Determining the Focal Mechanisms of Earthquakes
by Full Waveform Modeling

by
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ABSTRACT

Determining the focal mechanism of an earthquake helps us to better characterize reservoirs, define faults, and understand the stress and strain regime. The objective of this thesis is to find the focal mechanism and depth of earthquakes. This objective is met using a full waveform modeling method in which we generate synthetic seismograms using a discrete wavenumber code to match the observed seismograms. We first calculate Green’s functions given an initial estimate of the earthquake’s hypocenter, the locations of the seismic recording stations, and the velocity model of the region for a series of depths with intervals of 1 km. Then, we calculate the moment tensor for 6840 different combinations of strikes, dips, and rakes for each of those depths. These are convolved with Green’s function and with an assumed smooth ramp source time function to produce the different synthetic seismograms corresponding to the different strikes, dips, rakes, and depths. We use a grid search in order to find the synthetic seismogram, with the combination of depth, strike, dip, and rake, that best fits the observed seismogram. These parameters will be the focal mechanism solution of an earthquake. The whole procedure is repeated for a reduced number of recording stations in order to determine a minimum number of recording stations that is needed for a reliable source mechanism and depth solution.
We tested the method using two earthquakes in Southern California. Their locations, depths, and source mechanisms were determined using data from a multitude of stations. Southern California Seismic Network’s real-time solution of earthquake 9718013 puts the earthquake at a depth of 15.22 km. The moment tensor inversion method determines the depth of the earthquake to be 8 km with a strike, dip, and rake of 318, 33, -180, respectively. The same network determines the depth of earthquake 14408052 to be 7.3 km. The moment tensor solution determines the strike, dip, rake, and depth of earthquake 14408052 to be 162, 82, -167, and 5 km, respectively. In this study, we wanted to test our method using seismograms from a relatively few stations. We used five stations for each earthquake, then 3 stations for earthquake 9718013, and two stations for earthquake 14408052. When using five recording stations, the strike, dip, rake, and depth of earthquake 9718013 are 300, 60, -170, and 15 km, respectively. When using three recording stations for the same earthquake, the strike, dip, rake, and depth are 300, 60, -180, and 14 km, respectively. For earthquake 14408052, the strike, dip, rake, and depth are 160, 80, -170, and 7 km, respectively, when using five recording stations. The strike, dip, rake, and depth for this same earthquake are 160, 80, -160, and 8 km, respectively, when using only two stations. The results show that the ten best solutions for each earthquake are very similar, and identical in many cases, indicating that the method is robust and the solution is unique. This assures us that the full waveform modeling method is a fast and reliable way to find the focal mechanisms and depths of earthquakes using seismograms from a few stations when the velocity structure is known.

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Chapter 1

Introduction

1.1 Introduction

The objective of this thesis is to determine the focal mechanisms and depths of earthquakes by using full waveform synthetic seismograms. Studying earthquakes and determining their focal mechanisms is an active field of research in seismology. Many seismologists are determining focal mechanisms of earthquakes using various methods including the matching of full waveform seismograms. This research has different applications depending on the area or purpose for which these seismic event are studied. In earthquake-prone regions, such as Southern California where earthquakes are a great hazard, the focal mechanisms of seismic events are studied to better understand the stress and strain regime and earthquake hazard (Shen et al., 2007). In other regions, such as oil and gas fields, earthquakes are triggered by fluid injection and hydrocarbon extraction (Sarkar, 2008; Sze et al., 2005). Monitoring these earthquakes helps to better characterize reservoirs and also helps to determine where to produce or inject within a given hydrocarbon field (Maxwell, 2007).

Historically, studies of earthquake source mechanism have used short period body wave amplitude and polarity data. The most frequently used method is by studying the initial motion polarity of both primary (P) and secondary (S) waves from seismic recordings of many stations in order to draw the “beach ball” solution, which represents the focal mechanism of the earthquake (Nakamura, 2002). This method can be extended to include the amplitude data of P, SH, and SV with the polarity of the primary wave (Nakamura et al., 1999). Other approaches cross correlate the primary
waveforms to determine the focal mechanisms of earthquakes (Hansen et al., 2006). Another method that is used to determine the focal mechanism of earthquakes in Southern California involves using synthetic seismograms with relatively long periods to invert for the moment tensor (Qinya, 2006; Dreger, 2003).

The synthetic seismogram is the convolution of the source time function, Green’s function weighted by the moment tensor components, and the instrument response. The source time function is assumed to be known and Green’s function is calculated using information about the earthquake hypocenter, the seismic recording station location, and the velocity model of the subsurface. The instrument response can be found in the manual or by contacting the manufacturer of the instrument. The moment tensor is inverted for from the observed seismogram, Green’s function, and the instrument response to find the focal mechanism of a given earthquake (Stein and Wysession, 2003).

One shortcoming of the currently used moment tensor inversion method is that only long periods or relatively low frequencies are used in the inversion. This is not adequate when we want to study the focal mechanism of relatively small magnitude earthquakes, which typically have higher frequencies. One of the problems of studying first motion polarities of primary and secondary waves is that it requires a large number of receivers in order to roughly draw the “beach ball” for the earthquake in question, and even if we had many stations, there will always be a large amount of uncertainty in where to draw the fault planes on the “beach ball”. In principle, this shortcoming can be accounted for if we had a very large number of recording stations, which is not the case in reality.

In this thesis, I use synthetic seismograms and a grid search approach to find the best fit of the full waveforms in order to determine the source mechanism using short period data from a relatively few stations. In my approach, I generate full waveform synthetic seismograms and perform a grid
search to find the best combination of depth, strike, dip, and rake that will result in the synthetic seismogram with the best fit to the original observed seismogram. In order to do this, we have to know the velocity model of the subsurface and the epicenter of the earthquake of interest.

1.2 Thesis contents

The theoretical-numerical approach behind the full waveform modeling method is discussed in chapter two including an explanation of our grid search code that is used to find the best focal mechanism solution. Information about the earthquakes and the stations used to find their focal mechanism along with the Mojave and SoCal velocity models are included in chapter three. Results of this study and the discussion section are found in chapter four followed by conclusions in chapter five. More information about the code can be found in the appendix section of this thesis.
Chapter 2

Theoretical-Numerical Approach

2.1 Waveform modeling

Looking at P-wave first motions is in many cases inadequate to determine the focal mechanisms of earthquakes. Therefore, considering the full waveform solution is essential in understanding and analyzing earthquakes (Stein and Wysession, 2003). The seismic signal that travels from the source within the earth to a receiver can be modeled in a simple diagram as seen in Figure 2-1.

![Figure 2-1: System diagram of a seismogram (Scherbaum, 1994)](image)

The seismic source generates the signal, which travels through the earth and is influenced by the response of the instrument that records the seismic signal. Therefore, the resulting seismogram $u(t)$ can be written as in equation (1) and is shown in Figure 2-2.

$$u(t) = x(t) * e(t) * q(t) * i(t)$$  \(1\)

where: $x(t)$ is the source time function (the signal the earthquake puts into the ground)
$e(t)$ is the elastic effect of the earth structure

$q(t)$ is the anelastic effect of the earth structure

$i(t)$ is the instrument response of the seismometer

$e(t)$ and $q(t)$ are convolved to form Green's function $g(t)$

Convolution in the time domain is equivalent to multiplication in the frequency domain. So we apply the Fourier transform to equation 1 and rewrite it in the form:

$$U(\omega) = X(\omega) E(\omega) Q(\omega) I(\omega) \quad (2)$$

We will discuss each of these factors in the following sections to have a deeper understanding of the different components that make up a seismogram. There is one more component that affects a seismogram and that is noise. There are many known techniques in seismology to attenuate noise such as filtering, stacking and many others (Pujol, 2003; Kennett, 2001). We will not consider noise in this study because we will choose events with high signal to noise ratio and thus we can neglect the effect of noise.
2.2 The Source time function

The source time function denoted by $x(t)$ in equation (1) is the earthquake's source signal that is produced by the rupture of the fault. It is dependent on the derivative of the history of slip on the fault. To better understand this concept, try to think of an earthquake that results from a short fault that slips instantaneously. The seismic moment function of this earthquake is a step function. The derivative of this step function, which is a delta function, is the source time function for this particular earthquake. Relatively small earthquakes are considered to occur along short faults, which can be treated as a single point source. Displacements along these faults are approximated as a smooth ramp function and that is what we will assume for the source time function in this study (Clinton, 2004). Larger earthquakes occur along longer faults and thus have a more complicated source time functions.

