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A Dynamic Analysis of Short versus Long-Term Policies*

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Abstract

Recent anti-trust cases exacerbated the concerns of investors regarding the effects of a firm's monopoly power on its production choice, shareholder value, and the overall economy. We address this issue within a dynamic equilibrium model featuring a large monopolistic firm whose actions not only affect the price of its output, but also effectively influence the valuation of its stock. The latter renders time-inconsistency to the firm's dynamic production choice. When the firm is required to pre-commit to its strategy, the ensuing equilibrium is largely in line with the predictions of the textbook monopoly model. When the firm behaves in a time-consistent manner, however, the predictions are strikingly at odds. The trade-off between current profits and the valuation of future profits induces the firm to increase production beyond the competitive benchmark and cut prices. This policy may result in destroying shareholder value, and does indeed fully wipe out the firm's profit in the limit of the decision-making interval shrinking to zero, in line with the Coase conjecture.

JEL Classifications: D42, D51, D92, E20, G12

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1. Introduction

The pervasiveness of monopoly power, across many product categories in almost every part of the world economy, has long pre-occupied economists and lawmakers. There is also undisputed evidence that monopoly power is widespread amongst firms dominant enough to matter at an economy-wide level, the so-called “bellwethers.” This is evidenced by the series of ongoing anti-trust cases brought about by the U.S. government against, for example, IBM (1969 - 1982), AT&T (1974 - 1982), Microsoft (1994 - 1995, 1998 - present), and Bell Atlantic (1996 - 1997). Apparently, there is concern that these large monopolists may not just exert power in their own product markets, but their actions may impact the overall economy. For example, as argued in the Microsoft case, US consumers and businesses alike feared to become critically dependent on Microsoft products. There are well-known dominant players outside the US, too. For instance, OPEC’s production decisions appear to affect not only the oil producing countries’ economies, but also the overall world economy. While the surveys of imperfect competition by Hart (1985) and Bonanno (1990) have highlighted the importance of a general equilibrium analysis of such large monopolists, most studies of monopoly behavior have been undertaken at a partial equilibrium level assuming no impact on other markets. A true general equilibrium approach must face up to the challenge of accounting for the so-called “feedback” or “Ford effect,” as once argued by Henry Ford: through the influence of its own actions, a non-price-taking firm may affect the wealth of its customers, and hence the demand for the firm’s product. Finally, although there is growing work on general equilibrium asset pricing with market imperfections (e.g., see the survey by Sundaresan (2000)), the consideration of monopoly power is still missing in this literature.

Our primary objective in this paper is to investigate the optimal behavior of a monopolist who has sufficient power to impact economy-wide pricing. We model the extreme case of an economy containing a single monopolistic firm who then, at a general equilibrium level, impacts the overall price of consumption in the economy. Beyond the traditional assumption that its actions impact the price of its own good, the firm’s actions also influence the valuation of its stock. As will become evident from our analysis, the dynamic setting we employ leads to a distinction between short-term and long-term policies of the monopolistic firm. (For a competitive firm, there is no such distinction.) Part of our emphasis will be to study the differences between the ensuing equilibria under short versus long-term policies.¹

We adopt a familiar Robinson-Crusoe formulation of a discrete-time production finite-horizon

¹Firms’ short-termism is receiving increasing attention from both academics and practitioners. For example, *Business Week* (09/13/1999) voices the often-quoted complaint that Wall Street exerts increasing pressure on many firms for short-term results due to a shrinking of investors’ time horizons. The article points out that a share in AT&T is held for an average of 1.1 years, down from 3 years in 1990; a share in General Motors is held for an average of 1 year, down from 2 years in 1990. Firms’ short-termism is also frequently blamed on ever increasing managerial turnover. Anecdotal evidence and recent academic literature (Allen and Gale (2000), Palley (1997)) seem to agree that revolving management may be damaging for a long-lived firm.

economy populated by a representative consumer-investor-worker and a representative firm. The consumer derives utility from consumption and leisure, simultaneously invests in financial markets (including the firm's stock), and earns a labor income. The consumer's labor is demanded by the firm as the sole input to a stochastic non-constant-returns-to-scale production technology, producing the only good in the economy. We assume the objective of this firm is to maximize its market value, or the present value of its expected profits. The firm has monopoly power in the good market, in that it takes account of the impact (via market clearing) of its production plan on the price of its output. In our setting, the monopoly power manifests itself as an impact of the firm's labor demand on the state prices (or the pricing kernel). This manipulation of state prices results in time-inconsistency of the firm's production strategy, in that it has an incentive to deviate from the initial plan at a later date. To rule out time-inconsistency, we focus on two distinct types of monopolistic strategies: a "pre-commitment" strategy in which the firm initially chooses a plan to maximize its initial value, and thereafter cannot deviate from that plan; and a "time-consistent" strategy in which the firm chooses a plan each period, maximizing the value at that period, taking into account the re-adjustments that it will make in the future. Consistently with the literature (e.g., Blanchard and Fisher (1989, §11.4)), we interpret the time-consistent strategy as a short-sighted or short-term strategy, and the pre-commitment one as long-term.

Solving for the pre-committed monopolist's strategy reveals his optimal plan and the extent of his monopoly power to be driven by the concurrent profit, the marginal product of labor, and the consumer's attitude towards risk over consumption. The most immediate implications on the ensuing dynamic equilibrium are consistent with the predictions of textbook static monopoly models: lower good output and lower labor input, and higher price of consumption than the competitive counterpart. However, in contrast to the textbook case, profits and the firm's value can be either higher or lower. This arises because it is the firm's initial value the monopolist maximizes, not the profits nor the value at later times. By restricting production, he moves away from the competitive, profit maximizing, production plan.

The optimal behavior of the time-consistent monopolist contrasts sharply with that of the pre-committed monopolist. His optimal production plan and his monopoly power are driven by the negative of the ex-dividend stock price, in place of concurrent profits. In attempting to maximize the current stock valuation, this monopolist trades off between today's profit and the ex-dividend stock price; hence the appearance of the ex-dividend price. In direct opposition to the pre-commitment case, the equilibrium good output and labor demand are higher than the competitive case, while the price of consumption is lower. It is in this short-term monopolist's interest to depress the current price of consumption, so as to boost today's stock price. Yet more strikingly, the profits in every period are decreased in the monopolist's presence, and may even go negative, while the firm's value can be either lower or higher than in the competitive benchmark. We also argue that our main conclusions regarding the time-consistent equilibrium

hold true in the infinite horizon limit of our economy within a parametric example. To further explore the robustness of these results, we provide an alternative, albeit extreme, form of a production opportunity (constant-returns-to-scale) under which the monopoly power vanishes, and the solution coincides with the competitive.

It is well-recognized in monopolistic models of durable goods, that absent commitment the monopolistic firm may in a sense “compete with itself” across different time periods, and weaken its monopoly power. The Coase (1972) conjecture proposes that as the time interval between successive decisions is reduced to an infinitesimal length, this intertemporal competition will drive the firm’s profits to zero. Since our long-lived firm’s stock resembles a durable good, it is of interest to explore the limiting case of our economies where the decision-making time interval shrinks to zero. While the competitive and the pre-commitment monopolistic equilibrium retain their basic discrete-time structure and implications, we find the time-consistent equilibrium to tend to the limit of zero profits and hence zero firm’s value at all times. Hence, under the time-consistent scenario, monopoly power destroys shareholder value. However, within our framework, this zero profit limit does not coincide with the competitive solution.

The importance of imperfect competition is, of course, well-recognized in many areas of current theoretical and applied research, including international trade (Grossman (1992)), industrial organization (Tirole (1988)), corporate finance (Brander and Lewis (1986)); our interest is in the interplay of imperfect competition in product markets with the valuation of securities by the financial market. The standard textbook treatment of monopoly (Mas-Colell, Whinston and Green (1995, Chapter 12), Tirole (1988, Chapter 1)) considers a profit-maximizing firm which is the only producer of a good (and has no influence on other markets), in a static partial equilibrium setting. The primary implications of the textbook monopolist are restriction of production and a raising of the price of output in the market for its product. While this behavior is consistent with the equilibrium implications of our pre-committed monopolist, it is at odds with those of the time-consistent monopolist. The distinction is due to our accounting for the economy-wide impact of the firm’s decision and in particular for the impact on its stock price valuation.

In that regard, the most closely related work in a general equilibrium setting where the valuation of financial securities is affected by market power are the works of Basak (1997) and Kihlstrom (1998). These authors consider a single large consumer-investor acting as a non-price-taker in securities markets, in a Lucas (1978)-type pure-exchange setting.² While the monopolists in Kihlstrom and Basak select a consumption-investment plan to maximize utility, our monopolist selects a production plan to maximize the stock price of his firm. In this respect, ours is much closer to the standard textbook monopolist. Basak demonstrates that the non-price-taker acting as a price-leader in all markets manifests itself as a dependence of the state prices on the agent’s

²See also Lindenberg (1979) and Grinblatt and Ross (1985) for related analysis within a static mean-variance framework.

consumption choice. Kihlstrom relates the dynamic security price choice of the monopolist to the Coase (1972) conjecture, and shows that the inability of the monopolist to commit to the future (second period) price quote reduces his monopoly rents. Consequently, the first period security price is less than in the commitment scenario (but is still higher than the competitive price). In the spirit of Kihlstrom, but with an additional moral hazard problem of the monopolist, is the dynamic model of DeMarzo and Urosevic (2001).³ Absent commitment, they demonstrate an analog of the Coase conjecture in the continuous-time limit of their economy.

The rest of the paper is organized as follows. Section 2 describes the economy. Section 3 characterizes equilibrium in the economy with a monopolistic firm for the cases when the firm can commit to its future production plan and when it cannot. It also presents comparison of the resulting equilibrium quantities to those of a benchmark competitive economy. In Section 4, we consider the continuous-time limit of our economy and explore the Coase conjecture in our framework. Section 5 concludes and the Appendix provides all proofs as well as a discussion of the case of a monopolistic-monopsonistic firm.

2. The Economy

We consider a simple Robinson-Crusoe production economy with a representative firm and a representative consumer-investor-worker. We make the standard assumption that the consumer-investor-worker represents a continuum of identical atomistic agents who take prices as given and cannot act strategically. The economy has a finite horizon $[0, T]$, in which trading takes place at discrete times $t = 0, \dots, T$. There is a single consumption good serving as the numeraire (other choices of the numeraire are discussed in Remark 1). Uncertainty is represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t; t = 0, 1, \dots, T\}, \mathcal{P})$ generated by a production shock process ε . All stochastic processes are assumed adapted to $\{\mathcal{F}_t; t = 0, 1, \dots, T\}$, all stated (in)equalities involving random variables hold \mathcal{P} -almost surely. We assume all processes and expectations are well-defined, without explicitly stating the required regularity conditions.

