CAPABILITIES AND LIMITATIONS OF PHASE CONTRAST IMAGING
TECHNIQUES WITH X-RAYS AND NEUTRONS

By

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ABSTRACT

Phase Contrast Imaging (PCI) was studied with the goal of understanding its relevance and its requirements. Current literature does not provide insight on the effect of a relaxation in coherence requirements on the PCI capabilities of an imaging system. This problem is all the more important since coherent X-ray and Neutron sources are mostly unavailable.

We develop a model for PCI contribution to imaging for partially incoherent systems, and develop a methodology to identify a minimum and an optimum coherence length $\xi_{\text{min}}$ and $\xi_{\text{opt}}$. We propose a figure-of-merit $K_{PCI}$ that quantifies the PCI capabilities of an imaging system. Our calculations show that X-ray PCI systems based on free space propagation using microfocus X-ray tubes have little PCI capabilities. We develop a model to explain the edge enhancement observed with those systems; our results suggest that scatter reduction is the process responsible for the observed edge enhancement. We performed experiments that show good agreement with the model.

Coded Source Imaging (CSI) is proposed as a tool to produce highly coherent sources. The general theory of CSI is developed. We propose two possible systems: Fluorescent Coded Sources (FCS) and the AEB Encoded X-ray tube.

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1. Introduction

Non-destructive imaging of a sample is a common analysis technique used for applications as varied as radiography of a broken bone, routine mammography, a CT scan or a PET exam for tumor diagnosis, morphological ultrasound of a fetus, and security scanning of luggage at airports. The use of these imaging techniques is even more widespread than the immediately apparent applications; the same technology is used for scanning of containers at ports for homeland security reasons, neutron and X-ray imaging of the microstructure of mechanical parts such as turbines, and investigation of defects in industrially manufactured products. The list of applications for non-destructive testing is long.

![Figure 1: Progress in X-ray imaging. (a) Rontgen’s wife hand (1896); (b) Baggage scanner CT (2006); (c) Mammogram (2006).](image)

Of these techniques, the one with which the general public is probably most familiar is X-ray radiography. This technique is based on the ability of X-rays to be only partially absorbed as they travel through materials. Differences in absorption between various parts of a target object allow the imaging and identification of internal structural details of the object. Used for a variety of applications, X-ray radiography has a history of success. X-rays were discovered in 1896 by Röntgen, who famously took a picture of his wife’s hand to demonstrate an early application of X-ray radiography. Subsequent technical advancements enabled X-ray radiography to be used for high quality three
dimensional imaging, for instance for medical applications and for security screening at airports (see Figure 1).

Not surprisingly, X-ray imaging is generally used to image heavy, dense materials, which are opaque to X-rays, surrounded by soft, less dense materials, which are more transparent. A classical example is the imaging of bones in tissue: the radiation passes through tissue but is absorbed by bones, resulting in an image of the “shadow” of a bone on the X-ray film. The success of X-ray imaging has made it possible to extend the applicability of this technique beyond the relatively straightforward requirement to distinguish soft tissue from bone. For instance, most mammography examinations utilize X-rays; in this case, the goal is to identify cancerous soft tissue surrounded by healthy soft tissue. Clearly, applications such as mammography stretch the capabilities of X-ray imaging. In order to increase the quality of the image and to better distinguish various types of soft tissue, low-energy X-rays are used and the breast is squeezed. These procedures increase the dose to the patient and create discomfort. Nevertheless, X-ray based mammography continues to enjoy widespread use because, since mammography was introduced, early detection of breast tumors has dramatically reduced breast cancer mortality for women.

Recently, interest has grown in a family of imaging techniques that image the phase shift induced in X-rays passing through a sample rather than the absorption of the X-rays. These techniques, known as Phase Contrast Imaging (PCI), have been successfully demonstrated using synchrotron radiation. PCI techniques can provide increased contrast at edges between different materials in a sample, thus making them attractive for applications in which more contrast is needed than that which can be provided by traditional X-ray radiography. Many biological samples are “phase objects,” or objects that minimally absorb X-rays but are capable of inducing strong shifts in the phase of the incident radiation. For instance, it is generally believed that PCI of tissue can deliver better resolution for lower dose, making it potentially highly relevant in mammography applications. The price for this improvement in contrast is the need for coherent radiation, which imposes stringent requirements on the design of an imaging system.
The literature on PCI is vast and current research efforts continue to add to the body of literature. A variety of techniques have been proposed to perform PCI in a cost-effective manner so that its advantages can be exploited in common applications such as mammography. Early efforts to perform PCI focused on monochromatic, highly coherent radiation, which is expensive and difficult to achieve. More recent work has pointed to the successful use of polychromatic, low-coherence (or altogether incoherent) radiation, which could potentially make PCI a much more accessible technique. Edge enhancement has been observed in all techniques reported in the PCI literature.

The diversity of reported PCI techniques makes it difficult to directly compare the various systems and results. Considering the trend in recent literature toward more incoherent systems, the question of how these systems are able to deliver PCI arises. Coherence requirements are generally approached with a pragmatic view: if the system provides edge enhancement, it must be sufficiently coherent to perform PCI. The purpose of this thesis is to challenge this pragmatic approach and to more rigorously analyze the requirements for PCI. We believe that conclusions drawn in literature based on the pragmatic view of PCI coherence limitations are dubious, since the observed edge enhancement effects may arise as an unintended consequence related to the set-up of the PCI system instead of from phase information.

We are then set to explore the limitations of PCI and, if needed, to explain the edge enhancement effects seen in literature in systems that we believe are not capable of achieving PCI. We begin by discussing diffraction theory in Chapter 2. This discussion is based on treatments from various textbooks and literature sources, and provides the fundamental bedrock for understanding PCI effects. In Chapter 2, extreme care is taken to dispel any ambiguity regarding the definition of coherence. We contribute to the theoretical analysis of diffraction theory by discussing the physical interactions of photons with matter and their impact on the description of the index of refraction of the material, bringing together information that is not usually presented harmoniously. We also discuss the relevance of coherent and incoherent scattering in imaging.

Chapter 3 is concerned more specifically with PCI. We present a founding equation for PCI and use this equation to define PCI unambiguously. A thorough discussion, based on the fundamentals of diffraction theory presented in Chapter 2, of the relevance of the
physical interactions between radiation and matter in PCI will be presented. Based on this
discussion, we propose a quantitative methodology to analyze the PCI capabilities of an
imaging system. We discover that very high coherence is indeed required to perform PCI,
which implies that the low-coherence systems presented in literature must obtain their
observed edge enhancements through other phenomena. We propose a phenomenon that
we call Compton filtering as a possible reason behind these observed enhancement
effects. We propose a quantitative modeling of this effect and compare it to PCI. Chapter
4 provides an experimental validation of the theory developed in Chapter 3.

In Chapters 3 and 4, we demonstrated that high-coherence radiation is needed to
perform PCI; however, coherent sources are expensive and are limited by low flux. In
Chapter 5, we propose a new method, Coded Source Imaging (CSI), to obtain a coherent
source with an acceptable brightness. We first discuss the method in general and obtain
both the founding equation of CSI and the condition to apply CSI to PCI. We then
present four possible embodiments of PCI through CSI systems.

In Chapter 6 we present a critical review of PCI literature in view of the discussion
regarding coherence requirements presented in Chapter 3. Finally, we present the
conclusions of this work in Chapter 7.
2. Theoretical Background

The image formation process

Since this thesis is concerned with imaging, it is certainly appropriate to spend some
time describing the process by which radiation emitted by a source can cast an image of
an object on a detector. A simple way of approaching this problem is to assume the
radiation is composed of rays, and to use simple ray geometry to describe the image that
the radiation will cast on the detector. Under this framework, a ray may be obstructed by
parts of the object (e.g., opaque parts) but not by others (e.g., transparent parts), and the
resulting image on the detector is a shadow of the opaque parts of the object. The image
magnification is deduced by ray geometry. If we can assume that the object can assume
partial opacity, the prediction of this simplified theory turns out to be in striking
agreement with most radiographic experiments.

A ray geometry description of the image formation process is an intuitive way to
approach the problem. Its underlying assumption is that radiation can be represented by
particles moving in a straight line\(^1\) from the source to the detector. In general, X-ray and
visible light sources are two manifestations of the same kind of radiation; the ray
geometry approach assumes in both cases that photons emitted at the source travel
through the sample, and are then detected at the observation plane. This line of reasoning,
although very successful in some contexts, fails to explain results from experiments that
can be performed with both X-rays and visible light.

The most famous of these experiments for visible light is the Young double slits
gameometry. Ray geometry predicts in this case that the slits will cast their image on the
detector, where the apparent distance between the slits and the apparent size of the slits
will be magnified by the appropriate magnifying factor. Experiments show that a fringe
pattern is instead observed in the geometrical shadow of the slits. This result cannot be
explained without taking into account the wave nature of the radiation.

\(^1\) A simple and common extension of ray geometry takes into account particles moving in straight lines
between scattering events. The discussion in this section can be applied to this extended model as well.
A satisfactory representation of the image formation process needs to be rooted in the description of the evolution of the radiation wavefront, from the moment it is emitted at the source, to its propagation to and through the object, and finally to its propagation from the object to the observation plane. Such a description is usually referred to as diffraction theory.

We introduce here a standard representation of diffraction theory. The following derivation is usually applied to visible light impinging on apertures of various shapes, but can easily be applied to X-ray imaging. Figure 1 exemplifies the relevant geometry. The goal of the derivation presented below is to obtain an equation that describes how the perturbation introduced by an object O to the electromagnetic field (be it visible light or X-ray radiation) emitted by a source Q is propagated and recorded on a detector lying in the detector plane P. Such an equation will give us insight into the parameters that affect image quality in an imaging system.

Figure 1: Geometry of diffraction of radiation emitted by a source Q when passing through an object O, observed at the plane P. [5]

For ease of derivation, we will consider only the two extreme cases of perfect coherence and perfect incoherence of the source. A more thorough approach is to consider the theory of partial coherence in our derivation. The interested reader can study
the subject further [1]. The derivation presented in this chapter is a generalization of the presentation of diffraction theory found in textbooks [2,3,4].

**Coherent irradiation**

Let us consider a source $Q$ emitting a coherent radiation field $U_Q(x,y)$. The radiation impinges on an object $O$. Following the Huygens-Fresnel principle, we can write the wavefront after the object as if each point $O = (X,Y,0)$ of the object was an emitter of a new wavefront, where the new wavefront is proportional to (i) the radiation impinging on $O$, and (ii) a transmission function $q(X,Y)$ dependent on the specific interactions of the radiation with the object at each point $O$. The wavefront departing from each point $O$ is assumed to be spherical. The field of the radiation at a generic point $P = (x, y, R)$ at the observation plane is simply written, referring to Figure 1, as:

$$
\psi(x, y) = \frac{1}{i\lambda} \iint U_Q(X,Y)q(X,Y)e^{ikr}dXdY \left(\cos \hat{Z}r + \sin \hat{Z}r q \right).
$$

To simplify this equation, let’s assume that the source and the detector are both far enough from the object plane so that the obliquity factor $(\cos \hat{Z}r + \sin \hat{Z}r q)$ is unity. To further simplify the equation, we can focus on reasonable expansion of the term $e^{ikr}$, the spherical wave term. The distance $r$ between $O$ and $P$ is given exactly by the equation

$$
r = \sqrt{R^2 + (x-X)^2 + (y-Y)^2} = R \sqrt{1 + \left(\frac{x-X}{R}\right)^2 + \left(\frac{y-Y}{R}\right)^2}.
$$

Expanding this equation to the second order we find:

$$
r \approx R \left[1 + \frac{1}{2} \left(\frac{x-X}{R}\right)^2 + \frac{1}{2} \left(\frac{y-Y}{R}\right)^2\right].
$$

To further simplify (1), either the Fresnel approximation or the Fraunhofer approximation may be used.
a) Fresnel approximation

In evaluating the term \(e^{ikr}/r\), we must consider how many terms to use in the expansion of \(r\). For the denominator, it is safe to use only the first term under the condition that the observation plane is far away from the object. Such an assumption cannot in general be made for the exponential term, which varies rapidly with \(r\). For that term, a reasonable approximation, called the Fresnel approximation, takes into account the first two terms of the Taylor expansion. Under these assumptions, equation (1) will take the form of:

\[
\psi_R(x, y) = \frac{e^{ikr}}{i\lambda R} \iint U_Q(X, Y)q(X, Y)e^{i\frac{k}{2R}(x-X)^2+(y-Y)^2} q(X, Y) e^{i\frac{k}{2R}(xX+yY)} dXdY. \tag{3}
\]

Equation (3) gives the value of the radiation field at a distance \(R\) from the object plane under the assumption that the Fresnel approximation is valid. For the moment, it is worth trying to obtain a simplified form of equation (3) to gain some insight into the image formation process. Under this approximation, we can rewrite the exponential term in the integral by expanding the squared terms and regrouping:

\[
\psi_R(x, y) = \frac{e^{ikr}}{i\lambda R} \iint \left\{U_Q(X, Y)q(X, Y)e^{i\frac{k}{2R}(x^2+y^2)} + \frac{ik}{2R}(xX+yY)\right\} q(X, Y) e^{i\frac{k}{2R}(xX+yY)} dXdY \tag{4}
\]

Regrouping all the constants together and defining a new transmission function \(q'(X,Y) = q(X, Y)U_Q(X, Y)\) we then obtain the simplified equation for the field:

\[
\psi_R(x, y) = \frac{e^{i\frac{k}{2R}(x^2+y^2)}}{i\lambda R} e^{i\frac{k}{2R}(x^2+y^2)} \iint q'(X, Y)e^{i\frac{kr}{2R}(x^2+y^2)} e^{-i\frac{kr}{2R}(xX+yY)} dXdY. \tag{5}
\]
Equation (5) is readily interpreted as the Fourier transform of \( q'(X,Y) \frac{in(\lambda R)^2}{(X^2+Y^2)} \), aside from multiplicative amplitude and phase factors, both independent of \((X,Y)\). Introducing the frequency coordinates \( v_x = \frac{X}{\lambda R} \) and \( v_y = \frac{Y}{\lambda R} \), we obtain the equation:

\[
\psi_R(x,y) = A(R\nu_x, R\nu_y, R) F(q'(X,Y)e^{\frac{in(\lambda R)^2}{(X^2+Y^2)}})(\nu_x, \nu_y).
\] (6)

Equation (6) is the most compact representation of the Fresnel approximation of the diffraction of coherent radiation by a sample. In the derivation of the diffraction integral proceeding from equation (1) to equation (6), some simplifications have been made. The most important one is to assume that the object-to-detector distance satisfies the Fresnel approximation, that is, that we can expand the spherical wavefronts emitted by each point in the object in a series of polynomials up to the second power. This condition requires that the contribution of the higher order expansion terms will be less than one radian, or

\[
R^3 >> \frac{\pi}{4\lambda} [(X-x)^2 + (Y-y)^2]_{\text{max}}^2.
\]

Another more useful way to write the condition for the Fresnel approximation is by considering that the quantity on the right side of this inequality can be safely replaced by the square of the maximum transverse size of the object. Considering that the size of the sample is \( a \), such condition will read:

\[
R^3 >> \frac{a^4}{\lambda}.
\]

This is a sufficient but not a necessary requirement for the Fresnel approximation to be valid. The Fresnel approximation still holds whenever the higher order terms of the expansion do not change appreciably the value of the integral in equation (1).

Notice also how, while the Fresnel approximation may not hold for patterns that extend over the entire sample length \( a \), it may still be valid for smaller patterns \( \xi \). This is especially meaningful when the radiation source is not fully coherent, as in the majority
of cases, and the coherence length $\xi$ is of the order of $10^{-6}$ m, or much smaller than the sample size $a$.

The Fresnel approximation is in general considered valid, at least over limited features of the sample, whenever the object-to-detector distance exceeds a very small value – whenever we can assume that the object is not in contact with the detector. For the contact regime, no phase effect will be observable, since the spherical waves emitted by the object will not propagate and will not have a chance to interfere.

b) Fraunhofer approximation

The Fraunhofer approximation is a simpler approximation than the Fresnel one, where the expansion of $r$ can be arrested to the first order also in the evaluation of the exponential in equation (1). In this case, equation (6) simplifies so that $e^{\frac{i\pi}{2}(x^2+y^2)} \approx 1$, and we obtain:

$$\psi_R(x, y) = A(\lambda R v_x, \lambda R v_y, R) F(q(X, Y))(v_x, v_y). \quad (7)$$

Aside from a multiplicative factor, in the Fraunhofer approximation the radiation field is simply the Fourier transform of the transmission function $q'$. The condition for the Fraunhofer approximation to be valid is, similarly to the Fresnel approximation, that $R >> \frac{a^2}{\lambda}$. In practical terms, if the object size is of the order of 0.10 m, and the wavelength is $0.02*10^{-9}$ m, the condition for the Fraunhofer approximation is satisfied for $R$ greater than 5000 km. The Fraunhofer approximation is then rarely applicable to the entire extent of the object.

c) Contact, Fresnel and Fraunhofer regimes.

When evaluating the evolution of the radiation field in the region of space after the interaction with a sample, we can then speak of three different regions, or regimes, summarized in Table 1. At contact with the sample, the radiation has not propagated and
no interference effects have occurred. The contact regime is the classical radiographic regime – absorption of radiation in the sample casts an image on the detector.

Moving farther from the sample, phase effects contribute to the field distribution. Despite the fact that the intensity distribution does not directly carry any phase information, interference effects occurring in this region have a visible impact on the imaging of edges and other sharply changing details. The phase shift that the material imparts to the incoming radiation will be visible due to the interference effects that will occur in the wavefronts in the region ahead of the sample. This is the Fresnel regime, where the rapidly varying term $e^{i2R(X^2+Y^2)}$ in equation (5) enhances small features of $q'(X,Y)$, while slowly varying features across the entire sample will not be so enhanced. In the Fresnel regime it is then possible to select a distance $R$ small enough so that only edges and very fine details contribute to the interference, thus selecting which features to underline through edge enhancement. This property of the Fresnel regime is useful in the design of an imaging system that presents edge enhancement without corrupting the overall readability of the image.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Condition</th>
<th>Property</th>
<th>Imaging technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contact</td>
<td>$R \cong 0$</td>
<td>No interference effects; only absorption is visible</td>
<td>Standard Radiography</td>
</tr>
<tr>
<td>Fresnel</td>
<td>$\left(\frac{a^4}{\lambda}\right)^{\frac{1}{3}} \ll R &lt; \frac{a^2}{\lambda}$</td>
<td>Interference effects over small features</td>
<td>Phase Contrast Imaging</td>
</tr>
<tr>
<td>Fraunhofer</td>
<td>$R &gt;&gt; \frac{a^2}{\lambda}$</td>
<td>Interference effects over the entire sample, or large portions of it</td>
<td>Coherent Diffractive Imaging</td>
</tr>
</tbody>
</table>

Table 1: Summary of the three imaging regime encountered in diffraction theory.
Farther away from the sample, in the Fraunhofer regime, interference effects over a
large portion of the sample will in general be visible, and the contribution of the phase
effects to the diffraction integral will not be fast-varying. In this region, if the radiation
source is purely coherent, the radiation field will take the shape of the Fourier transform
of the transmission function of the sample. Reconstruction is then needed to obtain an
image of the object. Since the field is not an observable, and can be sampled only through
its intensity, in general it is not possible to obtain a perfect reconstruction of the object.

The different diffraction regimes have a relevance that depends on the imaging
methodology one chooses to adopt. We will discuss later the possible applications of the
three regimes, although a summary can be found in Table 1.

d) Observables

Equation (6) provides a tool to calculate the field generated by an object irradiated
with coherent radiation. Conversely, from the field distribution it is theoretically possible
to extract from (6) information about \( q' \), that is, information about the object.

To exemplify this point, we can simplify equation (5), in the Fraunhofer regime, to
\[
\psi(x) = \int q(X) e^{-iRX} dX,
\]
where we are considering a one-dimensional case, an object
illuminated by a plane wave (making \( q' = q \)), and we have set all the constants to unity.
Under these conditions, it is theoretically possible to extract information about \( q \) by the
inverse Fourier transformation
\[
q(X) = \int \psi(x) e^{iRX} dx,
\]
aside from some constants. It seems then possible to reconstruct completely \( q(X) \) from the knowledge of the field. Such
reconstruction would provide useful information about the structure of the object, which
as we shall see is accurately described by its transmission function \( q(X) \).

The problem is that the field \( \psi(x) \), which is the relevant parameter, is not an
observable. What a detector can record is the intensity of the field \( |\psi(x)|^2 \). Notably, the
intensity is real, while the field is in general a complex quantity. This means that an
intensity sampling provides only half of the information that would be contained in a
field sampling. For these reasons, a perfect reconstruction of \( q(X) \) is not in general possible, although we will present techniques to allow reconstruction in some cases.

It is now useful to write the equation giving the intensity of a diffracted coherent field. To do so, we restate the diffraction problem in time-dependent coordinates. Starting from equation (4), we define the "impulse response of free space" \( h(x,y) \)^2 as

\[
h(x, y) = \frac{2\pi i}{l \lambda R} e^{i2\pi(x^2+y^2)}.
\]

We can then assume that the source \( Q \) is time dependent, that is, \( U_Q \) is time dependent, and we reach the equation:

\[
\psi_R(x, y, t) = \iint q(X, Y)U_Q(X, Y, t)h(x-X, y-Y)dXdY.
\] (8)

This equation is equivalent to equation (5), but it is written so that the time dependent field is the convolution between the time dependent (through \( U_Q \)) transmission function and a time independent impulse function. The observable we are interested in is the intensity of the field, that is:

\[
I_R(x, y) = (\psi_R(x, y, t)\psi_R^*(\tilde{x}, \tilde{y}, t)).
\] (9)

The brackets ideally represent an infinite-time average. Combining equations (8) and (9), we obtain:

\[
I_R(x, y) = \iint dXdY:
\]

\[
\cdot \iint h(x-X, y-Y)h^*(x-\tilde{x}, y-\tilde{y})q(X, Y)q^*(\tilde{x}, \tilde{y})(U_Q(X, Y, t)U_Q^*(\tilde{x}, \tilde{y}, t))d\tilde{x}d\tilde{y}
\]

Since we are considering the case of purely coherent illumination, the values of \( U_Q \) at two points in the object plane \((X,Y)\) and \((\tilde{x}, \tilde{y})\) are perfectly correlated. A change over time of \( U_Q(X, Y, t) \) is perfectly reproduced in the same change over time of \( U_Q(\tilde{x}, \tilde{y}, t) \), so that at any point in time the two quantities differ only by the complex

---

2 The impulse response of free space depends also on \( R \). For clarity, this dependency is omitted in our formalism.
factors $U_Q(X, Y)$ and $U_Q(\bar{X}, \bar{Y})$. Under these conditions, the term in brackets in equation (10) is simply:

$$\left\langle U_Q(X, Y, t)U_Q^*(\bar{X}, \bar{Y}, t) \right\rangle^{coh} = U_Q(X, Y)U_Q^*(X, Y).$$  \hfill (11)

Combining equations (10) and (11), and noticing that the two integrals separate, we can rewrite equation (10) in an intuitive manner:

$$I_p(x, y) = \left| \iint h(x - X, y - Y)q(X, Y)U_Q(X, Y)dXdY \right|^2.$$  \hfill (12)

It is important to notice how the field at a distance $R$ is a linear combination of the fields emitted at the object plane, as shown in equation (4). Equation (12), in contrast, shows how the intensity of the field is not a linear combination of either the fields or the intensities at the object plane. A coherent imaging system is then a linear system in the complex value of the field, which is not an observable, but not in the intensity of the field.

e) Properties of a perfectly coherent imaging system.

In summary, if an imaging system is devised using coherent radiation, such a system will have the following properties:

1. The field at the detector plane is proportional to the Fourier transform of a quantity proportional to the transmission factor of the object;

2. The object-to-detector distance affects drastically the imaging properties of the system. Three regimes are found. The contact regime is where classical radiography happens. In the Fresnel regime, the wavefronts interfere to enhance edges and small features of the object. The Fraunhofer regime is when the interference pattern concerns the entire object, and the produced image is dictated by the Fourier transform of the object.

3. The produced image is a mapping of the intensity of the field in the image plane, and is not a linear combination of any properties of the radiation in the
object plane. For this reason, when image reconstruction is needed to reconstruct
the transmission function $q$ of the object, the reconstruction problem is often not
solvable.

**Incoherent irradiation**

We have studied the subject of an imaging system using perfectly coherent
irradiation. Such a system is of theoretical interest, but it is seldom encountered in
experimental practice. For visible light, even a laser will have a spatial extent that will
reduce its transverse coherence length. For X-rays, the most coherent source that comes
readily to mind is synchrotron radiation, but this kind of source has a finite spatial extent
and is not perfectly monochromatic. Most sources have only limited coherence and for
this reason it is important to understand the behavior of incoherent imaging systems.

a) Perfectly incoherent radiation

Since so far we have discussed perfectly coherent radiation, it seems instructive to
explore the opposite case of perfectly incoherent radiation. In this case, the quantity
$U_Q(X,Y,t)$ and $U_Q(\bar{X},\bar{Y},t)$ are assumed to vary perfectly randomly in time. No
correlation exists then between the field at different points in space and time. While it is
still true that the field at a point in the detector plane is at each moment $t$ the sum of the
fields emitted by the object, the same is not true for the time independent case. This
means that while equations (8), (9) and (10) still hold$^3$ in the incoherent case, all other
equations presented in the previous section are valid only for coherent irradiation.

Starting from equation (8), we can obtain the equation for the intensity of the field in
a perfectly incoherent imaging system. To do so, we write the value for the term in
brackets of equation (8) in the incoherent case:

$$
\langle U_Q(X,Y,t)U_Q^*(\bar{X},\bar{Y},t) \rangle^{incoh} = cI_Q(X,Y)\delta(X-\bar{X},Y-\bar{Y}).
$$ (13)

$^3$ Equation (8) does not take into account different travel times of the radiation from different points in the
object to the detector. Technically speaking, a more thorough equation would be:

$$
\psi_R(x,y,t) = \iint q(X,Y)U_Q(X,Y,t)(X,Y,x,y))h(x-\bar{X},y-\bar{Y})dx dy.
$$
IQ(X,Y) is the intensity that, following the Huygens-Fresnel principle, each point in the area occupied by the object would emit in the absence of the object; c is a real constant. The presence of the object introduces a modulation of the intensity that is related to the transmission function q. Using (13) in (10) we find:

\[ I_R(x, y) = c \iint |h(x - X, y - Y)q(X, Y)|^2 I_Q(X, Y) dX dY. \tag{14} \]

Equation (14) is the equivalent of equation (12) for the coherent case, but it can be compared to equation (4) in that while a coherent system is linear in the complex field value, an incoherent system is linear in intensity.

b) Properties of a perfectly incoherent imaging system

We can now summarize the properties of a perfectly incoherent imaging system:

1. The intensity of the field at the detector plane is proportional to the intensity of the field at the object plane modulated by the transmission function of the object;

2. The object-to-detector distance does not affect drastically the behavior of the imaging system. In all regimes no interference effects are observed

3. The produced image or the mapping of the intensity of the field in the detector plane is a direct image of the object, according to property 1. For this reason, no reconstruction is generally necessary.

c) Validity of the perfectly incoherent assumption

We have discussed at the beginning of this section how it is technically impossible to achieve perfect coherence in a real source. It is appropriate to notice how a perfectly incoherent system is also not a real system, in that every source has a coherence length that is at least equal to one wavelength. Equation (14) can be considered valid in all cases where the smallest features to be imaged are larger than one wavelength.

d) Comparison between coherent and incoherent irradiation
Until now, we have explored the two extreme cases of a perfectly coherent and a perfectly incoherent imaging system. Imaging systems of interest to us will behave in a hybrid way, depending on the level of coherence of the radiation being used. For a matter of comparison, it is instructive to keep the arbitrary distinction between the two extreme cases and consider the different imaging capabilities of the two systems.

i. Resolution

The quality of an imaging system can be estimated by its ability to differentiate between two points in an object. Let us then consider an object which is composed of two points in a perfectly opaque background. Let the two points be so close as to be “barely resolved”, where by this we loosely mean that the imaging system would not be able to resolve any two points that are closer. The two points in the object, following our derivation based on the Huygens-Fresnel principle, act as two point sources. Figure 2a shows how the two points are resolved when incoherent radiation is used to illuminate them. The question arises of whether the resolution changes when coherent radiation is used instead.

