Borehole Resistivity Inversion

by

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Abstract

In this thesis we develop a procedure for performing the inversion of borehole resistivity data using the software package developed by Western Atlas Logging Services, Houston, TX. Direct current resistivity methods, namely lateral sounding and conventional laterolog methods, are the main interest in this thesis. In resistive formations drilled with a conductive mud, where induction methods are not logged, it becomes imperative to combine these two methods in order to provide a reliable solution to the inversion problem. Lateral sounding provides comprehensive information about the resistivity distribution away from the borehole, while the higher resolution of the laterolog allows for detailed delineation of the formation.

Computationally, the inversion is performed using the constrained least-squares Marquardt algorithm combined with singular value decomposition. The nonlinear inversion problem is linearized after each iteration of the Marquardt method. One of the main benefits of the algorithm is its ability to incorporate all resistivity/conductivity methods into a unique solution that is able to explain and satisfy all measurements. Several levels of inversion analysis are considered, from one-dimensional inversion to a rigorous and comprehensive two-dimensional approach. With the two-dimensional approach, the data need not be corrected for borehole and shoulder bed effects.

We demonstrate the method with multiple synthetic examples in which the algorithm successfully recovers the formation parameters. Different noise levels, resistivity contrasts, depths of invasion, and initial guesses are considered. The method is then applied to field data consisting of lateral sounding logs and laterologs. The inversion results provide the resistivity variation away from the borehole. In most cases, the true to invaded zone resistivity ratio and the depth of invasion clearly indicate the layers with the best reservoir properties. In general, the field data inversion results agree very well with the available perforation data.

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Chapter 1

Introduction

Rock resistivity is one of the most important logging measurements because it is generally a function of rock porosity and the resistivity of the fluid occupying the pore space, the key rock properties in the search for hydrocarbons. Modern logging technologies allow for the recording of enormous amounts of data, which generally leads to accurate analysis of formation properties. In practice, resistivity measurements typically provide apparent values of the formation resistivities associated with particular formation volumes that depend on the characteristics of the measurement devices. These apparent values depend on the variation of physical and/or geological characteristics of the surrounding formations. When interpreting the data, one usually defines an earth model, which is a simplified earth structure described by a number of parameters, in our case, layer thicknesses and resistivities. The interpretation of the data consists of extracting the information from the apparent resistivities to derive the actual formation parameters.

One of the earliest methods for borehole resistivity data interpretation is the use of conventional correction charts. The correction charts represent the dependences and relationships among different earth model parameters. Various assumptions, such as layer boundary positions or the presence of drilling mud invasion in each layer, have to be made prior to interpretation. With chart interpretation, one usually attempts to introduce various corrections to the data, i.e. to exclude the influence of certain parameters on the tool response, and afterwards to determine the rest of the parameters. Therefore, correction charts rely heavily on the log analyst’s experience, consume much time, and most importantly, rarely provide accurate and comprehensive information which would combine and explain all measurements. Faster and more powerful data processing and interpretation techniques, such as inversion, are necessary for extracting the required information from such large data collections.

Geophysical inversion attempts to find the best fitting earth model for the field data by minimizing the misfit (error) between the data and the theoretical responses obtained by forward modeling. In nonlinear problems, we start with defining the initial set of parameters (initial guess) for which the theoretical tool responses are linearized in the vicinity of the model. A parameter
change is calculated in order to reduce the difference between the measured data and the theoretical response. If this difference can be further decreased by varying the parameters, the procedure is repeated iteratively until the algorithm converges to a solution. Convergence means that the misfit cannot be further reduced by changing the parameter vector. The obtained solution is not necessarily the correct one because local minima of the misfit function may exist. However, in many cases such minima can be avoided by using robust algorithms and choosing an appropriate initial guess and parameter constraints based on the “physical meaningfulness” of the solution for a particular problem.

Inversion techniques to solve for parameters of underground formations have been used in surface geophysics for many years. Inversion has been applied in borehole resistivity more recently. In 1984, Yang and Ward introduced the inversion of borehole normal resistivity logs via ridge regression. Their earth model is horizontally layered with radially homogeneous and isotropic layers and no borehole effect, which allows the use of the analytical solution for the potential distribution of a point source in an arbitrary layer. Thus, Yang and Ward's earth model is one-dimensional and has no vertical boundaries. Other studies on the 1D inversion of borehole resistivity data include Dyos (1987), Spalburg (1989), Wallase et al. (1991).

Whitman et al. (1989, 1990) introduced a more complex model which accounted for the borehole effect and zones saturated with borehole fluid (the invaded zones). They used the finite difference approximation approach on an exponentially expanding grid in both vertical and horizontal directions. Therefore, the accuracy of the results is largely determined by the rate of grid expansion. Whitman (1989) considered both normal and lateral logs; however, each of the logs is treated separately. However, if the data recorded with different tools are treated and inverted separately, we may have as many solutions (earth models) for the inversion problem as there are measurements. Ultimately we would like to combine and invert all measurements simultaneously, providing for an improved solution.

In 1994, Mezzatesta et al. (Western Atlas Logging Services) introduced a two-dimensional joint inversion of borehole resistivity measurements using the Marquardt algorithm combined with singular value decomposition. With the joint inversion approach, one can simultaneously invert as many logs as there are available and hence satisfy and explain all resistivity measurements at once. In other words, the algorithm provides the means for integrating all available resistivity methods regardless of their physical principles.
Perhaps one of the main advantages of the algorithm developed by Western Atlas Logging Services is its "comprehensiveness" in terms of the number of tools and measurements that can be used in the inversion. The incorporation of normal and lateral resistivity devices into the software have allowed us to apply an entirely new approach in dealing with electrical logging data.

Normal and lateral logs are widely and successfully applied in Russia; moreover, some Western companies have recently made attempts to make use of the latest technologies and developments and revive nonfocused logging (Vallinga and Yuratich, 1993). Considering the growing need in the industry for processing and interpreting nonfocused borehole resistivity data, this thesis is devoted to the solution of the inverse problem for lateral resistivity sounding. The joint inversion software, along with the eXpress™ system, has been generously provided to us for this project by Western Atlas.

With lateral sounding, we have several resistivity measurements providing different vertical and radial resolutions. Each measurement is to a certain extent affected by the presence of the borehole fluid and in some cases by its penetration into the formation, as well as by the so called shoulder effect, which results from the finite layer thickness. The integration of these measurements allows us to yield a single distribution of resistivities away from the borehole consistent with all measurements. The inversion process enables the tool behavior to be simulated and simultaneously accounts for the borehole, invasion and shoulder effects.

The depth of investigation of any resistivity tool is largely defined by the tool geometry, especially by the spacing between the receiving and transmitting electrodes. Therefore, in a set of resistivity measurements with different depths of investigation, the shallower measurements are considered to be influenced mostly by the zones near the borehole wall, while the deeper measurements reflect the apparent resistivity far from the borehole. This means that, in a layer invaded with the drilling mud, shallow devices provide the apparent resistivity closest to the invaded zone resistivity. The deepest measurements will sample the resistivity of the uncontaminated part of the formation. Figures 1-1 through 1-5 show the results of the inversion of five resistivity logs with different depths of investigation (lateral sounding logs). A simple three-layer model was used to simulate and invert the synthetic data. Only the middle layer is invaded (depth of invasion $L_{xo} = 1$ m), with the invaded zone resistivity of 5 ohm-m and true resistivity of 100 ohm-m. The upper and lower layer resistivities are 3 ohm-m, and the mud is 1 ohm-m with a borehole diameter of 0.2 m.
Figure 1-1. Results of separate inversion of the lateral sounding log: L 0.45. (Synthetic and theoretical curves overlap.)
Figure 1-2. Results of separate inversion of the lateral sounding log: L 1.05.
Figure 1-3. Results of separate inversion of the lateral sounding log: L 2.25.
Figure 1-4. Results of separate inversion of the lateral sounding log: L 4.25.
Figure 1-5. Results of separate inversion of the lateral sounding log: L 8.50.

<table>
<thead>
<tr>
<th>Model Lxo</th>
<th>Model Rm</th>
<th>Inverted Lxo</th>
<th>Model Rxo</th>
<th>RXO from L 8.50</th>
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<th>Theor. L 8.50</th>
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Model parameters: Lxo, Rm, Rxo, RT.
Figure 1-1 shows the inversion results for the shallowest measurement (L 0.45); the size of the device (and hence the depth of investigation) increases from Figure 1-1 to 1-5. The leftmost tracks in Figures 1-1 - 1-5 show the model invasion profile (invasion only in the middle layer) and the depths of invasion resulting from the inversion. The synthetic responses of the inverted logs and the theoretical logs resulting from inversion are shown in the right tracks. Finally, the middle tracks show the model and the inverted resistivities. It is clear from the figures that even when the error between the “real” and the theoretical data is very small (e.g. Figure 1-1), the earth model parameters in all cases \( R_{xo}, L_{xo}, R_t, \) and layer thickness) have been recovered incorrectly for the middle layer. This results from the fact that the information contained in only one log is not enough to describe the resistivity variation away from the borehole. Furthermore, the inversion results strongly depend on the initial earth model parameters, as illustrated in Figure 1-6. The figure shows two inversion results for the log of medium depth of investigation using slightly different initial resistivities (initial \( R_{xo} = R_t = 8 \) ohm-m for Figure 1-6a and \( R_{xo} = 1, R_t = 5 \) ohm-m for Figure 1-6b, the rest of the parameters are the same). The two results are different even though the data misfits in both cases do not exceed 10%. This indicates that there exist many solutions to our inversion problem that would satisfy the data misfit criterion, or in other words, that the problem is nonunique, or ill-posed. The probability of the algorithm leading to any particular solution largely depends on the initial guess.

Figure 1-7 shows the results of a simultaneous inversion of all five logs, shown separately in Figures 1-1 through 1-5, using the same synthetic data and initial earth model. One can see clearly in Figure 1-7 that the simultaneous inversion produced a virtually exact solution (the earth model and inversion results completely overlap), thus proving that only the entire information from all five logs allows us to solve for the resistivity distribution around the borehole. Not only did we obtain a very accurate result, we also matched and satisfied all resistivity data simultaneously. (The theoretical logs are not shown in the figure for the reason of complete overlapping with the synthetic logs.)

The accuracy of the results is a very important aspect of the inversion algorithm. Prior to applying the inversion algorithm to field data, we have to define the range of resistivity ratios between the borehole and the adjacent formations that can be resolved with a satisfactory degree of accuracy. An extensive study of the invasion on resistivity tool responses is also needed.
Figure 1-6. Comparison of two separate inversion runs of the lateral sounding log L 2.25 using different initial guesses.
Figure 1-7. Results of simultaneous inversion of 5 sounding logs.
Earth model: $R_m = 1$, $R_{sh} = 3$, $R_{xo} = 5$, $R_t = 100$ ohm-m, $L_{xo} = 1$ m.
and becomes available using this software.

The level of noise in the data associated with the tool itself is another vital issue in electrical sounding. Therefore, considerable work is needed to study the error propagation from the data to the parameter space.

Finally, several field cases are presented that allow us to estimate the inversion algorithm in practice. The results of the field inversion are compared with the available perforation data in order to make sure that our solutions are not only mathematically accurate but also physically meaningful. Both logging and perforation data have been generously contributed for this project by the Central Geophysical Expedition, Moscow, Russia. The inversion produces a layered resistivity structure. In general, it provided a reliable indication of the reservoir layers with high formation resistivities and considerable invasion. The parameter confidence intervals indicate how well the parameters are resolved and to what extent the responses are affected by a particular parameter. The inversion results agreed very well with the perforation data. In one field case, some a priori information, such as auxiliary borehole logs, had to be incorporated in order to improve the interpretation.

To summarize, the present study will allow us to:

- combine different resistivity measurements to yield a single and accurate resistivity distribution away from the borehole;
- analyze the ranges of the parameter variations that can be successfully inverted using the presented software;
- examine the influence of borehole and formation parameters, e.g. mud resistivity or invaded zone length, on the tool responses;
- study the influence of noise level on the accuracy of inversion results, in other words, the error propagation from the data space to the parameter space;
- apply the inversion algorithm to the field data and analyze the algorithm behavior in different formation sequences.
Chapter 2

Borehole resistivity logging

2.1 Earth model parametrization

The formation model geometry used throughout this thesis is shown in Figure 2-1. The major assumptions regarding the model are the following:

- it is symmetric with respect to the borehole axis;
- the model is two-dimensional: the resistivity changes both vertically (from layer to layer) and radially (borehole, invaded zone, undisturbed formation);
- in the vertical direction, the model consists of layers of different thicknesses parallel to the surface and perpendicular to the borehole axis;
- radially, the borehole and invaded zones within layers represent concentric cylinders around the borehole axis, perpendicular to the layer boundaries (i.e. the resistivity variation within a layer is determined not by a different formation mineral content but by a variation in fluid saturation).

Figure 2-1 shows parametrization of the subsurface as applied in the inversion. The following notation is used in this figure and throughout this thesis:

- **borehole**: \( R_m \) = mud resistivity, ohm-m;
  - \( BHD \) = borehole diameter, m;
- **invaded zone**: \( R_{xo} \) = resistivity of invaded zone, ohm-m;
  - \( L_{xo} \) = depth of invasion (\( BHD + 2*L_{xo} = \) diameter of invasion), m;
- **uncontaminated zone**: \( R_t \) = uncontaminated zone, or true, resistivity, ohm-m;
  - \( R_{sh} \) = shoulder bed resistivity, ohm-m.

The figure shows the simplest two-dimensional earth model consisting of three layers, two of them being “shoulders” described by a single resistivity, and the middle layer being the “target” layer. The layer of interest is described by the resistivities of the invaded and uncontaminated zones and the diameter of invasion.
Figure 2-1. 2-Dimensional earth model and its parametrization.

borehole: $R_m = \text{mud resistivity};$
BHD = borehole diameter, m;
invaded zone: $R_{xo} = \text{resistivity of invaded zone};$
$L_{xo} = \text{depth of invasion, m};$
uncontaminated zone: $R_t = \text{uncontaminated zone, or true, resistivity};$
$R_{sh} = \text{shoulder bed resistivity}.$
2.2 Borehole resistivity sounding

*Apparent resistivity concept*

Electric resistivity of the rocks depends on many factors, such as rock mineral content, porosity, temperature and pressure conditions, formation water mineralization, and the content of the fluid saturating the rock. Hence, it can help us determine the section lithology, structure of the rocks, hydrocarbon content, estimated formation net pay, etc.

