RESEARCH OBJECTIVES

In this research we seek to establish theoretical models for the recognition process, using as a basis the fact that it seems possible to describe recognition in terms of generalized symmetry considerations.

To illustrate this statement, consider the concomitant question of "likeness" as it seems to be handled by the brain. In doing so, we can only observe the external results that are due to the functional behavior of the brain, since we do not know the process, nor the specific criteria by means of which the brain reaches the conclusion that two things are alike or not alike. The brain bases recognition primarily on subjective elements, and secondarily on objective ones. This makes the study of recognition both difficult and challenging. Considering the accuracy with which the brain resolves questions of likeness, we naturally conclude that recognition is indeed a remarkable process and of the utmost importance to study. By observing how the brain resolves questions of this kind and how the results seem to be interrelated, we may be able to make an abstract characterization of the process involved. A moment's reflection tells us that "likeness," as conceived by the brain, satisfies the abstract postulates of group theory; therefore, the process is symmetric. One of the fundamental motivations of this research is to investigate the validity of this postulation and to determine its usefulness. Moreover, this symmetric conception of recognition may provide a fundamental means of visualizing the nature of "subjectivity."

In very few cases do the elements that we observe contain all of the symmetries characterizing a recognition process. In general, the observed elements contribute only partially in supplying the required symmetries; that is, in producing the group structure that characterizes the process. The brain itself produces the rest. Our research aims are the following: (a) to produce experimental evidence demonstrating the fact that the process of recognition can be completely characterized by a set of symmetries; (b) to isolate the elements under which the symmetrization should fall; (c) to find group structures of a process of recognition as a whole; and (d) to find the way in which the symmetries of the process are decomposed into those that come from external observation and those that are supplied by the brain. Our goal is to produce a plausible model of a process of recognition based on external observation, rather than on internal brain function.

The desire to study specific examples of recognition processes has motivated a number of experiments, one of which concerns handwriting. The analysis and synthesis of handwriting signals have both been examined in some detail in order to obtain an understanding of the handwriting mechanism. Accordingly, a system for measuring the displacement, velocity, and acceleration of handwriting movements has been developed. Samples of handwriting processed by this system indicate that the acceleration waveforms of uninterrupted handwriting approximate multilevel trapezoidal time functions. Electronic simulation of the measured displacement, velocity, and acceleration waveforms of handwriting has been accomplished. The "handwriting" generated by the electronic simulator can be made to duplicate human handwriting to a high degree of "likeness."

The handwriting simulator, in effect, represents a point-mass driven by a trapezoidal "force" function. Although the biological system that produces handwriting is highly complex, the motions involved can be duplicated in terms of an extremely simple model.

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Some preliminary results that are directed toward the establishment of relations between the model and the biological system responsible for handwriting have been obtained. These will be pursued further. Applications to neurological studies and medical diagnostics are being considered, as well as problems related to the recognition process.


M. V. Cerrillo, J. S. MacDonald, H. J. Zimmermann

A. INVESTIGATION OF HANDWRITING SIGNALS

In Quarterly Progress Report No. 72 (pages 187-191) a simple simulator that was capable of producing handwriting-like outputs was described. Since that report, improvements have been made to the simulator and measurements have been made of actual handwriting samples which indicate that the acceleration waveforms of uninterrupted handwriting approximate multilevel trapezoidal time functions.

The system that is used to obtain the vertical and horizontal projections of the displacement, velocity, and acceleration waveforms of handwriting motions is shown in the block diagram of Fig. XVIII-1.

![Block Diagram of the Handwriting Signal Analyzer](image)

Fig. XVIII-1. The Handwriting Signal Analyzer.

The displacement transducer produces electrical signals proportional to vertical and horizontal projections of displacement of the pencil point. These signals are lowpass-filtered (the 3-db point of the filter is 100 cps) and differentiated to produce the velocity signals. The velocity signals are again lowpass-filtered (3-db point, 100 cps) and differentiated to produce the acceleration signal. The 180-cps notch filter was necessary
to cut out the strong component at that frequency which was present in the laboratory at the time the measurements were made.