![Figure 2: Resolution of two “barely resolved” points in the case of a) incoherent illumination and b) coherent illumination. [2]](image)

4 A more rigorous definition exists for optical systems [2] where the specific case of a diffraction-limited optical system is described. We will refer to that discussion with the caveat that the imaging systems we are interested in are not in general diffraction-limited, and our conclusion will apply only in general terms.
In Figure 2b, we see that the answer depends on the phase mapping of the object. If the points each generate the same phase shift, the resolution of the coherent system will be worse than the resolution of the incoherent system. The two systems show a comparable behavior if the two phase shifts are in quadrature. Coherent irradiation of the sample gives better results if the two phase shifts are in opposition. The property of the object, i.e. its phase behavior, dictates which imaging system performs better.

ii. Contrast

Another relevant property of an imaging system is the response to a step function, that is, how an edge in the object is imaged. Figure 3 is again taken from a visible light, diffraction limited system [2]. We see how the intensity profile of an edge varies drastically from the coherent to the incoherent case (Figure 3a). This effect is visible in phase contrast experiments, where the coherence length of the radiation used is larger than the dimension of an edge feature. In this case, the contrast, or the difference between the intensity at the two sides of the edge, is higher for coherent radiation. It is important to notice, however, that in the coherent case the position of the edge is at a quarter of “steady state” response, and not at the more common middle point in the step profile, as in the incoherent case. Figure 3b shows how the edge would appear in the image, where the interference fringes of the coherent case are clearly visible.
iii. Speckle

If a visible light imaging experiment is carried out with laser light, a speckled image will result. The same effect is encountered in X-ray imaging. To understand the reason behind this behavior, let us assume that we irradiate a sample composed of a disordered group of scatterers (e.g., atoms) [6]. The relevant dimensions in the set-up, as shown in Figure 4, are the wavelength $\lambda$ of the radiation, the size $a$ of the sample, and the average distance $d$ between scatterers. The pattern cast by the radiation on the detector must depend on either $\frac{\lambda}{a}$ or $\frac{\lambda}{d}$. In the case of incoherent irradiation of the sample, the
dependency cannot depend on $a$, since the size of the sample only increases the number of atoms over which to average the intensity. Hence, the pattern cast by coherent illumination will be on the length scale of $\frac{\lambda}{d}$. If coherent illumination is used, alongside the incoherent behavior, the dependency on $\frac{\lambda}{d}$ will be visible due to interference between points in the entire length of the sample. Since $\frac{\lambda}{a} \ll \frac{\lambda}{d}$, the size of the features in the coherent image will be much smaller than the incoherent case. These fine features that are usually manifested as a granularity of the image are characteristic of the speckle effect.

Figure 4: The speckle effect. a) Incoherent radiation does not creates speckles. b) Speckles generated by coherent radiation. [6]

The speckle effect is in general an unwanted behavior of coherent systems$^5$. In most cases it results in a degradation of the image quality that has to be reduced by introducing some incoherence in the system. In visible light experiments, this can be achieved by using a ground glass in the optics of the imaging system. The speckle effect can in some cases

---

$^5$One of the most common experiences with speckles effects arise when an observer look at a laser spot passing through a lens and projected over a wall. Speckle is visible, but it is worth noticing that the speckle is formed on the retina of the observer and is not on the wall. It is in fact a feature of the imaging with coherent light of the non-uniformities in the wall.
cases be a useful feature, notably when the transmission function of the object can be reconstructed from the intensity of the field at the detector. As we already discussed, this is a special case and in general we can consider speckles to be a nuisance.

**Conclusion**

Diffraction theory provided us with some tools to evaluate the image formation process at the observation plane. We predict different behaviors for an imaging system if coherent or incoherent radiation is used. Since the coherence of the source has such an important impact on the imaging process, we will discuss the topic in more detail in the next section.

In general, we can assume that all sources will have a degree of coherence. The greater the coherence of the source, the farther the behavior of the imaging system will be from a ray-geometry assumption. Interference effects can be exploited to generate enhancements on parts of the image, or to try to reconstruct the complex transmission function of the object. A discussion of various imaging systems will follow in the next chapter, and such a discussion will find its roots in the behavior contained in the equations derived in this section.

**A Discussion about Coherence**

The term “coherence” can generate some confusion as it can apply to very different aspects of an imaging system. It can be used to describe the property of a radiation source, as we have done so far. It is important to keep in mind that the same term can be used to describe a type of scattering that radiation incurs – regardless of whether the radiation itself is coherent or not. For clarity, we will refer to this scattering as coherent scattering.

**Formal definition of coherence**

For the moment, we are concerned with the first meaning of coherence, that is, as an attribute of the source, and by extension of the radiation emitted by that source. In the previous section we discussed the case of a perfectly coherent or perfectly incoherent
source. We discussed how these extreme cases are not encountered in experimental practice.

A radiation source is defined as coherent if it emits photons highly correlated in space or time. Classical sources, such as an incandescent lamp in the case of light, or an x-ray tube in the case of x-rays, emit photons in a chaotic way; thus generally they are incoherent sources. Lasers are a typical source of coherent light, while coherent x-ray sources are usually obtained by filtering of incoherent sources.

Equation (9) gives us an idea for a practical definition of the complex degree of coherence [1]. If we call $U_1$ a field emitted at a point $P_1 = (x, y)$, and $U_2$ the field emitted at a point $P_2 = (\bar{x}, \bar{y})$, the complex degree of coherence between the two points can be defined as:

$$\gamma_{12}(\tau) = \frac{\langle U_1(t+\tau)U_2^*(t) \rangle}{\sqrt{\langle U_1(t)U_1^*(t) \rangle \langle U_2(t)U_2^*(t) \rangle}}. \quad (15)$$

Equation (15) is intuitively connected to the bracket term in equation (9), and its time invariant formulation $\gamma_{12}(0)$ is a widely used measure for the coherence existing between two generic emitting points in a source. Equation (11), which refers to a perfectly coherent case, can be simply stated as $\gamma_{12}(0) = 1$. Similarly, equation (13) is a clever way of writing the condition of perfect incoherence, $\gamma_{12}(0) = 0$, since the delta function in (13) ensures that the equation is identically zero except in the case where $P_1$ and $P_2$ coincide. In theory, the knowledge of the value of equation (15) for any two points of a partially coherent source $Q$ allows one to obtain through equation (10) the intensity of the field at the observation plane.

More practically, the coherence of a source is often described by its coherence length, that is, a distance over which the radiation can be considered coherent. This quantity is of more immediate concern in the design of an imaging system, since it allows us to consider the image formation process as coherent over features of an extent smaller than the coherence length, and incoherent otherwise.
Practical definition of coherence

Two kinds of coherence are possible: transverse and longitudinal.

a) Transverse coherence

A source is transversally coherent if the emitted wavefront is coherent in a plane transverse to the direction of propagation of the radiation. Let us consider [6] a point-like monochromatic source illuminating a Young’s double slit. It is well known [3] that an interference pattern will be detectable at a distance. In the small angle approximation, that is, when the detector or screen is far away from the slits, at an angle \( \alpha = m\lambda/d \) off the optical axis (with \( d \) being the separation between the slits), fully developed maxima (at integer \( m \)) and minima (at half integer \( m \)) will be visible.

Figure 5: Diffraction pattern generated by two slits at a distance \( d \) equal to the transverse coherence length of the source. In the text, we renamed the quantity \( R \) appearing in this figure as \( R_{QO} \), to avoid confusion to the distance \( R \) between object and detector defined in the previous section of this chapter. [6]

**Figure 5** shows the same principle for a source of extension \( w \). The source can be thought as a collection of infinitesimal sources, each emitting photons independently. The resulting fringes are blurred due to the incoherent summation of the fringes produced by each infinitesimal source. A measure of the transverse coherence of the incoming radiation is the separation \( \xi_d \) between the two slits at which the maxima generated from
the central element of the source coincide with the minima generated from the edge element of the source. This distance is normally called transverse coherence length.

Considering that the maxima generated by the central element are located at \( \alpha' = \lambda/d \), while the minima of the edge element are at \( \alpha'' = \lambda/2d + w/2R_{QO} \), where \( R_{QO} \) is the source to object distance, equating \( \alpha' = \alpha'' \) one finds:

\[
\xi_t = \frac{\lambda R_{QO}}{w}
\]  

(16).

In a general case, we will be interested in the properties of the radiation emitted from a source of extension \( w \), impinging on an object of transverse extension \( a \) at a distance \( R_{QO} \) from the source. The beam can be considered coherent over the extension of the object only if \( a < \xi_t \). Using (16) and writing the divergence of the beam as \( \Delta \theta = w/R_{QO} \), we find the condition for beam coherence over the transversal distance \( a \) as:

\[
\Delta \theta < \frac{\lambda}{a}
\]  

(17).

b) Longitudinal coherence

Let us consider a pointlike source emitting two simultaneous photons, one at a wavelength of \( \lambda \), and the other at a wavelength of \( \lambda + \Delta \lambda \). Similarly to the transverse coherence length, we define the longitudinal coherence length \( \xi_l \) as the distance over which the two wavefronts are in antiphase. If \( d' = N'\lambda \) is the distance traveled by the first wavefront after \( N' \) oscillations, and \( d'' = N''(\lambda + \Delta \lambda) \) the distance traveled by the second wavefront after \( N'' \) oscillations, by definition we have \( d' = d'' = \xi_l \) if \( N'' = N' - 1/2 \). Solving \( N\lambda = (N-1/2)(\lambda + \Delta \lambda) \) for \( N \), and remembering that \( d = N\lambda \), we find:

\[
\xi_l = \frac{1}{2} \frac{\lambda^2}{\Delta \lambda}
\]  

(18).
Equation (18) shows how only a strictly monochromatic source can be considered fully longitudinally coherent. The larger the bandwidth of the source, the shorter the longitudinal coherence length will be.

We are now interested in finding the equivalent of condition (17) for longitudinal coherence, that is, what requirement on the bandwidth of the source shall be imposed in order to ensure coherence of the radiation in a region of space. We refer to the argument reported in van der Veen [6].

Let us consider a fully transversally coherent polychromatic radiation impinging on a sample of transverse extent \(a\) and longitudinal extent \(l\). We are interested in evaluating the path length difference \((PLD)\) of the radiation after interaction with different parts of the sample, and ensuring that for all points the condition \(\xi_l > PLD\) is met. In Figure 6a we see that the PLD between the center and the edge of the transverse direction of the sample is \(a \sin(2\vartheta)/2\). The value of \(\vartheta\) depends on the momentum transfer \(q\) at the interaction with the sample. In reference to Figure 6c, we see how \(q = k \sin \vartheta\). With known trigonometric formulas we obtain \(\sin(2\vartheta) = \frac{q}{k} \sqrt{1 - \frac{q^2}{4k^2}}\), and since it is safe to assume that the momentum transfer \(q\) is much smaller than the overall momentum \(k\), neglecting higher order terms in \(q/k\) we obtain the condition \(\xi_l > aq/k\). This condition needs to hold for the maximum momentum transfer \(q\). The momentum transfer is related to the resolution \(s\) to be achieved in the experiment by the equation \(q = 2\pi/s\). Knowing that \(k = 2\pi/l\) and considering equation (18) we can formulate the condition \(\xi_l > aq/k\) as:

\[
\frac{\Delta \lambda}{\lambda} < \frac{s}{a}
\]  

(19).

It should be noted that longitudinal coherence has to be assessed also in the longitudinal direction, that is, the condition \(\xi_l > PLD\) has to be met also for path length differences along the propagation of radiation. The concept is exemplified in Figure 6b. In this case, \(PLD = l(1 - \cos(2\vartheta)) = 2l \sin^2 \vartheta\), which in most cases will be smaller than the \(PLD = \frac{1}{2} a \sin(2\vartheta)\) obtained for the previous case. Therefore, condition (19) is the
most stringent condition and is sufficient to ensure longitudinal coherence in most samples.

Figure 6: Diffraction pattern generated by two slits at a distance d equal to the transverse coherence length of the source. [6]

c) Practical considerations

Conditions (17) and (19) ensure, across a sample, a degree of coherence that is in general considered good. It is interesting to note that both conditions describe a characteristic of the radiation source that is possible to modify. For transverse coherence length, the most common way to improve the coherence of the beam is to use a pinhole. By restricting the spatial extent of the source, i.e. reducing the beam divergence, we can theoretically achieve an arbitrarily long coherence length. This approach can be very costly in terms of the available flux, and with most x-ray sources it is in general infeasible to achieve a transverse coherence length in the sub-micrometer range. It is worth noticing though that most applications do not require a coherence length that extends to the entire sample. Imaging techniques using coherent beams are aimed at enhancing edges between materials. An edge is a step function and as such does not have an extent. Nevertheless, for the physical nature of the contact between two materials, we can consider the edge to have an extent, usually in the micrometer to sub-micrometer scale.
By properly restricting the x-ray source with a pinhole, and by positioning the sample at a suitable distance, suppose we build a system that, according to condition (17), has a $\Delta \theta$ narrow enough to ensure transverse coherence over features extending up to 5 $\mu$m. Condition (19) provides us then with a limit to the resolution of the imager when a polychromatic source is used. For instance, if the relatively narrow fractional bandwidth of $10^{-2}$ is used, a resolution greater (that is, worse) than 50 nm is available. This value falls significantly short of the resolution that is available in most microscopy imagers, such as an electron microscope. It is important to consider the wider scope of applications of x-ray imaging, due to the higher penetration of photons as compared to electrons, allowing the non-destructive probing of deeper samples. Furthermore, sample preparation is much simpler for x-ray imaging than for many electron-based imaging techniques.

The conditions discussed so far are independent of the sample and of the specific imaging technique where coherent radiation is required. Different techniques might have more specific requirements for the positioning of the sample and of the detector, and the overall resolution of an imager can vary depending on the specific design.

**The transmission function $q(x,y)$**

The discussion of the image formation process relied heavily on the transmission function $q(x,y)$ as the term where all interactions between incoming radiation and the sample are accounted for. In other words, the ultimate goal of the image formation process is to provide us with a relationship between the image on the detector and the transmission function. Such a relationship can be as simple as a magnifying factor when ray-geometry can be considered valid, or as complicated as the interference patterns arising in the Fresnel regime of a coherent imager. The interest of such imaging systems lies in their ability to provide us insight into the properties of $q(x,y)$.

**A definition of $q(x,y)$**

A simple example of a transmission function is the description of a square aperture in an opaque screen in an optics experiment. Light impinges on the dark screen, and no
transmission happens except at the location of the square aperture. If the aperture has the width of 2a, we can write the transmission function as:

\[
q(x, y) = \begin{cases} 
1 & \text{if } -a < (x, y) < a \\
0 & \text{otherwise}
\end{cases}
\]  \hspace{1cm} (20)

This formulation of the transmission function is common in optics problems involving apertures of various shapes restricting an incoming light source.

In the absence of the object the point \((x, y)\) would emit a perfectly spherical wavefront \(e^{ikr}/r\), as per the Huygens-Fresnel principle. An object, however, would emit a modified wavefront such as \(A\frac{e^{ikr+\phi}}{r}\). From these descriptions, we obtain a generic expression for the transmission function:

\[
q(x, y) = A(x, y)e^{i\phi(x, y)}.
\]  \hspace{1cm} (21)

If we allow the term \(\phi\) to be complex, we can simplify the equation to:

\[
q(x, y) = e^{i\phi(x, y)}.
\]  \hspace{1cm} (22)

The term contains all information about the interactions that the radiation will incur in the sample in its path from the source to the detector. We can expect that such interactions will have the effect of changing both the amplitude and the phase of the radiation. If we aggregate all phase changing effects in a single term \(\delta\), and all absorption effects in a single term \(\beta\), we can write:

\[
\phi = \int (\delta + i\beta) dZ,
\]  \hspace{1cm} (23)

where the integration is over the path of the photon in the sample. The quantities \(\delta\) and \(\beta\) are also used to define the index of refraction of a material:
\[ n = \frac{k_{\text{in}}}{k_{\text{out}}} = 1 - \delta + i\beta. \]  

(24)

We can see from equations (23) and (24) that there is a deep connection between the index of refraction of a material and its transmission function. This should not surprise us, since both quantities depend on the interaction of the radiation with matter. Before going further in the study of the terms \( \delta \) and \( \beta \), we should look further into the interaction that can happen in an imaging system. Since we are primarily concerned with X-ray systems for medical diagnostics applications, we will focus on X-ray interactions with soft tissues.

**Interaction of X-rays with matter**

Our interest is in diagnostic radiography, that is, interaction of low energy photons (in general \(< 100\text{keV}\)) with low Z (atomic number) materials. In this range, three interaction modes of photons with matter are to be considered: photoelectric effect, Compton scattering and Rayleigh scattering.

a) **Photoelectric effect**

The photoelectric effect is described as the absorption of a photon by an atom. Upon absorption, the atom emits an electron and subsequently a characteristic photon (fluorescence). The absorption can happen only if the incoming photon has an energy equal to or higher than the binding energy of the electron to be ejected, most commonly the K-edge electron. The probability of the interaction between the incoming photon and the electron is maximal if the incoming photon has an energy that is close to that of the electron to be ejected, that is, close to the K-edge (or L-edge). This behavior is clearly visible in the discontinuous shape of the absorption coefficient close to the edge energies. Far from the edges, the probability of interaction is proportional to \( 1/E^3 \), where \( E \) is the energy of the incoming photon. The probability of interaction is also proportional to \( Z^3 \), where \( Z \) is the atomic number of the material.
In diagnostic radiography, the photoelectric effect is not the predominant interaction, except for x-rays of very low energy. For these applications, we can consider the fluorescence yield so low as to be neglected, and the photoelectric effect can be assumed to be purely absorption of photons in matter. Figure 7 shows how the importance of the photoelectric effect as compared to other effects decreases as the energy of the radiation increases in tissues. It also shows how in tissue and bone the K-edge is too small to be observable.

![Graph showing mass attenuation coefficient in tissue and bone.](image)

**Figure 7:** Mass attenuation coefficient in tissue and bone. [7]

b) Compton scattering

Compton, or incoherent, scattering is usually described as the inelastic interaction of a photon with a free electron. Electrons in materials are in general bound to an atom, but if the energy of the incoming photon is much higher than the binding energy of the electron, the electron will “appear” free to the photon. Since the total scattering
process obeys conservation of energy and momentum, the scattered photon will be at a different wavelength than the incoming photon. The relationship between the wavelength of the scattered photon $\lambda'$, that of the incoming photon $\lambda$, and the scattering angle $\theta$ can be obtained through conservation equations, and is:

$$\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta).$$

The probability of interaction via Compton scattering is described by the Klein-Nishina formula. For non-relativistic photon energies, that is, much smaller than 511 keV, the Klein-Nishina formula converges with the Thompson formula, aside for a form factor that is due to the fact that the interacting electron is not free but is bound. We can then write the cross section for Compton scattering in the non-relativistic region as:

$$\frac{d\sigma}{d\Omega} (\theta) \propto F_{\text{coh}} \left(x(\theta), Z\right) (1 - \cos^2 \theta),$$

where $x$ is the momentum transfer of the scattered photon, and $F$ is a form factor that takes into account the fact that the interacting electron is bound to an atom that is in general also not free. The differential cross section as a function of $\theta$ in the non-relativistic case is relatively flat, that is, photons are approximately as likely to be scattered at a small forward angle as they are to be scattered at a large angle. The total cross section, obtained by integration of the Klein-Nishina formula over the entire solid angle, varies as $1/E$ and is independent from the atomic number of the interacting element, but is proportional to electron density of the material. Figure 7 shows how Compton scattering is the dominant interaction in tissue x-ray energies between 50keV and 100keV.

c) Rayleigh scattering

Coherent (Rayleigh) scattering is an elastic interaction between the photon and an atom where only the direction of the photon is affected, but not its energy. This apparent violation of conservation of energy is explained by the fact that the incoming photon is absorbed by the atom, which becomes unstable. The atom subsequently emits a photon of
the same energy as the photon that was absorbed. The scattered radiation is coherent, that is, it has the same wavelength as the incoming radiation, hence the name coherent scattering.

The differential scattering cross section for Rayleigh scattering can be written, similarly to non-relativistic Compton scattering, as:

$$\frac{d\sigma}{d\Omega}(\vartheta) \propto F_{coh}^2(x(\vartheta), Z)(1 - \cos^2 \vartheta),$$

where $x = \frac{1}{\lambda} \sin\left(\frac{\vartheta}{2}\right)$ is the momentum transfer. Since coherent scattering is by and large an interaction involving the entire atom, the form factor is related to the electron charge distribution of the entire atom by means of a Fourier transform. For high momentum transfer $x$, that is, large angle scattering, only the most localized electrons contribute to the form factor. Since these electrons are not involved in interatomic bonding, a free atom approximation can be assumed. For low momentum transfer $x$, in general $<0.25$, the form factor will depend heavily on the bonding of the atoms with its neighbors. Thus, a measurement of $F_{coh}$, or more simply of $d\sigma/d\Omega$, provides information about the structure of the material.

The differential scattering cross section has a dependence on $\vartheta$ that favors small angles over large angles. At small angles the form factor is in general larger, and the differential scattering cross section is similar in shape to the Thompson scattering cross section. The Rayleigh differential cross section vanishes at large $\vartheta$.

The total scattering cross section for Rayleigh scattering is for most radiological applications an order of magnitude lower than Compton scattering. Nevertheless, it would not be correct to discard the contribution of Rayleigh scattering to the interaction of x-ray radiation with matter. In fact, if we restrict our interest to small angle scattering of photons in the forward direction, Rayleigh scattering is much more prominent than Compton scattering. This is due to the fact that photons undergoing Compton interactions are scattered more or less uniformly over a large solid angle, while photons undergoing Rayleigh scattering are localized in a small cone around the forward direction. Figure 8 shows the various contributions of scattering in water for a 60 keV x-ray beam.
Figure 8: Monte Carlo modeling of the contribution to scatter in the interaction of 60 keV pencil photon beam on water. The distance $r$ represent the distance from the axis of the beam, and correlates with the angle $\theta$, that is the momentum transfer $x$. [8]

As will be stressed other times in this thesis, it is important to keep clear the difference between coherent scattering and coherent radiation. Any photon can undergo Rayleigh (coherent) scattering, irrespective of whether it belongs or not to a coherent beam of photons.

d) A similarity to optics

Coherent scattering of X-rays has an interesting similarity to the classical optics problem of diffraction of light through a small slit. If coherent light, such as light from a small laser, is shined through a small slit, a diffraction pattern is cast in the geometrical shadow of the screen. The light is not absorbed by the slit, its energy is not affected by the interaction, but shape of the wavefront is perturbed by the shape of the slit, that is, the photons are scattered. Similarly, photons in an X-ray beam are scattered by Rayleigh scattering without loss or change in energy – the scattering is visible only through the
small angle deviation of the photons, which is tantamount to a deformation of the radiation wavefront. Pushing the similarity forward, we can consider light illuminating two Young slits, as described earlier in this chapter, as a phenomenon similar to the coherent scattering of X-ray by two adjacent atoms. If the X-ray source is coherent, interference effects will be visible.

The similarities between optics and X-ray imaging extend beyond single atom interactions. This is not surprising in that X-rays and visible light are two manifestations of the same physical phenomenon. The macroscopic difference between the interactions of the two types of radiation with matter comes from the fact that photons at X-ray wavelengths are less interacting with matter than photons in visible light.

**The expression for β and δ.**

Equation (23) summarizes all the interactions of radiation with matter in two terms. Aside from a unity factor, we see in equation (24) that these terms are the real and imaginary terms of the index of refraction.

a) An expression for δ

The real term, δ, introduces a phase shift in the radiation wavefront, but does not directly affect its intensity. Such a disturbance in the wavefront can be accounted for only by a coherent scattering of the photon in the object. Only the Rayleigh effect contributes to this term. The full derivation of the phase term is beyond the scope of this section. Referring to the derivation found in James [9], we give the relationship for the index of refraction n in absence of absorption:

\[
\frac{\beta}{N^2} = 1 - \frac{N_2 \epsilon^2}{2\pi mc^2} F_{coh},
\]

\[
n = 1 - \delta = 1 - \frac{N_2 \epsilon^2}{2\pi mc^2} F_{coh},
\]

where \( N \) is the number of atoms per unit volume. Equation (25) does not coincide with the equation for the δ term that is usually found in literature [6, 10, 11]. The usual expression is:
The difference between the two expressions is that in equation (26), $F_{\text{coh}}$ equals the atomic number $Z$, so that the atomic density is replaced by the electron density $N_e = ZN$. This assumption is suspect. The form factor for Rayleigh scattering in the almost forward direction, i.e. at very low momentum transfer, should depend on the properties of the entire atomic electron cloud, hence showing a dependency on the bonding between atoms. This dependency is not seen in equation (26). Equation (26) is an approximation of (25) where we assumed a free electron approximation. For this approximation to be valid, the energy of the incoming photons has to be much higher than the binding energy of the electrons. The photon then does not "see" the electrons as bound, and the free electron approximation holds. Moreover, under this assumption we can neglect the variation in energy level of the electron cloud caused by the presence of other atoms, i.e. the molecular properties of the material.

The applications we are concerned with involve X-ray sources of energies in the 10 to 100 keV range. The materials we are interested in imaging are low $Z$ materials, usually encountered in biological tissue, such as hydrogen, oxygen, carbon, and in some applications calcium. Calcium (Ca), which is the higher $Z$ material we are concerned with, is the litmus test for our approximation. We find in literature [12] that the most tightly bound electrons in Ca have a binding energy of 4keV. This value is smaller than the photon energies. If the requirement for the free electron approximation is written as $B_e < 10E_{\text{X-ray}}$, the approximation can be held valid for most imaging systems. Also, it is worth mentioning that the binding energy of the outer electrons, that are the ones involved in inter-atomic interactions, is much smaller than 4keV, thus ensuring that the free electron approximation is valid.

b) An expression for $\beta$

Photoelectric effects and Compton scattering contribute to the absorption term. This is the imaginary term of the index of refraction. It may seem unusual to attribute a scattering term to the absorption of photons. The reason behind this lies in the effect that
Compton scattering induces in the wave associated with the scattered photon. If we consider a train of photons represented by a wave of wavelength $\lambda$ traveling in a sample, the presence of Compton scattering in the sample removes some of the photons from the wave we are following, and generates a secondary wavefront at a different wavelength. For our purposes, the net effect is that the original wavefront is diminished in intensity.

We need to briefly mention the effect that the secondary wavefront has on the image formation process of an imaging system. We know from our discussion on Compton scattering that the scattered photons are distributed in the entire space; this implies that only a small portion of the photons are comprised in a small solid angle around the forward direction. When the observation plane is in contact or very close to the object plane, the solid angle between the scatterer and the detector is large; hence many scattered electrons will hit the detector. The number of photons hitting the detector this way does not in general exceed half the number of photons undergoing Compton scattering, since we assumed that the angular distribution of the scattering over the entire solid angle is more or less flat. As we move the detector further away from the object, the number of Compton scattered photons hitting the detector decreases. In the paraxial approximation, when the distance $R$ between object and detector is very large compared to the size of the object, we can neglect the contribution of Compton scattering to the imaging process. In the paraxial approximation, Compton scattering can be strictly considered an absorption term.

For object-to-detector distances shorter than the paraxial approximation, the portion of Compton scattered photons hitting the detector can be considered a noise added to the image. Since the contribution of Compton scattering is either absent or equated to noise, we consider Compton scattering effects as absorption effects for all object-to-detector distances. We should keep in mind though that the greater the object-to-detector distance, the lower the Compton noise will be, thus resulting in an improvement in the quality of the image. A similar result can be obtained using a grid or collimator at the observation plane to reduce the amount of scattering hitting the detector.

We expect $\beta$ to be proportional to the absorption coefficient of X-rays in matter $\mu$. It is beyond the scope of this section to derive the exact dependency, which we give as a result [6]:

$$\beta \propto \mu$$
In equation (27), the absorption coefficient \( \mu \) contains the most relevant information about the absorption of X-ray in a sample. This term depends on the energy of the incoming radiation and on the nature of the interacting material. Values of \( \mu \) for different materials and different energies can be found in literature.

\[ \beta = \frac{\mu \lambda}{4\pi}. \] (27)

c) Relative importance of \( \delta \) and \( \beta \).