The financial investment opportunities are represented by: a risky stock of the firm in constant net supply of 1 share that pays out dividends $\pi(t)$, $t = 1, \dots, T$; and enough zero net supply securities to dynamically complete the market. π is endogenously determined via the firm's optimization problem. Dynamic market completeness allows the construction of a unique system of Arrow-Debreu securities consistent with no arbitrage. Accordingly, we may define the *state price density process* ξ (or the pricing kernel $\xi(s)/\xi(t)$, $s \geq t$), where $\xi(t, \omega)$ is interpreted as the Arrow-Debreu price (per unit probability \mathcal{P}) of a unit of consumption good in state $\omega \in \Omega$ at

³See also DeMarzo and Bizer (1993), who were the first to point out the connection between durable goods and securities markets. DeMarzo and Urosevic (2001) also provide a comprehensive review and classification of the literature.

time t . The process ξ is to be determined in equilibrium. The time- t (cum-dividend) value $V(t)$ of the stock (of the firm) is then given by

$$V(t) = E \left[\sum_{s=t}^T \frac{\xi(s)}{\xi(t)} \pi(s) \mid \mathcal{F}_t \right]. \quad (1)$$

As evident from the subsequent analysis, our main results are valid in a deterministic version of our model, where ξ has only one value in each period. We introduce uncertainty to make our formulation comparable to financial markets models, standard in the literature.

2.1. The Consumer's Preferences and Optimization Problem

A representative consumer-investor-worker is endowed at time 0 with 1 share of the stock, and at each time t with $\bar{\ell}$ units of available labor, to allocate between leisure $h(t)$, and labor $\ell(t)$, for which he is paid a wage at rate $w(t)$. The consumer intertemporally chooses a non-negative consumption process c , labor process ℓ , and portfolio (of securities) process so as to maximize his lifetime utility.⁴ The consumer derives a separable von Neumann-Morgenstern time-additive, state-independent utility $u(c(t)) + v(h(t))$ from consumption and leisure in $[1, T]$. The functions u and v are assumed three times continuously differentiable, strictly increasing and strictly concave, and to satisfy $\lim_{c \rightarrow 0} u'(c) = \infty$, $\lim_{h \rightarrow 0} v'(h) = \infty$.⁵

Under the complete markets assumption, the consumer-worker's dynamic optimization problem can be cast in its Arrow-Debreu formulation as a static variational problem with a single budget constraint:

$$\max_{c, \ell} E \left[\sum_{t=1}^T u(c(t)) + v(\bar{\ell} - \ell(t)) \right] \quad (2)$$

$$\text{subject to } E \left[\sum_{t=1}^T \xi(t) (c(t) - w(t) \ell(t)) \right] \leq E \left[\sum_{t=1}^T \xi(t) \pi(t) \right]. \quad (3)$$

We do not explicitly apply the nonnegativity constraints $c(t) \geq 0$, $\ell(t) \leq \bar{\ell}$, $\ell(t) \geq 0$, because the conditions $\lim_{c \rightarrow 0} u'(c) = \infty$ and $\lim_{h \rightarrow 0} v'(h) = \infty$ guarantee $c(t) > 0$, $\ell(t) < \bar{\ell}$, while (in the equilibrium provided) the firm's production technology (Section 2.2) guarantees $\ell(t) \geq 0$.

The first-order conditions of the static problem (2)–(3) are

$$u'(c(t)) = y \xi(t), \quad (4)$$

$$v'(\bar{\ell} - \ell(t)) = y \xi(t) w(t), \quad (5)$$

⁴We introduce labor as one of the simplest, most commonly-adopted choices of input to the firm's production function; employing labor does not introduce the intertemporal complexity of employing capital.

⁵We make the assumption of separable utility for simplicity, given our intention is to focus on a comparison of monopolistic behavior in a particular good market. Our analysis readily extends to the general non-separable case $u(c, h)$. Our major comparative statics results (Propositions 2 and 4) remain valid under the assumption $u_{ch} \geq 0$ (which includes separable utility and Cobb-Douglas $u(c, h) = \frac{1}{\gamma} (c^\rho h^{1-\rho})^\gamma$, for $\gamma \in (0, 1)$.)

where the Lagrangian multiplier y satisfies

$$E \left[\sum_{t=1}^T \xi(t) (c(t) - w(t) \ell(t)) \right] = E \left[\sum_{t=1}^T \xi(t) \pi(t) \right]. \quad (6)$$

Consequently,

$$\frac{v'(\bar{\ell} - \ell(t))}{u'(c(t))} = w(t). \quad (7)$$

2.2. The Firm's Production and Optimization

The representative firm in this economy faces the same information structure and set of securities as the consumer. At each time $t = 1, \dots, T$, the firm uses labor, $\ell^D(t)$, as its only input to a production technology, f , which provides consumption good as output.⁶ The technology is stochastic, driven by a shock process ε , assumed (without loss of generality) to be strictly positive. The firm's output at time t is given by $f(\ell^D(t), \varepsilon(t))$. We assume f is increasing and strictly concave in its first argument and that $\lim_{\ell^D \rightarrow 0} f(\ell^D, \varepsilon) = \infty$ and $\lim_{\ell^D \rightarrow 0} f(\ell^D, \varepsilon) \geq 0$. The firm pays out a wage $w(t)$ for each unit of labor it utilizes, so its time- t profit is

$$\pi(t) = f(\ell^D(t), \varepsilon(t)) - w(t) \ell^D(t), \quad (8)$$

all of which it pays out as dividends to its shareholders. The firm's objective is to maximize its market value, or the present value of its expected profits under various market structures.

The firm's behavior is the main focus of this work. For the purpose of comparison, we first consider the optimal choice of a competitive firm, and the resulting competitive equilibrium (Section 2.3), and then turn to an economy where a firm exercises monopoly power in the market for the consumption good (Section 3). In the latter set-up our assumption of the firm's maximizing its value is prone to the criticism that applies to all equilibrium models with imperfect competition. To this day, the issue whether value maximization is the appropriate objective is still open (see Remark 1). Our viewpoint in this paper is to simply adopt the most well-understood equilibrium concept, the Cournot-Walrasian equilibrium, and despite its possible criticisms, focus on the implications. In particular, we interpret our representative consumer-shareholder as a standard Walrasian price-taking agent. As such, he perceives his decisions as having no effect on the valuation of his consumption/labor stream, given by the left-hand side of the budget constraint (3). On the other hand, as a shareholder, he can affect the quantity on the right-hand side of (3), his initial wealth, which is equal to the value of the firm. Monotonicity of his indirect utility of wealth, then, justifies the firm's value maximization as a desired objective of its shareholder.

⁶For simplicity, we do not model the time-0 consumption/leisure and production choice. All our results for $t = 1, \dots, T$ quantities, in the propositions of the paper, would remain valid if the time-0 choice were additionally modeled.

2.3. Benchmark Competitive Equilibrium

The objective of the competitive firm is choose the input ℓ^D so as to maximize its time-0 value, $V(0)$, taking the state prices and wage as given. The first-order conditions for a competitive firm's problem are given by

$$f_\ell(\ell^D(t), \varepsilon(t)) - w(t) = 0 \quad t = 1, \dots, T. \quad (9)$$

Definition (Competitive Equilibrium). *An equilibrium in an economy of one competitive firm and one representative consumer-worker is a set of prices (ξ^c, w^c) and choices $(c^c, \ell^c, \ell^{D^c})$ such that (i) the consumer chooses his optimal consumption/labor policy given the state price and wage processes, (ii) the firm makes its optimal labor choice given the state prices and wage, and (iii) the consumption and labor markets clear at all times:*

$$c^c(t) = f(\ell^{D^c}(t), \varepsilon(t)) \quad \text{and} \quad \ell^c(t) = \ell^{D^c}(t). \quad (10)$$

It is straightforward to show from (7)–(10) that in the competitive equilibrium, the equilibrium labor $\ell^c = \ell^{D^c}$ is given by

$$f_\ell(\ell^c(t), \varepsilon(t)) - \frac{v'(\bar{\ell} - \ell^c(t))}{u'(f(\ell^c(t), \varepsilon(t)))} = 0 \quad (11)$$

and the equilibrium consumption and profit processes by

$$c^c(t) = f(\ell^c(t), \varepsilon(t)), \quad \pi^c(t) = f(\ell^c(t), \varepsilon(t)) - f_\ell(\ell^c(t), \varepsilon(t)) \ell^c(t). \quad (12)$$

The equilibrium state price density, wage and firm value processes are given by

$$\xi^c(t) = u'(f(\ell^c(t), \varepsilon(t))), \quad w^c(t) = f_\ell(\ell^c(t), \varepsilon(t)), \quad (13)$$

$$V^c(t) = E \left[\sum_{s=t}^T \frac{u'(f(\ell^c(s), \varepsilon(s)))}{u'(f(\ell^c(t), \varepsilon(t)))} \left\{ f(\ell^c(s), \varepsilon(s)) - f_\ell(\ell^c(s), \varepsilon(s)) \ell^c(s) \right\} \middle| \mathcal{F}_t \right]. \quad (14)$$

The above conditions present a fully analytical characterization of the equilibrium, with (11) determining the labor as a function of the shock ε and then (12)–(14) determining all other quantities. Equation (11) states that the marginal rate of substitution between consumption and leisure is equated to the marginal product of labor.

3. Monopolistic Equilibrium

In this section, we assume the consumption good market to be imperfect, in that the firm has monopoly power therein. We take the firm to act as a non-price-taker in its output market, taking into account the impact of its production plan choice on the price of output. The firm is still a price-taker in its input/labor market, taking the wages w as given. (The extension to the

case where the firm is additionally a non-price-taker in the labor market is straightforward and is discussed in Appendix B.)

We will observe that the firm’s production strategy is time-inconsistent, in the sense that a monopolist has an incentive to deviate from his time-0 plan at a later date. When the monopolist gets to time t , he no longer cares about the time-0 value of the firm; rather, he would like to revise the production plan so as to instead maximize the time- t value of the firm. In Section 3.1, we consider the optimal choice of a monopolistic firm that chooses an initial strategy so as to maximize its time-0 value and “pre-commits” to that strategy, not deviating at subsequent times. In Section 3.2, we solve for the time-consistent strategy of a monopolist who re-optimizes his production plan to maximize the firm’s current value at each date t , taking into account the fact that he is not restricted from revising this plan at the future dates $s = t + 1, \dots, T$. The former can be thought of as a “long-term” strategy, providing the first-best solution to the problem of maximizing the firm’s initial value. The latter can be interpreted as a “short-term” or “short-sighted” strategy since the firm continually re-optimizes every period to boost current value. We present the equilibrium for both cases.

