td>
<td>1.995</td>
<td>.102</td>
<td>-.007</td>
<td>-.034</td>
</tr>
<tr>
<td>.6789</td>
<td>2.306</td>
<td>2.256</td>
<td>2.286</td>
<td>2.301</td>
<td>.050</td>
<td>.020</td>
<td>.005</td>
</tr>
<tr>
<td>.7287</td>
<td>2.736</td>
<td>2.764</td>
<td>2.719</td>
<td>2.718</td>
<td>-.028</td>
<td>.017</td>
<td>.018</td>
</tr>
<tr>
<td>.7862</td>
<td>3.241</td>
<td>3.347</td>
<td>3.254</td>
<td>3.234</td>
<td>-.106</td>
<td>-.014</td>
<td>.007</td>
</tr>
<tr>
<td>.8525</td>
<td>3.902</td>
<td>4.022</td>
<td>3.924</td>
<td>3.891</td>
<td>-.120</td>
<td>-.022</td>
<td>.011</td>
</tr>
<tr>
<td>.9311</td>
<td>4.771</td>
<td>4.821</td>
<td>4.788</td>
<td>4.764</td>
<td>-.050</td>
<td>-.017</td>
<td>.007</td>
</tr>
<tr>
<td>1.0163</td>
<td>5.833</td>
<td>5.687</td>
<td>5.810</td>
<td>5.851</td>
<td>.146</td>
<td>+.023</td>
<td>-.018</td>
</tr>
<tr>
<td>System</td>
<td>Percent Component</td>
<td>Figure</td>
<td>Curve</td>
<td>Log $\eta_0$</td>
<td>$E$ kcal/mol</td>
<td>Log $\eta'$</td>
<td>$E'$ kcal/mol</td>
</tr>
<tr>
<td>-------------------------</td>
<td>-------------------</td>
<td>--------</td>
<td>-------</td>
<td>--------------</td>
<td>--------------</td>
<td>-------------</td>
<td>---------------</td>
</tr>
<tr>
<td>Diopside-Albite-Anorthite</td>
<td>Di-Al-An</td>
<td>A.5</td>
<td>1</td>
<td>-7.245</td>
<td>75</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>40-60-0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>20-80-0</td>
<td>A.5</td>
<td>2</td>
<td>-7.061</td>
<td>81</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>20-60-20</td>
<td>A.5</td>
<td>3</td>
<td>-9.204</td>
<td>96</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orthoclase-Albite</td>
<td>Or-Al</td>
<td>A.6</td>
<td>1</td>
<td>-10.376</td>
<td>125</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>60-40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0-100</td>
<td>A.6</td>
<td>2</td>
<td>-8.526</td>
<td>105</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lava Mt. Vesuvius</td>
<td></td>
<td>A.7</td>
<td>1</td>
<td>-5.118</td>
<td></td>
<td>-.253</td>
<td>8.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A.7</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>A.7</td>
<td>3</td>
<td>-6.289</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>A.7</td>
<td>4</td>
<td></td>
<td>-.983</td>
<td>7.9</td>
<td></td>
</tr>
</tbody>
</table>
### Table A.4

**Comparison Between Fit of Viscosity as Function of Temperature For Two Natural Lavas by Two Forms of Temperature Dependence**

