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AN OVERVIEW
OF VEHICULAR SCHEDULING PROBLEMS

by

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ABSTRACT

This survey presents the state of the art of vehicular scheduling problems. The basic problems, node and branch routing, are defined. Extensions, generalizations and results for such cases are given.

I. Problem Definition and General Background

In most large scale endeavors it is essential to either deliver goods or service a set of prespecified tasks in a given area. Problems of such generality are quite common in large organizations of both government and private enterprise. As a result, an effort has been directed to classify such problems according to necessary assumptions and desired goals. In addition to establishing a general theoretical foundation, researchers have directed themselves to real problems to increase operating capacity while decreasing costs for particular users. While certain smaller problems have been essentially solved by hand; computer codes as the IBM VSPX [3] have been developed to handle more extensive problems with various options.

We shall consider problems of routing and scheduling with a non-real time demand. Routing will be defined as the aggregation of a collection of pickup or delivery points which a vehicle must traverse in a specific order. Scheduling is the aggregation of a set of feasible routes.

The basic problem is to have a specified depot from which all vehicles will depart and return, that is, all journeys will begin and end at one point for all vehicles. All outlets or service points are serviced by the vehicles originating from the depot and terminate there after the final call, although any particular vehicle may be required to make more than one call. It will be further assumed that the average quantity to be delivered at a call is less than the size of the smallest vehicle employed. These specifications and assumptions while quite general are typical of a good number of vehicular scheduling problems. Yet one must keep in mind that they are by no means final. Later we shall mention variations and extensions to the basic problem which have been treated. There are also possibilities for other variations as the application may dictate. In general we are directing ourselves to problems of distribution of goods and services.

Typically there are three types of problems, node or discrete routing, branch or continuous routing, and the general routing problem, a combination of node and branch routing. In routing vehicles through a set of nodes, node routing, a map of the region (network) to be considered is transferred to a set of nodes N and a set of arcs A. The objective is to determine a set of routes to service (pick up or deliver), that is, a set of nodes subject to capacity and other physical constraints, while minimizing the number of vehicles needed. For the one vehicle case we have the Traveling Salesman Problem to be considered in more detail later. In routing vehicles along the branches of a network, branch routing, our objective is to minimize the total time vehicles need for traversing the branches of a network more than the required number of times again subject to certain capacity constraints. Equivalently we must minimize deadheading (or deadhitting) of a vehicle which is the time a vehicle is being routed over the branches in a network more than the required number of times. For a discussion of the general branch routing problem see Orloff [27].

In addition to defining the problem we must decide upon some set of operating criteria which translated in terms of a cost will be desirable to minimize (or maximize in terms of profits). Many different applications have similar expenses. First we must determine a set of calls, service points or routing tours so that the route of each vehicle used is well-defined. An optimal or desirable set of calls is one which minimizes the number of vehicles or yields a reasonable number of vehicles so all points can be serviced while minimizing the mileage of each vehicle. Obviously one could use an enormous fleet of vehicles to satisfy any servicing objective at the expense of the cost of vehicles, cost of manpower and any other established work standards. Clearly this is undesirable. In addition, we must consider the less apparent costs as the fixed costs of licenses, rents, wages, insurance, and interest plus operating costs of fuel, lubricants, tires and various parts, maintenance, and depreciation. Thus beginning with numerous if not an infinite number of possible ways to service points, our objective to provide low operating costs will permit only a few such routes. Furthermore, it is our objective to efficiently find such an optimal routing tour.

Perhaps the most common scheduling problems are the single and multi-route truck dispatching problems or equivalently referred to as the vehicle scheduling problem, delivery problem, cloverleaf problem or the truck routing problem. Essentially we wish to schedule vehicles of a given capacity with non-uniform demands on the vehicles. In addition, there may be constraints on demands for service.