3.1. The Pre-Commitment Case

We now formulate the monopolist’s optimization problem. Recalling the price-taking consumer’s demand (4), clearing in the consumption good market implies

$$\xi(t) = u'(f(\ell^D(t), \varepsilon(t)))/y. \tag{15}$$

The monopolist’s influence in the good market manifests itself, via (15), as a (non-linear) impact of its input demand on state prices. Accordingly, the pre-committed monopolist solves the following optimization problem at time 0:

$$\max_{\ell^D, \xi} V(0) \quad \text{subject to } \xi(t) = u'(f(\ell^D(t), \varepsilon(t)))/y, \quad \forall t = 1, \dots, T, \tag{16}$$

where y satisfies (6).

Proposition 1 presents the optimal solution to this problem, assuming it exists.⁷

Proposition 1. *The pre-commitment monopoly optimal labor demand ℓ^D , $t = 1, \dots, T$ satisfies*

⁷It is well-known that the objective function in models with monopolistic firms may not necessarily be concave and hence not satisfy the second-order conditions without additional assumptions (e.g., Tirole (1988, Chapter 1)). In the pre-commitment case a sufficient condition for concavity is $u'(c)f_{\ell\ell}(\ell, \varepsilon) + 2u''(c)f_{\ell}(\ell, \varepsilon)(f_{\ell}(\ell, \varepsilon) - w) + u''(c)f_{\ell\ell}(\ell, \varepsilon)(f(\ell, \varepsilon) - w\ell) + u'''(c)f_{\ell}^2(\ell, \varepsilon)(f(\ell, \varepsilon) - w\ell) < 0$, $c = f(\ell, \varepsilon)$; $\forall \varepsilon, \ell, w$. Examples of utilities and production functions that satisfy this condition include the commonly employed power preferences over consumption $u(c) = c^\gamma/\gamma$ with $\gamma \in (0, 1)$ and power production $f(\ell, \varepsilon) = \varepsilon\ell^\nu$, $\nu \in (0, 1)$ (no additional restriction on $v(h)$ is required).

$$f_\ell(t) - w(t) = A(t) f_\ell(t) \pi(t) > 0, \quad (17)$$

where

$$A(t) \equiv -\frac{u''(t)}{u'(t)},$$

and $f(t)$, $u(t)$ and their derivatives are shorthand for $f(\ell^D(t), \varepsilon(t))$, $u(f(\ell^D(t), \varepsilon(t)))$ and their derivatives.

The structure of the first-order conditions bears resemblance to that of the textbook single-period monopolist. Any direct increase in profit due to an extra unit of input must be counterbalanced by an indirect decrease via the impact of that extra unit on the concurrent price system. In our set-up, the extent of monopoly power is driven by the current profit, the marginal product of labor and the (positive) quantity A , which captures the consumer's attitude toward risk over consumption. (The quantity A can also be restated in terms of the textbook "monopoly markup.") The higher the marginal product the more responsive is output to an extra unit of input. The more "risk-averse" the consumer, the less his consumption reacts to changes in the state price, or conversely, the more the state price reacts to changes in his consumption, and so the more incentive the monopolist has to deviate from competitive behavior. In the limit of a risk-neutral investor (preferences quasi-linear with respect to consumption), the monopolist cannot affect the state price at all and so the best he can do is behave competitively.

We now turn to an analysis of equilibrium in this economy.

Definition (Monopolistic Pre-Commitment Equilibrium). *An equilibrium in an economy of one monopolistic firm and one representative consumer-worker is a set of prices (ξ^*, w^*) and choices (c^*, ℓ^*, ℓ^{D*}) such that (i) the consumer chooses his optimal consumption/labor policy given the state price and wage processes, (ii) the firm makes its optimal labor choice in (16) given the wage, and taking into account that the price system responds to clear the consumption good market, and (iii) the price system is such that the consumption and labor markets clear at all times:*

$$c^*(t) = f(\ell^{D*}(t), \varepsilon(t)) \quad \text{and} \quad \ell^*(t) = \ell^{D*}(t).$$

The fully analytical characterization of the equilibrium in the monopolistic pre-commitment economy is given by (18)–(21). Equilibrium is determined by computing the supply and demand for labor from the consumer's and firm's first-order conditions, and then applying labor market clearing $\ell^* = \ell^{D*}$ to yield the labor as a function of the shock ε (18).⁸ The remaining quantities are then straightforward to determine; we list them in (19)–(21). The equilibrium labor is given

⁸See Lemma A.1 of Appendix A for the existence of an interior solution $\ell^*(t) \in (0, \bar{\ell})$ to equation (18).

by

$$f_\ell(\ell^*(t), \varepsilon(t)) - \frac{v'(\bar{\ell} - \ell^*(t))}{u'(f(\ell^*(t), \varepsilon(t)))} = A(t)f_\ell(\ell^*(t), \varepsilon(t)) \left[f(\ell^*(t), \varepsilon(t)) - \frac{v'(\bar{\ell} - \ell^*(t))}{u'(f(\ell^*(t), \varepsilon(t)))} \ell^*(t) \right] \quad (18)$$

and the equilibrium consumption and profit processes by

$$c^*(t) = f(\ell^*(t), \varepsilon(t)), \quad \pi^*(t) = f(\ell^*(t), \varepsilon(t)) - \frac{v'(\bar{\ell} - \ell^*(t))}{u'(f(\ell^*(t), \varepsilon(t)))} \ell^*(t). \quad (19)$$

The equilibrium state price density, wage and firm value processes are given by

$$\xi^*(t) = u'(f(\ell^*(t), \varepsilon(t))), \quad w^*(t) = \frac{v'(\bar{\ell} - \ell^*(t))}{u'(f(\ell^*(t), \varepsilon(t)))}, \quad (20)$$

$$V^*(t) = E \left[\sum_{s=t}^T \frac{u'(f(\ell^*(s), \varepsilon(s)))}{u'(f(\ell^*(t), \varepsilon(t)))} \left\{ f(\ell^*(s), \varepsilon(s)) - \frac{v'(\bar{\ell} - \ell^*(s))}{u'(f(\ell^*(s), \varepsilon(s)))} \ell^*(s) \right\} \middle| \mathcal{F}_t \right]. \quad (21)$$

Proposition 2 summarizes the comparison of pertinent quantities across the monopolistic and competitive economies.

Proposition 2. *The equilibrium labor, output, state price and wage in the pre-commitment monopoly economy and competitive economy, satisfy for all $t = 1, 2, \dots, T$, all $\varepsilon(t)$:*

$$\ell^*(t) < \ell^c(t), \quad f(\ell^*(t), \varepsilon(t)) < f(\ell^c(t), \varepsilon(t)), \quad c^*(t) < c^c(t) \quad (22)$$

$$\xi^*(t) > \xi^c(t), \quad w^*(t) < w^c(t). \quad (23)$$

The firm's initial value satisfies,

$$V^*(0) > V^c(0).$$

However, the time- t profit $\pi^*(t) > 0$ and firm's value $V^*(t)$ can be either higher or lower than $\pi^c(t)$ and $V^c(t)$, respectively, $\forall t = 1, \dots, T$.

The monopolist has an incentive to manipulate the price of consumption by producing less than his competitive counterpart. Reducing the supply raises the price at which clearing occurs, thereby increasing the valuation of the monopolist's intertemporal profits along with his stock price. The firm needs to use less input and, accordingly, the wage rate decreases. These results are consistent with the textbook static monopoly analysis. A difference in our model is that the flow of profits, $\pi(t)$, in the monopolistic equilibrium may be lower than that in the competitive benchmark (as illustrated in Example 1 of Section 3.2); this is because it is the present value of profits that our monopolist is striving to maximize, not profits per se. By restricting production, the monopolist moves away from the competitive, profit maximizing, production plan. Since the monopolist is maximizing the firm's initial value and since he has market power, it is intuitive

that the firm’s initial value will be higher than in the competitive case. However, we have not forced him to maximize later values of the firm, so it is not surprising that these later values may lie higher or lower than the competitive case (as illustrated in Example 1).

The optimal policies and equilibrium in this subsection rely on the ability of the monopolist to pre-commit to his time-0 plan. Without this commitment, the monopolist would later want to deviate from his initial plan; that is, at any date $t > 0$, the solution to the firm’s problem

$$\max_{\ell^D(s), \xi(s), s \geq t} V(t) \quad \text{subject to } \xi(s) = u' \left(f(\ell^D(s), \varepsilon(s)) \right) / y, \quad \forall s = t, \dots, T$$

is different from his time-0 plan, unless $V(t) = 0$. Due to the intertemporal dependence of V on the price of consumption, if the monopolist solves his problem at the initial period, he would change his mind at a later stage about the optimal ξ process. In other words, his production plan is not time-consistent. A similar problem arises in the context of a durable good monopoly (e.g., Tirole (1988, Chapter 1)), or in the context of dynamic securities markets with non-price-taking investors (Basak (1997), Kihlstrom (1998)). The firm’s stock in our model is similar to a durable good since it provides value over many periods. If there were a credible mechanism for the monopolist to commit to his time-0 plan, the time-0 share price of the firm that he owns would be *ceteris paribus* the highest possible; if however such commitment is impossible, we will show that the monopolist might be better off behaving competitively (Example 1). Possible pre-commitment mechanisms may include (i) employing a production technology with a time-to-build feature, and following Tirole (1988), (ii) handing the firm over to a third party instructed to implement the optimal strategy with a penalty for deviating from it (although renegotiation would be a potential issue), (iii) long-term relationships/contracts, (iv) “money-back guarantee” penalizing deviations from a targeted labor demand. Modeling additionally a pre-commitment mechanism is beyond the scope of our analysis. However, we provide the pre-commitment solution since we view it as a valuable yardstick against which we compare the time-consistent solution, reported below. Moreover, the pre-commitment solution also resembles the standard textbook monopoly behavior.