<table>
<thead>
<tr>
<th>$X = 1/t = 10^3/T$</th>
<th>Viscosity Measured</th>
<th>Viscosity Calculated From</th>
<th>Error $= f_m - f_{cal}$</th>
<th>Functional Form $X = 10^3/T$, $Y = 10^3/T-C$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f$</td>
<td>$f_1$</td>
<td>$f_2$</td>
<td>$f_m - f_1$</td>
</tr>
<tr>
<td>Lava from Mt. Vesuvius</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.296</td>
<td>4.452</td>
<td>4.359</td>
<td>4.412</td>
<td>.098</td>
</tr>
<tr>
<td>2.140</td>
<td>3.441</td>
<td>3.567</td>
<td>3.479</td>
<td>-.121</td>
</tr>
<tr>
<td>2.004</td>
<td>2.869</td>
<td>2.900</td>
<td>2.852</td>
<td>-.031</td>
</tr>
<tr>
<td>1.884</td>
<td>2.398</td>
<td>2.326</td>
<td>2.413</td>
<td>.022</td>
</tr>
<tr>
<td>Tachylite (1924 Mauna Ulu Flow, Kilauea)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.34</td>
<td>3.695</td>
<td>3.621</td>
<td>3.680</td>
<td>.074</td>
</tr>
<tr>
<td>2.22</td>
<td>2.903</td>
<td>2.993</td>
<td>2.918</td>
<td>-.090</td>
</tr>
<tr>
<td>2.07</td>
<td>2.176</td>
<td>2.238</td>
<td>2.200</td>
<td>-.062</td>
</tr>
<tr>
<td>1.97</td>
<td>1.885</td>
<td>1.812</td>
<td>1.858</td>
<td>.073</td>
</tr>
</tbody>
</table>
Fig. A1. Energy of activation for viscous flow in binary liquid silicates.²
Fig A.2. Relation between the pre-exponential constant $\nu$ and composition for binary liquid silicates.
Figure A.3. Plot of log viscosity in poises of a silicate glass (sample #8198, Shartris and Spinner 1951) vs. 1/T, 1/T-273, 1/T-473 where T is in K.
Figure A.4. Plot of log viscosity in poises for a silicate glass (sample #8189, Shartris and Spinner 1951) vs. $1/T$, $T$ in $\text{K}$. Three curves were fitted by least square technique.
Figure A.5. Plot of log viscosity in poises for indicated system vs. 1/T, T in °K. Data is from Birch (1942). Curves \( \log \eta = \log \eta_0 + E/kT \) fitted by method of least squares.
Figure A.6. Plot of log viscosity in poises vs. $1/T$, $T$ in $\text{K}$, for the indicated system. Data is from Birch (1942). Curve, $\log \eta = \log \eta_0 + E/kT$, is fitted by least square technique.
Figure A.7. Plot of log viscosity in poises vs. $1/T$, $T$ in $^\circ$K, for two natural lavas. Curves are fitted by least squares technique. Curves 1 and 2 are for a lava from Mt. Vesuvius, and curves 3 and 4 are for a tachylite lava from Mauna Iki flow, Kilaweia. The data is from Birch (1942).

**Log Viscosity**

CURVE EQUATION

1. \( \log = -5.118 + 10.426X \)
2. \( \log = -0.253 + 1.866Y \)
3. \( \log = -6.289 + 10.645X \)
4. \( \log = -3.076 + 4.439Y \)

where \( X = 10^3/T \), \( Y = 10^3/T - 700 \)
APPENDIX B

ENERGY SOURCES

Possible Sources

The possible sources or sinks of energy which may be important in the model are the release of gravitational potential energy as the material separates, chemical and phase changes, PdV work, radioactive decay, and strain energy which is locally converted to heat by dissipation. An order of magnitude estimation of the relative importance of the various possible sources is given in order to determine which sources may be neglected and to indicate the error introduced by their neglect.

Gravitational Potential Energy

The change in the gravitational energy as a volume $V$ of the basic phase is replaced by acidic material from depths is

$$V \Delta \rho g \Delta Z = M g \Delta Z$$

where $\Delta Z$ is the distance over which the transfer is made, and $M$ is the net mass transferred. In B.1, it is assumed that $g = \text{constant}$, $\Delta = 0$ and compressibility effects are negligible. These last two assumptions are justified in the next section. For the adiabatic case, the temperature rise is

$$\Delta T = \frac{\Delta \rho g \Delta Z}{\rho c}$$
where \( c \) is the specific heat. If \( \Delta z = 90 \text{ km}, \Delta p = .5 \text{ gm/cm}^3, \rho = \rho_b - S \Delta p = 3.5 \text{ gm/cm}^3, \ c = .25 \text{ cal/gm } ^{0}C, \ \Delta T = 33^0 C. \) The other sources of energy are compared to this source in order to determine their relative importance. The term B.1 was included in the energy balance equation and was found to be negligible for reasonable values of the velocity of the acid fraction.