2.3 Green's function

The two terms $e(t)$ and $q(t)$ in equation (1) represent the elastic and anelastic effect of the earth's structure respectively. The elastic term $e(t)$ describes the wave propagation in the perfectly elastic medium. It also describes the reflections and refractions at the different boundaries. The other term, namely, $q(t)$ describes the attenuation of the seismic waves. This happens when part of the mechanical energy of the seismic waves is lost to the medium by converting to heat during propagation. This attenuation can be described as a function of time ($t$) and angular frequency ($\omega$) as follows:

$$f(t,\omega) = Ae^{i\omega t}e^{-t/2Q}$$  \hspace{1cm} (3)

The term Q in the equation above is the quality factor and it quantifies the amplitude decay with time. Few important points to observe from equation (3) is that generally waves with higher frequency get attenuated faster than waves with lower frequency. We also note that the larger the
quality factor \((Q)\), the slower the decay, and thus the less the attenuation. The values of \((Q)\) for primary and shear waves are smaller for sediments and the waves get attenuated more in such mediums (Stein and Wysession, 2003). I want to point out here that the quality factor affect on our seismograms is negligible at the frequency ranges we will be working with, that is, 0.1 to 2.0 Hz, and the relatively short distances between the source and stations.

The elastic and anelastic terms \(e(t)\) and \(q(t)\), respectively can be convolved to get the Green’s function in time domain \(g(t)\) as seen in equation (4), or in frequency domain as seen in equation (5):

\[
g(t) = e(t) * q(t) \tag{4}
\]

\[
G(\omega) = E(\omega) Q(\omega) \tag{5}
\]

Green’s function is the signal that would arrive to the seismometer if the source were a point source (e.g. explosion) and the source time function was a delta function. Green’s function can be expressed in the Cartesian coordinate system as a double integral over frequency and horizontal wavenumber for elastic layered medium with the origin of the coordinate system at the source location (Bouchon, 2003):

\[
\phi(x,y,z;\omega) = \frac{iV_s(\omega)}{8\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-ivz} e^{-ik_x x} e^{-ik_y y} dk_x dk_y \tag{6}
\]

with

\[
v = \sqrt{\frac{\omega^2}{\alpha^2} - k_x^2 - k_y^2}, \quad \text{Im}(v) < 0 \tag{7}
\]

where: \(\phi\) is the displacement potential

\(\omega\) is the angular frequency

\(\alpha\) is the P wave velocity

\(V_s\) is the volume change at the source (Note: \(V_s = 0\) for an earthquake)

\(k_x, k_y\) are components of the horizontal wavenumber
The integral in equation (6) can be discretized due to the interference of the waves and expressed as in equation (8) (Bouchon, 1981). This equation describes both the geometrical spreading of the wave and the attenuation. The reflectivity and transmissivity matrices of the layered medium are calculated for each wavenumber.

\[
\phi_a(r,z;\omega) = -\frac{i\pi}{L} \sum_{n=0}^{\infty} \epsilon_n \frac{k_n}{\nu_n} J_0(k_nr)e^{-i\nu_n|z|} \tag{8}
\]

with

\[
k_n = \frac{2n\pi}{L}; \quad \nu_n = \sqrt{k_\alpha^2 - k_n^2} \tag{9}
\]

where: \( \phi_a \) is the displacement potential

\[
k_\alpha = \frac{\omega}{\alpha}
\]

\( \omega \) is the angular frequency

\( r \) is the distance between the source and the observation point

\( z \) is the depth

\( k \) is the wavenumber

\( L \) is source array spacing

\( J_0 \) is the zero order Bessel function

\( \epsilon_n \) is Neumann’s factor and it is defined as

\[
\epsilon_n = 2 \quad \text{if } n \neq 0
\]

\[
\epsilon_n = 1 \quad \text{if } n = 0
\]
2.4 Instrument response

One of the factors that determine a seismogram is the instrument response $i(t)$ as can be seen from equation (1). This will of course differ depending on the type of seismometer used. The instrument response is given by the manufacturer. The response of a seismometer changes with frequency and thus we have to account for that difference when trying to determine the focal mechanism of an earthquake. If the instrument response is flat at the frequency band we are interested in, then we can ignore the instrument response because it will have a small affect on the shape of the seismogram. Broadband seismograms have a flat response through most frequencies (Havskov and Alguacil, 2004). The seismic recording stations used in this study are all STS-2 broadband stations with flat instrument response at the frequency band we are interested in, namely, from 0.1 to 2.0 Hz as can be seen in Figure 2-3 (Southern California Earthquake Data Center). Therefore, we will ignore their responses. Figure 2-3 shows the frequency responses of broadband seismometers.

Figure 2-3: Velocity Response vs. Frequency (Southern California Seismic Network)
2.5 Full waveform synthetic seismograms

The full waveform synthetic seismograms seen in this thesis will be generated using a modified version of a discrete wavenumber code written originally by Michel Bouchon as part of his MIT Ph.D. thesis (Bouchon, 1976). The code is based on a discrete wavenumber representation of elastic waves. The user specifies some parameters of the earthquake, the location of the stations, and the velocity model of the medium. The code calculates the moment tensor and weights it with Green’s function, then convolves it with the source time function to produce the synthetic seismogram. Please refer to appendix (A) for more details about executing the code and the input and output files.

2.6 Matlab grid search code for source parameters

We developed a Matlab code version of the discrete wavenumber code with the improvement of optimizing the code so that it will perform a grid search through all the combinations of depths, strikes, dips, and rakes in a reasonable time frame on the order of a few minutes to a few hours depending on how small we choose our search intervals. The code was modified to calculate Green’s functions for five different depths with the midpoint being the initial estimate of the depth and 2 km above and below that depth with a 1 km grid interval. This operation is performed only once since Green’s function depends only on the source and receiver locations and the velocity structure, which is constant for a single pair of earthquake and seismic recording station. The code then calculates the moment tensor, which is dependant on the focal mechanism of the earthquake. This is done for different combinations of strikes, dips, and rakes. We specify the search interval to be 10° and start from 0° to 350°, 0° to 90°, and -90° to 90° for the strike, dip, and rake, respectively. This means that we have 36 x 10 x 19 = 6840 different combinations of strikes, dips, and rakes. Green’s functions are weighted by the components of the moment tensor and the results are convolved with a smooth ramp.
source time function to produce synthetic seismograms. This whole operation results in \(36 \times 10 \times 19 \times 5 = 34200\) different synthetic seismograms for one seismic station. A Butterworth band pass filter with frequency band of 0.1 to 2.0 Hz is applied to these seismograms and they are cross-correlated and normalized with the observed seismogram. \(L_2\) norm is added to the code in order to consider the amplitude match when performing the cross correlation and not only the phases of the waveforms. This is done independently for each component of each seismic station to allow for differences that arise from anisotropy and heterogeneity in the subsurface of the earth (Sileny and Vavrycuk, 2002).

The code performs a joint grid search by considering a group of stations and not only one station, and finds the best fit between the filtered observed and filtered synthetic seismograms that suit all three components of all the seismic stations used to study a single earthquake. The best fit is calculated using the following objective function (Li et al., 2009):

\[
\text{maximize } (J(\text{strike}, \text{dip}, \text{rake}, \text{depth})) = \sum_{n=1}^{N} \sum_{j=1}^{3} \{A_1 \max(d^n_j \otimes v^n_j) - A_2 \| d^n_j \otimes v^n_j \|_2 \}
\]

The objective function \((J)\) seen above consists of \(A_1\) through \(A_2\), which are predetermined weights for each of the two terms. The first term evaluates the maximum cross correlation between the normalized (relative to maximum amplitude) observed data \((d^n_j)\) and normalized synthetic waveforms \((v^n_j)\), where \(n\) denotes the station number, \(\otimes\) denotes cross correlation, and \(j\) denotes the component (north, south, vertical). The second term evaluates \(L_2\) norm, the direct difference between the synthetic and observed seismograms.

The corresponding strike, dip, and rake is the focal mechanism of the earthquake we are investigating, and the depth determines the hypocenter of the earthquake since we assume that the epicenter is known.
Chapter 3

Earthquake Data

3.1 Earthquakes

We test out method using seismograms from two earthquakes in Southern California. The epicenters of the events and the stations used to invert for the focal mechanism are shown in Figures 3-1 and 3-3. Maps with the major faults in the area of interest are shown in Figure 3-2 and 3-3 and a table with the source location parameters is shown in Table 3-1.

Figure 3-1: Earthquake 9718013 (red star) and 5 recording stations (yellow circles)
Figure 3-2: Earthquake 9718013 (cross sign) and 5 recording stations (black squares) (Southern California Earthquake Data Center)

Figure 3-3: Earthquake 14408052 (red star) and 5 recording stations (yellow circles)
3.2 Velocity models

The Southern California Earthquake Data Center (SCEC) released version 4, which is a new 3-D velocity model of Southern California, in 2005. The Mojave and SoCal velocity models seen below are good approximations but are not exactly identical to the most recent SCEC velocity model. This is expected because the SCEC velocity model is too smooth to generate any meaningful seismograms using the discrete wavenumber (DWN) code. Another point to clarify here is that the velocity model varies from one location to the other. In other words, the layers are not horizontal. This presents a
problem because the code used here only deals with horizontal layers. Therefore, the SoCal and Mojave velocity models seen below are used because they overcome the issues mentioned above and at the same time are good approximations to the SCEC velocity model.