3.2. The Time-Consistent Case

We now turn to the time-consistent monopolist’s optimization problem. Formally, the time-consistency requirement imposes an additional restriction on the firm’s production choice: at no time t can the monopolist be willing to deviate from his time-0 strategy. If the firm is not restricted from revising its strategy at dates $t = 1, \dots, T$, it should optimally choose the intertemporal production plan taking into account that it will always act optimally in the future. We proceed to find the monopolist’s subgame perfect strategy by backward induction; this is a time-consistent strategy. Specifically, the monopolist chooses the current input and price of consumption to maximize the firm’s current value given the firm’s future maximized value $\forall t = 0, \dots, T$, i.e.,

solves the dynamic program:

$$\bar{V}(t) \equiv \max_{\ell^D(t), \xi(t)} V(t) \quad \text{subject to } \xi(t) = u'(f(\ell^D(t), \varepsilon(t)))/y, \quad (24)$$

$$V(s) = \bar{V}(s), \quad (\ell^D(s), \xi(s)) = \operatorname{argmax} V(s), \quad \forall s = t+1, \dots, T,$$

where y satisfies (6).

Proposition 3 presents the optimal solution to this problem, assuming it exists.⁹

Proposition 3. *The time-consistent monopoly optimal labor demand ℓ^D , $t = 1, \dots, T-1$ satisfies*

$$f_\ell(t) - w(t) = -A(t) f_\ell(t) V_{ex}(t) < 0, \quad (25)$$

where $V_{ex}(t)$ denotes the ex-dividend value of the firm given by

$$V_{ex}(t) \equiv E \left[\sum_{s=t+1}^T \frac{u'(s)}{u'(t)} [f(s) - w(s) \ell(s)] \middle| \mathcal{F}_t \right] \quad \forall t = 1, \dots, T-1; \quad V_{ex}(T) = 0.$$

At time T , the time-consistent monopoly optimal labor demand ℓ^D satisfies $f_\ell(T) - w(T) = 0$.

The ex-dividend value of the firm's stock at time T is zero; hence the terminal first-order condition and optimal choice of the firm coincide with those of the competitive firm. The first-order condition at time T is the terminal condition for the backward induction: the remaining labor choices are determined by solving (25) backwards.

Similarly to the pre-commitment case, at the optimum for the time-consistent monopolist the increase in profit due to an extra unit of input used must counteract the indirect decrease via the impact of that extra unit on the price system. However, the extent of this monopoly power is now driven by the negative of the ex-dividend stock price in place of the current profit. (The consumer's attitude toward risk for consumption and the marginal product of labor appear as in the pre-commitment case.) The short-sighted monopolist only cares about (and tries to manipulate) the current valuation of the stock, which is made up of current profit plus the ex-dividend value of the firm. Given the time-consistency restriction, this monopolist takes the future value of profits as given and can only boost the firm's ex-dividend value by depressing the current price of consumption. So, while it was in the pre-committed monopolist's interest to boost the current price of consumption, it is in the time-consistent monopolist's interest to depress it; hence the negative term in (25). The ex-dividend value of the firm appears because this is what he manipulates; the higher is $V_{ex}(t)$, the stronger the incentive to cut concurrent

⁹In the time-consistent case a sufficient condition for concavity of the objective function is $-2 \frac{u''(c)}{u'(c)} + \frac{u'''(c)}{u''(c)} < -\frac{f_{\ell\ell}(\ell, \varepsilon)}{f_\ell^2(\ell, \varepsilon)}$, $c = f(\ell, \varepsilon)$; $\forall \varepsilon, \ell$, satisfied, for example, by power preferences $u(c) = c^\gamma/\gamma$ with $\gamma \in [0, 1)$, which includes $u(c) = \log(c)$ (no additional restriction on the production function $f(\ell, \varepsilon)$ is required).

profit to manipulate the firm's valuation. Hence, the firm's value serves as an extra variable in determining the optimal policy of the firm.

We now define an equilibrium in a monopolistic economy containing the time-consistent firm.

Definition (Monopolistic Time-Consistent Equilibrium). *An equilibrium in an economy of one monopolistic firm and one representative consumer-worker is a set of prices $(\hat{\xi}, \hat{w})$ and choices $(\hat{c}, \hat{\ell}, \hat{\ell}^D)$ such that (i) the consumer chooses his optimal consumption/labor policy given the state price and wage processes, (ii) the firm chooses a time-consistent strategy so as to maximize its objective in (24) given the wage, and taking into account that the price system responds to clear the consumption good market, and (iii) the consumption and labor markets clear at all times:*

$$\hat{c}(t) = f(\hat{\ell}^D(t), \varepsilon(t)) \quad \text{and} \quad \hat{\ell}(t) = \hat{\ell}^D(t).$$

The fully analytical characterization of the monopolistic time-consistent equilibrium is presented in (26)–(29). Again, from labor market clearing, we first determine the labor as a function of the shock ε (26).¹⁰ Then, (27)–(29) give the remaining equilibrium quantities. The equilibrium labor is given by

$$f_{\ell}(\hat{\ell}(t), \varepsilon(t)) - \frac{v'(\bar{\ell} - \hat{\ell}(t))}{u'(f(\hat{\ell}(t), \varepsilon(t)))} = -A(t) f_{\ell}(\hat{\ell}(t), \varepsilon(t)) V_{ex}(t) \quad (26)$$

and the equilibrium consumption and profit processes by

$$\hat{c}(t) = f(\hat{\ell}(t), \varepsilon(t)), \quad \hat{\pi}(t) = f(\hat{\ell}(t), \varepsilon(t)) - \frac{v'(\bar{\ell} - \hat{\ell}(t))}{u'(f(\hat{\ell}(t), \varepsilon(t)))} \hat{\ell}(t). \quad (27)$$

The equilibrium state price density, wage and firm value processes are given by

$$\hat{\xi}(t) = u'(f(\hat{\ell}(t), \varepsilon(t))), \quad \hat{w}(t) = \frac{v'(\bar{\ell} - \hat{\ell}(t))}{u'(f(\hat{\ell}(t), \varepsilon(t)))}, \quad (28)$$

$$\hat{V}(t) = E \left[\sum_{s=t}^T \frac{u'(f(\hat{\ell}(s), \varepsilon(s)))}{u'(f(\hat{\ell}(t), \varepsilon(t)))} \left\{ f(\hat{\ell}(s), \varepsilon(s)) - \frac{v'(\bar{\ell} - \hat{\ell}(s))}{u'(f(\hat{\ell}(s), \varepsilon(s)))} \hat{\ell}(s) \right\} \middle| \mathcal{F}_t \right]. \quad (29)$$

Proposition 4 presents a comparison of the monopolistic time-consistent and the competitive economies.

Proposition 4. *The equilibrium labor, output, state price and profit in the time-consistent monopolistic and the competitive economies satisfy, for all $t = 1, \dots, T - 1$, all $\varepsilon(t)$:*

$$\hat{\ell}(t) > \ell^c(t), \quad f(\hat{\ell}(t), \varepsilon(t)) > f(\ell^c(t), \varepsilon(t)), \quad \hat{c}(t) > c^c(t) \quad (30)$$

$$\hat{\xi}(t) < \xi^c(t), \quad \hat{w}(t) > w^c(t), \quad \hat{\xi}(t)\hat{\pi}(t) < \xi^c(t)\pi^c(t), \quad \hat{\pi}(t) < \pi^c(t). \quad (31)$$

¹⁰See Lemma A.3 of the Appendix for the existence of an interior solution $\hat{\ell}(t) \in (0, \bar{\ell})$ to equation (18).

The firm's initial value satisfies,

$$\hat{V}(0) < V^c(0).$$

At time T , all the equilibrium quantities coincide with those of the competitive benchmark. The firm's value, $\hat{V}(t)$, can be either higher or lower than in the competitive economy, $V^c(t)$, $t = 0, \dots, T - 1$.

The short-sighted monopolist's behavior is exactly opposite to that of the monopolist who pre-commits. He desires to depress the current price of consumption (to boost the current stock price); hence the lower level of state price density. He achieves this by increasing output, hence demanding more labor input, and in return pushing up the wage rate. Although this monopolist moves his labor demand in the opposite direction to the pre-committed monopolist, he also moves away from the maximum profit point and so profits are reduced relative to the competitive case. In his attempt to manipulate the stock price, concurrent profit and its value are cut. This type of behavior may overall result in a higher or lower value of the firm's stock price. By consistently reducing the value of concurrent profit, the short-sighted owners of the monopolistic firm may, then, cause damage to the firm and its stock price. This is so even though the time-consistent monopolist is restricted to maximize the firm's value at each point in time. For the pre-committed monopolist, we could explain this result by his not attempting to maximize this quantity, but here the explanation must be more complex (see Examples 1 and 2).

We now present three examples which deliver additional insights into the equilibrium quantities. The first example demonstrates that the monopolistic firm's profits can go negative in equilibrium. The second example allows an investigation of the relationship between the monopolistic and the competitive firm values, as well as the infinite horizon limit. The third example shows how under an alternative, albeit extreme, form of a production opportunity the time-consistent monopolist's profits go to zero, while the pre-committed monopolist's profits do not.

Example 1 (Negative Profits). Here, we provide a numerical example in which the time-consistent monopolistic firm's equilibrium profits $\hat{\pi}(t)$ are sometimes negative, and its stock price is always lower than that of a competitive firm. Consider the following parameterization:

$$u(c) + v(\bar{\ell} - \ell) = \frac{c^\gamma}{\gamma} + \frac{(\bar{\ell} - \ell)^\rho}{\rho}, \quad \gamma \in (0, 1), \quad \rho < 1; \quad f(\ell, \varepsilon) = \varepsilon + \ell^\nu, \quad \nu \in (0, 1); \quad T = 2.$$

The productivity shock enters the technology additively; this type of shock can be interpreted as providing an additional endowment as in a Lucas (1978)-type pure-exchange economy. We set $\bar{\ell} = 1$, $\gamma = 0.5$, $\rho = \nu = 0.9$, $\varepsilon(1) = 0.1$, $\varepsilon(2) = 0.5$ (deterministic for simplicity of exposition). The resulting equilibrium labor choices, profits and stock prices are reported in Table I.

	Competitive Equilibrium	Monopolistic Pre-Commitment Equilibrium	Monopolistic Time-Consistent Equilibrium
$\ell(1)$	0.60	0.41	0.84
$\ell(2)$	0.37	0.01	0.37
$\pi(1)$	0.16	0.22	-0.27
$\pi(2)$	0.54	0.51	0.54
$V(0)$	0.75	1.02	0.54
$V(1)$	0.65	0.75	0.52
$V(2)$	0.54	0.51	0.54

Table I. Equilibrium labor choice, profits and firm value in the competitive, monopolistic pre-commitment and monopolistic time-consistent economies. The reported values are for the economies parameterized by $u(c) + v(\bar{\ell} - \ell) = c^\gamma/\gamma + (\bar{\ell} - \ell)^\rho/\rho$, $f(\ell, \varepsilon) = \varepsilon + \ell^\nu$, $T = 2$, with $\bar{\ell} = 1$, $\gamma = 0.5$, $\rho = \nu = 0.9$, $\varepsilon(1) = 0.1$, $\varepsilon(2) = 0.5$.