**PdV Work**

The PdV work done by interchanging equal volumes of acidic and basic material through a distance \( \Delta z \) is

\[
(PdV)_{acidic} - (PdV)_{basic} = P V_0 \left\{ \frac{\Delta T}{\Delta z} (\kappa^a - \kappa^b) - \frac{\Delta P}{\Delta z} (\beta_T^a - \beta_T^b) \right\}
\]

where \( V_0 \) is the volume exchanged, \( P \) is the pressure, \( \kappa^i \) is the coefficient of thermal expansion of the \( i \) phase, and \( \beta_T^i \) is the isothermal compressibility. The relative importance of this term is found by taking the ratio of equations B.5 and B.1. Giving

\[
\frac{P}{\Delta \rho g} \left\{ \frac{\Delta T}{\Delta z} (\kappa^a - \kappa^b) - \frac{\Delta P}{\Delta z} (\beta_T^a - \beta_T^b) \right\}
\]

If \( P = -\rho g z, \frac{\Delta T}{\Delta z} = 10^{-5} ^{0}C/cm, \kappa^a - \kappa^b = 10^{-5}/^{0}C, \beta_T^a - \beta_T^b = 3 \times 10^{-13} \text{ cm}^2/\text{dyne}, \rho = 3.5 \text{ gm/cm}^3 \) and \( \Delta \rho = .5 \text{ gm/cm}^3 \), this ratio becomes \( 7 \times 10^{-4} z/km \). The PdV work is seen to be relatively unimportant if the changes occur above \( z = 10^3 \text{ km} \).
Chemical and Phase Changes

In a phase change, the amount of energy involved in the reversible conversion of mass \( M = \rho V \) is

\[
\Delta H_M = \rho V \Delta H
\]

where \( \Delta H \) is the heat of reaction/gm for the transition.

Forming the ratio of equations B.1 and B.2 gives

\[
\frac{\rho \Delta H}{\Delta \rho \Delta L}
\]

\( \Delta \rho \) is the density difference between the acidic and basic phases before the phase changes occur.

The transitions

\[
\text{albite + nepheline} = 2 \text{ jadeite}
\]

and

\[
\text{quartz} = \text{coesite}
\]

are typical of the transitions which might occur in a segregation process. If the ratio in equation B.3 is set equal one, a value of \( \Delta L \) can be found where the energy released by both processes are equal.

For the first relationship, Robertson et al (1957) gives \( \Delta H = -3.6 \text{ kcal/mol} \). Using \( \rho_{\text{jade}} = 3.3 \text{ gm/cm}^3 \), \( \Delta \rho = 0.5 \text{ gm/cm}^3 \), and \( g = 10^3 \text{ cm/sec} \) gives \( \Delta L = 49 \text{ km} \).

For the second relationship, MacDonald (1956) gives \( \Delta H = -0.22 \text{ kcal/mol} \) and \( \rho_{\text{coes}} = 3.02 \text{ gm/cm}^3 \). This gives \( \Delta L = 9 \text{ km} \).
The value of $\Delta z$ found for the first transition is comparable to the distances material is transferred in the earth. The energy requirements of the phase changes are of the same order of magnitude as the potential energy released by the separation of the phases, and can be neglected in the energy balance equation.

**Volume Changes**

Volume changes are found to be negligible in the energy balance consequently, there is no provision for volume changes in the model. This leads to an unrealistic final configuration, for the surface of the granitic material would be the same as the original surface of the ocean bottom. There are large volume changes associated with the phase changes that would occur in the acid fraction during separation. A brief discussion is given to indicate the magnitude of the possible increases in volume in the acid fraction. The changes in the basic fraction are neglected due to the deep depths at which such transitions would occur.