Considerable work in vehicular scheduling has been directed towards public service routing (Marks and Stricker [22]). In the course of a day in our typical cities, trash must be collected, streets cleaned, and mail delivered. The objective for each of the above problems is to find an optimal route for vehicles which must service every street in a network. Graph theoretically the problem is known as the Chinese Postman Problem: Trace the shortest continuous path through a network so that every arc is covered at least once. In the case of trash collection (Marks and Stricker [22]), not only do vehicles have a common origin point, but there are also capacity constraints on the vehicles; that is, one truck cannot service an entire city in any feasible period of time. So it is necessary to find distinct continuous tours for vehicles since the trucks that fill up must return to some origin and unload before resuming service. This type of public service problem may have other conditions to be met as the size of the crew or frequency of collection. It must be mentioned that although the truck delivery problem and the waste collection problem are quite different in objectives, theoretically they are essentially the same. The dump site is replaced by the depot and pickup points become delivery points, so similar techniques are used for both problems.

II. The Fundamental Problems

In this section we shall define and discuss the two fundamental problems in vehicular scheduling. They are most commonly referred to as the Traveling Salesman Problem and the Chinese Postman Problem. For the general arbitrarily large network these problems remain unsolved. Although optimal solutions cannot be guaranteed, there are numerous procedures which shall be discussed later which are believed to give, if not optimal, near optimal solutions in a relatively short amount of time. Many of these procedures are based on heuristics and are easier for one vehicle.

IIA. The Traveling Salesman Problem

Perhaps the most well-known node routing problem is the Traveling Salesman Problem: Given N cities with pairwise distances between each city known, find the tour that permits one to visit each city and return to the starting (terminal) point while minimizing the total distance traveled. This is considered a single route problem, that is, one salesman visits all the cities. Otherwise we have the m-salesman problem. Given N cities with pairwise distances between each city known and a terminal where each salesman starts and ends his tour and assume that each of the m salesmen can only visit k (k N) cities before returning to the terminal. Find the m tours the salesmen must make so that all the cities are visited and the distance traveled is minimized. Each tour must therefore have k nodes, plus the terminal, so m = N/k. If N/k is not integral, we must add or delete cities to maintain integrality.

Here we shall state the mathematical formulation of the Traveling Salesman Problem (Garfinkel and Nemhauser [16]).

$$\text{MIN} \quad \sum_{i=1}^{N} \sum_{j=1}^{N} C_{ij} \, x_{ij} \tag{1}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} x_{ij} = 1 \qquad j=1,\ldots,N \tag{2}$$

$$\sum_{j=1}^{N} x_{ij} = 1 \qquad i=1,\ldots,N \tag{3}$$

$$\sum_{i \in Q} \sum_{j \in \bar{Q}} x_{ij} \geq 1 \qquad \text{for every } Q \subset A, \; Q \neq \emptyset \tag{4}$$

$$x_{ij} = 0,1 \qquad i,j = 1\ldots,N \tag{5}$$

where

N is the number of nodes in the network

A is the set of arcs

x_{ij} is the number of times arc (i,j) is traversed

C_{ij} is the length of arc (i,j)

Q is a subset of nodes

$\bar{Q}$ is the complement of Q, i.e., A-Q

Equation (1) represents the distance to be minimized. Equation (2) states that for every node i exactly one edge (i,j) must be in every tour, i.e., a city is left only once. Similarly equation (3) states that for every node j, exactly one edge (j,i) must be in every tour, i.e., a city is entered only once.

Equation (4) eliminates possible subtours among a subset of the total number of cities. For example, it eliminates one solution for a five-city problem drawn in Figure 1 which is clearly unacceptable. $X_{ij} = 0$ implies arc (i,j) is not traversed while $x_{ij} = 1$ implies arc (i,j) is traversed. A branch and bound algorithm is usually used to solve this problem.

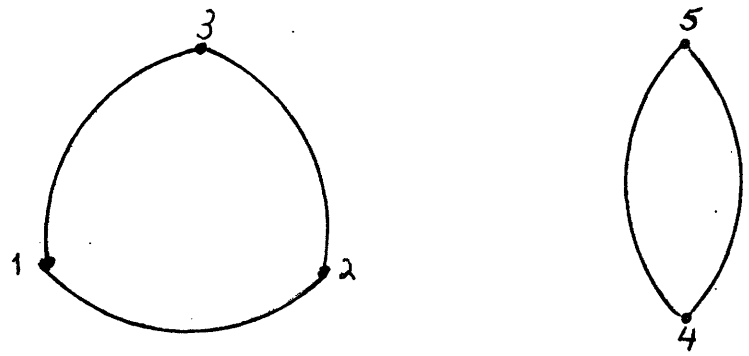

Figure 1: This solution to the traveling salesman problem will satisfy constraints (2), (3) and (5) but will violate constraint (4).