The apparent resistivity of the rocks around the borehole is usually determined by the measurements of potential difference $\Delta U$ or electric field $E$ caused by the current source of magnitude $I$. In an isotropic medium, the differential form of Ohm's Law takes the form:

$$ j = \sigma E, \quad (2.1) $$

where $j$ is current density, $\sigma$ is the conductivity of the medium, and $E$ is the gradient of a scalar potential:

$$ E = -\text{grad} U. \quad (2.2) $$

The relationship between the electric resistivity (conductivity) of the isotropic medium, current density, electric field, and potential can then be written as:

$$ j = \sigma E = \frac{1}{\rho} \cdot \frac{\partial U}{\partial r}, \quad (2.3) $$

where $r$ is the distance between the current source and the point of measurement. In an isotropic medium, the magnitude of $\rho$ in the formula above is the true resistivity of the medium, while in a nonisotropic medium it is its apparent resistivity $\rho_a$.

Let us consider a point current source of magnitude $I$ at a point $A$ in a homogeneous isotropic medium of resistivity $\rho$. For a sphere of arbitrary radius $r$ with the center at the current source, the current is evenly distributed on the surface of a sphere and its density is

$$ j = \frac{I}{S} = \frac{I}{4\pi \cdot r^2}, \quad (2.4) $$

where $S = 4\pi r^2$ is the surface area of the sphere.
Figure 2-2 shows the current lines and equipotential surfaces for a homogeneous formation of infinite thickness. From equation (2.3) it follows that

$$ E = \frac{\partial U}{\partial r} = j\rho . $$

(2.5)

Substituting (2.4) in (2.5), integrating and neglecting the integration constant (I=0 when r approaches infinity), we obtain:

$$ U = \frac{\rho \cdot I}{4\pi r} . $$

(2.6)

Let the electric field potentials be measured at the electrodes M and N at the respective distances from the source $r_1 = AM$ and $r_2 = AN$. If the current return electrode is considered to be infinitely far from the point of measurement, then the potential difference is

$$ \Delta U = \frac{\rho \cdot I}{4\pi} \left( \frac{1}{AM} - \frac{1}{AN} \right) = \frac{\rho \cdot I}{4\pi} \cdot \frac{MN}{AM \cdot AN} . $$

(2.7)

The resistivity or, in a nonhomogeneous case, the apparent resistivity is then

$$ \rho_a = \frac{K \cdot \Delta U}{I} , $$

(2.8)

where

$$ K = 4\pi \cdot \frac{AM \cdot AN}{MN} $$

is the sonde geometric factor usually referred to as the K-factor.

**Lateral sounding**

Normal and lateral sondes were the first devices invented for measuring formation resistivity. They appeared both in Russia and the U.S. in the late 1920's - early 1930's. (C. Schlumberger conducted the first electrical logging runs in 1926 - 1928.) Both sondes consist of four electrodes, one on the surface and three downhole with different arrangements. One pair of electrodes emits and receives the survey current and the other pair measures the potential drop in the formation.
Figure 2-2. Computed current patterns (arrows) and equipotential surfaces for a normal sonde in a homogeneous formation. (After Gianzero, S., Anderson, B., Introduction, 1992)
The current electrode on the surface is often considered to be at infinite distance from the point of measurement, and one usually speaks of a three-electrode nonfocused measurement scheme. With a scaling change, the potential difference is then presented as a log of apparent resistivity (see formula (2.8)).

Figure 2-3 shows the electrode arrangements for both normal and lateral sondes. The current and measuring electrodes are usually denoted as pairs: (A and B) and (M and N), respectively. For a normal sonde, the distance between the two “nonpaired” electrodes is considerably smaller than the distance between the “paired” ones (AM<<MN or AM<<AB), and vice versa for a lateral sonde (MN<<AM or AB<<AM).

Utilization of the lateral sounding technique for analysis of radial resistivity distribution represents the key difference of Russian resistivity logging compared to the West. The essence of the method consists in measuring apparent resistivity opposite to the interval being studied by means of lateral and normal sondes of various spacings. It allows us to discover the existence of strata penetrated by the drilling mud, as well as the resistivities of the invaded zone and the undisturbed part of the formation and the depth of penetration with sufficient accuracy.

In general, both normal and lateral sondes also include those with two current and one measuring electrodes in the borehole. However, they are rarely used and are not shown in Figure 2-3. The main reason for placing the current return electrode on the surface is to avoid the Delaware effect. If the current return electrode B is downhole, in the presence of very resistive formations, the path of the least resistance to the return is through the borehole. Since B is a current sink, it drives the potential of the reference electrode N below the value it would have had in the absence of the resistive formation. The result is a gradual and substantial increase in measured resistivity as B and N enter the resistive medium, even though the sonde itself is far removed from the resistive medium.

The size (or length) of the normal sonde is the distance between the nonpaired electrodes $L_{\text{norm}} = AM$. The recording point is referred to as O and is considered to be at half-distance between A and M (Figure 2-3a). For a lateral sonde, the size L is the distance from the nonpaired distant electrode to half distance between the paired electrodes, the latter being also the measurement point $L_{\text{lat}} = AO$ (Figure 2-3b). Figure 2-3c shows a lateral sonde in the borehole.
Figure 2-3. Electrode arrangements for (a) normal sondes; (b) lateral sondes; (c) borehole example (lateral sonde).
The suite of the so-called “standard logging” curves that are run in most boreholes includes five direct (bottom) lateral sondes of the following sizes: \( L = 0.45, 1.05, 2.25, 4.25, \) and \( 8.50 \) m; one inverted (top) lateral sonde \( L = 2.25 \) m; and one normal sonde \( L = 0.5 \) m. Some sondes or their sizes may vary depending on a specific borehole environment or on a particular problem. The normal sonde has a symmetric response and provides the apparent resistivity measurements which are very close to the true resistivity in resistive layers of great thickness. However, they prove highly unreliable in thin layers, as we will show later. The lateral sondes, having various depths of investigation, provide the information about the resistivity variation away from the borehole. The top lateral sonde is often used in manual interpretation for adjusting the layer boundaries (if the log is well differentiated) as well as in quantitative interpretation.

**Theoretical responses: lateral sondes**

Figure 2-4 shows an example of the theoretical response of an “ideal” lateral sonde (the distance between the measuring electrodes \( MN \rightarrow 0 \)) against resistive layers of different thicknesses surrounded by conductive shoulders. The solid lines represent the responses without borehole influence. Figure 2-4b also shows an example of borehole influence on the response in a dashed line.

For a layer of large thickness (thickness considerably larger than the tool size, \( h > AO \)), the response consists of five intervals corresponding to the process of moving the tool along the borehole (Figure 2-4a). Intervals \( ab, de \) and \( gh \) correspond to the period of time when all electrodes are in the same layer (lower shoulder 3, resistive layer 2, or upper shoulder 1, respectively). For intervals \( bc \) and \( ef \), the tool is positioned in such a way that the current and potential electrodes are in different layers and hence are divided by a layer boundary.

Figure 2-4 shows that the responses of lateral sounding tools are asymmetric with respect to the center of layer 2 even if \( \rho_1 \) and \( \rho_3 \) are equal. Note that in layer 2 the apparent resistivity above the middle of the layer is lower than the true resistivity \( \rho_2 \), while below the middle of the layer the apparent resistivity is greater than \( \rho_2 \). It is caused by the change in current density when crossing the layer boundaries. The current tends to flow downward below the middle of layer 2 (since \( \rho_2 > \rho_3 \)), and upward above it (\( \rho_2 > \rho_1 \)).
Figure 2-4. Theoretical responses of the bottom lateral sondes against resistive layers of different thicknesses surrounded by conductive layers ($\rho_1 < \rho_2 > \rho_3$).
The ratio of apparent to true resistivity measured between the electrodes M and N is determined by the ratio of potential gradients between M and N in nonhomogeneous and homogeneous media:

\[
\frac{\rho_a}{\rho_{MN}} = \frac{E}{E_H} = \frac{j \rho_{MN}}{j_H \rho_{MN}} = \frac{j}{j_H}, \quad \text{and} \quad (2.9)
\]

\[
\rho_a = \frac{j}{j_H} \rho_{MN}, \quad (2.10)
\]

where \( j_H \) is the current density in a homogeneous formation.

Hence, for a bottom lateral sonde until the current electrode A crosses the middle of the resistive layer, in the lower part \( j \) is greater than \( j_H \) and \( \rho_a > \rho_2 \), and in the upper part \( j \) is lower than \( j_H \) and \( \rho_a < \rho_2 \). Based on this fact, the layer boundaries for an ideal lateral sonde are picked at the maximum (bottom) and minimum (top) apparent resistivity points.

Interval \( ef \) (\( ef = AO \)) corresponds to the tool positions when potential electrodes MN are crossing the upper boundary and until the electrode A enters the resistive layer. The ratio of the current reflected by the upper boundary into layer 1 to the current transmitted into layer 2 is then determined by the ratio \( \rho_2/\rho_1 \). When \( j \ll j_H \), \( \rho_a = (j/j_H)\rho_2 \), and \( \rho_a \ll \rho_2 \).

When the thickness of the layer is approximately equal to the tool size (\( h \sim AO \)), the resistivity \( ef \) increases (Figure 2-4b). This results from the influence of not only the upper but also the lower shoulder, where the current density is higher.

For a very thin layer and large \( AO \) (\( h < AO \)), the maximum apparent resistivity of layer 2 decreases dramatically (Figure 2-4c). This results from the strong influence of the shoulders. One can also see a distinct second local maximum (point \( b \)) at the distance \( AO \) below the layer 2. The decrease in \( \rho_a \) in the lower halfspace from \( b \) to \( e \) occurs in the interval equal in length to the thickness of layer 2, with the current electrode A moving from the bottom to the top of layer 2. From \( e \) to \( c \), the current electrode is already above layer 2 and \( j \ll j_H \), \( \rho_a = (j/j_H)\rho_3 \) and \( \rho_a \ll \rho_3 \).

The theoretical responses for the top lateral sondes (when the current electrode is placed below the measuring ones) can be explained in the same way and will represent approximately the mirror images of the bottom sondes' theoretical responses.
The above discussion is valid for an “ideal” sonde (MN -> 0). In theory, we would like MN to approach zero to obtain the most accurate potential difference log. However, in a real case the signal level of the tool does not allow us to register the difference in potential change over very small distances. For the largest commonly used lateral sonde (AO = 8.5 m), the distance MN is equal to 1 m. Figure 2-5 corresponds to the theoretical response in such a case. The local anomaly in the lower halfspace stays unchanged since it is caused by the moving of the same current electrode A across the boundaries. The response is different from an “ideal” case only in the intervals where the boundaries are being crossed by the measuring electrodes. The magnitude of $\rho_{MN}$ and hence, $\rho_a$, changes more gradually, effectively smoothing out the curve. The extremum points corresponding to the layer boundaries move up by the distance MN/2. (For a top sonde they will move down by the distance MN/2.)

Theoretical responses: normal sondes

Figure 2-6 shows the theoretical responses of the normal sondes of different sizes for a high resistivity layer surrounded by conductive shoulders. “Ideal” normal sonde responses are always symmetric with respect to the center of the layer, because a bottom sonde AM is absolutely equivalent to a top sonde MA. Hence, in our discussion of normal theoretical responses, we can omit either half of the layer.

For a layer of a large thickness (h > AO), the response in the lower half of the layer consists of three intervals (Figure 2-6a). Intervals $ab$ and $cd$ correspond to the sonde positions where all electrodes are in the same layer (lower shoulder 3 or resistive layer 2). For the interval $bc$, the current and potential electrodes are divided by a layer boundary.

The formulas (2.9) and (2.10) for a normal sonde become:

\[ \frac{\rho_a}{\rho_M} = \frac{U_M}{U_H}, \text{ and} \]

\[ \rho_a = \frac{U_M}{U_H} \rho_M, \]

(2.11)
Figure 2-5. Theoretical responses for ideal and realistic lateral sondes.
Figure 2-6. Theoretical responses of normal sondes against resistive layers of different thicknesses surrounded by conductive layers ($\rho_1 < \rho_2 > \rho_3$).
where $U_H$ is the potential in a homogeneous formation. In the interval $ab$, the apparent resistivity increases with the sonde approaching the bottom of the resistive layer as the potential in the lower halfspace increases. For a thick layer, the resistivity in the interval $bc$ hardly changes (Figure 2-6a); however, with the size of the tool approaching the layer thickness, it decreases slightly (Figure 2-6b). This is caused by the current leaking into the upper halfspace and thus decreasing the potential at the measuring electrode M. The smaller the layer thickness, the sharper the potential drop.

In the interval $cd$, the apparent resistivity increases sharply and in thick layers approximately reaches the true resistivity in layer 2. For AM approaching the layer thickness ($AM \sim h$), the readings at the middle point are considerably lower than the true resistivity because of the current leakage into the upper halfspace (Figure 2-6b). Again, the thinner the layer, the greater the influence of the shoulder layers on the readings in resistive layer 2.

When $AM < h$ (Figure 2-6c), the normal sonde response is distorted due to the fact that the current does not tend to flow into the resistive layer. Only the $ab$ interval stays similar to that in the above figures. After the current electrode A crosses the upper boundary of layer 2, the two electrodes are divided by a thin resistive layer. This results in the minimum readings against the resistive layer until electrode M reaches the bottom of it, producing the interval $cc'$ whose length is equal to $(AM - h)$.

Due to such a high uncertainty in the readings of normal sondes in thin resistive layers, we did not use normal measurements in the inversion process. However, normal logs can be taken into account by comparing them with the theoretical responses calculated for the earth models resulting from inversion. The quality of normal logs can also be estimated from such a comparison.

**Effect of formation parameters on lateral theoretical responses**

Let us now consider the responses affected by the changes in resistivity and depth of invasion from layer to layer. Figures 2-7a-c show the current patterns calculated for the earth models described with different numbers of parameters. Figure 2-7a shows the current and equipotential lines for the case of a homogeneous halfspace influenced only by borehole drilling mud; Figure 2-7b is also influenced by the invaded zone resistivity; Figure 2-7c illustrates a realistic case also affected by the layer thickness.
Figure 2-7. Computed current patterns and equipotential surfaces for a normal sonde showing:
(a) borehole effect;
(b) invasion;
(c) a thin bed with invasion
\[ R_{xo} = 10 \text{ ohm-m}, \ R_t = 50 \text{ ohm-m}, \ R_{sh} = 5 \text{ ohm-m} \]
Figures 2-8 through 2-12 show the theoretical responses of the five lateral measurements described above for the same resistivity earth model, but for different layer thicknesses and invaded zone diameters. The earth model parameters are the following: $R_{sh} = 3$ ohm-m, $R_{xo} = 5$ ohm-m, $R_t = 100$ ohm-m, $L_{xo} = 0.5$ m (a resistive invaded layer surrounded by conductive shoulders). The borehole diameter is 0.2 m with the mud resistivity $R_m = 1$ ohm-m. From Figure 2-8 to 2-10 the thickness of the layer decreases from 10 m to 2 m to 0.5 m. The shaded area shows the invasion profile. For the thick layer (Figure 2-8), it is evident that the deeper the measurement, the less it is affected by the invaded zone, and consequently the closer its readings are to the true formation resistivity.

