AN INVESTIGATION OF THE UPPER MANTLE P-VELOCITY MODELS

USING SYNTHETIC SEISMOGRAMS

by

Yoichi Harita

B.S., University of California at Berkeley (1967)

submitted in partial fulfillment of the requirements for the degree of Master of Science

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Signature of Author.

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Abstract

A computer program written on the basis of the Cagniard-de Hoop's method to calculate the response from flat layered media to a unit impulse source was used to examine the upper mantle P-velocity structure of the earth, in the southern part of the United States.

Two existing models relevant to our locality were examined and found unsatisfactory on the basis of the synthetic seismogram they generated.

Several P-velocity models were constructed and examined. A model which gives correct arrival times and satisfactory synthetic seismograms has been found. This model includes a low-velocity zone similar to that of other models. A new feature of this model is a rapid increase in velocity near a depth of 500 km.

Thesis Supervisor: M. Nafi Toksoz
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Notation

c_i : P-Velocity in the ith layer

d_i : Density in the ith layer

\( \gamma \) : \( d_i/d_{i+1} \)

f(p) : Integrated Transmission-Reflection Coefficient

h : Vertical Distance to the Source from the Station

H : Step Function

K_0 : Modified Bessel Function of the Second Kind, of Order 0

L : \( dp/dt \)

p : \( \sin(i_c)/c_i \), where \( i_c \) is the Angle of Incident

P_0 : Critical p at Reflection

P : Pressure

r,x : Horizontal Distance between the Source and the Station

R(p) : Reflection Coefficient

k_i : \( -\frac{1}{2} \rho_j/(\mu_j - \mu_i) \)

k_r : \( k_i + k_j \)

R : \( \sqrt{r^2 + z^2} \)

s_i : Shear Velocity in the ith layer

t : Time

t_c : Refraction Time

t_o : Reflection Time

T(p) : Transmission Coefficient

\( \eta_i \) : \( \sqrt{1/c_i^2 - \rho^2} \)

\( \eta_i' \) : \( \sqrt{1/s_i^2 - \rho^2} \)

z : Depth

\( \mu, \lambda \) : Lame Parameters

\( \rho \) : Density
I. INTRODUCTION

The best known property of the interior of the earth is the seismic velocity profile as a function of depth. Any theory of the earth's structure must, therefore, satisfactorily predict this velocity profile as closely as possible. The usual method of determining velocity is to use the Wiechert-Herglotz equation [See Bullen (1965)] which involves the integration of the \( \frac{dt}{d\Delta} \) curve obtained experimentally from the travel time-distance information. The method cannot use the valuable information such as wave shape of the seismogram or varying amplitude with time in a seismogram and with the distance from the source. Also the method fails to determine a low velocity structure which is believed to exist at the depth of 100 km. Moreover, obtaining travel time and \( \frac{dt}{d\Delta} \) curve requires a great deal of data - many stations - in order to approximate a smooth \( \frac{dt}{d\Delta} \) curve.

In spite of these difficulties, numerous attempts have been made in the past to determine the velocity structure, and there are widely varying models [See, for example, Julian and Anderson (1968)] which give approximately correct arrival times. Of course, the variety may be due to the lateral heterogeneity of the earth. However, since Cagniard developed the revolutionary technique for computing the response from flat layered media, followed by de Hoop's (1960) modification, a more sophisticated method to determine the velocity structure has become possible.

In this thesis, the technique developed by those mentioned above and by others [Strick (1959) and Helm-
berger (1965 & 1967) is discussed and applied to the spherical earth. The validity of the spherical approximation has not yet been confirmed. However, this technique is known to work to the travel time computation, and the relatively small curvature of the earth in the upper mantle should not cause significant errors in computing transmission coefficients and reflection coefficients. The advantage of this new method are that it generates synthetic seismograms and that it enables us to examine the models from many more standpoints - the first and the following arrivals, amplitudes and wave forms.

The Cagniard-de Hoop technique and the theory for the synthetic seismogram computations are presented in Chapter II. The entire chapter is a summary of Dr. Donald Helmberger's contributions and included in this thesis for the sake of completeness in presentation.

In Chapter III, the computer program, originally written by Dr. Helmberger, modified and improved in the course of the research for this thesis by him and by the author, is described.

Chapter IV presents the result of this study - the P-velocity structure along an east-west profile in the southern United States. To determine the velocity structure, first we examine two existing models proposed for this region. One model is by Dowling and Nuttli (1964), based on the travel time data from the underground nuclear explosion BILBY (1963), which we use in our investigation. The other model is by Johnson (1967), and was obtained, using the dt/dΔ data from the Tonto Forest Seismological Observatory in Arizona. After the exam-
ination of these models, we construct some new models. In so doing, we use a conventional method for computing travel times for a given model. [See Bullen (1965)]

Then, we compute the synthetic seismograms and compare them with the records from the BILBY event. The criteria for comparison are the travel times of various P-arrivals, amplitudes and the wave forms of P-wave arrivals.
II. THEORY

In this chapter we present the theory which is the basis of the technique of computing the synthetic seismograms. Also shown in this chapter are several approximations we make in our computation.

1. Response from an Infinite Medium

For an infinite fluid with a unit pulse source at \( r = 0, z = 0 \), the Laplace-transformed pressure is:

\[
\overline{P}(r,z,s) = -\frac{i}{\pi} C_0 \int_{\rho} \kappa_0(s\rho r) e^{-s\eta_1 |z|} \frac{\rho}{\eta_1} d\rho.
\]

Due to the symmetry of the integrand with respect to the real \( p \) axis, (1) can be rewritten as:

\[
\overline{P}(r,z,s) = \frac{2}{\pi} J_m \int_{0}^{\infty} \kappa_0(s\rho r) e^{-s\eta_1 |z|} \frac{\rho}{\eta_1} d\rho.
\]

\( C_0 \) is a constant with dimensions of pressure times length and assumed unity from here on. Back-transforming (2), we obtain:

\[
P(r,z,t) = \frac{2}{\pi} J_m \int_{0}^{\infty} \frac{\rho}{\eta_1} \frac{H(t-\rho r - \eta_1 |z|)}{\sqrt{(z-\eta_1 |z|)^2 - \rho^2 r^2}} \, d\rho,
\]

where \( H(t) \) is a step function. Since the argument of a step function must be real, we must have the path such that:

\[
\tau = \rho r + \eta_1 |z|
\]
is real and positive. Due to de Hoop's modification of Cagniard's method (1960), we solve (4) for $p$ to obtain:

$$p = \frac{1}{R^2} \tau + i \frac{|z|}{R^2} \sqrt{\tau^2 - \frac{R^2}{c_1^2}}$$

(5)

In order that $\tau^2 - \frac{R^2}{c_1^2}$ be positive, we must have $\frac{R}{c_1} < \tau < \infty$. And this is the path of the contour $\Gamma$. Now, (3) is equivalent to:

$$P(r,z,t) = \frac{2}{\pi} \int_{\Gamma} \frac{H(t-\tau - \eta|z|)}{\sqrt{\eta(t-\tau)(t-\tau + 2\eta)}} \frac{dp}{d\tau} d\tau$$

(6)

and differentiating (5), we get:

$$\frac{dp}{d\tau} = \frac{i \eta}{\sqrt{\tau^2 - \frac{R^2}{c_1^2}}}$$

(7)

For $\frac{R}{c_1} < \tau$, the integrand of (6) is real, and we rewrite (6) as:

$$P(r,z,t) = \frac{2}{\pi} \Re \int_{t_0}^{t} \frac{\varphi(\tau)}{\sqrt{(t-\tau)(t-\tau + 2\eta)(\tau^2 - \frac{R^2}{c_1^2})}} d\tau$$

(8)

Since $H(t-\tau) = 0$ for $t < \tau$, and for $\tau < \frac{R}{c_1}$, the denominator of the integrand in (8) becomes complex. We define the reflection time $t_0 = R/c_1$ and simplify (8). Let

$$\theta = \arcsin \frac{\tau - \tau_0}{\sqrt{\tau - \tau_0}}$$

(9)

and with the transformation, (8) becomes:

$$P(r,z,t) = \frac{4}{\pi} \Re \int_{0}^{\pi/2} F(\theta) d\theta$$

(10)
where
\[ F(\theta) = \frac{\varphi(\theta)}{\sqrt{(t(\theta) + t_0) \cdot (t - t(\theta) + 2 \varphi(\theta)r)}} \]

\[ \varphi(\theta) = \frac{1}{R^2} \left[ (t - (t - t_0) \sin^2 \theta) + i \frac{121}{R} \sqrt{[t - (t - t_0) \sin^2 \theta]^2 - t_0} \right] \]

\[ t(\theta) = t - (t - t_0) \sin^2 \theta \]

In order to perform the integration (10), we must find \( F(\theta) \) numerically for each \( t \). This costly procedure can be avoided by the following approach. Define

\[ \varphi_R \equiv \Re(\varphi) \]

\[ \varphi_I \equiv \Im(\varphi) \]

\[ \varphi_0 \equiv \frac{r}{RCi} \]

We choose a set of \( \varphi_R \)'s starting at \( p_0 \) and increasing on some small interval \( \delta \), to \( p_t \equiv \varphi_R(t) \). We find, for each \( \varphi_R \), \( \varphi_I \) such that \( \Im(t) \) is zero. To clarify the situation, the limits are:

\[ \varphi_0 < \varphi_R < \varphi_t \]

\[ t_0 < t < t \]

\[ 0 < \theta < \pi \]

As \( t \to t_0 \), \( p \to r/RC_1 = p_0 \) (The First Motion Approximation).

And also

\[ t(t(\theta)) \Rightarrow t_0 = \frac{R}{C_1} \]

\[ F(\theta) \Rightarrow \frac{r}{RC}, \frac{1}{(2t_0 R)^{11}} \Rightarrow \frac{1}{2R} \]
and

$$P(r, z, t) = \frac{1}{R} H(t - t_0)$$

[Dix (1953)] (11)

This is the exact solution to (10).

2. Response from a Two-layered Medium

For a problem involving two-layered media, let suffixes 1 and 2 denote the upper and the lower layers, respectively. We have, for the equivalence of (6),

$$P(r, z, t) = \frac{2}{i} \sum \int \frac{R(p) H(t - z)}{p' \eta_i \sqrt{(t - z)(t - z + 2p')}} \frac{d p}{dz} dz$$

Note the quantity $R(p)$ in the integrand, which is:

$$R(p) = \frac{\eta_i \left\{ (1 - 2s^2 p^2) + 2 s_2^4 \rho \eta_2 \eta_2' \right\} - \delta \eta_2}{\eta_i \left\{ (1 - 2s^2 p^2) + 4 s_2^4 \rho^2 \eta_2 \eta_2' \right\} + \delta \eta_2}$$

and

$$T = \rho r + (z + h) \eta_i$$

$$R = \sqrt{r^2 + (z + h)^2}$$

Assuming $c_1 < s_2 < c_2$, $p$ leaves the real axis at $p = p_0$ and $1/s_2 < p_0 < 1/c_1$. We can obtain the S-refracted time, the reflected time ($t_0$) and the P-refracted time ($t_c$) by substituting $p = 1/s_2$, $p_0$, $1/c_2$ into (13), respectively. For example, let $p = 1/c_2$ and from (13) we get:

$$t_c = \frac{r}{c_2} + (z + h) \sqrt{\frac{1}{c_2^2} - \frac{1}{c_i^2}}$$
When we perform the numerical integration (12), we break $\mathcal{P}$ into two parts - from $t_c$ to $t_0$ where $p$ is real, and from $t_0$ onward where $p$ is complex. Thus, (12) can be rewritten as:

$$\mathcal{P}(r,z,t) = \mathcal{P}_1(r,z,t) + \mathcal{P}_2(r,z,t),$$

(14)

where

$$\mathcal{P}_1(r,z,t) = \frac{2}{\pi} \text{Im} \int_{t_c}^{t_0} \frac{p}{\eta} \frac{R(p)}{\sqrt{\eta(t-z)(t-z+2pr)}} \frac{d\rho}{dz} \, dz$$

and

$$\mathcal{P}_2(r,z,t) = \frac{2}{\pi} \text{Im} \int_{t_0}^{t} \frac{z}{\eta} \frac{R(p)}{\sqrt{\eta(t-z)(t-z+2pr)}} \frac{d\rho}{dz} \, dz$$

By letting

$$\theta = \arcsin \sqrt{\frac{z}{t}}$$

(15)

we change variables in (14) to

$$\mathcal{P}_1(r,z,t) = \frac{4}{\pi} \int_{\theta_0}^{\theta} F(\theta) \, d\theta,$$

(16)

where

$$F(\theta) = \text{Im} \left\{ R(p) \frac{\rho}{\eta} \frac{d\rho}{dz} \int \sqrt{\frac{z}{t-z+2pr}} \right\}$$

$$\theta = \arcsin \sqrt{\frac{z}{t}}$$
The integration can be carried out by trapezoidal rule. Thus, \( P_1(r,z,t) \) can be numerically evaluated, and \( P_2(r,z,t) \) is evaluated by the same method described in (10).