At time 1 the time-consistent monopolist sacrifices his concurrent profit in order to boost the time-1 value of the stock. Indeed, consistent with Proposition 4, depending on the parameterization, nothing prevents the sign of $\hat{\pi}(1)$ from becoming negative. This is a notable feature of our solution. Although real-world firms frequently announce losses, this phenomenon cannot be generated in a competitive model where production occurs within the decision period (provided the firm has an option to costlessly shut down during that period). Negative profits can obtain in models of predation, where rivalrous producers are competing for market share (e.g., Tirole (1988, Chapter 9)); in our model, in contrast, the monopolist controls the entire market and does not fear entry, yet his profit may still go negative.

A further striking conclusion revealed by Table I is that non-competitive behavior of the firm does not necessarily result in a higher value of the firm; in fact, the time-consistent monopolistic firm's stock price is lower than its competitive counterpart at all times. The recognition of his monopoly power actually damages the time-consistent monopolist at all points in time, and he would be better off behaving competitively. For completeness and comparison we also provide the equilibrium quantities for the monopolistic pre-commitment economy. Example 2, having a longer time horizon, and yielding an explicit solution, allows us to investigate this stock price effect more closely.

Example 2 (Firm Value Comparison and Time Horizon Effects). Since it appears that the intertemporal nature of the problem is critical, in this example we extend the horizon beyond

$T = 2$. It would also be important to discuss the limiting case of our model as $T \rightarrow \infty$, since in some models (e.g., repeated principal-agent), the equilibrium changes dramatically on transitioning from a finite to infinite horizon. In order to obtain explicit formulae for all equilibrium quantities, we simplify beyond the general specification of the utility function employed so far. The utility we adopt has the form $u(c(t), t) - v(\ell(t), t) = \beta^t(\log c(t) - \ell(t)^\rho/\rho)$, $\beta \in (0, 1)$, $\rho > 1$, where β is the time discount factor, required to make pertinent quantities well-defined in the limiting case of $T \rightarrow \infty$, and the second term is the disutility of labor. We show equilibrium labor to still be bounded from above by $\bar{\ell} = 1$. The firm's technology is represented by $f(\ell(t), \varepsilon(t)) = \varepsilon(t) \ell(t)^\nu$, $\nu \in (0, 1)$. To ensure that the equilibrium quantities are well-defined in the infinite horizon limit, we assume that ε is bounded, as well as assume away speculative bubbles. The resulting exact formulae for the equilibrium labor choices, profits and stock prices are reported in Table II, with the comparative statics as in Proposition 4.¹¹ The time-consistent labor choice monotonically increases as $T - t$ increases, converging to a limit (and diverging away from the competitive labor, which is constant). As $T - t$ increases, time-consistent profits fall, although do not vanish. At this point, the parallel with to the Coase (1972) conjecture is unavoidable: *ceteris paribus*, the more future production decisions the time-consistent firm can make, the lower its profits today. Does the firm erode its profits by allowing for more future decisions? We revisit this issue in Section 4.

	Competitive Equilibrium	Monopolistic Time-Consistent Equilibrium
$\ell(t)$	$\nu^{1/\rho}$	$\left[\nu \frac{1 - (\beta(1-\nu))^{T-t+1}}{1 - \beta(1-\nu)} \right]^{1/\rho}$
$\pi(t)$	$\varepsilon(t) \nu^{\nu/\rho} (1 - \nu)$	$\varepsilon(t) \left[\nu \frac{1 - (\beta(1-\nu))^{T-t+1}}{1 - \beta(1-\nu)} \right]^{\nu/\rho} \frac{(1-\nu)(1-\beta) + \nu(\beta(1-\nu))^{T-t+1}}{1 - \beta(1-\nu)}$
$V(t)$	$\varepsilon(t) \nu^{\nu/\rho} (1 - \nu) \frac{1 - \beta^{T-t+1}}{1 - \beta}$	$\varepsilon(t) \nu^{\nu/\rho} (1 - \nu) \left[\frac{1 - (\beta(1-\nu))^{T-t+1}}{1 - \beta(1-\nu)} \right]^{\nu/\rho + 1}$
$\lim_{T-t \rightarrow \infty} \ell(t)$	$\nu^{1/\rho}$	$\left[\frac{\nu}{1 - \beta(1-\nu)} \right]^{1/\rho}$
$\lim_{T-t \rightarrow \infty} \pi(t)$	$\varepsilon(t) \nu^{\nu/\rho} (1 - \nu)$	$\varepsilon(t) \left[\frac{\nu}{1 - \beta(1-\nu)} \right]^{\nu/\rho} \frac{(1-\nu)(1-\beta)}{1 - \beta(1-\nu)}$
$\lim_{T-t \rightarrow \infty} V(t)$	$\frac{\varepsilon(t) \nu^{\nu/\rho} (1-\nu)}{1-\beta}$	$\frac{\varepsilon(t) \nu^{\nu/\rho} (1-\nu)}{[1 - \beta(1-\nu)]^{\nu/\rho + 1}}$

Table II. Equilibrium labor choice, profits, and firm value and their limits as $T - t \rightarrow \infty$ in the competitive and monopolistic time-consistent economies.

The reported formulae are for the economies parameterized by $u(c(t), t) - v(\ell(t), t) = \beta^t(\log c(t) - \ell(t)^\rho/\rho)$, $\beta \in (0, 1)$, $\rho > 1$; $f(\ell(t), \varepsilon(t)) = \varepsilon(t) \ell(t)^\nu$, $\nu \in (0, 1)$.

¹¹We do not report the monopolistic pre-commitment equilibrium quantities since for logarithmic utility over consumption, the monopolist's problem is not well-defined, as is the case in the standard textbook treatment of monopoly (Mas-Colell, Whinston and Green (1995, p.429)). Moreover, one may extend our earlier analysis of the monopolistic pre-commitment and competitive equilibria to the case of infinite horizon without major difficulties.

Of special interest is the value of the firm's stock. In Figure 1, we plot the firm value in the competitive and monopolistic equilibria as a function of time. The monopolistic firm value can be either higher or lower than that of the competitive firm, depending on the age of the firm. At first blush, this result seems surprising, because the time-consistent monopolist is optimizing the firm's value at each point in time. Since he has more market power than a competitive firm, how can his firm's value come out lower? At time t he solves the problem

$$\hat{V}(t) = \max_{\ell^D(t), \xi(t)} f(\ell^D(t), \varepsilon(t)) - \hat{w}(t)\ell^D(t) + \frac{1}{u'(f(\ell^D(t), \varepsilon(t)))} E \left[\hat{\xi}(t+1) \hat{V}(t+1) | \mathcal{F}_t \right].$$

The competitive firm solves the same problem except that: it has no power over $\xi(t)$; and that $\hat{w}(t)$, $\hat{V}(t+1)$, $\hat{\xi}(t+1)$ are replaced by their respective values in the competitive equilibrium. This latter distinction explains why the monopolist's firm value may come out lower: he faces both a different equilibrium wage and different choices of the future state prices and firm value.

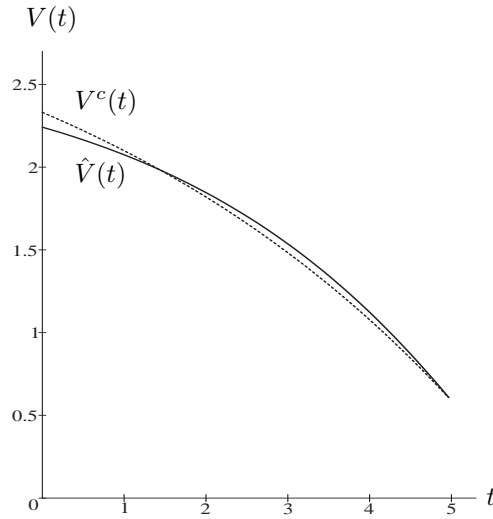


Figure 1. Equilibrium firm value versus time in the competitive and monopolistic time-consistent economies. The dotted plot is for the competitive economy and the solid plot is for the monopolistic time-consistent. The economies are parameterized by $u(c(t), t) - v(\ell(t), t) = \beta^t(\log c(t) - \ell(t)^\rho/\rho)$, $\beta = 0.9$, $\rho = 1.05$; $f(\ell(t), \varepsilon(t)) = \varepsilon(t) \ell(t)^\nu$, $\nu = 0.2$, $\varepsilon(t) = 1$, $\forall t$; $T = 5$.

Figure 1 also reveals that in this example the monopolistic firm increases the firm's value later in its lifetime and decreases it earlier in life. This is because the monopolistic firm competes with itself in future time periods; earlier in life it faces more competition, which adversely affects its value. In the limit of $T - t \rightarrow \infty$, the competitive firm value is unambiguously higher (Table II).

Example 3 (Constant Returns to Scale Technology). A constant returns to scale technology is widely employed in both the monopoly literature (e.g., Tirole (1988)), and asset pricing

literature (e.g., the workhorse production asset pricing model of Cox, Ingersoll and Ross (1985)). Thus far, we have been assuming strictly decreasing returns to labor; in this example, we extend our analysis to the case of constant returns, $f(\ell, \varepsilon) = \varepsilon\ell$. The firm's profit is now given by $\pi(t) = \varepsilon(t)\ell(t) - w(t)\ell(t)$.

For the competitive firm to demand a finite positive amount of labor, the equilibrium wage $w^c(t)$ must equal $\varepsilon(t)$; the equilibrium labor $\ell^c(t)$ is then determined from the consumer's optimization (7):

$$\frac{v'(\bar{\ell} - \ell^c(t))}{u'(f(\varepsilon(t), \ell^c(t)))} = \varepsilon(t). \quad (32)$$

The remaining equilibrium quantities, for all $t = 1, \dots, T$, are

$$c^c(t) = f(\ell^c(t), \varepsilon(t)), \quad \pi^c(t) = 0, \quad \xi^c(t) = u'(f(\ell^c(t), \varepsilon(t))), \quad V^c(0) = V^c(t) = 0.$$

Adaptation of our analysis in Section 3.1 shows that the pre-commitment monopolistic equilibrium labor demand $\ell^*(t)$ (assuming it exists and yields a maximum in the firm's problem) solves

$$-\frac{u''(f(\ell^*(t), \varepsilon(t)))}{u'(f(\ell^*(t), \varepsilon(t)))} \varepsilon(t)\ell^*(t) = 1, \quad (33)$$

implying $\ell^*(t) < \ell^c(t)$. The remaining equilibrium quantities are obtained from (19)–(21). The pre-commitment equilibrium, then, retains the main implications derived in Section 3.1; in particular, the firm cuts production and raises the price of output. The main difference from Section 3.1 is that monopoly profits are now always higher than the competitive ones (which are zero), consistent with the prediction of the textbook monopoly model. The firm's value is also higher.