The coefficients \( \delta \) and \( \beta \) have very different impacts on the imaging formation process, and are then difficult to compare. We find claims in literature that in biological samples, while \( \beta \) can be as small as \( 10^{-9} \), \( \delta \) is in the order of \( 10^{-6} \) [6]. Such claims are surprising, since, even neglecting the photoelectric effect, Compton scattering has a higher total cross section than Rayleigh scattering. Since, in equation (27), the information about the interaction is hidden in the absorption coefficient, it is not very practical to compare directly equations (26) and (27). The interested reader can investigate this matter further [9]; for the time being, we will consider such estimates correct.

No matter the relative value of \( \delta \) and \( \beta \), it is a fact that X-rays interact weakly with tissues and low Z materials. If an imaging system can obtain a mapping not only of \( \beta \), as in standard radiography, but also of \( \delta \), this will greatly improve the imaging capabilities of the system. The improvement is made more visible by the fact that detection of \( \delta \) relies on interference of coherent radiation, resulting in maxima and minima that are easily detectable.

**Conclusion**

This chapter established some basic facts that will be used in the following parts of this thesis. In particular, the basic equations for the image formation process under coherent and incoherent illumination have been established. The two equations (12) and
(14) can be used alternatively, and the discussion about the significance and extent of the coherence of a source will give insight on when to use (12) and when to use (14). Finally, the study of the transmission function q helps us ground the result of the imaging process to the object to be imaged, through the interaction of the radiation with the materials in the object.

References

3. Relevance of Phase Contrast Imaging and Coherence Requirements

Introduction

Recently, there has been an increased interest in alternative imaging techniques known as Phase Contrast Imaging (PCI), which can be performed with X-rays and Neutrons. The common feature of these methods is an emphasis on edge enhancement effects. These effects are usually ascribed to the ability to detect, under the proper experimental conditions, the information contained in the real part of the index of refraction of the material under investigation. In classical mechanics, the process is described as refraction which produces a distorted wave front.

PCI derives its name from the fact that refraction effects do not affect the amplitude of the radiation at the interaction point, inducing instead a phase shift $\varphi$ in the local radiation field. The radiation field itself is not an observable though, and the information contained in the phase term thus is not directly available. There are several ways of exploiting phase information as a source of image contrast. These fall into three broad categories: interferometry, diffractometry, and in-line holography, and are associated with directly measuring $\varphi$, $\nabla \varphi$, and $\nabla^2 \varphi$, respectively. Here, $\varphi$ is the phase change introduced in the incident radiation upon passing through the sample and is given by integrating over the ray path $\varphi = -\frac{2\pi}{\lambda} \int \delta(s) ds$, where $\lambda$ is the wavelength of the radiation, $\delta$ is the real part of the index of refraction, and $s$ is the integration variable along the ray path.

Interferometry and diffractometry techniques rely on the precise measurement of different path lengths and therefore require a high precision set-up of perfect crystals or gratings. Moreover, these techniques require essentially monochromatic X-rays. Thus, they have almost always been performed in laboratory environments using synchrotron X-ray sources. Little effort has been made to exploit these techniques for broad imaging.
applications such as security or medical screening, with some notable exceptions [1,2,3]. The technical difficulty involved in a broader application of interferometric and diffractometric methods to imaging is twofold: achieving the required mechanical precision in the alignment of crystals and gratings in a cost-effective way, and obtaining a monochromatic source bright enough to allow imaging with reasonable exposure times.

In-line holography, more commonly referred to as free space propagation, or off-focus imaging, relies on a simpler set-up. It is based on the fact that phase differences in the transmitted radiation will result in interferences after interaction with the sample. The interference pattern can be detected at a distance with a standard intensity measure of the radiation field. This allows for the use of conventional detectors with no need for crystals or gratings and thus no requirement for mechanical precision. The literature on free space propagation phase contrast imaging also suggests that polychromatic X-ray tubes or nuclear reactor neutron beamlines are suitable sources, the coherence requirement being essentially only transversal. In-line holography thus appears to be the most practical approach for real systems: provided that a suitable coherence is obtained in the source, an imaging system can be used as a phase contrast device by placing the detector at a distance from the focal point (at contact with the sample).

The term phase contrast imaging is in general used for a variety of techniques that provide edge enhancement. In this chapter, we will focus on a definition of phase contrast imaging that is both based on experimental applications and rooted in the physics of the phenomenon. We will concern ourselves only with those methods that lend themselves to broad imaging applications, such as medical diagnostics and security screening. The implication is that our definition of phase contrast includes only in-line holography methodologies. It is then tempting to refer to all edge enhancement obtained through free space propagation as phase contrast, advancing a purely empirical definition of the term. But we believe that this approach would be misleading, since multiple phenomena contribute to edge enhancement. Furthermore, such an inclusive definition of phase contrast would have little practical value, since it would be impossible to define the design parameter for a phase contrast imaging system without restricting our attention to a single physical phenomenon behind the edge enhancement effect.
Referring to our discussion of diffraction theory in Chapter 2, we here approach the more specific problem of PCI. In this chapter, we first discuss the nature of phase induced edge enhancement with X-rays and Neutrons. We then explore the requirements for PCI in terms of longitudinal and transversal coherence of the source and the geometry of the set-up. Finally, we then advance a hypothesis on a competing edge enhancement effect in X-ray free space propagation imaging and compare this effect to the expected PCI enhancement.

**Phase Contrast Imaging**

Experimentally, a phase contrast imaging system produces an edge enhanced image of the sample. For in-line holography, this happens when the source is coherent and the detector is placed in a region of space called near-Fresnel region. Diffraction theory suggests that no notable edge enhancement can be observed in the region of space closer to the detector. For coherent sources, in the region of space past the near-Fresnel region, especially entering the Fraunhofer region, the intensity field of the radiation gives an image that does not immediately represent the object, but instead represents its Fourier transform. In this region, a decoding technique has to be used; imaging performed in this region is generally called diffraction enhanced imaging rather than phase contrast imaging.

A good starting point to understand the physics of phase contrast imaging is a set-up in which a monochromatic, coherent source is used and the detector is located in the near Fresnel region. Still, we need to focus only on edge enhancement due to effects linked to phase shifts induced by the sample. To do so, we introduce an ideal object called a phase object. The amplitude of the radiation is unaffected by the object, whereas the phase is shifted, and this phase shift varies within the object. The relevance of such an object is that it is invisible for standard radiography, allowing us to focus only on the phase contrast effect in its imaging. Such an ideal object does not exist in nature, although biological samples can have characteristics that approach those of this ideal object.

We will discuss in general terms the visibility of a phase object in the near-Fresnel region and the definition of PCI that can be advanced based on this study. We will than
consider more particularly the physics of PCI performed with X-rays and neutrons so as to understand the similarities and the differences between the two techniques. Finally, we will make some remarks on the implication of the energy dependence of PCI in medical applications.

Visibility of a phase object in the near-Fresnel region

Our discussion of the theory of diffraction provided us with equations capable of describing the imaging process for both coherent and incoherent X-ray imaging. Focusing on imaging with coherent radiation, we presented by general considerations three possible imaging techniques depending on the distance between the sample and the detector: standard radiography, done at contact, that is at focus; Fresnel zone, slightly out-of-focus, where local edge enhancements are visible; and Fraunhofer zone, far out-of-focus, where global interference effects occur and decoding is required. We focus now on the Fresnel zone, and more specifically on the near end of the Fresnel zone, where phase contrast occurs through free space propagation.

We want to obtain a simplified representation of the intensity of the field produced by coherent radiation impinging on a phase object and measured at a distance $R$ from the object in the near-Fresnel region. We already presented a mathematical representation of such an object using its transmission function:

$$q(x) = e^{i\varphi(x)}.$$  (1)

As we discussed, such an object is impossible to image in standard radiography, since the intensity profile it generates is $I(x) = |q(x)|^2 = 1$, that is no contrast is generated. Imaging such an object without staining of the sample requires a means to obtain contrast using phase information. In this section, we will follow a theoretical approach found in literature [4] to obtain a simplified equation of the intensity profile generated by the interaction of the radiation with the phase object. Figure 1, already used in Chapter 2 in discussing diffraction theory, exemplifies the geometry of this problem.
Figure 1: Geometry of the diffraction of radiation emitted by a source Q when passing through an object O, observed at the plane P. [5]

Despite the fact that the radiation field is not an observable, it is easy to calculate through diffraction theory. At a distance \( R \) from the object, the radiation field is given by Equation (2.5), which we can rewrite more simply as the convolution between the transmission function of the phase object \( q(x) \) and a free space propagation term (the exponential term) that describes the evolution of the field after the interaction in free space propagation geometry:

\[
\psi(x) = q(x) * e^{-\frac{2\pi i}{R \lambda}}. \tag{2}
\]

We obtain the intensity of the field as

\[
I(x) = |\psi(x)|^2 = |q(x) * e^{-\frac{2\pi i}{R \lambda}}|^2,
\]

but this formulation does not provide any insight into the phase contrast image formation process. A mathematical manipulation of Equation (2) allows an easier understanding of the nature of the edge enhancement effect to be expected in phase contrast imaging. We know from experimental evidence that the edge enhancement is
due to a relation between the intensity of the field and the derivative of the transmission function across the transversal direction [6]. We will now outline the derivation of this relation.

The transmission function of the phase object can be rewritten as

\[ q(x) = \int Q(u) e^{-\pi u x} du , \]  \hspace{1cm} (3)

where \( Q(u) \) is the Fourier transform of \( q(x) \). Equation (3) simply states the definition of a Fourier transform, but it will be useful in the following derivation. As we mentioned, we expect to see a correlation between the derivative of some properties of the sample and the intensity profile of the radiation. The second derivative of \( q(x) \) in the transverse direction \( x \) can be written by direct integration of (1) as:

\[ \frac{d^2}{dx^2} [e^{i\varphi(x)}] = -e^{i\varphi(x)} [\varphi'(x)^2 + i\varphi''(x)] . \]  \hspace{1cm} (4)

Another expression for the same quantity can be obtained by direct derivation of Equation (3).

\[ \frac{d^2}{dx^2} [e^{i\varphi(x)}] = \int (-4\pi^2 u^2) Q(u) e^{-\pi u x} du . \]  \hspace{1cm} (5)

The relevance of these two equations to our derivation will be apparent later. For the moment, we focus on the equation of the field (2), into which we incorporate the expression of the transmission function given in Equation (3). By the definition of convolution we obtain the integral:

\[ \psi(x) = \int dX \int du Q(u) e^{-\pi u x} e^{-2\pi i u (x-X)} . \]  \hspace{1cm} (6)
Equation (6) expresses the field in terms of the Fourier transform of the transmission function. This equation can be simplified by direct integration over the variable $X$ to obtain:

$$\psi(x) = \int du Q(u) e^{i\pi Rxu^2} e^{-2\pi iux}. \quad (7)$$

This expression of the field is exact and is valid independently of the particular transmission function or of the geometry. Thanks to our definition of phase contrast imaging, we can now make use of the assumption of a geometry where the intensity measurement at $R$ will happen in the near-Fresnel region. In this region, we can expand the exponential in (7) and truncate the series to the first order:

$$e^{i\pi Rxu^2} = 1 + i\pi R\lambda u^2. \quad (8)$$

Equation (7) becomes then

$$\psi(x) = \int du Q(u) e^{-2\pi iux} + i\pi R\lambda \int duu^2 Q(u) e^{-2\pi iux}. \quad (9)$$

The first term is simply the transmission function, as described in Equation (3). The radiation field at a distance $R$ from a phase object, where $R$ is in the near-Fresnel region, is then given by:

$$\psi(x) = e^{i\psi(x)} + i\pi R\lambda \int duu^2 Q(u) e^{-2\pi iux}. \quad (9)$$

So far, we have manipulated the radiation field integral and used the assumptions of near-Fresnel geometry and an ideal phase object. It is interesting to notice the relation between Equation (5), giving the second derivative of the transmission function across the transverse direction, and the second term of the field in Equation (9). The two quantities are proportional, so it is possible to make Equation (9) more readable by writing the field in terms of the second derivative of the transmission function:
\[ \psi(x) = e^{i\phi(x)} - \frac{iR\lambda}{4\pi} \frac{d^2}{dx^2} [e^{i\phi(x)}]. \]  

(10)

Using the direct derivation of the transmission function as obtained in Equation (4), we can finally write an expression of the radiation field at \( R \) in terms of the phase shift \( \phi \) and its derivatives:

\[ \psi(x) = e^{i\phi(x)}[1 + \frac{iR\lambda}{4\pi} \phi'(x)^2 + \frac{R\lambda}{4\pi} \phi''(x)]. \]  

(11)

The field is not an observable, and we are looking for an equation for the intensity. The square of the field as expressed in Equation (11) contains terms in powers of \( \phi', \phi'', \) and \( \phi'\phi'' \). The expression of the intensity is greatly simplified by noting that in our region of interest \( R \) is sufficiently small as to neglect terms in \( R\lambda \) of order higher than the first. We thus obtain:

\[ I(x) = |\psi(x)|^2 = 1 + \frac{R\lambda}{2\pi} \phi''(x). \]  

(12)

Equation (12) shows how a pure phase object, although invisible in images taken at focus, becomes visible off-focus. This visibility is due to interference effects arising from different shifts in the phase of the radiation induced in different part of the phase object.

This phenomenon is what we call phase contrast imaging, that is the ability of a system to provide edge enhancement that is solely due to coherent scattering / interference effects. It is an intrinsically quantum mechanical phenomenon, since it cannot be explained through ray geometry. Its relevance extends beyond the visibility of a phase object; such enhancement exists in principle whenever coherent radiation is used to image an object where the object presents a variability in the phase shift induced in the radiation. The importance of deriving Equation (12) for a phase object lies in the fact that in this case the phase contrast effect is the only edge enhancing effect taking place. In the
case of a real object, other phenomena can induce a similar edge enhancing effect that may mask the phase-related enhancement.

**Definition of phase contrast**

Despite the interest in Phase Contrast Imaging, there is still some ambiguity in literature on what constitutes PCI. This is due to a double confusion. On one hand, there are multiple techniques that exploit phase information for sample analysis, such as Coherent Diffractive Imaging [7-9] or Coherent Neutron Scattering [10-12]. When used in standard imaging applications, the requirements in terms of coherence and reconstruction do not make these techniques interesting for standard imaging applications. Phase Contrast Imaging directly provides a better image and has requirements that can in principle be met outside a laboratory. On the other hand, phenomena that are not directly correlated to a phase shift can also provide edge enhancement effects, which are clearly not Phase Contrast Imaging. To properly classify various imaging techniques, we must define what constitutes Phase Contrast Imaging.

Equation (12) is a founding equation for phase contrast imaging. It shows that phase induced contrast can be obtained without interferometric methods. Taking an image at the focal plane, that is at contact \(R = 0\), no edge enhancement due to phase contrast effects is possible. By taking an image at a distance from the focal plane, the term in (12) containing the phase shift \(\phi\) becomes visible and its importance increases linearly with \(R\). It is important to recall, though, that despite the linearity shown in (12) between phase contrast enhancement effects and the object-to-detector distance \(R\), the entire derivation is based on the assumption of Fresnel zone. In particular, in Equation (8) a simplification was made based on \(R\) being at the near end of the Fresnel zone. We can then infer from Equation (12) that phase contrast imaging can be achieved when off-focus images are taken at a distance from the sample provided that distance is in the early Fresnel zone. We also can assume that increasing the distance beyond that zone would result in a higher sensitivity to edge enhancement, but also to a progressive corruption of image quality due to global interference effects. A competing effect on the image taken at a distance is the blurring due to lack of focus.
In Chapter 2, we discussed the physical interactions of photons with matter in order to understand the real processes that produce X-ray images. Following our definition of phase contrast imaging, only photons that are coherent at the place of interaction can contribute to the effect. Since the effect is due to the difference in phase shift between photons hitting nearby regions of the sample, a strong correlation between the phases of those photons at the places of interaction is necessary. If this correlation is loose, the overall effect will be an average of random phases and will not produce an observable effect. In general, for extended sources, the number of photons that meet this coherence requirement is low and limited to very narrow regions of space.

In the coherence region, that is a region of space that is small enough so that we can assume that the photons hitting the sample inside this region are mutually coherent, photons undergo the photoelectric effect, Compton scattering and Rayleigh scattering. Of these processes, only Rayleigh scattering is a coherent process, and it is therefore the only one that can contribute to the interference of the interacting photons in the region of space between the sample and the detector, providing phase contrast enhancement. Among the three listed interactions, Rayleigh scattering is the least likely to happen, usually by a factor of an order of magnitude. It seems then that in the overall imaging process, phase contrast is a minority effect. Absorption due to photoelectric effect and fogging of the film due to Compton scattering involve a much higher number of interacting photons, and it is tempting to assume that phase contrast effects are negligible.

**Phase contrast with X-rays**

The key to the importance of phase contrast imaging is the fact that, as shown in Equation (12), the contrast is dependent on the derivative of the phase shift in the transverse direction, as opposed to the phase shift itself. To understand the relevance of this observation we need to remind ourselves of the nature of this phase shift and how it correlates with properties of the material. In Chapter 2, we investigated this relationship, summarized in Equations (2.23) and (2.26) here reported:

\[
\varphi = \int (\delta + i\beta) \, dZ
\]  

(2.23)
If we still assume that the object under investigation is a phase object, we can usefully rewrite Equation (12) as:

\[ I(X) = |\psi(x)|^2 = 1 + R \frac{\lambda^3}{4\pi^2 \frac{e^2}{mc^2}} N_e''(x). \]  

Equation (13) pinpoints how the phase contrast enhancement is in effect a representation of the derivative of the electron density \( N_e \) in the material. This explains why phase contrast effects, despite being a minority effect, can provide very visible edge enhancement. At an edge, the difference between the electron densities of two materials might not be large, but the sharp nature of an edge makes the derivative across it extremely high. In many cases, this provides enough amplification to the enhancement to make the effect comparable if not more important than edge visibility due to much more likely phenomena such as photoelectric effect.

In Table 1 we find a useful summary of the imaging properties of coherent X-rays. The most straightforward process involved in X-ray radiography is the photoelectric effect. The interacting photon is absorbed and thus removed from the incoming beam. This effect results in a decrease in detected photons along the path of regions of the sample where photoelectric effect is more likely, namely regions of higher atomic density or higher Z. The principle behind this imaging effect is independent of the distance between the sample and the detector, aside from geometrical considerations such as magnification and focusing.

Compton scattering is often regarded as a nuisance in radiography. Photons interact with the electrons in the material and are scattered at an angle. When those photons are collected on the detector, it is not practical to distinguish them from photons coming directly from the source without having interacted in the sample. Therefore, the overall effect is to create fogging of the image, since spurious counts coming from Compton
scattering in the sample are added to valid photon counts (that is, photons passing through the sample without interaction). The noise level associated with Compton scattering is a problem in standard radiography and it is usually reduced by the use of a Bucky grid. A Bucky grid is a collection of lead stripes positioned between the sample and the detector. Photons leaving the sample at angles outside the acceptance angle of the slits between the lead stripes will be shielded and thus will not reach the detector. In this way, scatter above a set threshold can be filtered out. Due to the flat nature of the angle distribution of Compton scattering, a majority of scattered photons can be filtered this way. This filtering mechanism is not without drawbacks, however. To avoid the appearance of the lead stripes in the image, the grid is usually moved during the image acquisition. This introduces a moving part in the imaging system that can cause the sample to vibrate, corrupting the quality of the image. Also, the total photon count at the detector is diminished by the presence of the lead, resulting in poorer statistics.

<table>
<thead>
<tr>
<th></th>
<th>At Contact</th>
<th>At Distance</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Photoelectric Effect</strong></td>
<td>$I \propto f(N_a, Z)$</td>
<td>$I \propto f(N_a, Z)$</td>
<td>Standard radiography.</td>
</tr>
<tr>
<td><strong>Compton Scattering</strong></td>
<td>noise</td>
<td>$I \propto f(N_e, R)$</td>
<td>Fogging in standard radiography; improved absorption if filtered.</td>
</tr>
<tr>
<td><strong>Rayleigh Scattering</strong></td>
<td>$I \propto 1$</td>
<td>$I \propto f(N_e^r, R)$</td>
<td>Inconsequential in standard radiography; phase contrast imaging at distance.</td>
</tr>
</tbody>
</table>

*Table 1:* Summary of the effects of the three main physical effects on imaging with coherent X-ray radiation.

The impact of Compton scattering on imaging changes when in-line holography is used. At a distance, only photons scattered into the acceptance solid angle of the field of view at the detector will contribute to fogging. Photons scattered outside of this solid
angle will be considered absorbed, de facto increasing the absorption coefficient of the sample and improving the quality of the image. Free space propagation can then be considered as a filtering technique for Compton scattered photons. The effect on the film will depend on the number of photons that underwent Compton scattering and were scattered at an angle larger than the acceptance angle. This depends in general on the electron density (the higher, the more probable the interaction) and on the sample-to-detector distance \( R \). The dependency on \( R \) is nonlinear, since both the acceptance angle and the size of the field of view change with \( R \), producing a net effect that favors Compton scattering filtering at high \( R \). The energy of incoming photons is also a parameter, since the lower the energy, the flatter the angle distribution of the scatter, while a high photon energy results in a more forward oriented scatter. A flatter angle distribution gives more efficient filtering obtained through free space propagation.

Considerations that are similar to those made for Compton scattering can be applied to Rayleigh scattering, but both the fogging effect at contact and the filtering effect at a distance are much weaker for Rayleigh scattering. This is due to the fact that Rayleigh scattering is forward oriented, and most of the scattered radiation is contained in a small solid angle around the forward direction. If this has the advantage of resulting in small fogging at contact, also it makes it difficult to filter by both Bucky grids and free space propagation. Since Rayleigh scattering is in any case less probable than Compton scattering by about one order of magnitude, the number of photons that affect the image in the way described before is minimal and is usually neglected.

The phase contrast effect of Rayleigh scattering for images taken at a distance cannot be neglected, as discussed previously. As we can see from Table 1, this effect is the only one containing a dependence on the derivative of a physical quantity (the electron density). For this reason, Rayleigh scattering plays an important role in the image formation process at a distance despite the low fraction of total photons involved.
Phase contrast with Neutrons

The theoretical framework developed so far to describe phase contrast imaging and phase enhancement is valid independently of the kind of radiation used. In other words, the derivation of Equation (12) did not depend on the particular kind of radiation, and it is equally valid for neutron phase contrast as it is for X-ray imaging. As a reminder, Equation (12) predicts an image enhancement effect, as compared to a standard radiography image, for sample-to-detector distances greater than zero but within the Fresnel zone. It also predicts that the enhancement is proportional to the second derivative in the transverse direction of the phase shift that the sample locally imparts to the incoming radiation. We can then reason that the enhancement is particularly strong at edges in the sample, where the derivative of the phase shift assumes very high values.

Interestingly, despite the fact that Equation (12) is applicable to both X-ray and neutron phase contrast, and thus that an edge enhancement effect is to be expected in both cases, Equation (13) does not describe the physics of edge enhancement for neutrons. The reason behind this apparent paradox is that while diffraction theory, on which the derivation of Equation (12) is based, is general and valid for neutrons as well as photons, the interaction of neutrons with matter is fundamentally different than the interaction of photons with matter, so that Equation (2.26) is not valid for neutrons.

A thorough discussion of the nature of the interaction of neutrons with matter is beyond the scope of this section, but it is instructive to point out the similarities and differences between neutron and photon interactions. For both types of radiation, we are primarily concerned with coherent scattering phenomena; we thus focus only on this aspect of the interaction. If we consider a generic interaction, the interaction depends on the density $\rho$ of interacting particles multiplied by the coherent scattering length $\sigma_i$ of the physical object with which the interaction takes place. We can rewrite Equation (13) in a more general way as

$$I(x) = 1 + R \frac{\lambda^3}{4\pi^2} \frac{\partial^2}{\partial x^2} (\rho \sigma_i).$$

(14)
We have seen in Chapter 2 how coherent scattering of photons arises from Rayleigh scattering of a photon off of the electron cloud of an atom. For this interaction, the coherence length $\sigma_{\gamma}(x)$ is independent of the shape of the electron cloud and is equal to the classical radius of the electron, $\sigma_{\gamma}(x) = r_e$, a constant. The relevant density is the electron density, so that $\rho(x) = N_e(x)$. Under these considerations, for photon interactions, Equation (14) collapses to Equation (13), where the coherence length is a constant and does not contribute to edge enhancement. In X-ray imaging, two different materials with the same electron density would look identical.

Neutrons do not interact with a material at the atomic level, but rather at the nuclear level. Coherent scattering of neutrons is similar to coherent scattering of X-rays, but nucleons rather than electrons are involved. This means that for neutron interactions, $\rho(x) = \rho_n(x)$ in Equation (14). A more profound difference between neutron and X-ray interactions lies in the fact that for neutron interactions the coherent scattering length depends on the specific assortment of neutrons and protons in the nucleus. This implies that, for neutron interactions, materials with similar nuclear number can present very different coherent scattering lengths. Mathematically, the scattering length $\sigma_l = \sigma_n(x)$ is a function of the material at the specific position $x$, and as such cannot be taken out of the derivative operator in Equation (14).

The dependence of the coherent scattering length on the assortment of protons and neutrons in the nucleus of the target material has important consequences in imaging. The edge enhancement effect of neutron phase contrast imaging reveals not only sharp, sudden differences in nuclear densities, but also sharp, sudden differences in coherent scattering lengths. This implies that phase contrast imaging using neutrons can provide access to an additional physical property of materials. Performed with X-rays, phase contrast amplifies differences between materials without accessing any properties not already accessible by standard radiography.

This is not to say that neutron phase contrast imaging is inherently better than X-ray phase contrast imaging. As we discussed, the two techniques find different applications, and are usually complementary. Also, many practical aspects make neutron phase contrast harder to achieve than X-ray phase contrast. As we discussed, neutron sources are weaker, which translates to exposure times considerably longer than those for X-ray
phase contrast. Neutron sources are also less readily available and, in general, not portable. Furthermore, the design of a neutron phase contrast imager is restricted in that it may be more difficult to obtain the geometrical set-up needed for free space propagation in a nuclear reactor due to the limitations given by the reactor containment.

**Discussion on limits of phase contrast imaging**

Although PCI is an imaging technique based on coherent radiation, it is believed that a certain level of incoherence may be acceptable for PCI. We are here set to define the limit of coherence that allows PCI. The common approach to this question is to perform an experimental test: an incoherent system is used in a free space propagation set-up, and if edge enhancement is observed it can be inferred that the level of coherence of the system is acceptable. We take exception to this approach, since as discussed before, a free space propagation system can provide edge enhancement that is not phase induced. In this section, we explore theoretically the effect of a relaxation of the coherence of the system in theory, and we will propose an acceptable level of incoherence based on a statistical analysis of the physical processes involved in PCI. In Chapter 4, we will seek an experimental validation for our theory.

Phase contrast imaging using free space propagation techniques can be performed without using a phase detection device (e.g. an interferometer) and with a geometrical set-up that is only a trivial modification of a standard radiography set-up. The coherence requirement of the source is the most noticeable difference between a standard imager and a phase contrast imager. The calculations performed in this chapter have assumed a pointlike source emitting monochromatic radiation. In reality, perfect coherence, either longitudinally (polychromaticity) or transversally (source size), is unattainable: X-ray lasers are not currently available, and all sources possess a spatial extent.

Another parameter that should not be overlooked is the Fresnel limit. In our derivation we made an approximation that required the source-to-detector distance to be in the Fresnel zone. This imposes lower and upper limits on the sample-to-detector distance in addition to the lower limit on the source-to-sample distance dictated by coherence requirements. From Chapter 2, we know that:
\[ \left( \frac{a^4}{\lambda} \right)^{\frac{1}{3}} \ll R < \frac{a^2}{\lambda} \]  

where \( a \) is a size corresponding to the level of detail we are interested in. For instance, for medical applications, choosing \( a = 1 \text{ mm} \), we obtain \( 30 \text{ cm} \ll R < 50 \text{ km} \).

Proximity to the lower end of the Fresnel zone does not imply that no edge-enhancement will be observed, but it will not follow Equation (12). For the purpose of the following discussion of the limits of PCI, we will consider the lower end of Fresnel zone as our limit for PCI, and we set \( R_{\text{Fresnel}} = 30 \text{ cm} \).