1. Measurement of Simple Movements

The first experiments consisted in measuring the displacement, velocity, and acceleration of simple movements, such as straight to-and-fro movements and rotary movements. Examples of these appear in Figs. XVIII-2, XVIII-3, and XVIII-4. The effects of friction and relative movement of the pencil with respect to the fingers are in evidence here. Examination of these waveforms reveals that the accelerations are closely approximated by trapezoidal time functions. The displacement transducer was an electrolytic tank that afforded a convenient means of studying the effect of friction between the pencil and the writing surface. Figure XVIII-2a is a record of the displacement, velocity, and acceleration of a simple to-and-fro motion. The gross structure of the acceleration waveform is that of a trapezoidal wave, but the fine structure is too large to be ignored. Dominant among this fine structure are the large spikes occurring midway between the obvious discontinuities in the acceleration waveform (one of these is circled in Fig. XVIII-2a). That these spikes are caused by friction is demonstrated by the fact that they line up with zero crossings of velocity, and disappear when the pencil is lifted from the bottom of the tank as shown in Fig. XVIII-2b.

Figure XVIII-3 demonstrates further that the spikes of Fig. XVIII-2a are caused by static friction, since they do not appear in the vertical projection of the acceleration of rotary motion, regardless of whether the pencil bears on the bottom of the tank.

Figure XVIII-4 shows that quite a bit of the remaining noise on the acceleration waveform is caused by the mechanical factors involved in gripping the pencil with the fingers. Figure XVIII-4a shows the waveforms associated with simple to-and-fro motion. Figure XVIII-4b shows the resulting waveforms for the same motion when the pencil is clamped tightly to the fingers. Static friction spikes appear in both records, since the pencil rested on the bottom during both of these experiments. Aside from the static-friction spikes, it can be seen that the acceleration waveform that appears when the clamp is on is much cleaner than with the clamp off. The order of magnitude of force represented by the changes of acceleration referred to as fine structure is not large enough to be sensed as a pressure change on the pencil grip.

2. Handwriting Movements

Figures XVIII-5, XVIII-6, and XVIII-7 are typical examples of waveforms associated with handwriting movements. In each figure, the acceleration waveforms are shown twice, once with trapezoidal approximations superimposed, and once without. In making the trapezoidal approximations, the small variations in the acceleration are ignored, as
Fig. XVIII-2. Vertical motion – effect of friction.
Fig. XVIII-3. Rotary motion – vertical projection.
Fig. XVIII-4. Vertical motion.
Fig. XVIII-5. Handwriting sample.
Fig. XVIII-6. Handwriting sample.
Fig. XVIII-7. Handwriting sample.
are the spikes caused by static friction.

3. Handwriting Simulation

A block diagram of the handwriting simulator appears in Fig. XVIII-8. The simulator consists of two principal parts: a linear system that simulates a point mass possessing a small amount of viscous friction, and the trapezoidal wave generator that provides the "force" signal for the linear system. The equation simulated is

$$m \frac{d^2x}{dt^2} + k \frac{dx}{dt} = f(t),$$

where $f(t)$ is trapezoidal.

The use of an oscilloscope for displaying the output of the system requires two linear systems and two trapezoidal wave generators, one for vertical deflection, the other for horizontal deflection. A synchronizing system provides the start pulse for the trapezoidal wave generators and the reset signals for discharging the integration capacitors in the linear system.

Operation of the simulator is as follows: The synchronizing system contains an oscillator that generates integrator reset signals at a frequency that can be set by a front-panel control. The integrator reset signal is of 5-msec duration (reset interval or clamp interval). The start pulse for the trapezoidal wave generators occurs at the end of the reset interval. (See timing diagram, Fig. XVIII-9.) The start pulse initiates a rectangular-wave burst which is shaped into trapezoidal form. As described
previously,\(^1\) each segment of the trapezoidal wave can be varied by two controls on the generator, one of which adjusts the amplitude of the segment, the other its duration.