### 3.2.1 Mojave velocity model

<table>
<thead>
<tr>
<th>Thickness (km)</th>
<th>P (km/s)</th>
<th>S (km/s)</th>
<th>Density (g/cm³)</th>
<th>Q_p</th>
<th>Q_s</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>5</td>
<td>2.6</td>
<td>2.4</td>
<td>70</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>5.5</td>
<td>3.45</td>
<td>2.4</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>22.5</td>
<td>6.3</td>
<td>3.6</td>
<td>2.67</td>
<td>150</td>
<td>75</td>
</tr>
<tr>
<td>Half space</td>
<td>7.85</td>
<td>4.4</td>
<td>3.42</td>
<td>200</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 3-2: Mojave velocity model (Jones and Helmberger, 1998), and quality factor (Erickson et al., 2004)

### 3.2.2 SoCal velocity model

<table>
<thead>
<tr>
<th>Thickness (km)</th>
<th>P (km/s)</th>
<th>S (km/s)</th>
<th>Density (g/cm³)</th>
<th>Q_p</th>
<th>Q_s</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5</td>
<td>5.5</td>
<td>3.18</td>
<td>2.4</td>
<td>70</td>
<td>40</td>
</tr>
<tr>
<td>10.5</td>
<td>6.3</td>
<td>3.64</td>
<td>2.67</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>16</td>
<td>6.7</td>
<td>3.87</td>
<td>2.8</td>
<td>150</td>
<td>75</td>
</tr>
<tr>
<td>Half space</td>
<td>7.85</td>
<td>4.5</td>
<td>3.42</td>
<td>200</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 3-3: Southern California standard velocity model (Dreger and Helmberger, 1993), and quality factor (Erickson et al., 2004)

### 3.3 Seismic Recording Stations

The five stations used to find the focal mechanism and depth of earthquakes 9718013 and 14408052 are listed in Tables 3-4 and 3-5, respectively. These stations were chosen from a larger set of stations available in Southern California so that we will have relatively good azimuth coverage. Then the method is repeated using a subset of these stations with relatively poor azimuth coverage.
3.3.1 Stations used for event 9718013

<table>
<thead>
<tr>
<th>Station</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Distance (km)</th>
<th>Azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>AZ.KNW</td>
<td>33.7141</td>
<td>-116.7119</td>
<td>29.34</td>
<td>321.40</td>
</tr>
<tr>
<td>CI.JCS</td>
<td>33.0859</td>
<td>-116.5959</td>
<td>47.54</td>
<td>189.23</td>
</tr>
<tr>
<td>AZ.CRY</td>
<td>33.5654</td>
<td>-116.7373</td>
<td>21.65</td>
<td>287.20</td>
</tr>
<tr>
<td>CI.DGR</td>
<td>33.6500</td>
<td>-117.0094</td>
<td>48.54</td>
<td>289.12</td>
</tr>
<tr>
<td>AZ.RDM</td>
<td>33.6300</td>
<td>-116.8478</td>
<td>33.77</td>
<td>293.77</td>
</tr>
</tbody>
</table>

Table 3-4: List of seismic stations used for event 9718013

3.3.2 Stations used for event 14408052

<table>
<thead>
<tr>
<th>Station</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Distance (km)</th>
<th>Azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>CI.DSC</td>
<td>35.1425</td>
<td>-116.1039</td>
<td>46.54</td>
<td>37.98</td>
</tr>
<tr>
<td>CI.GMR</td>
<td>34.7845</td>
<td>-115.6599</td>
<td>69.38</td>
<td>92.39</td>
</tr>
<tr>
<td>CI.GSC</td>
<td>35.3017</td>
<td>-116.8057</td>
<td>64.75</td>
<td>327.17</td>
</tr>
<tr>
<td>CI.JVA</td>
<td>34.3662</td>
<td>-116.6126</td>
<td>52.74</td>
<td>199.69</td>
</tr>
<tr>
<td>CI.MCT</td>
<td>34.2264</td>
<td>-116.0407</td>
<td>73.85</td>
<td>151.90</td>
</tr>
</tbody>
</table>

Table 3-5: List of seismic stations used for event 14408052

3.4 Seismograms

3.4.1 Raw seismograms of event 9718013

In Figures 3-5 to 3-9, we show the three components (N, E, Z) of each of the five stations along with their frequency spectra. The sampling interval of the original seismograms is 0.05 seconds, and the sampling frequency is $1/0.05$ seconds = 20 Hz. The following seismograms have a Nyquist frequency of 10 Hz.
Figure 3-5: Earthquake recording at station AZ.CRY and frequency spectrum

Figure 3-6: Earthquake recording at station AZ.KNW and frequency spectrum
EQ ID = 9718013
Station AZ.RDM.BH dis=33.771km azm=293.7767

[Graph of earthquake recording and frequency spectrum for station AZ.RDM.BH]

Figure 3-7: Earthquake recording at station AZ.RDM and frequency spectrum

EQ ID = 9718013
Station CI.DGR.BH dis=48.5401km azm=289.1215

[Graph of earthquake recording and frequency spectrum for station CI.DGR.BH]

Figure 3-8: Earthquake recording at station CI.DGR and frequency spectrum
3.4.2 Filtered seismograms of event 9718013

The filtered observed seismograms are shown in Figures 3-10 to 3-14. At most stations, the amplitude spectra have high values at frequencies lower than 2.0 Hz. The observed seismograms are re-sampled to 20/256 = 0.0781 seconds to match the sampling interval of the synthetic seismograms of event 9718013, which will be shown in the results and discussion section in chapter four. We didn’t do the opposite, namely, calculate the synthetic seismograms every 0.05 seconds because it slows down the calculation time significantly. A Butterworth band pass filter with frequency band of 0.1 to 2.0 Hz is applied to these re-sampled observed seismograms and the resultant five seismograms are shown below.
Figure 3-10: Filtered earthquake recording at station AZ.CRY

Figure 3-11: Filtered earthquake recording at station AZ.KNW
Figure 3-12: Filtered earthquake recording at station CI.RDM

Figure 3-13: Filtered earthquake recording at station CI.DGR
3.4.3 Raw seismograms of event 14408052

In Figures 3-15 to 3-19, we show the three components (N, E, Z) of motion recorded at each of the five stations along with the frequency spectra. The sampling interval of the original seismograms is 0.05 seconds, and the sampling frequency is $1/0.05 = 20$ Hz. The following seismograms have a Nyquist frequency of 10 Hz.
Figure 3-16: Earthquake recording at station CI.GMR and frequency spectrum

Figure 3-17: Earthquake recording at station CI.GSC and frequency spectrum
EQ ID = 14408052
Station CI.JVA.BH dis=52.7478km azm=199.6934

Figure 3-18: Earthquake recording at station CI.JVA and frequency spectrum

EQ ID = 14408052
Station CI.MCT.BH dis=73.8569km azm=151.9083

Figure 3-19: Earthquake recording at station CI.MCT and frequency spectrum
3.4.4 Filtered seismograms of event 14408052

The filtered observed seismograms are shown in Figures 3-20 to 3-24. At most stations, the amplitude spectrum has high values at frequencies lower than 2.0 Hz. The observed seismograms are re-sampled to $\frac{23}{256} = 0.0703$ seconds to match the sampling interval of the synthetic seismograms of event 14408052, which will be shown in the results and discussion section in chapter four. We didn’t do the opposite, namely, calculate the synthetic seismograms every 0.05 seconds because it slows down the calculation time significantly. A Butterworth band pass filter with frequency band of 0.1 to 2.0 Hz is applied to these re-sampled observed seismograms and the resultant five seismograms are shown below.

Figure 3-20: Filtered earthquake recording at station CI.DSC
Figure 3-21: Filtered earthquake recording at station CI.GMR

Figure 3-22: Filtered earthquake recording at station CI.GSC
Figure 3-23: Filtered earthquake recording at station CI.JVA

Figure 3-24: Filtered earthquake recording at station CI.MCT
Chapter 4

Results and Discussion

4.1 Earthquake ID 9718013

4.1.1 Joint modeling using 5 stations

We use the joint grid search code to get the best combination of depth, strike, dip, and rake that results in the best fit between the filtered synthetic and filtered observed seismograms for all three components of motion and for all the five stations. The best fit is determined using cross correlation and $L_2$ norm, which are built-in Matlab functions and this is explained in further detail in section 2.6.

The time duration used in the matching of the synthetic and observed seismograms for stations CRY, KNW, RDM, DGR, JCS are 13, 14, 15, 18, and 18 seconds, respectively. The length of the synthetic seismogram is determined by observing the initial match between the synthetic and observed seismograms, then determining where the fit of the synthetic seismogram starts to deteriorate. We choose different time duration for each station depending on our observations of the match between the synthetic and observed seismogram.