The monopolistic time-consistent equilibrium coincides with the competitive one. To see why, recall from Proposition 4, that at the final time, $\hat{\pi}(T) = \pi^c(T)$. Since $\pi^c(T) = 0$, the time- $(T-1)$ ex-dividend value of the monopolistic firm, $\hat{V}_{ex}(T-1)$, is zero; hence by backward induction it can be shown that $\hat{\ell}(t) = \ell^c(t) \forall t$, and the competitive equilibrium obtains:

$$\hat{c}(t) = f(\ell^c(t), \varepsilon(t)), \quad \hat{\pi}(t) = 0, \quad \hat{\xi}(t) = u'(f(\ell^c(t), \varepsilon(t))), \quad \hat{V}(0) = \hat{V}(t) = 0, \quad t = 1, \dots, T.$$

In contrast to the decreasing returns to scale case, the time-consistent monopolist loses all his monopoly power. Accordingly, the value of his stock, as well as profit, are strictly lower than in the pre-commitment equilibrium. It is quite clear that the short-sighted behavior has a devastating effect on the firm's valuation. Ironically, it benefits the representative consumer, who, in effect, sets the labor demand of the firm so as to maximize his own expected lifetime utility.

Remark 1 (Price Normalization in Monopolistic Models). Ours is a general equilibrium model with imperfect competition; accordingly, it is not immune to the general criticism of all such models initiated by Gabszevicz and Vial (1972) (see also Mas-Colell (1982)). The criticism

has to do with the so-called “price-normalization problem” which arises in the presence of multiple goods because Walrasian (and Cournot-Walrasian) equilibrium theory determines relative prices but has little to say about nominal price formation. In the perfectly competitive benchmark, this deficiency is not an issue, since the nominal price indeterminacy does not affect real quantities. In the monopolistic (imperfectly competitive) equilibrium, however, the real quantities become dependent on the choice of price normalization. Indeed, for some choices of normalization, Dierker and Grodal (1986) show that no equilibrium exists, while for others it does exist. In an earlier version of this paper (Basak and Pavlova (2002)), we demonstrate the robustness of our main results to some alternative specifications of the numeraire. In particular, we consider a basket of commodities consisting of α units of the consumption good and $(1 - \alpha)$ units of labor, $\alpha \in (0, 1)$, and set it to be the numeraire.¹²

Related to the price-normalization problem, is the issue of whether maximizing the firm’s value through time is the correct objective. From the point-of-view of realism, it is generally accepted that this is what a firm should do, so the problem we solve is consistent with conventional thinking. However, the literature such as Dierker and Grodal (1986) (see also Dierker and Grodal (1999)), clearly casts some doubt on this approach. Hart (1985) suggests that profit maximization may be inappropriate in a monopolistic world because “the owners of the firm are interested not in monetary profit per se but in what that profit can buy”. Although Hart discusses maximization of owner’s utility as an alternative objective, he points out that the question of how to aggregate is not resolved when there are multiple owners. Similarly, Bonanno (1990) (in his survey) remarks, “unfortunately we are still far from a satisfactory theory of general equilibrium with imperfect competition”. To this day, the price normalization and the objective of the firm under imperfect competition in general equilibrium remain open issues. Based on this, we have simply adopted the most commonly employed price normalization, as well as the most conventionally considered objective function as the best starting point, and we focus on the implications.

4. Connection to the Coase Conjecture

As we discussed in Section 3, the time-inconsistency of the firm’s production plan in our monopolistic pre-commitment equilibrium is similar to the time-inconsistency arising in the context of a durable good monopoly (e.g., Tirole (1988, Chapter 1) and references therein). The long-lived firm’s stock in our model is similar to a durable good in that its value is durable over many periods. The durability of goods implies that a monopolistic firm in a sense competes with itself in pricing goods that it produces at other times and must take this into account in its production behavior. Similarly, in our context, “current owners” of the firm can be thought of as competing with “future

¹²Such a basket is a natural and realistic numeraire to adopt, as argued, for example, by Pavlova and Rigobon (2003) (for more discussion of prices indexes, see Schultze (2003)).

owners” in maximizing the value of the firm’s stock; they must account for possible revisions to today’s intertemporal production plan by the future owners. This effective competition between past and future sales in the durable good problem tends to weaken the monopoly power and may ultimately enforce competitive behavior. This observation is the Coase (1972) conjecture: as the time interval between successive decision points shorten, the price of the durable good charged by the monopolist converges to the competitive price (marginal cost), and the monopoly profits are driven to zero.

To examine the extension of Coase’s intuition to the case of a long-lived monopolistic firm manipulating the price of its stock, we explore the limiting case of our economy as the decision-making interval shrinks.¹³ We partition the time horizon $[0, T]$ in our economy into n small intervals of length Δ , so that $t = 0, \Delta, 2\Delta, \dots, n\Delta = T$. All flow variables in the economy ($\ell(t)$, $c(t)$, $f(t)$ and $\pi(t)$) are now interpreted as “flows over the interval t to $t + \Delta$ ”. We then take the limit as $n \rightarrow \infty$ ($\Delta \rightarrow 0$). Proposition 5 reports the resulting optimality conditions and equilibrium characterization for the competitive, monopolistic pre-commitment and monopolistic time-consistent economies, assuming appropriate regularity conditions are satisfied.

Proposition 5. *In the continuous-time limit, as $n \rightarrow \infty$, $\forall t$:*

(i) *The competitive firm’s optimal labor demand ℓ^D satisfies*

$$f_\ell(\ell^D(t), \varepsilon(t)) - w(t) = 0. \tag{34}$$

Consequently, the competitive equilibrium characterization is given by the continuous-time analogs of equations (11)–(14).

(ii) *The pre-commitment monopolistic firm’s optimal labor demand ℓ^D satisfies*

$$f_\ell(\ell^D(t), \varepsilon(t)) - w(t) = A(t) f_\ell(t) \pi(t) > 0. \tag{35}$$

Consequently, the monopolistic pre-commitment equilibrium characterization is given by the continuous-time analogs of equations (18)–(21).

(iii) *Assume further that $\varepsilon(t) \in [k, K]$, $0 < k < K < \infty$. Then the continuous-time limit exists for the monopolistic time-consistent equilibrium. The time-consistent monopolistic firm’s optimal labor demand ℓ^D satisfies*

$$\pi(\ell^D(t), \varepsilon(t)) = f(\ell^D(t), \varepsilon(t)) - w(t)\ell^D(t) = 0, \quad \forall t. \tag{36}$$

¹³Our goal here is to merely explore what happens to equilibrium quantities as the firm’s decisions become more frequent, and to not present a continuous-time extension of our model. Similar discrete-time models with $T = \infty$ have been argued to exhibit multiple equilibria as the decision-making interval shrinks, according to the Folk Theorem (see also Ausubel and Deneckere (1989)), which motivated us to keep the horizon T finite.

Consequently, the equilibrium labor for $\forall t$ is given by

$$f(\hat{\ell}(t), \varepsilon(t)) - \frac{v'(\bar{\ell} - \hat{\ell}(t))}{u'(f(\hat{\ell}(t), \varepsilon(t)))} \hat{\ell}(t) = 0. \quad (37)$$

Furthermore, for $\forall t$,

$$\begin{aligned} \hat{c}(t) &= f(\hat{\ell}(t), \varepsilon(t)), \quad \hat{\pi}(t) = 0, \quad \hat{\xi}(t) = u'(f(\hat{\ell}(t), \varepsilon(t))), \\ \hat{w}(t) &= \frac{v'(\bar{\ell} - \hat{\ell}(t))}{u'(f(\hat{\ell}(t), \varepsilon(t)))} \quad \text{and} \quad \hat{V}(t) = 0. \end{aligned}$$

The equilibrium characterization of the competitive and the monopolistic pre-commitment economies are exactly analogous to the discrete-time characterizations. Our main focus is on the monopolistic time-consistent equilibrium. There, the firm's profit and value indeed shrink to zero, as the monopolist's decision interval becomes arbitrarily small – in line with the Coase conjecture. In other words, the competing forces between current profits and the valuation of future profits may diminish shareholder value, fully destroying it in the continuous-time limit. Note, however, that our monopolist's profits are not equated to the competitive ones. This is simply because, in contrast to the textbook Marshallian framework adopted by Coase where competition implies free entry of firms, competitive behavior in our setting does not yield zero profits in equilibrium. This distinction is due to our Arrow-Debreu-McKenzie setting; in particular, to the assumptions that there is a fixed number of firms, and that the technology of each firm is convex.

5. Conclusion

In this paper, we model a production economy which, along with a standard representative consumer, includes a large value-maximizing monopolistic firm. The firm manipulates its valuation as well as the price of the good that it produces. This feature makes its time-0 production plan time-inconsistent. We address the time consistency problem in two polar ways: first, we assume the firm can credibly commit to never revoking its time-0 decision (long-term policy); and second, we assume that the firm takes into account that it will revise its production plan at each decision point (short-term policy). At a general equilibrium level, we show that the long-term policy is largely consistent with the implications of the textbook static monopoly model; as compared to the competitive economy, the output is decreased and the price of consumption is increased, yet the profits and the firm's value can be either increased or decreased. The short-term policy, however, is at odds with the static model; the output is increased while the price is decreased. More strikingly, under the short-term policy, the profits in every period are decreased, and may even go negative, while the firm's value can drop below than in the competitive benchmark. The distinction between the long- and short-term policies becomes even sharper in the continuous-time limit of our economy: while the pre-commitment equilibrium retains its basic discrete-time

structure and implications, the time-consistent equilibrium tends to the limit of zero profits and hence zero firm's value at all times. Since our primary focus is comparison between the long- and short-term approaches, and comparison with the textbook model, we have omitted an analysis of the volatility and risk premium of the monopolistic firm's stock price, and of the impact of monopoly power on the equilibrium interest rate, market price of risk, output and consumption variabilities. This analysis would be straightforward in the continuous-time limit of our economy, but less tractable in discrete-time.