The pressure response to a delta function source is:

\[
P(r,z,t) = \frac{2}{\pi} \frac{\partial}{\partial t} \int_0^t R(p) \frac{\partial}{\partial z} \frac{d\tau}{(t-z)(t-z+2\rho)}.
\]  

(17)

3. High Frequency Approximation

When we deal with sources of high frequency or of a short duration, the following approximation holds:

\[
t - \zeta + 2\rho r \approx 2\rho r
\]  

(18)

From (17) we get by substituting (18):

\[
P(r,z,t) = \frac{2}{\pi} \frac{\partial}{\partial t} \left( \frac{1}{V} \ast \Im \left\{ R(p) \frac{\partial}{\partial z} \frac{1}{\sqrt{2\rho}} \right\} \right).
\]  

(19)

Define

\[
\psi(t) = \Im \left\{ R(p) \sqrt{\frac{p}{2\rho}} \frac{1}{\eta} \frac{d\rho}{dz} \right\}
\]  

Thus (19) can be written as:

\[
P(r,z,t) = \frac{2}{\pi} \frac{\partial}{\partial t} \frac{2}{\pi} \left( \frac{1}{V} \ast \psi(t) \right)
\]  

(20)
4. A Multi-layered Case

For a multi-layered case, we must consider multiple reflections in addition to refractions. We have the Laplace-transformed pressure:

\[ \tilde{P}(r, z, s) = -\frac{i}{\pi} \int_{\eta_0}^{\pi} K_0(spr) \mathcal{R}(\rho, s) e^{-s\eta_1(z+h)} \frac{\rho}{\eta_1} d\rho, \]  

(21)

where

\[ \mathcal{R}(\rho, s) = R_{12}(\rho) + \sum_{m} (-1)^{m+1} R_{23}^{m} R_{12}^{m-1} \frac{e^{-2\rho R_{12}}}{1 - R_{12}^2}. \]

Or (21) can be written as:

\[ \tilde{P}(r, z, s) = -\frac{i}{\pi} \int_{\eta_0}^{\pi} K_0(spr) \frac{\rho}{\eta_1} R_{12}(\rho) e^{-s\eta_1(z+h)} d\rho \]

\[ -\frac{i}{\pi} \sum_{m} \int_{\eta_0}^{\pi} K_0(spr) R_{23}^{m} \frac{e^{-2\rho R_{12}}}{1 - R_{12}^2}, \]

\[ \times e^{-s[\eta_1(z+h) + 2\rho R_{12}]} d\rho. \]  

(22)

Further:

\[ \tilde{P}(r, z, s) = \tilde{P}_0(r, z, s) + \sum_{m} \tilde{P}_m(r, z, s), \]

(23)

where

\[ \tilde{P}_m(r, z, s) = -\frac{i}{\pi} \int_{\eta_0}^{\pi} K_0(spr) \mathcal{P}_m(\rho) \frac{\rho}{\eta_1} e^{-s\mathcal{G}_m(\rho)} d\rho, \]

\[ \mathcal{P}_m(\rho) = R_{23}^m R_{12}^{m-1} (1 - R_{12}^2)(-1)^{m+1}, \]

\[ \mathcal{G}_m(\rho) = \eta_1(z+h) + 2\rho R_{12} \].
As before, $\bar{p}_m(r,z,s)$ can be transformed back to:

$$\bar{p}_m(r,z,t) = \frac{2}{\pi} \int_0^\infty \int_0^\infty \frac{d\rho}{\rho} \frac{\bar{p}(\rho)}{\eta_i} \sqrt{-(t-\tau_m)(t-\tau_m+2\rho r)} H(t-\tau_m) \, dz$$

where

$$\tau_m = \rho r + \eta_i (z+h) + 2 \, \text{Th} \rho i \eta z .$$

If layers have different thicknesses and if we change the variables of integration to $\tau$, as before, we get

$$\bar{p}_m(r,z,t) = \frac{2}{\pi} \int_0^\infty \int_0^\infty \frac{d\rho}{\rho} \frac{\bar{p}(\rho)}{\eta_i} \sqrt{-(t-\tau)(t-\tau+2\rho r)} H(t-\tau)$$

where

$$\tau = \rho r + 2 \sum_{j=1}^{m} \eta_j \text{Th} j ,$$

$$\frac{d\rho}{d\tau} = \left( \gamma - 2 \rho \sum_{j=1}^{m} \frac{\eta_j}{\eta_i} \right)^{-1} ,$$

$m$ is the last layer that the ray penetrates. Therefore, the refracted wave begins at $t_c$ which corresponds to $p_c = 1/c_{m+1}$ and the reflection occurs at $p_0$ corresponding to $t_0$. As before $1/c_{m+1} < p_0 < 1/c_m$ and as $p$ increases from $p_0$, it goes into the complex plane. At this point, we again apply the high frequency approximation (18) to (25) and obtain, for a delta function source:

$$\bar{p}_m(r,z,t) = \frac{2}{\pi} \frac{2}{2} \left[ \frac{1}{\bar{z} t} \times \psi_m(t) \right]$$

where

$$\psi_m(t) = \int_0^\infty \frac{\bar{p}(\rho)}{\eta_i} \frac{d\rho}{d\tau} \left\{ \sqrt{2\pi r} \frac{1}{\eta_i} \frac{d\rho}{dz} \right\} .$$
The merit of employing the high frequency approximation is, when we add rays (indexed \(m\)), from (23),

\[
P(r,z,t) = \sum_{m} P_{m}(r,z,t)
\]

\[
= \sum_{m} \frac{2}{\pi} \frac{\partial}{\partial t} \left( \frac{1}{\sqrt{t}} \ast \psi_{m}(t) \right).
\]  \hspace{1cm} (27)

Since the sum of convolved quantities such as in (27) is the convolved sum, i.e.,

\[
P(r,z,t) = \frac{a}{\pi} \frac{\partial}{\partial t} \left( \frac{1}{\sqrt{t}} \ast \sum_{m} \psi_{m}(t) \right).
\]  \hspace{1cm} (28)

Thus, we need not perform convolution for \(m\) times, but only once.

5. Generalized Transmission Coefficient

\(f_{m}(p)\) in (26) is called the generalized transmission coefficient. It can be written as the product of reflection coefficients \(R_{ij}(p)\) and transmission coefficients \(T_{ij}(p)\).

\(R_{ij}(p)\) is the reflection coefficient when the ray is reflected at the boundary of the \(i\)th and \(i+1=j\)th layers. Its explicit form is:

\[
R_{ij}(p) = \frac{A - \eta_{2}B}{A' - \eta_{2}B'}
\]  \hspace{1cm} (29)

where

\[
A = -a + b, \quad B = a + b, \quad A' = -a' + b', \quad B' = a' + b',
\]
\[ a = \rho^2 (\kappa_r - \rho^2)^2 \]
\[ b = \eta_i \eta_i' (\kappa_j - \rho^2)^2 - k_i k_j \eta_i \eta_j' \]
\[ a' = \eta_i \eta_i' \rho^2 \]
\[ b' = \eta_j' (\kappa_i - \rho^2)^2 - \eta_i' k_i k_j \]

\[ T_{ij}(p) \] is the transmission coefficient for the case the ray is refracted from the ith to the i+1=jth layers. Its explicit form is:

\[
T_{ij}(p) = \frac{2k_i \eta_i [\eta_i'(\kappa_i - \rho^2) - \eta_i'(\kappa_j - \rho^2)]}{D} \tag{30}
\]

where

\[
D = \rho^2 (\kappa_r - \rho^2)^2 + \eta_i \eta_j \eta_i' \eta_j' \rho^2 + \eta_i \eta_j' (\kappa_j - \rho^2)^2 \\
+ \eta_j \eta_i' (\kappa_i - \rho^2)^2 + \eta_i' \eta_i k_i k_j - \eta_i' \eta_j k_i k_j
\]

6. First Motion Approximation

As \( t \) approaches \( t_o \), the term \( \frac{dp}{dt} \) in (28) goes to infinity, as

\[
\frac{dp}{dt} = \left( \tau - 2p \sum_{j=1}^{n} \frac{Th_j}{\eta_j} \right)^{-1}
\]

is not defined at \( t = t_o \). In order to evaluate \( \psi_m(t) \) at \( t = t_o \) (which happens to be the most significant point of the ray), we make use of the Simpson's rule.
Figure 2.1

Pressure near \( t = t_0 \)

\[
\int_{t_0-h}^{t_0+h} f(t) \, dt = \frac{h}{3} \left[ f(t_0-h) + 4f(t_0) + f(t+h) \right] \\
= \frac{h}{3} \left[ f_1 + 4f_2 + f_3 \right] = \phi . \tag{31}
\]

From (31) we obtain:

\[
f_2 = \frac{1}{4} \left( 3\phi - (f_1 + f_3) \right) , \tag{32}
\]

which is what we wish to calculate. \( \phi \) is calculated partly numerically \((t_0-h < t < t_0-\delta, t_0+\delta < t < t_0+h)\).

For \( t_0-\delta < t < t_0+\delta \), we can show that:

\[
\int_{t_0-\delta}^{t_0+\delta} f(t) \, dt = 2f(t_0) \sqrt{\delta} . \tag{33}
\]

In order to show (33), we consider the Taylor expansion:

\[
t = t_0 + \frac{dt}{dp}(p-p_0) + \frac{d^2t}{dp^2}(p-p_0)^2 \frac{1}{2!} + \ldots . \tag{34}
\]
And since $dt/dp \approx 0$ at $t = t_0$, we have:

$$\frac{dt}{dp} \approx \sqrt{2(t-t_0)} \frac{d^2t}{dp^2} \tag{35}$$

From (25), we obtain by differentiating,

$$\frac{d^2t}{dp^2} = -2 \sum_j \frac{T_{h_j}}{\eta_j^3 C_j^2} \tag{36}$$

Substituting (36) into (35) and then into (26), we get:

$$P_m(r, z, t) = \frac{2}{n} \frac{r}{d^2} \left[ \frac{1}{\eta_i^2} \times \frac{1}{r-t_0} \int \left\{ \frac{\mathcal{F}_m(\rho)}{\sqrt{\eta_i \sqrt{-\sum_j T_{h_j} / \eta_j^3 C_j^2}}} \right\} \right] \tag{37}$$

Now let:

$$\theta = \frac{d^2t}{dp^2} \quad , \quad \psi_1(t) = \int \left\{ f_m(\rho) \frac{\mathcal{F}}{\eta_i \sqrt{\theta}} \right\} \frac{1}{\sqrt{t_0-t}} \quad , \quad \text{for } t < t_0 ,$$

$$\psi_2(t) = \int \left\{ f_m(\rho) \frac{\mathcal{F}}{\eta_i \sqrt{\theta}} \right\} \frac{1}{\sqrt{t_0-t}} \quad , \quad \text{for } t > t_0 ,$$

$$F_1(t) = \int \left\{ f_m(\rho) \frac{\mathcal{F}}{\eta_i \sqrt{\theta}} \right\} \quad \text{for } t < t_0 ,$$

$$F_2(t) = \int \left\{ f_m(\rho) \frac{\mathcal{F}}{\eta_i \sqrt{\theta}} \right\} \quad \text{for } t > t_0 .$$

Consider

$$\int_{t_0-s}^{t_0} \psi_1(t) dt = \int_{t_0-s}^{t_0} \frac{F_1(t)}{\sqrt{t_0-t}} dt$$

Since $F_1(t)$ varies slowly near $t = t_0$, we get:

$$\approx 2 F_1(t_0) \sqrt{s} \quad \tag{39}$$
Similarly:
\[ \int_{t_0}^{t_0+h} \psi_2(t) \, dt \approx 2F_2(t_0) \sqrt{\delta} . \]
Thus, we have shown for \( t_0 - \delta < t < t_0 + \delta \), the part of is
\[ 2 \int F_1(t_0) + F_2(t_0) \sqrt{\delta} . \] (40)

For the rest of \( \phi \), we use the trapezoidal rule to integrate. We have values of \( \psi_n(t) \) computed for these outer regions and we divide \( t_0 - h \) to \( t_0 - \delta \) to five parts (chosen rather arbitrarily), interpolate \( \psi_n(t) \) for each \( p \), and do the same for \( t_0 + \delta \) to \( t_0 + h \). This result and (40) are added to obtain finally \( \phi \). Then \( f_2 \) can be computed from (32).

We now consider the case in which \( t_c \) approaches \( t_0 \). We can show that
\[ \frac{df_n}{dt} \Rightarrow g(p) \left( \frac{1}{\sqrt{t_0 - t_c}} \right) , \]
where \( g(p) \) and \( h(p) \) are smooth functions of \( p \). Hence, we may delete the pressure from our consideration since it does not vary in any extraordinary way. Consider:
\[ \int_{t_c}^{t_0} \sqrt{\frac{t-t_c}{t_c-t}} \, dt = \int_{t_c}^{t_0} \frac{t-t_c}{\sqrt{(t-t_c)(t_0-t)}} \, dt . \] (41)
Now let
Therefore, (41) can be written as:

\[
\int_{t_c}^{t_o} \frac{\tau - t_c}{\sqrt{\tau}} \, d\tau = \frac{\sqrt{\pi}}{c} \left[ -\frac{b}{ac} \int_{t_c}^{t_o} \frac{d\tau}{\sqrt{\tau}} - t_c \int_{t_c}^{t_o} \frac{d\tau}{\sqrt{\tau}} \right] = \frac{\pi}{2} (t_o - t_c).
\]

Thus we have shown that when \(\psi_m(t)\) is integrated, it behaves linearly with interval \(t_o - t_c\). The justification for integrating \(\psi_m(t)\) is that since the source function is very flat and when it is convolved with \(\psi_m(t)\), \(\psi_m(t)\) is virtually integrated over time. We see now that as \(t_o - t_c\) becomes arbitrarily small, the contribution of \(\psi_m(t)\) from this part is essentially negligible.

7. Directivity Function

Instead of solving for pressure in the preceding discussion, we may replace it by the displacement potential \(\phi(y, z, t)\) assuming a step function source. And we use the same equations.