The applications of this problem statement are numerous. Job shop scheduling, aircraft routing, vehicle routing, and production planning models can be formulated in this mathematical framework. There have been three fundamental approaches to the Traveling Salesman Problem. The first is a tour to tour improvement technique based on heuristics which gives a good but usually suboptimal solution. The order which cities are visited are switched while maintaining feasibility; that is, no city is visited more than once. Termination occurs when the analyst feels there is no significant improvement. Although a solution is suboptimal it is obtained rather quickly. Secondly, the tour building technique first presented by Little et al.[21] provides a means of successively choosing the next city to be visited. Heuristics as choosing the nearest city have been used for this selection. Pierce and Hatfield [29] have introduced an interesting application to a production sequencing problem with additional constraints dealing with job deadlines. Pierce [28] also extended his work to the single and multi-route truck dispatching problem to be discussed later. Lastly, subtour elimination is one where a less constrained problem is solved which may not be feasible, that is, all cities may not be visited in one connected tour. To insure feasibility, constraints are added to eliminate disconnected subtours.

For routing more than one vehicle the above procedures can be augmented by other heuristics. There are generally two basic choices in node routing procedures.

First one may divide a network into subgraphs and route one vehicle over each subgraph, cluster first -- route second. Then one exploits optimization techniques as previously mentioned for the smaller more tractable problem. Usually the number of required vehicles in unknown, so we usually parametrize on this number, i.e., the number of necessary subgraphs. Secondly, one may initially simplify the problem by disregarding for the moment, time and capacity constraints of the vehicles, then form a giant tour through all the nodes (or branches) in the network. In order to satisfy the necessary constraints this great tour is subdivided into subtours, route first -- cluster second (Newton and Thomas [26]). In choosing which of the above two procedures one must consider the number of pick-up or delivery points for the tour of a vehicle, the number of routes to be created, and the travel time between service points,and service points and the terminal point. Usually for few routes to be created and many service points for each route it is more effective to form a giant tour which is then partitioned into smaller subtours. For many tours with few service points per tour it is generally more effective to form the routes first. In Newton and Thomas [26] this analysis is carried out for different problems. See Beltrami and Bodin [6] and Bodin [7] for further discussion s.

IIB. The Chinese Postman Problem

The second problem we shall consider is the continuous counterpart of the Traveling Salesman Problem, and is called the Chinese Postman Problem. Here we wish to find a tour on a connected, undirected graph G that includes each edge of G at least once and minimizes the total distance traveled, where the length of each edge is nonnegative. Symbolically we have,

$$\text{MIN} \quad \sum_{i=1}^{N} \sum_{j=1}^{N} C_{ij} \, x_{ij} \tag{6}$$

$$\text{s.t.} \quad \sum_{k=1}^{N} x_{Ki} - \sum_{K=1}^{N} x_{iK} = 0 \qquad i = 1,\ldots,N \tag{7}$$

$$x_{ij} + x_{ji} \geq 1 \qquad \text{for all } (i,j) \varepsilon \; A \tag{8}$$

$$x_{ij} \geq 0 \tag{9}$$

$$x_{ij} \quad \text{integer} \tag{10}$$

where

N is the number of nodes in the network

A is the set of arcs

x_{ij} is the number of times arc (i,j) is traversed

C_{ij} is the length of arc (i,j)

Equation (6) represents our objective, to minimize the distance traveled subject to conditions stated in equations (7) - (10). Equation (7) is the continuity equation, i.e., every time we enter a node, we must leave that node. Equation (8) states that each edge is covered at least once while equations (9) and (10) state that an edge cannot be traversed a negative nor a nonintegral number of times. Thus we have again formulated a pure integer linear programming problem. Traditional techniques as branch and bound or cutting plane algorithms however, have not proven to be adequate because of the size of most problems. There are usually hundreds of arcs and the number of variables and constraints is generally twice the number of arcs. For all branches either directed or undirected and only one vehicle, exact procedures for the solution of this problem are known (Leibling [19], Edmonds and Johnson [14], Christofides [8]). For both directed and undirected arcs we have heuristic methods (Edmonds and Johnson [14]). Generally for more than one vehicle two choices are available as in the Traveling Salesman Problem. One can cluster nodes into subgraphs or partition a giant tour. Forming a giant tour first usually results in a better solution; that is, less deadheading. Yet from an administrative view this technique is usually undesirable. Partitioning first gives non-overlapping regions as opposed to routes that may interact Although non-overlapping routes are easier to administer, the deadheading time is usually greater.