I will use the Southern California standard velocity model (SoCal), which is defined earlier in this thesis for this earthquake. I will show plots of the best-fit solution of the filtered synthetic seismograms overlaying the filtered original seismogram for each component, that is, the north-south, east-west, and vertical components, and I will do so for each of the five stations.
Figure 4-1: Station CRY with band pass filter applied

Figure 4-2: Station KNW with band pass filter applied
Figure 4-3: Station RDM with band pass filter applied

Figure 4-4: Station DGR with band pass filter applied
It is obvious from the above five Figures that our joint grid search code did a very good job in finding a solution especially if we considered that the solution managed to fit all three components of motion recorded at each station and did so for all five stations, which gives us a great amount of confidence in the solution. The parameters of this best solution puts the depth of the earthquake at 15 km with a strike, dip, and rake of 300, 60, and -170, respectively. The best ten solutions (Table 4-1) all give similar focal mechanisms. This is all done with a filtering frequency band of 0.1 to 2.0 Hz. The results can be compared to Southern California Earthquake Data Center’s moment tensor solution, which puts the earthquake at a depth of 8 km with a strike, dip, and rake of 318, 33, and -180, respectively. The Southern California Earthquake Data Center puts the earthquake at a depth of 15.22 km, which is consistent with our results (Southern California Earthquake Data Center). The focal mechanism as we can see comparing the two sets of solutions are reasonably identical except for the dip. The dip of strike-slip faults in Southern California are closer to vertical and a dip of 60 is more
reasonable for faults in Southern California than a much shallower dip of 33 (Toköz, M. Nafi, personal communication, March 2009).

The “beach ball” solution representing this focal mechanism is shown below in Figure 4-6. Looking at it, it is clear that this is a right lateral strike slip fault. The strike of the fault is consistent with the northwest, southeast orientation of the faults in the region as seen in Figure 3-2. It is also worth noting that stations that are located to the west or east of the earthquake are expected to have a more pronounced P-wave first arrival in the east-west component compared to the north-south component. That explains the relatively small amplitude of the P-wave first arrival in the north-south component of station CRY compared to the east-west component as seen in Figure 4-1. Similarly, the amplitude of the P-wave first arrival in the north-south component is relatively large compared to the east-west component for station JCS, shown in Figure 4-5, because it lies almost directly to the south of the earthquake. The P-waves first arrivals are expected to be relatively weak in amplitude when the station’s azimuth lies close to the nodal plane of the “beach ball” solution. This is due to the fact that the stress switches from compression, represented by an upward polarity for the P-wave first arrival, to dilation, represented by a downward polarity for the P-wave first arrival. This can be seen in all components of station RDM shown in Figure 4-3 because it has an azimuth of 293.7°, which is close to the strike of the fault.

![Figure 4-6: Best solution beach ball representation for earthquake 9718013 using 5 stations](image-url)
<table>
<thead>
<tr>
<th>Solution #</th>
<th>Strike</th>
<th>Dip</th>
<th>Rake</th>
<th>Depth (km)</th>
<th>Goodness of fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>300</td>
<td>60</td>
<td>-170</td>
<td>15</td>
<td>37.89</td>
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<tr>
<td>2</td>
<td>300</td>
<td>60</td>
<td>-160</td>
<td>15</td>
<td>37.15</td>
</tr>
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<td>3</td>
<td>300</td>
<td>50</td>
<td>-170</td>
<td>15</td>
<td>37.00</td>
</tr>
<tr>
<td>4</td>
<td>300</td>
<td>50</td>
<td>-180</td>
<td>15</td>
<td>36.77</td>
</tr>
<tr>
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<td>70</td>
<td>-160</td>
<td>15</td>
<td>36.64</td>
</tr>
<tr>
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<td>50</td>
<td>170</td>
<td>15</td>
<td>36.29</td>
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<td>50</td>
<td>-160</td>
<td>15</td>
<td>36.23</td>
</tr>
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<td>8</td>
<td>300</td>
<td>60</td>
<td>-180</td>
<td>15</td>
<td>36.19</td>
</tr>
<tr>
<td>9</td>
<td>310</td>
<td>50</td>
<td>160</td>
<td>15</td>
<td>36.09</td>
</tr>
<tr>
<td>10</td>
<td>310</td>
<td>50</td>
<td>170</td>
<td>15</td>
<td>35.86</td>
</tr>
</tbody>
</table>

Table 4-1: Best 10 solutions in descending order for event 9718013 using 5 stations

Histograms of strike, dip, rake, and depth of the best 200 solutions for event 9718013 using five stations are shown in Figure 4-7. Depth is the most constrained parameter because it has the smallest standard deviation percentage. Notice that the depth values do not look like a typical bell shaped curve because we choose the depth interval in the grid search to be constrained between 11 km and 15 km. If we had allowed the depth to have larger limits, we would expect the depth histogram to look more symmetric. Dip values are spread out over most possible values and have a relatively large standard deviation percentage compared to the other three parameters, which makes the dip the least constrained parameter. This could be due to the fact that all five recording stations used to model the focal mechanism of this earthquake are to the northwest and south of the earthquake’s epicenter with no station coverage on the east as shown in Figure 3-1. The values of strike that represent the same nodal plane are taken to be the larger strike value. For example if the strike is 310, then it is represented on the histogram as 310. However, if the strike is 130, which is equivalent to a strike of 310, then the strike value is converted from 130 to 310 and represented as such in the histogram. Similarly, values of rake are converted to the larger positive equivalent to make it easier to see in the histogram. For example, if the rake is 170 then it is represented as 170. However, if the rake is -170,
then we add 360 to that to arrive to the equivalent positive rake of 190 and it is represented as such in the histogram.

![Histograms of the best 200 solutions for earthquake 9718013 using 5 stations](image)

Figure 4-7: Histograms of the best 200 solutions for earthquake 9718013 using 5 stations

We used five stations to find the focal mechanism of earthquake 9718013 and obtained a very good fit between the observed and synthetic seismograms. The fact that all the best ten solutions are consistent makes us confident in our results and in our full waveform modeling method.

### 4.1.2 Joint modeling using 3 stations

We will test our method using three stations with poor azimuth coverage, shown in Figure 4-8, and compare our results with the results we obtained using five stations.
Stations used for event 9718013 (3 stations)

Figure 4-8: Earthquake (red star) and 3 recording stations (yellow circles)
Station: AZ.KNW; Filter [0.1-2.0]; SoCal velocity model

Figure 4-9: Station KNW with band pass filter applied
Figure 4-10: Station RDM with band pass filter applied

Figure 4-11: Station DGR with band pass filter applied
Once again our grid search found a very good match between the filtered observed and filtered synthetic seismograms for all three components of the three stations as shown in Figures 4-9 to 4-11. The ten best solutions are shown in Table 4-2 and they all show similar focal mechanisms. This gives us confidence in our modeling method. The focal mechanism here is similar to the one we got previously when we used five seismic stations. The depth is 14 km with a strike, dip, and rake of 300, 60, and -180, respectively. The “beach ball” solution is shown in Figure 4-12. This suggests that the fault is a right lateral strike slip fault.

![Figure 4-12: Best solution beach ball representation for earthquake 9718013 using 3 stations](image)

We show the histograms of the best 200 solutions of strike, dip, rake, and depth for event 9718013 using three stations in Figure 4-13. The strike has the smallest standard deviation percentage.
and thus is the most constrained parameter. The dip still has the largest standard deviation percentage and thus is again the least constrained parameter. The strike, rake, and depth are still relatively well constrained. Notice that we have converted the values of strike and rake exactly as we did in the previous example. For example, we added 360 to all negative values of rake to obtain the equivalent positive rake value so that it will be easier to draw the histogram.

![Histograms of the best 200 solutions for earthquake 9718013 using 3 stations](image)

Figure 4-13: Histograms of the best 200 solutions for earthquake 9718013 using 3 stations

4.2 Earthquake ID 14408052

4.2.1 Joint modeling using 5 stations

We will test our method with another earthquake in Southern California. We use the joint grid search code to get the best combination of depth, strike, dip, and rake that results in the best fit
between the filtered synthetic and filtered observed seismograms for all three components and for all the five stations. The best fit is determined using cross correlation and $L_2$ norm, which are built-in Matlab functions. Further details on how the best fit is determined can be found in section 2.6. The time duration used in the matching of the synthetic and observed seismograms for stations DSC, GMR, GSC, JVA, MCT are 17, 22, 22, 18, and 20 seconds, respectively. I use the Mojave velocity model, which is defined earlier in this thesis, for this earthquake. In Figures 4-14 to 4-18, I show plots of the best-fit solution of the filtered synthetic seismogram overlaying the filtered original seismogram for each component, namely, the north-south, east-west, and vertical components, and I will do so for each of the five stations.