When simulating a specific sample of handwriting, the sample to be imitated is recorded on transparency film or thin paper, and inserted into an optical device that superimposes its image on that from the cathode-ray tube screen of the simulator. The various segments of the trapezoidal waves of the simulator are adjusted until the two images coincide. Figure XVIII-10 shows three steps in setting up a simulation of the word "brew." The displayed waveforms are, from top to bottom: vertical output \(y(t)\); vertical trapezoidal wave; horizontal output \(x(t)\); and horizontal trapezoidal wave. The display period for a typical four-letter word is approximately 40-50 msec, which represents a time scaling approximately 20-30 times faster than actual handwriting.

Figures XVIII-11 and XVIII-12 give examples of typical simulations carried out by using the simulator just described. In each of these, the trapezoidal waveforms were set up to match as closely as possible the trapezoidal approximations to the actual acceleration waveforms (see Figs. XVIII-5 and XVIII-6). The trapezoidal waves were then trimmed to give close match between the output and the original sample.

The leading and trailing slopes of the trapezoidal waves are independently adjustable, but there is no independent adjustment for each segment of the wave. Inspection of
Fig. XVIII-10. Setting up the simulator.
Fig. XVIII-11. Simulation of the sample of Fig. XVIII-5.
Fig. XVIII-12. Simulation of the sample of Fig. XVIII-6.
Figs. XVIII-5, XVIII-6, and XVIII-7 reveals that the slopes on the various segments of the trapezoidal waves are not the same. This, together with the fact that the effect of static friction is neglected, accounts for the most noticeable discrepancies between the original and simulated handwriting.

Figure XVIII-11 is a simulation of the word "brew" displayed in Fig. XVIII-5. In Fig. XVIII-11a, the top word is the simulation, and the bottom one is the original word written by the subject. Figure XVIII-11b shows waveforms taken from the simulator which correspond to those of Fig. XVIII-5a. The effect of the uniform leading slope required by the simulator can be seen as a more rounded tip on the first negative peak in the vertical velocity waveform of Fig. XVIII-11b than appears on the same waveform in Fig. XVIII-5a. This happens because the slope on the simulated vertical acceleration waveform is too low at this point. On the other hand, the fifth positive segment of the vertical acceleration waveform of Fig. XVIII-11b is wider than it should be because the slope at this point is too large compared with the actual waveform of Fig. XVIII-5a. Figure XVIII-11c shows the set of simulator waveforms corresponding to Fig. XVIII-5b. Figure XVIII-12 is a simulation of the word "fell" displayed in Fig. XVIII-6.

The work summarized here indicates that in spite of the complicated structure of the biomechanical system responsible for producing handwriting, it is possible to model that system insofar as its output signals are concerned as

\[ \frac{d^2x}{dt^2} + a \frac{dx}{dt} = f(t), \]

where \( f(t) \) is a trapezoidal time function. Our measurements set an upper bound on the value of the damping parameter

\[ a \leq \frac{0.3}{T}, \]

where \( T \) is the length of the longest segment of the trapezoidal wave. Although the noise in the system prevented a more exact determination, this bound is felt to be very conservative. Even this bound is such that the damping term can be neglected.

3. Further Observations

Although we have not yet probed into the biological implications of the measurements described above, several interesting phenomena have appeared: The first of these concerns the nature of the acceleration waveform for varying speeds of movement. Figure XVIII-13 shows various sections of a continuous record of movement with steadily increasing speed. The entire record was taken with the pencil lifted to eliminate the effects of static friction. Figure XVIII-13a and 13b shows the displacement, velocity,
Fig. XVIII-13. Waveforms for various speeds of movement.
and acceleration of wrist movement for relatively slow movement. Notice that the acceleration is very "pulselike," and the control is apparently discontinuous. In fact, the acceleration waveforms in these two records are quite amenable to trapezoidal approximation, although each section of the trapezoid is very short and contributes very little to the displacement, as contrasted with the trapezoidal sections shown previously.