In Figures 2-9 and 2-10, one can clearly see the local maxima below the resistive layer in cases when the sonde size is larger than the layer thickness. For a 2-meter layer, only the two deepest measurements have these second maxima, while for a 0.5-meter layer all measurements except for the shallowest one (LO.45) are considerably affected by the layer thickness. For the 10-meter layer, any visible influence of the bed thickness is reflected below the resistive layer only on the deepest measurement (L8.50), since its depth of investigation is of the order of the layer thickness.

Figure 2-11 illustrates the theoretical responses for a multi layer model. The resistivities in the high resistive layers are the same as above, while in conductive layers $R_t$ (or $R_{sh}$) = 3 ohm-m. Note that in the resistive layers the shallow measurements (for LO.45 and L1.05, $AO < h$) are roughly the same, while the deeper measurements differ, resulting from the effect of the current electrode crossing the boundaries as described above. In the deepest response, one can also see the superposition of the second maxima resulting from the two resistive layers.

Figure 2-12 shows the influence of the depth of invasion on the tool responses for a 2-meter layer (resistivities are the same as above). The depth of invasion changes from 0.2 to 0.5 to 1 m (the shaded area is the invasion profile), and obviously the larger the depth of invasion, the harder it is for the tool to "reach" the true formation resistivity. This results in the curves being less differentiated since the resistivities of the shoulders and the invaded zone are considerably lower than that of the uncontaminated formation.
Figure 2-8. Lateral sounding theoretical responses for a 10-meter layer. Model: $R_{sh} = 3$, $R_{xo} = 5$, $R_t = 100$ ohm-m, $L_{xo} = 0.5$ m.
Figure 2-9. Lateral sounding theoretical responses for a 2-meter layer.

Model: $R_{sh} = 3$, $R_{xo} = 5$, $R_t = 100$ ohm-m, $L_{xo} = 0.5$ m.

<table>
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<table>
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<tr>
<td>4.25</td>
</tr>
<tr>
<td>8.50</td>
</tr>
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</table>

Model: 
- $R_{sh} = 3$
- $R_{xo} = 5$
- $R_t = 100$ ohm-m
- $L_{xo} = 0.5$ m
Figure 2-10. Lateral sounding theoretical responses for a 0.5-meter layer. Model: \( R_{sh} = 3, R_{xo} = 5, R_t = 100 \text{ ohm-m}, L_{xo} = 0.5 \text{ m}. \)
Figure 2-11. Lateral sounding theoretical responses for a multi layer model. Conductive layers: $R_t = 3$ ohm-m; resistive layers: $R_{xo} = 5$, $R_t = 100$ ohm-m, $L_{xo} = 0.5$ m.
Figure 2.12: Lateral sounding theoretical responses for different invasion diameters.

Model: $R_h = 3$, $R_{x0} = 5$, $R_t = 100$ ohm-m.
2.3 Laterolog

One of the major disadvantages of lateral sounding is its relatively low vertical resolution. It results from the fact that the current from the point source flows in the borehole in all directions, leading to borehole and shoulder effects. The idea of forcing the current to flow into the formation against which the potential difference is being measured has been implemented in the laterolog device (Doll, 1951). The simplest laterolog device (LL3) is comprised of three electrodes: a survey current electrode located in the center of the array surrounded by two elongated guard electrodes, forcing the emitted currents to be perpendicular to the surface of the mandrel as shown in Figure 2-13. The smaller the center electrode, the more highly resolved the measurement.

In LL3, a constant current is applied to the center electrode. An auxiliary current of the same polarity is applied to the guard electrodes. The guard electrode current is automatically and continuously adjusted to maintain a zero potential difference between the center electrode and the guard electrodes. This forces the current emanating from the current electrode to flow into the formation. A drop in potential is caused by the flow of the current through the surrounding formation to a remote current return electrode. This potential difference is related to the resistivity of the formation.

The accuracy of the measurement is greatly enhanced by designing the array to focus on the zone of interest. With a laterolog, we still need a conductive borehole environment in order to transmit the current from the electrodes to the formation. However, the mud column has much less influence on laterolog readings than with lateral nonfocused methods.

The depth of investigation of LL3 is approximately 1-2 meters. With very shallow depths of invasion, the laterolog provides the apparent resistivity close to the true formation resistivity. One of the disadvantages of the LL3 lies in its limited depth of investigation. Having a single resistivity measurement (or even two measurements, shallow and deep, as in a Dual laterolog, which is not considered in this thesis), it is difficult to determine the resistivity distribution away from the borehole as well as the depth of invasion.

Figure 2-14 shows theoretical LL3 responses for a complicated earth model. We have three invaded layers divided by conductive shoulders. The borehole diameter is 0.2 m with the mud \( R_m = 1 \) ohm-m. In a resistive layer with \( R_{xo} \ll R_t \) the response is determined by the depth of invasion. With shallow depth of invasion, \( L_{xo} = 0.1 \) m (layer 2), the LL3 response is
Figure 2-13. Laterolog (LL3) electrode configuration.
Figure 2-14. Laterolog theoretical responses for a sequence of layers. $R_{sh} = 3$, $R_m = 1$ ohm-m, 
layer 1: $R_{xo} = 5$, $R_t = 100$ ohm-m, $L_{xo} = 0.5$ m;
layer 2: $R_{xo} = 5$, $R_t = 100$ ohm-m, $L_{xo} = 0.1$ m;
layer 3: $R_{xo} = 100$, $R_t = 5$ ohm-m, $L_{xo} = 0.5$ m.
approaching $R_t$. However, with deeper invasion, $L_{xo} = 0.5$ m (layer 1), the influence of invaded resistivity zone is substantial. In layer 3, the formation is more conductive than the invaded zone which leads to the characteristic “horns” (sharp decreases of resistivity) on the layer boundaries. The latter situation is not favorable for laterologs.

2.4 Summary

In this thesis, we are aiming to combine lateral sounding and laterolog techniques in order to provide both a mathematically accurate and physically meaningful solution. Lateral sounding measurements enable us to thoroughly examine the resistivity distribution away from the borehole and solve for both the invaded zone and the undisturbed zone parameters. However, vertical resolution of sounding measurements, especially the deeper ones, is not very high. Since the current is allowed to flow from the emitting electrode in all directions, the lateral sounding measurements are not very sensitive to layer boundaries with low layer resistivity contrasts. On the other hand, the laterolog uses guard electrodes held at the same potentials as the current electrode, to force the current to flow into the formation. This reduces the influence of shoulder beds and allows us to read the apparent resistivities very close to the true undisturbed formation resistivities in zones with shallow invasion. The laterolog also has a higher vertical resolution of the boundaries than the nonfocused measurements. The combination of both methods provides us with a most comprehensive and detailed picture of the resistivity distribution in both radial and vertical directions.
Chapter 3

Forward modeling and inversion

3.1 General considerations
The mathematical basis for any inversion is finding the best fitting earth model for the field data by means of minimizing the difference between the data and model theoretical responses. With the earth parametrized by layer thicknesses and resistivities, as shown in Figure 2-1, we first choose the initial earth model. The initial guess is a set of particular physical and geological conditions for which synthetic tool responses are generated and compared to the field data. The more information we have about such conditions for the data, the more educated is our initial guess, and hence, the faster we get to the solution. If we are not satisfied with the result, we keep changing the model parameters and simulating the responses until a predefined condition of convergence is met. Convergence criteria allow us to evaluate the misfit between the theoretical and field data. The process of varying the model parameters to achieve an acceptable data approximation is called optimization. Thus, in general, inversion depends on and is limited by

- forward modeling algorithms;
- optimization strategy;
- types and quality of the data available.

The principles of the forward modeling and inversion algorithms, as well as the optimization technique employed in the inversion software developed by Western Atlas Logging Services, are described in this chapter. The types and quality of data used in both synthetic and field data examples are discussed in Chapters 4 and 5.

3.2 Forward modeling principles
An effective forward modeling algorithm is always one of the main challenges in any iterative procedure. The technique of fast numerical simulation of induction and resistivity logs has been developed by Tamarchenko and Druskin (1993). A hybrid approach, consisting of a combination of integral equations and finite-difference schemes, is applied to a Green’s function problem. An analytical representation of the solution is obtained in each layer in the vertical direction, and the radial distribution of the field is approximated on an expanding grid. The
solution is then matched on the layers boundaries, leading to a tri-diagonal system of linear algebraic equations, where each element is a matrix. What follows is the description of the main principles employed in the numerical simulation algorithm. For detailed information on integral equations and numerical implementation, the reader is referred to Tamarchenko and Druskin (1993).

Figure 3-1 (a) shows the subsurface geometry for numerical simulation. The model is axially symmetric, with horizontal layers, intersected by a vertical cylindrical borehole. Each layer contains, or may contain, an invasion zone with cylindrical boundaries coaxial with the borehole.

Let us explain the theoretical principles of the forward modeling code using the example of a simplified three-layer model intersected by the insulating mandrel as shown in Figure 3-1 (b). Note that the borehole and invaded zone are not included.

The electromagnetic field distribution is described by a partial differential equation. For a DC current, the function $u(x, s)$ represents a potential of a circular electrode positioned at a point $s = (r_s, z_s)$ and measured at the point $x = (r, z)$. The function $u(x, s)$ satisfies the following equation:

$$
a \frac{1}{r} \frac{\partial}{\partial r} r \sigma(r, z) \frac{\partial u}{\partial r} + \frac{\partial}{\partial z} r \sigma(r, z) \frac{\partial u}{\partial z} = \delta(r - r_s, z - z_s), \tag{3.1}
$$

where $\sigma(r, z)$ is the conductivity of the medium and $\delta(r - r_s, z - z_s)$ is the delta function.

Let us consider an $i$-th layer surrounded by the shoulders of different resistivities. In order to find the analytical solution for potential $u$ at a point $(r_0, z_0)$, we introduce an auxiliary point source $V$ in the layer $i$. The source potential $v$ satisfies the following equation:

$$
a \frac{1}{r} \frac{\partial}{\partial r} r \sigma(r, z) \frac{\partial v}{\partial r} + \frac{\partial}{\partial z} r \sigma(r, z) \frac{\partial v}{\partial z} = -\delta(r - r_0, z - z_0). \tag{3.2}
$$

We can now express the electric potential $u(r_0, z_0)$ in the layer $i$ using Green's theorem:

$$
u(r_0, z_0) = \oint_{S} \left( -u \frac{\partial v}{\partial n} + v \frac{\partial u}{\partial n} \right) dS. \tag{3.3}
$$

In the above equation, the potential of the auxiliary source $v$ represents the Green's function and
Figure 3-1. Forward modeling:
(a) 2-dimensional earth model with the tool positioned in the borehole;
(b) simplified layered structure without borehole.
\[ \frac{\partial}{\partial n} \] is the normal derivative at the surface \( S \) directed outwards. If we assume a homogeneous environment for the Green’s function everywhere outside the \( i \)-th layer (i.e. \( \rho_{i-1} = \rho_i = \rho_{i+1} \)), then the equation (3.2) for the auxiliary potential \( v \) can be simplified to:

\[ \frac{\partial^2 v_k}{\partial z^2} - \lambda_k^2 v_k = -\delta(z-z_0), \quad k = 1, 2, \ldots, \infty, \]  

(3.4)

where \( \lambda_k^2 \) are the eigenvalues of the operator \( \frac{\partial}{\partial r} \sigma(r,z) \frac{\partial}{\partial r} \), and \( v_k \) is the corresponding harmonic of the potential \( v \) from the following equation:

\[ \frac{\partial}{\partial r} \sigma(r) \frac{\partial v_k}{\partial r} + \lambda_k^2 \sigma(r) v_k = 0, \quad k = 1, 2, \ldots, \infty. \]  

(3.5)

The formula (3.3) for an \( i \)-th layer becomes:

\[ u_i(r_0, z_0) = u_i^0(r_0, z_0) + \int_{S_i} \left( -u_{i+1} \frac{\partial v_i}{\partial n} + v_i \frac{\partial u_i}{\partial n} \right) dS. \]  

(3.6)

The \( u_i^0(r_0, z_0) \) term in equation (3.6) appears only in the case when the current electrode is located in the same layer where the potential \( u_i \) is being determined.

On each horizontal boundary, both the potential and the current normal to the boundary are continuous, which can be expressed for an \( i \)-th layer in the following form:

\[ u_i = u_{i+1}, \quad \text{and} \quad \sigma_i \frac{\partial u_i}{\partial n} = \sigma_{i+1} \frac{\partial u_{i+1}}{\partial n}. \]

Equating the potentials on each boundary (left-hand side of equation (3.3)), we end up with a system of integral equations. The potential \( u_i \) satisfies the Newman boundary condition on the mandrel surface:

\[ \frac{\partial u_i}{\partial n} = 0. \]

The selection of the Newman boundary condition for the Green’s function \( v \) on the surface of the mandrel.
\[ \frac{\partial v_i}{\partial n} = 0 \]

allows us to reduce the surface of integration, i.e. to eliminate the interval (b) of the surface \( S_i \) as shown in Figure 3-1 (b).

Thus, applying Green's theorem in each layer and matching the potentials and current fluxes on the plane boundaries, we obtain a blocked tri-diagonal system of integral equations. The vertical direction is treated analytically, which proves to be computationally effective when dealing with hundreds of meters of data involving large numbers of layers. Radially, the differential operator in equation (3.5) is approximated with a finite-difference scheme on an expanding grid. Depending on the desired accuracy of the solution and the depth of investigation of a particular tool, it is possible to adjust the number of nodes in the grid as well as the distance between them.

Note that this method can be used for any type of radial conductivity distribution. The only requirement is to have enough nodes in the radial grid to approximate the field behavior properly. With such considerations, the borehole and invaded zones can be easily incorporated into our simplified formation model.

### 3.3 Inverse problem

**General concepts**

In this thesis, we shall consider only the discrete case of the inversion problem. The detailed description of the continuous case can be found in Mezzatesta (1996). In general, an inversion problem can be stated as:

\[ d = F(x) + e, \quad (3.7) \]

where the data vector \( d = \text{col} (d_1, d_2 \ldots d_n) \) of \( N \) observations is a function of the model parameters vector \( x = \text{col} (x_1, x_2 \ldots x_m) \) of \( M \) parameters, and \( e \) is the error vector due to the finite accuracy of the measurements.