Figure 4-14: Station DSC with band pass filter applied
Figure 4-15: Station GMR with band pass filter applied

Figure 4-16: Station GSC with band pass filter applied
Figure 4-17: Station JVA with band pass filter applied

Figure 4-18: Station MCT with band pass filter applied
It is obvious from the above five Figures that our joint grid search code once again did a very good job in finding the focal mechanism, especially if we considered that the solution managed to fit all three components of each station and did so for all five stations, which gives us a great amount of confidence in the method and resultant solution. The best solution determined that the depth of the earthquake is at 7 km with a strike, dip, and rake of 160, 80, and -170, respectively. The strike of the fault is consistent with the orientation of faults in the region as can be seen in Figure 3-4. All ten best solutions resulted in the same focal mechanism as shown in Table 4-3. This is all done with a filtering frequency band of 0.1 to 2.0 Hz. The results can be compared to Southern California Earthquake Data Center’s moment tensor solution, which puts the earthquake at a depth of 5 km with a strike, dip, and rake of 162, 82, and -167, respectively. The Southern California Earthquake Data Center puts the earthquake at a depth of 6 km, which is close to our results. The focal mechanisms as we can see comparing the two sets of solutions are similar.

The “beach ball” solution representing this focal mechanism is shown below in Figure 4-19. This suggests that this is a right lateral strike slip fault. The amplitude of the P-wave first arrival in station GMR, shown in Figure 4-15, is relatively small in the north-south component compared to the east-west component because station GMR is to the east of the earthquake epicenter.

![Figure 4-19: Best solution beach ball representation for earthquake 14408052 using 5 stations](image)
Table 4-3: Best 10 solutions in descending order for event 14408052 using 5 stations

<table>
<thead>
<tr>
<th>Solution #</th>
<th>Strike (°)</th>
<th>Dip (°)</th>
<th>Rake (°)</th>
<th>Depth (km)</th>
<th>Goodness of fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>160</td>
<td>80</td>
<td>-170</td>
<td>7</td>
<td>43.29</td>
</tr>
<tr>
<td>2</td>
<td>170</td>
<td>70</td>
<td>-160</td>
<td>7</td>
<td>43.09</td>
</tr>
<tr>
<td>3</td>
<td>160</td>
<td>80</td>
<td>-160</td>
<td>8</td>
<td>43.07</td>
</tr>
<tr>
<td>4</td>
<td>170</td>
<td>70</td>
<td>-170</td>
<td>7</td>
<td>42.78</td>
</tr>
<tr>
<td>5</td>
<td>160</td>
<td>80</td>
<td>-160</td>
<td>7</td>
<td>42.52</td>
</tr>
<tr>
<td>6</td>
<td>170</td>
<td>80</td>
<td>-170</td>
<td>7</td>
<td>41.72</td>
</tr>
<tr>
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<td>80</td>
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<td>40.99</td>
</tr>
<tr>
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<td>170</td>
<td>70</td>
<td>-160</td>
<td>8</td>
<td>40.64</td>
</tr>
<tr>
<td>9</td>
<td>160</td>
<td>90</td>
<td>-170</td>
<td>7</td>
<td>40.48</td>
</tr>
<tr>
<td>10</td>
<td>340</td>
<td>90</td>
<td>170</td>
<td>7</td>
<td>40.48</td>
</tr>
</tbody>
</table>

Histograms of strike, dip, rake, and depth are shown in Figure 4-20. The dip is the least constrained parameter with the largest standard deviation percentage. However, it is less than the values obtained for earthquake 9718013. This could be due to the fact that stations used for earthquake 14408052 have better azimuth coverage as can be seen comparing Figures 3-1 and 3-3. The depth is not the most constrained parameter as was the case for earthquake 9718013 and this is because the stations used for this earthquake are further away from the earthquake’s epicenter and further stations are less dependent on depth. Notice that strike values that represent the same nodal plane are converted to the smaller value to be consistent with the best ten solution seen in Table 4-3. Rake values have been converted just like in previous examples to the equivalent positive rake value. It is also worth mentioning that the histogram showing depth values is not symmetrical because we constrained the depth grid search interval to be between 4 km and 8 km. If we allowed the depth to have values larger than 8 km, we would expect the histogram to look more symmetrical than it looks now with the current depth constraint.
We used five stations to find the focal mechanism and depth of earthquake 14408052. In the next section, we will choose a subset of the recording stations and use our method to try to find the focal mechanism and depth of the same earthquake. These results are compared with those obtained using five stations.

4.2.2 Joint modeling using 2 stations

We will test our method using two stations with poor azimuth coverage, shown in Figure 4-21, and compare our results with the results we obtained using five stations.
Figure 4-21: Earthquake (red star) and 2 recording stations (yellow circles)

Figure 4-22: Station DSC with band pass filter applied
It is obvious from the above two Figures that our joint grid search code once again did a very good job in finding the focal mechanism and depth of the earthquake. The parameters of the best solution puts the depth of the earthquake at 8 km with a strike, dip, and rake of 160, 80, and -160, respectively. This is all done with a filtering frequency band of 0.1 to 2.0 Hz. The results are consistent with the previous results obtained using five seismic stations.

The “beach ball” solution representing this focal mechanism is shown below in Figure 4-24. This once again suggests that the fault is a right lateral strike slip fault. The best ten solutions in descending order are shown in Table 4-4. These results once again are consistent with the results obtained using five recording stations, and are consistent with each other, which give us confidence once again in the method used to find the focal mechanism and depth of an earthquake. It should be noted here that the code was optimized and thus it choose 160 for the strike and not 340 because the rake search interval
is from -90 to 90. We will have to change this interval to be from -180 to 180 to get a strike of 340 as a solution. However, this would be the same nodal plane and the code would be consuming more time to obtain the exact same focal mechanism.

![Strike = 160; Dip = 80; Rake = -160](image)

Figure 4-24: Best solution beach ball representation for earthquake 14408052 using 2 stations

<table>
<thead>
<tr>
<th>Solution #</th>
<th>Strike</th>
<th>Dip</th>
<th>Rake</th>
<th>Depth (km)</th>
<th>Goodness of fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>160</td>
<td>80</td>
<td>-160</td>
<td>8</td>
<td>24.66</td>
</tr>
<tr>
<td>2</td>
<td>330</td>
<td>90</td>
<td>160</td>
<td>8</td>
<td>24.11</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
<td>90</td>
<td>-160</td>
<td>8</td>
<td>24.11</td>
</tr>
<tr>
<td>4</td>
<td>160</td>
<td>90</td>
<td>-160</td>
<td>8</td>
<td>24.05</td>
</tr>
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<td>5</td>
<td>340</td>
<td>90</td>
<td>160</td>
<td>8</td>
<td>24.05</td>
</tr>
<tr>
<td>6</td>
<td>150</td>
<td>80</td>
<td>-150</td>
<td>8</td>
<td>23.86</td>
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<tr>
<td>7</td>
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<td>80</td>
<td>160</td>
<td>8</td>
<td>23.84</td>
</tr>
<tr>
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<td>320</td>
<td>80</td>
<td>170</td>
<td>8</td>
<td>23.57</td>
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<tr>
<td>9</td>
<td>160</td>
<td>90</td>
<td>-170</td>
<td>8</td>
<td>23.45</td>
</tr>
<tr>
<td>10</td>
<td>340</td>
<td>90</td>
<td>170</td>
<td>8</td>
<td>23.45</td>
</tr>
</tbody>
</table>

Table 4-4: Best 10 solutions in descending order for event 14408052 using 2 stations

Histograms of the strike, dip, rake, and depth are shown below in Figure 4-25. The strike has a larger standard deviation percentage compared to the previous case where we used five stations. That is expected since we are using only two stations and they both are not close to the nodal plane. The depth is well constrained but is not symmetrical as we would expect due to the fact that we limited the depth grid search interval to be between 4 km and 8 km. Had we allowed the depth to have values larger than 8 km, we would have seen a more symmetrical distribution of the values of depth.
Figure 4-25: Histograms of the best 200 solutions for earthquake 14408052 using 2 stations
Chapter 5

Conclusions

In this thesis we developed and tested an approach for determining focal mechanisms of local earthquakes by calculating synthetic seismograms for a complete suite of source models and finding the one that matches the observed seismograms. The primary goal was to obtain the mechanism from seismograms at a few (2 to 5) stations using broadband data that include high frequencies. The algorithm is a grid-search approach that finds the source mechanism by matching synthetic seismograms from all possible source models with the observed seismograms. Two criteria are used for the match: (a) similarity of waveforms obtained by correlations and (b) the amplitudes. All three components (N, E, Z) are used for matching. The method was tested using two earthquakes in Southern California.