The feldspars at depth would be in a form similar to jadeite (Robertson et al 1957). A 28% volume increase is associated with the transition 2 jadeite $\rightarrow$ albite $+$ nepheline at 1 bar and $25^\circ C$. The transition coesite $\rightarrow$ quartz gives a 14% volume increase (MacDonald 1956) under the same conditions. The percent change in volume is little affected by pressure.
If 30 kms of acidic material is converted from the high density phase stable in the mantle to the low density phase stable in the crust, the resulting increase in elevation would be between 4 and 9 kms. The loss of volatiles to form the oceans and atmosphere might decrease this by about 1 km.

**Radioactivity**

If $A$ is the rate of radioactive heat production/\(\text{cm}^3\).sec for a granite, the radioactive source term for the model is obtained by assuming that the distribution of the radioactive material is proportional to the distribution of the acidic phase. This gives

$$J_R = A\alpha^2 N$$

where for a granite $\alpha^2 N = 1$. Bullard (1954) gives $A = 5.3 \times 10^{-13}$ cal/cm$^3$.sec, and this is the value used in this thesis.

**Strain Energy**

The question of how much strain energy is released annually is not settled. However, it seems certain that all of the strain energy released in a local region is not carried away as seismic energy. It is equally certain that the deep and intermediate-depth seismic faulting is not a brittle fracture. Consequently, the energy carried by seismic waves must be considered as a lower limit to the elastic energy released by failure.
Estimations of the amount of energy released each year by earthquakes as determined by relating the measured amplitudes of the seismic waves to the energy of the earthquake are not at all certain. Gutenberg and Richter (1956) estimate $10^{24}$ ergs/yr. Their previous estimation was $10^{26}$ ergs/yr.

The potential energy lost in a cooling earth is a source of energy which is stored as elastic energy and may subsequently be converted into heat energy. To estimate the magnitude of this source, a rough treatment of the potential energy lost in a cooling earth is given.

The common assumption made in treating the energy released in an earth which undergoes some sort of failure is to assume that the initial and final stress states are hydrostatic. With this assumption, the potential energy released in a cooling earth can be expressed by calculating the difference in potential energy between two spheres in a hydrostatic state of stress that have the same mass but a different radius. In terms of the change of radius, the difference is

$$
\Delta w = \frac{1 - 2 \frac{\pi}{3} \rho g}{35E} (\rho g)^2 R^4 \Delta R
$$

where $\Delta R$ is the difference in radius and $\Delta R \ll R$. If we assume that the release of the potential is more or less continuous, this can be written as rate of energy release by replacing $\Delta w$ by $\Delta w/\Delta t$ and $\Delta R$ by $\Delta R/\Delta t$. 
Hales (1953) gives $\Delta H/\Delta t = 10^{-11}$ cm/sec for a cooling earth. Taking

\[ R = 6.4 \times 10^8 \text{ cm} \quad E = 10^{12} \text{ dynes/cm}^2 \]

\[ \rho = 5.5 \text{ gm/cm}^3 \quad \gamma = 1/4 \]

\[ g = 10^3 \text{ cm/sec}^2 \]

gives $\Delta \omega/\Delta t = 2 \times 10^{25}$ ergs/yr. This is 10 times the observed seismic energy and $10^{-2}$ of the total heat flow out of the earth ($8 \times 10^{27}$ ergs/yr.).

Due to the uncertainty in the rate of strain energy released as heat, the value of this term (called $Q_E$) will be assumed constant with depth and some multiple of the rate of energy release as measured by seismic wave energy. The expression is

\[ Q_E = \frac{7.56n}{V} \times 10^8 \text{ cal/sec} \]

where $n$ is a multiple of the energy contained in seismic waves and $V$ is the volume in which this energy is dissipated. $V$ is taken as a cube $100 \times 1000 \times H$ km$^3$ where 100 and 1000 km are the area of the region and $H$ is the depth to the bottom of the undifferentiated region. The values used in the calculations are $n = 10$ and 1 and $H = 90$ km giving $Q_E = 8 \times 10^{-13}$ and $8 \times 10^{-14}$ cal/sec/cm$^3$ respectively.

