XIX. SPONTANEOUS RADIOFREQUENCY EMISSION FROM
HOT ELECTRON PLASMAS*

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RESEARCH OBJECTIVES

The production of a hot plasma usually results in an anisotropic velocity distribution
function for the constituent particles. Such distribution functions are known to be unstable
(velocity-space instabilities), and experimentally are often identified by the radiofre-
quency emissions that they produce.

The pulsed electron-cyclotron discharge, which we have studied in the past, produces
a hot electron plasma. In the "afterglow" (in between the applied microwave pulses) we
have observed strong emission near the electron-cyclotron frequency and its harmonics.

We plan to continue our theoretical studies on velocity-space instabilities, and the
experimental study of the enhanced cyclotron emission observed in the afterglow of our
electron-cyclotron discharge.

A. Bers

A. ELECTROSTATIC PLASMA INSTABILITIES AT CYCLOTRON HARMONICS

We have continued to study instabilities in a neutral unbounded magnetized plasma
composed of infinitely massive ions and electrons that are assumed to have the relativ-
istic velocity distribution

\[ f_0(v) = \delta(v_\perp - v_\perp 0) \delta(v_\parallel)/2\pi v_\perp 0, \]

in which the subscripts \( \perp \) and \( \parallel \) stand for perpendicular and parallel to the static mag-
netic field. Under the assumption of \( \exp\{\omega - jk \cdot r\} \) dependence for the field quantities,
a simultaneous solution of the linearized Vlasov equation and Maxwell's equations has
been shown to result in the electrostatic (\( k \) along \( \vec{E} \)) dispersion relation

\[ \left( \frac{k_\parallel}{k_\perp} \right)^2 = \frac{1 - \frac{1}{2} \left( \frac{\omega}{\omega_b} \right)^2 \sum_{n=-\infty}^{\infty} \frac{J^2_{n-1} - J^2_{n+1}}{J_n^2} + \left( \frac{\beta_\parallel \omega}{\omega_b} \right)^2 \sum_{n=-\infty}^{\infty} \frac{n^2 J_n^2}{\left( \frac{\omega}{\omega_b} - n \right)^2}}{1 - \left( \frac{\omega}{\omega_b} \right)^2 \sum_{n=-\infty}^{\infty} \frac{J_n^2}{\left( \frac{\omega}{\omega_b} - n \right)^2}}, \]

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where $\omega_p$ and $\omega_b$ are the relativistic plasma and cyclotron frequencies, respectively, $\beta_\perp$ is the ratio of the electron velocity to the speed of light in free space, and $J_n$ is the $n^{th}$ order Bessel function of argument $p = k_1 v_0 / \omega_b$. The nonrelativistic form of this equation, which is obtained by setting $\beta_\perp = 0$, has been studied by Bers and Gruber who investigated the small-signal energy implications of the resulting waves, and by Gruber, Klein, and Auer who showed that for large enough $\omega_p / \omega_b$ waves propagating nearly along the static magnetic field were unstable. Dory, Guest, and Harris have claimed that the nonrelativistic equation predicts that waves propagating perpendicular to the magnetic field are stable unless $\omega_p / \omega_b$ exceeds 2.7. As we have shown in a previous report, these instabilities occur near $\omega / \omega_b = n + \frac{1}{2}$, where $n$ is an integer. Our present work, including the relativistic term, predicts unstable waves propagating nearly perpendicular to the magnetic field at values of $\omega_p / \omega_b$ considerably less than the threshold stated by other authors. In contrast to the previous reported instabilities, these new modes, which are purely relativistic in origin, occur at frequencies close to integer multiples of the cyclotron frequency. Figure XIX-1 illustrates the results of a numerical analysis of the dispersion relation. The curves were obtained for $\omega_p / \omega_b = 0.5$ and $v_0 / c = 0.1$. Note that near each cyclotron harmonic there are two branches of the dispersion relation. At a given value of $k_1 v_0 / \omega_b$, the curves are similar to those reported by Bers and Gruber,

![Fig. XIX-1. Dispersion diagram.](image)
except that near some cyclotron harmonics the two branches comprising the particular mode have coupled so as to require complex frequency for all \( k_\parallel /k_\perp \) less than a critical value. The figure illustrates that the particular modes that are unstable depend upon the value of \( k_\perp v_{0\parallel}/\omega_b \). For convenience, the root locus of purely real roots has been drawn on the \( k_\parallel /k_\perp = 0 \) plane. We have reported a more complete analysis of these roots previously.\(^5\)

Physically, the instabilities have been interpreted in terms of small-signal energy arguments. Evaluation of the small-signal time-average energy density\(^6\) has shown that the lower frequency wave at each uncoupled harmonic mode carries negative energy, whereas the higher frequency wave carries positive energy. The instabilities occur when the plasma parameters are such as to result in an internal coupling of the two waves.

We are now investigating an exact electromagnetic solution of the waves in this plasma and are considering the physical mechanism that results in the relativistic instability.

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References