Determining the focal mechanism of earthquakes by the full waveform modeling method is fast and reliable. The discrete wavenumber method calculates the entire frequency spectrum and thus we can choose a suitable frequency band when attempting to find the best-fit solution. Not only did the best solution result in a good match, but also all top ten solutions are very similar, and in many cases, identical. Results obtained in this thesis prove that the solutions of the focal mechanisms are robust and consistent. We were able to perform the correlation using a relatively high frequency range, that is, from 0.1 to 2.0 Hz, which helps to constrain the model. This makes the approach ideally suited for application to induced seismic events in oil and gas fields where: (1) the velocity structure is known, (2) events are relatively small and are of high frequency, and (3) station coverage is sparse.

In our tests, we found that the full waveform modeling method works well with data from five stations and with as few as two stations. Synthetic seismograms are sensitive to focal depth and their
computation requires a good knowledge of seismic velocity structure(s) between the source and receivers. The focal mechanisms obtained for both test earthquakes are consistent with the moment tensor solution of Southern California Earthquake Data Center, with the exception of the dip value of earthquake 9718013, we obtained a steeper dip and argued that the steeper dip is more probable for strike slip faults in Southern California. The strikes of the faults are consistent with the orientation of faults in the region. Depth values of earthquakes obtained using full waveform modeling are consistent with the Southern California Earthquake Data Center solutions, but different in one case from that obtained by the moment tensor solutions obtained using low frequency data.
Appendix A: Discrete Wavenumber Calculations:

A.1 Input file

The input file to the discrete wavenumber code is shown in Figure A-1. This is just an example and some of the parameters will change throughout the different cases. The first line specifies the number of layers of the velocity model, which is four. The next four lines correspond to information about each layer. Each of these rows have six columns which corresponds to the thickness of the layer in km, the primary wave velocity in km/second, the shear wave velocity in km/second, the density in g/cm$^3$, the quality factor of p-waves, and the quality factor of s-waves. Two different velocity models will be used and this is just one of them.

```
4
5.500  5.500  3.180  2.40  70.000  40.000
10.500  6.300  3.640  2.67 100.000  50.000
16.000  6.700  3.870  2.80 150.000  75.000
 0.000  7.850  4.500  3.42 200.000 100.000
13.000
10.0  10.0  30.0
1.00
1.000
5  0.000
29.3438  321.4061
47.5486  189.2344
21.6583  287.2009
48.5401  289.1215
33.771  293.7767
256  18.00
 0.40  0.000
20000  500
```

Figure A-1: Input file for the discrete wavenumber code

The line right after the velocity model is the hypocentral depth of the earthquake, which is 13 km in this example. The next line specifies the strike, dip, and rake. This doesn’t matter here because
we will perform a grid search through all combinations of strikes, dips, and rakes, as we will see in the Matlab grid search section. The next line specifies the slip amplitude, and then on the next line is the length of the fault in km and this is used to calculate the amplitude of the seismogram. We used one km as the length of the fault because that is the typical length of faults on which earthquakes with magnitudes 5 occur. The line that follows specifies the number of receivers and the depth of the receivers in km. All the receivers are assumed to be at the same depth and that is zero. The next line lists the distance in km and the azimuth, measured clockwise from the north, of each receiver. The first column in the next line specifies the number of points of each seismogram, and this number must be a multiple of 2. The second column of the same row is the length of the time window in seconds. The following line specifies the rise time of the slip in seconds and the starting time of the seismogram. The last line specifies ‘M’ or the maximum value of K allowed, which should always be larger than K. This is followed by ‘XL’, or the periodicity length, which should always be larger than or equal to the length of the time window in seconds times the largest velocity plus the distance between the receiver and the seismic event.

The moment tensor is calculated using the values of strike (φ), dip (δ) and rake (λ) as shown in the following equations (Udías, 1999):

\[ m_{11} = - \sin \delta \cos \lambda \sin 2\phi - \sin (2\delta) \sin^2 \phi \sin \lambda \]
\[ m_{22} = \sin \delta \cos \lambda \sin (2\phi) - \sin (2\delta) \cos^2 \phi \sin \lambda \]
\[ m_{33} = \sin (2\delta) \sin \lambda \]
\[ m_{12} = \sin \delta \cos \lambda \cos (2\phi) + \frac{1}{2} \sin (2\delta) \sin (2\phi) \sin \lambda \]
\[ m_{13} = - \sin \lambda \sin \phi \cos (2\delta) - \cos \delta \cos \lambda \cos \phi \]
\[ m_{23} = \cos \phi \sin \lambda \cos (2\delta) - \cos \delta \cos \lambda \sin \phi \]
Green’s function is weighted by the moment tensor and convolved with a smooth ramp function to generate the synthetic seismograms.

A.2 Executing the code

The names of the input and output files are specified in the executable Fortran file YIBAL_VELOCITY.f. The code is executed by typing in the following command in the terminal window:

```
> g77 -o yibal YIBAL_VELOCITY.f
> yibal
```

This will cause the code to run and the output will appear in the specified directory.

A.3 Output file

The output of the code is called “output” and it contains three columns as we can see in Figure A-2. The first column is “1”, which corresponds to the North–South component of the synthetic seismogram. The second and third columns correspond to the time and amplitude respectively. The terms “2” and “3” seen later in the first column corresponds to the East – West, and vertical components, respectively. It is important to note here that the rake in the discrete wavenumber code is defined differently than the conventional definition of rake. In order to convert the rake back to the conventional definition we add 180° if the rake is negative, and subtract 180° if the rake is positive. For example, if the code gives us a rake value of -10, then the value of the rake using the conventional definition is 170°. All rake values obtained are converted to the conventional definition of the rake.
<table>
<thead>
<tr>
<th></th>
<th>0.0</th>
<th>0.0</th>
</tr>
</thead>
<tbody>
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<td>0.00906</td>
<td>4.51E-07</td>
</tr>
<tr>
<td>1</td>
<td>0.07813</td>
<td>1.44E-07</td>
</tr>
<tr>
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<td>0.11719</td>
<td>-4.75E-07</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
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<td>0.0</td>
</tr>
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</tr>
<tr>
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<td>-4.24E-07</td>
</tr>
</tbody>
</table>

Figure A-2: Output file from the discrete wavenumber code
Appendix B: Grid Search Code:

```matlab
%% read in parameters
clear all
inp_id = fopen('model_homo.inp','r'); %modified
cnt = fscanf(inp_id, '%d', 1);
laystruct = fscanf(inp_id, '%f', [6 cnt]);
depth = fscanf(inp_id, '%f', 1);
str = fscanf(inp_id, '%f', 1);
dip = fscanf(inp_id, '%f', 1);
rake = fscanf(inp_id, '%f', 1);
slip = fscanf(inp_id, '%f', 1);
faultl = fscanf(inp_id, '%f', 1);
NR = fscanf(inp_id, '%d', 1);
ZR = fscanf(inp_id, '%f', 1);
for inr=1:NR
    RO(inr) = fscanf(inp_id, '%f', 1);
    AZ(inr) = fscanf(inp_id, '%f', 1);
end
ntime = fscanf(inp_id, '%d', 1);
TL = fscanf(inp_id, '%f', 1);
tsource = fscanf(inp_id, '%f', 1);
T0 = fscanf(inp_id, '%f', 1);
M = fscanf(inp_id, '%d', 1);
XL = fscanf(inp_id, '%d', 1);
gf_flag = fscanf(inp_id, '%d', 1);
gf_ind=fscanf(inp_id, '%d', 1);
dep_schrange=fscanf(inp_id, '%d', 1);
dep_intv=fscanf(inp_id, '%f', 1);
%f search distance is -dep_schrange*dep_intv:dep_schrange*dep_intv
floese(inp_id);
disp(['NR=' num2str(NR)]);
tic;
%% load SAC file
lowc = 0.1; highc=2.0;
sratio=1;
xcsh = 5; % this is tolerance = x*0.05=shift allowed while correlating in sec
[b,a]=butter(2, [lowc highc]/(1/0.05/2)); %define lowpass filter
load(['/scratch/busfarha/scedc/14408052/14408052'_eqhdr.mat']);
load(['/scratch/busfarha/scedc/14408052/14408052' '_rcvsigN.mat']);
rcvsigN=rcvsig;
load(['/scratch/busfarha/scedc/14408052/14408052' '_rcvsigE.mat']);
rcvsigE=rcvsig;
load(['/scratch/busfarha/scedc/14408052/14408052' '_rcvsigZ.mat']);
rcvsigZ=rcvsig;
clear rcvsig
```

66
staids = [4 5]; % change stations if needed

for staind = 1:size(staids,2)
    # code...
end

%% calculate the source time function in freq. domain

Q=TL/2;
PIL=pi*2/XL;
AIP1=i*PIL;
DT=TL/ntime;
DFREQ=1/TL;
NFREQ=ntime/2;
AW=-pi/Q;
FREQ=0;
source = zeros(1,NFREQ);
    for FI=1:NFREQ
    RW=pi*2*FREQ;
    OMEGA=RW + i*AW;