**Longitudinal coherence**

Monochromatic radiation is extremely hard to obtain. Because of the problems involved in obtaining monochromatic coherent radiation, it is important to understand what level of incoherence / polychromaticity is acceptable in a phase contrast imager. Indeed, phase contrast imaging first became viable as a real imaging technique after experiments were successfully performed using polychromatic radiation [13]. Since Equation 12 was derived under the assumption of radiation of a single wavelength \( \lambda \), considerations of the effects of longitudinal incoherence are in order. From Chapter 2, we know that diffraction theory suggests a limitation on the intrinsic resolution of an imager whenever polychromatic radiation is used (see Equation 2.19). This is in general of little relevance in standard imaging, since the image that is obtained is an incoherent average of many different contributions. For phase contrast imaging, the image is a superposition of a standard radiograph (although at a distance) and a coherence-induced image containing the phase enhancement information.

To evaluate the limitations on resolution, let us assume that at each point of the sample we can identify a region of space of radius \( \xi_i \), inside of which the radiation can be considered transversally coherent. In this region, for the purpose of evaluating the impact of coherent processes such as Rayleigh scattering on the image, we need to evaluate the
maximum resolution made available to the system. Equation (2.19) can be usefully rewritten as:

$$s > \xi, \frac{\Delta \lambda}{\lambda}$$

(16)

where $s$ is the resolution of the system. To realistically evaluate the resolution limit imposed by the polychromaticity of the system, let us assume a transversal coherence length of 10 $\mu$m and a fractional bandwidth of $10^{-2}$. Under these conditions, the maximum resolution achievable by the system is 100 nm. This figure is better than the resolution available to most commonly used detectors, so this limitation is inconsequential. In cases where a higher resolution detector, such as a Fuji film imaging plate, is used, care should be taken to evaluate the transversal coherence length and the fractional bandwidth of the source. A higher resolution is accessible for the standard radiography contribution to the image, but the phase enhancement contribution might not enjoy the full resolution of the system. In general, it is difficult to improve on the resolution limit. The fractional bandwidth of the system is intrinsic to the source and can be changed only at the price of drastically limiting the photon output. A very small coherence length allows a better resolution, but at the price of compromising the phase contrast capabilities of the system. Nevertheless, for most applications, the limitation on the resolution is not particularly punishing.

A second implication of using polychromatic radiation for phase contrast imaging can be inferred by a study of Equation (12). Let us assume that the net effect of polychromatic radiation impinging on a sample is equivalent to the effect of multiple monochromatic sources with wavelengths dispersed over the polychromatic source bandwidth. The net effect then consists of a superposition of edge enhanced images, all of them depending on the derivative of the electron density (for X-ray PCI) or of the nuclear density and coherent scattering length (for Neutron PCI) across the edge, but each amplified by a different factor proportional to $\lambda^3$. The resulting image still enjoys an edge enhancement. Ultimately, for most applications, an X-ray source with a reasonably
narrow fractional bandwidth of 10\(^{-2}\) can be used for phase contrast imaging with little inconvenience.

The most common source of neutrons is a nuclear reactor. Neutrons are plentifully produced during the fission process, and beamlines can be fabricated in the core to allow these neutrons to travel outside the reflector and the shielding and be used for research purposes, like imaging. The neutrons produced during fission span a wide range of energies, and before reaching the beamline they usually undergo a thermalization process that reduces their mean energy and further broadens their spectrum. The fractional bandwidth spectrum of a neutron beamline from a reactor source is in general broader than that of common X-ray sources. Methods exist to reduce this bandwidth by discriminating against neutrons outside of a given energy band, but by doing so a sizable part of the available signal is lost. In general, the limit on resolution imposed by the polychromaticity of the beam is not a concern in neutron imaging. It is to be noted though that neutron sources usually have a broader energy spectrum compared to X-ray sources.

Transversal coherence

The second coherence requirement to perform phase contrast imaging is transversal. This quantity is important because it defines the size of the ensemble of atoms from which phase contrast phenomena can arise. Perfect transversal coherence can be achieved with a pointlike source, but such a source is not available. In Chapter 2 the topic of transversal coherence was discussed and condition (2.16), here renumbered as Equation (17) for ease of reference, was established:

\[
\xi_r = \frac{\lambda R_{\text{co}}}{W}.
\]

The transversal coherence at the sample is a measure of the maximum distance between two points for which interference can occur. Applied to phase contrast imaging, a system with a low transversal coherence length gives an edge enhancement effect arising from photons interacting with material immediately in the vicinity of an edge;
material further from the edge is involved for a system with a higher transversal coherence length. For a polychromatic source, it is convenient to use an average $\lambda$ in Equation (17). X-ray sources, for example X-ray tubes, have a spatial extent $w$ of tens of micrometers to a few millimeters; the geometrical arrangement of the experiment usually allows a source-to-sample distance $R_{QO}$ of a few centimeters to a few meters. This results in a less-than-perfect transversal coherence. For instance, the coherence length for a tungsten-based microfocus X-ray tube with focal spot $w = 10 \, \mu m$ and a source-to-sample distance of $R_{QO} = 5 \, m$, emitting photons of $\lambda = 0.2*10^{-10} \, m$, is $\xi_t = 10 \, \mu m$. In literature, such a system is considered to have very good coherence and to be suitable to perform phase contrast imaging. A standard radiography system, using a Tungsten-based X-ray tube with a focal spot $w = 1 \, mm$ and a source-to-sample distance $R_{QO} = 10 \, cm$, provides a coherence length of only $\xi_t = 2 \, nm$. Let us consider now a thermal neutron source, with mean $\lambda = 1.8*10^{-10} \, m$, restricted with a $w = 0.1 \, mm$ Gadolinium pinhole, and a source-to-sample distance $R_{QO} = 3 \, m$. The transversal coherence length is $\xi_t = 5 \, \mu m$. If a standard system with an opening $w = 5 \, mm$ and a source-to-sample distance $R_{QO} = 1 \, m$ is used, the transversal coherence length is $\xi_t = 4*10^{-8} \, m$.

It is evident that the transversal coherence length can vary widely among systems. In the remainder of this section, we will propose a model to determine the PCI properties of a system. Specifically, the model identifies, for a given situation, a minimum transversal coherence length in order for the system to provide PCI capabilities. We will also discuss how the PCI capabilities of a system are affected by its geometrical properties and the energy of the radiation.

Let us focus on evaluating the impact of the transversal coherence length of the system on its PCI capabilities. For ease of terminology, we will often refer to X-ray PCI, but the conclusions we will draw are independent of the kind of radiation that is used. The nature of the problem is statistical: the higher the transversal coherence length, the larger the sample of atoms that, locally, can contribute to phase contrast effects. For
instance, a transversal coherence length\(^6\) of 10 \(\mu\)m provides \(10^{10}\) atoms for possible contribution to edge enhancement effects. A coherence length of 2 nm allows only 400 atoms across the edge to have a possible contribution\(^7\).

To better understand what we mean by atoms contributing to PCI, a reminder of the physical process involved in phase contrast enhancement is in order. We consider an extremely thin sample, so as to be considered two dimensional. Let us consider a photon undergoing Rayleigh (coherent) scattering with an atom in the sample. In order for enhancement to happen, interference with a second photon undergoing coherent scattering with a second atom located within the transversal coherence length from the first one must occur. The probability \(p^{PCI}\) of having an event contributing to phase contrast is given by:

\[
p^{PCI} = p_{coh} \cdot p(n > 1, N_{\xi})
\]

where \(p_{coh}\) is the probability of a Rayleigh scattering event occurring with any atom, while \(p(n > 1, N_{\xi})\) is the probability of at least one Rayleigh scattering event occurring in the ensemble of \(N_{\xi}\) atoms contained in an area of diameter \(\xi_{t}\). \(p_{coh}\) depends on the material and on the energy of the radiation, and can be inferred from the coherent scattering cross section \(\sigma_{coh}\). The second quantity depends on \(p_{coh}\) and \(N_{\xi}\) and is given by \(p(n > 1, N_{\xi}) = 1 - B(1, \mu, \sigma)\), where \(B\) is the binomial distribution\(^8\) with mean \(\mu = p_{coh} \cdot N_{\xi}\) and standard deviation \(\sigma = \mu \cdot (1 - p_{coh})\).

We calculated the value of \(p^{PCI}\) as a function of scattering length \(\xi_{t}\) for various interaction probabilities \(p_{coh}\). The results are shown in Figure 2. For very large transversal coherence lengths, the probability of a photon contributing to phase contrast imaging is equal to the probability of a photon undergoing coherent scattering. As the transversal coherence length decreases, a strongly non-linear behavior is observed so that

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\(^6\) We assume here that the system offers a constant transversal coherence length in all directions. Some systems, like grating-based coded source systems (see Chapter 5) offer a coherence length in only one direction.

\(^7\) Atoms are assumed here to be circles \(10^{-10}\) m in diameter.

\(^8\) In the following calculation, we assumed the size of the ensemble to be large enough to allow the use of a normal distribution instead of the binomial, as per the central limit theorem.
the probability of obtaining an event contributing to phase contrast imaging falls quickly to zero. We can infer that for each $p_{coh}$, we can define a practical lower limit $\xi_{min}$, under which PCI effects do not deliver a meaningful contribution to the image:

$$p^{PCI}(\xi_{min}) = \frac{p_{coh}}{2}.$$  \hspace{1cm} (19)

We can also define an optimum length $\xi_{opt}$ which is the threshold above which ulterior improvements in transversal coherence length are inconsequential to the PCI capabilities of the system.

$$p^{PCI}(\xi_{opt}) = p_{coh} \cdot 0.9.$$  \hspace{1cm} (20)

The spread between $\xi_{min}$ and $\xi_{opt}$ can be small, and for the design of a PCI system it is important to deliver a transversal coherence length $\xi_t$ that is as close as possible to $\xi_{opt}$. For each material, for a given radiation at a given energy we can calculate $p^{PCI}$ from the coherent scattering cross section $\sigma_{coh}$, and we can then infer an optimal value $\xi_t$ for the imaging system.

For instance, if we consider X-ray imaging of tissue with 60 keV photons, a coherence length of $\xi_{opt} = 17 \mu m$ allows optimal exploitation of PCI, a coherence length of $\xi_{min} = 9.25 \mu m$ allows just partial exploitation of PCI, while a transversal coherence length of less than 5 \mu m does not allow PCI. Thus, based on Figure 2, for an X-ray system devoted to medical applications, we set the limiting coherence length at about $\xi_{min} = 10 \mu m$. As we discussed previously, a transversal coherence length of $\xi_t = 10 \mu m$ is an upper limit of what can be achieved with microfocus X-ray tube systems, suggesting that PCI with such sources will be difficult to achieve.
Figure 2: Probability $p_{PCI}$ of obtaining a phase contrast enhancement event per photon as a function of the transversal coherence length $\xi_t$ for photons of wavelength $\lambda = 0.2 \times 10^{-10}$ m, per atom in Tissue, Aluminum and Calcium.

We have shown how we can define an $\xi_{\text{min}}$ for each material and for each photon energy. For example, based on Figure 2, we find $\xi_{\text{min}} = 3.35$ µm and $\xi_{\text{opt}} = 6.05$ µm for imaging of Aluminum with 60 keV X-rays. It is worth noting that this value gives a necessary condition to have PCI effects. The magnitude of the intrinsic PCI capabilities of an imaging system is given by $p_{PCI}$ and it can in some cases be low even for $\xi_t > \xi_{\text{opt}}$.

The value of $p_{PCI}$ does not in itself give insight into the relevance of the PCI capabilities of the system to the final image. If we assume that a standard radiography is obtained through photoelectric absorption events, the probability of photoelectric effect is a natural benchmark to evaluate PCI. We then introduce:

---

9 We use the composition for the A-150 tissue equivalent plastic, that is, H: 10.1%, C: 77.68%, N: 3.5%, O: 5.21%, F 1.7%, Ca 1.8%. The density is assumed to be 1.127 g/cm$^3$. 

66
\[ \kappa^{PCI} = \frac{D^{PCI}}{P_{p.e.}} \]  

(21)

Although the values of \( \xi_{\text{min}} \) and \( \xi_{\text{opt}} \) calculated through both \( \kappa^{PCI} \) and \( D^{PCI} \) are identical, the information contained in \( \kappa^{PCI} \) is more interesting for imaging purposes. \( \kappa^{PCI} \) provides information on the relative weight of PCI compared to standard radiography for a specific material at a specific energy. In Figure 3, we show the dependency of \( \kappa^{PCI} \) on \( \xi_\tau \). From Figure 3 it is clear that PCI is more relevant for Tissue than for Aluminum, since the image of a Tissue sample would be more affected by the availability of PCI. This information was not available in Figure 2.

**Figure 3**: \( \kappa^{PCI} \) as a function of the transversal coherence length \( \xi_\tau \) for photons of wavelength \( \lambda = 0.2*10^{-10} \) m, per atom in Tissue\(^{10}\), Aluminum, and Calcium.

\(^{10}\) We use the composition for the A-150 tissue equivalent plastic, that is, H: 10.1%, C: 77.68%, N: 3.5%, O: 5.21%, F 1.7%, Ca 1.8%. The density is assumed to be 1.127 g/cm\(^3\).
We show in Figure 4 the value of $\kappa^{PCI}$ for Tissue as a function of photon energy for different $l/d = R_{\infty}/w$. Small differences in the $l/d$ of the system significantly decrease its PCI capabilities. It is worth noticing that for most X-ray tube based imaging systems designed with medical diagnostic applications in mind, a high $l/d$ fraction is on the order of magnitude of $l/d \sim 10^4$. Figure 4 suggests that these systems will not be able to perform PCI with X-rays exceeding a few keV. Since the dose to the patient is inversely related to the energy of the X-rays, it is impractical to use low energies for in-vivo imaging. A higher $l/d$ system can operate at higher energies where the PCI can give a stronger contribution to the image, with the additional benefit of reducing the dose to the patient.

![Figure 4: $\kappa^{PCI}$ as a function of the photon energy for various $l/d = R_{\infty}/w$. The maximum theoretical value is given as a reference.](image-url)
For any given radiation energy, type of radiation and material, we can define a minimum coherence length necessary to perform phase contrast imaging, as exemplified in Figure 2. In Figure 4, we link the phase contrast capabilities of a system for medical diagnostics to operating parameters such as \( l/d \) ratio and photon energy \( E \). This methodology can be repeated for any material. For instance, in Figure 5 we give a similar graph for Aluminum.

![Graph](image)

**Figure 5:** \( \kappa^{PCI} \) as a function of the photon energy for various \( l/d \) ratio. The maximum theoretical value is given as a reference.

\( \kappa^{PCI} \) is a useful indicator of the absolute intrinsic phase contrast capabilities of a system. It is sometimes useful to evaluate the PCI capabilities of an imaging system compared to its maximum theoretical capabilities. To do so, we propose a quality factor for the PCI capabilities of a system:
The quantities $p^{PCI}$ and $q^{PCI}$ provide useful insight into the intrinsic PCI capabilities of an imaging system. The quantity $\kappa^{PCI}$ is a figure of merit to evaluate the impact of PCI on image quality. Equation (12) shows that the enhancement is also proportional to the sample-to-detector distance $R$, with a cut-off value at the Fresnel limit $R_{Fresnel}$. We then propose a new figure of merit of the PCI capabilities of an imaging system:

$$K^{PCI} = \kappa^{PCI}(R_{QO}, w, \lambda, \sigma_{coh}) \cdot (R - R_{Fresnel}) \cdot \alpha$$

(23)

where $\alpha$ is a factor, with dimensionality m$^{-1}$, that describes the proportionality of PCI edge enhancement to the sample-to-detector distance. The exact value of $\alpha$ is unknown, and its study will require experimental investigation with high-coherence imaging systems. Based on Equation (12), it is reasonable to expect that $\alpha$ is inversely proportional to the photon energy. We also know that $\alpha = 0$ for $R < R_{Fresnel}$. For the moment, we will approximate $\alpha$ as,

$$\alpha = H(x - R_{Fresnel})$$

(24)

where $H$ is the Heaviside step function. The figure $K^{PCI}$ will be useful to compare phase contrast edge enhancement to other edge enhancement effects that might also be dependent on the sample-to-detector distance $R$.

In this section, we introduced four parameters to evaluate the PCI capabilities of an imaging system: $\xi_{min}$, $\xi_{opt}$, $\kappa^{PCI}$ and $K^{PCI}$. $\xi_{min}$ and $\xi_{opt}$ provide the requirements for transversal coherence length; $\kappa^{PCI}$ and $K^{PCI}$ provide figures to evaluate the relevance of PCI in a specific application.

In literature, many systems have been used to perform phase contrast imaging. The topic of PCI in literature is discussed in Chapter 6, and the parameters of those systems
will be assessed there. It is enough to mention here that most efforts done to date to perform PCI through microfocus X-ray tube technology failed to produce systems with significant PCI capabilities, according to our calculations. Despite this fact, an edge enhancement effect has been observed in these systems, and erroneously ascribed to PCI. In the next section, we propose a theory to explain the observed edge enhancement.

**Further discussion on limits of phase contrast imaging**

We have discussed the fact that a coherence length of 10 μm can be achieved in an imaging system built with medical applications in mind, for use in a hospital setting. This system, though, is the upper limit of what is possible to build. The small microfocus X-ray tube required is expensive and cannot provide a flux high enough to perform examination in a timely fashion. Also, the geometrical arrangement requiring 5 m of space between the source and the patient requires large spaces that are effectively an additional cost for the hospital.

Assuming a system providing a transversal coherence length of 10 μm, we can now analyze in more detail what is the real requirement for polychromaticity. From Equation (15), we know that the maximum resolution available to phase contrast depends on the transversal coherence length and on the fractional bandwidth. Assuming that for medical applications the lowest resolution of interest is in the tens of micrometers, the required fractional bandwidth is 1, which is largely available. Only for more specific applications, where better resolutions are needed, must ways of reducing the fractional bandwidth of the system be found.

Finally, while it seems possible to use polychromatic X-ray sources to perform phase contrast imaging, the transversal coherence length required imposes a severe limitation on the spatial extent of the source. Standard sources, such as a standard X-ray tube, are not immediately suitable for phase contrast imaging. State-of-the-art microfocus X-ray tubes in a geometry allowing a long distance between source and sample narrowly meet the minimum requirements for phase contrast imaging of Tissue. In Chapter 5 we will investigate alternative ways to provide high coherence lengths at reasonably high flux using off-the-shelf equipment that can expand the applicability of phase contrast imaging.
**A model for Compton scattering reduction through free space propagation**

In the previous section we obtained quantitative limits on the parameters of an imaging system in order to perform phase contrast imaging. Interestingly, edge enhancement has been observed in literature with free space propagation X-ray systems with no PCI capabilities (see Chapter 6 for a more detailed discussion). This edge enhancement must be due to other effects.

From our discussion of the various phenomena involved in the image formation process, we expect that Compton scattering is the effect involved in the improvement observed in incoherent systems operated in free space propagation geometry. To validate this assumption, let us investigate in more detail the statistical relevance of scattering reduction due to free space propagation, to which we will refer as Compton Filtering (CF), in a very thin sample. Compton scattered photons hitting the field-of-view of the sample inside the detector, creating fogging, can be filtered out of the image by increasing the sample-to-detector distance. This filtering effect reduces fogging of the film and results in an enhancement of the imaging capabilities of the system.

Let us consider a point source illuminating a two dimensional circular sample of radius $R_s$. An incoming photon has a probability $p_{\text{incoh}}$ of undergoing Compton scattering in the sample. A scattered photon has a probability $p_{\text{miss}}$ of being scattered at such an angle that it would not hit the detector in the geometrical shadow of the object (here called Field-of-View (FoV)). The probability per photon per atom of having an event that would contribute to CF signal is:

$$P_{\text{CF}} = P_{\text{incoh}} \cdot P_{\text{miss}}.$$  \(25\)

While $p_{\text{incoh}}$ is only a function of the cross section $\sigma_{\text{incoh}}$ for Compton scattering in the material, $p_{\text{miss}}$ depends on the particular geometry of the set-up. For the calculation of this last parameter, we will refer to [Figure 6](#). We are set to calculate the probability $p_{\text{hit}}$ of a scattered photon hitting the FoV, from which we will derive $p_{\text{miss}} = 1 - p_{\text{hit}}$. 
Let us consider the probability \( p(r_i \rightarrow P) \) that a photon scattered at a generic point \( r_i \) of the sample hits the FoV at a generic point \( P \). This probability is proportional to the differential cross section for Compton scattering, so that

\[
p(r_i \rightarrow P) = \frac{1}{\sigma_{\text{inc}}(2\theta)} \cdot \partial \sigma_{\text{inc}} \bigg|_{\theta, \phi}.
\]

Equation (26) expresses the probability of a scattering event happening in an infinitesimal solid angle around the direction \((\theta, \phi)\) and is given by the Klein-Nishina formula. \( \theta \) is the angle between the direction of propagation of the photon before scattering, and the direction of propagation of the photon when scattered towards \( P \). The probability does not depend on \( \phi \), due to the symmetry of Compton scattering. \( P \) is identified by the distance \( r_z \) from the perpendicular to the FoV plane passing through \( r_i \), and by the angle \( \phi_z \) (Figure 6). The infinitesimal area in the FoV seen by the scattered photon is \( dA_z = r_z dr_z d\phi_z \cdot \cos \theta_1 \), where \( \theta_1 \) is the angle between the direction of propagation of the photon scattered towards \( P \) and the infinitesimal area around \( P \). The probability of a scattered photon in \( r_i \) hitting the infinitesimal area around \( P \) is \( p(r_i \rightarrow P) \cdot dA_z \). We find the probability \( p(r_i \rightarrow \text{FoV}) \) of a photon being scattered from \( r_i \) to the FoV by integration:

\[
p(r_i \rightarrow \text{FoV}) = 2 \cdot \int_0^{r_{\text{max}}} dr_z \int_{\phi_{\text{min}}}^{\phi_z} \frac{1}{\sigma_{\text{inc}}(2\theta)} \cdot \partial \sigma_{\text{inc}} \bigg|_{\theta, \phi} \cdot r_z \cdot \cos \theta_1.
\]

The integration limits are between 0 and \( r_{\text{max}} \) for \( r_z \), with \( r_{\text{max}} = r_{\text{fov}} + r_1 \), and between \( \phi_{\text{min}} \) and \( \phi_z \) for \( \phi_z \). All quantities are obtained through trigonometric considerations. For instance:

\[
\phi_{\text{min}} = \cos^{-1}\left\{ \frac{r_z^2 + (r_{\text{fov}} - r_i)^2 - 2 \cdot r_{\text{fov}}^2 \cdot (1 - \frac{r_{\text{fov}}^2 + r_z^2 - r_i^2}{2 \cdot r_{\text{fov}}^2 \cdot r_i})}{2 \cdot r_z \cdot (r_{\text{fov}} - r_i)} \right\}.
\]
To determine the probability $p_{\text{hit}}$ that any one photon scattered in the sample will hit the detector we need to integrate Equation (23) over the sample, to obtain:

$$p_{\text{hit}} = \frac{1}{\pi \cdot r^2} \int_0^{2\pi} \int_0^{r_{\text{FoV}}} \rho(r_i \rightarrow \text{FoV}) \cdot \cos \phi \, dr_i \, d\phi_i$$

(29)

This integral cannot be solved in closed form due to the complicated nature of the expression for $\rho(r_i \rightarrow \text{FoV})$ in Equation (27). In order to calculate $p_{\text{miss}} = 1 - p_{\text{hit}}$ we
solved the integrals in Equations (27) and (29) numerically through a Matlab routine. It is worth noticing that the derivation of Equations (27) and (29) is general, but any solution is specific to the sample material and to the photon energy through the Compton scattering differential cross section in Equation (27). We can argue that the term $\frac{\sigma_{\text{diff}}}{\sigma}$ in Equation (26) is to a first approximation only a function of the photon energy, so that $p(I_j \rightarrow P)$ and $p_{\text{miss}}$ would not depend on the nature of the sample. An example of the angular shape of Equation (26) can be found in Figure 7. The value of $P^{CF}$ will always be sample dependent due to the presence of $p_{\text{incoh}}$.

![Figure 7](image)

**Figure 7:** Angular distribution of the Compton scattering differential cross section for three different energies. [14]

We show in Figure 8 the results of this calculation for an imaging system where a tissue sample has been positioned at $R_{OO} = 3$ m from the source, and the detector can be positioned freely in a space 3 m wide ($R < 3$ m). Since $P^{CF}$ does not give information about the relative importance of CF compared to PCI or contact radiography, we propose in analogy with PCI the figure of merit:
\[ K^{CF} = \frac{P^{CF}}{P_{p.e.}} \]  \hspace{1cm} (30)

As we can see, CF contributions increase non-linearly with R, while PCI contributions increase linearly after the Fresnel limit \( R_{Fresnel} = 0.3 \) cm. The PCI contribution is strongly dependent on the available coherence length, that is on the source size. Only systems with extremely small focus can provide a PCI contribution to the image. This contribution can be substantial thanks to the linear dependency on R. At the distances under investigation in Figure 8, CF is dominant over PCI for imaging of Tissue.

Figure 8: Figures of merit \( K^{CF} \) and \( K^{PCI} \) for three focus sizes, as a function of sample-to-detector distance \( R \). Source-to-sample distance \( R_{SO} = 3 \) m. Tissue sample 5 cm in diameter. \( E = 60 \) keV.
We show in Figure 9 similar results for an Aluminum sample. The geometrical parameters and photon energy are similar to the previous case. A similar behavior is observed in this case, with the difference that slightly smaller transversal coherence lengths are needed (i.e., slightly larger source sizes). In this case PCI gives a more important contribution as compared to CF, although both effects are less prominent (lower value of $K$) compared to the Tissue example.

To investigate the prominence of CF at different energies, let us consider the case of a source-to-sample distance $R_{OO} = 3$ m and a sample-to-detector distance of $R = 2$ m. In Figure 10, we show the ratio $P^{CF}/P^{PCI}$ for Tissue for three source sizes[^11]. We notice how a source size $w < 10$ μm is needed in this set-up to obtain meaningful PCI at energies $E > 30$ keV. At small energies, PCI contributes significantly to the image relative to CF, but

[^11]: This is equivalent to showing $K^{CF}/K^{PCI}$
as the energy increases the contribution becomes negligible unless the transversal
coherence length of the system is exceptionally good.

The same study in Aluminum shows (Figure 11) a different behavior. PCI dominates
CF at low energy, and the system is slightly less susceptible to a deterioration of PCI
capabilities due the increase of source size.

![Figure 10: $P_{CF}/P_{PCI}$ as a function of photon energy in Aluminum. $R_{QO} = 3$ m, $R = 2$ m.](image)
Finally, we conclude from this study that PCI can give an important contribution to the imaging capabilities of the system, but that high transversal coherence lengths are needed. It is important to notice though that once the optimum coherence length $\xi_{opt}$ is met, no additional benefit is obtained from higher coherence lengths. Even in those cases, CF outperforms PCI for short sample-to-detector distances.

This analysis was performed for two specific cases, but general conclusions can be drawn. PCI is a powerful technique that can provide edge enhancement thanks to its dependency on the derivative of the electron density. However, there must be sufficient contribution to the phase contrast image in order for the edge enhancement to be visible. We developed a method to evaluate the relevance of PCI and CF in an imaging system, and we applied this method to the case of Tissue and Al imaging in a set-up that can be reproduced in a laboratory. We also have been aggressive in our choice of source size, and our analysis already assumes imaging systems that push the state-of-the-art in coherent imaging.
The results obtained with Tissue and Aluminum show that CF is in general responsible for the bulk of the enhancement contribution for short sample-to-detector distances. Despite this fact, the PCI contribution can become more meaningful than standard absorption even at short distances, as it is in the case of PCI Tissue imaging. In these cases, extremely coherent systems can provide a PCI enhancement that impacts the image quality.

Let us define a low-coherence system as an imaging system that provides a transversal coherence length on the order of a few micrometers and is therefore considered coherent in most literature, but does not meet the required limit $\xi_{\text{min}}$ or have a low $K^{PCI}$. Our study shows that contrary to common wisdom, such a system does not provide any visible PCI effects, and that edge enhancement effects observed in free space propagation set-ups are due to CF. We will experimentally support this conclusion in Chapter 4.

**An optimization problem**

Let us evaluate the impact of PCI and CF in the case of a microfocus X-ray tube used to image a circular sample 5 cm in radius. The total available space for imaging is 5 m, which is the size of a large room. The experimenter positions the detector 5 m from the source. We want to explore the optimum placement of the sample for edge enhancement purposes. The photon energy is 60 keV and the sample is made of Tissue. We assume an $R_{\text{Fresnel}}$ of 30 cm.