However when we measure body wave amplitude on the
surface of the earth, we must be concerned with the conversion of the measured amplitude to the actual amplitude because of the P-SV interaction at the reflecting surface. The conversion factor is given by L. Knopoff, et. al. (1960) as:

For the P-wave,

$$D_P(p) = \frac{-\beta_i^{-2} \eta_P (2p^2 - \beta_i^{-2})^2}{R(p)}$$

For the S-wave,

$$D_S(p) = \frac{-4p \eta_P \eta_S \beta_i^{-2}}{R(p)}$$

where $\beta_i$ is the shear velocity in the surface layer,

$$\eta_P = \sqrt{\frac{1}{c_i^2} - p^2} \quad ; \quad \eta_S = \sqrt{\frac{1}{s_i^2} - p^2}$$

$$R(p) = (2p^2 - \beta_i^{-2})^2 + 4p^2 \eta_P \eta_S$$

8. Spherical Layer Approximation

The method described above is valid for flat horizontal layers. We know that equations for distance and travel time [Grant and West (1965)]:

$$\Delta = 2p \int_0^h \frac{v(z) dz}{\sqrt{1 - p^2 v^2(z)}}$$

$$t = 2 \int_0^h \frac{dz}{v(z) \sqrt{1 - p^2 v^2(z)}}$$
where \( p = \sin(i) / V(z) \) (ray parameter),

\[ h : \text{thickness of the layer,} \]

for horizontal layers. And for spherical layers [Bullen (1965)]:

\[
\Delta = 2\pi \int_{r_p}^{r_o} \frac{dr}{r \sqrt{\eta^2 - p^2}},
\]

\[
t = 2 \int_{r_p}^{r_o} \frac{\eta^2 dr}{r \sqrt{\eta^2 - p^2}},
\]

where \( \eta = r/v, \ p = r \sin(i) / V, \ r_o: \text{radius of the earth,} \)

\[ r_p: \text{radius of the deepest point of penetration}. \]

We see immediately that the equations (43) will be equivalent to (44) if the quantities \( p \) and \( V \) are multiplied by \( 1/r \).

In our computation, the compressional velocities, the shear velocities, the layer thicknesses and the densities are multiplied by \( r_o / r, \) where \( r_o = 6371 \text{ km} \)

(the radius of the earth), the justification being the compatibility of the assumption with the equations (43) and (44), and the additional factor of \( r_o \) (a constant) is simply to normalize the quantities to the proper dimensions. Thus,

\[
c'_i = c_i Q
\]

\[
s'_i = s_i Q
\]

\[
\rho'_i = \rho_i Q
\]

\[
T'_i = T_i Q,
\]

where \( Q = r_o / r \).
The operation above is performed at the very beginning of the computation scheme and hence all the following computation is done with the normalized quantities (45).

9. Transfer Function

In order to generate synthetic seismograms from theoretical responses, we must devise a system function that takes the theoretical response as the input and generates the synthetic seismogram as the output. Such a transfer function is defined as a convolution of the source function with the instrumentation response. However, we know that at large ranges, 3000 km for example, the earth returns a step function if the input is a step function omitting the Q effects. That is, if we treat the mantle as a simple velocity gradient, then the displacement looks like the input. [See Figure 2.2]

Let \( R(t) \) be the output at the range 3376 km. Then,

\[
R(t) = \frac{3}{\sqrt{t}} [ T(t) * \phi(t) ]
\]

where \( \phi(t) \) is approximately a step function. In such a case \( R(t) = T(t) \). \( R(t) \) is used as the transfer function throughout this study. In Chapter IV, we will see the actual wave form of the transfer function. A synthetic seismogram is complete when we convolve the pressure (or displacement) [See Eqn. (28)] with the transfer function. In our method using the high frequency approximation, however, we convolve the transfer function with \( 1/\sqrt{t} \) to obtain the modified transfer function. [See Figures 4.2 and 4.3]
Figure 2.2

Theoretical Response from a Nearly Linear Gradient

This response was obtained from the Nuttli's model at 3376 km. The peak at 1.4 seconds was caused by a slight decrease in gradient at the depth 850 km.
Since we convolve $1/ \sqrt{t}$ with the transfer function which is nearly flat at $t = 0$, we are essentially integrating $i/ \sqrt{t}$ in the neighbourhood of $t = 0$. Let $f(t) = 1/ \sqrt{t}$. And let $h$ denote some small number. Using the Simpson's rule of integration, we obtain

$$\int_{0}^{2h} \frac{1}{\sqrt{t}} \, dt = \int_{0}^{2h} f(t) \, dt = \frac{h}{3} \left[ f(0) + 4f(h) + f(2h) \right]$$

But analytically, it is equal to $2 \sqrt{2h}$. Thus,

$$f(0) = \frac{11\sqrt{2} - 4}{2} \frac{1}{\sqrt{h}} \approx \frac{9.56}{\sqrt{h}}$$

In the digital computation, we let $h$ be the digitization interval and thus, $f(0)$ is obtained.
II. METHOD OF COMPUTATION

The procedure for computing the theoretical response to a unit impulse source is described below. The program was originally written for the Control Data 3600 computer at the University of California at San Diego by Dr. Helmberger. It has been converted so that it is compatible with the M.I.T. IBM 360-65/40 computer system in the course of the research for this thesis by Dr. Helmberger and by the author. It will be instructive to refer to Chapter II and the Appendix for the theory and the program listing, as the reader follows this chapter.

In computing the response, we must specify the characteristics of rays we are interested in. We, therefore, specify the layer $k$ to which the ray reaches without reflection and the manner that the ray reflects in the layers beneath $k$. The number of layers involved in internal reflections can be at most four (neighbouring ones). If more than four layers are involved, the reflection coefficients become exceedingly small and negligible. We name various configurations of internal reflections for the purpose of computation. There are two subroutines that define various constants for each configuration. One (CON) is used for configurations involving only two layers (This is the more usual case than the latter). The other (CONN) is for cases involving four layers. Cases for one and three layers are special cases of two-layer and four-layer cases, respectively. These subroutines define $MT(J)$, the number of transmissions from the $J$th to the $J+1$th layers; $LTP(J)$, the number of times
the ray travels through the Jth layer; \( NN(J) \), the number of reflections on the \( J+1 \)th layer from the Jth layer; 
\( MM(J) \), the number of reflections on the \( J-1 \)th layer from the Jth layer; and \( NF \), the number of possible ways the ray can travel with the same set of constants above. \( N \) is used to describe the rays. For CON, the various \( N \)'s correspond to the following figures [Figures 3.1]. 
And also for CONN, another set of figures are drawn. Though there are infinitely more configurations, they are not considered here because the intensity of the ray becomes negligible, as the values of the constants go up. The choice of using CON or CONN is made by the parameter \( MF \). That is,
\[ MF \begin{cases} 1 & \text{for CON,} \\ 1 & \text{for CONN.} \end{cases} \]

The program first computes the modified velocities, densities and layer thicknesses according to the spherical layer approximation with \texttt{SUBROUTINE CURAY} [See (43)-(45)]. The \texttt{MAIN} program gives the control to \texttt{SUBROUTINE SETUP} after defining constants and executing \texttt{CURAY}. For each call of SETUP, a response of a particular ray is computed and later all the responses are added. On the argument list of SETUP are:

- \( K \) : First Layer Involved in Internal Reflection
- \( NS \) : Starting Ray
- \( NO \) : Ending Ray
- \( MO \) : 0 for Finding the First Arrival Time
  \( \begin{cases} 2 & \text{for Subsequent Calls} \end{cases} \)
- \( MPLOT \) : 0 for No Plot of Theoretical Response
  \( \begin{cases} 2 & \text{for Plot} \end{cases} \)
Figures 3.1 Ray Configurations

for Two-layered Cases
Using the notation, the generalized transmission coefficient \( f_\text{m}(p) \) is:

\[
\begin{align*}
\mathcal{R}_{\text{mm}(k)} \cdot \mathcal{R}_{\text{m}(k+1)} \cdot \mathcal{R}_{\text{n}(k+1)} \cdot \mathcal{N}_{\text{n}(k+2)} \\
\mathcal{R}_{\text{b}(k)} \cdot \mathcal{R}_{\text{b}(k+1)} \cdot \mathcal{R}_{\text{b}(k+1)} \\
x \\
\mathcal{T}_{\text{b}(k+1)} \cdot \mathcal{T}_{\text{b}(k+1)}
\end{align*}
\]

\[ (46) \]
Figures 3.2
Ray Configurations for Four-layered Cases
MPUNCH : 0 No Punched Output of Theoretical Response
2 Punch Wanted.

SUBROUTINE SETUP first finds the first arrival of any ray among all the possible rays considered - either reflection or refraction. This is done by FUNCTION TS, which finds the largest $c_i$'s for $1 \leq i \leq n$, $KST \leq n \leq KEND$. For each $n$, SUBROUTINE FIND2 is called to compute reflection time $t_o$ and corresponding $p_o$ by letting $dp/dt$ go to $\infty$ or in the actual case, minimizing $|dt/dp|$ [See (25)]. TS also finds the refraction time $t_c$ using FUNCTION PTIM. $t_c$ is simply $t$ that corresponds to $p = \sin \frac{\theta}{c_{n+1}} = 1/c_{n+1}$ [See (25)]. $t_o$ and $t_c$ for each $n$ are stored in $T(n)$. TS is set to the minimum of all $T(n)$ and returned to SETUP. Upon returning, $TT(1)$ is set equal to $TS$, which is the first arrival counting from $t = 0$, when the source explodes. Then, the array $TT$ is defined by incrementing by DEL, which is defined in MAIN. TS is called only once in a series of calls of SETUP.

The reflection and refraction constants described above are defined by calling either CONN, or CONSTN and CON. Then, SUBROUTINE HIGH is called. First, by examining the transmission constant LTP, the deepest layer of penetration is determined and stored in KM. That is, if $LTP(J) = 0$ for some $J$, then $KM = J - 1$, after $LTP(J)$ have taken non-zero values for all $I < J$. Again FIND2 computes the reflection time $t_o$ for the particular ray. This time,

$$\frac{dt}{dp} = x - p \sum_{j=1}^{KM} \frac{Th_j LTP_j}{\sqrt{\frac{1}{c_j^2} - p^2}}$$
and \( p_0 \) is the value of \( p \) that makes \( dt/dp = 0 \) as before. Consequently,

\[
  t_o = p x + \sum_{j=1}^{KM} T_{hj} LT_{p_j} \sqrt{\frac{1}{c_j^2} - \rho_o^2}.
\]

RG is the difference between \( t_o \) and \( t_c \). SUBROUTINE HELP is then called to find \( t_c \) and \( dp/dt \) for \( p = 1/c_{KM+1} \), corresponding to refraction at the KM+1th layer boundary. TG = \( t_c - t_o \). To determine how the problem of evaluating \( \psi_m(t) \) near \( t_o \) should be dealt with, a series of tests are performed on the magnitude and the sign of TG with four constants defined in MAIN - TN's. If \( p_0 < p_c \), then we only evaluate \( \psi_m(t) \) for complex \( p \). Also if TG is large, then we do not want to divide the interval \( t_c - t_o \) too closely. Thus, SUBROUTINE DELPS is called to set the interval (DELP) so that the divisions are closer as \( p \) approaches \( p_0 \) and \( p_c \), and wider in the middle. DELPS performs this operation with the trigonometric sine function. NO is the dimension of DELP, or the number of partitions on the real \( p \) axis. However, if TG is not very large, then DELP is defined with equal intervals. After the real \( p \)'s (DELP) are defined, HIGH calls PLN1 and PLN2 to compute \( f_m(t) \). [See (14)] PLN1 computes \( \psi_m(t) \) for \( t_c < t < t_o \), and in this interval \( p \) only takes real values. In case of \( t_o < t_c \), PLN1 is not called since \( t_c \) is fictitious.

The procedure of computing \( \psi_m(t) \) as described in (26) is as follows:

1) Take \( p_c = 1/c_{KM+1} \) and increment it by DELP for
each \( p \); HELP is called to find corresponding time \( t_c \) (TT(I)) and \( \frac{dp}{dt} \) (DTP).

ii) For each \( p \), call ROC to compute part of \( f_m(p) \) [See (46)] using the constants defined in either CON or, CONN and CONSTN. (RPR)

iii) GENCC computes the product of transmission coefficients through layers 1 to \( K \), in which the ray simply travels forth once and back one. (TOT)

iv) CRSTPP computes the transmission coefficient [See (30)]; RET computes the reflection coefficient [See (29)]; and \( f_m(p) \) is the product of RPR and TOT above, which is set equal to \( TQ \), and PLN1 stores the imaginary part of \( TQ \) in \( RP \).

iv) \( \varphi_i(EA) \) is computed.

v) If the directivity function is desired [See (42)], then \( \text{NDIRT} > 1 \) and (42) is computed.

vi) \( \psi_m(t) \) as described in (19) and (20) are then computed and stored in PHI(I).

vii) Above six steps are repeated for increasing \( p \) till DELP(NO) is exhausted, and if the time corresponding to the last \( p \) used is reasonably far from \( t_o \) (criterion is DLTP) then the procedure is repeated for \( p \) incremented by some small real number till \( t \) reaches \( t_o \).

viii) At the end of PLN1, \( \text{TT}(1) = t_c \) and \( \text{PHI}(1) = 0 \) are set. This is to save computation time, since obviously the starting time is the refraction time and \( \psi_m(t_o) = 0 \) if PLN1 is ever called.