Let us denote \( f = \text{col} (f_1, f_2 \ldots f_n) \) the model theoretical response vector of size \( N \) for a particular set of parameters. A perturbation of the model response about the initial set of parameters \( x_0 \) can be represented by the first order Taylor expansion:
\[ f(x) = f(x_0 + \Delta x) = f_0 + \sum_{j=1}^{M} \frac{\partial f}{\partial x_j} \Delta x_j + \text{HOT} , \]  

(3.8)

where \( \Delta x_j = x_j - x_{j,0} \) represents the formation parameter change vector. Neglecting higher order terms (H.O.T.), equation (3.8) can also be written as:

\[ \Delta f(x_0) = \sum_{j=1}^{M} \frac{\partial}{\partial x_j} f(x_0) \Delta x_j , \]  

(3.9)

where \( \Delta f \) is the absolute change in tool responses to variations in formation parameters. In matrix form, we write:

\[ \Delta f = J \Delta x . \]  

(3.10)

In this equation, \( J \) is the \( NxM \) Jacobian matrix of derivatives evaluated at \( x_0 \) for each data point:

\[
J = \begin{bmatrix}
\frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_N}{\partial x_1} \\
\vdots & \ddots & \vdots \\
\frac{\partial f_1}{\partial x_M} & \cdots & \frac{\partial f_N}{\partial x_M}
\end{bmatrix}.
\]  

(3.11)

**Logarithmic rescaling**

In many cases it is convenient to have the Jacobian in dimensionless form. For that, we have to consider relative increments \( \delta f_i \) and \( \delta x_j \), defined as:

\[
\delta f_i = \frac{\Delta f_i}{f_i} , \text{ and } \delta x_j = \frac{\Delta x_j}{x_j} , \quad i = 1, 2, \ldots N; \quad j = 1, 2, \ldots M.
\]  

(3.12)

Defining the diagonal matrices,

\[
F_{ij} = \begin{cases}
1, & i = j \\
0, & i \neq j
\end{cases}, \quad f_i \neq 0 , \quad i, j = 1, 2, \ldots N, \text{ and }
\]  

(3.13)
\[ X_{ij} = \begin{cases} \frac{1}{x_i} & i = j \\ 0 & i \neq j \end{cases}, \quad x_i \neq 0 \quad , i, j = 1, 2, \ldots M, \quad (3.14) \]

the following equation holds:

\[ FAf = FG(X^{-1}X)\Delta x = (FGX^{-1})X\Delta x. \quad (3.15) \]

According to the definition of relative increments, the following relations apply:

\[ \delta f = FAf \quad \text{and} \quad \delta x = X\Delta x. \quad (3.16) \]

Then the relationship between the relative increments and the earth model parameters becomes:

\[ \delta f = (FGX^{-1})\delta x = \Gamma \delta x. \quad (3.17) \]

where \( \Gamma \) is the matrix \( J \) in dimensionless form, containing the bilogarithmic derivatives as entries, i.e.:

\[ \Gamma_{ij} = \frac{x_i}{f_i} \frac{\partial f_i}{\partial x_j} = \frac{\partial \ln f_i}{\partial \ln x_j}, \quad i = 1, 2, \ldots N; \quad j = 1, 2, \ldots M. \quad (3.18) \]

We can now rewrite equation (3.17) in vector form:

\[ \delta f = \sum_{j=1}^{M} \frac{\partial}{\partial \ln x_j} \ln f \cdot \delta x_j. \quad (3.19) \]

Equations (3.17) and (3.19) represent the total relative variation of the response vector \( f \) due to individual relative variation of the parameter vector \( x \), in the vicinity of an initial model \( x_0 \).

Throughout the following discussion, we will keep the more familiar \( J \) notation for the Jacobian matrix, bearing in mind that it is in fact the dimensionless form of it.

**Singular Value Decomposition (SVD)**

In order to solve equation (3.10) as well as any inversion problem, we need to invert matrix \( J \). Singular value decomposition (SVD) is one of the most powerful methods for matrix inversion. It
provides the orthogonal decomposition of the Jacobian matrix into a product of three matrices:

\[ J = U \Lambda V^T . \]  

(3.20)

where \( U \) is an \( N \times M \) matrix whose columns contain \( M \) orthogonal data eigenvectors \( u_i \) associated with the eigenvalues of \( J J^T \):

\[ J J^T u_i = \lambda_i^2 u_i, \quad (i = 1, 2 \ldots n), \quad (\lambda_{p+1} = \ldots = \lambda_n = 0) , \]  

(3.21)

\( V \) is an \( M \times M \) matrix whose columns contain the \( M \) orthonormal "model parameter" eigenvectors \( v_i \), satisfying

\[ J^T J v_i = \lambda_i^2 v_i , \]  

(3.22)

and \( \Lambda \) is an \( M \times M \) diagonal matrix of positive square roots of the eigenvalues of \( J^T J \). \( \Lambda \) is also ordered in such a way that: \( \lambda_1 > \lambda_2 > \lambda_3 \ldots > \lambda_p \), and \( \lambda_{p+1} = \ldots = \lambda_m = 0 \). Note that \( U^T U = V^T V = V V^T = I_m \), but we do not necessarily have \( U U^T = I_n \) (Lines and Treitel, 1984).

3.4 Employed inversion techniques

The inversion algorithm employs the Marquardt method with singular value decomposition (SVD). Specifically, we use damped constrained least-squares iteratively to update the parameter vector for a given model. The introduction of damping allows us to account for the level of noise in the data, while by using the constraints of the earth model parameters, we can assign a physically and geologically meaningful range to our solution. We keep updating the parameters and computing a new model response estimate until the problem converges. Convergence criterion is user defined: either a specified maximum number of iterations or a minimum acceptable relative root mean squared error can be used for this purpose. Singular value decomposition is used after each iteration for matrix inversion.


The basic idea of least-squares inversion is to minimize the sum of squares of the errors between the model response and the observed data. Let us recall the notation:
\( \mathbf{d} = \text{col} (d_1, d_2 \ldots d_N) \) is the data vector of \( N \) observations,  
\( \mathbf{f} = \text{col} (f_1, f_2 \ldots f_N) \) is the model response vector of \( N \) observations,  
\( \mathbf{x} = \text{col} (x_1, x_2 \ldots x_M) \) is the model parameters vector of \( M \) parameters,  
\( \mathbf{e} = \mathbf{d} - \mathbf{f} \) is the error vector between the observed and the modeled data,  
\( \mathbf{x}_0 \) is the initial estimate of the parameters, and  
\( \mathbf{f}_0 \) is the corresponding initial model response.

Once again, a perturbation of the model response about \( \mathbf{x}_0 \) can be represented by the first-order Taylor expansion:

\[
\mathbf{f} = \mathbf{f}_0 + \sum_{j=1}^{M} \frac{\partial f_j}{\partial x_j} (x_j - x_{j,0}) + \text{HOT},
\]

or, in matrix notation, neglecting the higher order terms,

\[
\mathbf{f} = \mathbf{f}_0 + \mathbf{J} \Delta \mathbf{x},
\]

where \( \mathbf{J} \) is the \( N \times M \) Jacobian matrix:

\[
\mathbf{J} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \ldots & \frac{\partial f_1}{\partial x_M} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_N}{\partial x_1} & \ldots & \frac{\partial f_N}{\partial x_M} \end{bmatrix},
\]

and \( \Delta \mathbf{x} = \mathbf{x} - \mathbf{x}_0 \) is the parameter change vector.

The least-squares, or Gauss-Newton, approach aims to minimize the cumulative squared error:

\[
S = \mathbf{e}^T \mathbf{e} = \| \mathbf{d} - \mathbf{f} \|^2.
\]

The error vector can be represented as

\[
\mathbf{e} = \mathbf{d} - \mathbf{f} = \mathbf{d} - (\mathbf{f}_0 + \mathbf{J} \Delta \mathbf{x}) = \Delta \mathbf{d} - \mathbf{J} \Delta \mathbf{x},
\]
with the discrepancy vector $\Delta d$ being the misfit between the observed data and the initial model response. Then the cumulative squared error is

$$S = (\Delta d - J \Delta x)^T (\Delta d - J \Delta x).$$  \hspace{1cm} (3.28)

Minimization of $S$ with respect to $\Delta x$ requires that

$$\frac{\partial S}{\partial \Delta x} = 0, i = 1, 2, \ldots, N.$$  \hspace{1cm} (3.29)

Differentiating as described by Graybill (1969), we obtain

$$\frac{\partial}{\partial \Delta x} (\Delta x^T J^T J \Delta x - \Delta d^T J \Delta x - \Delta x^T J^T \Delta d + \Delta x^T \Delta x) = 0,$$

from which

$$J^T J \Delta x = J^T \Delta d,$$

so that the condition for parameter change vector is then

$$\Delta x = (J^T J)^{-1} J^T \Delta d.$$  \hspace{1cm} (3.32)

In nonlinear inversion, the new set of parameters resulting from each iteration is substituted into the parameter difference vector (in place of $x_0$) and the procedure is repeated until the convergence criterion is satisfied.

**Weighted least-squares**

Weighting the measurements allows for assigning different importances to the measurements recorded with different accuracy. The new objective function to minimize is then defined as

$$S = \|W \cdot e\|^2,$$  \hspace{1cm} (3.33)

with $W$ being a diagonal matrix containing the weights on its diagonal:
\[ W = \text{diag}(w) \] \hspace{1cm} (3.34)

Equation (3.33) now becomes:

\[ S = \|W(d - f)\|^2 = \|W(\Delta d - J\Delta x)\|^2 = [W(\Delta d - J\Delta x)\]'}(3.35)

Minimization of \( S \) with respect to \( \Delta x \) as described above requires that

\[ \frac{\partial S}{\partial \Delta x} = 0, i = 1, 2...N \]

which leads to

\[ J^TW^2J\Delta x - J^TW^2\Delta d = 0, \text{ or} \]

\[ \Delta x = (J^TW^2J)^{-1}J^TW^2\Delta d \] \hspace{1cm} (3.37)

(3.37) is also known as the Gauss-Newton theorem.

**Regularization methods. Marquardt algorithm.**

The inversion of the matrix \( JJ \) (or \( JTW^2J \)) requires that the matrix be nonsingular. In order to overcome the problem of singularity, regularization methods are applied. These methods stabilize the solution by adding a priori information to the equations.

Suppose we need to solve an equation:

\[ \Delta d = J\Delta x \] \hspace{1cm} (3.38)

and impose some a priori information on \( \Delta x \):

\[ \Delta x = \Delta x_{apr} \] \hspace{1cm} (3.39)

where \( \Delta x_{apr} = x_{apr} - x_0 \). Using matrices of weight for both equations \( W_d \) and \( W_p \), respectively, we can rewrite the equations as
\[ W_d J \Delta x = W_d \Delta d, \text{ and} \]
\[ \beta W_p \Delta x = \beta W_p \Delta x_{\text{apr}}, \]  
where \( \beta \) is the regularization factor. We can combine these equations into a single set, yielding:

\[ \begin{pmatrix} W_d J \\ \beta W_p \end{pmatrix} \Delta x = \begin{pmatrix} W_d \Delta d \\ \beta W_p \Delta x_{\text{apr}} \end{pmatrix}. \]  

Applying the least-squares solution, which implies premultiplying it by \( \begin{pmatrix} W_d J \end{pmatrix}^T \), we get:

\[ \begin{pmatrix} W_d J^T W_d J \\ \beta W_p \end{pmatrix} \Delta x = \begin{pmatrix} W_d J^T W_d \Delta d \\ \beta W_p \Delta x_{\text{apr}} \end{pmatrix}, \]  

or in compact form,

\[ (J^T W_d^2 J + \beta^2 W_p^2) \Delta x = J^T W_d^2 \Delta d + \beta^2 W_p^2 \Delta x_{\text{apr}}. \]  

Finally, solving for the parameter change vector, we get:

\[ \Delta x = (J^T W_d^2 J + \beta^2 W_p^2)^{-1} (J^T W_d^2 \Delta d + \beta^2 W_p^2 \Delta x_{\text{apr}}). \]  

The term \( \beta^2 W_p^2 \) increases the stability of the system. When \( W_d = W_p = I \) and \( x_{\text{apr}} = 0 \), the method becomes the unweighted Marquardt method:

\[ \Delta x = (J^T J + \beta I_N)^{-1} J^T \Delta d. \]  

When \( J^T J \) matrix is singular, the least-squares solution does not exist. Moreover, when it is nearly singular, i.e. \( \det(J^T J) \ll 1 \), the solution oscillates. In other words, small changes in parameter values cause large variations in the data misfit. By adding a damping, or Marquardt, factor \( \beta \) to the main diagonal of the matrix \( J^T J \), we avoid having zero or near zero eigenvalues in it.
Note that, in general, in the case of nonlinear inversion, the objective function to minimize is:

\[ S = \|W_d(\Delta d - J\Delta x)\|^2 + \beta^2\|W_p(\Delta x - \Delta x_{apr})\|^2. \tag{3.47} \]

**Method of steepest decent**

The steepest descent solution is normal to any given misfit function contour:

\[ \Delta x_g = -\lambda \nabla S(x), \tag{3.48} \]

where \( \lambda \) is some positive number. At a starting point \( x_0 \) the gradient of the error function \( S \) is calculated as:

\[
\frac{\partial S}{\partial x_j} = \frac{\partial}{\partial x_j} (e^T e) = \sum_{i=1}^{N} \frac{\partial}{\partial x_j} (e_i^2) = 2 \sum_{i=1}^{N} \frac{\partial e_i}{\partial x_j} e_i, (j = 1, 2 \ldots M) \tag{3.49}
\]

Substituting \( e = d - f \), we have

\[
\frac{\partial S}{\partial x_j} = (-2) \sum_{i=1}^{N} \frac{\partial f_i}{\partial x_j} e_i, (j = 1, 2 \ldots M) \tag{3.50}
\]

or in vector form:

\[
\nabla S(x) = (-2)(W_dJdx)^T (W_d\Delta d + \beta^2(W_pJdx)^T(W_p\Delta x_{apr})) \tag{3.51}
\]

from which,

\[
\delta S(x) = (-2)\delta x^T J^T W_d^2 \Delta d + \beta^2 W_p^2 \Delta x_{apr} \tag{3.52}
\]

Hence, the steepest descent direction is given by

\[
I(x) = J^T W_d^2 \Delta d + \beta^2 W_p^2 \Delta x \tag{3.53}
\]

This vector defines the search direction for the steepest descent method.
The Levenberg-Marquardt algorithm is a combination of Gauss-Newton and steepest descent methods. The parameter change vector for the Marquardt algorithm is:

\[ \Delta x = \left( J^T W_d^2 J + \beta^2 I_n \right)^{-1} J^T W_d^2 \Delta d . \]  \hspace{1cm} (3.54)

When no regularization is required, the parameter change in the steepest descent method is:

\[ \Delta x = -\lambda J^T W_d^2 d , \]  \hspace{1cm} (3.55)

and the Marquardt algorithm changes the parameters according to the steepest descent method for large \( \beta \)'s, when the current parameters are far from the solution. The Gauss-Newton method gives:

\[ \Delta x = (J^T W_d^2 J)^{-1} J^T W_d^2 \Delta d . \]  \hspace{1cm} (3.56)

and as \( \beta \) gets smaller, the Marquardt method approaches the Gauss-Newton algorithm, which is more effective when the parameters are close to the solution.