III. Further Research and Results

In this section we examine in more detail the different vehicular scheduling problems considered to date. Again the problems are fundamentally of two types, the node covering problems, represented by the Traveling Salesman Problem and the arc or branch covering problems or the Chinese Postman Problem.

IIIA. Node Covering Research

The fundamental node routing problem is the Traveling Salesman Problem. Many variations of this problem have been considered and solved. An extensive bibliography of this problem is given in Bellmore and Nemhauser [5] and Eilon, Watson-Gandy, Christofides [15].

The Truck Dispatching Problem is a generalization of the Traveling Salesman Problem. Dantzig and Ramser [10] consider N points (cities) with demands q_i for deliveries and a terminal point with no demand. Let C = the capacity of each vehicle and

$$\max_{1 \leq i \leq N} < q_i < C < \sum_{i=1}^{N} q_i$$

Suppose further that the shortest routes between any two cities is given. The problem is to find routes for all vehicles such that the city's demands are satisfied and the total distance traveled by the vehicle is minimum. The formulation can be simply stated as the following integer program:

$$\text{MIN} \sum_{i=1}^{N} \sum_{j=1}^{N} C_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{j=1}^{N} x_{ij} = 1 \qquad i=1,\ldots,N$$

$$x_{ij} = x_{ji}$$

$$x_{ij} = (0, 1)$$

where C_{ij} is the distance between cities i and j

x_{ij} = 1 if city i and j are paired
0 otherwise

The method of solution is to synthesize a sequence of solutions in a number of stages of aggregations. Once cities are paired, they are considered as a single point for the next iteration, that is, they remain paired throughout. Suboptimization is taken on pairs or groups of points and routes are built up until the capacity constraint of a vehicle is met. The problem is solved as a linear program and heuristics are used to treat fractional x_{ij}.

Dantzig and Ramser also suggest and treat various extensions of this problem. One may stipulate that the vehicle must return to the terminal when it has reached m of the $N-1$ remaining cities where m is a division of $N-1$. For small m, optimal routes can easily be determined by sight. They present near optimal solutions for m large. The particular extension is referred to as the Clover Leaf Problem. Furthermore, these results can be extended from one product delivered by trucks all with the same capacity to many carriers with different capacities, i.e., multiple truck capacities. Multiple-product demand, demands for several products at each city, can be treated.

Clarke and Wright [9] extend the work of Dantzig and Ramser and seek the optimum routing of a fleet of trucks with different capacities from a terminal to a number of cities or delivery points. Similar assumptions of Dantzig and Ramser are also made. Locations to be serviced are paired and ranked according to a savings in time associated with each pair. Points are combined while maintaining feasibility until all the points are used. Essentially they use a cluster first-route second approach. Altman et al.[1] consider a further extension where towns are assigned on certain days of the week. Beltrami and Bodin [6] discuss other variants of the Clarke and Wright algorithm.

Clarke and Wright's work provide a basis for IBM's scheduling software package VSPX (3). This package provides for a network analysis which computes travel distances and times between delivery points and then considers only dominating schedules. An algorithm based on Mills [24] is used to find the shortest route through a network. Additional constraints can be included in the routing problem, as priority ratings, earliest and latest starting times, different vehicle capacities and several commodities carried by a vehicle. At present VSPX is the most extensive software package of its kind in existence.

Balinski and Quandt [4] consider a set covering formulation which is applicable for small multi-route truck dispatching problems. Single vehicle schedules are enumerated and then an optimal route is chosen. So for large problems we must first generate many schedules which can be very time consuming. Yet for a highly constrained problem, the number of feasible routes may be small and this method is then better.