After PLN1 computes \( \psi_m(p) \) for \( t_c < t < t_o \), PLN2 computes \( \psi_m(p) \) for \( t < t_o \) including \( \psi_m(t_o) \) [See the first motion approximation]. As PLN1 returns to HIGH, HIGH computes
the complex path of \( p \) such that, as in (25), the imaginary part of \( t \) is zero. To accomplish it, \texttt{CONTOR} is called. It takes \( p_o \) and increment it by \( \text{DELP} \), and for each \( p \), \texttt{TIME2} is called to find the corresponding imaginary part of \( p \) such that \( \text{Im}(t) \leq \mathcal{S} \). If so, complex \( p \) and \( dp/dt \) and real \( t \) are stored in arrays \( \text{PP} \), \( \text{DDPT} \) and \( \text{TT} \), respectively, to be used in PLN2. After \( p \) reaches the end specified by \( \text{DELP} \), \texttt{CONTOR} repeats the operation till \( t \) reaches \( t_o + \text{TMX} \) or \( t_c + \text{TMX} \) whichever is the smallest. \( \text{TMX} \) is defined in \texttt{MAIN}. Upon \texttt{CONTOR}'s return to \texttt{HIGH}, PLN2 computes \( \Psi_m(p) \). It is very similar to PLN1, but the starting point of \( \text{PHI}(I) \) is now \( \text{MO} \) and the ending point is \( M \), both of which are defined in \texttt{CONTOR}. After \( \Psi_m(p) \) is computed for \( t_o < t < t_o + \text{TMX} \), PLN2 deals with the problem of \( \Psi_m(p) \) as described in the section, the first motion approximation.

\texttt{FUNCTION SF2} computes Equation (37). To perform the integration by the trapezoidal rule, we utilize the value of \( \Psi_m(p) \) already obtained for \( t_o - h < t < t_o + h \); \texttt{SUBROUTINE INTERP} is called to obtain \( \Psi_m(p) \) at five points between \( t-h \) and \( t-\mathcal{S} \), and \( t+\mathcal{S} \) and \( t+h \). In the program, \( t_o = \text{TTT(NO+1)} \), \( t_o - \mathcal{S} = \text{TTT(NO)} \), \( t_o + \mathcal{S} = \text{TTT(MO)} \) and \( h = \text{DP} \). \( \text{DELL} \) is the partition. \( t \) is increased by \( \text{DELL} \) starting from \( t_o - h \), \( \Psi_m(p) \) is obtained (interpolated) for each \( t \) by \texttt{INTERP}, and finally (39) is used to obtain \( p \). At the very end of PLN2, (31) is applied to get \( \Psi_m(t) \) at \( t = t_o \). As PLN2 returns to \texttt{HIGH}, the maximum indices of the computed time and pressure, \( \text{TD} \) and \( \text{PHI} \) in \texttt{COMMON /EXACT/}, are set equal to \( M \), defined in PLN2. When \texttt{HIGH} returns to \texttt{SETUP}, \texttt{SUBROUTINE ADJUST} is called to
locate the index I of the array T in COMMON /THY/ defined in SETUP to be equidistance apart, which corresponds to 
TD(I) in COMMON /EXACT/. NFIX = I and returned to SETUP. Then, TD is shifted in such a way that \( t_o \) lands on some 
T. \( (\psi_m\{t_o\}) \) is the most important point of all) Since 
TD is so closely spaced near \( t = t_o \) that shifting makes 
little difference in actual response time. Then, INTERP 
is used again to interpolate PHI for each T (equispaced) 
and the values are stored in PP after being multiplied 
by NF - symmetry constant defined in CONN or in CONSTN 
and CON. In other words, if there are more than two 
rays that can be specified by the same values of param-
eters, then the response must be multiplied by the number 
of such rays.

After PP is filled, if there are no more rays to 
consider (that is, if \( KO > 1 \) then SUBROUTINE SETT is called 
to perform the last operation [See (27)]. SETT reads 
the transfer function \( SS(KO) \), FUNCTION CONVS convolves 
SS and PP point by point and the result is stored in 
CC. Then, the derivative is taken and the result is 
plotted and printed (P).

This is the end of the program.
IV. DISCUSSION AND RESULTS

In this chapter some of the results obtained are discussed, the main object being to examine two of the existing models of the upper mantle [Dowling & Nuttli (1964) and Johnson (1967)] and to find a better model for the southern United States using the synthetic seismogram discussed in the previous chapters.

The upper mantle P-models by Nuttli and Johnson are shown on Figure 4.1. Both have a low velocity zone under the Mohorovicic discontinuity, but in the Nuttli's model, the velocity increases almost linearly with depth after the low velocity zone, whereas in the Johnson's model, there are two pronounced changes in the gradient - one at around 450 km and the other at around 700 km. We shall examine synthetic seismograms generated by these models, and compare them with actual records obtained in the Bilby underground explosion (1963).

But, first it may be interesting to look at some sample output of the computer program. [See Figures 4.2-4.7] In Chapter II we referred to the high frequency approximation [See Eqn. (18)]. Another computer program had been written without the approximation. Clearly, this program is more exact, but slower. Figure 4.2 is the transfer function that each response is to be convolved with. Figure 4.3 shows the transfer function convolved with $1/\sqrt{t}$, which we use in our approximate method. Figure 4.4 is the theoretical response to a unit pulse. Figure 4.5 is the response from the approximate method. Note that, in effect, the exact response is the convolution of the approximate response with
Figure 4.1
P-Velocity Models of the Upper Mantle
Figure 4.2
Transfer Function

Figure 4.3
Modified Transfer Function

Figure 4.4
Exact Response

Figure 4.5
Approximate Response
Figure 4.6 shows the approximate synthetic seismogram, whereas Figure 4.7 shows the exact synthetic seismogram. In terms of amplitude, phase and frequency, the two synthetic seismograms look nearly the same, and provide some evidence to the validity of the approximation.

In the following figures (4.8-4.12), records of the Bilby event are shown. The time scale is the same in all these figures and in all the others that will follow. But the amplitude is not absolute and only significant in one record. The relative amplitudes and the arrival times of these records are tabulated in Table 4.1.

Table 4.1 Bilby Event (1963)

<table>
<thead>
<tr>
<th>STATION</th>
<th>LATITUDE (°')</th>
<th>LONGITUDE (°')</th>
<th>RANGE (km)</th>
<th>TIME (sec.)</th>
<th>MAGNITUDE (A/T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source</td>
<td>37 03 38</td>
<td>116 01 18</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Raton N.M.</td>
<td>36 43 46</td>
<td>104 21 37</td>
<td>1039</td>
<td>136.7</td>
<td>11</td>
</tr>
<tr>
<td>Shamrock Texas</td>
<td>35 04 58</td>
<td>100 21 50</td>
<td>1426</td>
<td>187.7</td>
<td>165</td>
</tr>
<tr>
<td>Durant Okl.</td>
<td>34 02 11</td>
<td>96 13 04</td>
<td>1831</td>
<td>231.8</td>
<td>374</td>
</tr>
<tr>
<td>Liddieville La.</td>
<td>32 08 10</td>
<td>91 52 30</td>
<td>2274</td>
<td>280.7</td>
<td>915</td>
</tr>
<tr>
<td>Orlando Fla.</td>
<td>28 28 01</td>
<td>81 13 17</td>
<td>3376</td>
<td>375.0</td>
<td>265</td>
</tr>
</tbody>
</table>

(all locations are in the Northern and the Western Hemisphere)

Date: 13 Sept. 1963 ; Time: 17:00:00.13 oz
Magnitude: m=5.8
Figure 4.6
Approximate Synthetic Seismogram

Figure 4.7
Exact Synthetic Seismogram
Figure 4.8 Bilby Event

Range 1039 km
Figure 4.9  Bilby Event

Range 1426 km
Figure 4.10  Bilby Event

Range 1831 km
Figure 4.11 Bilby Event

Range 2274 km
Figure 4.12  Bilby Event

Range 3376 km
Note that these amplitudes are determined for the first arrivals, not the maximum amplitude in general. Thus, in the short ranges - 1039 km and 1426 km - the amplitudes are small because they are of Pn not of P, which arrives later. Notice that the amplitude reaches the maximum at the range of 2274 km. We will be much concerned with this fact when we derive a suitable model.

In examining the models by Nuttli and by Johnson for this region of the North America, the following procedure was used. First we ran a simple program which only computed the arrival times - refraction and reflection - from each layer by the method described in the previous chapters. We found the earliest arrival time at a certain range and found which layer it came from. Then, we examined the arrivals from the surrounding layers, and the responses that came in within so many seconds (usually 6 seconds for the sake of economy) after the first arrival were noted. We, then, ran the synthetic seismogram program to compute the synthetic seismogram considering only those responses found earlier.

We first applied our method to the Nuttli's model. Table 4.2 shows the first arrivals and magnitudes computed by our program. The synthetic response from the first arrival to six seconds later is shown in Figure 4.13. The synthetic seismogram, Figure 4.13 convolved with 4.3, is Figure 4.14. Compare the record (Fig. 4.8) with this figure. The difficulty is that at this range, the ray travels at very shallow depth so that the subsurface structure, which we expect to be extremely non-uniform,
Figure 4.13

Theoretical Response Nuttli 1039 km
Figure 4.14 Synthetic Response Nutli 1039 km
Table 4.2
First Arrival Times and Magnitudes for the Nuttli's Model

<table>
<thead>
<tr>
<th>Range (km)</th>
<th>Time (sec.)</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1039</td>
<td>136.0</td>
<td>15</td>
</tr>
<tr>
<td>1426</td>
<td>184.0</td>
<td>---*</td>
</tr>
<tr>
<td>1831</td>
<td>232.6</td>
<td>180</td>
</tr>
<tr>
<td>2274</td>
<td>278.0</td>
<td>140</td>
</tr>
<tr>
<td>3376</td>
<td>373.0</td>
<td>34</td>
</tr>
</tbody>
</table>

(* As shown in Figure 4.15, the magnitude for this range was very small)

greatly affects the ray paths. [See S. W. Smith (1962)]

Our assumption is a lateral homogeneity, which may be violated here. Other reasons may be that the model has a negative discontinuity at about 75 km deep which caused the negative peak in the theoretical response and that we neglected all the rays with weak response (N ≥ 5); if more rays had been added, the response would have been smoother. Similarly, synthetic seismograms were generated for the other ranges (Figures 4.15-4.18). The author reminds the reader that the amplitude between any two seismograms - real or synthetic - may not be compared.

We expected a gradual decay in the amplitude with increasing range, since the Nuttli's model has no gradient variations. At the range of 2274 km, therefore, the model does not give a large amplitude and does not give a small first arrival that the record shows. On this ground, we concluded that this model does not fit the actual record in this range, though travel times do fit.
Synthetic Response Nuttli 1426 km

Figure 4.15

Synthetic Response Nuttli 1831 km

Figure 4.16
Synthetic Response Nuttli 2274 km

Figure 4.17

Synthetic Response Nuttli 3376 km

Figure 4.18
Next we examined the Johnson's model [See Table 4.3].

Table 4.3
First Arrivals and Amplitudes for the Johnson's Model

<table>
<thead>
<tr>
<th>Range (km)</th>
<th>Time (sec.)</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1039</td>
<td>137.0</td>
<td>---</td>
</tr>
<tr>
<td>1426</td>
<td>186.9</td>
<td>80</td>
</tr>
<tr>
<td>2274</td>
<td>279.2</td>
<td>50</td>
</tr>
</tbody>
</table>

Again a difficulty arises at the short ranges. The refracted arrivals are much too early and last too long in the record, perhaps due to destructive interference caused by complex subsurface structure. We ran his model only at three ranges (Figures 4.19-4.21), the most important one being at 2274 km. The Johnson's model generated a small first arrival, but the time between the arrivals of the first ray and the large second ray is much too long, and though the largest amplitude in this synthetic seismogram (Fig. 4.21) is very large, there is little resemblance to the actual record at the range.

We, then, constructed some models with the effort concentrated on the seismogram at 2274 km. The models, along with the Nuttli's, are shown on Figure 4.22. We took the negative discontinuity at 75 km deep. The first model (Model I) has very small change in the slope from the Nuttli's model. The seismogram and the theoretical response at 2274 km are shown on Figures 4.23 and 4.24. Note that the period of the transfer function (Figure 4.3) is 1.15 seconds, and the distribution of
Synthetic Response Johnson 1039 km

Figure 4.19

Synthetic Response Johnson 1426 km

Figure 4.20
Figure 4.21 Synthetic Response Johnson 2274 km

![Graph showing synthetic seismic response over 11 seconds]
Figure 4.22
Models I, II and III
Figure 4.23: Synthetic Response Model I 2274 km

Figure 4.24: Theoretical Response Model I 2274 km
the peaks in Figure 4.24 seemed to interfere destructively with each other to generate a poor seismogram. (The synthetic seismograms for all these models are similar to the Nuttli's at other ranges, with the exception of the one at 1039 km without the negative peak. The amplitude at 1831 km is 0.001).

Model II has a more pronounced gradient change at 510 km. This gradient gave a more prominent second arrival starting at 1.1 seconds after the first arrival [See figures 4.25 and 4.26]. Note that the significant factor is the area under the theoretical response, not simply the height of the peaks. In this model the separation of the peaks helped to generate a better synthetic seismogram, but the fit with the record, especially in the first second is not satisfactory.

Model III has another high gradient region at 430 km in addition to the one at 510 km. The first arrival is much smaller and the amplitude in the synthetic seismogram improved [See Figures 4.27 and 4.28]. The wave shape improved as well as the amplitude, somewhat. But a close examination of the actual record indicates that we need a small peak for the first arrival then a much larger peak at 1.5 seconds later. In order to obtain a response like this, Model II is the best of all we have tried, the difficulty being that the first and the second arrivals were too close. To attain a wider separation, further models were studied. Models IV and V are shown on Figure 4.29. The theoretical responses and the synthetic seismograms are on Figures 4.30-4.33. The continued high gradient at 300 km and flat gradient following
Figure 4.25

Theoretical Response Model II 2274 km

Figure 4.26

Synthetic Response Model II 2274 km
synthetic Response  Model III  2274 km

Figure 4.27

Theoretical Response  Model III  2274 km

Figure 4.28
in Model IV gave too late second arrival as shown in Figure 4.31.