Singular value decomposition

The singular value decomposition (SVD) of an \( NxM \) rectangular matrix \( J \) provides the orthogonal decomposition of the Jacobian matrix into a product of three matrices:

\[ J = U \Lambda V^T , \]  \hspace{1cm} (3.57)

where matrices \( U, \Lambda \) and \( V \) are defined in the previous section.

For an overdetermined problem, when \( N >> M \), that is, with the number of data points exceeding the number of model parameters, some singular values \( \lambda_i \) may become small, which will greatly affect the solution. In order to avoid that, the SVD method is combined with the Marquardt algorithm, as illustrated below.

Substituting (3.57) into the Marquardt solution,\n
\[ \Delta x = (J^T J + \beta^2 I_N)^{-1} J^T \Delta d , \]  \hspace{1cm} (3.58)
we obtain:

\[ \Delta x = (V \Lambda U^T U \Lambda V^T + \beta^2 I_N)^{-1} V \Lambda U^T \Delta d , \]

\[ \Delta x = (V \Lambda^2 V^T + \beta^2 I_N)^{-1} V \Lambda U^T \Delta d , \]

\[ \Delta x = V(\Lambda^2 + \beta^2 I_N)^{-1} V^T V \Lambda U^T \Delta d , \]

\[ \Delta x = V(\Lambda^2 + \beta^2 I_N)^{-1} \Lambda U^T \Delta d , \]

and finally,

\[ \Delta x = V \text{diag} \left( \frac{\lambda_j}{\lambda_j^2 + \beta^2} \right) U^T \Delta d . \] (3.59)

By applying the rotation matrix \( V \), changes in eigenparameters are converted into changes in actual parameters.

If we rearrange the term in the brackets in equation (3.59), such that

\[ \lambda'_{ii} = \begin{cases} \frac{1}{\lambda_{ii}} & \text{for } \lambda_{ii} > 0 \\ 0 & \text{otherwise} \end{cases} \] (3.60)

and define the matrix \( B \) with the entries satisfying

\[ B_{ij} = \frac{\lambda_{ij}^2}{\lambda_{ij}^2 + \beta^2} . \] (3.61)

then \( \Delta x = VB \Lambda' U^T \Delta d \). The matrix \( B \) is very important in the progress of inversion. If we normalize \( B_{ii} \) by the largest eigenvalue,

\[ \tilde{\lambda}_{ii} = \frac{\lambda_{ii}}{\lambda_{11}} \] (normalized eigenvalue) and \[ \tilde{\beta} = \frac{\beta}{\lambda_{11}} \] (normalized damping factor),

then for \( B_{ii} \) we obtain:
These values of $B_{ii}$ control the changes of the transformed parameters and depend on the ratio of the normalized eigenvalue and normalized damping parameter. The normalized damping factors, $B_{ii}$, control the changes in parameters at each iteration level. There are three possible cases for this ratio:

1. $\tilde{\beta} > \tilde{\lambda}$ - $B_{ii}$ is approaching 1 and the respective parameter combination is well resolved.

2. $\tilde{\beta} < \tilde{\lambda}$ - $B_{ii}$ becomes small, which means that the parameter $x_i$ is changed by only a small fraction. Such poorly resolved parameters lead to a stronger damping.

3. $\tilde{\beta} \approx \tilde{\lambda}$ - $B_{ii}$ is approaching 0.5 and the respective parameter is barely damped.

The above means that $\tilde{\beta}$ effectively controls the relative eigenvalues as a threshold value. In practice, it is commonly set to 0.1 for the first iteration in the inversion process, which means that the parameter combination with eigenvalues less than 10% of the maximum eigenvalue are being damped. Thus at the first stage of inversion, only well-resolved parameters are varied in order to obtain fast improvement of the fit between field and synthetic data. Between the iterations, $\tilde{\beta}$ is decreased in order to allow for the influence of the less-resolved parameters on the result. The lower limit of the normalized damping factor is often set at 0.01, leading to parameter combinations with eigenvalues less than 1% of the maximum to be considered irrelevant.

Jupp and Vozoff (1975) define a class of inversion procedures by setting:

$$B_{ii}^{(N)} = \frac{\tilde{\lambda}_{ii}^{2N}}{\tilde{\lambda}_{ii}^{2N} + \tilde{\beta}^{2N}}.$$  \hspace{1cm} (3.63)

For $N = 1$, the method described above is obtained. In the inversion software used throughout this thesis, $N = 2$ is used and the method is often called second-order Marquardt-Levenberg. If we assume $\tilde{\beta} = 1$, then with the increase of $N$, the function
tends to become a step profile and gradually changes to a sharp cutoff at 1 as $N \to \infty$ (see Jupp and Vozoff, 1975).

Another advantage of using singular value decomposition is that many important statistics are output as a by-product. One of the important statistical characteristics is the parameter importance. The damping factors of the transformed parameters are projected back in order to provide an indication of how well the actual parameters are resolved:

$$R = VB.$$  \hspace{1cm} (3.64)

The diagonal elements of $R$ are the damping factors of the original parameters. They lie between 0 and 1. An entry approaching 1 indicates that the parameter has a strong influence on the fit of the model response. A damping factor close to 0 reflects little influence of a particular parameter on the fit.

**Introduction of constraints**

The mapping from data space into model space is not unique and any information on the ranges of parameter variations will help us choose a petrophysically meaningful solution in case the misfit function has local minima. Such a priori information can be incorporated into an inversion process in the form of constraints.

Two kinds of constraints are defined: equalities and inequalities. A set of equality constraints can be expressed in the following form:

$$g(x) = a,$$  \hspace{1cm} (3.65)

and a set of inequality constraints:

$$h(x) \leq b,$$  \hspace{1cm} (3.66)

where $g(x)$ and $h(x)$ are the vector functions of parameters and $a$ and $b$ are constant vectors. For
a number of constraints \( N_c \), an inequality constraint means that

\[
h_j(x) \leq b_j, \quad j = 1, 2, \ldots, N_c.
\]

When inequality constraints in the form (3.66) are not satisfied, they can always be reduced to the case of equality constraints, which we consider below.

The object function we now want to minimize is:

\[
S = \| W_d (f(x) - d) \|^2 + \beta^2 \| W_p (x - x_{apr}) \|^2 + \| W_c (g(x) - a) \|^2,
\]

where the last term is the penalty to the objective function due to unsatisfied constraints. Linearization of the function leads to the following:

\[
S(\Delta x) = \| W_d (\Delta d - J \Delta x) \|^2 + \beta^2 \| W_p (\Delta x - \Delta x_{apr}) \|^2 + \| W_c (G \Delta x - \Delta a) \|^2,
\]

where \( G \) is the matrix of derivatives of the matrix \( g(x) \), and \( \Delta a = a - g(x_0) \) is the error vector due to the unsatisfied constraints. Taking the derivative of (3.68) and solving for \( \Delta x \), we get:

\[
\Delta x = (J^T W_d^2 J + G^T W_c^2 G + \beta^2 W_p^2)^{-1} (J^T W_d \Delta d + G^T W_c \Delta a + \beta^2 W_p \Delta x_{apr}).
\]

Note that if \( W_c = I \) and \( \Delta x_{apr} = 0 \), this equation reduces to the constrained version of the Marquardt method.

3.5 Summary

To summarize this chapter, let us emphasize again that the inversion method used in this thesis employs the Marquardt algorithm combined with singular value decomposition for matrix inversion according to the governing equations (3.54) and (3.61). The a priori information and constraints on all inversion parameters can also be introduced, leading to the general equation (3.69).
Chapter 4

Synthetic data inversion

4.1 Earth model parameters

In order to justify the results of inversion of any data set, multiple tests on synthetic data have to be performed. The following parameters have been under consideration in the process of choosing the appropriate sets of earth models for the inversion of synthetic data.

- **Mud resistivity.** All resistivity devices employ electrodes to emit the current and record the measurements. Therefore, the borehole environment has to be conductive in order to transfer the current into the surrounding formation. For that reason, oil-based resistive muds are not considered in this thesis. Water-based mud resistivities generally range from $0.01$ to $10$ ohm-m, while lateral sounding logging is normally done with the muds of resistivity ranging from $0.05$ to $2-3$ ohm-m (and most often with $R_m = 0.1 - 1$ ohm-m). Note also that the larger the resistivity contrast between the mud and the surrounding formation, the greater the signal distortion. In any borehole we are trying to minimize its effect on the measurements. For the reasons explained above, we find it sufficient to use two mud resistivities for testing of the inversion algorithm: $R_m = 0.1$ ohm-m and $R_m = 1$ ohm-m.

- **Resistivity contrasts.** (1) A resistive invaded layer surrounded by conductive shoulders and (2) a conductive invaded layer surrounded by resistive shoulders have been the two basic earth models used in the inversion of synthetic data. In all cases, the mud resistivity has been lower than the surrounding formation resistivities. The resistivity contrasts ranged from $R_t/R_m = 10$ to $R_t/R_m = 10,000$.

- **Layer thickness.** Most experiments have been done for two layer thicknesses: $2$ m and $0.5$ m. A $2$-meter layer is considered to be a relatively thick layer that can be accurately resolved by most lateral sondes. For a $0.5$-meter layer, the effect of surrounding (shoulder) formations is so great that only the shortest spacing measurement can in most cases reliably pick the layer boundaries. Also, the depth of investigation of the shortest spacing sounding device is $\sim 0.5$ m (approximately equal to the tool size $L = 0.45$ m). Therefore, the thickness of $0.5$ m is considered to be an extreme case which is very hard to resolve.

- **Depth of invasion.** Depth of invasion is one of the most important parameters we are aiming
to resolve with our sounding technique. Therefore, it is extremely important to know how it affects the responses and the results of invasion. Three different depths of invasion have been chosen in order to study this effect: \( L_{xo} = 0.2 \text{ m} \), \( 0.5 \text{ m} \), and \( 1 \text{ m} \). The case when \( L_{xo} = 0.2 \text{ m} \) is very close to the case without invasion and is therefore the easiest to resolve. The invasion of \( 1 \text{ m} \) is considered to be fairly deep and difficult to resolve, especially in high resistivity layers.

Note that all tests have been run for only one borehole diameter of \(-0.2 \text{ m (8")}\), for practical reasons. Lateral sounding is done in boreholes of that diameter in the overwhelming majority of the cases.

The following table shows all earth model parameter sets used in the inversion tests of synthetic data for a mud resistivity of 1 ohm-m.

**Table 1: Earth model parameters for the inversion of synthetic data, \( R_m = 1 \text{ ohm-m} \)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>BHD, m</th>
<th>( R_{sh} )</th>
<th>( R_{xo} )</th>
<th>( R_t )</th>
<th>( L_{xo}, \text{ m} )</th>
<th>( h, \text{ m} )</th>
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<tbody>
<tr>
<td>1</td>
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Table 1: Earth model parameters for the inversion of synthetic data, $R_m = 1 \text{ ohm-m}$

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Each inversion experiment from Table 1 has been done for the cases of

- clean data;
- 5% noise data
- 10% noise data.

In most cases, the results are given below only for the 10%-noise data inversion tests, since those are obviously the most difficult to resolve.

Based on the analysis of inversion results from the Table 1, the following model parameter sets have been chosen for inversion for a mud resistivity of 0.1 ohm-m. The parameter sets given below in Table 2 represent the cases most difficult to resolve with lateral sounding tools (e.g. the deepest invasion). Each inversion test for $R_m = 0.1 \text{ ohm-m}$ has been run for clean data and 10% noise data.
Table 2: Earth model parameters for the inversion of synthetic data, $R_m = 0.1 \text{ ohm-m}$

<table>
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<tr>
<th>Parameter</th>
<th>BHD, m</th>
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<td>0.5</td>
</tr>
<tr>
<td>7</td>
<td>0.2</td>
<td>1000</td>
<td>3</td>
<td>10</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>0.2</td>
<td>1000</td>
<td>3</td>
<td>10</td>
<td>1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

4.2 Sampling rates
The sampling rate most frequently used in lateral sounding is 10 samples per meter. However, depending on various factors, such as local stratigraphy or specific tasks on the site, the sampling rates of 5 samples per meter and 20 samples per meter are sometimes used. All synthetic data inversion tests have been done for the rate of 10 samples per meter as that is used in recording the field data.

4.3 Initial guesses
The maximum of four parameters are allowed to change for each layer in the inversion process: layer thickness (h), resistivities of invaded and uncontaminated zones ($R_{xo}$ and $R_t$, respectively), and depth of invasion ($L_{xo}$). All of them were allowed to change simultaneously when inverting the synthetic data. The information about the borehole (i.e. its diameter and mud resistivity) is often provided in the form of caliper, mud resistivity, and other logs, and is considered accurate and reliable.

In most cases, from the borehole sounding logs, one can easily estimate whether the resistivity increases or decreases away from the borehole, and approximately evaluate at least the invaded zone resistivity from the readings of shallow sondes. Nevertheless, when inverting synthetic data, we tried to make the resistivity initial guesses even "worse" than the log.
estimates. Depending on the lateral sounding readings, the initial resistivities have been picked in the range of 50% to 2000% off the true resistivity. The “best” initial guesses corresponded to the earth models with conductive layers, which are the easiest to resolve with the sounding technique. The “worst” initial guesses corresponded to a very resistive layer which are hard to detect with nonfocused resistivity methods. For example, the initial resistivity of 50 ohm-m has been picked as the initial guess for the uncontaminated zone resistivity of 1000 ohm-m because even the deepest measurements in such a resistive layer do not exceed a few dozen ohm-m.

The next initial guess parameter is the invasion profile. Because it is often impossible to estimate the presence of invasion in any particular layer from the borehole logs, it has been assumed that in synthetic cases we also do not have information about the invasion profile. In other words, each layer is assumed to contain an invaded zone until it is proven otherwise.

Finally, the initial guesses for layer boundaries have been picked depending on layer thicknesses. In most cases, each boundary was initially perturbed such that the layer thickness was changed by 50%. In the process of inversion, each boundary was then allowed to move by a maximum of 45% of the layer thickness (the layer itself or the adjacent layer). The following example explains this procedure.

Suppose we have a 2-meter layer surrounded by 3-meter and 4-meter shoulders above and below, as shown in Figure 4-1. The initial perturbation moves the layer boundaries, for example, to 2.5 m and 5.5 m (from 2 m and 5 m, respectively). Then the minimum allowed thickness of the middle layer is determined as the current thickness minus the maximum allowed percent of change (45%) of its thickness for each boundary: $h_{\text{min}} = 3\text{m} - (0.45 + 0.45) \times 3\text{m} = 0.3 \text{m}$. The maximum allowed thickness is determined as the current thickness plus the maximum percentage of change of the adjacent layer thickness for each boundary: $h_{\text{max}} = 3\text{m} + 0.45 \times 2.5 + 0.45 \times 3.5 = 5.7 \text{m}$.