Figure XVIII-13b shows the beginning of the trend of the small trapezoidal segments of the acceleration waveform to form a larger trapezoidal pattern, in that they alternately predominate above and below the zero axis. Figure XVIII-13c shows the transition to this situation in which, near the end of this record, the fine structure in the acceleration waveform alternates above and below the axis to give the over-all appearance of a large trapezoidal wave. As the speed of movement continues to increase, there is a quite sudden transition to a "clean" trapezoidal wave. This transition is shown in Fig. XVIII-13d in which, toward the end of this record, the acceleration waveform is of the nature of those found in handwriting. Figure XVIII-13e shows the transition from conventional trapezoidal form to a more triangularly shaped waveform as the frequency of the movement reaches its highest value. The frequency at this point is approximately 4 cycles, and although it is possible to go a cycle or two faster than this, there is no change in the basic shape of the waveforms beyond this point. Movement of the type shown in Fig. XVIII-13a and 13c will be referred to as "continuously controlled movements" (that is, continuous control in the context of a sampled system), while those in Fig. XVIII-13d and 13e will be called "stroke-controlled" movements. The inference is that slower movements are controlled in a semicontinuous fashion, whereas fast movements are controlled on a stroke-by-stroke basis.

Examples of both types of control of movement are sometimes found in the same handwriting sample. Such a sample appears in Fig. XVIII-13f. Here the subject tried to write "John" as slowly as possible while maintaining continuous movement. The vertical projections of displacement and acceleration are shown. Continuous control is evident in the first part of the sample, while the last part is stroke-controlled. Section "a" of the stroke-controlled portion represents the transition from one type of control to the other (as in Fig. XVIII-13d) as the speed of the writing increased slightly throughout the execution of the sample. Figure XVIII-13g shows the same word written as fast as possible by the same person. The last part of the word is segmented to correspond to the segmentation of the stroke-controlled portion of Fig. XVIII-13f. The two extra segments occur because the last letter turned out to be an "m" instead of an "n," but that is what happens when writing is hurried.

The records presented in Fig. XVIII-13 show that the acceleration of carefully controlled movements can be approximated by trapezoidal waveforms independently of the speed of the movement, but there are two distinct types of that movement: one that we have called "continuous," and the other that we have called "stroke."
Fig. XVIII-14. Waveforms from action of different muscle groups.
Correlation has been observed between the size of the member (and hence the musculature) that is moved in a particular motion and the rise-time of the trapezoidal acceleration waveform.

Figure XVIII-14 illustrates this very well. The four sets of waveforms were made by movement of certain specific parts of the anatomy. Figure XVIII-14a shows to-and-fro motion of the fingers holding the pencil. Figure XVIII-14b is a record of motion of the fingers from left to right while holding a pencil. In Figure XVIII-14c motion is about the wrist joint; that is, with the fingers held rigid, the hand is swung from left to right. In Fig. XVIII-14d waveforms obtained when the entire arm is moved to and fro (that is, rotation about the shoulder joint) are shown.

The rise-time of the acceleration waveforms is seen to increase with increasing size of the moving member. The rise-time of the acceleration in Fig. XVIII-14d, which results from the large muscles acting upon the shoulder joint, is approximately 0.1 second. This rise-time is so long that for the repetition rate of the movement shown, the trapezoidal wave degenerates into a triangular wave.

Details concerning the work summarized here can be found in the author's doctoral thesis. The research is at present continuing toward the development of an improved simulator, and investigation of possible relations between the properties of handwriting signals and the functional principles of the biological system that produces them. We have also begun to attempt to apply the ideas expressed in our research objectives to the recognition of handwriting.

J. S. MacDonald

References

1. J. S. MacDonald, Rectangular wave generator, Quarterly Progress Report No. 73, Research Laboratory of Electronics, M. I. T., April 15, 1964, pp. 135-139.