On the other hand Model V gave a better fit. For some reason the first arrival in this model is too large, and when experimentally the interfering phases were eliminated - Model V' - [See Figures 4.34-4.35] the synthetic response looks much better.

We conclude that Model V gives the best correlation to the actual record. Perhaps the first arrival can be made smaller by less velocity contrast at around 260 km where the gradient changes. Because of the sensitivity of the program to any minor change, an adjustment of an order of a few thousands of the total velocity made a significant difference in the shape of the synthetic seismogram. The author believed that making adjustments of such an order to achieve a better fit was only tedious and achieved little. Therefore, the author claims that Model V or a model extremely similar to it can generate satisfactory synthetic seismograms. [See Table 4.4 for the P-velocity] Though Model V has been claimed satisfactory, we have yet no way to prove the uniqueness or otherwise. However, beside the conventional method of determining the P-velocity structure using only the travel time information, we now have a much powerful method - synthetic seismogram - to determine more delicate structure variations.

For the completeness of the study, the synthetic seismograms of Model V at ranges 1831 km and 3376 km are shown on Figures 4.37-4.40. As mentioned earlier, since the gradient is nearly flat near the depth where
Figure 4.29
Models IV and V

Depth (km) vs. P-Velocity (km/sec) graph showing the comparison between Models IV and V. The graph indicates that Model IV and Model V have different profiles, with Model V generally having a slightly higher velocity at most depths compared to Model IV. There is also a notable feature labeled "Nuttli" on the graph, which appears to be a specific point of interest.
Synthetic Response Model TV 2274 km

Figure 4.30

Theoretical Response Model TV 2274 km

Figure 4.31
Figure 4.32

Synthetic Response Model V 2274 km

Figure 4.33

Theoretical Response Model V 2274 km
Figure 4.34

Synthetic Response Model $V' \ 2274 \ km$

---

Figure 4.35

Theoretical Response Model $V' \ 2274 \ km$
Figure 4.36  Model V
Reduced Time vs. Range

Time (sec) = 10.9 x Range (deg)
Synthetic Seismogram Model V 1831 km

Figure 4.37

Theoretical Response Model V 1831 km

Figure 4.38
Figure 4.39

Synthetic Seismogram Model V 3376 km

Figure 4.40

Theoretical Response Model V 3376 km
### Table 4.4 Model V P-Velocity

<table>
<thead>
<tr>
<th>Depth (km)</th>
<th>Velocity (km/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.59</td>
</tr>
<tr>
<td>40</td>
<td>6.61</td>
</tr>
<tr>
<td>40</td>
<td>7.93</td>
</tr>
<tr>
<td>60</td>
<td>7.93</td>
</tr>
<tr>
<td>80</td>
<td>7.80</td>
</tr>
<tr>
<td>110</td>
<td>7.80</td>
</tr>
<tr>
<td>150</td>
<td>7.93</td>
</tr>
<tr>
<td>260</td>
<td>8.64</td>
</tr>
<tr>
<td>400</td>
<td>9.10</td>
</tr>
<tr>
<td>508</td>
<td>9.41</td>
</tr>
<tr>
<td>525</td>
<td>9.80</td>
</tr>
<tr>
<td>650</td>
<td>10.55</td>
</tr>
<tr>
<td>850</td>
<td>10.98</td>
</tr>
<tr>
<td>1000</td>
<td>11.29</td>
</tr>
</tbody>
</table>

[See also Figure 4.36]  

The first ray bottoms, the theoretical response takes a similar form to a step function, and hence the synthetic seismogram is very similar to the transfer function. The reason why the record does not resemble the corresponding synthetic seismogram after a few seconds may be that there is a complex interaction due to the Moho or due to very shallow structure of the earth.
V. CONCLUSION

In this thesis, we attempted to investigate the P-velocity structure in the upper mantle of the earth in the southern part of the United States, using a newly developed technique of synthetic seismograms, instead of the conventional Wiechert-Herglotz method.

Although the validity of this new method is yet to be proven for the case of spherical layers, the method is known to give correct travel times. The method computes synthetic seismograms which can contain all the compressional response from all the depths - not only the travel times and $\frac{d^2 t}{d \Delta^2}$, which were the only information obtained from the conventional method. Due to the increased amount of information, we were able to examine models more accurately with added criteria such as amplitude as functions of time and of range, or the period and the phase of seismograms, which had seldom been considered before in the study of velocity structures.

Among two existing models we examined, the model by Dowling and Nuttli (1964) reached from the same data [BILBY Report (1963)] as we used, did not give satisfactory results. Due to the linear increase of the P-velocity with depth, the seismogram generated by this model were decreasing in amplitude with range. On the other hand the record from the Bilby event showed clearly that, at the range of 2274 km, there was a small first arrival about 1.5 seconds before a very large major P-arrival. The other model we tested was the Johnson's (1967) calculated from the data obtained at the Tonto
Forest Seismological Observatory in Arizona. Due to its two prominent steps in velocity, the synthetic seismogram at the range 2274 km showed a small first arrival followed too late by a major P-arrival. Therefore, it showed little resemblance to the record at this range. We then constructed some models with the correct arrival times, and tested them, with the main emphasis on the range 2274 km. Among the five models tested, one showed a satisfactory fit to the record. This model, Model V, has a pronounced step in velocity at the depth of 500 km and gives a strong arrival at the range 2274 km from this step.

Though there are more improvements to be made and approximations to be shown valid rigorously, we believe that this method of computing synthetic seismograms is extremely useful and convincing because of the remarkable resemblance between the synthetic seismograms and the records, which we demonstrated in this thesis, at least for the upper mantle where the curvature is small.

The uniqueness of the solution cannot be proven at this stage, and therefore, the structure proposed as Model V in this thesis may not be the only representation of the upper mantle. There may be other models that give similarly good seismograms. Nonetheless, by adding more criteria to these provided by the conventional method of computing velocity as a function of depth, we may be able to limit the possibilities.
References


Helmberger, Donald V., The Relationship between Point & Line Seismic Sources, Doctoral Oral Examination Paper at the University of California at San Diego, 1965.

Helmberger, Donald V., Head Waves from the Oceanic Mohorovicic Discontinuity, Ph.D. Thesis at the University of California at San Diego, 1967.


APPENDIX

On the following pages, the computer program discussed in the thesis in Chapter III is listed.
C MAIN PROGRAM
COMMON/TINP/DELTM,DLTM,MTD,DLTP,JO,NDIRT
COMMON/STUFF/C(100),S(100),D(100),TH(100),X
COMMON/TFIX/TN1,TN2,TN3,TN4,JN1,JN2,JN3,JN4
COMMON/CONFIX/DEL,NN,NDP,TMX,XYDIM,YDIM,DP,KO
COMMON/THY/TT(8000),PP(8000)
COMMON/SYTH/XD11,YD11,XD22,YD22,XD33,YD33
COMMON/FOURCT/MF,NMF,KMF,KNMF
COMMON/PLOTCT/CON,NNF,NPT
COMMON/LPRINT/PRNT,PRNTS,KST,KEND
LOGICAL PRNT,PRNTS
PRNT = .FALSE.
PRNTS = .TRUE.
JN1=12
JN2=10
JN3=8
JN4=100
TN1=.8
TN2=.2
TN3=.1
TN4=.01
NDIRT=0
NNP=1
NP =2
X = 2274.0
CO=(2./X)**5/(3.1416)
CON=CO
MTD=3
DLTM=.5
DELTMDT=1.E-6
JO = 45
CALL CURAY(JO)
DLTP=.04
XD11=2.
YD11=2.
XD22 = 4.
XD33 = 4.
XDIM = 4.
YD22 = 4.
YD33 = 4.
YDIM=2.
DP=.04
DEL=DP
NDP=15
TMX = 6.
NN = 150
MF=0
CALL NEWPLT('M5207','6267','WHITE','BLACK')
KC = 0
KST = 17
KE'4D = 20
CALL CLOK1
CALL SETUP (18,0,1,2,0,2,0)
CALL CLOK2
CALL CLOK1
CALL SETUP (24,0,1,1,2,2,0)
CALL CLOK2
CALL SETUP (29,0,1,2,2,0,2)
KO = 75
CALL CLOK1
CALL SETUP (29,0,1,2,2,0,2)
CALL CLOK2
CALL ENDPLT
* Subroutines CLOK1 and CLOK2 are to find the time spent between the two calls; they are not listed in the Appendix.
COMMON/STUFF/C(100),S(100),D(100),TH(100),X
DATA TH/0.0,40.0,20.0,2*10.0,30.0,20*20.0,2*10.0,22*20.0/
DATA C/0.0001,6.2,7.93,7.88,7.84,7.8,7.84,7.88,8.00,8.13,
A8.25,8.38,8.505,8.61,8.677,8.743,8.81,8.877,8.943,9.01,
B9.06,9.12,9.17,9.23,9.28,9.37,9.58,9.79,9.9,10.02,
C10.14,10.25,10.37,10.49,10.58,10.63,10.68,10.72,10.77,10.82,
D10.87,10.92,10.96,11.01,11.06,11.11,11.16,11.21,11.25,11.37
/ DATA S/0.0001,3.7,4.494,4.2,4.36,4.3,4.36,4.42,4.46,4.50
A4.55,4.69,4.74,4.84,4.85,4.94,4.95,5.00
B5.04,5.09,5.13,5.18,5.22,5.31,5.42,5.53,5.64,5.73,
C5.83,5.92,6.02,6.11,6.16,6.21,6.24,6.28,6.33,6.37,
D6.41,6.45,6.49,6.53,6.57,6.61,6.66,6.7,6.74,6.78/
DATA D/0.0001,2.84,3.44,3.442,3.444,3.45,3.455,3.475,3.495,3.52,
A3.54,3.57,3.59,3.6,3.62,3.63,3.65,3.66,3.68,3.71
B3.71,3.72,3.74,3.75,3.76,3.78,3.82,3.86,3.89,3.92,
C3.95,3.98,4.01,4.04,4.05,4.06,4.07,4.08,4.09,4.13
D4.11,4.12,4.13,4.14,4.15,4.16,4.17,4.18,4.19,4.20/
SUBROUTINE ADJUST(NFIX)
COMMON/THY/T(8CO),PHI(8000),FF(600)
COMMON/EXACT/PHI(500),TD(500),NEND,NM
N=NM+1
TR=TD(M)
I=0
80  I=I+1
   IF(T(I).GT.TR) GO TO 81
   GO TO 80
81  DNE=TR-T(I-1)
   DPL=T(I)-TR
   IF(ABS(DNE).GT.ABS(DPL)) GO TO 83
   DELTA=-DNE
   NFIX=I-1
   GO TO 85
83  DELTA=DPL
   NFIX=I
85  DO 84 J=1,NEND
     TD(J)=TD(J) DELTA
84  CONTINUE
END
SUBROUTINE CON(N, K1, K2)
COMMON/CFINX/NT, KT, N, LT, LTP(100), NF
COMMON/CN/ANT(100), MT(100), MB(100), NNB(100), LU(100), LB(100),
2 LL(100), NNF(100)
DO 5 J = 1, 100
  5 LTP(J) = 0.0
     NT = ANT(N)
     KT = MT(N)
     MB = MB(N)
     NNB = NNB(N)
     LT = LL(N)
     LTP(K1) = LU(N)
     LTP(K2) = LB(N)
     NF = NNF(N)
     RETURN
END
SUBROUTINE CONN(N,K,KN)

COMMON/CFIX/NT,KT,MB,NB,LT,LTP(100),NF
COMMON/NFIX/MM(100),NN(100),MT(100)

DO 25 J=1,100
   PM(J) = 0
   NN(J) = 0
   MT(J) = 0
   LTP(J) = 0
25 CONTINUE

K1 = K & 1
K2 = K & 2
K3 = K & 3
K4 = K & 4

PM(K1) = 1
GO TO (1, 2, 3, 4, 5, 6, 7, 8, 9, 10), N
1
   PM(K1) = 0
   PM(K4) = 1
   NN(K1) = 1
   LTP(K1) = 4
   LTP(K2) = 2
   LTP(K3) = 2
   LTP(K4) = 2
   NF = 2
   MT(K2) = 1
   MT(K3) = 1
   MT(K4) = 1
   GO TO 30
2
   PM(K1) = 0
   PM(K3) = 1
   NN(K1) = 1
   LTP(K1) = 4
   LTP(K2) = 2
   LTP(K3) = 2
   NF = 2
   MT(K2) = 1
   MT(K3) = 1
   GO TO 30
3
   PM(K2) = 1
   PM(K4) = 1
   NN(K1) = 1
   LTP(K1) = 4
   LTP(K2) = 4
   LTP(K3) = 2
   LTP(K4) = 2
   NF = 2
   MT(K2) = 2
   MT(K3) = 1
   MT(K4) = 1
   GO TO 30
4
   PM(K2) = 1
   PM(K4) = 1
   NN(K2) = 1
   LTP(K1) = 2
   LTP(K2) = 4
   LTP(K3) = 2
   LTP(K4) = 2
   NF = 2
MT(K2) = 1
MT(K3) = 1
MT(K4) = 1
GO TO 30

5
PM(K2) = 1
PM(K3) = 1
NN(K1) = 1
LTP(K1) = 4
LTP(K2) = 4
LTP(K3) = 2
NF = 2
PM(K2) = 2
MT(K3) = 1
GO TO 30

6
PM(K3) = 2
NN(K1) = 1
LTP(K1) = 4
LTP(K2) = 4
LTP(K3) = 4
NF = 1
MT(K2) = 2
MT(K3) = 2
GO TO 30

7
PM(K3) = 1
PM(K4) = 1
NN(K1) = 1
LTP(K1) = 4
LTP(K2) = 4
LTP(K3) = 4
LTP(K4) = 2
NF = 2
MT(K2) = 2
MT(K3) = 2
MT(K4) = 1
GO TO 30