When the dimension of the region is decreased to lengthen the time scale as described in Chapter III, the amount of elastic energy released in the region is kept constant by increasing the value of $Q_E$ used in the later runs.
These values of $n$ were selected because the calculations in Chapter IV indicate that they are the minimum values of $Q_E$, which can cause the near surface differentiation. These values are higher than can be easily explained. Unless some mechanism of failure can concentrate such large amounts of energy locally, it is difficult to see how the near surface differentiation could occur. The following two mechanisms are mentioned in order to indicate possible means of achieving this concentration.

Treitel (1958) suggests that if shock waves occur in the region of failure, the energy carried in small amplitude seismic waves is less than 0.1 of the energy released with the remainder dissipated as heat. This would provide a mechanism for concentrating such large amounts of energy.

Orowan (1960) discusses the stresses required for brittle failure to occur even at relatively moderate depths in the earth and finds that the stress differences required for this type of failure are prohibitively large ($\sim 400,000$ bars). He then suggests that earthquakes (i.e., faulting) may be due to a gradual concentration and local acceleration of creep arising from an inherent physical instability.

In any mechanism of failure similar to that suggested by Orowan, a large part of the elastic energy released locally is converted into heat. An equally interesting feature of this type of mechanism of failure is that it
provides a means of concentrating the strain energy from a much larger region into the region of failure.

It is difficult to visualize any mechanism of failure under high hydrostatic confining pressure which takes into account the observed plastic flow at high temperatures under long period stress differences that would not convert a large part of the released elastic energy into local heat energy.
APPENDIX C
NUMERICAL APPROXIMATION

General
The set of simultaneous nonlinear equations (2.1, 2.5, and 2.12) which form the mathematical model cannot be solved in closed form. Consequently, numerical techniques are used to obtain a solution. The partial differential equations were approximated by a set of finite difference equations using central differences to replace the differential operators. This set of difference equations was solved on the IBM 704 at the M. I. T. Computational Center. The method of solution used is a simple forward marching scheme in the time variable.

There is a danger that answers obtained from such approximations will bear little or no relationship to the solution of the differential equations. This can occur due to the amplification of round off error, to excessive truncation error, or to blunders. The net spacing and the order of the differences used determine the relationships that must exist between the parameters in the problem in order that the solution to the difference equations approximates the solution of the differential equations. In certain linear and quasilinear cases, an error analysis can determine the required relationships between the parameters for a given net and order of differences. For most nonlinear systems, it is not
possible to perform an error analysis. In such cases, it is necessary to experiment numerically in order to establish the required relationships between the parameters. The set of equations that form the mathematical model in this thesis is such a nonlinear system.

**Stability and Convergence Conditions**

From analysis of finite difference approximations to a quasi-linear parabolic partial differential equation, John (1952) derived requirements that the space and time net spacings and the coefficient of the highest order derivative must satisfy if the solution of the difference approximation is to be stable and convergent to the solution of the differential equation. In the difference net, these requirements take the form

\[ 0 < \frac{R \tau}{\rho c h^2} < \frac{1}{2} \]

In addition to this requirement, the conservation of energy equation in the model requires an additional constraint. This is due to the term giving the rate of conversion of the gravitational potential energy into heat energy.

Since in finite difference schemes, the value of a function over an interval in space and time is approximated by its value at a point. There are selections of the size of the time and space increments that satisfy
the requirement in equation 0.1 for which the estimation of the concentration of the acidic phase bears little or no relation to its mean value over the interval selected. This leads to difficulty in the potential energy conversion term which involves a product of the velocity and the concentration of the acidic phase. The volume selected by the space interval may be swept clean of the acidic phase in a very small fraction of the time interval selected by equation C.1. The conversion of potential energy term uses the non-zero concentration for the whole time interval. The amount of energy that is apparently converted to heat energy by viscous dissipation can be several orders of magnitude greater than the decrease in the potential energy. In extreme cases, this can lead to apparent temperature rises of order $10^5$ K.