Our analysis involving a single monopolistic firm, a single consumption good, and a representative consumer is, admittedly, simplistic. Our goal in this paper has been to develop the minimal setting possible capturing the mechanism through which the firm's market power may impact valuation in the economy, and not to produce the most empirically plausible model. Throughout the paper, we also discuss the robustness of our implications to alternative modeling approaches. For realism, however, one would need to extend this work to include multiple imperfectly competitive firms producing homogeneous, or differentiated, goods, or multiple consumers. The former (multiple firms) would draw from the results of the oligopoly literature, while the latter (multiple consumers) would involve aggregation of the consumers' preferences into a representative agent.

Appendix A. Proofs

Proof of Proposition 1. Since there is no consumption or production at time 0, we can set $\xi(0) = 1/y$ w.l.o.g. Upon substitution of (15) into (1), and using the definition (8), we obtain an equivalent representation of (16):

$$\max_{\ell} E \left[\sum_{t=1}^T u'(f(\ell(t), \varepsilon(t))) \left\{ f(\ell(t), \varepsilon(t)) - w(t)\ell(t) \right\} \right], \quad (\text{A.1})$$

where the firm's objective is now a function of $\{\ell(t); t = 1, \dots, T\}$ only. The first-order conditions for (A.1) are given by

$$\begin{aligned} u''(f(\ell(t), \varepsilon(t))) f_{\ell}(\ell(t), \varepsilon(t)) (f(\ell(t), \varepsilon(t)) - w(t)\ell(t)) \\ + u'(f(\ell(t), \varepsilon(t))) (f_{\ell}(\ell(t), \varepsilon(t)) - w(t)) = 0, \quad \forall t = 1, \dots, T. \end{aligned}$$

Using the definition of $A(t)$ and rearranging above yields (17). The sufficient condition for concavity in footnote 7 guarantees that the labor solving (17) is the maximizer for (16). Since the firm has the option to shut down (set $\ell^D(t) = 0$) during any period $[t, t+1]$ in this static optimization, $\pi(t) \geq 0$ and $u'(f(\ell(t), \varepsilon(t)))\pi(t) \geq 0$. This together with strict concavity of each term in (A.1) (by assumption) guarantees that $u'(f(\ell^D(t), \varepsilon(t)))\pi(t) > 0$ at the optimum, and hence $\pi(t) > 0$. $\pi(t) > 0$ together with $A(t) > 0$, $f_{\ell}(\ell, \varepsilon(t)) > 0$ (by assumptions on preferences and technology), ensures that the expression on the right-hand side of (17) is strictly positive. *Q.E.D.*

The following Lemmas A.1 and A.2 are employed in the proofs below. Lemma A.1 shows that under regularity conditions (satisfied, for example, by power preferences over consumption $u(c) = c^{\gamma}/\gamma$ with $\gamma \in (0, 1)$ and power production $f(\ell, \varepsilon) = \varepsilon\ell^{\nu}$, $\nu \in (0, 1)$; no additional restriction on $v(h)$ is required) both equilibrium ℓ^c and ℓ^* belong to the interior of $[0, \bar{\ell}]$. Lemma A.2 is employed in the proofs of Propositions 2, 4 and 5.

Lemma A.1. *Under the standard assumptions on preferences and production (see Sections 2.1 and 2.2), there exists a unique solution, $\ell^c(t) \in (0, \bar{\ell})$, to (11). Assume further that $\lim_{\ell \rightarrow 0} u'(f(\ell, \varepsilon)) f_{\ell}(\ell, \varepsilon) < \infty$, $\lim_{\ell \rightarrow 0} u'(f(\ell, \varepsilon)) \ell < \infty$, $\lim_{\ell \rightarrow 0} -u''(f(\ell, \varepsilon)) f_{\ell}(\ell, \varepsilon)/u'(f(\ell, \varepsilon)) < 1$ and $\lim_{\ell \rightarrow 0} (u''(f(\ell, \varepsilon))) f_{\ell}(\ell, \varepsilon) \ell + u'(f(\ell, \varepsilon)) < \infty \forall \varepsilon$. Then there exists a solution, $\ell^*(t) \in (0, \bar{\ell})$, to (18).*

Proof of Lemma A.1. Since $\lim_{\ell \rightarrow 0} u'(f(\ell, \varepsilon)) f_{\ell}(\ell, \varepsilon) = \infty > \lim_{\ell \rightarrow 0} v'(\bar{\ell} - \ell)$, and $\lim_{\ell \rightarrow \bar{\ell}} v'(\bar{\ell} - \ell) = \infty > \lim_{\ell \rightarrow \bar{\ell}} u'(f(\ell, \varepsilon)) f_{\ell}(\ell, \varepsilon)$, and since $u' f_{\ell}$ is decreasing in ℓ while v' is increasing in ℓ , there exists a unique solution, $\ell^c \in (0, \bar{\ell})$, to (11). Since $\lim_{\ell \rightarrow 0} u'(f(\ell, \varepsilon)) f_{\ell}(\ell, \varepsilon) - v'(\bar{\ell} - \ell) - A(t) f_{\ell}(\ell, \varepsilon) (u'(f(\ell, \varepsilon)) f_{\ell}(\ell, \varepsilon) - v'(\bar{\ell} - \ell) \ell) = \infty$ and $\lim_{\ell \rightarrow \bar{\ell}} u'(f(\ell, \varepsilon)) f_{\ell}(\ell, \varepsilon) - v'(\bar{\ell} - \ell) - A(t) f_{\ell}(\ell, \varepsilon) (u'(f(\ell, \varepsilon)) f_{\ell}(\ell, \varepsilon) - v'(\bar{\ell} - \ell) \ell) = -\infty$, continuity of $u'(f(\ell, \varepsilon)) f_{\ell}(\ell, \varepsilon) - v'(\bar{\ell} - \ell) - A(t) f_{\ell}(\ell, \varepsilon) (u'(f(\ell, \varepsilon)) f_{\ell}(\ell, \varepsilon) - v'(\bar{\ell} - \ell) \ell)$ on $(0, \bar{\ell})$ together with the boundary behavior at $\ell \rightarrow 0$ and $\ell \rightarrow \bar{\ell}$ ensures existence of a solution, $\ell^* \in (0, \bar{\ell})$, to (18). *Q.E.D.*

Lemma A.2. $f_\ell(\ell, \varepsilon) - \frac{v'(\bar{\ell} - \ell)}{u'(f(\ell, \varepsilon))}$ is decreasing in ℓ for all ε .

Proof of Lemma A.2. Assumptions on preferences and production and straightforward differentiation deliver the result. *Q.E.D.*

Proof of Proposition 2. Since the right-hand side of (18) is strictly greater than zero due to Proposition 1, and the right-hand side of (11) is zero, it follows from Lemma A.2 that $\ell^*(t) < \ell^c(t)$. The remaining inequalities in (22)–(23) are then straightforward to derive: they are due to $f(\ell, \varepsilon)$ and $w = v'(\bar{\ell} - \ell)/u'(f(\ell, \varepsilon))$ being increasing in ℓ , $\forall \varepsilon$, the good market clearing, and $\xi = u'(f(\ell, \varepsilon))$ being decreasing in ℓ , $\forall \varepsilon$. Equation (6) is automatically satisfied in equilibrium due to clearing in the good and labor markets, hence y is indeterminate and we can normalize $y = 1$. Together with our earlier normalization $\xi(0) = 1/y$, this yields $\xi(0) = 1$ in both the competitive and monopolistic pre-commitment equilibria. For $w = w^*$, the function $\xi\pi(\ell, \varepsilon, w) \equiv u'(f(\ell, \varepsilon))(f(\ell, \varepsilon) - w\ell)$ achieves its maximum at ℓ^* . $\xi\pi$ is strictly decreasing in w , hence for any $w > w^*$, $u'(f(\ell, \varepsilon))(f(\ell, \varepsilon) - w\ell) < u'(f(\ell^*, \varepsilon))(f(\ell^*, \varepsilon) - w^*\ell^*)$, for all ℓ . This together with $w^c(t) > w^*(t)$ implies that for each term in the expression for $V(0)$ (1), we have $\xi^*(t)\pi^*(t) > \xi^c(t)\pi^c(t)$, $\forall t$. Hence $V^*(0) > V^c(0)$. Example 1 provides evidence for the last assertion of the Proposition. *Q.E.D.*

Proof of Proposition 3. Substituting (15) into (1) and solving (24) at time $t = T$, we obtain the optimality condition $f_\ell(T) - w(T) = 0$, identical to that of the competitive firm. Consequently, $\bar{V}(T) = f(T) - w(T)\ell^D(T) > 0$ and $V_{ex}(T) = 0$. At time $t = T - 1$, the first-order condition for (24) is

$$f_\ell(\ell(T-1), \varepsilon(T-1)) - w(T-1) - \frac{u''(f(\ell(T-1), \varepsilon(T-1))) f_\ell(\ell(T-1), \varepsilon(T-1))}{u'(f(\ell(T-1), \varepsilon(T-1)))^2} \\ * E[u'(f_\ell(\ell(T), \varepsilon(T))) \{f(T) - w(T)\ell^D(T)\} | \mathcal{F}_{T-1}] = 0.$$

Using the definition of V_{ex} and rearranging above yields (25) at time $T - 1$. Continuing the backward induction, we obtain (25) for all $t = 1, \dots, T - 1$. The sufficient condition for concavity in footnote 9 guarantees that the labor solving (25) is the maximizer for (24). $\bar{V}(t)$, obtained on each step of the backward induction, is strictly positive because the firm's objective in (24) is strictly concave and $V(t) \geq 0$ ($V(t) < 0$ is ruled out since the firm can shut down at any time and thus increase its value to zero). Consequently, $V_{ex}(t) = E[\frac{\xi(t+1)}{\xi(t)} V(t+1) | \mathcal{F}_t]$ is strictly positive. This together with $A(t) > 0$ and $f_\ell(\ell, \varepsilon(t)) > 0$ implies that the right-hand side of (25) is strictly negative. *Q.E.D.*

Lemma A.3. Under the standard assumptions on preferences and production (see Sections 2.1, 2.2), and the regularity condition $-2 \frac{u''(c)}{u'(c)} + \frac{u'''(c)}{u''(c)} < -\frac{f_{\ell\ell}(\ell, \varepsilon)}{f_\ell^2(\ell, \varepsilon)}$, $c = f(\ell, \varepsilon)$; $\forall \varepsilon, \ell$, (footnote 9), there exists a unique solution, $\hat{\ell}(t) \in (0, \bar{\ell})$, to (26).