% source is the Fourier transform of a smooth ramp function of
% rise time equal to tsource:
% tstart=T0-tsource*2.;
    tstart=T0;
    c1=exp(OMEGA*pi*tsource/4.);
    source(FI)=-i *pi*tsource/2./(c 1-1/c1) *exp(i*OMEGA*tstart)*slip;

% IF what is desired is ground velocity instead of ground displacement,
% then add :
    source(FI)=source(FI)*i*OMEGA;
    FREQ=FREQ+DFREQ;
end
source = (source)';

%% load in obs and Green's funs

green11 = zeros(NFREQ*3, (dep_schrange*2+1)*NR);
green12 = zeros(NFREQ*3, (dep_schrange*2+1)*NR);
green13 = zeros(NFREQ*3, (dep_schrange*2+1)*NR);
green22 = zeros(NFREQ*3, (dep_schrange*2+1)*NR);
green23 = zeros(NFREQ*3, (dep_schrange*2+1)*NR);
green33 = zeros(NFREQ*3, (dep_schrange*2+1)*NR);
green11_1 = zeros(NFREQ, (dep_schrange*2+1)*NR);
green11_2 = zeros(NFREQ, (dep_schrange*2+1)*NR);
green11_3 = zeros(NFREQ, (dep_schrange*2+1)*NR);
green12_1 = zeros(NFREQ, (dep_schrange*2+1)*NR);
green12_2 = zeros(NFREQ, (dep_schrange*2+1)*NR);
green12_3 = zeros(NFREQ, (dep_schrange*2+1)*NR);
green13_1 = zeros(NFREQ, (dep_schrange*2+1)*NR);
green13_2 = zeros(NFREQ, (dep_schrange*2+1)*NR);
green13_3 = zeros(NFREQ, (dep_schrange*2+1)*NR);
green22_1 = zeros(NFREQ, (dep_schrange*2+1)*NR);
green22_2 = zeros(NFREQ, (dep_schrange*2+1)*NR);
green22_3 = zeros(NFREQ, (dep_schrange*2+1)*NR);
green23_1 = zeros(NFREQ, (dep_schrange*2+1)*NR);
green23_2 = zeros(NFREQ, (dep_schrange*2+1)*NR);
green23_3 = zeros(NFREQ, (dep_schrange*2+1)*NR);
green33_1 = zeros(NFREQ, (dep_schrange*2+1)*NR);
green33_2 = zeros(NFREQ, (dep_schrange*2+1)*NR);
green33_3 = zeros(NFREQ, (dep_schrange*2+1)*NR);
for inr=1:NR
    for idep_sch = -dep_schrange:dep_schrange
        g_ind = idep_sch+(dep_schrange*2+1)*(inr-1) + dep_schrange+1;

        green11_tmp = load(['./GREENFCN/green_fun11_' num2str(idep_sch) '_' num2str(inr)]);
green12_tmp = load(['./GREENFCN/green_fun12_' num2str(idep_sch) '_' num2str(inr)]);
green13_tmp = load(['./GREENFCN/green_fun13_' num2str(idep_sch) '_' num2str(inr)]);
green22_tmp = load(['./GREENFCN/green_fun22_' num2str(idep_sch) '_' num2str(inr)]);
green23_tmp = load(['./GREENFCN/green_fun23_' num2str(idep_sch) '_' num2str(inr)]);
green33_tmp = load(['./GREENFCN/green_fun33_' num2str(idep_sch) '_' num2str(inr)]);

        green11(:,g_ind) = green11_tmp(:,1) + i*green11_tmp(:,2);
green12(:,g_ind) = green12_tmp(:,1) + i*green12_tmp(:,2);
green13(:,g_ind) = green13_tmp(:,1) + i*green13_tmp(:,2);
green22(:,g_ind) = green22_tmp(:,1) + i*green22_tmp(:,2);
green23(:,g_ind) = green23_tmp(:,1) + i*green23_tmp(:,2);
green33(:,g_ind) = green33_tmp(:,1) + i*green33_tmp(:,2);

        green11_1(:,g_ind) = green11(1:3:end, g_ind);
green11_2(:,g_ind) = green11(2:3:end, g_ind);
green11_3(:,g_ind) = green11(3:3:end, g_ind);
green12_1(:,g_ind) = green12(1:3:end, g_ind);
green12_2(:,g_ind) = green12(2:3:end, g_ind);
green12_3(:,g_ind) = green12(3:3:end, g_ind);
green13_1(:,g_ind) = green13(1:3:end, g_ind);
green13_2(:,g_ind) = green13(2:3:end, g_ind);
green13_3(:,g_ind) = green13(3:3:end, g_ind);
green22_1(:,g_ind) = green22(1:3:end, g_ind);
green22_2(:,g_ind) = green22(2:3:end, g_ind);
green22_3(:,g_ind) = green22(3:3:end, g_ind);
green23_1(:,g_ind) = green23(1:3:end, g_ind);
green23_2(:,g_ind) = green23(2:3:end, g_ind);
green23_3(:,g_ind) = green23(3:3:end, g_ind);
green33_1(:,g_ind) = green33(1:3:end, g_ind);
green33_2(:,g_ind) = green33(2:3:end, g_ind);
green33_3(:,g_ind) = green33(3:3:end, g_ind);
end
end

%% do the calculation
[c, d] = butter(2, [lowc highc]/(256/40/2)); %define lowpass filter
cnter = cos(2*pi/(256*4)*[0:256-1]);
cnt = 0;
pi180 = pi/180;
n3 = ntime + ntime + 3;
tex1 = -AW*DT;
tex1 = exp(tex1);
yyy = zeros(ntime, 1);
ex7 = 1;
for i = 1:ntime
    yyy(i) = ex7;
    ex7 = ex7 * tex1;
end
sy = zeros(ntime, 3);
strarr = 0:10:350;  \% define search spaces
diparr = 10:10:90;
rakearr = -90:10:90;
deparr = -dep_schrange:dep_schrange;
t_len = length(strarr) * length(diparr) * length(rakearr) * length(deparr);
rm5 = zeros(t_len, 3 + NR * 3 + 1 + 1);  \%[str dip rake xcorrs sum depdepth]

for idep_sch = deparr
    fprintf('idep_sch=%d\n', idep_sch);
    for str = strarr
        fprintf('strike=%3d', str);
        for dip = diparr
            for rake = rakearr
                cnt = cnt + 1;
                CS = cos(str*pi180);  \% calculate each moment component
                SS = sin(str*pi180);
                CDI = cos(dip*pi180);
                SDI = sin(dip*pi180);
                CR = cos(rake*pi180);
                SR = sin(rake*pi180);
                AS1 = CR*CS+SR*CDI*SS;
                AS2 = CR*SS-SR*CDI*CS;
                AS3 = -SR*SDI;
                AN1 = -SDI*SS;
                AN2 = SDI*CS;
                AN3 = -CDI;
                CM11 = -2.*AS1*AN1;
                CM22 = -2.*AS2*AN2;
                CM33 = -2.*AS3*AN3;
                CM12 = -(AS1*AN2+AS2*AN1);
                CM13 = -(AS1*AN3+AS3*AN1);
CM23 = -(AS2*AN3 + AS3*AN2);

for inr = 1:NR
    cutlen = cut_table_t*256/40;  % defines cutting of synthetic data
    g_ind = idep_sch+length(deparr)*(inr-1) + dep_schrang+1;
    green_total(:,1) = CM11*green11_1(:,g_ind) + CM22*green22_1(:,g_ind) +
        CM33*green33_1(:,g_ind) + CM12*green12_1(:,g_ind) +
        CM13*green13_1(:,g_ind) + CM23*green23_1(:,g_ind);
    green_total(:,2) = CM11*green11_2(:,g_ind) + CM22*green22_2(:,g_ind) +
        CM33*green33_2(:,g_ind) + CM12*green12_2(:,g_ind) +
        CM13*green13_2(:,g_ind) + CM23*green23_2(:,g_ind);
    green_total(:,3) = CM11*green11_3(:,g_ind) + CM22*green22_3(:,g_ind) +
        CM33*green33_3(:,g_ind) + CM12*green12_3(:,g_ind) +
        CM13*green13_3(:,g_ind) + CM23*green23_3(:,g_ind);
    if inr == 1
        syn_cut = 102;
    end
    if inr == 2
        syn_cut = 140;
    end
    if inr == 3
        syn_cut = 108;
    end
    if inr == 4
        syn_cut = 121;
    end
    if inr == 5
        syn_cut = 140;
    end