Let us discuss the results, presented in Figure 12, considering first the behavior of CF enhancement contributions as the sample-to-detector distance $R$ increases. For $R = 0$, the sample is positioned at 5 m, at contact with the detector. This is the standard radiography case, and no enhancement is visible. CF does not occur. As $R$ increases, that is as the sample is positioned closer to the source, the CF enhancement contribution increases sharply, reaching its maximum after about 40 cm. After this point, further movement of the sample does not change the CF contribution. When the sample is positioned extremely close to the source, the CF effect falls due to the large size of the
Figure 12: An optimization problem, with total space $R + R_{QO} = 5$ m. 60 keV X-rays, 5 cm sample.

FoV. In this region, our model is not accurate because the assumption of a point-like source becomes invalid.

It is more convenient to discuss the PCI enhancement of the system starting with the sample extremely close to the source, that is exploring Figure 12 right-to-left. For low $R_{QO}$ (high $R$), the system does not provide a transversal coherence length that would allow PCI, and no effect is visible. As $R_{QO}$ increases ($R$ decreases), the system with a source size $w = 1 \mu m$ reaches a level of coherence that enables PCI. As $R_{QO}$ increases further ($R$ decreases further), the $p^{PCI}$ of the system increases sharply, as seen in Figure 2. The figure of merit $K^{PCI}$ (Figure 12) increases as well, although less sharply due to the decrease of $R$. At the point $R_{QO} = 1$ m ($R = 4$ m), the 1 μm focus system has reached its optimum transversal coherence length. Notice how at this point neither of the other two source sizes has provided an $\xi_t > \xi_{\text{min}}$ yet. As $R_{QO}$ increases beyond this point, higher transversal coherence lengths become available but do not provide any additional advantage to the PCI enhancement contribution. After this point $\kappa^{PCI}$ plateaus at its
maximum level, but $K^{PCI}$ decreases linearly with increasing $R_{QO}$ due to the decreasing sample-to-detector distance $R$. When $R$ is below the Fresnel limit, no PCI enhancement is possible.

For a source size $w = 10 \ \mu$m, a very small PCI edge enhancement effect is available but it is not enough to provide a visible contribution. No PCI enhancement is predicted in this case for a source size of $w = 25 \ \mu$m. CF is dominant in all cases. The $w = 1 \ \mu$m system is able to provide a significant PCI contribution to imaging, equal roughly to 3 times the contribution coming from absorption, for $R_{QO} = 1 \ \text{m}$ and $R = 4 \ \text{m}$.

As we mentioned previously, some phase contrast experiments described in literature fall short of the requirement to perform PCI. Some of these experiments cannot meet the transversal coherence length because a small enough focal spot cannot be provided. Most experiments have a much smaller geometrical space available compared to what we described in this section, making it impossible to reach $\xi_{opt}$. This in turn prevents the sample-to-detector distance from meeting the Fresnel zone limit and does not allow a high enough $K^{PCI}$ to be achieved. We believe that enhancement effects found in literature in those cases have been erroneously ascribed to PCI and are actually edge enhancement effects due to CF (see Chapter 6).

In general, PCI can be performed and is capable of delivering a significant contribution to image quality, but very high transversal coherences are needed. For X-ray imaging, the only systems to our knowledge that can meet such a limitation are synchrotron radiation systems.

**Conclusions**

Propelled by the strong interest in literature for a class of edge enhancing techniques known by the umbrella name of PCI, we set in this chapter to develop an operational definition of what is phase contrast and to describe its function. Following already published mathematical derivation stemming from diffraction theory, we derived the equation that justifies the edge enhancement capabilities of a coherent imaging system using free space propagation. We proposed a non ambiguous definition of PCI based on this founding equation. Moreover, we explored the physical processes involved in
imaging formation with X-rays, detailing the effects that result in phase contrast imaging, and expanding on the known equations to obtain a dependency of edge enhancement on physical properties of the sample. We found that the interest in this imaging technique is justified by its edge enhancement capabilities. We also briefly explored the nature of phase contrast imaging with neutrons.

A thorough analysis of the requirements of a phase contrast imaging system has been carried out, revealing that while the use of a polychromatic beam is not particularly detrimental, the requirement on transversal coherence is more stringent than what appears in literature. We developed a model to evaluate the PCI capabilities of a partially coherent imaging system. We described how transversal coherence length, energy of the radiation, type of material in the sample and geometrical factors of the imaging system contribute to the PCI capabilities of a particular set-up. An analysis of imaging systems found in literature has been performed (and will be presented more thoroughly in Chapter 6), and it is our conclusion that systems based on current technology of microfocus X-ray tubes do not in general have PCI capabilities. In Chapter 5, we will discuss some ideas to build an X-ray source able to provide the needed transversal coherence.

The fact that edge enhancement has been observed in literature with free space propagation microfocus X-ray tube systems that do not present an appropriate longitudinal coherence has been discussed. We developed a model to evaluate the Compton scattering reduction obtained through free space propagation, a technique we called Compton Filtering (CF). A quantitative discussion on the relative importance of CF and PCI showed that for those systems, CF provides an edge enhancement that can be misrepresented as PCI edge enhancement. We discussed the relevance of different parameters of the imaging system in the two systems, and discussed the optimal use of available geometrical space for maximizing the edge enhancement effect.

In conclusion, we have shown how we can obtain an image containing additional information not included in a standard radiography image. By moving the detector off-focus, we are able to make use of the phase shift that the sample imparts to the incoming radiation to obtain an edge enhancement effect. The distance between the sample and the detector is to be chosen such that it is large enough to allow the edge enhancement effect to be visible. This technique, called free space propagation, has the added benefit of
introducing Compton Filtering in the system, thus delivering additional edge enhancement. In most set-ups, Compton Filtering is the dominant edge enhancement effect. The most punishing condition to obtain phase contrast edge enhancement is the condition on the transversal coherence length of the system. In order to obtain a suitable coherence length, the spatial extent of the source has to be extremely small, thus reducing the available flux, and increasing the exposure times. Also, a long source-to-sample distance is needed. Despite these drawbacks, phase contrast imaging has the potential of providing a valuable imaging tool in medical imaging and in material analysis.

REFERENCES
4. Experimental validation

**Introduction**

In Chapter 3 we discussed in some detail the coherence requirements for Phase Contrast Imaging (PCI). According to our calculations, PCI edge enhancement can in most applications be obtained using polychromatic radiation. The transversal coherence length provided by the source is the limiting factor in the design of a PCI system. As specific examples, we considered imaging of two materials, Tissue and Aluminum, with 60 keV photons, and we determined that the coherence length required to perform PCI in those cases is not easily provided with normal sources, such as standard X-ray tubes. We will explore alternative methods to use a standard X-ray tube for phase contrast imaging in Chapter 5.

PCI edge enhancement has been obtained in literature using monochromatic low-energy X-rays provided by synchrotron sources, with typical a focal spot size of 50 μm. High transversal coherence lengths are obtained by extremely long sample-to-detector distances, typically between 40 m and 100 m. The logistical hurdles involved in setting up large spaces between source and sample, the problems involved in providing a synchrotron source, and the high dose delivered by low energy radiation makes these systems unsuitable for medical or industrial applications.

The use of polychromatic microfocus X-ray tubes has been proposed, and the results presented in literature seem encouraging. Nevertheless, a survey of these systems (presented in Chapter 6) shows that they generally fall short of the required $\xi_{\text{min}}$, and are therefore not capable of delivering PCI. Despite this shortcoming, an edge enhancement effect is observed. In Chapter 3, we advanced the interpretation that this enhancement is due to Compton Filtering (CF).

In this chapter we seek experimental validation for our theory. To do so, we built an imaging system capable of providing edge enhancement, with a transversal coherence length $\xi_t < \xi_{\text{min}}$. In Chapter 3 we defined such a system as a low-coherence system. Based
on results in literature, we expect to see an edge enhancement effect in images taken off-focus relative to images taken at contact. To explore the nature of the edge enhancement, we then shorten the transversal coherence length $\xi_t$ of the system to a value similar to that offered in standard radiography, while at the same time preserving the free space propagation geometry. We evaluate the edge enhancement for this incoherent system, and compare it to the edge enhancement obtained with the low coherence system. Finally, we will evaluate the edge enhancement as a function of the sample-to-detector distance and of the shape and features of the sample. This information will allow us to correctly ascribe the detected edge enhancement to the proper phenomenon.

Our set-up does not try to replicate any particular set-up described in literature. We built a low-coherence system with $1 \mu m < \xi_t < 5 \mu m$. As proposed in Chapter 3, we define an imaging system as low-coherent if $1 \mu m < \xi_t < \xi_{\text{min}}$. The system can also be operated as an incoherent system, with a transversal coherence length of a standard imaging system, that is with $\xi_t < 50 \text{ nm}$.

In order to perform experiments at two different transversal coherence lengths, we built two pinholes to be used with a standard X-ray tube. The resulting set-up uses only off-the-shelf equipment, and the distances used for the positioning of the sample and the detector are easily reproducible in a hospital or industrial setting.

In a preliminary experiment, multiple images of a cell phone and of a seashell were taken. A set of two images at different sample-to-detector distances $R$ were taken for each object: at contact ($R < 5 \text{ mm}$), where PCI effects would not be possible; and at a distance. The distance was chosen to match the typical sample-to-detector distance observed in microfocus X-ray tube PCI literature, that is between 20 cm and 1 m.

After replicating in the preliminary experiment observations in literature of edge enhancement with a low-coherence system, we proceed to perform a more quantitative experiment to understand the nature of the enhancement. For this experiment, we choose to image only the cell phone, for its richly detailed structure. We operate at a different voltage in order to achieve shorter exposure times than those typically used in standard imaging practice.

A set of images of the cell phone was taken at three sample-to-detector distances. Each set of images was taken twice, once with the system set as low-coherence and once
with the system set as incoherent. A comparison between the two images allowed us to draw conclusions concerning the nature of the edge enhancement.

Our results support our theory that the bulk of the edge enhancement observed in low-coherence systems is not due to PCI. In the remainder of this chapter we will give a more accurate description of the equipment used to perform the experiments. We will then discuss in detail the set-up of the experiments and the results.

**Available instrumentation**

In this section the interested reader can find information about the equipment used to perform the experiments described in the following section.

**Source**

An X-ray tube provided by American Science and Engineering and originally designed for homeland security applications was used. The tube has a Tungsten anode of unknown size. The X-ray tube window is a circular opening 2 cm in diameter, to which a lead collimator is attached. The collimator is composed of two parts, a first part 3.5 cm long with bore size of 1 cm, and a second part 2.5 cm long with bore size of 2 cm. Lead beam filters of 0.5 cm, 0.25 cm, 0.12 cm are provided.

The range of operation of this tube is very broad. It can reach a voltage of 250 kVp and deliver a current up to 13.5 mA. We measured the spectrum of the tube at two different voltages and with a lead filter 0.25 cm thick. The results are presented in Figure 1. For the 100 kVp curve, the filter has the effect of sharpening the spectrum. Unfortunately, the total number of counts is also reduced. In this measurement, the total number of counts (i.e. the integral under the peaks) for the 100 kVp case is 61230 counts, against 12260 counts for the filtered case, for a five-fold loss in flux. The use of a filtered tube for experiment can be advantageous in term of fractional bandwidth of the system, but it would drastically reduce the available flux.
Figure 1: Spectra of photon energy for operating voltages at 100 kVp and 225 kVp. The 100 kVp experiment was repeated with a lead filter 0.25 cm thick. The Germanium detector was calibrated with Co-57 and Cs-137 sources before the data collection. A benchmark Cs-137 (energy of 661.62 keV, out of the range showed in figure) was used during data collection and to normalize the number of counts.

Pinholes

To control the transversal coherence length $\xi_d$ of the imaging system, the radiation from the X-ray tube is restricted with two pinholes. One pinhole is a 2 mm hole drilled in the MIT Reactor Laboratory machine shop into a rectangular piece of lead 3 mm thick and 1 cm on a side. This pinhole was taped with double sided tape to the collimator on the side facing the X-ray tube window. The second pinhole is a 25 $\mu$m tapered hole in a Platinum disk 500 $\mu$m thick and 5 mm in radius, and was acquired from Ernst F. Fullam, Inc. This second pinhole was taped with standard tape to the lead pinhole.

The transmission of the Platinum disk is high, due to its small thickness. We calculated the transmission factor for various energies (Table 1); the expected transmission for 60 keV photons is on the order of $10^{-3}$. As a reference, the 3 mm lead
A pinhole presents a transmission factor of $10^{-9}$. As a result, a high noise level in the image is to be expected; no artifacts are introduced by the transmission. It is anyway possible to perform a comparative analysis of the images taken at contact and at a distance with this pinhole, since the noise affects both cases equally. Nevertheless, a more opaque pinhole disk would be desirable for pure imaging purposes. This is difficult to achieve since the thickness of the disk cannot be increased without introducing undesired collimating effects. An alternative option is to select a high X-ray tube voltage in order to have a mean photon energy of 80 keV, thus exploiting the K-edge of Platinum.

<table>
<thead>
<tr>
<th>E (keV)</th>
<th>$I/I_0$</th>
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</thead>
<tbody>
<tr>
<td>40</td>
<td>1.68E-06</td>
</tr>
<tr>
<td>50</td>
<td>5.79E-04</td>
</tr>
<tr>
<td>60</td>
<td>9.52E-03</td>
</tr>
<tr>
<td>78.39</td>
<td>9.45E-02</td>
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<td>8.58E-05</td>
</tr>
<tr>
<td>100</td>
<td>4.74E-03</td>
</tr>
</tbody>
</table>

**Table 1:** Transmission factor $I/I_0$ through the Platinum pinhole as a function of the radiation energy.

**Detectors**

The images presented in this chapter were recorded on two Fujifilm Phosphor Imaging Plates. The plates make use of the photostimulated luminescence (PSL) effect, a property of some phosphors which emit light if two excitations, the second at a higher wavelength than the first, occur. During the irradiation of a PSL phosphor, a valence electron is excited and trapped in the conduction band. If the phosphor is subsequently illuminated with a laser, the energy will facilitate the decay of the trapped electron in the valence band, causing emission of light. The plate is used during the experiment as an X-ray film, and it stores the information until it is scanned with a dedicated reader. An erasing step allows the plate to be reused multiple times. The operation of the imaging plate is exemplified in Figure 2, along with a schematic representation of the PSL phenomenon.
Fujifilm Imaging Plates have the advantage of being very sensitive and having a remarkable linear response to irradiation. They present an extremely wide dynamic range of $10^5$ and can achieve 50 μm resolution. The disadvantage lies in the need for scanning the plate after the experiment, a process that is impractical and can take up to a few minutes. Two imaging plates marked as ND were used for this experiment. The ND nomenclature indicates that the plates are covered with a Gd$_2$O$_3$ converter for neutron detection. This characteristic does not diminish the X-ray sensitivity of the plates. The Imaging Plates have an active area of 20 cm x 40 cm.

A Thales Flashscan-33, an amorphous silicon detector with a resolution of 127 μm and real time imaging capabilities, was also used during the experiment for the purpose of aligning the sample in the beam.

**Geometry**

The experiment was performed in a shielded room in the MIT Bitter Magnet Laboratory building. The geometry of the set-up is straightforward and is depicted in Figure 3. The pinhole is positioned as close as possible to the anode of the X-ray tube. The first position of the detector is at contact with the sample. In the preliminary
experiment, only a second position for the detector was used, whereas three positions were used for the main experiment.

![Experimental Set-up Diagram](image)

**Figure 3:** Top view and side view of the experimental set-up.

The axis of the beam is not level to the floor (Figure 3). Distances are measured following the axis of the beam. The sample and the detector are parallel to each other but are not perpendicular to the beam. The angle between beam and detector is small enough that no distortion is visible in the images.
Experiments and Results

Preliminary experiment

This preliminary experiment was performed with the purpose of reproducing the results obtained in PCI literature with a microfocus X-ray tube. The system offers a transversal coherence length at the sample similar to the one offered in literature, which is between $\xi_t = 1 \, \mu\text{m}$ and $\xi_t = 5 \, \mu\text{m}$. We use a pinhole of size $w = 25 \, \mu\text{m}$, with a distance between source and sample $R_{QO} = 300 \, \text{cm}$, thus ensuring a transversal coherence length of $3 \, \text{m}$.

Figure 4 shows two images of a seashell, taken at contact and at a sample to-detector distance of $144 \, \text{cm}$. Both images, taken at $50 \, \text{kVp}$ and $13 \, \text{mA}$, had an exposure time of $60 \, \text{minutes per image}$. For a Calcium based sample, using the parameters of this experiment, we calculate based on the model presented in Chapter 3 a $\xi_{min} = 3.1 \, \mu\text{m}$ and a $\xi_{opt} = 5.65 \, \mu\text{m}$. The figure of merit $K^{PCI} = 0.07$, while $K^{CF} = 0.132$. Our derivation predicts that PCI has a negligible contribution to the image, and CF has a small contribution. The image at a distance has been rescaled to match the size of the image at contact. It is unclear from Figure 4 that an edge enhancement effect has taken place in the image at a distance. A comparison of the plot profiles of a section of the two images show that a slight edge enhancement has occurred at a distance, but the results are not conclusive. The edge enhancement appears in the plot profile as a steeper line at the place where an edge occurs.

Figure 5 shows the image of a cell phone. The image was taken at the same time as the image of the shell, and therefore all the experimental parameters are the same. Assuming that the feature of the cell phone are mainly made of Aluminum, we calculate $\xi_{min} = 2.8 \, \mu\text{m}$ and $\xi_{opt} = 5.5 \, \mu\text{m}$. The figure of merit $K^{PCI} = 0.15$, while $K^{CF} = 0.65$. Also in this case, we expect CF to be prominent compared to PCI, and both effects to be small compared to absorption.
Figure 4: (a) A standard radiography image of a sea shell; (b) Image taken with free space propagation, \( R = 144 \text{cm} \); (c) Plot profile of a section of the shell, marked with a white dotted line in a-b. 60 min exposure per image, 50kVp 13mA, \( R_{\text{FO}} = 300 \text{ cm} \).

Again, the image at distance has been rescaled to match the size of the image at contact. In this case, we can notice an enhancement in the image taken at a distance. This increase in quality is achieved despite the loss of focus. More details of the circuitry of the cell phone are visible in the center of the image, and the edges appear to be sharper.

Figure 6 shows the intensity profile of the microphone of the cell phone. The detail has been chosen for its visibility in both images. In the profile, it is clear that the edges between the rings are sharper in the image at a distance, in that the intensity profile change value more abruptly. The different behavior observed in the edge enhancements for the sea shell and the cell phone is consistent with out expectation based on the figure-of-merit \( K \).
This preliminary experiment succeeded in showing an edge enhancement for systems with low transversal coherence length when free space propagation is used. We noticed a difference in behavior concerning this enhancement depending on the sample. This is due to different values of $K$ for the two cases. We also believe that part of the difference in enhancement can be ascribed to the level of details in the sample, that produce a damping effect on the estimated $K^{CF}$. In the cell phone image the details, clustered closely together, produce a fogging due to Compton scattering which clouds the visibility of nearby details. A scattering reduction technique, such as CF, can drastically improve the quality of the image. By comparison, a sea shell does not have the same level of details and the fogging of an edge due to nearby structure is less prominent. For this reason, scattering reduction techniques will have less of an impact on this image.
Finally, this preliminary experiment was useful to establish that an edge enhancement can be produced, and in raising questions on the nature of the enhancement. In view of the discussion just presented about CF, we deemed useful to repeat the experiment on the cell phone as a control for this experiment and to produce better quality images that can give a better insight on the enhancement process.

**Main experiment**

We repeat the preliminary experiment on the cell phone. We tested the free space propagation geometry at two distances, $R = 45$ cm and $R = 100$ cm. Moreover, while our preliminary experiment required long exposure times due to the lack of signal from the X-ray tube, in this experiment we are set to obtain a stronger signal thanks to a higher voltage in the tube. Working at 70kVp, with no lead filters in place, we were able to drastically reduce the exposure times to 5 minutes and to obtain much sharper images.

The transversal coherence length of this system is reduced compared to the one of the preliminary experiment. In this case, $\xi_t = 2.1 \, \mu m$. At this energy, $\xi_{min} = 3.75 \, \mu m$ and
opt = 6.8 μm. The figure of merit $K^{PCI} = 0.053$, while $K^{CF} = 1.52$. This system is the perfect example of a low-coherence system. Its transversal coherence is sufficiently large as to be considered good in literature, but the actual PCI capabilities of the system are negligible.

Due to the increase in quality of the image we can now notice the off-focus effect for the images taken at a distance. The quality of the contact image is already very good, and for this reason the enhancement in the cell phone image is not as immediately noticeable. Figure 7 shows the images of the cell phone. The sample was not moved in-between images, and the only operation done on the data is a rescaling of image (b) and (c) to compensate for magnification.

Figure 7: (a) Cell phone image at contact; (b) Cell phone image at $R = 45$ cm; (c) Cell phone image at $R = 100$ cm. 25μm pinhole, 5 min exposure per image, 70kVp 13.5mA, $R_{QD} = 300$ cm.
Although the off-focus effect gives the impression of a worsening of the image as the sample-to-detector distance $R$ increases, an analysis of the intensity profile (Figure 8) seems to suggest that an edge enhancement is obtained at a distance.

To evaluate the enhancement quantitatively, we can calculate an approximation of the first derivative of the intensity profile across the edges, and compare the value obtained at different distances. We choose 4 edges in Figure 8, at pixel 35, 85, 135 and 290. For each edge and for each distance, we calculated the ratio between the difference in intensity across the edge and the size of the edge in pixel$^{12}$. We then obtain the enhancement for an edge as the ratio of the derivative at a distance and the derivative at contact. We obtain that across the four edges, the average gain for $R = 45$ cm is 1.8, while the average gain for $R = 100$ cm is 1.7. These figures are close enough to be considered equivalent considering our measuring methodology.

We have thus proven that an edge enhancement is produced through free space propagation geometry. The PCI nature of the enhancement is put into question by the fact that no additional gain seems to be obtained by increasing the sample-to-detector distance $R$ from 45 cm to 100 cm, while PCI edge enhancement is proportional to $R$. This tapering of the edge enhancement with distance is compatible with the behavior of CF discussed in Chapter 3.

The analysis of the above results suggests that CF is responsible for the observed edge enhancement observed in the low-coherence system. We seek a validation of our hypothesis by repeating the experiment with an incoherent system. PCI enhancement would be strongly affected by a decrease in transversal coherence length, while scattering reduction enhancement would not be affected.

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$^{12}$ The process is admittedly somewhat arbitrary, since it there is no clear definition of the point where an edge start and end. We choose to focus on four edges whose borders we were able to distinguish with little ambiguity.
The set-up for this second experiment is identical to the previous one, with the exception of the size of the pinhole, which is \( w = 2 \text{ mm} \) (\( \xi_x = 0.03 \mu \text{m}, K^{PCI} = 0, K^{CF} = 1.52 \)). For all purposes, we can consider this system as incoherent. The increased size of the pinhole also made more flux available for imaging, thus allowing a further reduction in exposure times to 2 minutes.

Figure 9 shows the three images of the cell phone obtained with an incoherent system. A slightly different sample-to-detector distance (\( R = 40 \text{ cm} \) instead of 45 cm) has been used for one of the images at a distance. This difference does not affect the analysis of this experiment. The same cell phone detail studied for the previous case has been examined, and the results are presented in Figure 10. Again, we notice an edge enhancement effect occurring for the images at a distance, with no noticeable difference between the image taken at a sample-to-detector distance \( R = 40 \text{ cm} \) and \( R = 100 \text{ cm} \).
The same peaks studied for the previous experiment have been studied here. We observe in this case an average gain at $R = 40$ cm of 1.8, and at $R = 100$ cm of 1.9. Considering the ambiguity in the measurement methodology of the derivative of a peak, these two figures are to be considered equivalent. Moreover, the gain observed in the free space propagation experiment performed with an incoherent system is the same than the one observed with a low-coherence system.

The results of this study show that the same qualitative and quantitative enhancement can be obtained through free space propagation techniques with incoherent systems and with low-coherence systems. It also shows that the enhancement does not linearly increase with sample-to-detector distance. For the distances investigated in this study, no enhancement has been observed between $R = 40$ cm and $R = 100$ cm.
Figure 10: (a) Intensity profile for details of cell phone at three sample-to-detector distances; (b) Detail at contact; (c) Detail at R = 40 cm; (d) Detail at R = 100 cm. 2 mm pinhole, 2 min exposure per image, 70 kVp 13.5 mA, R_{oo} = 300 cm.

These experimental results agree with the models of PCI and CF developed in Chapter 3 and presented here. Also, we observe a tapering off of the CF enhancement effect at R = 40 cm, as expected for CF (Figure 3.9 in Chapter 3).

We believe that through theoretical analysis in Chapter 3 and through this experimental work we strongly supported our contention that edge enhancement in free space propagation low-coherence set-ups is not due to PCI but to CF. We believe that this result extend beyond the pinhole-based low-coherence systems, and include efforts in literature to perform phase contrast using microfocus X-ray tubes.

To further validate our theory we investigated a portion of the object where CF does not play a central role. If the object to be imaged is sparse, Compton fogging between adjacent details will be minimal. Our choice of a cell phone was dictated by the fact that the quantity and density of details lent itself to a study about CF and PCI. Interestingly, there is one detail in the image of the cell phone which is removed from the bulk of the image and thus is less affected by Compton fogging.
The image taken with the 25 µm set-up of the antenna of the cell phone is analyzed in Figure 11. The antenna, not being near the bulk of the cell phone, is a good example of a detail for which CF would not provide a noticeable improvement. On the other hand, if PCI enhancement is present, it will not be affected by the placement of the antenna.

Figure 11: (a) Detail at contact; (b) Detail at 45cm distance; (c) Detail at 100cm distance; (d) Intensity profile for details of cell phone at three sample-to-detector distances. 25 µm pinhole, 5 min exposure per image, 70kVp 13.5mA, \( R_{QD} = 300 \) cm.

In Figure 11, we observe that no edge enhancement is visible in the intensity profile of the antenna. A study of the gain shows no meaningful advantage at a distance. For completeness, the same study has been performed on the image taken with the 2 mm pinhole set-up (Figure 12). Also in this case, no edge enhancement is observed\(^{13} \).

\(^{13}\) The last data point in the \( R = 40 \) cm intensity profile is spurious and it is due to an imperfection occurred during the scanning of the imaging plate. The hair-like corruption is visible in Figure 12b.
Figure 12: (a) Detail at contact; (b) Detail at \( R = 40 \) cm; (c) Detail at \( R = 100 \) cm; (d) Intensity profile for details of cell phone at three sample-to-detector distances. 2 mm pinhole, 2 min exposure per image, 70kVp 13.5mA, \( R_{OO} = 300 \) cm.

**Conclusions and future work**

In conclusion, the results from the experiment with the incoherent system reinforce our argument that free space propagation techniques provide an edge enhancement that is not due to PCI. We notice that the enhancement obtained this way have a non linear dependency with the sample-to-detector distance, as expected following the theory in Chapter 3. The experiment performed with a low-coherent system shows that the behavior of the system is essentially identical to the behavior of an incoherent system, thus reinforcing our contention that PCI enhancement cannot be observed with this kind of systems. We identify this improvement as CF. This result dispute the classification of
low-coherence systems as PCI, contrary to standard practice in literature. In the Chapter 6, we will survey low-coherence systems in literature.

Future work on this topic is a further experimental evaluation of the dependency of CF with distance, via multiple experiments at short sample-to-detector distances, that is where CF experiences the most variability. A general problem of this kind of evaluation is that the effect is highly dependent on the shape and level of details of the sample, and it is not obvious how to generalize results for all classes of objects. Also, we focused in this thesis on free-space propagation techniques to perform CF edge enhancement. A common way of obtaining discrimination against Compton scattering in imaging is to position an absorbent grid, called Bucky grid, between sample and detector, in order to eliminate photons coming from outside a fixed acceptance angle. It is interesting to evaluate the relative effectiveness of CF and grid filtering. We performed an experiment with a Bucky grid that did not provide conclusive data due to the difficulty in obtaining precise motion of the parallel grid\(^1\). Although it seems that in our experiment CF more efficient system to provide scattering reduction than grid filtering, we do believe that a system using a grid with the required precision can be as efficient.

Finally, experiments comparing results obtained with a low-coherence or incoherent system with results obtained with a high coherence system (\(\xi \approx \xi_{\text{opt}}\)) are in order. It is currently a challenge to produce an imaging system with the required coherence length without resorting to synchrotron radiation facilities. We will discuss options to produce more practical high coherence sources in Chapter 5.