To insure that the amount of energy converted given by the approximation in equation C.1 does not exceed the amount of energy available, it is necessary to apply a condition which permits only one space interval to be swept clean in a time interval. The required condition is

\[( \tau^m_k )_{\text{max}} < (U^m_k)_{\text{max}} \quad \text{C.2} \]

For each iteration, the velocity at all points of the net is calculated and the space increment is divided by these velocities. The smallest quotient is selected. This is
the longest time increment that can be used if, at most, one space interval is to be swept clean in a time interval. This value of the time increment is compared to the value selected by the requirement in C.1, and the smaller of these two values is used as the length of the time increment for that iteration. This makes the time increment a variable which must be calculated at each step, but it insures that both equations 0.1 and C.2 are satisfied while using the longest possible time interval consistent with the two conditions.

The above is the original thinking which lead to the condition C.2. Calculations made using the condition C.2 indicates that the conversion of potential energy term is negligible in the energy balance and on these grounds could be omitted from the equation. However, the conservation of mass equation also requires a condition similar to equation C.2 for it involves the product NU and since the conversion of potential energy term is very sensitive to the violation of the constraint, it is retained in the calculations.

An observation based on a study of the literature concerning finite difference approximations to linear and quasi-linear equations is that the higher the order of the differences used in an approximation, the smaller is the range of the variables for which the solution given by the approximation represents the solution of the differential
equation. It is preferable in work of this type to use as low order differences as possible to approximate the differential operators and a fine mesh spacing in order to cover the widest possible range of variables.

Accuracy

A check on the accuracy of the approximation was to calculate the solution for two mesh spacings and compare the solutions. Table C.1 gives the solution for these runs. The parameters used were the same except for the number of mesh points and the length of a space increment. Column 1 is Case 1 described in Table 3.1. The difference between the two runs is insignificant.

The physical arguments that were applied to check for blunders in the formulation of the problem or the approximations used are (1) the velocity of the acidic phase must always be toward the surface, (2) the temperature cannot be negative when measured on the Kelvin scale, and (3) the possible range of the temperature variations can be estimated.

Difference Equations

The subscript \( l \) refers to the spacial location and the superscript \( m \) refers to the temporal location in the difference net. The space increment is selected by

\[
h = \frac{H}{L}
\]

where \( H \) is the length of the space and \( L \) is the number of
<table>
<thead>
<tr>
<th>Depth (h = 30 km, L = 10)</th>
<th>Temp. ( N(t)/N(0) )</th>
<th>Temp. ( N(t)/N(0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>300</td>
<td>1.000</td>
</tr>
<tr>
<td>15</td>
<td>699</td>
<td>1.000</td>
</tr>
<tr>
<td>30</td>
<td>1056</td>
<td>1.001</td>
</tr>
<tr>
<td>45</td>
<td>1371</td>
<td>1.009</td>
</tr>
<tr>
<td>60</td>
<td>1643</td>
<td>1.026</td>
</tr>
<tr>
<td>75</td>
<td>1873</td>
<td>1.044</td>
</tr>
<tr>
<td>90</td>
<td>2060</td>
<td>1.051</td>
</tr>
<tr>
<td>105</td>
<td>2206</td>
<td>1.041</td>
</tr>
<tr>
<td>120</td>
<td>2309</td>
<td>1.018</td>
</tr>
<tr>
<td>135</td>
<td>2371</td>
<td>.979</td>
</tr>
<tr>
<td>150</td>
<td>2392</td>
<td>.831</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Depth (h = 15 km, L = 20)</th>
<th>Temp. ( N(t)/N(0) )</th>
<th>Temp. ( N(t)/N(0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>300</td>
<td>1.000</td>
</tr>
<tr>
<td>15</td>
<td>699</td>
<td>1.000</td>
</tr>
<tr>
<td>30</td>
<td>1056</td>
<td>1.001</td>
</tr>
<tr>
<td>45</td>
<td>1371</td>
<td>1.009</td>
</tr>
<tr>
<td>60</td>
<td>1643</td>
<td>1.026</td>
</tr>
<tr>
<td>75</td>
<td>1873</td>
<td>1.044</td>
</tr>
<tr>
<td>90</td>
<td>2060</td>
<td>1.051</td>
</tr>
<tr>
<td>105</td>
<td>2206</td>
<td>1.041</td>
</tr>
<tr>
<td>120</td>
<td>2309</td>
<td>1.018</td>
</tr>
<tr>
<td>135</td>
<td>2371</td>
<td>.979</td>
</tr>
<tr>
<td>150</td>
<td>2392</td>
<td>.831</td>
</tr>
</tbody>
</table>

**Note:** \( N(t)/N(0) = \) Relative Number of Tubes/cm.\(^2\)
increments. The time increment Δ is selected as indicated above.