In general, when dealing with real borehole sounding data, the initial guesses for all parameters are usually picked straight from the logs. In fact, quite often it is the only available information about the formation resistivities. For example, after the initial interval delineation, initial $R_{xo}$ and $R_{t}$ values can be chosen as the pick values of the shallowest and deepest measurements in each layer. It should be noted that in most cases the initial guesses (especially for resistivities of invaded and uncontaminated zones) for the synthetic data have been chosen
Figure 4-1. Example of layer thickness change in the inversion process for a 2-meter layer.

\[ h_{\text{min}} = h_{\text{ini}} - (0.45 + 0.45)h_{\text{ini}} = 3m - 0.9*3m = 0.3 \text{ m} \]

\[ h_{\text{max}} = h_{\text{ini}} + 0.45h_1 + 0.45h_2 = 3m + 0.45*2.5m + 0.45*3.5m = 5.7 \text{ m} \]
even “further off” from the true model parameters than those we would pick from the logs, in
order to see how well the algorithm works.

Convergence criteria have been set to either one of the two following conditions (whichever
happens first):

(1) a maximum number of iterations has been reached;

(2) a predefined value for the relative RMS error has been reached (RMS error is virtually no
longer decreasing).

In all cases, no more than 10 iterations have been run in each test, and generally 7 iterations
have been sufficient to provide the desired accuracy. Ten iterations of a 10-meter interval
inversion (5 logs, 10 data points per meter) normally take 10-15 minutes of CPU time on an SGI
computer.

4.4 Inversion results

Figures 4-2 through 4-4 show the inversion results for one of the cases: a 2-meter thick
resistive layer with invasion and \( R_{sh} = 3 \) ohm-m, \( R_{xo} = 10 \) ohm-m, \( R_t = 100 \) ohm-m, and \( L_{xo} = 1 \)
m. As mentioned above, for each earth model, three sets of data have been inverted: clean data,
5%- and 10%-noise level data. Figure 4-2 illustrates the inversion results for the clean data and
10%-noise data. The left and middle tracks show the invasion profile (shaded) and resistivities,
respectively. The “true” model curves are shown in black, while the inversion results are in dark
grey. The right tracks illustrate the data fit (5 sounding logs with different spacings) for both
cases. The RMS error is on the order of \( 10^{-4}\% \) for the clean data and approaches 10% for 10%
noise data (which is a good indication that we do not fit the noise in this case).

Figure 4-3 shows the confidence intervals for the inverted parameters: \( L_{xo}, R_{xo}, \) and \( R_t \). The
upper part, again, is for the clean data, and the lower part for 10% noise data. Confidence
intervals are 95% and are shaded. The leftmost, middle and right tracks show the confidence
intervals for \( L_{xo}, R_{xo}, \) and \( R_t \), respectively. It can be seen from the figure that the three
parameters are well-resolved even for the noisy case. In our initial guess, we assumed that we
have no information about the invasion profile. In other words, we assumed that the shoulder
layers are invaded as well. This is exactly where the large confidence intervals on \( L_{xo} \) and \( R_{xo} \) in
the shoulders come from. It becomes obvious that in the shoulder layers there is no invasion
Figure 4-2. Inversion results of synthetic data. $R_{sh} = 3$, $R_{xo} = 5$, $R_t = 100$ ohm-m, $L_{xo} = 1$ m.

(a) clean data, $h = 2$ m

(b) 10% noise data, $h = 2$ m
Figure 4-3. Parameter confidence regions and importances. $R_{sh} = 3$, $R_{xo} = 5$, $R_t = 100$ ohm-m, $L_{xo} = 1$ m.

(a) clean data.

(b) 10% noise data
Figure 4-4. Parameter confidence regions for $R_t$, $R_{sh} = 3$, $R_{xo} = 5$, $R_t = 100$ ohm-m, $L_{xo} = 1$ m

- Clean data
- 5% noise
- 10% noise
since \( R_t = R_{x0} \) in both of them. The main quantitative proof for this conclusion comes from the parameter *importances* shown in the right half of the \( L_{x0} \) (left) tracks. \( R_{x0} \) and \( R_t \) importances approach 1 in all three layers, while the \( L_{x0} \) importance approaches 0 in the layers with no invasion. This means that \( L_{x0} \) is poorly resolved in those two layers, is not important and has no influence on the model responses in the shoulders.

Figure 4-4 illustrates the increase of the confidence intervals with the percentage of noise in the data. The three tracks show the confidence intervals for the true resistivity (the same earth model) for the clean, 5\% and 10\% noise data. The figure shows that in a layer without invasion, \( R_t \) is the only important parameter, and its confidence level is much higher than that of the layer with invasion.

The error bounds on the layer boundaries have been determined accurately in most cases, and virtually exactly in cases of clean data. Table 3 illustrates the dependence of layer thickness error bounds on the amount of noise in the data: the higher the noise level, the wider the parameters confidence regions.

**Table 3: Error bounds on layer thickness for the following earth model: \( R_m = 1, R_{sh} = 3, R_{x0} = 5, R_t = 100 \) ohm-m, \( L_{x0} = 1 \) m.**

<table>
<thead>
<tr>
<th>Noise level</th>
<th>Inverted layer thickness, m</th>
<th>Lower bound, m</th>
<th>Upper bound, m</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2-meter layer</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clean data</td>
<td>2.000007</td>
<td>2.000005</td>
<td>2.000008</td>
</tr>
<tr>
<td>5% noise</td>
<td>1.958387</td>
<td>1.949051</td>
<td>1.967768</td>
</tr>
<tr>
<td>10% noise</td>
<td>1.922409</td>
<td>1.903097</td>
<td>1.941918</td>
</tr>
<tr>
<td><strong>0.5-meter layer</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clean data</td>
<td>0.491595</td>
<td>0.491571</td>
<td>0.491618</td>
</tr>
<tr>
<td>5% noise</td>
<td>0.491865</td>
<td>0.491255</td>
<td>0.492476</td>
</tr>
<tr>
<td>10% noise</td>
<td>0.481597</td>
<td>0.480443</td>
<td>0.482754</td>
</tr>
</tbody>
</table>

Figures 4-2 through 4-4 and Table 3 illustrate the results of inversion for only one earth model and two noise levels (clean data and 10\%-noise data). As follows from Table 1, numerous experiments for different layer thicknesses, resistivity contrasts, depths of invasion, mud
resistivities, and noise levels have been run. In all clean data cases described in Tables 1 and 2 (with the exception of those mentioned below), all parameters have been recovered using inversion with high accuracy and RMS error equal to, and in most cases well below, $10^{-2}$-$10^{-3}$%. For this reason (and so as not to bore the reader with endless plots of inversion results), let us from now on concentrate on the 10%-noise data cases. The noise in borehole resistivity logs is normally estimated at around 10%. The latest tools generally provide an even better signal-to-noise ratio. However, keeping in mind that the data used in this thesis was collected some 20 years ago, we have chosen this noise level to be reasonably realistic.

The following plots of synthetic data inversion results are shown for the invaded zone diameter of 1 meter. Figure 4-5 illustrates parameter importances and confidence regions for a 0.5 meter layer with the same earth model parameters as in Figure 4-4. Again, large confidence intervals for the invaded zone parameters and low $L_{xo}$ importance in the shoulders reflect the absence of invasion in these layers. All target layer parameters are resolved with a very high accuracy.

Multiple tests have been run with the assumption that we have invasion in all layers. The results have shown that the algorithm can easily distinguish between invaded and noninvaded layers with a high degree of accuracy. From now on, we assume the presence of invasion only in the target layer, which will save computation time. In other words, we assume that we can make an educated guess about the invasion profile in the layers, which in most cases is true for synthetic data generated for such simple models.

Figure 4-6 shows the inversion results for an order of magnitude higher resistivity contrast: $R_{sh} = 1000$ ohm-m, the rest of the parameters as above; hence $R_{sh}/R_t = 100$ and $R_{sh}/R_m = 1000$. The parameters are resolved very well, with $R_t$ being slightly underestimated but with the true value lying within its confidence interval. Larger confidence intervals for $R_t$ result in lower parameter importance.

Figure 4-7 illustrates the inversion results for a reversed resistivity situation: conductive layer surrounded by resistive shoulders ($R_{sh} = 100$ ohm-m, $R_{xo} \approx R_t$ in the invaded layer). Figure 4-7a represents a 2 meter layer with the true resistivity of 5 ohm-m, while Figure 4-7b is for a 0.5 meter layer and $R_t$ of 10 ohm-m. The parameters are recovered very well. Note the low $L_{xo}$ importance in the conductive layer. It results from the fact that the resistivity ratio in the target
Figure 4-5. Inversion results and confidence intervals. Model: $R_{sh} = 3$, $R_{xo} = 5$, $R_t = 100$ ohm-m, $L_{xo} = 1$ m, $h = 0.5$ m.
Figure 4-6. Inversion results and confidence intervals. Model: \( R_{sh} = 1000, R_{xo} = 3, R_t = 10 \ \text{ohm-m}, \ L_{x0} = 1 \ \text{m}. \)

(a) \( h = 2 \ \text{m} \)

(b) \( h = 0.5 \ \text{m} \)
Figure 4-7. Inversion results & confidence intervals. Model: $R_{sh} = 100$, $R_{xo} = 3$ ohm-m, $L_{xo} = 1$ m.

(a) $h = 2$ m and $R_t = 5$ ohm-m

(b) $h = 0.5$ m and $R_t = 10$ ohm-m
layer \((R_t/R_{xo} = 10/3)\) is considerably lower than the contrast with the surrounding formations. In other words, the higher the resistivity contrast in the layer, the more important the invasion diameter.

This fact is supported by Figure 4-8a. The resistivity model is the following: \(R_{sh} = 1000\) ohm-m, \(R_{xo} = 10\) ohm-m, \(R_t = 100\) ohm-m, and the \(L_{xo}\) importance approaches 1 in the target layer. In Figure 4-8b, \(L_{xo}\) importance goes down again, since \(R_{xo} = 3\) ohm-m and \(R_t = 10\) ohm-m in the conductive layer. The confidence regions for all parameters include their true model values.

Table 4 summarizes the results of the inverted layer thickness and error bounds for different earth models and layer thicknesses for the case of 10%-noise data.

**Table 4: Error bounds on layer thicknesses for 10%-noise data and a constant depth of invasion \((L_{xo} = 1\) m). \(R_m = 1\) ohm-m.**

<table>
<thead>
<tr>
<th>Model resistivities, ohm-m</th>
<th>h, m</th>
<th>Inverted layer thickness, m</th>
<th>Lower bound, m</th>
<th>Upper bound, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_{sh} = 3, R_{xo} = 5, R_t = 100)</td>
<td>1.922409</td>
<td>1.903097</td>
<td>1.941918</td>
<td></td>
</tr>
<tr>
<td>(R_{sh} = 3, R_{xo} = 10, R_t = 1000)</td>
<td>2.017953</td>
<td>2.009099</td>
<td>2.026847</td>
<td></td>
</tr>
<tr>
<td>(R_{sh} = 100, R_{xo} = 3, R_t = 10)</td>
<td>2.003150</td>
<td>1.995785</td>
<td>2.010542</td>
<td></td>
</tr>
<tr>
<td>(R_{sh} = 1000, R_{xo} = 3, R_t = 10)</td>
<td>2.054890</td>
<td>2.038513</td>
<td>2.071399</td>
<td></td>
</tr>
<tr>
<td>(R_{sh} = 3, R_{xo} = 5, R_t = 100)</td>
<td>0.481597</td>
<td>0.480443</td>
<td>0.482754</td>
<td></td>
</tr>
<tr>
<td>(R_{sh} = 3, R_{xo} = 10, R_t = 1000)</td>
<td>0.550300</td>
<td>0.5627476</td>
<td>0.574112</td>
<td></td>
</tr>
<tr>
<td>(R_{sh} = 100, R_{xo} = 3, R_t = 10)</td>
<td>0.486091</td>
<td>0.473979</td>
<td>0.498511</td>
<td></td>
</tr>
<tr>
<td>(R_{sh} = 1000, R_{xo} = 3, R_t = 10)</td>
<td>0.478643</td>
<td>0.465081</td>
<td>0.492600</td>
<td></td>
</tr>
</tbody>
</table>

Figures 4-9 through 4-16 show the inversion results for the mud resistivity of 0.1 ohm-m. The influence of such a conductive mud on the tool responses and inversion results can be clearly seen. In some cases, the parameters (especially \(L_{xo}\) and \(R_t\)) are slightly underestimated. However, in general the results are more than acceptable, considering that the maximum resistivity contrasts for some models reach 10,000 \((R_t/R_m)\).
Figure 4-8. Inversion results and confidence intervals. Model: $R_{sh} = 1000$ ohm-m, $L_{xo} = 1$ m.

(a) $h = 2$ m, $R_{xo} = 10$, and $R_t = 100$ ohm-m

(b) $h = 0.5$ m, $R_{xo} = 3$, and $R_t = 10$ ohm-m
Figures 4-9 and 4-10 illustrate the inversion results, data misfits, and confidence intervals for a 2 meter and a 0.5 meter layer, respectively. The resistivities are the same as in Figure 4-2: $R_{sh} = 3$, $R_{xo} = 10$, $R_t = 100$ ohm-m. Note that all parameter importances are approaching 1. The target parameters are about 10% below the true model parameters, with the true parameters in most cases lying within the confidence regions. Note that the inversion results for a 0.5 meter layer (Figure 4-10) are more accurate than those for a 2 meter layer. The reason for this fact comes from a different noise distribution, which determines the accuracy of the inversion results. In numerous experiments with different noise distributions in the data we have run, in each case parameter estimates lie within a maximum of 10-15% interval of the true model values.

Figures 4-11 (2 m layer) and 4-12 (0.5 m layer) support this fact. The true resistivity of the target layer is now 1000 ohm-m, and the rest of the parameters are as above. Note that the theoretical responses in Figures 4-9 ($R_t = 100$ ohm-m) and 4-11 ($R_t = 1000$ ohm-m) are virtually the same with only the largest spacing (L8.50) being slightly affected by a higher formation resistivity. However, the parameter estimates in Figure 4-11 are even more accurate than those for the lower resistivity contrast. Again, the largest confidence intervals are observed for the true resistivity of the target layer, and in both figures (4-11 and 4-12) they include the true model parameter values.