Pierce [28] treats the single routing problem with additional constraints. He allows for earliest and latest arrivals at a specific point, optional deliveries, split deliveries, and constraints on vehicles as volume limit, maximum number of stops and time per trip. The solution is sought by a branch and bound tour building approach. Two approaches for branching are considered. The first is flooding, starting with many branches at once and choosing the next branch which is most profitable. Second, one may follow one specific branch to completion, i.e., feasibility or infeasibility. The first method usually requires a long time to prove optimality while the second generates feasible solutions quickly and a heuristic is used to terminate. This seems to be more successful.

Andrew and Hamann [2] extend Little et al.work and use a tour generating branch and bound technique. It is good to find a solution quickly and provide for unequal demands at cities with unequal vehicle route capacities.

Hausmann and Gilmour [18] treat the transport of a commodity or service by truck from a terminal to m customers. Each customer has a minimum required frequency of delivery which may be increased to take advantage of economics in truck routing. Two types of costs are considered, the cost per mile of truck travel and a fixed cost incurred each time a customer receives a delivery.

Customers are classified by groups so that when any customer in a group requires a delivery the entire group is serviced. In addition, Hausman and Gilmour treat the multi-period problem. An example is given for fuel oil delivery.

IIIB. Arc Covering Research

The first formal statement and significant result in the arc covering problem was by Euler. This is the Konigsberg Bridge Problem. Euler proved that it is not possible to trace a continuous path covering every edge in a graph without repeating any arcs unless the degree of every node is even, i.e., there are an even number of edges incident to every node. So an Euler tour (minimum path covering every edge exactly once) exists for an undirected graph if and only if the degree of every node is even. If odd nodes exist we must cover some arc at least once.

Mei-Ko [23] later considered the problem for graphs with odd nodes. He considered the equivalent problem to minimize the sum of the lenghths of the repeated edges where the repeated edges, when added to G, form a new graph G* with every node even. He shows that for 2n odd nodes, n paths must be duplicated between odd nodes such that the total length of the arcs traversed more than once is minimum. The necessary and sufficient condition for optimality is that the sum of the lengths of the respected arcs on every cycle of G* not exceed half the length of the cycle. His algorithm for finding the optimal tour is however not computationally efficient.

Edmonds [11], [12], [13] translates the Chinese Postman Problem into one where a matching algorithm becomes applicable. The matching algorithm is used to determine which nodes must be paired so that the lengths of the repeated arcs is minimum. Yet first one must find the shortest paths connecting every possible pair of odd nodes which can be computationally burdensome for larger networks. Further results on considering the one vehicle directed branch routing problem as a matching problem is given in Beltrami and Bodin [6].

Glover [17] takes another alternative approach to the problem. Given a graph G, Glover find a graph $H \leq G$ such that every node of $G' = G + H$ is even. H is a set of duplicated arcs with minimum total length. The arcs of H are called pseudo-edges. For a graph of 2n odd nodes Glover solves n shortest path problems in succession to obtain an optimal solution. Like Edmond's algorithm, Glover's work may become very difficult for large networks occuring in real routing problems.

Murty's [25] Symmetric Assignment Problem is also an alternative approach to the Chinese Postman Problem. A minimum edge covering tour is determined by solving a symmetric assignment problem whose cost matrix is composed of lengths of the shortest paths between every pair of odd nodes. A branch and bound technique is used to find the set of shortest paths connecting every pair of odd nodes whose total path is minimum. The problem is symmetric in that if arc (i,j) is in the optimal solution so is arc (j,i).

Lin [20] uses a branch exchange method to provide locally optimal solutions to many variants of one vehicle routing problems for either branch or node routing.

Striker [30] addresses the m-Postman Problem and solves it heuristically. Essentially the Chinese Postman Problem is solved for the original network and then the result is partitioned by several rules. Other approaches solve the districting problem first and then solve the Chinese Postman Problem for each district.

BIBLIOGRAPHY

1. Altman, S., Bhagat, N., and Bodin, L., "Extensions of the Clarke and Wright Algorithm for Routing Garbage Trucks," presented at the TIMS 8th International Meeting (Washington, D.C.), March, 1971.

2. Andrew, G.M., and Hamann, J.R., "Truck Routing: A Branch and Bound Solution," mimeographed notes presented at the meeting of the Operations Research Society of America, September, 1967.

3. Anonymous, Vehicle Scheduling Program, Application Description Manual H20-0464, IBM Corporation, White Plains, New York, 1968.