The approximations to the differential operators used are

\[ \frac{\partial f}{\partial x} = \frac{f^{m}_{l+1} - f^{m}_{l}}{h} \]

\[ \frac{\partial f}{\partial t} = \frac{f^{m-1}_{l} - f^{m}_{l}}{\Delta t} \]

\[ \frac{\partial^2 f}{\partial z^2} = \frac{f^{m}_{l+1} - 2f^{m}_{l} + f^{m}_{l-1}}{h^2} \]

where \( f^{m}_{l} \) is the value of the function at the point \((m, l)\) in the net.

The velocity equation in this approximation is

\[ U_{l}^{n} = -U_{0} \exp \left\{ \frac{E}{kT_{l}^{m}} (1 + \eta h l) \right\} \]  \[ \text{C.3} \]

where \( U_{0} = \Delta \rho g a^2 / c \eta_{0} \)

when the form \( \eta = \eta_{0} \exp \frac{E (1 + \eta z)}{h (T - C)} \) for the viscosity is used, \( T^{m}_{l} \) is replaced in equation C.3 by \( T^{m}_{l} - C \) and the calculations proceed as before unless \( T^{m}_{l} - C < 0 \), then the velocity, \( U_{n}^{m} = 0 \), is used.

The conservation of energy equation becomes

\[ T_{l+1}^{m+1} = T_{l}^{m} + \frac{c}{h} \left\{ \frac{R (T^{m}_{l+1} - 2T^{m}_{l} + T^{m}_{l-1})}{h^2} + U_{l}^{m+1} \Delta \rho \pi a^2 N^{m}_{l} \right\} \]

\[ \left[ \frac{c}{h} \left( T^{m}_{l+1} - T^{m}_{l} \right) - \frac{g}{b} \right] + \frac{A \pi a^2 N^{m}_{l}}{b} + Q_{E} \]  \[ \text{C.4} \]
and the conservation of mass equation is

\[ N_l^{m+1} = N_l^m - \frac{r}{h} (N_{l+1}^m U_{l+1}^m - N_l^m U_l^m) \]  

The boundary conditions on the energy equation are at \( Z = 0 \)

\[ T_0^m = T_0 \]

and to express the condition of no heat flow at \( Z = H \), it is necessary to extend the domain of definition of the temperature to include the point \( N + 1 \), and define

\[ T_{L+1}^m = T_L^m \]

The initial temperature distribution is

\[ \bar{T}_l^0 = -\frac{T_H - T_0}{H^2} (n l)^2 + 2 \left( \frac{T_H - T_0}{H} \right) n l + T_0 \]

The boundary condition on the conservation of mass equation at \( Z = 0 \) is

\[ U_0^m = 0 \]

The use of the equations are simpler if the condition of no mass transport across \( Z = H \) is satisfied by extending the domain of definition as in the temperature case and defining

\[ N_{L+1}^m = 0 \]

which insures that no mass is transported across the lower surface.

Equations C.3, C.4, and C.5 are the equations programmed and run on the computer.
J. Downs was born and attended school in Savannah, Georgia. He graduated from the Georgia Institute of Technology with highest honor in 1956, receiving a B. S. in Physics. At the Massachusetts Institute of Technology he had a two year tuition scholarship (1956 and 1957). He was a National Science Foundation Fellow for two years (1958 and 1959).
BIBLIOGRAPHY


MacDonald, G. J. F., 1956, Quartz-Coesite Stability Relations at High Temperatures and Pressures, American Journal of Science, 254, p.713.


Stewart, D., 1960, Personal Communication.