8
PM(K3) = 1
PM(K4) = 1
NN(K2) = 1
LTP(K1) = 2
LTP(K2) = 4
LTP(K3) = 4
LTP(K4) = 2
NF = 2
MT(K2) = 1
MT(K3) = 2
MT(K4) = 1
GO TO 30

9
PM(K4) = 2
NN(K1) = 1
LTP(K1) = 4
LTP(K2) = 4
LTP(K3) = 4
LTP(K4) = 4
NF = 1
MT(K2) = 2
MT(K3) = 2
MT(K4) = 2
SUBROUTINE CONSTN(NO)
COMMON/CN/NNT(100),MMT(100),MMB(100),NNB(100),LU(100),LB(100),
ILL(100),NNF(100)
DO 5 J=1,NO
NNT(J)=0
MMT(J)=0
MMB(J)=0
NNB(J)=0
LU(J)=0
LB(J)=0
NNF(J)=1
LL(J)=0
5 CONTINUE
N=1
MMT(N)=1
LU(N)=2
N=2
NNB(N)=1
LU(N)=2
LB(N)=2
LL(N)=1
N=3
MMT(N)=2
NNT(N)=1
LU(N)=4
N=4
NNB(N)=2
NNT(N)=1
LU(N)=4
LB(N)=4
LL(N)=1
N=5
NNB(N)=2
NNT(N)=1
LU(N)=4
LB(N)=4
LL(N)=1
N=6
NNB(N)=1
MMT(N)=1
NNT(N)=1
LU(N)=4
LB(N)=2
NNF(N)=2
LL(N)=1
N=7
NNB(N)=3
MMB(N)=2
LU(N)=2
LB(N)=6
LL(N)=1
N=9
NNB(N)=3
NNT(N)=2
LU(N)=6
N=10
NNB(N)=2
\begin{align*}
M_{MT}(N) &= 1 \\
L_{L}(N) &= 1 \\
M_{MB}(N) &= 1 \\
N_{NT}(N) &= 1 \\
L_{U}(N) &= 4 \\
L_{B}(N) &= 4 \\
N_{NF}(N) &= 2 \\
N &= 11 \\
N_{NB}(N) &= 2 \\
M_{MT}(N) &= 1 \\
L_{L}(N) &= 2 \\
N_{NT}(N) &= 2 \\
L_{U}(N) &= 6 \\
L_{B}(N) &= 6 \\
N_{NF}(N) &= 3 \\
N &= 12 \\
N_{NB}(N) &= 3 \\
M_{MB}(N) &= 1 \\
N_{NT}(N) &= 1 \\
L_{U}(N) &= 4 \\
L_{B}(N) &= 6 \\
N_{NF}(N) &= 2 \\
L_{L}(N) &= 2 \\
N &= 13 \\
N_{NB}(N) &= 1 \\
M_{MT}(N) &= 2 \\
N_{NT}(N) &= 2 \\
L_{U}(N) &= 6 \\
L_{B}(N) &= 2 \\
N_{NF}(N) &= 3 \\
L_{L}(N) &= 1 \\
N &= 14 \\
M_{MT}(N) &= 4 \\
N_{NT}(N) &= 3 \\
L_{U}(N) &= 8 \\
N &= 15 \\
M_{MT}(N) &= 3 \\
N_{NS}(N) &= 1 \\
N_{NT}(N) &= 3 \\
L_{U}(N) &= 8 \\
L_{B}(N) &= 2 \\
L_{L}(N) &= 1 \\
N_{NF}(N) &= 4 \\
N &= 16 \\
N_{NB}(N) &= 2 \\
M_{MT}(N) &= 2 \\
N_{NT}(N) &= 3 \\
L_{U}(N) &= 8 \\
L_{B}(N) &= 4 \\
N_{NF}(N) &= 6 \\
L_{L}(N) &= 2 \\
N &= 17 \\
M_{MB}(N) &= 1 \\
N_{NT}(N) &= 2 \\
N_{NB}(N) &= 2 \\
M_{MT}(N) &= 2 \\
L_{U}(N) &= 6 \\
L_{B}(N) &= 4 \\
N_{NF}(N) &= 3 \\
L_{L}(N) &= 1 \\
N &= 18 \\
M_{MT}(N) &= 1 \\
N_{NS}(N) &= 3
\end{align*}
FUNCTION CCNVS(FA, FP, DEL, NF, N) A-13

DIMENSION FP(1), FA(1)

COMPUTES CONVOLUTION OF FP AND FA T=DEL*2N

NF MUST BE ODD

NN=N
DN=DEL

IF(NN.LT.1) GO TO 2

NDO=MINO(NN, (NF-1)/2)

IP=2
NP=2*NN

EVEN=FP(IP)*FA(NP)

OCD=0

IF(NDO.LT.2) GO TO 11

DO 10 I=2, NDO

IP=IP+1

NP=NP-1

OCD=ODD*FP(IP)*FA(NP)

IP=IP+1

NP=NP-1

EVEN=EVEN*FP(IP)*FA(NP)

10 CONTINUE

11 CONTINUE

ENDS=FP(1)*FA(2*NN+1) & FP(IP+1)*FA(NP-1)

CONVS=DN*(ENDS&4.*EVEN&2.*OCD)/3.

RETURN

2 CONVS=0.

END
COMPLEX FUNCTION CR(P,C)
COMPLEX P,CZ

CZ=1./C**2-P*P
U=REAL(CZ)
X=AIMAG(CZ)
R=SQRT(X*X+U*U)
W1=ABS(R&U)/2.
W2=ABS(R-U)/2.
R1=SQRT(W1)
R2=SQRT(W2)
CR=R1-R2*(0.,1.)
END
COMPLEX FUNCTION CRSTPP(P, V1, S1, RHO1, V2, S2, RHO2)

COMPLEX C, P, E1, E1P, E2, E2P

COMPLEX E1, E2, C1, C2, C3, C4, C5, C6, T

COMPLEX A, B, AP, BP

REAL K1, K2, K3, K4

K4 = RHO2 * S2**2 / (RHO1 * S1**2)

B1 = .5 / (1 - K4)

B2 = .5 * K4 / (K4 - 1)

K1 = B1 / S1**2

K2 = B2 / S2**2

K3 = K1 + K2

E1 = CR(P, V1)

E2 = CR(P, V2)

E1P = CR(P, S1)

E2P = CR(P, S2)

C1 = (P**2) * (K3 - P**2)**2

C2 = P**2 * E1 * E1P * E2P

C3 = (E1 * E1P) * (K2 - P**2)**2

C4 = E2P * (K1 - P**P)**2

C5 = K1 * K2 * E1 * E2P

C6 = K1 * K2 * E1P

AP = C1 + C3 - C5

BP = C2 + C4 - C6

T = 2 * K1 * E1 * (E2P * (K1 - P**2) - E1P * (K2 - P**2))

B = AP + E2 * BP

CRSTPP = T / B

RETURN

END
SUBROUTINE CURAY(JC)
COMMON/STUFF/C(100),S(100),D(100),TH(100),X,RCSQ(100),RSSQ(100)
COMMON /SENSE/ CRCSQ(100)
DIMENSION DEPTH(100)
REAL*8 DRCSC
PRINT 2, X
2 FORMAT (1HI,10X"CURAY"/11X"RANGE"/16X"THICKNESS"/9X"DEPTH"/5X"P"
&"-VELOCITY"/5X"S-VELOCITY"/8X"DENSITY")
DEPTH(1) = TH(1) / 2.0
DO 10 J = 2,JO
10 DEPTH(J) = DEPTH(J-1) + (TH(J)-TH(J-1))/2.0
DO 5 J = 1,JO
Q = 6371.0 / (6371.0-DEPTH(J))
C(J) = C(J) * Q
S(J) = S(J) * Q
D(J) = D(J) * Q
TH(J) = TH(J) * Q
DRCSC(J) = 1.0 / DBLE(C(J))**2
RCSQ(J) = DRCSC(J)
5 RSSQ(J) = 1.0 / S(J) ** 2
PRINT 1, (J,TH(J),DEPTH(J),C(J),S(J),D(J),J=1,JO)
1 FORMAT (I5,5X,5G15.4)
RETURN
END
SUBROUTINE FIND2 (Q,K,DEL,DET,PQ,TQ,KN,N)
COMMON/STUFF/C(100),S(100),D(100),TH(100),X
COMMON /SENSE/ RCSQ(100)
COMMON/CFIX/NT,KT,PB,NB,LT,LTP(100),NF
COMMON /LPRINT/ PRNT,PRNTS
LOGICAL PRNT,PRNTS
REAL*8 E(100),BLTEM,TOTEM,PG,TOtBLPPSQRCSQ
KOUNT = 0
TDE = DEL
J1=K1
J2=K1\&KN
8 P = Q
KCOUNT = KOUNT \& 1
5 P=P&DEL
PSQ = P ** 2
BLTEM = 0.0
DO 10 J = 1,K
E(J) = DSQRT(DABS(RCSQ(J)-PSQ))
10 BLTEM = BLTEM - TH(J) / E(J)
BLTEM = 2.0 * BLTEM
DO 30 J = J1,J2
E(J) = DSQRT(DABS(RCSQ(J)-PSQ))
30 BLTEM = BLTEM-TH(J)*LTP(J)/E(J)
BL = X \& BLTEM\P
IF(ABS (DEL).LE.1.E-18) GO TO 1
6 IF (DABS(BL).LE.X/DET) GO TO 1
2 IF(BL)3,1,4
3 DEL=ABS (DEL*.5)
GO TO 5
4 DEL=ABS (DEL*.5)
GO TO 5
1 IF (DABS(BL).LT.0.0) GO TO 7
IF (KOUNT.LE.5) GO TO 7
Q = Q/10.0
DEL = TDE
GO TO 8
7 PQ = P
TOTEM = 0.0
DO 11 J = 1,K
11 TOTEM = TOTEM \& E(J) \& TH(J)
TOTEM = TOTEM * 2.0
DO 31 J=J1,J2
31 TOTEM = TOTEM \& E(J) \& TH(J) \& LTP(J)
TO = P*X \& TOTEM
PQ = PQ
TQ = TQ
IF (DABS(BL).LT.1.0E-6) RETURN
IF (.NOT.PRNT) RETURN
PRINT 17, PQ, TO, BL
17 FORMAT (1HO,4X*PO = 'G18.6,10X*TO = 'G18.6,10X*BL = 'G18.6)
RETURN
END
SUBROUTINE CELPS (NNN, RG, NN, N)
DIMENSION PP(50)
COMMON/SPE/CELP(400), DD1, DD2, DD3, DD4, NO
RG=RG-1. E-08
PI=3. 141593
AN=PI/(NNN*2.)
J=NN
A=AN
DELP(J)=RG*(SIN(A)**N)
TO=DELP(J)
A=A*AN
PP(1)=DELP(1)
1 J=J+1
PP(J)=RG*SIN(A)**N
DELP(J)=PP(J)-PP(J-1)
DELP(J)=ABS(DELP(J))
TO=TO+DELP(J)
A=A*AN
IF(TO.LT.RG) GO TO 1
2 NO=J-1
END
COMPLEX FUNCTION GENCC(P,N)
COMPLEX P, CRSTPP, GCD, GCU, TO
DIMENSION GCD(100), GCU(100)
COMMON/STUFF/C(100), S(100), D(100), TH(100), X
KK=N-1
IF(KK.LT.2) GO TO 40
DO 34 J=2, KK
   VV1=C(J)
   SS1=S(J)
   RR1=D(J)
   VV2=C(J-1)
   SS2=S(J-1)
   RR2=D(J-1)
   GCD(J)=CRSTPP(P, VV1, SS1, RR1, VV2, SS2, RR2)
   GCU(J)=CRSTPP(P, VV2, SS2, RR2, VV1, SS1, RR1)
34 CONTINUE
40 TO=1.
IF(KK.LT.2) GO TO 36
DO 35 J=2, KK
   TO=TO*GCD(J)*GCU(J)
35 CONTINUE
36 GENCC=TO
END
SUBROUTINE HELP(K,N,P,TTP,DTP,KN)  
COMMON/STUFF/C(100),S(100),T(100),X,RCSQ(100),NSSQ(100)  
COMMON/CFIX/NT,KT,M,NB,L,LT,LTP(100),NF  
J1=K+1  
J2=J1+KN  
PSQ = P*P  
TOTEM = 0.0  
DO 11 J = 1,K  
   E = SORT(ABS(RCSQ(J)-PSQ))  
   TOTEM = TOTEM + E*TH(J)  
11   LTEM = LTEM-TH(J)/E  
   TOTEM = TOTEM + 2.0  
   BLTEM = BLTEM + 2.0  
DO 31 J = J1,J2  
   E = SORT(ABS(RCSQ(J)-PSQ))  
   BLTEM = BLTEM - TH(J)*LTP(J)/E  
31   TOTEM = TOTEM + E*TH(J)*LTP(J)  
   BL = X - P*BLTEM  
   TO = P*X & TOTEM  
   DTP=1./BL  
   TTP=TO  
RETURN  
END
SUBROUTINE HIGH(NDP,TMX,K,KI,N)
COMMON/TFIX/TN1,TN2,TN3,TN4,JN1,JN2,JN3,JN4
COMMON/CFIX/NT,KT,MB,NB,LT,LTP(100),NF
COMMON/STUFF/C(100),S(100),D(100),TH(100),X
COMMON/EXACT/PHI(500),TD(500),NEND,NM
COMMON/MAGIC/PP(300),DDPT(300),TT(300)
COMMON/SPE/DELP(400),DD1,DD2,DD3,DD4,NO
COMMON/PATHC/PO,T0,KK
COMMON/INPR/DELTM,DLTM,MTD,DLTP,J0,NDIRT
COMMON/LPRINT/PRNT,PRNTS
LOGICAL PRNT,PRNTS
DIMENSION E(100)
COMPLEX PP, CDPT
KK=K
KM=K
J=K
17 J=J&+1
IF(LTP(J).LT.1) GO TO 16
KM=J
GO TO 17
16 CONTINUE
333 FORMAT(6110)
IF (.NOT. PRNT) GO TO 4
PRINT 1
1 FORMAT(5X'(LTP(J)),J=1,KM')
WRITE(6,333) (LTP(J),J=1,KM)
PRINT 3
3 FORMAT(9X'K'9X'N'8X'KM')
WRITE(6,333) K,N,KM
4 V2=C(KM&1)
XM=0.
DO 98 J=1,KM
XM=AMAX1(XM,C(J))
98 CONTINUE
DEL=1./XM
81 P=-1.E-9
DET=1.E12
KN=KM-K
KP=KN-1
CALL FIND2(P,KK,DEL,DET,PO,T0,KP,N)
RG=A S(PO-1./V2)
NK=2
NN=NDP
KP=KN-1
P=1./V2
CALL HELP(K,N,P,TTP,DTP,KP)
TC=TTP
TG=TO-TTP
IF (PO.LE.1./V2) GO TO 6
IF (TG.GT.TN1) GO TO 6
JN=JN1
IF (TG.GT.TN2) GO TO 8
JN=JN2
IF (TG.GT.TN3) GO TO 8
JN=JN3
8 QZ = RG/(JN&+1)
DO 15 J = 1,JN
DELP(J) = Q2