Figures 4-13 through 4-16 illustrate the case of a conductive layer surrounded by resistive shoulders. The theoretical responses in Figures 4-13 (2 m layer) and 4-14 (0.5 m layer) reflect that even when the resistivity of the shoulders is $R_{sh} = 100$ ohm-m, this case is difficult to resolve with lateral sounding tools. The current tends to flow into the conductive layer, which makes it harder to estimate the resistivity of the shoulders. This results in larger confidence intervals for all parameters, particularly $R_{sh}$ and $L_{xo}$. It is especially true for the 0.5 m layer (Figure 4-14), when even the shortest spacing measurement (L0.45) does not read close to the shoulder resistivity when all electrodes are well below the conductive layer.

Finally, Figures 4-15 and 4-16 illustrate the same situation with even more resistive shoulders: $R_{sh} = 1,000$ ohm-m and $R_{sh}/R_m = 10,000$. If we consider the theoretical responses, we can clearly see that this situation is almost impossible to resolve using only lateral sounding measurements. However, for a 2-m case the parameter estimates are very good except for the upper shoulder resistivity. Note that in the lower shoulder none of the measurements can "reach"
Figure 4-9. Inversion results: curve misfits and confidence intervals. $R_m = 0.1 \, \text{ohm-m}$. Model: $R_{sh} = 3, R_{xo} = 10, R_t = 100 \, \text{ohm-m}, L_{xo} = 1 \, \text{m}, h = 2 \, \text{m}$. 

Confidence intervals on $L_{xo}$, $R_{xo}$, and $R_t$. 

[Image of diagram showing model and inverted results with confidence intervals.]
Figure 4-10. Inversion results: curve misfits and confidence intervals. $R_m = 0.1$ ohm-m. Model:

$R_{sh} = 3$, $R_{xo} = 10$, $R_t = 100$ ohm-m, $L_{xo} = 1$ m, $h = 0.5$ m.

Confidence intervals on $L_{xo}$, $R_{xo}$, and $R_t$. 
Figure 4-11. Inversion results: curve misfits and confidence intervals. $R_m = 0.1$ ohm-m. Model: $R_{sh} = 3$, $R_{xo} = 10$, $R_t = 1000$ ohm-m, $L_{xo} = 1$ m, $h = 2$ m.
Figure 4-12. Inversion results: curve misfits and confidence intervals. $R_m = 0.1$ ohm-m. Model:

- $R_{sh} = 3$, $R_{xo} = 10$, $R_t = 1000$ ohm-m, $L_{xo} = 1$ m, $h = 0.5$ m.

Confidence intervals on $L_{xo}$, $Rxo$, and $Rt$. 

- Lower bound $L_{xo}$ Imp.
- Upper bound $Rxo$ Imp.
- $L_{xo}$ 10% noise Rt Imp.
- $Rxo$ 10% noise
- $Rt$ 10% noise
Figure 4-13. Inversion results: curve misfits and confidence intervals. \( R_m = 0.1 \) ohm-m. Model: \( R_{sh} = 100, R_{xo} = 3, R_t = 10 \) ohm-m, \( L_{xo} = 1 \) m, \( h = 2 \) m.
Figure 4-14. Inversion results: curve misfits and confidence intervals. $R_m = 0.1$ ohm-m. Model: $R_{sh} = 100$, $R_{xo} = 3$, $R_t = 10$ ohm-m, $L_{xo} = 1$ m, $h = 0.5$ m.
Figure 4-15. Inversion results: curve misfits and confidence intervals. $R_m = 0.1 \text{ ohm-m}$. Model:
$R_{sh} = 1000$, $R_{xo} = 3$, $R_t = 10 \text{ ohm-m}$, $L_{xo} = 1 \text{ m}$, $h = 2 \text{ m}$. 

Confidence intervals on $L_{xo}$, $R_{xo}$, and $R_t$. 

<table>
<thead>
<tr>
<th>Model $R_t$</th>
<th>Theor. $L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>2000</td>
</tr>
<tr>
<td>$L_{xo} = 1.2 \text{ m}$</td>
<td>$L_{xo} = 1.25 \text{ m}$</td>
</tr>
<tr>
<td>$R_{xo} = 2 \text{ ohm-m}$</td>
<td>$R_{xo} = 2.25 \text{ ohm-m}$</td>
</tr>
<tr>
<td>$R_t = 2000 \text{ ohm-m}$</td>
<td>$R_t = 4.25 \text{ ohm-m}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inverted $R_t$</th>
<th>Theor. $L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>2000</td>
</tr>
<tr>
<td>$L_{xo} = 1.2 \text{ m}$</td>
<td>$L_{xo} = 1.25 \text{ m}$</td>
</tr>
<tr>
<td>$R_{xo} = 2 \text{ ohm-m}$</td>
<td>$R_{xo} = 2.25 \text{ ohm-m}$</td>
</tr>
<tr>
<td>$R_t = 2000 \text{ ohm-m}$</td>
<td>$R_t = 4.25 \text{ ohm-m}$</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Model $R_{xo}$</th>
<th>Theor. $L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>2000</td>
</tr>
<tr>
<td>$L_{xo} = 1.2 \text{ m}$</td>
<td>$L_{xo} = 1.25 \text{ m}$</td>
</tr>
<tr>
<td>$R_{xo} = 2 \text{ ohm-m}$</td>
<td>$R_{xo} = 2.25 \text{ ohm-m}$</td>
</tr>
<tr>
<td>$R_t = 2000 \text{ ohm-m}$</td>
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<thead>
<tr>
<th>Inverted $R_{xo}$</th>
<th>Theor. $L$</th>
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<tbody>
<tr>
<td>2000</td>
<td>2000</td>
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<tr>
<td>$L_{xo} = 1.2 \text{ m}$</td>
<td>$L_{xo} = 1.25 \text{ m}$</td>
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<tr>
<td>$R_{xo} = 2 \text{ ohm-m}$</td>
<td>$R_{xo} = 2.25 \text{ ohm-m}$</td>
</tr>
<tr>
<td>$R_t = 2000 \text{ ohm-m}$</td>
<td>$R_t = 4.25 \text{ ohm-m}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lower bound $L_{xo}$</th>
<th>Upper bound $R_{xo}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2 m</td>
<td>1.25 m</td>
</tr>
<tr>
<td>2 ohm-m</td>
<td>2.25 ohm-m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lower bound $R_{xo}$</th>
<th>Upper bound $R_{t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 ohm-m</td>
<td>2.25 ohm-m</td>
</tr>
</tbody>
</table>

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Figure 4-16. Inversion results: curve misfits and confidence intervals. $R_m = 0.1$ ohm-m. Model: $R_{sh} = 1000$, $R_{xo} = 3$, $R_t = 10$ ohm-m, $L_{xo} = 1$ m, $h = 0.5$ m.

Confidence intervals on $L_{xo}$, $R_{xo}$, and $R_t$. 

Error bounds on $L_{xo}$, $R_{xo}$, and $R_t$. 

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the 1000 ohm-m resistivity of this layer. The closest measurement (L8.50) is a full order of magnitude lower than the true value, which results again from the fact that all the current is flowing into the conductive layer (and the borehole). Since the ratio of the resistivities in the conductive layer is considerably lower that the contrast with the surrounding formation, the diameter of invaded zone is underestimated. The situation gets even worse for the 0.5 m layer shown in Figure 4-16. As one can see, all inverted parameters, especially the layer thickness, are far away from the true earth model parameters, and such a case can hardly be resolved using only lateral sounding logs.

4.5 Summary

Multiple synthetic data for various earth models have been inverted in order to provide the basis for field data inversion. Different mud resistivities, depths of invasion, layer thicknesses, and resistivity contrasts between the target and the surrounding formations have been tested extensively. Systematic noise of 5% and 10% level has been introduced into the inverted data to account for the “real” borehole conditions.

As a whole, the inversion algorithm proved to work very well for the inversion tests described above. The formation parameters with the resistivity ratio \((R_t/R_m)\) of up to 10,000, layer thicknesses of 0.5 m, and depth of invasion of up to 1 m have been successfully recovered for the synthetic data with the noise level of up to 10%.
Chapter 5

Field data inversion

5.1 Types of inversion analyses

There are three types of analyses available in the inversion package. They all employ the same
inversion approach but differ in dimensionality, complexity, amount of data used, and number of
parameters to invert for. The most comprehensive inversion is usually based on complete, rigorous
analysis of all measurements to determine the layer boundaries and the resistivity variations
within each layer. This type of inversion has been used in all of the synthetic examples described
above for two reasons. First, we wanted to verify the algorithm’s ability to properly recover the
layer boundaries. Second, for synthetic data simulated for simple models, the time consumption is
not considerable. However, for the amount of data we have to deal with in the field, time may be a
prohibitive factor. This is where we need to make use of less accurate but faster analyses to pro-
vide an educated initial guess for the earth model, estimate the data quality, evaluate the borehole
environment, and sometimes even provide a reasonable solution. The three types of inversion
analysis are described below.

1D inversion provides a level-by-level one dimensional analysis with resistivity varying only
in the radial direction. Each depth point is treated independently of the others, thus shoulder bed
effects are not taken into account. In other words, the resistivities of the formations surrounding a
particular data point are not taken into account. For this reason, environmental and shoulder bed
corrections generally have to be introduced into the data prior to inversion. This algorithm is very
fast and usually takes minutes to run on a SPARC station 20.

Rapid 2D inversion provides a 2D model with resistivity varying both radially and vertically.
Shoulder bed effects are taken into account. However, the layer boundaries have to be selected
prior to inversion and cannot be changed. The processed interval is usually treated as a whole
object. After the layers have been selected, one “characteristic” data point is chosen in each layer
and used in inversion. For example, with the layer boundaries detected, as described in Chapter 2,
the characteristic point for laterologs is the median resistivity value in each layer (supposedly the
closest one to the “true” resistivity), while for lateral sounding measurements it is the peak value
in each layer. The rest of the data points in a particular layer are not taken into account; therefore,
an incorrectly picked characteristic point may result in the wrong estimation of formation parameters. Time consumption is much greater than for the 1D case but does not usually exceed a few hours on a SPARC station 20 depending on the complexity of the problem. Keeping the layer boundaries fixed greatly affects the accuracy of the solution. However, rapid inversion provides a reliable estimate of the resistivity distribution around the borehole. It is also possible to properly adjust the layer boundaries by comparing the shapes of the misfits of the real and synthetic curves.

*Enhanced 2D inversion,* in addition to the 2D model, provides the ability to change and redefine layer boundaries during the inversion. All data points are used in inversion, although it is possible to resample the data, i.e. decrease or increase the sampling rate by interpolation based on the desired accuracy. Enhanced inversion also allows for a more detailed analysis of different parts of the interval by means of “windowing.” Figure 5-1 shows the idea of windowing. The user can pick the constant number of layers in a window to be inverted at each step, and the thickness of the shoulders above and below the current window, whose resistivities are also taken into account. The number of overlapping layers, while moving down the borehole to the next window, can also be specified. This inversion provides the most accurate 2D interpretation but may take days to complete on a SPARC station 20.

There are different ways to combine these three techniques in order to reduce computational time. For example, 1D inversion can provide the information on the radial resistivity profile and determine the layers without invasion, which can reduce the number of parameters for further processing.

### 5.2 Data description and preprocessing

Full suites of lateral sounding and laterolog measurements were available for all field examples described below. The data were recorded with a very saline drilling mud (resistivity of about 0.5 ohm-m), in a borehole of a regular diameter of about 0.2 m (~8”). Gamma ray, neutron-gamma, spontaneous potential, and caliper logs are also available in most cases, providing valuable information for reservoir delineation. As we have mentioned above, such borehole conditions are not favorable for induction measurements. Even after producing inversion results using lateral methods, the theoretical induction logs simulated afterwards were corrupted in high resistivity zones.

The most important issues in inversion processes are the computation time, accuracy, and
Figure 5-1. Windowing for enhanced 2D inversion. The number of processed and overlapping layers and the shoulder thickness is user specified.
physical meaning of the result. When dealing with the inversion of hundreds of meters of real data, computation time becomes critical. For a particular problem, an enhanced 2D inversion may take from several hours to several days to complete, depending on the task. The main factors influencing the results and the computation time include the inversion technique used, the number of inverted curves and data points, and the number of parameters describing the resulting earth model. For large amounts of data, it becomes imperative not only to have a fast computer but also to reduce the influence of all these factors on the inversion process as much as possible.

As far as the number of logs to be inverted is concerned, lateral sounding and laterolog measurements effectively complement each other to provide for both radial investigation and vertical resolution. Inversion of only lateral sounding data has also been tested. The deepest sounding measurement \( L = 8.50 \, \text{m} \) has very poor resolution and in most boreholes tends to average the resistivities over large intervals. For this reason, based on the practical experience, the deepest measurement has not been used in the inversion process.

The quality of lateral sounding logs is average, with the estimated acceptable level of noise in Russian data generally being in the neighborhood of 10-15%. The laterolog quality is somewhat poorer, especially in high resistivity layers. In fact, in some intervals the laterolog has a saw-tooth shape so as to make any interpretation without filtering impossible. High noise levels in this data result from the fact that the logs were recorded in 1970’s - 1980’s in analog format and had to be digitized manually. Therefore, all data have been filtered prior to processing and interpretation, using a centered average filter. All absolute depths have been changed for the field data examples.

5.3 Layer boundaries detection and initial delineation

One of the most important parts of data interpretation is the correct detection of layer boundaries. A subroutine of the software called LAYER has been used for initial layer boundary detection. The resulting layered structure is used as input in the program, building the earth model for initial guesses.

LAYER finds the depths of potential bed boundaries using the rate of change with a depth of one or several curves. It determines the slope of the curve at each level by computing the slope of
a segment fitted to the curve within its corresponding moving window. The threshold value is used to specify the absolute value of the slope that must be exceeded for each level to be considered a layer boundary. "Boundary candidates" are determined at the levels with highest absolute values of slope, and those exceeding the threshold value are flagged as layer boundaries.

For reasonably accurate data, the levels with the highest absolute values of slope correspond to the inflection points. Consequently, the approach used in LAYER will give the best results when applied to symmetric logs, in particular focused laterologs, because the boundaries for symmetric curves are picked at the inflection points, as discussed in Chapter 2. The laterolog provides better delineation of the section due to higher vertical resolution and hence has been used for initial reservoir delineation. In most cases, the input logs have to be filtered prior to processing using LAYER. The algorithm is very sensitive to the rate of change of the input curve, and we need to ensure that noise spikes are not picked as boundary candidates.

The layer boundaries can be adjusted manually afterwards (and at any point in the inversion process) according to the log analyst’s experience. Therefore, the layer boundaries detection can be both automated and manual.

5.4 Depth matching
When dealing with different measurements that might have been recorded at different times, it is important to make sure that all data are on depth, i.e. that the absolute depth readings are the same for all logs. The disagreement between the absolute depth readings for different measurements may result, for example, from an accidental shift of "zero" depth between the runs.