**References**


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\(^1\) If the grid is composed of parallel slabs of absorbent material, a continuous transversal movement has to be imparted to the grid in order to obtain an image which does not contain the grid itself. The grid should travel uniformly during the exposure time, over a path equal to its period. In standard imaging practice, the grid is vibrated, which can in some case lead to vibration in the sample or the detector thus corrupting the image. In our experiment, we moved the grid of a random length at regular intervals.
5. Coded Source Imaging

Introduction

The most stringent limitation for Phase Contrast Imaging (PCI) is in the transversal coherence length $\xi_b$, which is required to be above a minimum value $\xi_{min}$ in order to obtain a statistically meaningful probability of having a coherent scattering event contributing to PCI. Three parameters determine the coherence length: the size of the radiation source $w$, the distance between source and sample $R_{QQ}$ and the wavelength of the radiation $\lambda$. The relationship between these parameters and the transversal coherence length $\xi_t$ was established in Chapter 2 and discussed further in Chapter 3. Specifically, the transversal coherence length $\xi_t$ is linearly proportional to $R_{QQ}$ and $\lambda$, and inversely proportional to $w$.

The source-to-sample distance $R_{QQ}$ is a parameter that can be changed within the constraints. In most settings, the space is limited. Hospitals do not in general offer large spaces for diagnostic X-rays, and such a requirement is considered an added cost. In laboratory, it may be possible to organize longer spaces, but still with the limiting factor of the size of the laboratory room and any radiation protection requirements. If Neutron PCI using a reactor source is considered, the available space is strictly limited by the size of the containment. Furthermore, with the use of common sources such as a standard X-ray tube or a nuclear reactor beamline, only extremely large distances can provide the transversal coherence length $\xi_t$ required for PCI without a modification of the source.

For a given radiation source, it is very difficult to select a specific wavelength $\lambda$ or to provide wavelengths not present in the original spectrum. As for the size of the focal spot $w$, it is easy to obtain a smaller source by restricting the radiation window using proper shielding material. A source as large as a few centimeters can be used to provide a focal spot of a few micrometers by the use of a pinhole, that is a small hole drilled in an
absorbent material. This is the most immediate way of increasing the transversal coherence of a standard imaging system.

With such a method, one can trade a higher coherence for a reduction of the available flux. To understand the magnitude of this reduction, let us consider a reactor beamline used for Neutron imaging providing a focal spot \( w = 1 \text{ cm} \) in radius and operating with a 1 minute exposure time. To upgrade this facility to PCI, let us assume we need a focal spot \( w = 50 \mu\text{m} \) to provide the required coherence length \( \xi_{\text{min}} \). With a focal spot of this size, we are in effect using only \( 2.5 \times 10^{-5} \) of the available Neutrons. The exposure time to obtain the same image is lengthened to months. Clearly, such an exposure time is unacceptable.

Let us now consider an X-ray system for medical applications, small focal spot of \( w = 200 \mu\text{m} \) delivering an image in 3 second exposures. We want to restrict the focal spot to \( w = 10 \mu\text{m} \) in order to obtain PCI. The flux is \( 2.5 \times 10^{-3} \) times the original flux. The exposure time granting the same statistics as a standard radiography system is 120 minutes. This figure is considered unacceptable for routine diagnostic exams such as mammograms\(^{15}\).

PCI has proven to be a promising technique to achieve high resolution, thanks to the edge enhancement effect associated with interference phenomena. As shown in the examples presented above, it is difficult to use this technique in standard medical and engineering practice due to the severe limitations arising from the coherence requirement. This is especially true for Neutron applications due to the low brightness of sources. In this chapter, we propose methods to modify or build radiation sources that will meet the coherence requirement of PCI while at the same time preserving the radiation flux. All our suggestions fall under the umbrella category of Coded Source Imaging (CSI) techniques: imaging using a source that has a large spatial extent and fine features, such as a collection of pinholes arranged in a particular pattern, or a sequence of slits.

The idea behind CSI is to exchange the pinhole source with another, brighter one. The goal is to improve the imager by increasing the ratio of the signal to the noise. A simple way to visualize CSI is to imagine substituting the single pinhole approach

\(^{15}\) The comparison is not totally accurate, since the phase contrast system would provide much higher resolution than the standard radiography system thanks to the very small focal spot.
described above with an N pinhole system. If the N pinholes are positioned such that the N images produced do not overlap, we can take the N different images and add them to generate a single image. It is straightforward to see that the statistics of the resulting image are equivalent to those of an image from a single pinhole taken with an exposure time N times longer. If we assume that the N pinholes cover half the size of the original source, we obtain a PCI system operating at exposure times comparable to a standard radiography system.

Unfortunately, such a gain is not realistic. In all practical cases, the images cast from each pinhole will partially overlap. In the overlap regions, it is impossible to distinguish the contribution of each pinhole to the total count. This situation results in two difficulties: the detected image must be reconstructed in a more involved way than just adding the different images and the statistical improvement due to the presence of multiple images is reduced by the fact that information about the origin of each count is lost in the overlapping.

We start this chapter by treating CSI as a pure imaging problem, without consideration for its use in PCI. We will first derive the mathematical foundation of the CSI formation process. We then prove that perfect reconstruction in an ideal case is possible. We will give the relation between mask pattern and decoding algorithm. Other considerations about the imaging capabilities of a CSI system will be presented [1]. It is important to notice that this work has broader applications than obtaining bright coherent sources for PCI. Under some conditions, standard radiography can benefit from the use of encoded sources.

We will then consider in more detail the implications of using a CSI system to perform PCI. We will delve into four practical ways of using a CSI system for PCI: grating-based PCI, N-pinhole systems, Fluorescent Coded Sources and Advanced Electron Beam Encoded X-ray tubes.

**Coded Source Imaging**

Spatially coded sources first appear in literature with reference to X-ray imaging [2]. Coded source X-ray tube design arises from the need for high resolution. Conventionally,
the ways to increase resolution in transmission imaging are either to decrease the focal-spot size or to increase the source-to-object distance. These measures are not very effective or feasible, since the exposure time becomes too long: in medical applications, this translates into blurring from patient movement, whereas in materials science this may result in an unbearable economic cost. A smaller focal spot is indeed a way of obtaining higher frequencies, but it is not the only way. If we do not consider simple spots but detailed structures, it is possible to obtain high spatial frequencies despite the large physical dimension of the source. The way to do this is to build a coded source that consist of a distribution of pinholes following a pattern whose Fourier representation has high frequencies.

The idea of using a coded source had some momentum in the 1970s, and then was discarded in favor of more conventional methods. Both the increasing quality of the available X-ray tube, allowing high enough resolution with standard techniques, and the lack of fast computers to perform the reconstruction of the encoded image in a reasonable time explains the loss of interest for this idea. The success obtained by coded apertures, a method similar to coded source, and the huge increase in computing power in recent years justify a renewed interest in this field.

**Encoding and Decoding**

An encoded image is obtained through CSI. The description of the encoding of the image formation process is straightforward. With reference to Figure 1, the recorded image on the detector can be written as:

\[
R(x_r) = \int S(x_s) O(x_o) dx_s = \int S(x_s) O(x_r - \frac{x_s - x_o}{a + b}) dx_s
\]  

(1)

where \(S\) is the function describing the coded source, \(O\) is the function describing the transmission of the object, \(x_s\) is the running variable along the source plane, \(x_r\) is the running variable along the detector plane, and \(x_o\) is the running variable along the object plane. In Equation (1) the simple geometrical relation
\[ x_0 = x_r - \frac{x_r - x_s}{a+b} b \]  

(2)

was used.

The recorded image can be written in a more useful form by the change of variable:

\[ \dot{x}_s = -\frac{b}{a} x_s; \]  

(3)

\[ R(x_r) \propto \int S\left(-\frac{a}{b} x_s\right) O\left(\frac{a}{a+b} (x_r - x_s)\right) dx_s. \]  

(4)

This form is immediately recognizable as a correlation, giving the set of equations that are cardinal to the encoding process:

\[ R(x_r) \propto S^m_\omega * O^{m_2} \]  

(5)

\[ S^m_\omega (\chi) = S\left(-\frac{a}{b} \chi\right) \]  

(6)

\[ O^{m_2} (\chi) = O\left(\frac{a}{a+b} \chi\right). \]  

(7)
The recorded image is the correlation of the coded source and of the object. It is tempting to propose a decoding technique involving the fact that the Fourier transform of a correlation is the product of the Fourier transform of the two terms.

\[ \hat{O} = F^{-1}\left(\frac{F(R)}{F(S)}\right) \]  \hspace{1cm} (8)

Unfortunately, the presence of noise in the real data makes this technique impractical. In the real case, Equation (8) would read

\[ \hat{O} = F^{-1}\left(\frac{F(R)}{F(S)}\right) - F^{-1}\left(\frac{F(N)}{F(S)}\right) \]  \hspace{1cm} (9)

where \( F(N) \) is constant at all frequencies, whereas \( F(S) \) typically has zeros. The noise term will then dominate the reconstructed image.

A better strategy for decoding is to use a decoding pattern \( G \).

\[ \hat{O} = (S^n_s \ast O^n_m) \times G = O^n_m \ast (S^n_s \times G) = O^n_m \ast PSF. \]  \hspace{1cm} (10)
In Equation (10) the recorded image is convoluted with the decoding pattern. The point spread function (PSF) is the response of the imager to a point-like object. It is easy to notice how the designer is free to set the PSF by choosing a couple $(S, G)$ so that:

$$S^m \times G = \delta.$$  

(11)

This is the condition for CSI to achieve perfect reconstruction. The reader will notice that in the presence of noise, Equation (11) reads:

$$\hat{O} = (S^m \ast O^n + N) \times G = O^n \ast (S^m \times G) + N \times G,$$  

where $N \times G$ is the convolution of a constant with a function, which is a constant no matter what the function is.

**Signal-to-Noise Ratio advantage over a single pinhole.**

The rationale for using a coded source in an imager is to improve the count statistics of the system. In this context, an accepted figure of merit is the ratio between throughput signal, i.e. the image of the object, and the noise associated with it. The ratio of the two gives a good indication of the quality of the imager and is called the Signal-to-Noise Ratio (SNR).

It is useful at this point to evaluate the SNR of a single pinhole and compare it to the SNR of a multiple pinhole system. Assuming ideal behavior of the detector and of the electronics associated with it, the source of noise is the radiation source itself. Independently of the kind of radiation being emitted, it is reasonable to assume that the emission process follows a Poisson distribution. Thus, for the properties of the distribution, if $s$ is the transmitted signal, $s$ is also its variance, and the noise associated to it is $\sqrt{s}$. The resulting SNR is $\sqrt{s}$.

The same calculation can be easily performed for coded sources, with a limiting assumption. In the presence of $N$ pinholes, the total signal is $N \cdot s$. Suppose that each pinhole casts an image of the object that does not overlap with the images cast by the
other pinholes. Under this assumption, it is possible to distinguish on the detector the radiation coming from each of the sources, and each image is independent of the others. The variance of a sum of independent sources is the sum of the variances, i.e. \( N \cdot s \), resulting in a noise term equal to \( \sqrt{N \cdot s} \). The resulting SNR is \( \sqrt{N \cdot s} \).

This simplistic argument (Table 1), shows a SNR advantage of the coded source over a single pinhole of \( \sqrt{N \cdot s} \). This result suggests that the higher the number of pinholes in a coded source, the better the count statistics will be. The problem with this line of reasoning is that a higher \( N \) reduces the validity of the assumption that the counts coming from different pinholes are independent. Also, this assumption cannot be a requirement for the design of a coded source, since a non-overlapping property will depend not only on the coded source pattern but also on the geometry of the object.

<table>
<thead>
<tr>
<th></th>
<th>Pinhole</th>
<th>N Pinholes</th>
<th>Coded Sources</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Signal</strong></td>
<td>( S )</td>
<td>( N \cdot S )</td>
<td>(&lt; N \cdot S )</td>
</tr>
<tr>
<td><strong>SNR</strong></td>
<td>( \sqrt{S} )</td>
<td>( \sqrt{N \cdot S} )</td>
<td>( \sqrt{N \cdot S} )</td>
</tr>
<tr>
<td><strong>SNR/Pinhole</strong></td>
<td>1</td>
<td>( \sqrt{N} )</td>
<td>( 1 &lt; \text{SNR} &lt; \sqrt{N} )</td>
</tr>
</tbody>
</table>

**Table 1:** SNR advantage for a single pinhole, for an ideal system of \( N \) pinholes and for a real coded source with \( N \) pinholes.

The question of what happens to the SNR advantage for a real coded source will be addressed later in this section. Suffice it here to say that the answer depends on the pattern of the coded sources, on its open fraction, and on the characteristics of the object being imaged.
Field-of-View and Resolution

Figure 2 demonstrates the geometry needed to calculate the Field-of-View (FoV) of a CSI imager. The calculation is straightforward and requires only simple geometrical considerations. The relationship between the FoV and the geometry of the imager system is

\[ l_1 : d_m = (a - l_1) : \text{FoV}, \]  

(13)

where \( l_1 \) is the focal distance to the source, \( d_m \) is the size of the coded source mask, and \( a \) is the source-to-detector distance. Considering also that

\[ l_1 : d_m = l_2 : d_d, \]  

(14)

with some algebra we obtain

\[ \text{FoV} = \frac{a \cdot d_d - b \cdot d_m}{a + b}. \]  

(15)

A simple geometrical approach is not sufficient to estimate the resolution of a CSI imager. Again, an analogy with the single pinhole case gives insight into the problem. In the pinhole case, the resolution is set by the size of the pinhole. A point-like object casts on the detector a shadow the size of the magnified pinhole. If the shadows of two point-like objects overlap, resolution is corrupted and the image blurred. More specifically, it is convenient to describe the size of the shadow of the pinhole on the detector in a statistical sense as the full width at half maximum (FWHM).
In the case of a coded source, it is necessary to establish a relationship between the shape of each pinhole and the projection of a point-like object on the detector. It is convenient to write the source $S$ in terms of the shape $h$ of each hole and of the pattern $S_d$.

\[ S(x_s) = \int S_d(x_s - x) \cdot h(x) \, dx \]  

(16)

For a point-like object, using Equations (6) and (7), we rewrite Equation (4) as:

\[ R(x_t) \propto S_{\delta}^m \ast \delta = S_{\delta}^m (x_t), \]  

(17)

and decoding according to Equations (10) and (11), we obtain:

\[ \hat{O}(x) \propto \int S_{\delta}^{m_m} (x_s - \xi) \cdot h^m (\xi) \cdot G(x + x_s) \cdot dx_s \cdot d\xi = \]

\[ = \int h^m (\xi) \cdot d\xi \int S_{\delta}^{m_m} (x_s - \xi) \cdot G(x + x_s) \cdot dx_s = \]

\[ = \int h^m (\xi) \cdot \delta(\xi + x) d\xi = h^m (-x) = h(-\frac{x}{b}) \]  

(18)
This is the proof that in CSI as well as in pinhole imaging, the size of the hole is the factor that establishes the resolution of the system. In the present case, assuming a circular hole with diameter $d_h$, the relation between the geometrical parameters and the resolution is

$$\lambda_g \cdot a = \frac{a}{b} \cdot d_h \cdot (a + b)$$  \hspace{1cm} (19)

This results in the expression:

$$\lambda_g = \frac{a^2}{ab + b^2} \cdot d_h.$$  \hspace{1cm} (20)

It is convenient to combine Equations (15) and (19), since it is in general possible to change the geometry in order to change the resolution and the Field-of-View.

$$\frac{FoV}{\lambda_g} = \frac{b}{a^2} \cdot a \cdot d_m - b \cdot d_h.$$  \hspace{1cm} (21)

**Coded Sources and Coded Apertures**

CSI is described by Equation (5). The decoding strategy is summarized in Equation (10), and the basic requirement for the design of a source mask and the associated decoding pattern is given in Equation (11). To some readers, these equations will look extremely familiar. Formally, the founding equations of CSI have been written to match the founding equations of Coded Aperture Imaging (CAI) [3].

CAI has a long tradition in astronomy, and more recently has been successfully used in near-field problems such as small animal imaging and medical applications. Being an emission imaging system, CAI relies on the encoding of a source (the object) via a mask (the coded aperture). Compared to CSI, the object and source exchange position.

Coded apertures have been introduced in emission imaging for the same reason coded sources are being proposed in transmission imaging, i.e. to obtain a SNR
advantage to achieve better resolution or shorter exposure. SNR and artifact reduction have been studied for CAI and results exist in literature [4, 5]. Those results can be ported to CSI thanks to the formal equivalence of the two techniques.

A short qualitative discussion of these two topics follows; a more rigorous analysis is beyond the scope of this thesis.

**SNR Advantage Artifacts**

The previous discussion on SNR pointed out that care should be put in the choice of the source pattern. Not only must it correlate with a suitable decoding pattern to result in a delta function in order to have perfect reconstruction according to Equation (11), but it must also have a pattern that ensures a sensitivity advantage of the system compared to the state-of-the-art. The signal-to-noise ratio must be studied in detail before choosing a specific design.

The main purpose of using a coded source is to increase the signal-to-noise ratio (SNR) of the imaging system. The SNR calculation depends on the coded source pattern and on the characteristics of object and of the background. Similar calculations performed for CAI [3] suggest that, for objects with an opaque background and slit or pinhole features that have enhanced transmission, many types of coded sources have a marked SNR advantage over the pinhole.

Every imaging device is affected by non-idealities. For CSI, since it involves sophisticated convolution and deconvolution of images, artifact reduction techniques are especially necessary. A common source of artifacts is the modulation of the projection of the source onto the object by a \( \cos^3(\theta) \) factor. The use of a decoding pattern that is independent of such modulation will generate distortions in the final images. This is not the only source of artifacts that is expected. For example, the designer of a coded source should use care in selecting the thickness of the mask: a thin mask will transmit radiation and generate a high background, a thick one will collimate the radiation at the holes, which would no longer act like ideal pinholes.

The same artifacts arise in CAI imaging, and the same techniques of artifact reduction encountered there can be used in CSI. Specifically, it is possible to reduce artifacts drastically by using a mask / anti-mask technique. The technique involves taking
two images of the object, where in the second image the transmitting spots have been exchanged with opaque ones and vice-versa. Patterns that allow the implementation of this technique are easily found. For example, the MURA (Modified Uniform Redundant Array) pattern is orthogonal. A simple rotation of the coded source between the two images is then required.

**Conclusion**

For the reader acquainted with the technique of CAI, CSI is its natural extension in transmission radiography. CSI provides an advantage in the signal-to-noise ratio (SNR) compared to the state of the art. This advantage can be traded off for a series of desired features. The most obvious of such features is better resolution, applicable, for example, to the detection of micron-size flaws in mechanical parts. It can also be traded for shorter exposures, a feature that is particularly desirable in medical imaging. Finally, a high SNR can be used to improve the spatial coherence of the radiation for PCI.

Generally speaking, CSI improves on existing techniques by using an extended source more efficiently. In Neutron imaging, where sources are typically a few centimeters in size but are not very bright, it is essential to maximize the use of the available neutron flux.

**Phase Contrast Imaging using Coded Sources**

In the discussion about CSI, we observed that the SNR advantage can be traded for a higher transversal coherence length in order to perform PCI. If we think of the coded source as a collection of different sources all of the same spatial extent, it is reasonable to assume that the transversal coherence length of the system is governed by the size of each of these sources, and not by the total size of the coded source. This assumption is in line with the reasoning that the images formed by the sources are independent from each other. While this is obvious for standard imaging, more care has to be put into the argument for phase contrast imaging.

Let us assume for the moment that the coded source is composed of $N$ independent point-like sources. Each source is perfectly coherent. Since each produces radiation
following a process that is totally independent from the others, the sources are mutually incoherent. For the sake of argument, let us assume that we can use each source independently, and we decide to do this sequentially. Each source will then cast a PCI enhanced image on the detector, which will be added to the images cast from all other sources. The images will superimpose on the detector, and decoding will be necessary as in standard radiography.

Statistically, there is no difference between the case where the sources are used sequentially and the case where all the sources emit simultaneously. Interference effects between mutually incoherent photons coming from different sources do not contribute to imaging. By extension, any system ensuring mutual incoherence between the sources can be used for PCI. The decoding algorithm for such a system is identical to the one used in standard radiography. The PCI contribution to the final image depends only on the coherence length that each source can provide.

In some cases it is not possible to assume that the CSI system is perfectly mutually incoherent. Let us assume that inside the \( N \) sources, a subgroups of \( N_c \) sources contained in a radius \( R_c \) are mutually coherent. Two figures for coherence length are now available: the single source transversal coherence length \( \xi_s = \frac{\lambda R_{oo}}{w} \), where \( w \) is the size of the single source, \( R_{oo} \) the distance between source and sample and \( \lambda \) the wavelength of the radiation; and the group transversal coherence length \( \eta_i = f(R_c) \). The condition \( \xi_s > \eta_i \) is considered to always hold true by construction. Interference effects among photons coming from different mutually coherent sources are a nuisance to the image, while interference effects among photons from a single coherent source can provide valuable PCI signal. Of the PCI events (defined in Chapter 3) in the region defined by the group transversal coherence length, only \( 1/N_c \) effectively contribute to PCI signal. Outside \( \eta_i \), PCI event will contribute to PCI signal, since no mutual coherence effect will arise.

Let us assume that all the PCI events occurring in an area given by the circle of radius \( \xi_s \) minus the circle of radius \( \eta_i \) constitute PCI signal, while only \( 1/N_c \) of the PCI

\[16] \text{The relationship depends on the particular embodiment of the system.}
events in an area given by the circle of radius \( \eta_t \) constitute PCI signal. We can propose a quality factor \( Q \) for CSI PCI as a function of the quantity \( \chi = \frac{\xi_t}{\eta_t} \): 

\[
Q = \frac{(\chi^2 - 1) + \frac{1}{N_c}}{\chi^2}.
\]  

(22)

\( Q \) has a value varying between \( 1/N_c \) (for \( \xi_t \approx \eta_t \)) and 1 (for \( \xi_t \gg \eta_t \)). The behavior of the quality factor \( Q \) is plotted in Figure 3. We see that \( Q \) is close to unity for \( \xi_t \geq 10\eta_t \), even with a large number \( N_c \) of mutually coherent sources. If \( \xi_t \approx \eta_t \), a small number \( N_c \) of mutually coherent sources can drastically corrupt the phase contrast capabilities of the CSI system.

**Figure 3:** Quality for phase contrast capabilities of a CSI system, as a function of the ratio \( \chi \) between the single source coherence length \( \xi_t \) and the group coherence length \( \eta_t \).
**Practical Phase Contrast Systems Involving CS**

CSI can in principle provide the transversal coherence length required to perform PCI. Each single source composing the CSI system needs to meet the $\xi_{opt}$ and $K_{PCI}$ requirements. As an additional condition, $Q$ should be close to unity.

PCI with gratings [6] has been proposed in the past. Gratings can be considered a special case of CSI. We will discuss this system first. In general, the most obvious embodiment of a CSI system is an absorbent mask with $N$ pinholes disposed according to a pattern. In this section, we will present this system and discuss its strengths and drawbacks. Finally, we will discuss two alternative systems to build a coded source for phase contrast imaging, the Fluorescent Coded Source and the AEB Encoded X-ray Tube.

**Gratings**

In a landmark article about the use of standard X-ray tube to perform PCI, Pfeiffer [6] proposes the idea of using a set of gratings to obtain phase detection and to achieve a high coherence length at the source. We will discuss this work in Chapter 6; here we are concerned only about the concept of using a grating made of absorbent material to obstruct the radiation coming from an X-ray tube. The radiation will pass through the slits of the grating, and the small dimension of the slit is supposed to provide the transversal coherence length necessary to perform phase contrast imaging.

A grating is not immediately classified as a coded source, but the principle behind the two methods is identical. The grating encodes the source so that a relatively high percentage of the available photons is available, and the radiation is delivered from a collection of small sources. In this methodology, there is no reconstruction algorithm thanks to the fact that the detector system is placed at the Talbot [7] distance, where the image will self-reconstruct.

Gratings provide high transversal coherence in only one direction. This asymmetry suggests that PCI enhancement will be direction-dependent, and in normal set-up different resolution will be available depending on the direction of the detail. In principle
it is possible to take a set of images (e.g. two orthogonal images) to obtain a more uniform edge enhancement effect.

The monodirectionality of the transversal coherence results in a smaller coherence area compared to isotropic system with \[ \xi = \xi_{mono} \]. This results in a drastic reduction of the size \( N_\xi \) of the ensemble of atoms used to calculate the probability \( p(n>1, N) \) in Equation 3.18. In Chapter 3, we studied the case of imaging of tissue with 60 keV photons. We noticed that the minimum and optimal coherence lengths of such a system are \( \xi_{min} = 9.25 \, \mu m \) and \( \xi_{opt} = 17 \, \mu m \). We repeated the calculation\(^{17} \) for a system delivering monodirectional coherence, with a constant transversal coherence length in the direction of the slits \( \xi_{incoh} = 40 \, nm \). We obtain a minimum and an optimal monodirectional coherence lengths of \( \xi_{mono} = 1.7 \, mm \) and \( \xi_{mono} = 5.6 \, mm \). If a grating system had a grating-to-sample distance \( R_{g0} = 10 \, m \), at 60keV it would still require a slit width \( w = 0.12 \, \mu m \) in order to achieve \( \xi_{opt} = \xi_{mono} \). Such small value is unrealistic, and we believe that grating based CSI is not a viable option.

Another parameter to be considered is the actual photon flux that is made available after the grating. This quantity depends on the thickness of the gratings, the distance between grating and focal spot and open fraction of the gratings. In Chapter 6, a simulation is performed using the parameters presented by Pfiiffer, and it shows how the available flux is a small fraction of the total flux made available at the source. This fact is explained by the low angular acceptance of a thick grating with very small.

In conclusion, the use of a grating can be advantageous in some case because of its use in combination with particular detection schemes, but it has very poor PCI capabilities.

**N-pinhole system**

The N-pinhole system (Figure 4) is the more intuitive embodiment of a coded source system. It is composed of a mask containing \( N \) pinholes positioned in front of a radiation source having a large focal spot. In principle, the use of the available flux depends on the open fraction \( \rho \) of the mask.

\(^{17} \) The calculations are analogous to the ones presented in Chapter 3. The details are omitted.
As we discussed previously, CSI theory can use many results obtained in the study of coded apertures. The similarity between the two systems is very striking for the N-pinhole system, since the embodiment itself is a coded aperture. We refer the reader to the very thorough work of Accorsi [3] for more information on the SNR of this kind of systems.

PCI coherence requirements demand the use of extremely small pinholes, possibly on the order of a few micrometers. To avoid collimation effects and low acceptance angle, the thickness of the mask should be limited to values on the order of tens of micrometers. A very shallow thickness results in a high mask transmission which increases the noise level and worsen the image quality.

For an N-pinhole mask positioned at a distance $R_{SO}$ from a source of size $w_s$, the coherence radius

$$R_c = \frac{\lambda R_{SO}}{w_s}. \quad (23)$$

The value of the group coherence length $\eta$, is given in this geometry by
\[ \eta_r = \frac{\lambda R_{SO}}{W_s} = R_c \frac{R_{SO}}{R_{SO}} = m \cdot R_c \]  

(24)

where \( m \) is the magnification factor of the CSI. A sufficient condition for assuring a high \( Q \) is \( \chi = \frac{\xi}{\eta_r} \geq 10 \). In our geometry, taking into consideration the first equivalence in equation (24), this results in the condition:

\[ W_s < 10 \]  

(25)

The size \( w \) of the pinholes in the N-pinhole mask needs to be of an order of magnitude smaller than the size of the radiation source used to illuminate the mask. Since large \( w_s \) and small \( w \) are desirable, condition (25) is met in most cases.

If \( w_s < 10 w \), we need to ensure that only one pinhole is contained in a radius \( R_c \). This results in a limitation on both the open fraction \( \rho \) of the mask, and on its pattern design. The open fraction is limited by the condition:

\[ \rho \leq \frac{w_s^2}{R_c^2} \]  

(26)

An additional limitation exists on the pattern shape, where it is required that the distance \( d \) between two pinholes in the mask follow the relation:

\[ d \geq R_c \]  

(27)

In conclusion, an N-pinhole system can be used to perform PCI, but a series of limitations are dictated by the transversal coherence and mutual incoherence requirements. Mutual incoherence is in general achieved in most geometry and for most masks. The real technical limitation of an N-pinhole system arises from the angular acceptance of the pinholes for thick masks, or the high noise level due to transmission through a thin mask. For these reasons, improvements on the CSI design are necessary.
Fluorescent Coded Source

The N-pinhole system is a very intuitive embodiment of a coded source, and it carries the advantage of using a simple set-up consisting of adding a coded mask to an existing radiation source, like an X-ray tube. As we have discussed, the N-pinhole CSI is an imperfect system for PCI applications.