CONTINUE

NO=JN
IF(TG.LT.TN4) GO TO 2
GO TO 19

6 CALL DELPS(ANN,RG,1,NK)
IF (.NOT.PRNT) GO TO 19
PRINT 7, V2, XM, PO, RG, TC, TO, (DELP(J), J=1, NO)

7 FORMAT (1HO,4X,V2 = 'G13.6,5X,XM = 'G13.6,5X,PO = 'G13.6/5X,RG = '
    &G13.6,5X,TC = 'G13.6,5X,TO = 'G13.6/5X,DELP'/((G15.6))

19 IF(PO.LE.1./V2) GO TO 2
CALL PLN1(PC,TO,K,N,TC,KN,V2)

2 MO=NO&2
   IF(TG.LT.TN4) MO=2
   IF(PO.LT.1./V2) MO=2
   CALL CONTOR(TMX,M,KN,N,MO)
   IF (.NOT.PRNT) GO TO 620
PRINT 5

5 FORMAT (1HO,13X,PP '27X,DDPT '24X,TT)
JJ=MO
WRITE(6,200) (PP(J),DDPT(J),TT(J), J=JJ,M)

200 FORMAT (5E15.4)

620 CALL PLN2(PC,TO,K,MO,M,KN)
NEND=M
NM=NO
IF(PO.LT.1./V2) NM=0
   IF(TG.LT.TN4) NM=0
   IF(PRNTS) PRINT 9, (TD(LLM),PHI(LLM), LLM=1,NEND)
9 FORMAT (1HO,14X,TD '23X,PHI'/ (IG25.7))
RETURN
END
SUBROUTINE INTERP(XP,YP,N,X,Y)
DIMENSION XP(N),YP(N)
REAL DIF1,DIF2,DIFY,DR
1 IF (X .GT. XP(N))GO TO 6
   IF (X .LT. XP(1)) GO TO 6
2 DO 10 I=1,N
   IF (XP(I) -X) 10,102,3
10 CONTINUE
3 K= I-1
   DIF1=XP(I) -XP(K)
   DIF2=XP(I) -X
   RATIO = DIF2/DIF1
   DIFY = ABS (YP(I) - YP(K))
   DR = DIFY*RATIO
5 Y = YP(I) & DR
   IF (YP(I) .GT. YP(K)) GO TO 4
   RETURN
4 Y = YP(I) - DR
   RETURN
102 Y=YP(I)
   RETURN
6 Y = 0.
   RETURN
END
SUBROUTINE PLNI(P0, TO, K, N, TC, KN, V2)

COMMON/TINP/DELTM, DLM, MTD, DLTP, JO, NDRT

COMMON/SPE/DELP(400), DD1, DD2, DD3, DD4, NO

COMMON/STUFF/C(100), S(100), D(100), TH(100), X

COMMON/EXACT/PHI(500), T(500), NEND, NM

COMMON/LPRINT/PRNT, PRNTS

LOGICAL PRNT, PRNTS

COMPLEX RPR, ROC, Q, TOT, GENCC, TQ

K1 = K61

K2 = K161

KP = KN - 1

P = 1. / V2

DO 80 I = 2, NC

J = I - 1

P = P & DELP(J)

Q = P0. ** (0., 1.)

CALL HELP(K, N, P, TTP, DTP, KP)

TT(I) = TTP

RPR = ROC(Q, K, KN)

TOT = GENCC(Q, K)

TQ = TOT * RPR

RP = AIMAG(TQ)

EA = (1. / C(2)) ** (2 - P * P)**.5

IF(NDRT, GT, 1) GO TO 1

R3 = 1. / EA

GO TO 2

1

EB = (1. / S(2)) ** (2 - P * P)**.5

R1 = EB**2 - P * P

R2 = R1**264. * P * P * EA * EB

R3 = R1 / (R2*S(2)**2)

IF(PRNT) PRINT 10, EA, EB, R1, R2

10 FORMAT (1HO, 'EA = ', G13.6, 5X, 'EB = ', G13.6, 5X, 'R1 = ', G13.6, 5X, 'R2 = ', G13.6)

2 PHI(I) = (RP * DTP * R3**P)**.5

IF (PRNT) PRINT 9, P, DELP(I), TTP, DTP, TOT, RPR, R3, PHI(I), RP

9 FORMAT (1HO, 'P = ', G15.6, 5X, 'DEL = ', G15.6, 5X, 'TTP = ', G15.6, 5X, 'DTP = ', G15.6, 5X, 'RPR = ', G20.6, 5X, 'R3 = ', G15.6, 5X, 'PHI = ', G15.6, 5X, 'RP = ', G15.6)

80 CONTINUE

4 IF(TO - TTP, LT, DLTP) GO TO 3

PP = PO - P

I = NO & 1

NO = I

Q = P

CALL HELP(K, N, P, TTP, DTP, KP)

TT(I) = TTP

RPR = ROC(Q, K, KN)

TOT = GENCC(Q, K)

TQ = TOT * RPR

RP = AIMAG(TQ)

EA = (1. / C(2)) ** (2 - P * P)**.5

IF(NDRT, GT, 1) GO TO 5

R3 = 1. / EA

GO TO 6

5 EB = (1. / S(2)) ** (2 - P * P)**.5

R1 = EB**2 - P * P

R2 = R1**264. * P * P * EA * EB
R3 = R1 / (R2 * S(2)**2)

IF (PRNT) PRINT 10, EA, E, R1, R2

PHI(I) = (RP * DTP * R3 * P**.5)

IF (PRNT) PRINT 9, P, DELP(I), TTP, DTP, TOT, RPR, R3, PHI(I), RP

GO TO 4

3 TT(I) = TC

PHI(I) = 0.

RETURN

END
SUBROUTINE PLN2(PO,TO,K,MO)
COMMON/TINP/DELTM,DLTM,MTD,DLTP,JO,NDIRT
COMMON/MAGIC/PP(300),DDPT(300),TT(300)
COMMON/EXACT/PHR(500),TTT(500),NEND,NM
COMMON/STUFF/C(100),S(100),D(100),TH(100),X
DIMENSION FF(50)
COMMON / LPRINT/ PRNTPRNTS
LOGICAL PRNTPRNTS
COMPLEX P,ROC,FM,E1
COMPLEX PP,BT,DDPT,RP,RPP,GC
COMPLEX RBT,TL,CR,RC,GENCC
COMPLEX EA,EB,R1,R2,R3,PH
KP=KN-1
K1=K+1
K2=K1+1
DC 5 I=M0,M
TTT(I)=TT(I)
P=PP(I)
RP=ROC(P,K,KN)
GC =GENCC(P,K)
EA =CR(P,C(2))
IF(NDIRT.GT.1) GO TO 32
R3=1./EA
GO TO 38
32 EB=CR(P,S(2))
R1=EB**2-P*P
R2=R1**2+4.*P*P*EA*EB
R3=R1/(R2**2)**2)
38 BT=CSQRT(P)
PH=R3*DDPT(I)*GC*RPT*BT
PHR(I)=AIMAG(PH)
IF(PRNT) PRINT 9, P, GC, RP, R1, R2, R3, EA, EB, PH
9 FORMAT (1HO,4X'P = '2G18.6/5X'R1 = ,2G18.6/5X'R2 = '2G18.6/5X'EB = '2G18.6/5X'PH = '2G18.6)
CONTINUE
P=PO*(1.0,0.0)+O. -- (0.91)
I=MO-1
Q=PO
SF=SF2(Q,K,KP,N)
TTT(I)=TO
GC=GENCC(P,K)
RP=ROC(P,K,KN)
RPP=SF*GC*RP*(X/2.)*2**5
IF(NDIRT.LT.1) GO TO 2
EA=CR(P,C(2))
EB=CR(P,S(2))
R1=EB**2-P*P
R2=R1**2+4.*P*P*EA*EB
R3=(R1*EA)/(R2**2)**2)
RPP*RPP*R3
2 PRE=REAL(RPP)
PIM=AIMAG(RPP)
IF(.NOT.PRNT) GO TO 3
PRINT 1, Q, SF, PRE, PIM
PRINT 9, P, GC, RP, R1, R2, R3, EA, EB, PH
1 FORMAT (5X'Q = 'G18.6,10X'SF = 'G18.6/
+5X'PRE = 'G18.6,10XPIM = 'G18.6)
3 NC=MO-2
DP *DLTP
F1=0.*
SUM=0.*
IF(MO.LE.3) GO TO 46
TNN3TO-DP
CALL INTERP(TTT,PHRM,TNN,Y)
F1=0
FF(1)=Y
SUM3SUM+2.*PIM*(TNN-TT(TT(NO))**.5
IF (PRNT) PRINT 4, SUM
4 FORMAT (5X'SUM = 'G18.6)
DELL=(TNN-TT(NO))/5.
TT(1)=TNN
DO 41 J=2,6
TT(J)=TT(J-1) +DELL
CALL INTERP(TTT,PHRM,TT(J),Y)
FF(J)=Y
SUM=SUM+(FF(J-1)+FF(J))/2.*DELL
IF (PRNT) PRINT 4, SUM
41 CONTINUE
46 TTP=TTT(MO)
IF(TTT(MO)-TNT.GT.DP) GO TO 43
SUM=SUM+2.*PRE*TNT(MO)-TNN)**.5
CALL INTERP(TTT,PHRM,TTP,Y)
FF(1)=Y
DELL=(TNT+DP-TTT(MO))/5.
DO 42 J=2,6
TT(J)=TT(J-1) +DELL
CALL INTERP(TTT,PHRM,TT(J),Y)
FF(J)=Y
SUM3SUM+(FF(J-1)+FF(J))/2.*DELL
IF (PRNT) PRINT 4, SUM
42 CONTINUE
F3=FF(6)
PHR(I)=(3.*SUM/DP-F1-F3)/4.
GO TO 44
43 TTT(MO)=TNT+DP
PHR(MO)=PRE/(DP**.5)
F3=PHR(MO)
SUM=SUM+2.*PRE*(DP)**.5
PHR(I)=(3.*SUM/DP-F1-F3)/4.
44 CONTINUE
END
FUNCTION FTIM(P,X)
COMMON/STUFF/CT(100),ST(100),DT(100),MT(100),XK(100)
FSW = .F. XX 2
RR = 0.0
UU = U = 1.0
i = 0
RR = RR + IN(J)*X
FTIM = FTIM + RR*2.0
RETURN
END
COMPLEX FUNCTION RET(P,V1,S1,D1,V2,S2,D2)  
COMPLEX E1P,P,A,B,AP,BP,BT,CR  
COMPLEX E1,E2,E2P,C1,C2,C3,C4,C5,C6  
REAL K1,K2,K3,K4  
D=D1/D2  
K4=S2**2/(S1**2*D)  
B1=.5/(1-K4)  
B2=.5*K4/(K4-1)  
K1=B1/S1**2  
K2=B2/S2**2  
K3=K1+K2  
E1=CR(P,V1)  
E2=CR(P,V2)  
E2P=CR(P,S2)  
E1P=CR(P,S1)  
C1=(P**2)*(K3-P**2)**2  
C2=P**2*E1*E1P*E2P  
C3=(E1*E1P)*(K2-P**2)**2  
C4=E2P*(K1-P*P)**2  
C5=K1*K2*E1*E2P  
C6=K1*K2*E1P  
AP=C1+C3-C5  
BP=C2+C4-C6  
A=-C1+C3-C5  
B=-C2+C4-C6  
BT=AP+BP*E2  
RET=(A-B*E2)/BT  
RETURN  
END
COMPLEX FUNCTION ROC(P, K, KN)

COMMON/STUFF/C(100), S(100), TH(100), X

COMMON/CFIX/NT, KT, MB, NB, LT, LTP(100), NF

COMMON/NFIX/MM(100), NN(100), MT(100)

COMPLEX Q, RF, RNT, RNB, TD, TU, T1, T2, TDU, RET, P, RMB, CRSTPP

Q=(1., 0.) & C.* (0., 1.)