The employed procedure for depth matching of the sounding and laterolog data is the following.¹ We first pick the shallowest curve (L = 0.45 m) and, using the (corrected) LAYER results, invert this single curve as if it were the only one available. We invert the shallowest measurement for only one parameter, the “true” resistivity, with the predefined layer boundaries. Using the resulting earth model, we then simulate the data for the rest of the tools (measurements). It is apparent that most of them should be underestimated and the misfits will be enormous, especially if the invaded zone resistivity is lower than the true resistivity (which is usually the case for our data). However, the shapes of the curves will generally reflect whether the simulated data extremum (minimum and maximum) points correspond to those in the field data. From such a compar-

¹. The original idea has been brought up in a private discussion with M. Frenkel and A. Mezzatesta.
ison, it is quite easy to detect which curves are off depth with the shallowest one and shift them accordingly. (The term “on-depth” is used in a relative sense, since we assume the shallowest curve to be fixed.)

5.5 Mud resistivity adjustment

Mud resistivity measurements are not available in our data sets. However, an approximate magnitude of the mud resistivity in the interval of interest is usually available. Not heavily relying on those numbers, as they are usually not associated with a particular depth but are given as approximate estimates, we have used the one-dimensional inversion in order to estimate $R_m$ in several intervals in each borehole. For this it is best to pick a shaly zone where the sounding curves practically overlap each other since there is no invasion. We solve for mud resistivity based on the four sounding measurements, assuming one-dimensional resistivity distribution in a layer. In all boreholes, the initial given values for mud resistivity agreed well with the inversion results. In all field examples, the mud resistivities ranged from 0.5 to 0.7 ohm-m.

5.6 Field data inversion

Field example 1

Figure 5-2 shows the first set of data: SP, gamma ray, neutron-gamma and caliper measurements on the left track, the sounding logs on the middle track, and the laterolog on the right track. It is a carbonate/shale sequence. The spontaneous potentials curve reflects a considerable negative anomaly in the interval 720 - 740, thus indicating a potential interval of interest. Note the considerable increase in resistivity in this interval of greater than an order of magnitude compared to the surrounding formation. The data have been filtered, checked for depth matching, and the initial layer boundaries have been picked, as described above.

Figures 5-3 and 5-4 illustrate the enhanced 2D inversion results. The data have been processed using the windowing technique described above, with each window containing 7 layers, 2 meter “shoulders” to account for the resistivities changes immediately above and below the window, and one layer overlap between the windows. The average layer thickness is about 1-1.5 meter; therefore, the shoulder thickness of 2 meters is considered sufficient to account for the resistivities of the layers above and below each window. The initial guesses for the invaded zone and the true resistivities have been set at the average values picked in each layer from the
Figure 5-2. Field example 1. Initial data set.
Figure 5-3. Field example 1. Curve misfits and inversion results: $L_{x0}$, $R_{x0}$, and $R_t$. 

<table>
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</tr>
<tr>
<td>Synthetic L 4.25</td>
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</tbody>
</table>

$R_{x0}$: 2 ohm-m

$R_t$: 2000 ohm-m

$L_{x0}$: 2 ohm-m

$R_{x0}$: 2000 ohm-m
Figure 5-4. Field example 1. Resulting parameters--$L_x$, $R_x$, and $R_t$--and their confidence intervals.
shallowest sounding (L 0.45) and laterolog (LL3) curves, respectively. Each layer contained invasion and the initial value for the depth of invasion was set to 0.1 m.

Figure 5-3 shows the curve misfits for four sounding logs (middle track) and the laterolog (right track). The right track also shows the resulting invaded zone and true resistivities. The left track illustrates the depth of invasion (Lxo) and the spontaneous potentials curve. The sounding and laterolog curve misfits averaged around 18%. In many cases the large misfits resulted from different vertical resolutions of the sounding logs and the laterolog, when the nonfocused logs do not “see” and cannot resolve thin layers whereas the laterolog can.

If we look at the Lxo curve on the left track, we can see that the depth of invasion in the interval 719 - 757 m is on average somewhat larger than in the surrounding formations. Also, note that some invasion is present virtually everywhere across the entire interval. Looking only at the Lxo, the “deeply invaded zones” and the noninvaded zones are indistinct, which is not unusual to encounter in carbonate sequences. However, if we look at the resistivities (right) track, there are two distinct zones where the invaded zone and true resistivities differ considerably: 720 - 742 m and 742 - 757 m. In the first interval, the Rt/Rxo ratio sometimes reaches more than two orders of magnitude. In the second interval, the ratio is lower. This allows us to assume that the upper interval is oil saturated and the lower water saturated with the oil-water contact being at about 743.5 m. This is also indicated by the substantial negative anomaly on the SP log in the interval 720 - 742 m.

Now let us go back to the Lxo curve and refer to Figure 4-3. Recall that the extremely large confidence intervals on Lxo and Rxo result from the assumption that all layers in the model are invaded. Since the shoulder layers in Figure 4-3 are actually not invaded (according to the initial model), the Rxo and Rt values in the shoulders are the same, and the invasion is unimportant and does not affect the theoretical responses. It is similar to saying that the invasion is not defined in those layers (the shoulders). Now look at Figure 5-4, which shows the resulting earth parameters: Lxo, Rxo, and Rt, and their confidence intervals (95%) for our first field example. Each track shows the error bounds for each of the above parameters, respectively. Note the extremely large confidence intervals for Lxo and Rxo above 720 m and below 759 m (left and middle tracks), which results from a decrease of the Rt/Rxo ratio. The smaller the difference between the two resistivities, the less important the invasion. The error bounds on the
resistivities are the smallest in the interval 720 - 744 m and it confirms our discussion on the importance of invasion in this region.

The perforation data collected in this well define the productive oil-saturated layer to be in the interval 720 - 742.8 m (indicated by shading on the depth track in Figure 5-3) with the debit of 201 tons per day. Thus, the inversion results perfectly match the perforation data.

**Field example 2**

Figure 5-5 shows the second set of data. The logs are plotted in the same order as in the previous example: SP, gamma ray, neutron-gamma and caliper measurements on the left track, the sounding logs on the middle track, and the laterolog on the right track. It is again a carbonate/shale sequence. The spontaneous potentials curve is less differentiated than in the previous example, but reflects a negative anomaly in the interval 109 - 123. The substantial increase in resistivity can be seen in approximately the same interval. The data have been filtered, checked for depth matching, and the initial layer boundaries have been picked, as described in the beginning of this chapter.

Figures 5-6 and 5-7 illustrate the enhanced 2D inversion results. The data have been processed using the same windowing parameters as in the example above. The same procedure was also used in choosing the initial guesses. Figure 5-6 shows the curve misfits for the sounding logs (middle track) and the laterolog (right track). The right track also shows the resulting invaded zone and true resistivities. Note the scale difference between the R_t and R_xo curves. The left track illustrates the depth of invasion (L_xo) and the spontaneous potentials curve. The sounding and laterolog curve misfits averaged around 17%.

The L_xo curve on the left track shows deeper invasion in the interval 107 - 128 m than in the surrounding formations, with the maximum invasion in the interval ~100 - 114 m. It is worth mentioning again that the initial guess for the depth of invasion in each layer has been set to 0.1 m, while the minimum value for invasion was constrained to 5 cm. In general, it is reasonable to assume that if the depth of invasion falls below 0.1 m in a particular layer, the invasion does not have a considerable effect on the tool responses and is not very important. Again, one can trace distinct zones where the invaded zone and true resistivities differ considerably and the zones where the R_t/R_xo ratio is low. Based on the output resistivity magnitudes, we can assume that the
Figure 5-5. Field example 2. Initial data set.
Figure 5-6. Field example 2. Curve misfits and inversion results: L_{XO}, R_{XO}, and R_t.
Figure 5-7. Field example 2. Resulting parameters—$L_{xo}$, $R_{xo}$, and $R_{t}$—and their confidence intervals.
producing layer is located in the interval 107 - 123 m. It is confirmed by the confidence intervals on the parameters in Figure 5-7. The confidence intervals on $L_{xo}$ in zones with shallow invasion (e.g. the lower part from 130 m and down) are not as large as in the previous example but are still considerably larger than in the upper interval. Note also that the lower bounds on $L_{xo}$ in zones with shallow invasion often fall below 0.1 m. The SP log gives the maximum negative anomaly in the interval of 110 - 114 m. The zone with maximum invasion, which in general means the increase in porosity and permeability (confirmed by the negative anomaly of the neutron-gamma log), is located around 110.5 - 114 m and indicates the best reservoir layer in this figure.

The perforation data have been collected in this well in the interval 100 - 136 m and it was discovered that the productive oil-saturated layer is in the interval 110 - 116 m (indicated by shading on the depth track in Figure 5-6) with the debit of 44 tons per day. This corresponds very well to the layer with maximum invasion from the inversion results and we find the match to be quite satisfactory.

Field example 3

Figure 5-8 shows the third set of data. The logs are plotted in the same order as in the previous examples: SP, gamma ray, neutron-gamma and caliper measurements on the left track, the sounding logs on the middle track, and the laterolog on the right track. It is again a carbonate/shale sequence. The spontaneous potentials curve reflects a considerable negative anomaly in the interval 752 - 767 m. The data have been filtered, checked for depth matching, and the initial layer boundaries have been picked, as described in the beginning of this chapter.

Figures 5-9 and 5-10 illustrate the enhanced 2D inversion results. The data have been processed using the same windowing parameters as in the previous examples. The same procedure was also used when choosing the initial guesses. Figure 5-9 shows the curve misfits for the sounding logs (middle track) and the laterolog (right track). The right track also shows the resulting invaded zone and true resistivities. Note again the scale difference between the $R_t$ and $R_{xo}$ curves. The left track illustrates the depth of invasion ($L_{xo}$) and the spontaneous potentials curve. The sounding and laterolog curve misfits averaged around 10%, providing the best of all examples fit of theoretical and actual logs.
Figure 5-8. Field example 3. Initial data set.
Figure 5-9. Field example 3. Curve misfits and results of inversion: $L_{x_0}$, $R_{x_0}$, and $R_t$.

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### Table 1: curve misfits and results

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### Figure 1: Graphical representation

Graph showing variation of $L_{x_0}$, $R_{x_0}$, and $R_t$ with depth.
Figure 5-10. Field example 3. Resulting parameters--L\(_{xo}\), R\(_{xo}\), and R\(_t\)--and their confidence intervals.
This data is quite different from the previous two examples. The $L_{x_0}$ curve on the left track shows the invasion present virtually everywhere in the interval. There are also no zones with considerable contrasts between $R_t$ and $R_{x_0}$, though $R_t/R_{x_0}$ ratio decreases in the middle part (740 - 752 m). Only the spontaneous potential log gives us a distinct negative anomaly as mentioned above, indicating a potential reservoir layer. Figure 5-10, which illustrates the confidence intervals on the parameters, does not provide sufficient information to interpret the inversion results alone as we did in the previous examples. The fact that the invasion is present in most of the layers results in more or less constant confidence intervals. Only considering the SP anomaly, as well as very low readings of gamma ray and neutron-gamma logs, can we make the assumption that the pay zone is located in the interval 753 - 764 m.

This assumption is confirmed almost exactly by the perforation data in this well which placed the pay zone in the interval 754 - 764 m and (indicated by shading on the depth track in Figure 5-9) with the debit of 82 to 100 tons per day. With this field example, use of the auxiliary logs (e. g. a priori information) available in this well helped the interpretation of the inversion results.
Chapter 6

Conclusions

In this thesis, we have applied an entirely new approach to the interpretation of borehole resistivity, in particular electrical sounding, data. The algorithm developed by Western Atlas Logging Services has allowed us to perform joint interpretation of all available resistivity measurements, providing a single resistivity distribution around the borehole, and the ability to explain all measurements simultaneously.

We combined two different resistivity techniques, the lateral sounding and laterolog, to provide both mathematically accurate and physically meaningful solutions to the inversion problem. Multiple depths of investigation of the lateral sounding measurements enable us to thoroughly examine the resistivity distribution away from the borehole. On the other hand, the laterolog has a higher vertical resolution of the boundaries than the lateral sounding logs, as well as the reduced influence of shoulder beds and the borehole environment. The combination of both methods provides us with the most comprehensive and detailed picture of the resistivity distribution in both radial and vertical directions.

Considerable studies have been conducted to test the software on multiple synthetic examples with different earth model parameters and levels of noise. We have defined the range of resistivity ratios between the borehole and the adjacent formations that can be resolved with a satisfactory degree of accuracy. In addition, we have studied extensively the influence of other formation parameters (e.g. invaded zone parameters) on resistivity tool responses and the results of inversion. Also, considerable work has been done on the influence of noise in the accuracy of the recovered parameters and the error propagation from the data to the parameter space.

The field data inversion is the best illustration of the abilities of the algorithm. In one case, the interpretation of the inversion results was complicated due to the presence of invasion in the entire inverted interval, which made it necessary to use auxiliary logs (gamma ray, SP) in the interpretation. In the rest of the cases, the results of the resistivity inversion agreed very well with the available perforation data. In general, it has been shown both in synthetic examples and in the field cases in carbonate sequences that inversion provides very satisfactory results of the resistivity distribution away from the borehole, which in most cases can be readily used in the
oil in place estimation.

One of the concerns with dealing with large amounts of field data in this thesis has been the considerable time consumption. The most time-consuming procedure in the flow of this inversion algorithm is the calculation of the Jacobian matrix for each combination of perturbed parameters. After each perturbation, a new matrix of derivatives has to be calculated and inverted. It would be beneficial to study the abilities of other inversion schemes, e.g., conjugate gradient algorithm, that do not require the computation and inversion of the entire Jacobian matrix, and compare the results with those from the Marquardt/SVD technique. The development of a possible combination of different inversion techniques to enhance the speed of the current inversion algorithm is certainly one of the interesting directions for future work.

Another area for the algorithm improvement and development is the earth model dimensionality. The model described in Chapter 2 is axisymmetric around the borehole with vertical and horizontal boundaries. Such an assumption can be made only if the borehole intersects the layer boundaries at the right angles (often it is a vertical borehole drilled through horizontally layered formations). However, quite often the borehole is substantially deviated from the vertical axis, especially at large depths. The layers within the earth are also hardly exactly parallel to each other. In such cases, the assumed earth model becomes invalid because we have to account for the third dimension: the angle at which the borehole intersects the layer boundaries. However, since the current is distributed in all directions around the borehole, it would be hard to account for different resistivities in the formations surrounding a particular data point without being able to take measurements at each data point in several directions. Having developed a mathematical basis for such a problem, one could study the applicability and the limitations of our assumption that the borehole always intersects the layer boundaries at the right angle. For the time being, this problem stays beyond the scope of this thesis.
References