A reasonable improvement is to provide a system where the N sources are independent radiation sources. Each source will then behave independently, and will be able to emit in the entire space instead of being constrained by its angular acceptance. Also, the independence of the sources implies that $\eta_i = \xi_i$, ensuring a quality factor $Q = 1$.

The main problem behind a system composed of multiple independent sources is how to obtain the sources and how to arrange them in a compact fashion. Clearly, positioning multiple X-ray tubes in an array is both unpractical and not cost-effective.

A Fluorescent Coded Source (FCS) makes use of a single X-ray tube that shines radiation against a coded target. The coded target is composed of a collection of dots of size $w$ made of fluorescent material. When photons from the X-ray tube hit the target, photoelectric effect in each dot would release fluorescent photons isotropically. We can position the rest of the imaging system at an angle from the direction of the X-ray tube beam, so that the direct beam does not influence the image.

A FCS can in principle solve the angular acceptance, high noise and mutual coherence problems that are inherent in an N-pinhole system. It can also provide with a lower fractional bandwidth compared to an X-ray source, thanks to the characteristic nature of fluorescent emission. This property can be used to overcome limitation on the resolution of PCI enhancement due to polychromaticity.
Figure 5: Fluorescence from a Gd target compared to the direct beam. The measurements have been done with a Germanium detector placed in the direct beam (without the target in place) and off the beam, a few centimeters from the Gd target. The counts are normalized against a Cs-137 reference source, and are presented in a logarithmic scale.

The main drawback of a FCS is the low fluorescent yield of most materials. We performed a trial experiment using a Gadolinium target. Figure 5 shows the relative strength of the fluorescent beam compared to the direct beam. The system is highly inefficient, and our preliminary attempts to build a FCS have so far been frustrated by the lack of available flux. Optimization of the choice of fluorescent material should improve the efficiency. Further work is needed to evaluate the feasibility of a FCS.
**AEB Encoded X-ray Tube**

In the previous section, we proposed the design of a FCS to overcome the restrictions of an N-pinhole system. An FCS is affected by low efficiency. Ideally, we would build an X-ray Coded Source as a collection of microfocus X-ray tubes.

We propose here a new concept for an X-ray radiation source, based on an Advanced Electron Beam device. With this proposal we are set to build cost effectively the ideal set-up we discussed previously: the fabrication of a source composed by a collection of very small microfocus X-ray tubes. The device is pictured in Figure 6.

The Advanced Electron Beam delivers electrons to a thin window 25 cm in radius. If a pattern of $N$ Tungsten beans each $w$ in size are deposited on the window, each will act like the anode of an independent X-ray tube. Bremsstrahlung radiation will be generated by the electrons stopped in the beans, and the system would act as a collection of microfocus X-ray tubes organized around a large surface. Each single source would have power limited by $w$, due to limitation on the heat removal at the window. The large available area allows the deposition of a high number of beans, thus overcoming the problem of low flux per bean.

![Figure 6: Exemplification of an AEB Encoded X-ray Tube.](image-url)
An AEB Encoded X-ray Tube can provide highly coherent radiation at high flux, $Q = 1$ and low noise. The fabrication of the pattern can be done with existing technologies and the Advanced Electron Bean is available on the market.

**Conclusions**

In Chapter 2 and Chapter 3 we discussed how PCI can provide edge enhancement and ultimately deliver a better quality image. The requirement for obtaining those results is a large transversal coherence length. Unfortunately, standard sources can provide high coherence only at the price of an unacceptable limitation on the available flux.

In this chapter we discussed the idea of using CSI to obtain high coherence length and high flux. We developed the mathematics of CSI reconstruction and we discussed the impact of CSI on image quality. We described an equivalence between CSI and coded aperture, so that reconstruction algorithms, artifact reduction techniques and SNR calculations originally developed for Coded Aperture can be ported to CSI.

We focused on the use of CSI for PCI, and we proposed a quality factor $Q$ as an additional PCI requirement for CSI systems. We found that a CSI system can be used to perform PCI provided that some care is taken in the design of the system.

Four CSI systems were discussed. The idea of using a grating was proposed by Pfiffer [6]. We showed that gratings do not provide sufficient PCI capabilities. An N-pinhole system can achieve good PCI capabilities, but it is adversely affected by requirements on mask thickness.

Finally, two innovative systems, still in the prototyping phase, are described: a FCS and an AEB Encoded X-ray tube. Both systems carry the promise to deliver high transversal coherence without the problems that arise in gratings and in N-pinhole systems. The AEB Encoded X-ray tube has the potential of offering unprecedented high flux with high coherence and quality factor equal to unity, in a compact source.

Future work need to be performed to ultimate a working prototype of the AEB Encoded X-ray tube. In this thesis, we provided the foundation for such an effort by developing the conceptual design of such a system, along with concrete mathematical proof of the feasibility of the CSI technique that is at its core.
References


6. Critical Review of PCI Literature

Introduction

In this chapter we will review and discuss the literature related to phase contrast imaging. The body of work on phase contrast imaging is very broad, and the purpose of this chapter is to explore the field without attempting to provide an exhaustive bibliography of the current efforts. Other reviews of phase contrast imaging have been presented in the past, notably by Van der Veen [1]. For a review of the theory underlying phase contrast image, a good reference article has been written by Pogany [2].

Phase contrast imaging is a term that has been used to describe multiple practical efforts to achieve better image quality through edge enhancement. The broad nature of the subject, along with a lack of precisely defined boundaries for the phase contrast imaging community, makes any classification of the existing literature difficult and certainly subjective. We operated with a summary distinction between X-ray phase contrast efforts and neutron phase contrast efforts. In both cases, a variety of methods have been presented in the literature, and we selected a representative sample in order to give an overview of the state of the art in phase contrast imaging.

X-ray phase contrast imaging

Two themes are present in the development of phase contrast imaging systems: the need for a coherent source, and the need for an effective way of extracting from the radiation field additional information that can provide the sought-after edge enhancement. Paradoxically, the community has moved from extremely coherent sources coupled with very precise interferometers for the detection of phase to largely incoherent sources coupled with systems that can provide phase information only indirectly. This backward development is justified by the impossibility of porting the early work on phase contrast imaging to everyday applications, due to the high-end nature of the technology involved. The development over time of X-ray phase contrast imaging techniques is not a
pursuit for technological perfection, but rather for affordable ways of performing experiments that could in principle be already performed in prohibitively expensive ways.

**Interferometric methods**

Standard imaging is commonly performed using an X-ray tube, but early explorations of phase contrast imaging commonly used synchrotron sources, which provide a strong, parallel, monochromatic photon beam with very high transversal coherence. Unfortunately, synchrotron sources are expensive, impractical and difficult if not impossible to come by in an industrial or hospital setting. The differences between standard X-ray radiography and the early attempts at phase-enhanced radiography are not limited to the source. All detectors, working as a recorder of a number of photon interactions with each detecting pixel, are intuitively suited to the detection of an absorption image of a sample; the detection of a map of phase shifts induced in the wavefront while traveling in the sample, as required in phase contrast imaging, is not straightforward. Phase detection can be achieved through interferometry, and early studies used crystal interferometers for this purpose. For phase detection, this choice is ideal, in that it provides the very information that is needed for phase imaging. At the same time, crystal interferometers are expensive, require a very high precision in their set-up, and have extremely low tolerance to vibration.

This class of efforts is based on the interferometric technique developed by Bonse and Hart in 1965 [3], which consists of splitting the radiation beam into two identical beams, one to be passed through the sample, and the second one to be used as a reference beam (Figure 1). The two beams are recombined before they hit the detector, and the resulting interference pattern provides an intensity profile that depends on the difference in the optical path experienced by the beams. If the geometrical path of the two beams is identical, the difference in optical path is only dependent on the phase shift that one of the beam experiences in the sample. Interferometric techniques thus provide images that are proportional to the phase shift in the material. This is a significant difference from the behavior we discussed in Chapter 3, where the image is proportional to the Laplacian of the phase shift.
The beam splitters that are at the base of the Bonse-Hart interferometer are made of partially transmitting Bragg crystals. Aside from difficulties and costs involved in the fabrication of the crystals, the main problem associated with this technique is the stringent requirements for the positioning and the mechanical stability of the system. The maximum allowable motion between the crystals needs to be lower than the spacing between the crystal lattice, that is, lower than $10^{-10}$ m. Such precision is impossible to obtain outside of a dedicated laboratory, and it is more difficult to maintain for large crystals. Since the field of view of the system is restricted by the size of the crystal, it is extremely impractical to use such a system for the imaging of biological samples of the size of a few centimeters, such as a breast.

Despite the prohibitively punishing requirements just described, this technique has been successfully used for imaging of tissues. In 1972, Ando and Hosoya used a Bonse-Hart interferometer with an X-ray tube source to perform imaging of a bone [5]. Subsequent interferometric efforts made use of a synchrotron radiation source to image soft tissues [6, 7]. Phase contrast X-ray computer tomography (CT) using synchrotron radiation and a Bonse-Hart interferometer was used to obtain images of cancer lesions in rabbit [8], of human tumor tissues [9], of human breast tumor [10] and of nerve tissue [11].

Figure 1: Explanatory scheme of a Bonse-Hart interferometer [4]
In conclusion, synchrotron radiation provides a highly coherent, monochromatic source, and direct phase measurement is possible through a Bonse-Hart interferometer. This set-up constitutes an imaging system that detects the phase shift introduced by the sample, and as such it is different from the phase contrast system that we defined in Chapter 3. The difference lies in the fact that for practical purposes we restricted our definition of phase contrast to differential phase contrast systems, i.e., where the intensity of the radiation field at the detector depends on the Laplacian of the phase shift. The reason behind this definition is that differential phase contrast is much more practical than direct phase contrast, and in some applications is also more desirable. It is to be noted that despite the successes of the Bonse-Hart interferometric approach, the cost associated with it makes it not competitive with standard imaging techniques.

Two limitations hamper the applicability of the technique just described: the use of synchrotron radiation, and the use of a Bonse-Hart interferometer. Inside the synchrotron radiation community, efforts have been made to develop an alternative, less demanding technique for phase detection.

![Figure 2: Explanatory scheme of a shearing interferometer [4](image)](image)

David [4] proposed the use of a shearing interferometer (Figure 2). A shearing interferometer is composed of three gratings: a phase grating positioned before the sample acts as a beam splitter, a second phase grating with half the pitch of the first one recombines the two beams. The interference of the two beams creates an intensity profile of fringes that depends on the difference in optical path of the two beams. Both beams receive a phase shift from the sample, although at slightly different geometrical locations. The interference pattern is then dependent not on the phase shift of the sample but on its...
gradient along the transversal direction. An amplitude grating, made of absorptive material, is positioned in the interference region. The position of the minima and maxima in the interference pattern relative to the absorption portion of the third grating determines the intensity of the radiation that will be collected at the detector. Ultimately, the intensity collected at the detector is proportional to the derivative across the transverse direction of the phase shift of the material. That is, a shearing interferometer, in contrast to a Bonse-Hart interferometer, provides differential phase contrast imaging.

The system based on shearing interferometry is extremely interesting. First, the fact that the intensity image is proportional to the derivative of the phase shift (that is, of the electron density in the material) is an advantage for obtaining edge enhancement. Second, the requirement on stability and mechanical precision for the operation of a shearing interferometer is less stringent than that for a Bonse-Hart interferometer. Alignments on the order of the pitch of the gratings, which usually is micrometers in size, are enough to ensure the phase detection capabilities of the system. As we shall discuss later in this chapter, though, this limitation is still very severe in a hospital setting.

The main limitation of this system comes from the difficulty of fabricating the gratings, especially the amplitude grating used to detect the variations in the interference pattern. To overcome this difficulty, David performed his experiments with a modified set-up, where the amplitude grating was exchanged with a phase grating coupled with a Bragg crystal (see Figure 3), which is used to select the 0th diffraction order of the radiation field after the analyzer phase grating. Moreover, the set-up does not use a pre-sample beam splitter. The high coherence length (higher than the pitch of the post-sample beam-splitter phase grating) provided by the synchrotron radiation makes the presence of the first beam splitter grating unnecessary.
Figure 3: Schematic of the actual set up used by David [4]

An image of polystyrene spheres taken by David with this methodology is shown in Figure 4. David performed his experiments using synchrotron radiation at the energy of 12.4 keV and 20 keV, a focal spot of 80 μm and a staggering source to sample distance of 40 m, ensuring a very good coherence length of 30 to 50 μm. Clearly, this set-up is capable of performing phase contrast imaging, but it is also clearly not portable to an industrial or hospital setting. Aside from the extremely large source-to-sample distance, which is certainly not reproducible in a hospital for logistical reasons, the use of a synchrotron as a source is also problematic.
Figure 4: Images of 100 and 200 mm polystyrene spheres obtained using synchrotron radiation and a shearing interferometer. [4]

Weitkamp [12] proposed the use of a slightly modified shearing interferometer. The grating interferometer is made of two gratings (see Figure 5). The first grating $G_1$ is a phase grating, deep-etched into silicon. It shows negligible absorption of radiation but introduces a phase shift of $\pi$. Illuminated with an undistorted photon beam, i.e. without a sample, the grating $G_1$ casts on the detector a fringe pattern whose pitch is half the pitch of the grating $G_1$. When a sample is introduced, the phase shift in the beam will appear as a shift of the fringe pattern on the detector.
The shift of the fringe pattern, and the fringe pattern itself, amounts to a few micrometers, well below the resolution of available detectors. To overcome this difficulty, an analyzer grating $G_2$ is used, along with a technique called "phase stepping". The second grating has the same period as the expected undistorted fringe pattern. Phase stepping involves taking multiple images of the sample with the grating $G_2$ translated along the $x_g$ axis. The total translation is of the size of the period of the grating, and it is achieved in 4, 8 or 12 steps. $G_2$ transforms local fringe positions into signal intensity variation.

Weitkamp demonstrated experimentally the use of this technique using a synchrotron radiation source. Figure 6 shows one result, where polystyrene spheres are imaged using a 4-step phase stepping technique. An image of the phase gradient, immediately available from the variation of the intensity profile obtained through the phase stepping, is given, along with a mathematical reconstruction of the phase profile. Finally, an image of the average intensity at each point over each step of the phase stepping is shown. This last
image is equivalent to an image obtained through free-space propagation. The coherence length of the system used is not known.

Figure 6: (a-d) Phase contrast images of two polystyrene spheres of diameter 100 μm and 200 μm, taken with a shearing interferometer, at four different position of the phase stepping sequence. (e) Intensity oscillations over the phase stepping sequence at two points of the image. The two points are marked with a white dot in the raw pictures a-d. From this graph, the average intensity \( a \) and the phase oscillation \( \varphi = \varphi_2 - \varphi_1 \) can be determined. (f) Phase gradient obtained from e. (g) Phase shift mapping obtained by integration of f. (h) Average intensity \( a \) of all pixels. This image is equivalent to the phase contrast image defined in Chapter 3 for free space propagation. [12]

The use of a shearing interferometer with phase stepping provides a method to detect phase that, if not devoid of limitations and drawbacks, can in theory be implemented in any setting. The need for micrometer scale precision mechanics is an issue for the cost-effectiveness of the system compared to standard radiography, but is a huge improvement over the Bonse-Hart set-up. The field of view of a shearing interferometer is limited by
the available size of the gratings, which at this point can be fabricated only up to 6 cm in size. Although small, this is anyway an improvement on the Bragg crystals.

Moreover, Weitkamp noticed that a phase contrast imaging system using a shearing interferometer can be used also with polychromatic radiation, such as radiation coming from an X-ray tube or neutron radiation produced in a nuclear reactor. The idea of obtaining phase contrast imaging using different source types is important, since synchrotron radiation requires expensive and cumbersome equipment. Nevertheless, care should be taken in making sure that the transversal coherence of the phase contrast imaging system is high, as described in Chapter 3. Very good coherence lengths have in general been observed in experiments involving monochromatic synchrotron radiation.

High coherence interferometric systems using synchrotron radiation have been used to test the applicability of phase contrast imaging to the field of mammography. This medical application seems especially promising, since by nature standard X-ray radiography is not well suited to detect a tumor, which is a small amount of soft tissue, embedded in the healthy soft tissue of the breast. In a study by Takeda [13] published in 2000, the possibility of using X-ray phase contrast for clinical imaging was explored. This work demonstrates the use of phase contrast X-ray CT images to reveal structure in in-vitro human carcinoma with 30 μm resolution, without the need for a contrast agent. Takeda further observes that the X-ray energy used for phase contrast imaging can be raised from 17.7 keV, as used in standard radiography, to 30 keV, thus reducing the dose to the patient. As shown in Figure 7, the refractive index of pathological tissue differs from that of healthy tissue and shows little variation within various pathological specimens. Application of X-ray phase contrast imaging to other kinds of cancer are then expected to be similar to the mammography application reported by Takeda.
Figure 7: Graph depicts the mean refractive index and specific gravity for various cancerous tissues. [13]

Many approaches have been tried to obtain a viable X-ray phase contrast radiograph. One of the most interesting is presented by Pfeiffer et al. [14]. This paper belongs to a wider group of efforts to obtain phase contrast imaging using only minor modifications to standard X-ray radiography set-ups. The aim is to use widely available standard X-ray tubes as sources, instead of expensive synchrotrons or microfocus tubes.

In this work, Pfeiffer proposes the use of a source composed of a standard X-ray tube and an absorption grating. This source grating creates a collection of individually coherent line sources. Despite the sources not being mutually coherent, each one produces a phase-enhanced image. The resulting superposition of images would normally require decoding to obtain a workable image. Here, however, the decoding stage is omitted thanks to the Talbot self-reconstructing properties of the system [15]. This
grating system, exemplified in Figure 8 along with the shearing interferometer used for phase detection, was discussed as a first example of coded source imaging in Chapter 5.

Figure 8: (a) Set-up of the grating based phase contrast imager developed by Pfeiffer et al. (b) Images of the gratings. [14]

Pfeiffer’s work is to our knowledge the first attempt at an efficient use of a standard X-ray source for phase contrast. Its advantage over the use of a single slit, or of a pinhole, lies in the better usage of available photons. An X-ray tube spot size can be as large as a few centimeters, and restricting it to dimensions of the order of tens of micrometers is extremely punishing on the available flux. Extended coded sources provide a way of
meeting the coherence requirement of phase contrast while preserving an efficient use of the available photon flux.

The coded source proposed by Pfeiffer was originally developed for use in a parallel beam. Its performance in a diverging beam, such as that provided by an X-ray tube, is hampered by a potentially low acceptance angle of the grating slits, as discussed in Chapter 5. The problem arises as the thickness of the grating becomes larger than the width of each slit. When the grating is located close to the source, only the slits lying in a maximum acceptance angle range contribute to the output flux. As the grating is moved farther away, more slits contribute to the output flux but the overall solid angle under which the source sees the gratings is reduced.

![Graph](image)

**Figure 9:** Efficiency of the grating system as a function of grating-anode distance. The distance is in logarithmic scale.

The article does not address questions about the efficiency of the system. We calculated the flux available with the geometry described in Pfeiffer [14] using a Matlab simulation. The simulation assumed ray geometry for the photons and further assumed that any photon hitting the massive part of the grating is absorbed. Under these
assumptions, using an open fraction of 40%, the effective available flux is 5.6%. Figure 9 shows the calculated flux as a function of the distance between the tube anode and the grating. Despite the low value of the effective available flux fraction compared to the open fraction of the grating, the system still outperforms the pinhole in efficiency of usage of the available extended source. Unfortunately, as discussed in Chapter 3, the effective phase contrast quality factor of a grating system is much worse than of a system made of pinholes, due to the monodimensionality of the coherence length.

The detection of the phase information in Pfeiffer’s experiment was performed via a phase stepping technique using a shearing interferometer. The set-up is exemplified in Figure 8; results of this imaging technique are exemplified in Figure 10. The images were taken using a Mo target X-ray tube operated at 40 kVp and 25 mA. 27 raw images of 40 s exposure each were recorded using a CCD camera with a 100 μm thick YAG screen, providing an available resolution of about 30 μm. The absorption grating had a period of 127 μm and an open fraction of 0.2, resulting in a series of openings about 25 μm. Given a distance of 1.765 m between source and sample, the expected coherence length in the transverse direction from the grating slits is about 3 μm.18

As was discussed in Chapter 5, this coherence length is too small to allow meaningful phase contrast enhancement to be observed in the image. Despite this, it is clear from Figure 9 that edge enhancement was achieved. One reason behind the improvement can be found in the detection technique that is used. Despite free-space propagation not being used in this case, the last stage of the shearing interferometer is essentially a very precise Bucky grid mechanism. This last grating can be described as a grid with a period of 2 μm with the absorbing structure having a height of 15 μm. Since the open fraction is about 40%, the width of each opening is 0.8 μm, with a resulting grid ratio of 19:1 and an lpi of 12700. These parameters allow for much better Compton discrimination than most commercially available Bucky grids. Moreover, the phase stepping mechanism provides an extremely accurate averaging effect, as we discussed in Chapter 5. Finally, we believe that most of the enhancement observed in Figure 10 is not due to phase contrast effects, which at the available coherence length should give a

18 Although the transverse coherence length is monodirectional, there is no evidence that there exists a difference in resolution from one direction to another.
minimal contribution to the final image, but rather to Compton filtering performed via a very sophisticated Bucky grid system.

Figure 10: (a) Standard radiography of a small fish. (b) Phase contrast enhanced picture of a small fish using a shearing interferometer. (c)-(h) Details of selected areas of the small fish. [14]

The same comment can be made for Figures 4 and 6, with the notable difference that those systems, thanks to the use of synchrotron radiation, present a much larger transversal coherence length. For this reason, we believe that the edge enhancement in Figure 4 is a combination of Compton filtering and phase contrast edge enhancement, where the phase contrast contribution is very significant. We cannot comment on the
implications for Figure 6 due to the lack of information on the coherence length of that system.

Non-interferometric methods

Davis [16] proposed a non-interferometric technique to provide phase contrast edge enhancement. The method requires incoming radiation in the form of a monochromatic plane wave. This is hard to obtain; the set-up used by Davis made use of a monochromator at the source and a monolithic silicon crystal to transform the radiation into a plane wave. As shown in Figure 11, the crystal reflects the radiation in the direction of the sample, illuminating it with a collimated plane wave. This treatment of the incoming radiation is needed to allow very precise discrimination of the perturbation incurred in the wave while passing through the sample.

Figure 11: Schematic of the method proposed by David. [16]

As was previously discussed, the interaction of the radiation with each point in the sample induces a modification, in general, of both the local amplitude and the phase of the wave. If we restrict our attention to weakly absorbing materials, we can consider a phase shift as the only effect of the interaction. In Figure 11, this effect is represented as an inhomogeneous distortion of the wavefront. Information about the sample is conveyed in the shape of the distortion.

Alternatively, we can consider the incoming radiation as a collection of photons that impinge on the sample, and as a result of interactions are scattered at small angles from the original path. Davis proposes a way of detecting the small angle scattering via a set of
two perfect crystals. The first crystal receives the radiation from the sample, and reflects it at a fixed angle to another crystal. This second crystal reflects the radiation to a detector, where the image is formed. The interesting aspect of this methodology is that, of the radiation hitting the first crystal, only the portion that is incident perpendicularly is reflected to the second crystal, thus providing a selection method for only the plane wave components of the incoming wave. By rocking the two crystals off axis, the experimenter can choose to filter out the direct part of the incoming radiation and to image only radiation that scattered to a certain angle.

Davis shows promising results through this technique. Three sets of images are presented, and each is compared with both standard radiography and "off-axis" phase contrast imaging. The samples are a leaf (Figure 12), a mosquito, and a test sample prepared with glue and glass fibers. The phase contrast images provide more details about the sample as compared to the standard radiography images. Also, by selecting different angles between crystals, different types of details can be enhanced.

Figure 12: Image of a leaf using (a) standard radiography and (b) phase contrast imaging [16].
This phase detection technique is very promising, but it is hampered by the requirement that the incoming radiation be monochromatic. Also, although the technique is non-interferometric, precise mechanical alignment and stability and a perfect crystal the size of the field of view are needed. For these reasons, it is hard to imagine such a technique being widely used in a hospital setting.

Although Davis reported this technique under the umbrella name of phase contrast imaging, it is unclear if this method belongs in this group. This work is based on imaging of radiation scattered by a sample, similarly to Harding’s work [17, 18], which is not usually considered as belonging to phase contrast imaging. Since small angle scattering is preferentially targeted, this technique is based on imaging capabilities associated with Rayleigh scattering, which is in general more probable than Compton scattering at very low angles. The common ground with phase contrast imaging ends here though, since this technique does not take advantage of interference phenomena and it is unclear what is the transversal coherence requirement.

There is abundant literature dealing with the use of free space propagation techniques with synchrotron radiation. Snigirev [19] used such an approach to image organic samples; in this work, a source-to-sample distance of 50 m with a focal spot of 100 μm was used, providing a coherence length at 30 keV of 20 μm. Clotens [20] used a 25 keV beam with focal spot of 40 μm and a source-to-sample distance of 150 m, thus delivering a transversal coherence length of 20 μm, for imaging of microstructure and damage in a material. An alternative source was used by Gurayev [21], who used a modified electron microscope to fabricate a polychromatic ultramicrofocus X-ray source. The electrons were projected against a tantalum foil, producing a beam centered at 1.7 keV with an effective focal spot of 0.12 μm. Given the source-to-sample distance of a few millimeters, the coherence length available to the system was about 10 μm. This is to our knowledge one of the few set-ups described in literature outside of the synchrotron radiation community that can provide the coherence length required to perform phase contrast imaging. However, the system is impractical for most uses due to the very low energy of the radiation, the difficulty in building the source, and the expense of modifying an electron microscope.
Wilkins [22] is the first to our knowledge to propose the use of a microfocus X-ray tube with free space propagation to perform phase contrast imaging. He used an X-ray tube with photon energies peaked at 60 keV, a focal spot of 20 μm, and a source-to-sample distance of 30 cm for imaging of a small aquarium goldfish (Figure 13). The transversal coherence length of the system was 0.4 μm.

![Figure 13: (a) Image at contact of a goldfish (2 min exposure); (b) Image at a distance (1.1 m) of a goldfish (110 min exposure) [22].](image)

Wilkins succeeded in making a case for the edge enhancement capabilities of this system. Microfocus X-ray tube phase contrast imaging has thus been studied further. For example, Donnelly [23] used a 10 μm focus system restricted to a geometrical space between the sample and the detector of 55 cm, thus realistically restricting the source-to-sample distance to 45 cm. This system presented an expected coherence length of 2.2 μm (at 25 keV of photon energy). The maximum current was 0.3 mA. In another study, Birch [24] used a similar system, with focal spot of 20 μm and an available source-to-sample distance of 58 cm, sporting a coherence length at 20 keV of 1.8 μm. The maximum current again was 0.3 mA.

As we can see from the examples presented above, set-ups used in literature that purport to perform phase contrast imaging do not actually provide the required transversal coherence length to take advantage of the phase information. Moreover, the sample-to-detector distance provided by the systems is not sufficient to meet the near-Fresnel zone requirement. For these reasons, we do not believe that the edge enhancement observed in these experiments is due to phase contrast. We instead believe
that Compton filtering is the main motor of the edge enhancement effect seen in these set-ups. This topic has been discussed in Chapter 3.

It is also worth noticing that if the above set-ups were modified to allow a much longer source-to-sample distance and a slightly longer sample-to-detector distance, they could in principle be used for phase contrast imaging. Nevertheless, the microfocus X-ray tubes used cannot provide the current necessary to perform imaging with reasonable exposure times.