IF(KN.GT.2) GO TO 90

K1=K&1
K2=K&2

V1=C(K1-1)
S1=S(K1-1)
D1=D(K1-1)
V2=C(K1)
S2=S(K1)
D2=D(K1)
V3=C(K2)
S3=S(K2)
D3=D(K2)
V4=C(K2&1)
S4=S(K2&1)
D4=C(K2&1)

IF(K.LT.2) GO TO 51

T1=CRSTPP(P, V1, S1, D, V2, S2, D2)
T2=CRSTPP(P, V2, S2, D2, V1, S1, D1)

GO TO 52

51
T1=Q
T2=C
CONTINUE

TDU=T1*T2

IF(NT.GT.0) GO TO 1

2
RNT=Q
GO TO 10

1
RNT=RET(P, V2, S2, D2, V1, S1, D1)

10
RMT=RET(P, V2, S2, C2, V3, S3, C3)

IF(LT.LT.1) GO TO 4

3
TC=CRSTPP(P, V2, S2, D2, V3, S3, D3)
TU=CRSTPP(P, V3, S3, D3, V2, S2, D2)

IF(MB.GT.0) GO TO 5

6
RMB=Q
GO TO 20

5
RMB=RET(P, V3, S3, C3, V2, S2, D2)

20
RNB=RET(P, V3, S3, D3, V4, S4, C4)

GO TO 30

4
TC=Q
TU=C
RMB=Q
RN =Q

ROC=RNT**NT*RMT**KT
GO TO 40

30
CONTINUE

ROC=RNT**NT*RMB**MB**RMT**KT*RNB**NB*(TC*TU)**LT

40
ROC=ROC*TDU

GO TO 91

90
TDU=Q
RNT=Q
RMT=Q
J2=K&KN
J1=K&1
DO 63 J=J1,J2
  N=J
  M=J&1
  IF(MM(J).GT.0) GO TO 61
  GO TO 62
61 T1=RET(P,C(N),S(N),C(M),S(M),D(M))
  RMT=RMT*T1**PM(J)
62 CONTINUE
63 CONTINUE
DO 73 J=J1,J2
  N=J-1
  M=J
  IF(AN(J).GT.0) GO TO 71
  GO TO 72
71 T1=RET(P,C(M),S(M),D(M),C(N),S(N),D(N))
  RNT=RNT*T1**NN(J)
72 CONTINUE
73 CONTINUE
DO 83 J=J1,J2
  N=J-1
  M=J
  IF(N.EQ.1) MT(J)=0
  IF(MT(J).GT.0) GO TO 81
  GO TO 82
81 TD=CRSTPP(P,C(N),S(N),D(N),C(M),S(M),D(M))
  TU=CRSTPP(P,C(M),S(M),D(M),C(N),S(N),D(N))
  T1=(TD*TU)**MT(J)
  TDU=TDU*T1
82 CONTINUE
83 CONTINUE
ROC=RMT*RNT*TDU
91 ENC
SUBROUTINE SETT(KO,DP,LN)
COMMON/SYTH/XD11,YD11,XD22,YD22,XD33,YD33
COMMON/THY/T(8000),PP(8000),RP(600)
DIMENSION P(1000)
COMMON/PLCTC/CON,NNF,NPT
COMMON / LPRINT/ PRNT,PRNTS
LOGICAL PRNT,PRNTS
DIMENSION SS(200),TT(200)
DIMENSION C(1000),TD(1000)
DIMENSION XL(2),YL1(4),YL2(4),YL3(4)
DATA XL/'TIME SEC'/
DATA YL2/'THEORETICAL PO'/
DATA YL3/'SYNTHETIC RESP'/
DATA YL1/'TRANSFER FTN'/
FORMAT(2E15.4)
READ(5,300) (SS(J),J=1,KO)
IF (PRNT) PRINT 1, (SS(J),J=1,KO)
FORMAT (1HO,'SOURCE FUNCTION'/(2G15.4))
TT(1)=0.
DO 16 J=2,KO
TT(J)=TT(J-1)+DP
CONTINUE
IF(NPT.LT.1) GO TO 31
LS1=0
CALL PICTUR(XD11,YD11,XL,-8,YL1,-16,
2 TT,SS,KO,0.,LS1)
31 CONTINUE
T(1)=0.
DO 10 J=2,LN
T(J)=T(J-1)+DP
CONTINUE
CALL PICTUR(XD22,YD22,XL,-8,YL2,-16,
2 T,PP,LN,0.,LS1)
L=0
NK=LN/2-1
DO 20 N=1,NK,NNF
L=L+1
C(L)=CONVS(PP,SS,DP,KO,N-1)
TD(L)=2.*DP*(N-1)
CONTINUE
DO 30 J=2,L
P(J)=(C(J)-C(J-1))/(DP*.2.)
P(J)=P(J)*CCN
CONTINUE
P(1)=0.
CALL PICTUR(XD33,YC33,XL,-8,YL3,-16,
2 TD,P,L,0.,LS1)
IF(PRNTS) PRINT 2, (TD(J),P(J),J=1,L)
2 FORMAT (1HO,10X'TIME',10X'PRESSURE'/2G18.4))
END
SUBROUTINE SETUP(KMP,NSNO,MOMPLOTMPUNCH)COMMON/CONFIX/DELT,NN,NDP,TMX,XDIM,YDIM,DP,KOCOMMON/CFIX/NT,KT,KB,NB,LT,LTP(100),NFCOMMON/FOURCT/MF,NMF,KMF,KNMFCOMMON/THY/T(8CCO),PP(8CCO),FF(600)COMMON/EXACT/PHI(500),TD(500),NEND,NMCOMMON/LPRINT/PRNT,PRNTSLOGICAL PRNT,PRNTS
DIMENSION XL(2),YL1(4),YL2(4),YL3(4)
DATA YL2/' THEORETICAL PO'/
DATA XL/I"TIME SEC"/
IF(MO.GT.1) GO TO 11
I=K
TT(I)=TS(I)
PP(I)=0.
DO 10 J=2,NN
TT(J)=TT(J-1)+CEL
PP(J)=0.
10 CONTINUE
11 CONTINUE
IF(MF.LT.1) GO TO 30
CALL CONN(NMFKMF,KNMF)
CALL HIGH(NDP,TMX,KMF,KNMF,NMF)
CALL ADJUST(NFIX)
M=NMF&I
N2=NFIX&I
N1=NFIX-1
IF(N1.LE.2) GO TO 41
DO 35 J=1,N1
CALL INTERP(TD,PHI,NEND,TT(J),Y)
PP(J)=PP(J)&*Y*NF
35 CONTINUE
41 CONTINUE
PP(NFIX)=PP(NFIX) &NF*PHI(M)
DO 36 J=N2,NN
CALL INTERP(TD,PHI,NEND,TT(J),Y)
PP(J)=PP(J)&*Y*NF
36 CONTINUE
GO TO 7
30 CONTINUE
CALL CONN(NO)
K1=K61
K2=K62
DO 32 N=NSNO
CALL CON(N,K1,K2)
CALL HIGH(NCP,TMX,K,KI,N)
CALL ADJUST(NFIX)
M=NMF&I
N2=NFIX&I
N1=NFIX-1
IF(N1.LE.2) GO TO 42
DO 31 J=1,N1
CALL INTERP(TD,PHI,NEND,TT(J),Y)
PP(J)=PP(J)&*Y*NF
31 CONTINUE
42 CONTINUE
PP(NFIX)=PP(NFIX) &NF*PHI(M)
DO 33 J=N2,NN
   CALL_INTERP(TD,PHI,NEND,TT(J),Y)
   PP(J)=PP(J)*Y*NF
   CONTINUE
32 CONTINUE

7 IF(.NOT.PRNT) GO TO 12
   PRINT 13, (TD(J),PHI(J),J=1,NEND)
13 FORMAT (1H0,15X,TD,15X,PHI/(2G18.6))
   PRINT 14, (TT(J),PP(J),J=1,NN)
14 FORMAT (1H0,15X,TT,15X,PP/(2G18.6))
12 IF(MPLOT.LT.1) GO TO 1
   CALL_PICTUR(XDIM,YDIM,XL,-8,YL2,-16,
2 TP,PP,NN,0.0)
1 CONTINUE
   IF(MPUNCH.LT.1) GO TO 2
   LN=NN
   NK=LN
   DEL=DP
   WRITE(7,100) TT(1),CP,DEL
   WRITE(7,200) NN,LN,NK
   WRITE(7,100) (PP(J),J=1,LN)
200 FORMAT(3I10C)
100 FORMAT (5E15.6)
   2 IF(KO.LT.1) RETURN
   CALL_SETT(KO,DP,NN)
   RETURN
END
FUNCTION SF2(P, K, KN, N)
COMMON/CFIX/NT, KT, M, N, LT, LTP(100), NF
COMMON/STUFF/C(100), S(100), D(100), TH(100), X, RCSQ(100), RSSQ(100)
PSQ = P ** 2
TE = 0.0
DO 5 J = 1, K
   ESQ = ABS (RCSQ(J) - PSQ)
   E = SQRT(ESQ)
5 TE = TE & TH(J) * RCSQ(J) / (ESQ * E)
   TE = TE * 2.0
   J1 = K & 1
   J2 = J1 & KN
   DO 10 J = J1, J2
      ESQ = ABS(RCSQ(J) - PSQ)
      E = SQRT(ESQ)
10 TE = TE & TH(J) * LTP(J) * RCSQ(J) / (ESQ * E)
SF2 = SQRT(P/(X * TE * ABS(RCSQ(2) - PSQ)))
RETURN
END
SUBROUTINE TIME2(P, PC, DPT, T, KN, N) A-36
COMMON/PATHC/P0, TO, K
COMPLEX E, P, T, PC, CR, BL, DPT, F
DIMENSION E(100), F(100)
COMMON/STUFF/C(100), S(100), D(100), TH(100), X
COMMON/CFIX/NT, KT, MB, NB, LT, LTP(100), NF
COMMON / LPRINT/ PRNT, PRNTS
LOGICAL PRNT, PRNTS
DL = PO * .5
KO = K
DET = 1.0E-10
K1 = KO * 0
K2 = KO * KN
P = P & DL * (0., 1.)
T = P * X
DO 1 J = 1, KO
   E(J) = CR(P, C(J))
   T = T * 2. * TH(J) * (E(J) * LP(J))
1 CONTINUE
DO 11 J = K1, K2
   E(J) = CR(P, C(J))
   T = T & 6. * TH(J) * (E(J) * LP(J))
11 CONTINUE
CT = AIMAG(T)
IF (ABS(DL) . LE. 1.0E-9) GO TO 2
IF (ABS(CT) . LE. DET) GO TO 2
IF (CT) 4, 2, 5
DL = ABS(DL) * .5
GO TO 6
5 DL = -ABS(DL) * .5
GO TO 6
2 CONTINUE
PC = P
L = X
DO 10 J = 1, KO
   BL = BL - 2. * P * TH(J) / E(J)
10 CONTINUE
DO 12 J = K1, K2
   BL = BL - P * TH(J) * (LTP(J) / E(J))
12 CONTINUE
DPT = 1. / BL
IF (CT . LT. 1.0E-5) RETURN
IF (.NOT. PRNT) RETURN
PRINT 110, P, E(1), T, DPT
110 FORMAT (1HO, 4X, P = '2G18.6/5X', E(1) = '2G17.6/5X', T = '2G18.6/5X', CPT = '2G18.6')
FUNCTION TS(K)

COMMON/CFIX/NT, NT, MB, NB, LT, LTP(100), NF
COMMON/STUFF/C(100), S(100), D(100), TH(100), X, RCSQ(100), RSSQ(100)
COMMON/LPRINT/PRNT, PRNTS, KST, KEND

LOGICAL PRNT, PRNTS

DIMENSION T(200)

DET=1.E-12

N=0

X1=0.

DO 98 M = 1, KST

X1=AMAX1(X1, C(M))

98 CONTINUE

DO 102 J=KST, KEND

X1 = AMAX1(X1, C(J))

P=-1.E-9

DEL=1./X1

LTP(J)=2

N=J-1

CALL FIND2(P, M, DEL, DET, PO, TO, 0, 1)

N=NC1

T(N)=TO

102 CONTINUE

DO 103 J=KST, KEND

N=NC1

PX=1./C(J+1)

TX=PTIM(PX, J)

T(N)=TX

103 CONTINUE

TS=1.E-6

DO 106 J=1, N

TS=AMIN1(T(J), TS)

106 CONTINUE

IF (PRNT) PRINT 1, (T(J), J=1, N)

1 FORMAT (5X'T(J),J=1,N'/(4G18.4))

RETURN

END
| Value           | 0.0          | 0.3446E 00 | 0.7517E 00 | 0.1268E 01 | 0.1878E 01 | 0.2117E 01 | 0.8531E 00 | -0.1227E 01 | -0.3050E 01 | -0.4445E 01 | -0.5364E 01 | -0.5003E 01 | -0.3913E 01 | -0.2629E 01 | -0.1274E 01 | 0.2053E 00 | 0.1729E 01 | 0.3290E 01 | 0.4036E 01 | 0.3901E 01 | 0.3193E 01 | 0.1849E 01 | 0.6724E-01 | -0.1483E 01 | -0.2708E 01 | -0.3658E 01 | -0.3944E 01 | -0.3847E 01 | -0.3524E 01 | -0.3029E 01 | -0.2367E 01 | -0.1529E 01 | -0.5685E 00 | 0.1828E-01 | 0.1832E 00 | 0.1525E 00 | 0.4464E-01 | -0.1481E 00 |
|----------------|-------------|------------|------------|------------|------------|------------|------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Description    | A-38        |            |            |            |            |            |            | (Transfer Function) |            |             |             |             |             |             |             |             |             |             |             |             |             |             |             |             |             |             |             |             |             |             |             |             |             |             |             |

- Values are in scientific notation.
- The table represents a set of values with corresponding exponents.