    for j = 1:3
        c0 = green_total(:,j).*source;
        c1 = [c0; 0; conj(c0(end:-1:2))];
        y = fft(c1);
        sy(:,j) = real(y(end:-1:1).*yyy-y(1));
        sy(syn_cut:end) = 1e7*eps;  % change this for syn seismogram (x/256*40 = length of syn
        sy = syfilt(c, d, sy(:,j));
        sy(:,j) = sy(:,j)/max(abs(sy(:,j)));
        sysq(:,j) = (abs(sy(:,j))).^sqratio.*(sy(:,j)./abs(sy(:,j)));
    end
    sysq(:,3) = -sysq(:,3);
    sy = sysq;
    [maxn inf] = max(xcorr(sac(inr).sqdatan, sysq(:,1), xch));
    [maxe ine] = max(xcorr(sac(inr).sqdatae, sysq(:,2), xch));
    [maxz iz] = max(xcorr(sac(inr).sqdataz, sysq(:,3), xch));
    rms(cnt, 3+(inr-1)*3+1) = maxn;  % this is the xcorrelation part
rms(cnt, 3+(inr-1)*3+2) = maxe; %this is the xcorrelation part
rms(cnt, 3+(inr-1)*3+3) = maxz; %this is the xcorrelation part
if in>xcsh
    syn = [zeros(in-xcsh-1, 1);sy(:,1)];
else
    syn = sy(xcsh+2-in:end, 1);
end
if ie>xcsh
    sye = [zeros(ie-xcsh-1, 1);sy(:,2)];
else
    sye = sy(xcsh+2-ie:end, 2);
end
if iz>xcsh
    syz = [zeros(iz-xcsh-1, 1);sy(:,3)];
else
    syz = sy(xcsh+2-iz:end, 3);
end
rms(cnt, 3+(inr-1)*3+1) = rms(cnt, 3+(inr-1)*3+1) - 1*norm(syn(1:floor(cutlen(inr))))-sac(inr).sqdatan(1:floor(cutlen(inr))),2);
rms(cnt, 3+(inr-1)*3+2) = rms(cnt, 3+(inr-1)*3+2) - 1*norm(sye(1:floor(cutlen(inr))))-sac(inr).sqdatae(1:floor(cutlen(inr))),2);
rms(cnt, 3+(inr-1)*3+3) = rms(cnt, 3+(inr-1)*3+3) - 1*norm(syz(1:floor(cutlen(inr))))-sac(inr).sqdataz(1:floor(cutlen(inr))),2);
end
rms(cnt, 1) = str;
rms(cnt, 2) = dip;
rms(cnt, 3) = rake;
rms(cnt, end) = idep_sch;
end
end
fprintf('');
end
end

rms(:, 3+NR*3+1) = sum(rms(:,4:NR*3+3), 2);

[a ind]=max(rms(:,3+NR*3+1));
rms(ind,1:3)
[aa ind2]=sort(rms(:,3+NR*3+1));
bestsol = rms(ind2(end-10:end,end),1:3)
bestdep = rms(ind2(end-10:end),end)
bestxcr = rms(ind2(end-10:end),3+NR*3+1)
% plot the best solution
clear synew
str = bestsol(end, 1);
dip = bestsol(end, 2);
rake = bestsol(end, 3);
idep_sch = bestdep(end);

%%
CS=cos(str*pi180); %calculate each moment component
SS=sin(str*pi180);
CDI=cos(dip*pi180);
SDI=sin(dip*pi180);
CR=cos(rake*pi180);
SR=sin(rake*pi180);
AS1=CR*CS+SR*CDI*SS;
AS2=CR*SS-SR*CDI*CS;
AS3=-SR*SDI;
AN1=-SDI*SS;
AN2=SDI*CS;
AN3=-CDI;
CM11=-2.*AS1*AN1;
CM22=-2.*AS2*AN2;
CM33=-2.*AS3*AN3;
CM12=-(AS1*AN2+AS2*AN1);
CM13=-(AS1*AN3+AS3*AN1);
CM23=-(AS2*AN3+AS3*AN2);

for inr=1:NR
    if inr==1
        syn_cut=102;
    end
    if inr==2
        syn_cut=140;
    end
    if inr==3
        syn_cut=108;
    end
    if inr==4
        syn_cut=121;
    end
    if inr==5
        syn_cut=140;
    end

    g_ind = idep_sch+(dep_schrange*2+1)*(inr-1)+dep_schrange+1;

    green_total(:,1) = CM11*green11_1(:,g_ind) + CM22*green22_1(:,g_ind) +
    CM33*green33_1(:,g_ind)+ CM12*green12_1(:,g_ind)+
    CM13*green13_1(:,g_ind)+ CM23*green23_1(:,g_ind);
green_total(:,2) = CM11*green11_2(:,g_ind) + CM22*green22_2(:,g_ind) +...
CM33*green33_2(:,g_ind)+ CM12*green12_2(:,g_ind)+...
CM13*green13_2(:,g_ind)+ CM23*green23_2(:,g_ind);

green_total(:,3) = CM11*green11_3(:,g_ind) + CM22*green22_3(:,g_ind) +...
CM33*green33_3(:,g_ind)+ CM12*green12_3(:,g_ind)+...
CM13*green13_3(:,g_ind)+ CM23*green23_3(:,g_ind);

for j=1:3
  c0 = green_total(:,j).*source;
  c1 = [c0; 0; conj(c0(end:-1:2))];
  y = fft(c1);
  sy(:,j) = real(y(end:-1:1).*yyy-y(1));

  sy(syn_cut:end,j) = 1e7*eps;
  sy(:,j) = filtfilt(c, d, sy(:,j));
  sy(:,j) = sy(:,j).*ctaper;
  sy(:,j) = sy(:,j)/max(abs(sy(:,j)));
  sysq(:,j) = (abs(sy(:,j))).^(sqratio).*(sy(:,j)/abs(sy(:,j)));
end
sysq(:,3) = -sysq(:,3);
sy = sysq;

for in
  [an in] = max(xcorr(sac(inr).sqdatan, sysq(:,1), xcsh));
  [ae ie] = max(xcorr(sac(inr).sqdatae, sysq(:,2), xcsh));
  [az iz] = max(xcorr(sac(inr).sqdataz, sysq(:,3), xcsh));
if in>xcsh
  syn = [zeros(in-xcsh-1, 1);sy(:,1)];
else
  syn = sy(xcsh+2-in:end, 1);
end
if ie>xcsh
  sye = [zeros(ie-xcsh-1, 1);sy(:,2)];
else
  sye = sy(xcsh+2-ie:end, 2);
end
if iz>xcsh
  syz = [zeros(iz-xcsh-1, 1);sy(:,3)];
else
  syz = sy(xcsh+2-iz:end, 3);
end
ddt =0.1562;
figure
subplot(3,1,1,'FontSize',15)
plot(linspace(0,length(sac(inr).redatan)*ddt, length(sac(inr).redatan)), sac(inr).redatan); hold
on; plot(linspace(0,length(syn)*ddt,length(syn)), syn,'r');

if inr==1;
title(' Station: CI.DSC ; Filter [0.1-2.0] ; Mojave velocity model','fontsize',20)
end

if inr==2;
title(' Station: CI.GMR ; Filter [0.1-2.0] ; Mojave velocity model','fontsize',20)
end

if inr==3;
title(' Station: CI.JVA ; Filter [0.1-2.0] ; Mojave velocity model','fontsize',20)
end

if inr==4;
title(' Station: CI.MCT ; Filter [0.1-2.0] ; Mojave velocity model','fontsize',20)
end

if inr==5;
title(' Station: CI.GSC; Filter [0.1-2.0] ; Mojave velocity model','fontsize',20)
end

xlabel('N Time(sec)','FontSize',15)
ylabel('Amp','FontSize',15)
legend('Observed','Synthetic')
axis([0 22 ylim])

subplot(3,1,2,'FontSize',15)
plot(linspace(0,length(sac(inr).red ata)*ddt,length(sac(inr).redatae)), sac(inr).redatae); hold on; plot(linspace(0,length(sye)*ddt,length(sye)),sy e,'r');

xlabel('E Time(sec)','FontSize',15)
ylabel('Amp','FontSize',15)
axis([0 22 ylim])

subplot(3,1,3,'FontSize',15)
plot(linspace(0,length(sac(inr).redataz)*ddt,length(sac(inr).redataz)), sac(inr).redataz); hold on; plot(linspace(0,length(syz)*ddt,length(syz)),syz,'r');

xlabel('Z Time(sec)','FontSize',15)
ylabel('Amp','FontSize',15)
axis([0 22 ylim])
end

toc;
References


Pujol, Jose (2003), Elastic Wave Propagation and Generation in Seismology, United Kingdom: Cambridge University Press.


Southern California Seismic Network, *Fig 4.b - Velocity Response vs. Frequency for STS-1/2, CMG3ESP, and CMG40T Instruments* [online], available: http://www.scsn.org/stationequip.html [accessed 7 April 2009].