Proof of Lemma A.3. Since on each step of the backward induction, $\lim_{\ell \rightarrow 0} u'(f(\ell, \varepsilon))f_\ell(\ell, \varepsilon) - v'(\bar{\ell} - \ell) + A(t)u'(f(\ell, \varepsilon))f_\ell(\ell, \varepsilon)V_{ex} = \infty$ and $\lim_{\ell \rightarrow \bar{\ell}} u'(f(\ell, \varepsilon))f_\ell(\ell, \varepsilon) - v'(\bar{\ell} - \ell) + A(t)u'(f(\ell, \varepsilon))f_\ell(\ell, \varepsilon)V_{ex} = -\infty$ and since $u'(f(\ell, \varepsilon))f_\ell(\ell, \varepsilon) - v'(\bar{\ell} - \ell) + A(t)u'(f(\ell, \varepsilon))f_\ell(\ell, \varepsilon)V_{ex}$ is decreasing in ℓ , there exists a unique solution, $\hat{\ell} \in (0, \bar{\ell})$, to (26). *Q.E.D.*

Proof of Proposition 4. Since the right-hand side of (26) is strictly less than zero due to Proposition 3, and the right-hand side of (11) is zero, it follows from Lemma A.2 that $\hat{\ell}(t) > \ell^c(t)$. Consequently, $f(\ell, \varepsilon)$ and $w = v'(\bar{\ell} - \ell)/u'(f(\ell, \varepsilon))$ being increasing in ℓ , $\forall \varepsilon$, and the good market clearing yield the comparisons on f , c , w . The comparison on ξ follows from $\xi = u'(f(\ell, \varepsilon))$ being decreasing in ℓ , $\forall \varepsilon$.

To prove the remaining statements, define the equilibrium profit function $\Pi(\ell(t), \varepsilon(t)) \equiv f(\ell(t), \varepsilon(t)) - \frac{v'(\bar{\ell} - \ell(t))}{u'(f(\ell(t), \varepsilon(t)))} \ell(t)$. The first-order condition for maximization of $\Pi(\ell(t), \varepsilon(t))$ with respect to ℓ is satisfied by $\tilde{\ell}(t)$ such that

$$f_\ell(\tilde{\ell}(t), \varepsilon(t)) - \frac{v'(\bar{\ell} - \tilde{\ell}(t))}{u'(f(\tilde{\ell}(t), \varepsilon(t)))} = -\frac{v''(\bar{\ell} - \tilde{\ell}(t))}{u'(f(\tilde{\ell}(t), \varepsilon(t)))} + \frac{A(t) f_\ell(\tilde{\ell}(t), \varepsilon(t))}{u'(f(\tilde{\ell}(t), \varepsilon(t)))} v'(\bar{\ell} - \tilde{\ell}(t)) > 0. \quad (\text{A.2})$$

Due to Lemma A.2, $\tilde{\ell}(t) < \ell^c(t)$, $\forall t, \varepsilon(t)$. It is straightforward to verify that for any ℓ such that $f_\ell - \frac{v'(\bar{\ell} - \ell)}{u'(f(\ell, \varepsilon))} \leq 0$, $\forall \varepsilon$, $\Pi_\ell < 0$. This together with (26) implies that $\Pi_\ell(\ell^c(t)) < 0$ and $\Pi_\ell(\hat{\ell}(t)) < 0 \forall t, \varepsilon(t)$. Continuous function $\Pi_\ell(\cdot; \varepsilon)$ does not change sign on $[\ell^c(t), \hat{\ell}(t)]$, because if it did, there had to be a point $\check{\ell}(t)$ on $[\ell^c(t), \hat{\ell}(t)]$ satisfying $\Pi_\ell(\check{\ell}(t)) = 0$, which is not possible for we have shown that any such point has to lie to the left of $\ell^c(t)$. Consequently, $\Pi_\ell(\cdot; \varepsilon)$ is monotonically decreasing on $[\ell^c(t), \hat{\ell}(t)]$, and hence $\hat{\pi}(t) < \pi^c(t)$, $\forall t$. It then follows that since $\hat{\xi}(t) < \xi^c(t)$, $\hat{\xi}(t)\hat{\pi}(t) < \xi^c(t)\pi^c(t)$, and hence from (1) that $\hat{\xi}(t)\hat{V}(t) < \xi^c(t)V^c(t)$. This together with the argument behind the normalization adapted from the proof of Proposition 2, yields $\hat{V}(0) < V^c(0)$.

Since at time T the equilibrium conditions (11) and (26) of the competitive and the monopolistic time-consistent economies coincide, $\hat{\ell}(T) = \ell^c(T)$, yielding equalization of the remaining equilibrium quantities at time T . Finally, an example can be constructed to verify that $\hat{V}(t)$ can be lower or higher than $V^c(t)$. Example 2 of Section 3.2 demonstrates this under a modified set of assumptions on $v(h)$. *Q.E.D.*

Proof of Proposition 5. Consider an arbitrary partition $t = 0, \Delta, 2\Delta, \dots, n\Delta = T$ of $[0, T]$. The consumer-worker's problem is now given by

$$\begin{aligned} & \max_{c, \ell} E \left[\sum_{t=1}^T \left(u(c(t)) + v(\bar{\ell} - \ell(t)) \right) \Delta \right] \\ \text{subject to} & \quad E \left[\sum_{t=1}^T \xi(t) \left(c(t) - w(t) \ell(t) \right) \Delta \right] \leq E \left[\sum_{t=1}^T \xi(s) \pi(s) \Delta \right], \end{aligned}$$

where the flow of utility is defined over a rate of consumption and leisure. The above yields the first order conditions

$$\begin{aligned} u'(c(t))\Delta &= y \xi(t) \Delta, \\ v'(\bar{\ell} - \ell(t))\Delta &= y \xi(t) w(t) \Delta. \end{aligned} \quad (\text{A.3})$$

For any Δ , the first-order conditions are equivalent to (4)–(5), consequently, the consumer demand facing the firm is the same as in the discrete case.

The value of the firm is given by

$$V(t) = E \left[\sum_{s=t}^{T-\Delta} \frac{\xi(s)}{\xi(t)} (f(\ell(s), \varepsilon(s)) - w(s)\ell(s))\Delta \middle| \mathcal{F}_t \right]. \quad (\text{A.4})$$

The competitive firm is choosing ℓ^D to maximize $V(0)$ in (A.4) taking ξ as given; accordingly, its labor demand ℓ^D satisfies for all $t = \Delta, \dots, T - \Delta$

$$f_{\ell}(t) \Delta - w(t) \Delta = 0.$$

The pre-committed monopolist is choosing ℓ^D and ξ so as to maximize $V(0)$ in (A.4) subject to $\xi(t) = u'(f(\ell(t), \varepsilon(t)))/y$ (as implied by (A.3)). His labor demand ℓ^D satisfies

$$f_{\ell}(t) \Delta - w(t) \Delta = A(t)f_{\ell}(t)\pi(t)\Delta.$$

Since the equations above are independent of Δ , equations (34) and (35) obtain; in addition, the continuous-time limits of the discrete-time competitive and monopolistic pre-commitment equilibria are now given by the continuous-time analogs of (11)–(14) and (18)–(21), respectively, for all $t \in [0, T]$.

Similarly, the problem of the time-consistent monopolist is given by (24) with $V(t)$ specified in (A.4). The backward induction solution yields the following for the firm's labor demand ℓ^D

$$(f_{\ell}(t) - w(t)) \Delta = -A(t) f_{\ell}(t) V_{ex}(t) \leq 0, \quad t = \Delta, \dots, T - \Delta. \quad (\text{A.5})$$

$\ell(t)$ is bounded from above by $\bar{\ell}$, and, anticipating equilibrium, since the right-hand of (A.5) is nonpositive, it is bounded from below by $\ell^c(t) > 0$ due to Lemma A.2. $\varepsilon(t)$ is bounded by assumption, hence $f(\ell, \varepsilon)$, $f_{\ell}(\ell, \varepsilon)$, $u'(f(\ell, \varepsilon))$ and $A(t)$ are bounded. As we take the limit as $\Delta \rightarrow 0$ in (A.5), given the boundedness, its left-hand side tends to zero, hence, so must the right-hand side. Given the boundedness of all quantities except $V_{ex}(t)$ on the right-hand side of (A.5), we must have $V_{ex}(t) \rightarrow 0$. Consequently, $\pi(t) \rightarrow 0$; in addition, given our boundedness argument, $V(t) \rightarrow 0$, and the firm's labor demand has to be such that $\ell^D(t)$ yields zero profit $\pi(t) = f(\ell^D(t), \varepsilon(t)) - w(t)\ell^D(t) = 0$, as is stated in (36). In the resulting equilibrium, $\hat{\pi}(t) = 0$ as well, hence (37). (37) yields the equilibrium labor, which in turn determines the remaining equilibrium quantities. *Q.E.D.*

Appendix B. The Case of a Monopolistic-Monopsonistic Firm

Let us suppose we allow the firm to also have power over the wage rate, i.e., to solve for all $t = 0, \dots, T$:

$$\begin{aligned} \max_{\ell^D(s), \xi(s), w(s); s \geq t} V(t) \quad & \text{subject to } \xi(s) = u'(f(\ell^D(s), \varepsilon(s)))/y, \\ w(s) = & \frac{v'(\bar{\ell} - \ell^D(s))}{u'(f(\ell^D(s), \varepsilon(s)))}, \quad \forall s = t, \dots, T. \end{aligned}$$

The pre-committed monopolist-monopsonist (hereafter “monopsonist”) only solves this problem at $t = 0$, while the time-consistent solves it backwards for $t = 0, \dots, T$. The first-order conditions for the pre-committed monopsonist are

$$f_\ell(t) - w(t) = A(t)f_\ell(t)\pi(t) + (A^\ell(t) + A(t)f_\ell(t))\ell(t)v'(t) > 0, \quad (\text{A.6})$$

and for the time-consistent monopsonist are

$$f_\ell(t) - w(t) = -A(t)f_\ell(t)V_{ex}(t) + (A^\ell(t) + A(t)f_\ell(t))\ell(t)v'(t), \quad (\text{A.7})$$

where

$$A^\ell(t) \equiv -\frac{v''(t)}{v'(t)} > 0.$$

All other things (A , f_ℓ , π) being equal, (A.6) suggests that the wage effect acts in the same direction as the price effect, implying that the pre-committed monopsonist will decrease his labor (and output) even more than the monopolist, and in turn the value of the firm will increase more. For the time-consistent case, however, (A.7) suggests that the wage effect counteracts the price effect, implying again a lower labor input (and output) than the monopolist. If the price effect dominates, comparative static comparisons with the competitive case will be as in Proposition 4; if the wage effect dominates, they will be as in Proposition 2. The intuition for this extra term is quite clear; it is in the firm’s interest to reduce wages, and to do so it will reduce its labor demand.

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