4. Balinski, M.L. and Quandt, R.E., "On an Integer Program for a Delivery Problem," Operations Research, Vol. 12, 1964, pp 300-304.

5. Bellmore, M. and Nemhauser, G.L., "The Traveling Salesman Problem: A Survey," Operations Research, Vol. 21, No. 2, 1973, pp 498-516.

6. Beltrami, E. and Bodin, L., "Networks and Vehicle Routing for Municipal Waste Collection," to appear in the Journal of Networks.

7. Bodin, L., "A Taxonometric Structure for Vehicle Routing and Scheduling Problems," to appear in the Journal of Networks.

8. Christofides, N., "The Optimum Traversal of a Graph," Report 71/16 of Imperial College of Science (London), November, 1971.

9. Clarke, G., and Wright, J.W., "Scheduling of Vehicles from a Central Depot to a Number of Delivery Points," Operations Research, Vol. 12, 1964, pp 568-581.

10. Dantzig, G.B., and Ramser, J.H., "The Truck Dispatching Problem," Management Science, October, 1959, pp 81-91.

11. Edmonds, J., "Paths, Trees and Flowers," The Canadian Journal of Mathematics, Vol. 17, 1965, pp 449-467.

12. Edmonds, J., "Maximum Matching and a Polyhedron with (0,1) Vertices," Journal of Research National Bureau of Standards, Vol. 69B, 1965.

13. Edmonds, J., "Optimum Branchings," Journal of Research National Bureau of Standards, Vol. 71B, 1967, pp 223-240.

14. Edmonds, J. and Johnson, E., "Matching Euler Tours and the Chinese Postman Problem," to appear in Mathematical Programming.

15. Eilon, S., Watson-Gandy, Christofides, N., Distribution Management, Hafner Publishing Co., (New York) 1971.

16. Garfinkel, R.S. and Nemhauser, G.L., Integer Programming, Wiley, 1972.

17. Glover, F., "Finding an Optimal Edge-Covering Tour of a Connected Graph," ORC 67-13, Operations Research Center, University of California, Berkeley, California, 1967.

18. Hausman, W.H., and Gilmour, P., "A Multi-Period Truck Delivery Problem," Transporation Research, Volume 1, Pergamon Press, 1967, pp 349-357.

19. Leibling, T., "Graphen Theorie in Planungs - und Touren Problemen," Lecture notes in Operations Research, Vol. 21, Springer-Verlag, 1970.

20. Lin, S., "Computer Solution of the Travelling Salesman Problem," Bell System Technical Journal, Vol. 44, 1965, pp 2245-2269.

21. Little, J.D.C., Murty, K., Sweeney, D., Karel, C., "An Algorithm for the Travelling Salesman Problem," Operations Research, Vol. 11, No. 6, 1963, pp 972-989.

22. Marks, D.H. and Stricker, R., "Routing for Public Service Vehicles," Journal of the Urban Planning and Development Division, ASLE, Vol 97, No. UP2, December, 1971, pp 165-178.

23. Mei-Ko, Kwan, "Graphic Programming Using Odd or Even Point," Chinese Mathematics, No. 1, 1962, pp 273-277.

24. Mills, G. A, "A Decomposition Algorithm for the Shortest-Route Problem," Operation Research Society of America, Vol. 14, 1966, pp 279-291.

25. Murty, K.G., "The Symmetric Assignment Problem," ORC 67-12, Operations Research Center, University of California, Berkeley, 1967.

26. Newton, R. and Thomas, W., "Developing a Computer Program for Bus Routing: Final Report," State University of New York Research Foundation, Albany, New York, July, 1970.

27. Orloff, C., "A Fundamental Problem in Vehicle Routing," to appear in the Journal of Networks.

28. Pierce, J.F., "On the Truck Dispatching Problem - Part I," IBM Cambridge Scientific Center Report 320-2018, Cambridge, Massachusetts, July, 1967. Revised December, 1967.

29. Pierce, J.F. and Hatfield, D.J., "Production Sequencing by Combinatorial Programming," IBM Cambridge Scientific Center Report 320-2000, Cambridge Massachusetts, 1966.

30. Stricker, R., Public Sector Vehicle Routing: the Chinese Postman Problem, thesis presented to M.I.T. in September, 1970, in partial fulfillment for the degree of Master of Science.