<table>
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<th>$\xi_t$ (m)</th>
<th>$\xi_{min}$ (m)</th>
<th>$\xi_{opt}$ (m)</th>
<th>$K_{PCI}$</th>
<th>$K_{CF}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wilkins</td>
<td>3.72E-07</td>
<td>7.80E-06</td>
<td>1.40E-05</td>
<td>0.00E+00</td>
<td>4.82E+00</td>
</tr>
<tr>
<td>Donnelly ($R_{QQ} = 0.41$ m)</td>
<td>1.69E-06</td>
<td>5.00E-06</td>
<td>9.15E-06</td>
<td>1.66E-04</td>
<td>9.96E-01</td>
</tr>
<tr>
<td>Donnelly ($R_{QQ} = 0.14$ m)</td>
<td>5.79E-07</td>
<td>5.00E-06</td>
<td>9.15E-06</td>
<td>0.00E+00</td>
<td>9.96E-01</td>
</tr>
<tr>
<td>Birch ($R_{QQ} = 0.116$ m)</td>
<td>7.19E-07</td>
<td>2.30E-06</td>
<td>4.15E-06</td>
<td>4.31E-05</td>
<td>2.83E-02</td>
</tr>
<tr>
<td>Birch ($R_{QQ} = 0.464$ m)</td>
<td>2.88E-06</td>
<td>2.30E-06</td>
<td>4.15E-06</td>
<td>3.39E-03</td>
<td>2.83E-02</td>
</tr>
<tr>
<td>Snigarev</td>
<td>6.20E-05</td>
<td>2.30E-06</td>
<td>4.15E-06</td>
<td>6.66E-02</td>
<td>2.83E-02</td>
</tr>
</tbody>
</table>

Table 1: Comparison of PCI and CF contribution to Tissue imaging using typical microfocus X-ray tube systems found in literature (Wilkins [22], Donnelly [23], Birch [24]) and a typical synchrotron radiation set-up (Snigarev [19]).

In Table 1, we summarize the survey of PCI capabilities on Tissue for typical PCI imaging systems presented in literature making use of microfocus X-ray tube technology. Of the surveyed systems, we found that Wilkins and Donnelly have no appreciable PCI capabilities. Significant edge enhancement is expected at a distance thanks to a large $K_{CF}$; no edge enhancement due to phase effect is expected. Of the set-ups proposed by Birch, low PCI capabilities are expected for the geometry with maximum $R_{QQ}$. This is the only geometry of the ones surveyed that presents a $\xi_t > \xi_{min}$. Nevertheless, the contribution of
PCI to imaging is inferior to the contribution of CF. In general, due to the low photon energy used in Birch, the total edge enhancement effect is modest compared to the only-CF edge enhancement in Wilkins and Donnelly.

As a comparison, data from a synchrotron radiation set-up is presented. In Snigarev, $\xi_t > \xi_{opt}$, and the PCI capabilities of this system exceed the CF capabilities. The total edge enhancement obtained in Snigarev is superior to that achieved by Birch, which uses similar photon energies. Both system are capped in the maximum enhancement by the use of soft X-rays. It is worth noticing that the system proposed by Snigarev has a transversal coherence length exceeding $\xi_{opt}$ by about 60 μm. This extra coherence does not contribute to the imaging capabilities of the system, and a lower $\xi_t$ could have been be used without loss of $K^{PCI}$. In Snigarev, a higher photon energy could have been selected. The reduction of coherence length due to this increase is inconsequential until $\xi_{opt}$ is reached, while $K^{PCI}$ would increase.

<table>
<thead>
<tr>
<th>E (keV)</th>
<th>$\xi_t$ (m)</th>
<th>$\xi_{min}$ (m)</th>
<th>$\xi_{opt}$ (m)</th>
<th>$K^{PCI}$</th>
<th>$q^{PCI}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>6.20E-05</td>
<td>2.30E-06</td>
<td>4.15E-06</td>
<td>6.66E-02</td>
<td>100%</td>
</tr>
<tr>
<td>20</td>
<td>3.10E-05</td>
<td>3.60E-06</td>
<td>6.60E-06</td>
<td>2.23E-01</td>
<td>100%</td>
</tr>
<tr>
<td>30</td>
<td>2.07E-05</td>
<td>5.00E-06</td>
<td>9.15E-06</td>
<td>4.13E-01</td>
<td>100%</td>
</tr>
<tr>
<td>40</td>
<td>1.55E-05</td>
<td>6.40E-06</td>
<td>1.20E-05</td>
<td>6.15E-01</td>
<td>98%</td>
</tr>
<tr>
<td>50</td>
<td>1.24E-05</td>
<td>7.80E-06</td>
<td>1.40E-05</td>
<td>7.20E-01</td>
<td>83%</td>
</tr>
<tr>
<td>60</td>
<td>1.03E-05</td>
<td>9.20E-06</td>
<td>1.70E-05</td>
<td>6.64E-01</td>
<td>59%</td>
</tr>
<tr>
<td>80</td>
<td>7.75E-06</td>
<td>1.20E-05</td>
<td>2.20E-05</td>
<td>2.98E-01</td>
<td>18%</td>
</tr>
</tbody>
</table>

Table 2: PCI parameters for Snigarev's set-up operated at different energies.

The PCI capabilities of Snigarev's set-up for Tissue imaging as a function of photon energy are summarized in Table 2. We notice how, according to our theory, there is large room for improvement in this set-up. Figure 14 shows $K^{PCI}$ as a function of energy. A ten-fold improvement in PCI capabilities of this system can be obtained by better exploiting the available coherence.
In conclusion, according to our theory, systems based on microfocus X-ray tube
cannot in general provide the coherence length that is required to perform PCI.
Synchrotron radiation systems do provide sufficient coherence, and are PCI capable.
Nevertheless, according to our study of PCI requirements, synchrotron radiation systems
do not optimally exploit the available coherence length.

Free space propagation techniques produce an image that is proportional to the
derivative of the phase shift, and for this reason they are sometimes called differential
phase contrast methods. For edge enhancement purposes, the dependency of the
irradiance of the field on the derivative of the phase image of the object is advantageous,
since the derivative of an edge is in general more visible than the edge itself. For more
complicated features, the lack of a direct phase image is a nuisance, since reconstruction
is needed to interpret the phase contrast signal.

The main difficulty in obtaining a pure phase image of a sample is in the fact that
detectors provide irradiance measurements, and do not record the phase of the incoming
radiation. Teague [25] proposes a method to determine the phase map from two irradiance measurements at different planes. The reasoning begins by manipulating the parabolic equation satisfied by the field in order to obtain an equation containing only the phase \( \phi \) and the intensity \( I \) of the field. Such equation is the two-dimensional Poisson equation:

\[
\nabla^2 \psi = \frac{-2\pi}{\lambda} \frac{\partial}{\partial z} I
\]

where

\[
\nabla \psi = I \nabla \phi.
\]

It is reasonable to assume that if \( I \) and its derivative along the longitudinal axis \( z \) are known, the phase \( \phi \) can be obtained. The value of \( I \) and its derivative can be obtained by standard irradiance measurements at two planes. Indeed, Allman [26] reports having obtained phase information from a set of two neutron radiography images at different distances from the sample. Unfortunately, such an approach is not valid in general.

The solution of the Poisson equation and its integration to obtain the phase is not trivial. A general solution using Green’s function \( G \) is proposed. Assuming a standard Green’s function for a two dimensional Poisson problem, the general solution for the phase is:

\[
\phi(\vec{r}) = \int d\vec{r}' G(\vec{r}, \vec{r}') \frac{-2\pi}{\lambda I} \frac{\partial I(\vec{r}')}{\partial z} + \int ds' \phi(\vec{r}') \frac{\partial G}{\partial n}(\vec{r}, \vec{r}') .
\]

To solve for the phase, it is not sufficient to have the value of \( I \) and of its derivative. In general, the knowledge of the phase over a perimeter of the region of interest is needed. Such a measurement is usually impractical to obtain. It is worth noticing that for circularly symmetric phase maps, the integral over the perimeter is zero, and the measurement of the perimeter phase is not necessary. We expect that the more the phase
map of a sample varies from circularly symmetric, the higher the error in estimating the phase from irradiance measurements.

Teague [25] proposes an interesting method to obtain phase information via non-interferometric imaging techniques. The validity of the approach is general only under the condition that limited phase information, that is, phase values along a perimeter, is available either a priori or by measurement. This is usually not the case. The approach developed in Teague can still provide a useful approximation of the phase mapping of a sample, but the quality of such an approximation depends on the symmetries present in the sample itself.

Olivo [27, 28] points out the difficulty of obtaining an imaging system suitable for clinical use when using free space propagation, grating interferometry, or crystal interferometry. A competing technique, based on the use of coded apertures, is proposed (Figure 15). This approach makes use of a post-sample coded aperture mask, at contact with the detector, to enhance the quality of the image. The mask is positioned so that each absorbing element of the mask covers a part of a detector pixel, thus reducing the size of the active area. Partially blocking a detector pixel is a way to enhance the contrast due to refractive effects in the sample: after refraction, photons that would without interaction have hit the active area of the detector instead fall in the obstructed area, and photons that would have hit the obstructed area are recorded. This way, negative and positive peaks are recorded in the image. Since this technique effectively uses only a fraction of the available photons, a pre-sample coded aperture mask is also used to reduce the dose delivered to the sample.
The coded aperture technique has many merits. The pre-sample coded mask can be built so that the angular acceptance is significant. Moreover, since the proposed system does not rely on interferometric methods, vibration reduction and mechanical precision are not a problem. Also, Olivo and Speller propose to use coded aperture masks with open fractions up to 50%, which is a relatively efficient use of the radiation source.

This system is superficially similar to those we advocated in Chapter 4. In reality, the coded source systems proposed by us, and more specifically the N-pinhole system, are very different in nature from the system championed by Olivo and Speller. The difference lies in the fact that while the N-pinhole system we described has the goal of providing coherent radiation from an incoherent source, the only purpose of Olivo and Speller’s coded mask at the source is to reduce the dose to the sample by delivering only the radiation that is used in the detection stage.

Olivo and Speller inscribe their methodology in the group of phase contrast imaging techniques, but it is unclear how phase contrast plays a role in this system. While Olivo and Speller propose a clever way of exploiting refraction effects to enhance the image, it is not evident how the method takes advantage of post-sample interference effects. Moreover, it seems to us that the method would not be able to produce a direct or differential phase mapping of the sample.
Olivo and Speller validate their technique by a computer simulation and by experiment. The simulation is based on ray-tracing of photons from the source to the detector. Assuming a pointlike monochromatic source, a photon is generated traveling in a random direction. If the photon hits the pre-sample mask, it is considered absorbed and a new photon is generated. Otherwise, its path is followed. If it intersects the sample, a random number generator tells if the photon is absorbed, refracted or if it passes undisturbed, depending on the properties of the sample at the interception point and the energy of the photon. The path is followed to the post-sample mask, and if the photon hits the mask it is absorbed and a new photon is generated. If it hits the active area of a detector pixel the photon is added to the counts for that pixel, and a new photon is generated. The entire cycle has to be repeated for multiple pointlike sources to simulate a source of finite size, and for multiple photon energies to simulate polychromaticity.

![Diagram](image)

**Figure 16:** (a) Relative intensity profiles for various pixel exposure fractions; (b) Effect on contrast of different fractions of pixels exposed to radiation. [27]

The results obtained through this simulation are encouraging, and indeed show an increase in contrast as the fraction of pixels exposed to radiation decreases (Figure 16). More fundamentally, the structure of the simulation underlines the fact that the methodology presented by Olivo and Speller is not a phase contrast technique, in that it can be simulated without taking into account interference effects.

An experimental validation has also been presented. The experiment was performed with a 100 μm focus X-ray tube run at 35 kVp and with 130 μm thick gold masks. The source-to-detector distance is 2 m and the sample-to-detector distance is 0.4 m. The
configuration of the system suggests a transversal coherence length of 0.7 μm. The low value of the transversal coherence length of the system confirms that the edge enhancement does not come from phase contrast effects.

**Conclusion**

Early phase contrast imaging systems were developed using monochromatic radiation and crystal interferometry to detect the phase map of the sample. Very high transversal coherence lengths were made available using a synchrotron radiation source and a geometry allowing tens of meters of source-to-sample distance. Despite the prohibitive cost and technical difficulty of such systems, experiments showing an increased image quality have been successfully performed. Crystal interferometry can provide directly the phase map of an object, but at the cost of a mechanical stability requirement on the order of a fraction of an Ångstrom. Gratings interferometry can only directly provide the differential phase map, but the mechanical requirements are much less stringent (on the order of microns, usually). This type of interferometry has been used both with highly coherent set-ups involving a synchrotron radiation source and with less coherent set-ups involving the use of X-ray tubes coupled with a special case of a coded source, that is an additional absorption grating. An analysis of the systems at lower coherence shows that they do not provide a coherence length large enough to perform phase contrast. The grating interferometer still provides an efficient method to reduce the Compton fogging of the film, thus explaining the observed edge enhancement.

Non-interferometric methods have also been used with both high coherence radiation in the synchrotron radiation community, and with lower coherence radiation using microfocus X-ray tubes. A review of microfocus systems shows that in most cases the coherence length provided is not large enough to provide phase contrast imaging capabilities. An edge enhancement is anyway observed thanks to the fact that the techniques used to induce phase contrast also provide scattering reduction capabilities. Some systems, despite classification as phase contrast imaging systems, make explicit use of scattering reduction and discrimination in order to achieve edge enhancement, disregarding altogether requirements on transversal coherence length.
A summary of the coherence length of the various techniques plotted against a usage factor, which includes cost and ease of use considerations, is shown in Figure 17.

![Figure 17: Usage analysis of various techniques described in literature that claim phase contrast edge enhancement.](image)

**Neutron Phase Contrast Imaging**

We have so far focused on literature concerned with X-ray phase contrast techniques. Neutron phase contrast imaging can be performed in a similar fashion, with the caveat that bright, coherent and monochromatic sources are not available. Also, crystal interferometry for neutrons is harder to perform than for X-rays. For these reasons, the first successful attempt at neutron phase contrast imaging occurred much later than the first successful attempt using X-rays.
Figure 18: (a) Standard neutron radiography of a yellow-jacket wasp; (b) Phase contrast image of a yellow-jacket wasp. [26]

Allman [26], at NIST, obtained a phase enhanced image of a yellow-jacket wasp, shown in Figure 18, and of a lead sinker. A monochromatic ($\lambda = 4.43$ Å) neutron beam was restricted via a pinhole to obtain a focal spot size of 0.4 mm. The distance between the pinhole and the sample was 1.8 m, and a detector was placed at two locations, at contact and at 1.8 m from the sample. Based on these figures, the transversal coherence length provided by the system was 2 μm. This coherence length is small for X-ray phase contrast imaging, but it is appropriate for neutron imaging where coherent scattering is more likely relative to other interactions compared with X-ray imaging. Phase detection was performed through free space propagation; a phase map of the sample was obtained from the contact image, and a differential phase contrast image was obtained at a distance via a method similar to the one described in Teague [25].

Allman’s work demonstrated the feasibility of neutron phase contrast imaging, but the requirement of a monochromatic beam is very punishing on the available neutron flux. The exposure time for Figure 18 was not reported, but exposure times of hours or days have been reported elsewhere for neutron phase contrast imaging. An improvement on this system was proposed by McMahon [29], who explored theoretically and experimentally the possibility of using a polychromatic neutron beam for phase contrast imaging. The experimental validation was performed with a beam of cold neutrons, using a pinhole of 200 μm, a source-to-sample distance of 1.5 m, and a sample-to-detector distance of 1.42 m. The expected coherence length of the system is 3 μm.
McMahon showed the possibility of performing phase contrast imaging using neutron beams without the inclusion of a monochromator. Karjilov [30] made use of this idea to perform a wide array of experiments. The experiments were performed with a thermal beam obstructed by a 500 mm pinhole, and a source-to-sample distance of 6.5 m. The available coherence length was 2 μm. Phase detection was performed using free space propagation, with no reconstruction of the direct phase map. The first of the experiments is the imaging of two edge profiles, one made of Al and the second one made of Ti. The interest in using these two materials is that they have coherence scattering lengths that are similar in absolute value, but opposite in sign ($\sigma_{\text{Al}} = 3.5$ fm, $\sigma_{\text{Ti}} = -3.4$ fm). Thanks to the dependency of neutron phase contrast imaging on the derivative of the coherent scattering length, the two edges will produce opposite phase shifts, as shown in Figure 19. The sample-to-detector distance in this case was 1 m.

Figure 19: Left: edge of a 5 mm thick Al plate; Right: edge of a 3 mm thick Ti plate. Exposure times 120 min. [30]

The importance of the sample-to-detector distance in the relevance of the phase contrast induced edge enhancement is explored. Images of a set of stainless steel syringes of various radius were taken at sample-to-detector distances ranging from 0.3 m to 0.9 m.

19 Karjilov incorrectly reports a transversal coherence length of 5 μm.
Figure 20 shows the result of this experiment, which validates our expectation based on Equation 3.12 that phase contrast enhancement increases with the sample-to-detector distance. Other objects (a cogwheel, a piece of lead foam) were imaged, and the use of smaller pinhole apertures to increase coherence was attempted. For a pinhole size of 100 μm, the signal-to-noise ratio of the system was corrupted and the overall quality of the image is inferior to the one taken with the larger 500 μm pinhole.

![Intensity profiles of a set of stainless steel syringe needles, taken at various sample-to-detector distances.](image)

**Figure 20:** Intensity profiles of a set of stainless steel syringe needles, taken at various sample-to-detector distances. Exposure time (per image): 180 min. [30]

The neutron phase contrast imaging experiments discussed so far were performed using a pinhole and free space propagation techniques. An alternative approach is proposed by Pfeiffer [31]. Borrowing the shearing interferometer approach developed for X-ray phase contrast imaging [4, 12, 14] and discussed here in the previous section, Pfeiffer used a cold neutron source, with a fractional bandwidth reduced to 25% by a thick Beryllium filter, to perform a tomographic image of two rods, one of lead and the other of titanium (Figures 21 and 22). The absorption grating had a period of 1.08 mm and an open fraction of 0.5. The source-to-detector distance was 5.23 m, allowing the system to reach a coherence length of 4.25 μm. This experiment is relevant due to the difference in sign of the coherent scattering length of the two materials ($\sigma_{\text{Pb}} = 9.28 \text{ fm}$,
σ_{Ti} = -3.4 \text{ fm). This difference is observable only through neutron phase contrast and is not available by X-ray phase contrast imaging.

In conclusion, phase contrast imaging with neutrons is more practically difficult due to the lower availability of coherent sources, but it has been successfully performed and it is a promising technique for material studies. Compared to X-ray phase contrast imaging, slightly inferior transversal coherence requirements are needed, as discussed in Chapter 3. Phase contrast induced edge enhancement has been observed with transversal coherence lengths as low as 2\mu m. Moreover, the peculiar characteristic of neutron phase contrast, its sensitivity to variation in coherence scattering length, has been exploited successfully.

![Sample composed of Pb wire wrapped around a Ti rods.](image)

**Figure 21:** Sample composed of Pb wire wrapped around a Ti rods. (a) Differential phase contrast image; (b) Reconstructed phase map; (c-d) Tomographic slices at the planes marked by broken white lines in a-b. Exposure time: 240 min [31].
Figure 22: Tomographic image of a Pb wire wrapped around a Ti rod (three view angles). [32]

References

[32] Picture taken from Physical Review Focus Website
   http://focus.aps.org/story/v17/st20 on December 2008
7. Conclusions and Future Work

In this thesis, we explored the theory of Phase Contrast Imaging (PCI) with the goal of understanding its relevance and its requirements. The literature on PCI is vast, and multiple experiments have pointed at possible advantages in using PCI for applications such as mammography. Early work focused on using extremely coherent radiation, and it was assumed that the PCI requirements for longitudinal (that is, chromatic) and transversal (that is, spatial) coherence were extremely high. Phase detection was achieved either with interferometric methods or with free-space propagation set-ups.

Knowledge about the effectiveness and the requirements of PCI are based on diffraction theory. Through derivations that are essentially independent from the physics of the interaction of radiation with matter, it is possible to establish the image formation process for an imaging system relying on coherent radiation, and more specifically the fundamental equation of PCI. This was done in literature, and we presented those results in Chapter 2 and Chapter 3 of this thesis. PCI with perfectly coherent radiation provides an edge enhancement in a region of space called Fresnel zone, at a distance from the sample. The enhancement is proportional to the derivative of the real part of the index of refraction of the material, and it is also proportional to the sample-to-detector distance. These results do not give us a specific tool to evaluate the relevance of PCI enhancement in the final image. Moreover, this theory, developed with perfectly coherent radiation in mind, is not helpful in understanding how a relaxation in coherence requirements affects the PCI capabilities of an imaging system.

This problem is all the more important since coherent X-ray and Neutron sources are mostly unavailable. Synchrotron radiation has been used to provide a very coherent X-ray source: the radiation was selected at a specific wavelength to ensure longitudinal coherence, and an extremely long space was allocated between source and sample to ensure transversal coherence. A similar option in Neutron imaging is altogether not available to flux limitations and space constraints.

Early work on PCI relied on extremely expensive and impractical equipment to obtain a coherent source and to demonstrate the advantage of PCI in imaging (and specifically in mammography). The advantage observed in these experiments matched...
the predictions of the theory of PCI with coherent sources: an edge enhancement was observed, thus producing a higher resolution image. Most recent work is focused on obtaining the same edge enhancement without the need for interferometric methods, and with the use of polychromatic radiation source with lower transversal coherence. In those low-coherence cases an edge enhancement was indeed observed, and it was therefore concluded that it was possible to perform PCI with low-coherence sources.

In this thesis, we contend that this conclusion is unjustified. Edge enhancement effects can be a by-product of the particular geometry of a PCI experiment, without being related to phase contrast effects. We believe that a theoretical investigation of the relevance of PCI at low coherence is in order to validate the assumptions found in literature. For this purpose, we took care in the discussions on diffraction theory and PCI in Chapters 2 and 3 to describe the physical phenomena that are at the base of the image formation process. Starting from our physical understanding of PCI, we proposed a model to evaluate the transversal coherence requirement of a PCI system and to predict what level of PCI is possible with a system that does not meet the requirements. We also discussed the relevance of polychromaticity for PCI.

Our results confirm that a polychromatic imaging system can be used without a severe loss of PCI capabilities. However, we also found that the requirement on transversal coherence is more stringent, and a reduction of the transversal coherence length of an imaging system can severely compromise its PCI capabilities. Specifically, depending on the application and on the energy of the radiation, a minimum coherence length $\xi_{\text{min}}$ exists under which the PCI capabilities of the imaging system are very low or nonexistent. An optimal coherence length $\xi_{\text{opt}}$ exists for which the system reaches its maximum PCI capabilities. There is no advantage in using a coherence length higher than $\xi_{\text{opt}}$, and in some extreme cases this can be detrimental to the quality of the image due to interference effects across large features of the sample. We also proposed a figure-of-merit $K^{PCI}$ to evaluate the relevance of PCI enhancement on the image as compared to other effects.

Our contention is that a class of PCI experiments presented in the literature do not actually provide edge enhancement through phase contrast effects. To explain the nature of the edge enhancement observed in those cases, we developed a model for a competing
edge enhancing mechanism that we called Compton Filtering (CF). CF is based on the property of free space propagation systems to reject of Compton scattering events due to the fact that a scattered photon does not hit the geometrical shadow of an image sample. A figure-of-merit $K^{CF}$ equivalent to $K^{PCI}$ was proposed and the two effects were compared. Our calculations show that it is likely that the edge enhancement effect observed in free space propagation low-coherence set-ups is to be ascribed to CF and not to PCI. Our calculations also show that when proper coherence is provided, PCI effects can contribute noticeably to the quality of an image.

In Chapter 4 we described an experimental validation of our theory that free space propagation, low-coherence systems do not provide PCI edge enhancement, but they do provide an enhancement due to CF. We developed an imaging system that allowed us to easily modify the available coherence length and performed a set of experiments with both a low-coherence set-up and an incoherent set-up. We observed edge enhancement in both cases. We also observed that the edge enhancement does not increase linearly with the sample-to-detector distance. CF enhancement is expected to increase for short distances, and to remain constant after that; PCI enhancement is expected to increase linearly with distance. Finally, the presence of the edge enhancement seems to depend on the geometry of the sample. CF enhancement depends on the distribution of scatterers in the Field-of-View; PCI enhancement should be independent from the geometry of the sample. Our results are compatible with our calculation for $K^{CF}$ and $K^{PCI}$, and this experimental work thus provided further validation to our theory.

We demonstrated that PCI requires high coherence systems, and that if such a system is available, PCI can provide higher quality images than those available with standard radiography or with CF alone. Still, any application of PCI in medical diagnostics or industrial non-destructive testing requires the design of a cost- and flux- effective way of producing spatially coherent radiation. In Chapter 5, we explored a new methodology, Coded Source Imaging (CSI), which we believe can deliver high spatial coherence with a radiation flux similar to that of standard sources. The theory of image formation for CSI was developed and a methodology for image reconstruction was presented. Advantages in Signal-to-Noise ratio and possible methods for reducing artifacts were discussed, along
with a similarity with an existing technique in emission imaging called Coded Aperture Imaging (CAI).

We believe that CSI has the potential of becoming an interesting imaging methodology. More specifically, in this thesis we discussed the design of a CSI system to perform PCI. The problem of single source coherence and group coherence was identified and modeled. Our calculation showed that it is possible to build a CSI system that would enable PCI. We presented four possible embodiments of this methodology.

The grating CSI is an idea that has been used in PCI literature, although it has not been presented as a CSI technique. We discussed the relevance of the existence of a preferential direction in the source coherence and the implication on the value of $\xi_{\text{min}}$ and $\xi_{\text{opt}}$ for such a system. Our calculation showed that such a system is not suitable for PCI since the coherence requirements are much higher than for a system providing coherence isotropically. We also discussed the angular acceptance of a grid system and the fact that the flux available after the grid can be drastically reduced due to geometry. The N-pinhole system is designed to provide a practical technique to convert a standard source to a PCI source. Design issues of this system were discussed, in particular the relevance of PCI corruption due to mutual coherence of the sources. The angular acceptance of the system was addressed. The N-pinhole system is more apt than a grating system to perform PCI, but low flux due to low angular acceptance is a limiting factor. It does not seem from our calculation that the mutual coherence of the sources imposes cumbersome mask design limitations.

We proposed two possible systems that would not be affected by low angular acceptance problems: Fluorescent Coded Sources (FCS) and the AEB Encoded X-ray tube. FCS are based on the emission of fluorescent radiation by a target composed by a pattern of fluorescent material. The emission is isotropic, and is therefore not affected by the geometrical arrangement of source and mask. Preliminary experiments with FCS resulted in an extremely low flux due to poor fluorescent yield of the target. An FCS is highly affected by the low efficiency of the fluorescence process, and extreme care will have to be devoted in the choice of material. An AEB Encoded X-ray tube is effectively a design for a new generation X-ray tube exploiting an electron generating device called Advanced Electron Beam. The interest in this device arises from the large window it
offers, effectively providing a large area to deposit a number of very small targets for Bremsstrahlung radiation production.

In Chapter 6 we described the literature on PCI, and we gave a summary of the systems that, in our belief, do not provide enough coherence to perform PCI. We analyzed those works with the theoretical framework developed in Chapter 3.

In conclusion, we contributed to the theory of PCI by developing a model for PCI contribution from low-coherence systems. We explored edge enhancement effects due to PCI geometry but not to phase contrast itself, and identified CF as the main source of enhancement in low-coherence systems. We obtained experimental validation for our theory. We believe that PCI can produce dramatic enhancement in imaging if the appropriate radiation source is provided. We then proposed the CSI methodology to deliver the appropriate coherence without loss of flux and without resorting to synchrotron radiation. Finally, we produced a critical review of the current literature on PCI.

Much work is left to be done on PCI. Our theory of PCI is based on some arbitrary assumptions, notably concerning the exact proportionality of PCI enhancement to the sample-to-detector distance. Only experimental investigation with a high coherent system can give us more insight on the modeling of this property. For CSI imaging, we developed the mathematical tools to design a system and proposed embodiments of CSI systems that would enable PCI in a cost-effective way and with high fluxes. More efforts are needed to bring these systems from prototyping or the design phase to be applicable to important problems such as mammography.

It is our belief that it is possible to build a cost-effective system to perform PCI based mammography. An increase in the detectability of small tumors has the proven effect of drastically reducing the mortality of women by breast cancer. This thesis provides the tools to design a PCI system and to evaluate its PCI capabilities. We have shown that the production of a PCI system with off-the-shelf equipment is extremely difficult, and claims in literature of a PCI system based on microfocus X-ray tube are in
general not accurate. We propose CSI as a possible path towards the construction of this system.