XVII. CIRCUIT THEORY[*]

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A. HIGH-ORDER FREQUENCY MULTIPLICATION WITH VARACTORS

Frequency multiplication with 100 per cent efficiency has been predicted by the Manley-Rowe power relation for a nonlinear reactance (1). Indeed, an efficiency of approximately -1 db for a frequency doubler using the varactor (nonlinear reactance) in the pure capacitive mode has been reported (2, 3). However, for high-order multipliers, the efficiency drops off rapidly and eventually falls well below the $1/N^2$ limit for an ideal rectifier (4). On the other hand, with the varactor used as a rectifier, efficiencies close to the $1/N^2$ limit have been attained (3). Besides the filter losses, the other source of loss can be attributed to the inherent series resistance of the varactor. It was felt that the loss caused by this series resistance is so serious in the capacitive mode that the rectifier mode would be more desirable in the high-order case.

Theoretical investigations have been carried out to determine the effect of the series resistance on the efficiency of the varactor multiplier. In particular, the relative merits of the pure capacitive mode, the rectifier mode, and the capacitive-rectifier mode have been investigated.

An equivalent-circuit model for the varactor is shown in Fig. XVII-1. From energy conservation it can be shown that the efficiency is of the form

$$\eta = \frac{1}{N^2} + \left[(H_c - H_r) - \left(P_{s1} + N^2 P_{sN}\right)\right] \frac{1}{N^2 W} \tag{1}$$

where N is the harmonic number that is of interest; W is the input power at the fundamental frequency $f_1 = (\omega_1)/(2\pi)$; P_{s1} and P_{sN} are the power dissipated by the series resistance at the fundamental and at the Nth harmonic, respectively; and H_c and H_r are two quantities associated with the nonlinear capacitance and the nonlinear resistance, respectively. These quantities have the form:

$$H_c = \frac{\omega_1}{2\pi} \int_0^{2\pi} v_c[q_c(y)] \frac{d^3 q_c(y)}{dy^3} \, dy \tag{2}$$

$$H_r = \frac{1}{2\pi} \int_0^{2\pi} \frac{dv_r(i_r)}{di_r} \left[\frac{di_r(y)}{dy} \right]^2 dy \tag{3}$$

where $y = \omega_1 t$; and $v_c(q_c)$ and $v_r(i_r)$ are the nonlinear characteristics for the capacitor and the resistor. For positive capacitance and positive resistance, it can be shown that both H_c and H_r are positive.

With reference to Eq. 1, it is seen that in order for $\eta > 1/N^2$, the condition

$$H_c - H_r > P_{s1} + N^2 P_{sN} \tag{4}$$

must be satisfied.

These conclusions can be deduced from this condition. First, the presence of the term H_r can only render the inequality diffic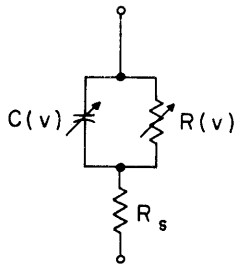ult to meet. Thus, the capacitive mode of operation is the more desirable. Second, the rectifier mode is made more effective by the presence of the nonlinear capacitance ($H_c > 0$). Third, the N^2 term on the right-hand side of Eq. 4 reveals the rapid deterioration of the high-order multiplier in the presence of the varactor series resistance.

Fig. XVII-1. Equivalent-circuit model for a varactor.

However, since the quantities in Eq. 4 are all functions of the actual circuit parameters, the series resistance, the input power, and the harmonic number, realizability of the condition of optimal operation can best be investigated by an analysis of the actual circuit. This investigation is now proceeding.

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References

1. J. M. Manley and H. E. Rowe, Some general properties of non-linear elements. Part I. General energy relations, Proc. IRE 44, 903-913 (1956).

2. D. B. Leeson and S. Weinreb, Frequency multiplication with nonlinear capacitors — A circuit analysis, Proc. IRE 47, 2076 (1959).

3. Interim Progress Report on Contract No. NAsw-33, Research Laboratory of Electronics, M.I.T., Dec. 1, 1959; see Sections 2 and 3.

4. C. H. Page, Harmonic generation, Proc. IRE 46, 1738 (1958).